Energy-energy correlations:
from QCD to $\mathcal{N} = 4$ SYM and back

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"Current Themes in High Energy Physics and Cosmology"

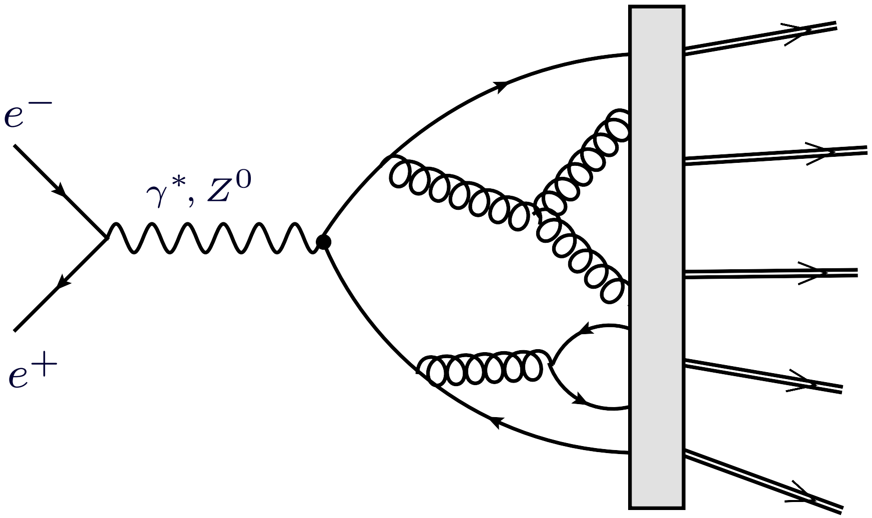
Why we like $\mathcal{N} = 4$ SYM

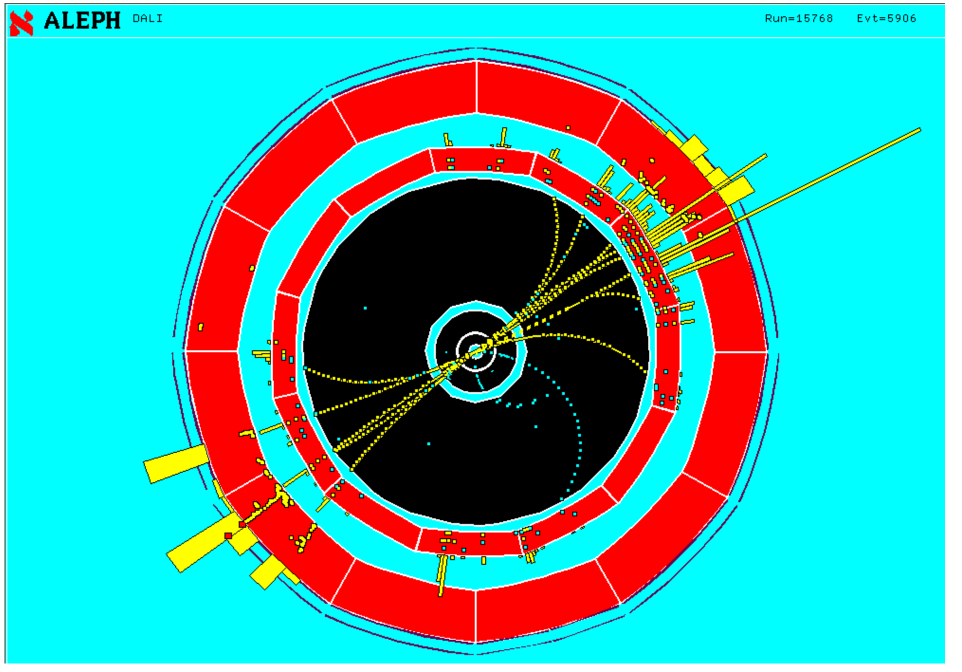
- ✔ Maximally supersymmetric, conformal four-dimensional gauge theory
- ✔ Is believed to be integrable, in the planar limit at least
- ✔ Remarkable relations between various quantities:
  - ✗ Scattering amplitudes: $A_n(p_i) = \langle p_1, p_2, \ldots, p_n | S | 0 \rangle$
  - ✗ (Light-like) Wilson loops: $W_n = \langle \mathrm{tr}\, P \exp\left(i \oint_{C_n} dx \cdot A(x) \right) \rangle$
  - ✗ Correlation functions: $G_n(x_i) = \langle \mathcal{O}(x_1)\mathcal{O}(x_2) \ldots \mathcal{O}(x_n) \rangle$
- ✔ Scattering amplitudes suffer from IR divergences and require a regularisation
- ✔ Exact scattering matrix $S = 1$

How much physics can we learn from scattering amplitudes in $\mathcal{N} = 4$ SYM?

e^+e^- annihilation in QCD

- PETRA (1978-1986) and LEP (1989-2010)





- A virtual photon or Z^0-boson decay into quarks and gluons that undergo a hadronization process into hadrons

- Final states can be described using the class of *infrared finite* observables (event shapes): energy-energy correlations (EEC), thrust, heavy mass, . . .

