Quantum gravity and the equivalence principle

Pierre Vanhove

Current Themes in High Energy Physics and Cosmology

Niels Bohr Institute, Copenhagen

based on work 1309.0804 and work in progress

N.E.J. Bjerrum-Bohr, John Donoghue, Barry Holstein, Ludovic Planté

Motivations

Quantum gravity is still rather poorly understood although it expected to play a fundamental role in structure of our present universe.

Motivations

Quantum gravity is still rather poorly understood although it expected to play a fundamental role in structure of our present universe.

Gravity is intrinsically non-linear, with a dimensional coupling constant and non-renormalisable

Motivations

Quantum gravity is still rather poorly understood although it expected to play a fundamental role in structure of our present universe.

Gravity is intrinsically non-linear, with a dimensional coupling constant and non-renormalisable

Recent on-shell S-matrix computations in pure and extended supergravity showed that *many* simplifications take place leading to *surprisingly simple* results compared to the Feynman graph approach

Motivations

Quantum gravity is still rather poorly understood although it expected to play a fundamental role in structure of our present universe.

Gravity is intrinsically non-linear, with a dimensional coupling constant and non-renormalisable

Recent on-shell S-matrix computations in pure and extended supergravity showed that *many* simplifications take place leading to *surprisingly simple* results compared to the Feynman graph approach

In this talk we want to explain that the remarkable properties of on-shell gravity amplitudes allow to perform concrete physically motivated computations in pure gravity coupled to various kind of massive matter

Quantum gravity as an effective field theory

[Donoghue] has explained that one can evaluate some long-range infra-red contributions in any quantum gravity theory and obtain reliable answers independent of the UV completion.

Some physical properties of quantum gravity are *universal* being independent of the UV completion

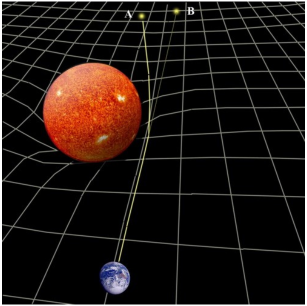
We are interested in quantum gravity contributions at loop order that depend only on the structure of the effective tree Lagrangian

These will be infra-red contributions involving only the structure of the tree amplitudes and independent of the UV completion

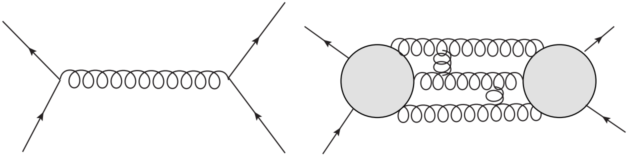
Physics of the effective field theory approach

Using the effective field theory approach to gravity one can compute

- the classical (post-Newtonian) and quantum contributions to the gravitational potential between masses
- Quantum corrections to the bending angle of massless particle by a massive classical object



Classical physics from loops



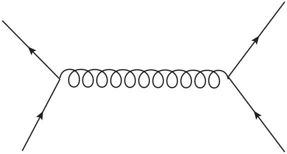
We will be considering the pure gravitational interaction between massive and massless matter of various spin

$$\mathcal{L}_{\text{EH}} \sim \int d^4x \left(-\frac{2}{\kappa^2} \mathcal{R} + \kappa h_{\mu\nu} T^{\mu\nu}_{\text{matter}} \right) ,$$

We will be considering perturbative computations $\kappa^2 = 32\pi G_N$

$$\mathfrak{M} = \frac{1}{\hbar} \mathfrak{M}^{\text{tree}} + \hbar^0 \mathfrak{M}^{1-\text{loop}} + \cdots .$$

Classical physics from loops



The tree-level contribution is the 1-graviton exchange giving the classical Newtonian potential in the non-relativistic limit

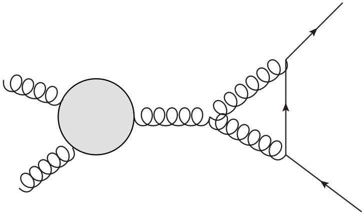
$$\mathfrak{M}^{\text{tree}} \propto G_N \frac{(m_1 m_2)^2}{\vec{q}^2}$$

The potential is obtained by

$$V(r) = \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{1}{4m_1 m_2} \mathfrak{M}(\vec{q})\, e^{i\vec{q}\cdot\vec{r}}$$