- Can be computed in perturbative QCD, hadronisation corrections are ‘small’ at high energy

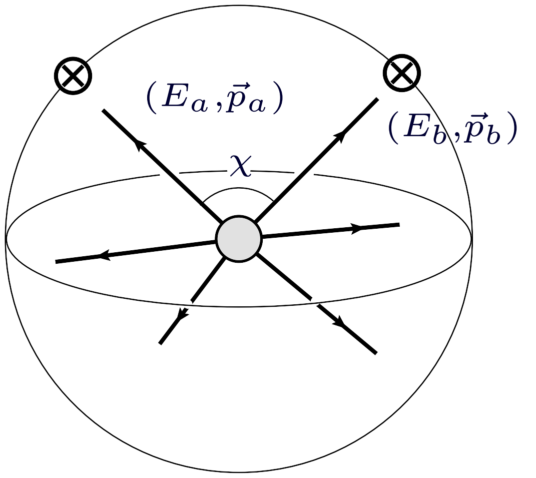
Energy-energy correlations

- Function of the angle $0 \le \chi \le \pi$ between detected particles

  [Basham,Brown,Ellis,Love]

$$\mathrm{EEC}(\chi) = \left\langle \frac{1}{\Delta\chi} \sum_{a,b} \frac{E_a E_b}{Q^2} \theta\big(\Delta\chi - |\cos\theta_{ab} - \cos\chi|\big) \right\rangle_{\text{events}}$$

  Total energy $\sum_a E_a = Q$



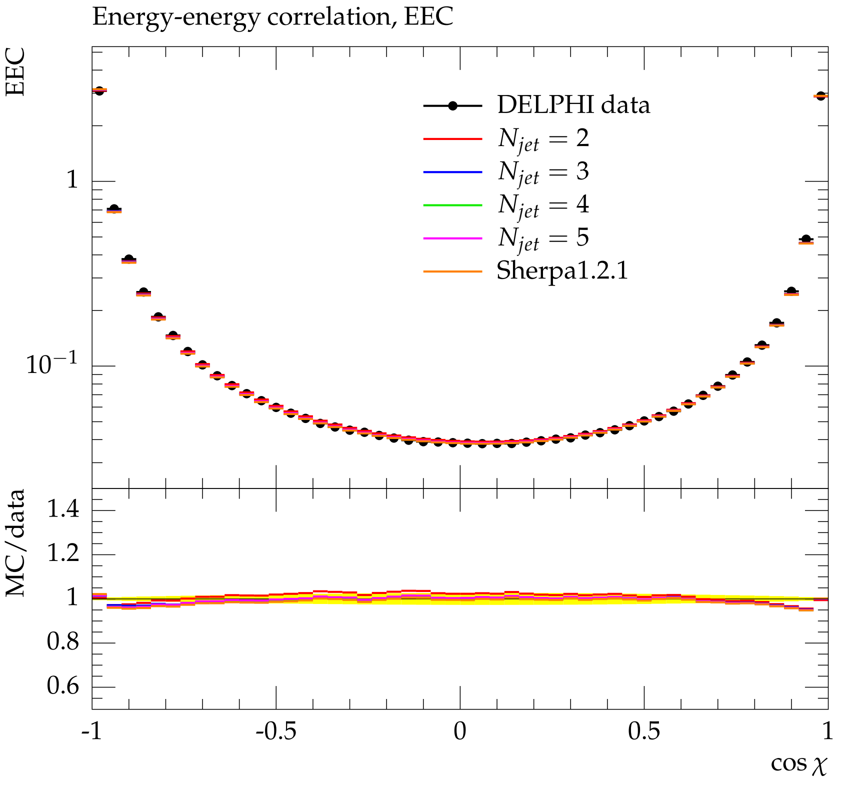
- Conventional ('amplitude') approach

$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\text{tot}}} \sum_{a,b} \int d\sigma_{a+b+X} \frac{E_a E_b}{Q^2} \delta(\cos\theta_{ab} - \cos\chi)$$

- Weak coupling expansion in QCD

$$\mathrm{EEC}(\chi) = a_{\mathrm{S}} A(\chi) + a_{\mathrm{S}}^2 B(\chi) + O(a_{\mathrm{S}}^3)$$

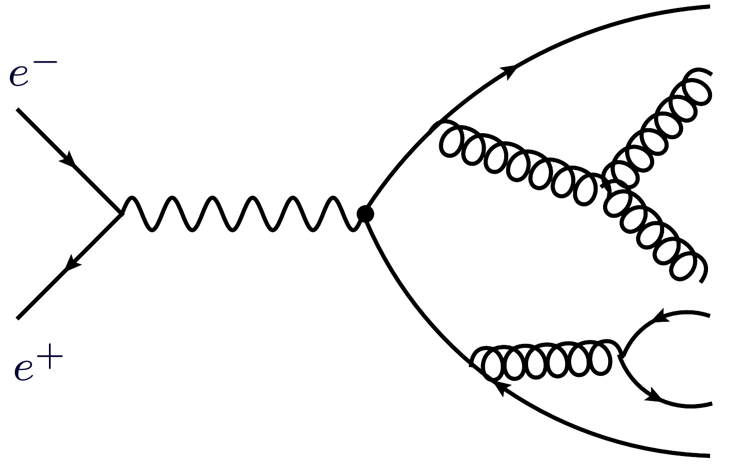
- Current status (1978 – today):
  - ✗ Very precise experimental data
  - ✗ Poor analytical control, $B(\chi)$ is known numerically