Classical physics from loops

Let's consider the one-loop contribution for a say a massive scalar of mass m



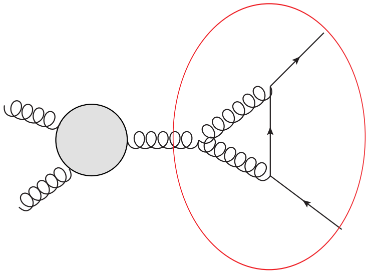
Putting back the factors of $\hbar$ and c the Klein-Gordon equation reads

$$(\Box - \frac{m^2c^2}{\hbar^2})\phi = 0$$

Notice that the $\hbar$ dependence on the mass term

Classical physics from loops

Let's consider the one-loop contribution for a say a massive scalar of mass m



The triangle contribution with a massive leg $p_1^2 = p_2^2 = m^2$ reads

$$\int \frac{d^4\ell}{\ell^2((\ell+p_1)^2 - \frac{m^2c^2}{\hbar^2})((\ell-p_2)^2 - \frac{m^2c^2}{\hbar^2})}\Bigg|_{\text{finite part}} \sim \frac{1}{m^2}\left(\log(s) + \frac{\pi^2 m}{\hbar\sqrt{s}}\right)$$

Classical physics from loops

The $1/\hbar$ term at one-loop contributes to the *same* order as the classical tree term [Donoghue; Bjerrum-Bohr, Donoghue, Holstein; Donoghue, Holstein; Bjerrum-Bohr, Donoghue, Vanhove]

$$\mathfrak{M} = \frac{1}{\hbar}\left(\frac{G_N(m_1m_2)^2}{\vec{q}^2} + \frac{G_N^2(m_1m_2)^2(m_1+m_2)}{|\vec{q}|} + \cdots\right) + \hbar^0 G_N^2\, O(\log(\vec{q}^2)) + \cdots$$

For the scattering between a massive matter of mass m and massless matter of energy E one gets

$$\mathfrak{M} \sim \frac{1}{\hbar}\left(G_N\frac{(mE)^2}{\vec{q}^2} + G_N^2\frac{m^3E^2}{|\vec{q}|}\right) + \hbar\, G_N^2 O\left(\log(\vec{q}^2), \log^2(\vec{q}^2)\right) .$$

The mechanisms generalizes to higher loop-order amplitudes to leads to the higher order post-Newtonian corrections

Classical physics from loops

The one-loop contribution is a small correction if and only if

$$G_N m|\vec{q}| \ll 1 \,.$$

This condition implies that the impact parameter b has to be much larger that the Schwarzschild radius of the massive scalar

$$b \gg r_S \,.$$

For the evaluation of the post-Newtonian corrections this correction is clearly satisfied has the potential is evaluation far away from the massive source but when massless particles are involved we will need to rethink about this condition

Corrections to Newton's potential

One-loop corrections to Newton's potential can be calculated using effective field theory approach to gravity [Donoghue; Bjerrum-Bohr, Donoghue, Holstein; Bjerrum-Bohr, Donoghue, Vanhove]

$$V(r) = -\frac{G_N m_1 m_2}{r} \left(1 + C\,\frac{G_N(m_1+m_2)}{r} + Q\frac{G_N \hbar}{r^2}\right) + Q' G_N^2 m_1 m_2 \delta^3(\vec{x})$$

- C is the classical correction and Q and Q' are quantum corrections

Corrections to Newton's potential

One-loop corrections to Newton's potential can be calculated using effective field theory approach to gravity [Donoghue; Bjerrum-Bohr, Donoghue, Holstein; Bjerrum-Bohr, Donoghue, Vanhove]

$$V(r) = -\frac{G_N m_1 m_2}{r}\left(1 + C\,\frac{G_N(m_1+m_2)}{r} + Q\frac{G_N\hbar}{r^2}\right) + Q' G_N^2 m_1 m_2 \delta^3(\vec{x})$$

- C is the classical correction and Q and Q' are quantum corrections

- If $\lambda = \hbar/(m_1+m_2)$ is the Compton wavelength

$$C\,\frac{G_N m_1 m_2 (m_1+m_2)}{(r\pm\lambda)^2} \simeq C\,\frac{G_N m_1 m_2 (m_1+m_2)}{r^2} \pm C\,\frac{G_N m_1 m_2 \overbrace{(m_1+m_2)\lambda}^{\hbar}}{r^3}$$