- *Final goal: develop more efficient method to computing EEC*

e^+e^- annihilation in $\mathcal{N}=4$ SYM

- ✔ Define EEC in $\mathcal{N}=4$ SYM and evaluate it at weak/strong coupling



- ✔ From QCD to $\mathcal{N}=4$ SYM: introduce an analog of the electromagnetic current
  - ✗ (protected) half-BPS operator built from the six real scalars Φ^I (with $I=1,\dots,6$)

$$O^{IJ}_{\mathbf{20'}}(x) = \mathrm{tr}\left[\Phi^I\Phi^J - \tfrac{1}{6}\delta^{IJ}\Phi^K\Phi^K\right]$$

  - ✗ To lowest order in the coupling, $O_{\mathbf{20'}}(x)$ produces a pair of scalars out of the vacuum
  - ✗ For arbitrary coupling, the state $O_{\mathbf{20'}}(x)|0\rangle$ can be decomposed into an infinite sum over on-shell states with an arbitrary number of scalars (s), gauginos (λ) and gauge fields (g)

$$\int d^4x\, \mathrm{e}^{iqx}\, O_{\mathbf{20'}}(x)|0\rangle = |ss\rangle + |ssg\rangle + |s\lambda\lambda\rangle + \dots$$

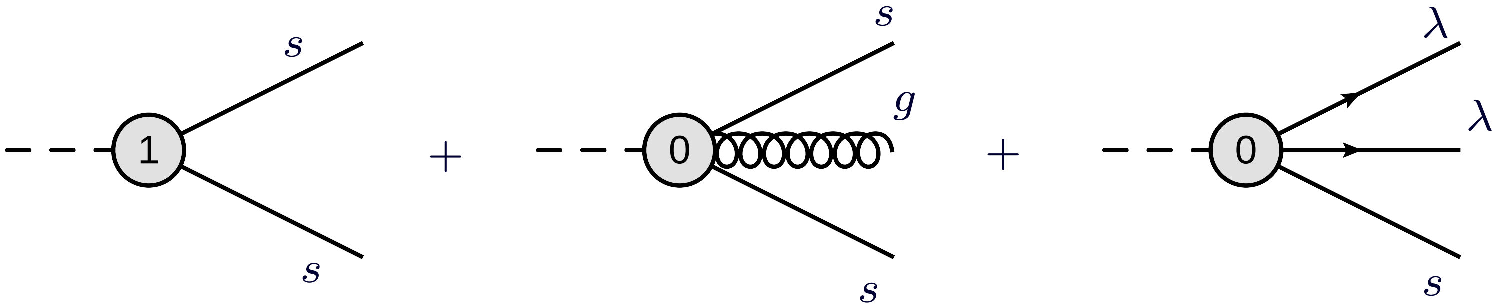
EEC in $\mathcal{N} = 4$ SYM

- ✔ Conventional approach

$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\rm tot}} \sum_{a,b,X} \int \mathrm{dLIPS}\, \left|\mathcal{A}_{a+b+X}\right|^2 \frac{E_a E_b}{Q^2} \delta(\cos\chi - \cos\theta_{ab})$$

- ✔ The amplitude of creation of the final state $|a, b, X = \text{everything}\rangle$

$\mathcal{A}_{a+b+X} =$



$+ \ldots$

- ✔ Matrix elements ($s_{ij} = (p_i + p_j)^2$ with $p_i^2 = 0$)

$$|\mathcal{A}_{ss}|^2 = \left|\langle s(p_1)s(p_2)|O_{\mathbf{20'}}|0\rangle\right|^2 = \frac{2}{s_{12}}\left[1 + aF_{\rm virt}(q^2)\right]$$

$$|\mathcal{A}_{ssg}|^2 = \left|\langle s(p_1)s(p_2)g(p_3)|O_{\mathbf{20'}}|0\rangle\right|^2 = a\,\frac{s_{12}}{s_{13}s_{23}}$$

$$|\mathcal{A}_{s\lambda\lambda}|^2 = \left|\langle \lambda(p_1)\lambda(p_2)s(p_3)|O_{\mathbf{20'}}|0\rangle\right|^2 = a\,\frac{2}{s_{12}}$$

't Hooft coupling $a = g_{\rm YM}^2 N/(4\pi^2)$

EEC from amplitudes I

- The total cross section

$$\sigma_{\text{tot}}(q) = \int \text{dLIPS}_2\, |\mathcal{A}_{ss}|^2 + \int \text{dLIPS}_3\, \left(|\mathcal{A}_{ssg}|^2 + |\mathcal{A}_{s\lambda\lambda}|^2\right) + O(a^2)$$

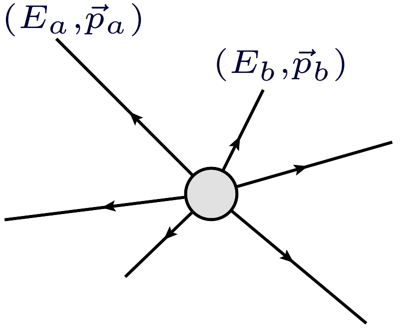
$$= \frac{N^2-1}{16\pi}\left[1 + aF_{\text{virt}}(q^2)\right] + a\int \text{dLIPS}_3\, \frac{s_{12}^2 + 2s_{13}s_{23}}{s_{12}s_{13}s_{23}} + O(a^2) = \frac{N^2-1}{16\pi} + 0\cdot a + O(a^2)$$