- Q in the potential $V(r)$ is ambiguous but $V(r)$ is not observable

Corrections to Newton's potential

One-loop corrections to Newton's potential can be calculated using effective field theory approach to gravity [Donoghue; Bjerrum-Bohr, Donoghue, Holstein; Bjerrum-Bohr, Donoghue, Vanhove]

$$\mathfrak{M}^{1-\text{loop}}(q^2) = \frac{G_N (m_1 m_2)^2}{q^2} + C\frac{G_N^2 (m_1 m_2)^2 (m_1+m_2)}{|q|} + \hbar \left(Q G_N^2 (m_1 m_2)^2 \log(-q^2) + Q' G_N^2 (m_1 m_2)^2 \right)$$

The coefficients of $1/\sqrt{-q^2}$ and $\log(-q^2)$ in the amplitude are unambiguously defined and depend on the long range physics

Corrections to Newton's potential

One-loop corrections to Newton's potential can be calculated using effective field theory approach to gravity [Donoghue; Bjerrum-Bohr, Donoghue, Holstein; Bjerrum-Bohr, Donoghue, Vanhove]

$$\mathfrak{M}^{1-\text{loop}}(q^2) = \frac{G_N(m_1m_2)^2}{q^2} + C\frac{G_N^2(m_1m_2)^2(m_1+m_2)}{|q|} + \hbar\left(QG_N^2(m_1m_2)^2\log(-q^2) + Q'G_N^2(m_1m_2)^2\right)$$

- Q' is the short distance UV divergences of quantum gravity: need to add the R^2 term ['t Hooft-Veltman]

$$S = \int d^4x|-g|^{\frac{1}{2}}\left[\frac{2}{32\pi G_N}\mathcal{R} + c_1\mathcal{R}^2 + c_2R_{\mu\nu}R^{\mu\nu} + \cdots\right]$$

Corrections to Newton's potential

One-loop corrections to Newton's potential can be calculated using effective field theory approach to gravity [Donoghue; Bjerrum-Bohr, Donoghue, Holstein; Bjerrum-Bohr, Donoghue, Vanhove]

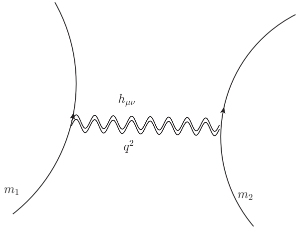
$$\begin{aligned}\mathfrak{M}^{1-\mathrm{loop}}(q^2) = \frac{G_N(m_1m_2)^2}{q^2} + C\frac{G_N^2(m_1m_2)^2(m_1+m_2)}{|q|} \\ + \hbar\left(QG_N^2(m_1m_2)^2\log(-q^2) + Q'G_N^2(m_1m_2)^2\right)\end{aligned}$$

The coefficients C and Q are independent of the UV completion and any quantum gravity theory should give these computations

Perturbative technics

Classical Newton's potential is obtained in the non-relativistic limit

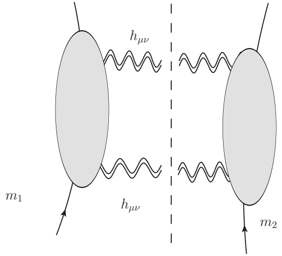
$$V(|\vec{q}|) = \frac{G_N m_1 m_2}{\vec{q}^2} \qquad V(r) = -\frac{G_N m_1 m_2}{r}$$



is derived by a tree-level graph exchanging a graviton

Loop amplitude

Since we are only interested in the long range graviton exchange, it is enough to just evaluate the gravitons cut

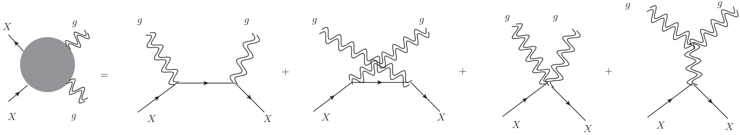


we need to know the gravitational Compton amplitudes on a particle of spin s with mass m

$$X^{s,m} + \text{graviton} \to X^{s,m} + \text{graviton}$$

Gravitational compton scattering

Gravitational Compton scatting off a massive particle of spin $s = 0, \frac{1}{2}, 1$



using Feynman rules and DeWitt or Sannan's 3- and 4-point vertices this is a big mess but this will be simplified using the momentum kernel formalism to gravity amplitude