- Energy-energy correlations

$$\text{EEC} = \Big[\int \text{dLIPS}_2\, w(p_1,p_2)\, |\mathcal{A}_{ss}|^2 + \int \text{dLIPS}_3\, w(p_1,p_2,p_3)\left(|\mathcal{A}_{ssg}|^2 + |\mathcal{A}_{s\lambda\lambda}|^2\right) + O(a^2)\Big]/\sigma_{\text{tot}}$$

Weight factors for EEC

$$w(p_1,p_2,\dots) = \sum_{a,b} \frac{E_a E_b}{q^2}\,\delta(\cos\theta_{ab} - \cos\chi)$$



- One-loop calculation (unprotected quantity) [Zhiboedov],[Engelund,Roiban]

$$\text{EEC}_{\mathcal{N}=4} = \frac{a}{4z^2(1-z)}\ln\frac{1}{1-z} + O(a^2)$$

IR finite, positive definite function of $z = (1-\cos\chi)/2\,, \qquad 0 < z < 1$

- Two-loop correction is hard to compute ($\sim 10^2$ diagrams)

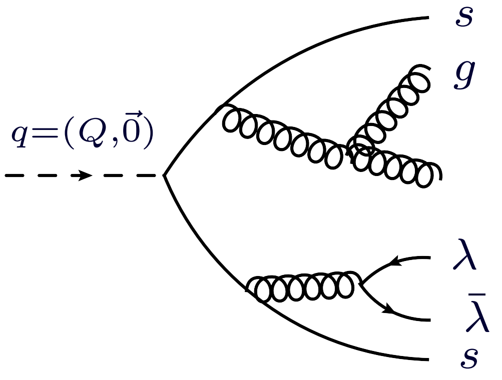
EEC from amplitudes II

- ✔ Conventional approach

$$\mathrm{EEC}(\chi) = \frac{1}{\sigma_{\mathrm{tot}}} \sum_{a,b,X} \int \mathrm{dLIPS}\, \left|\mathcal{A}_{a+b+X}\right|^2 \frac{E_a E_b}{Q^2} \delta(\cos\chi - \cos\theta_{ab})$$

The amplitude of creation of the final state $|a, b, X = \mathrm{everything}\rangle$

$$\mathcal{A}_{a+b+X} = \int d^4x\, \mathrm{e}^{iqx} \langle a, b, X | O_{\mathbf{20'}}(x) | 0 \rangle$$



- ✔ Main disadvantages:
  - ✗ presence of infrared divergences in transition amplitudes $\mathcal{A}_{a+b+X}$
  - ✗ integration over the Lorentz invariant phase space of the final states $d\mathrm{LIPS}$
  - ✗ necessity for summation over all final states $\sum_X$
  - ✗ no analytical results beyond one loop
- ✔ New approach: EEC can be computed from *correlation functions of energy flow operators*

EEC from correlation functions

✔ Total cross section from the optical theorem

$$\sigma_{\rm tot}(q) = \sum_X (2\pi)^4 \delta^{(4)}(q - p_X) |\mathcal{A}_{O_{\mathbf{20'}} \to X}|^2$$
$$= \int d^4x \ {\rm e}^{iqx} \sum_X \langle 0|O^\dagger(0)|X\rangle \, {\rm e}^{-ixp_X} \, \langle X|O(0)|0\rangle$$
$$= \int d^4x \ {\rm e}^{iqx} \, \langle 0|O^\dagger(x) O(0)|0\rangle = \frac{1}{16\pi}(N^2-1)\theta(q^0)\theta(q^2)$$

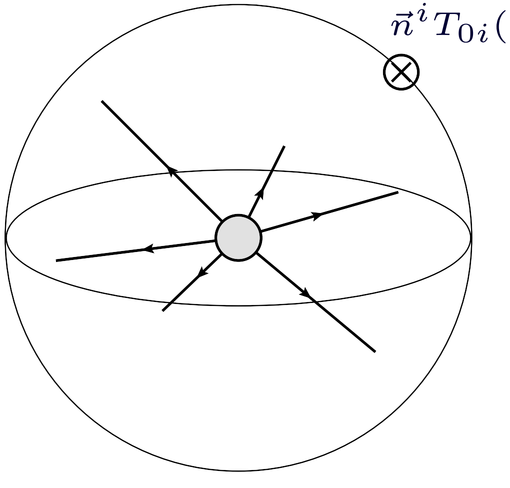
Wightman correlation function, protected for 1/2-BPS operators

✔ Generalization to EEC

$$\text{EEC} \sim \sum_X \langle 0|O^\dagger(x)|X\rangle w(X) \langle X|O(0)|0\rangle = \langle 0|O^\dagger(x) \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2) O(0)|0\rangle$$

Energy flow operator

$$\mathcal{E}(\vec{n})|X\rangle = \sum_a E_a \, \delta^{(2)}(\Omega_{\vec{p}_a} - \Omega_{\vec{n}})|X\rangle$$



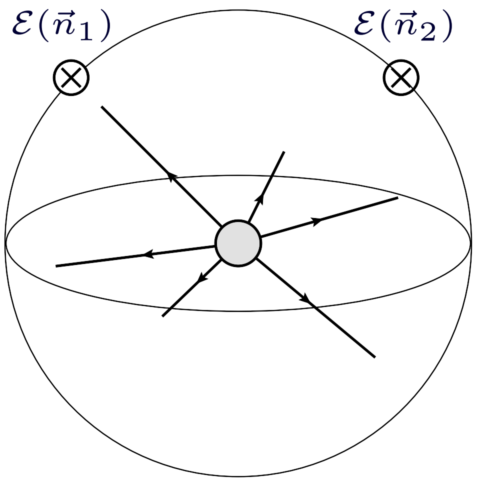
✔ Relation to the energy-momentum tensor in $\mathcal{N}=4$ SYM

[Sveshnikov,Tkachov],[GK,Oderda,Sterman]

$$\mathcal{E}(\vec{n}) = \int_0^\infty dt \lim_{r\to\infty} r^2 \, \vec{n}^i T_{0i}(t, r\vec{n})$$

EEC from correlation functions II

- Energy flow correlations [GK,Sterman],[Belitsky,GK,Sterman],[Hofman,Maldacena]



$$\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle_q = \sigma_{\rm tot}^{-1}\int d^4x\, \mathrm{e}^{iqx}\langle 0|O^\dagger(x)\,\mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\,O(0)|0\rangle$$

Energy flow in the direction of $\vec{n}_1$ *and* $\vec{n}_2$

- Average over the orientations $\vec{n}_1$ and $\vec{n}_2$ with the relative angle χ kept fixed

$$\mathrm{EEC} = \int d\Omega_1 d\Omega_2\, \delta(\vec{n}_1\cdot\vec{n}_2 - \cos\chi)\langle \mathcal{E}(\vec{n}_1)\mathcal{E}(\vec{n}_2)\rangle_q/q^2$$

- Multi-fold integral of *Wightman* 4pt function

$$\mathrm{EEC} \sim \underbrace{\int d^4x\,\mathrm{e}^{iqx}}_{\text{Fourier}}\underbrace{\int_0^\infty dt_1 dt_2 \lim_{r_i\to\infty} r_1^2 r_2^2}_{\text{Detector limit}}\underbrace{\langle 0|O^\dagger(x)\,T_{0\vec{n}_1}(x_1)T_{0\vec{n}_2}(x_2)\,O(0)|0\rangle}_{\text{Wightman corr. function}}\Big|_{x_i=(t,r\vec{n}_i)}$$

  - ✗ Compute corr.function $\langle O^\dagger(x)T(x_1)T(x_2)O(0)\rangle$ in Euclid
  - ✗ Continue to Minkowski with Wightman prescription
  - ✗ Take detector limit + perform Fourier

Correlation functions in $\mathcal{N}=4$ SYM

- Quantum corrections to various correlation functions are determined by the same scalar function

$$\langle O(x_1)O(x_2)O(x_3)O(x_4)\rangle_E = \frac{1}{x_{12}^2 x_{23}^2 x_{34}^2 x_{41}^2}\Phi(u,v;a)$$

$$\langle O(x_1)T(x_2)T(x_3)O(x_4)\rangle_E = \frac{1}{(x_{12}^2 x_{23}^2 x_{34}^2)^2}P(\partial_u,\partial_v)\Phi(u,v;a)$$

Conformal ratios

$$u = x_{12}^2 x_{34}^2/(x_{13}^2 x_{24}^2)\,, \qquad v = x_{23}^2 x_{41}^2/(x_{13}^2 x_{24}^2)$$

- Universal function in $\mathcal{N}=4$ SYM at weak coupling [Eden,Schubert,Sokatchev],[Bianchi et al]

$$\begin{aligned}\Phi(u,v) =& a\,\Phi^{(1)}(u,v) + a^2\bigg(\frac{1}{2}(1+u+v)\left[\Phi^{(1)}(u,v)\right]^2 \\ &+ 2\left[\Phi^{(2)}(u,v) + \frac{1}{u}\Phi^{(2)}(v/u,1/u) + \frac{1}{v}\Phi^{(2)}(1/v,u/v)\right]\bigg) + O(a^3)\end{aligned}$$

$\Phi^{(1)}(u,v)$ ‘box’ integral, $\Phi^{(2)}(u,v)$ ‘double’ box integral

- $\mathcal{N}=4$ superconf. symmetry allows us to determine $\Phi_{\rm weak}(u,v)$ to six loops [Eden,Heslop,GK,Sokatchev]
- AdS/CFT correspondences predicts $\Phi(u,v)$ at strong coupling [Arutyunov, Frolov]

From Euclid to Minkowski

- ✔ Brute force method: compute anew using Schwinger-Keldysh technique (too hard)
- ✔ Better method: analytically continue correlation functions from Euclid to Minkowski+Wightman
- ✔ Warm-up example: free scalar propagator $D_{\rm Euclid}(x) = \langle \phi(x)\phi(0)\rangle \sim 1/x^2$

$$\langle 0|\phi(x)\phi(0)|0\rangle = \sum_n \langle 0|\phi(x)|n\rangle\langle n|\phi(0)|0\rangle$$

$$= \sum_{E_n>0} \mathrm{e}^{-iE_n(x^0-i0)+i\vec{p}\vec{x}}\langle 0|\phi(0)|n\rangle\langle n|\phi(0)|0\rangle \sim \frac{1}{(x^0-i0)^2-\vec{x}^2}$$