The Momentum Kernel formalism Gravity amplitude

The KLT relation allow to express the field theory multi-particle tree-level amplitudes as bilinear of color ordered Yang-Mills amplitudes

$$\mathfrak{M}_n^{\text{tree}} = (-1)^{n-3} \sum_{\sigma,\gamma\in\mathfrak{S}_{n-3}} \mathcal{S}[\gamma(2,\dots,n-2)|\sigma(2,\dots,n-2)]_{k_1} \times \mathcal{A}_n(1,\sigma(2,\dots,n-2),n-1,n)\widetilde{\mathcal{A}}_n(n-1,n,\gamma(2,\dots,n-2),1)$$

The color ordered Yang-Mills amplitudes satisfy the annihilation relation $\forall\beta \in \mathfrak{S}_{n-2}$

$$\sum_{\sigma\in\mathfrak{S}_{n-2}} \mathcal{S}(\sigma(2,\dots,n-1)|\beta(2,\dots,n-1))|_{k_1}\mathcal{A}(1,\sigma(2,\dots,n-1),n) = 0$$

[Bern, Carrasco, Johansson] [Kawai,Lewellen, Tye; Tye, Zhang;Bjerrum-Bohr, Damgaard, Feng, Søndergaard; Bjerrum-Bohr, Damgaard, Søndergaard, Vanhove; Stieberger]

The Momentum kernel in field theory

The $\alpha' \to 0$ limit of the monodromy relations between string theory amplitudes lead to an object named momentum kernel $\mathcal{S}$

$$\mathcal{S}[i_1,\ldots,i_k|j_1,\ldots,j_k]_p := \prod_{t=1}^{k}\left(p\cdot k_{i_t} + \sum_{q>t}^{k}\theta(t,q)\,k_{i_t}\cdot k_{i_q}\right)$$

$\theta(t,q) = 1$ if $(i_t - i_q)(j_t - j_q) < 0$ and 0 otherwise

[Bern, Carrasco, Johansson; Bjerrum-Bohr, Damgaard, Vanhove; Stieberger; Mafra, Schlotterer]

[Bjerrum-Bohr, Damgaard, Feng, Søndergaard; Bjerrum-Bohr, Damgaard, Søndergaard, Vanhove]

Tree amplitudes with massive external legs I

We are interested into *pure gravity* amplitudes of gravitons scattering off *massive particles*, the relation between gravity and YM amplitude stays the same [Bjerrum-Bohr, Donoghue, Vanhove]

We remark

- The amplitude relation is valid in any dimensions
- The momentum kernel is a function of the scalar products $k_i \cdot k_j$
- Massive particle in 4 dimensions are massless particle in higher dimensions

This implies that the expression for the gravity amplitudes as bilinear of YM amplitudes will apply as well with massive matter external states

Tree amplitudes with massive external legs II

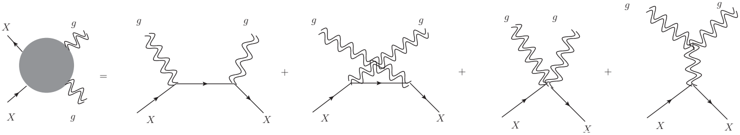
Another argument goes back to the way the relation is derived from the properties of the string theory amplitudes

$$\mathcal{A}^{\text{vector}}(\sigma(1,\ldots,n)) = \int_{x_{\sigma(1)}<\cdots<x_{\sigma(n)}} d^{n-3}x\, f(x_i - x_j) \prod_{1\leqslant i<j\leqslant n} (x_i - x_j)^{2\alpha' k_i\cdot k_j}$$

- Massive state vop are of the form $V =: (\partial X)^{n+1} e^{ik\cdot X} :$ with $\alpha' k^2 = n$
- The OPE between the plane-wave still gives $(x_i - x_j)^{2\alpha' k_i\cdot k_j}$
- The function $f(x_i - x_j)$ develops new poles $1/(x_i - x_j)^m$ with m integer to accommodate for the masses $\alpha'(k_i^2 + k_j^2) = m$

But the momentum kernel and the amplitudes relations arises from the phases of $(x_i - x_j)^{2\alpha' k_i\cdot k_j}$ they are still valid in the same form for massive external states

Gravitational compton scattering

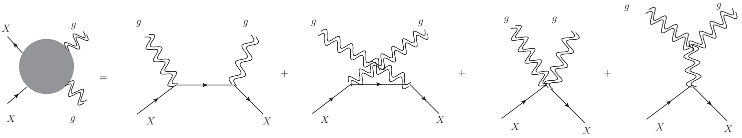


We express the gravity Compton scattering as a product of two Yang-Mills amplitudes

$$\mathfrak{M}(X^s g \to X^s g) = G_N \times (p_1 \cdot k_1)\, \mathcal{A}_s(1234) \tilde{\mathcal{A}}_0(1324)$$

$\mathcal{A}_s(1234)$ is the color ordered amplitudes scattering a gluon off a massive spin s state $X^s g \to X^s g$

Gravitational compton scattering

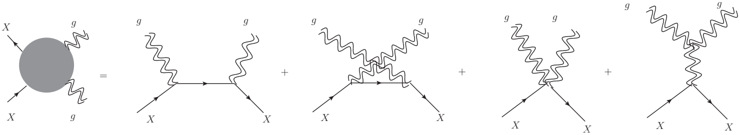


We express the gravity Compton scattering as a product of two QED Compton amplitudes using the monodromy relations

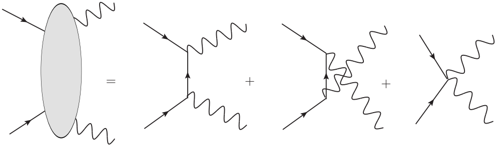
$$(k_1 \cdot k_2)\, \mathcal{A}_s(1234) = (p_1 \cdot k_2)\, \mathcal{A}_s(1324)$$

$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{(p_1 \cdot k_1)(p_1 \cdot k_2)}{k_1 \cdot k_2}\, \mathcal{A}_s(1324) \tilde{\mathcal{A}}_0(1324)$$

Gravitational compton scattering



The gravity Compton scattering is expressed as the square of QED (abelian) Compton amplitudes



$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{(p_1 \cdot k_1)(p_1 \cdot k_2)}{k_1 \cdot k_2} \mathcal{A}_s(1324) \tilde{A}_0(1324)$$

Gravity Low-energy theorems from QED

A first physical consequence of the relation between the gravitational Compton amplitudes and the QED amplitudes are low-energy theorem for gravity

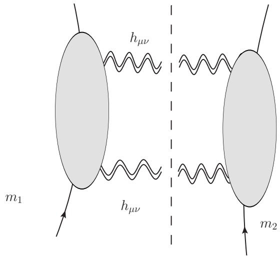
These low-energy theorem are important for determining the long range - small momentum transfer - contributions and for making the connection with classical GR

$$\mathfrak{M}(X^s g \to X^s g) = G_N \frac{p_1 \cdot k_1 \, p_1 \cdot k_2}{k_1 \cdot k_2} \mathcal{A}_s(1324) \tilde{\mathcal{A}}_0(1324)$$

The QED Compton amplitude is exact at fixed angle up to order p^2 so this immediately leads to the fact that the Compton gravity amplitude is exact up to order p^4

The relation provides a much simpler expression for the soft graviton behaviour than the derivation by [Gross, Jackiw; Jackiw]

The one-loop amplitude between massive particles



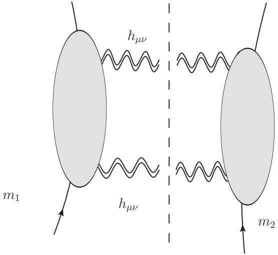
We are only interested in the $1/\sqrt{-q^2}$ and $\log(-q^2)$ terms since the terms of $(q^2)^n/\sqrt{-q^2}$ and $(q^2)^n \log(-q^2)$ are negligible in the non-relativistic limit. Only the massless graviton cut is enough.