- ✔ How to get Wightman correlation functions ('magic' recipe) [Mack]
  - ✗ Go to Mellin space:

$$\Phi_{\rm Euclid} = \int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1\, dj_2}{(2\pi i)^2} M(j_1,j_2;a)\, u^{j_1} v^{j_2}\,, \qquad u = \frac{x_{12}^2 x_{34}^2}{x_{13}^2 x_{24}^2}\,, \qquad v = \frac{x_{23}^2 x_{41}^2}{x_{13}^2 x_{24}^2}$$

  - ✗ Substitute $x_{ij}^2 \quad\to\quad x_{ij,+}^2 = x_{ij}^2 - i0\cdot x_{ij}^0$

$$\Phi_{\rm Wightman} = \int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1\, dj_2}{(2\pi i)^2} M(j_1,j_2;a)\left(\frac{x_{12,+}^2 x_{34,+}^2}{x_{13,+}^2 x_{24,+}^2}\right)^{j_1}\left(\frac{x_{23,+}^2 x_{41,+}^2}{x_{13,+}^2 x_{24,+}^2}\right)^{j_2}$$

- ✔ $M(j_1,j_2;a)$ is known both at weak and strong coupling in planar $\mathcal{N}=4$ SYM

All-loop prediction for EEC

Master formula

$$\mathrm{EEC}(\chi) = \frac{1}{4z^2(1-z)} \int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} \underbrace{M(j_1,j_2;a)}_{\text{corr.function}} \underbrace{K(j_1,j_2)}_{\text{detector}} \underbrace{\left(\frac{1-z}{z}\right)^{j_1+j_2}}_{\text{angular dependence}}$$

The dependence on the angle χ enters through

$$z = (1-\cos\chi)/2\,, \qquad\qquad 0 < z < 1$$

Detector function is independent on the coupling

$$K(j_1,j_2) = \frac{2\,\Gamma(1-j_1-j_2)}{\Gamma(j_1+j_2)[\Gamma(1-j_1)\Gamma(1-j_2)]^2}$$

The dependence on the coupling constant resides in the Mellin amplitude

$$\Phi(u,v;a) = \int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} M(j_1,j_2;a)\, u^{j_1} v^{j_2}$$

$$M(j_1,j_2;a) = \underbrace{aM^{(1)}(j_1,j_2) + a^2 M^{(2)}(j_1,j_2)}_{\text{are known}} + \dots$$

Warm up exercise

- ✔ Master formula at one loop

$$\mathrm{EEC}^{(1-\mathrm{loop})} = \frac{a}{4z^2(1-z)} \int_{-\delta-i\infty}^{-\delta+i\infty} \frac{dj_1 dj_2}{(2\pi i)^2} M^{(1)}(j_1, j_2; a) K(j_1, j_2) \left(\frac{1-z}{z}\right)^{j_1+j_2}$$

Mellin amplitude

$$M^{(1)}(j_1, j_2) = -\frac{1}{4} \left[\Gamma(-j_1)\Gamma(-j_2)\Gamma(1+j_1+j_2)\right]^2$$

$$K(j_1, j_2) = \frac{2\,\Gamma(1-j_1-j_2)}{\Gamma(j_1+j_2)[\Gamma(1-j_1)\Gamma(1-j_2)]^2}$$

- ✔ Change integration variable $j_1 + j_2 \to j_1$

$$\begin{aligned}
\mathrm{EEC}^{(1-\mathrm{loop})} &= -\frac{a}{4z^2(1-z)} \int \frac{dj_1 dj_2}{(2\pi i)^2} \frac{j_1^2}{2(j_1-j_2)^2 j_2{}^2} \frac{\pi}{\sin(\pi j_1)} \left(\frac{1-z}{z}\right)^{j_1} \\
&= \frac{a}{4z^2(1-z)} \int \frac{dj_1}{2\pi i} \frac{\pi}{j_1 \sin(\pi j_1)} \left(\frac{1-z}{z}\right)^{j_1} \\
&= \frac{a}{4z^2(1-z)} \sum_{k=-1}^{-\infty} \frac{(-1)^k}{k} \left(\frac{1-z}{z}\right)^{k} \\
&= \frac{a}{4z^2(1-z)} \ln\frac{1}{1-z}
\end{aligned}$$

EEC at two loops

Final result for EEC

$$\text{EEC}_{\mathcal{N}=4} = \frac{1}{4z^2(1-z)}\left\{aF_1(z) + a^2\left[(1-z)F_2(z) + \tfrac{1}{4}F_3(z)\right] + O(a^3)\right\}, \qquad z = \tfrac{1}{2}(1-\cos\chi)$$