- The cut contributions

$$\mathfrak{M}|_{\text{singlet cut}} = \int \frac{d^{4-2\epsilon}\ell}{\ell_1^2 \ell_2^2 \prod_{i=1}^4 \ell_1 \cdot p_i}$$

$$\mathfrak{M}|^{\text{non-singlet cut}} = \int d^{4-2\epsilon}\ell \frac{\mathfrak{Re}\left(\text{tr}_-(\not\ell_1 \not p_1 \not\ell_2 \not p_2)\right)^4}{\ell_1^2 \ell_2^2 \prod_{i=1}^4 \ell_1 \cdot p_i}$$

The one-loop amplitude between massive particles

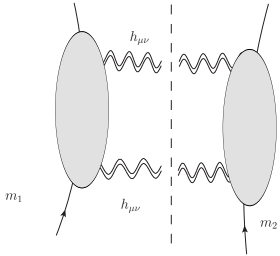


We are only interested in the $1/\sqrt{-q^2}$ and $\log(-q^2)$ terms since the terms of $(q^2)^n/\sqrt{-q^2}$ and $(q^2)^n \log(-q^2)$ are negligible in the non-relativistic limit.
Only the massless graviton cut is enough.

- In the non-relativistic limit the amplitude decomposes

$$\mathfrak{M} \simeq G_N^2 (m_1 m_2)^4 (I_4(s,t) + I_4(s,u)) + G_N^2 (m_1 m_2)^3 s (I_4(s,t) - I_4(s,u))$$
$$G_N^2 (m_1 m_2)^2 (I_3(s,m_1) + I_3(s,m_2)) + G_N^2 (m_1 m_2)^2 I_2(s)$$

The one-loop amplitude between massive particles



We are only interested in the $1/\sqrt{-q^2}$ and $\log(-q^2)$ terms since the terms of $(q^2)^n/\sqrt{-q^2}$ and $(q^2)^n \log(-q^2)$ are negligible in the non-relativistic limit. Only the massless graviton cut is enough.

- The result is given by

$$\mathfrak{M} \simeq G_N^2 (m_1 m_2)^2 \left(\underbrace{6\pi}_{C} \frac{m_1 + m_2}{\sqrt{-q^2}} \underbrace{-\frac{41}{5}}_{Q} \log(-q^2) \right)$$

Universality of the result I

In the non-relativistic limit the second order potential reads

$$\mathfrak{M}^{(2)}(q^2) \simeq G_N^2 (m_1 m_2)^2 \left(C \frac{(m_1 + m_2)}{\sqrt{-q^2}} + Q\hbar \log(-q^2) \right)$$

The coefficient C and Q have a spin-independent and a spin-orbit contribution

$$C, Q = C, Q^{S-I} \langle S_1 | S_1 \rangle \langle S_2 | S_2 \rangle + C, Q_{1,2}^{S-O} \langle S_1 | S_1 \rangle \vec{S}_2 \cdot \frac{p_3 \vec{\times} p_4}{m_2} + (1 \leftrightarrow 2)$$

This expression is generic for all type of matter, the numerical coefficients are the same for all matter type

Universality of the result II

The universality of the coefficients with respect to the spin of the external states is a consequence of

- The reduction to the product of QED amplitudes
- the low-energy theorems of [Low, Gell-Mann, Goldberger] and [Weinberg]

In the non-relativistic limit the QED Compton amplitudes reads

$$\mathcal{A}(X^s\gamma \to X^s\gamma) \simeq \langle S|S\rangle\, \mathcal{A}(X^0\gamma \to X^0\gamma) + \frac{\vec{S}\cdot\hat{\mathcal{A}}}{m}$$

The KLT formula gives that the tree gravity amplitude reads

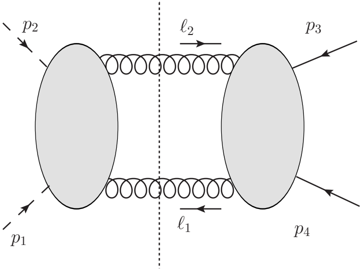
$$\mathfrak{M}(X^s g \to X^s g) \simeq \langle S|S\rangle\, \mathfrak{M}(X^0 g \to X^0 g) + \frac{\vec{S}\cdot\hat{\mathfrak{M}}}{m}$$

The low-energy theorem imply that $\hat{\mathcal{A}}$ and $\hat{\mathfrak{M}}$ are independent of the spin s

Universality of the result III

- In the cut this leads to universality of the result [Bjerrum-Bohr, Donoghue, Vanhove]
- This is totally what one expects from the equivalence principle and the multipole expansion of the gravitational interaction between massive states
- The long range quantum correction involves low-energy gravity degrees of freedom and is independent of any microscopic high-energy model dependent contributions

The one-loop amplitude for massless particles

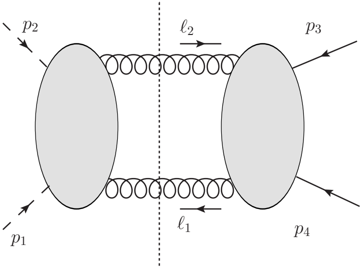


The gravitational one-loop amplitude between a massless and a massive particle $E \ll m$ and $G_N m|q| \gg 1$

$$\mathfrak{M}^{\text{classical}}(s) = \frac{1}{\hbar}\left(G_N\frac{(mE)^2}{q^2} + G_N^2\frac{m^3E^2}{\sqrt{-q^2}}\right)$$

The result is independent of the spin as in the previous case

The one-loop amplitude for massless particles

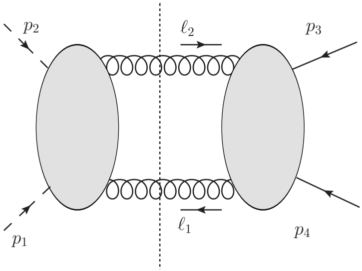


The gravitational one-loop amplitude between a massless and a massive particle $E \ll m$ and $G_N m|q| \gg 1$

$$\mathfrak{M}^{\text{quantum}}(s) = (G_N m E)^2 \left(\left(c_{\text{UV}} + \frac{mE}{q^2}\right) \log s + \log^2 s \right)$$

The coefficient c_{UV} depends on the high-energy UV behaviour.
The rest of the coefficients are universal

The one-loop amplitude for massless particles

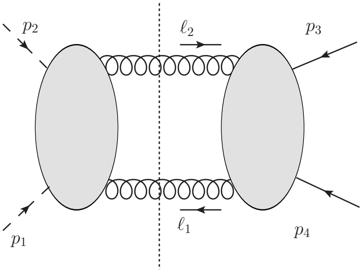


The gravitational one-loop amplitude between a massless and a massive particle $E \ll m$ and $G_N m|q| \gg 1$

$$\mathfrak{M}^{\text{quantum}}(s) = (G_N m E)^2 \left((c_{\text{UV}} + \frac{mE}{q^2}) \log s + \log^2 s \right)$$

These taking the soft graviton emission into account one is left with non vanishing quantum correction

The one-loop amplitude for massless particles

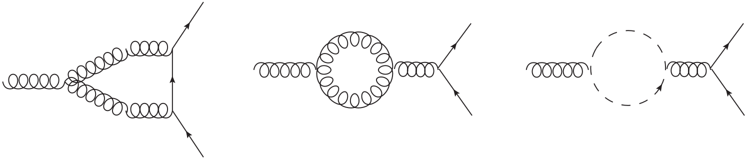


The gravitational one-loop amplitude between a massless and a massive particle $E \ll m$ and $G_N m|q| \gg 1$

$$\mathfrak{M}^{\text{quantum}}(s) = (G_N m E)^2 \left(\left(c_{\text{UV}} + \frac{mE}{q^2}\right) \log s + \log^2 s \right)$$

There is no birefringence effects to contrary to case with electrons loops contributing to the interaction [Drummond, Hathrell;Berends, Gastmans]

The bending angle



The geodesic equation in a background metric where we have quantized the interaction of the graviton with the massive source including the light fields

$$b^2 = r_S^2 \left(\frac{4}{\theta^2} + \frac{15\pi}{8\theta} + c_{\text{vac}} \left(\frac{\ell_P}{r_S} \right)^2 \right)$$

No violation of the equivalence principle but the vacuum contribution depends on the massless matter in the loop

Outlook

Recent progresses from string theory technics, on-shell unitarity, double-copy formalism simplifies a lot perturbative gravity amplitudes computations

- The amplitudes relations discovered in the context of massless supergravity theories extend to the pure gravity case with massive matter
- The use of quantum gravity as an effective field theory allows to compute universal contributions from the long-range corrections
- We can reproduce the classical GR post-Newtonian corrections to the potential and understand some generic properties using low-energy theorems
- We can evaluate various interesting quantum corrections to physically observable quantities (to appear [Bjerrum-Bohr, Donoghue, Holstein, Planté, Vanhove])