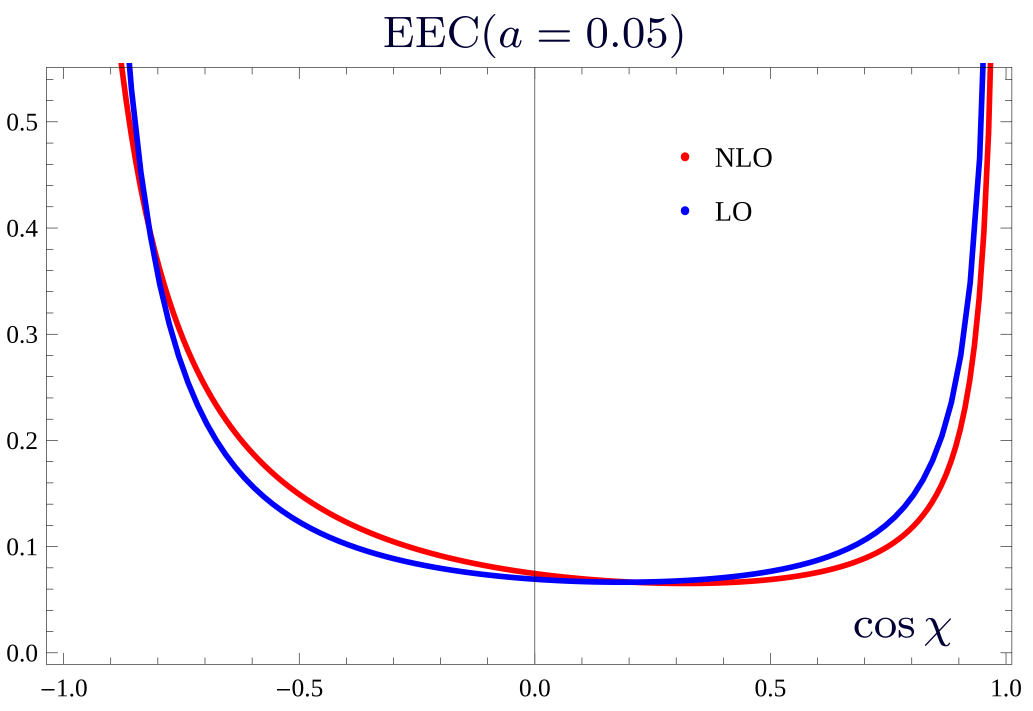
$F_w(z)$ are linear combinations of functions of homogenous weight $w = 1, 2, 3$

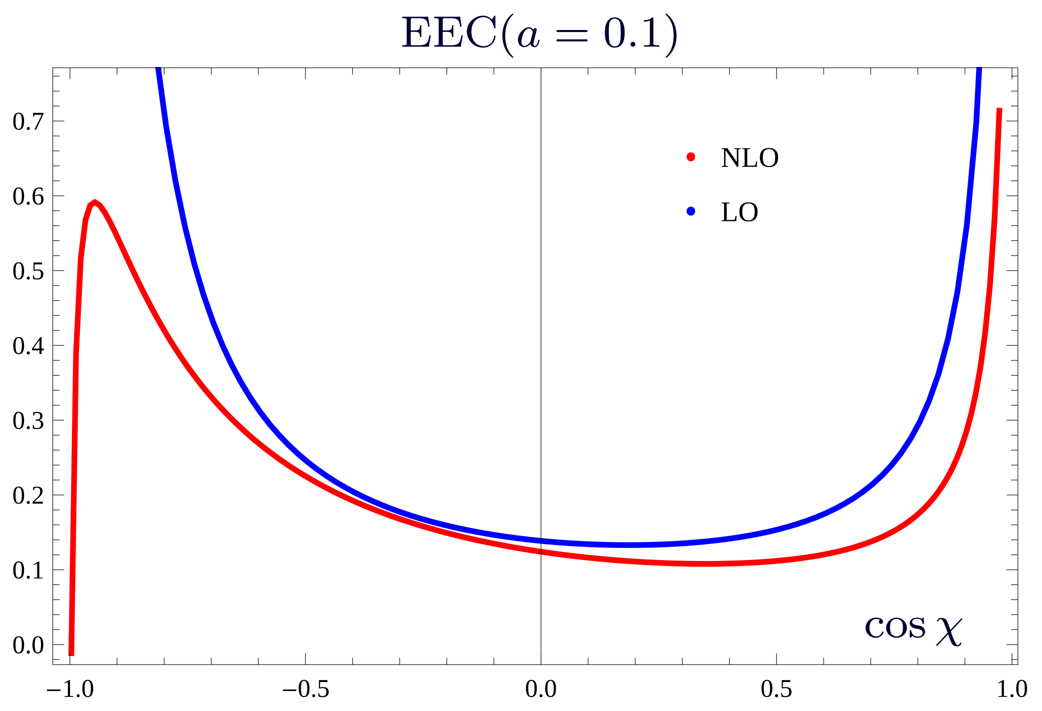
$$F_1(z) = -\ln(1-z)$$

$$F_2(z) = 4\sqrt{z}\left[\text{Li}_2\left(-\sqrt{z}\right) - \text{Li}_2\left(\sqrt{z}\right) + \frac{1}{2}\ln z \ln\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\right] + (1+z)\left[2\text{Li}_2(z) + \ln^2(1-z)\right] + 2\ln(1-z)\ln\left(\frac{z}{1-z}\right) + z\frac{\pi^2}{3},$$

$$F_3(z) = (1-z)(1+2z)\left[\ln^2\left(\frac{1+\sqrt{z}}{1-\sqrt{z}}\right)\ln\left(\frac{1-z}{z}\right) - 8\text{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}-1}\right) - 8\text{Li}_3\left(\frac{\sqrt{z}}{\sqrt{z}+1}\right)\right] - 4(z-4)\text{Li}_3(z) + 6(3+3z-4z^2)\text{Li}_3\left(\frac{z}{z-1}\right) - 2z(1+4z)\zeta_3 + 2\big[(3-4z)z\ln z + 2(2z^2-z-2)\ln(1-z)\big]\text{Li}_2(z) + \frac{1}{3}\ln^2(1-z)\left[4(3z^2-2z-1)\ln(1-z) + 3(3-4z)z\ln z\right] + \frac{\pi^2}{3}\left[2z^2\ln z - (2z^2+z-2)\ln(1-z)\right]$$

From weak to strong coupling





- ✔ At weak coupling $\mathrm{EEC}_{\mathcal{N}=4}$ has a shape which is remarkably similar to the one in QCD
- ✔ Going from one to two loops, EEC flattens
- ✔ This agrees with strong coupling prediction for EEC in planar $\mathcal{N}=4$ SYM [Hofman,Maldacena]

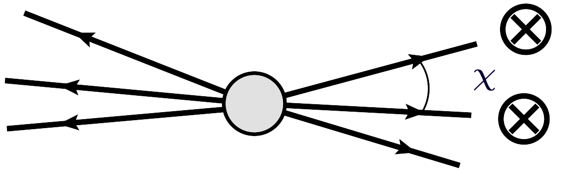
$$\mathrm{EEC}_{\mathcal{N}=4} \overset{a\to\infty}{\sim} \frac{1}{2}\left[1 + a^{-1}\left(1 - 6z(1-z)\right) + O(a^{-3/2})\right]$$

No jets at strong coupling

End-point asymptotics I

Small angle correlations $\chi \to 0$ (or $z \sim \chi^2 \to 0$): calorimeters measure nearly collinear particles

$$\text{EEC} \overset{z\to 0}{\sim} \frac{a}{4z}\left[1 + a\left(\ln z - \tfrac{1}{2}\zeta_3 + \zeta_2 - 3\right)\right]$$



- ✔ Corrections are enhanced by $\ln z$, no homogenous transcedentality
- ✔ Resummation of leading log's $a(a\ln z)^k$ using the "jet calculus" [Konishi,Ukawa,Veneziano]

$$\text{EEC} \overset{z\to 0}{\sim} \frac{a}{4z}\int_0^1 dx\, x^2 D(x, Q^2/S_{ab})$$
$$= \frac{a}{4z}(Q^2/S_{ab})^{-\gamma_T(3)} = \frac{a}{4} z^{-1+\gamma_T(3)}$$

$\gamma_T(3) = a + O(a^2)$ – twist-2 *time-like* anomalous dimension of spin $S = 3$

$D(x, Q^2/S_{ab})$ probability to fragment into a pair of partons with $S_{ab} = 2E_aE_b(1-\cos\chi) \sim Q^2 z$

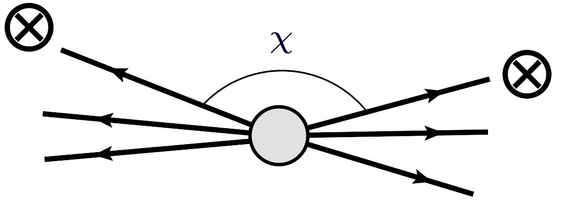
- ✔ Resummation weakens singularity of EEC for $\chi \to 0$, jets at weak coupling

$$\int_0^{\chi_0} d\cos\chi\, \text{EEC} \sim \frac{a}{\gamma_T(3)} \sim 1\,, \qquad (\chi_0 \ll 1)$$

End-point asymptotics II

EEC in the back-to-back kinematics $\chi \to \pi$ (or $y \equiv 1 - z \sim (\pi-\chi)^2 \to 0$)

$$\text{EEC} \stackrel{z\to 1}{\sim} \frac{1}{4y}\left\{a\ln(1/y) - \frac{a^2}{2}\left[\ln^3(1/y) + \frac{\pi^2}{2}\ln(1/y)\right]\right\}$$



- ✔ Large (Sudakov) corrections $a^k(\ln y)^n$ come from the emission of soft and collinear particles
- ✔ All order resummation [Collins, Soper]

$$\text{EEC} \sim \frac{1}{8y} H(a)\int_0^\infty db\, b\, J_0(b) S(b^2/y;a)$$

$J_0(b)$ Bessel function; $S(b^2/y;a)$ the Sudakov form factor (with $b_0 = 2\,\mathrm{e}^{-\gamma_\mathrm{E}}$)

$$S = \exp\left[-\frac{1}{2}\Gamma_{\text{cusp}}(a)\ln^2\left(\frac{b^2}{yb_0^2}\right) - \Gamma(a)\ln\left(\frac{b^2}{yb_0^2}\right)\right]$$

Dependence on the coupling constant is encoded in three functions

$$\Gamma_{\text{cusp}}(a) = a - \tfrac{1}{2}\zeta_2 a^2\,, \qquad \Gamma(a) = -\tfrac{3}{2}\zeta_3 a^2\,, \qquad H(a) = 1 - \zeta_2 a$$

- ✔ Perturbative corrections to $\text{EEC}(z\to 1)$ have homogeneous transcedentality

$$\left[\text{EEC}_{\text{QCD}}(z\to 1)\right]_{\text{maximal transcedentality}} = \text{EEC}_{\mathcal{N}=4}(z\to 1)$$

Conclusions and open questions

- ✔ Energy correlations are good/nontrivial physical observables in $\mathcal{N} = 4$ SYM
- ✔ Relation to energy flow correlations in QCD (most complicated part)?
- ✔ All symmetries of $\mathcal{N} = 4$ SYM are preserved, what is the manifestation of integrability?
- ✔ Interpolation between weak and strong coupling?
- ✔ Other proposals for ‘good’ observables?