Hybrid Inflation with Planck Scale Fields

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Small vs. Large Field Inflation

Categories of Inflationary models:

1. Large Field: $\phi \gg M_p$. Chaotic inflation, natural inflation, monodromy inflation. Predict observable gravity waves (consistent with possible result reported by BICEP2). $r = 0.2 \Rightarrow \epsilon = 0.012; \quad V = (2 \times 10^{16}\mathrm{GeV})^4$.
2. Small Field: $\phi \ll M_p$. Associated with "hybrid inflation". No observable gravity waves. For hybrid, challenges to understand n_s as reported by Planck ($n_s = 0.9603 \pm 0.0073$.)

Theoretical Challenges in Each Framework

1. Large field:
   a. Chaotic inflation: typically monomial potentials. Require small coefficients (why?). Suppression of $\frac{\phi^n}{M_p^{n-4}}$. Why?
   b. Natural inflation: requires $f_a \gg M_p$, doesn't seem to be realized in string theory. Alternative: monodromy inflation, multi-natural inflation (roughly equivalent to mondromy inflation.
2. Small field: Planck scale corrections still important. Very tiny couplings to account for $\frac{\delta\rho}{\rho}$. (More precise shortly).

More generally, in any framework: challenging to make predictions which would tie to a detailed microscopic picture.

Non-Compact String Moduli as the Arena for Inflation

By itself not a new idea (e.g. Banks, 1990's). But will add some new elements.

Will assume some degree of low energy supersymmetry, to justify existence pseudomoduli space; scalar partners of compact moduli (axions).

Large/small field inflation: close parallels to large/small field solutions of strong CP problem.

Outline

1. Review of (small field) hybrid inflation – general lessons.
2. Large/small field solutions of the strong CP problem
3. Inflation with non-compact moduli
4. Ingredients for successful modular inflation
5. Large field excursions in the moduli space
6. Small field excursions
7. Implications

Hybrid Inflation: Small Field

Often described in terms of fields and potentials with rather detailed, special features, e.g. so-called waterfall field.

Can be characterized in a more conceptual way. Inflation occurs in all such models on a pseudomoduli space, in a region where supersymmetry is badly broken (possibly by a larger amount than in the present universe) and the potential is slowly varying

Essentially all hybrid models in the literature are small field models; this allows quite explicit constructions using rules of conventional effective field theory, but it is not clear that small field inflation is selected by any deeper principle.

BICEP2 result, if confirmed, would rule out these models; many were already ruled out by the Planck result for n_s. [The Planck theory paper indeed said hybrid inflation ruled out.]

Simplest (supersymmetric) hybrid model:

$$W = I(\kappa\phi^2 - \mu^2) \tag{1}$$

ϕ is known as the waterfall field.

Classically, for large I, the potential is independent of I;

$$V_{cl} = \mu^4 \ (\phi = 0)$$

independent of I. The quantum mechanical corrections control the dynamics of the inflaton:

$$V(I) = \mu^4(1 + \frac{\kappa^2}{16\pi^2}\log(|I|^2/\mu^2)). \tag{2}$$

κ is constrained to be extremely small in order that the fluctuation spectrum be of the correct size; κ is proportional, in fact, to V_I, the energy during inflation. The quantum corrections determine the slow roll parameters.

$$V_I = 2.5 \times 10^{-8} \epsilon^2 M_p^3 \tag{3}$$

$$\kappa = 0.17 \times \left(\frac{\mu}{10^{15}\mathrm{GeV}}\right)^2 = 7.1 \times 10^5 \times \left(\frac{\mu}{M_P}\right)^2 . \tag{4}$$

Kahler Potential Corrections

One expects corrections in powers of M_p. Organize the effective field theory in powers of I (Bose,Monteux,Stephenson-Haskins,M.D.). The quartic term in K,

$$K = \frac{\alpha}{M_p^2} I^\dagger I I^\dagger I \tag{5}$$

gives too large an η unless $\alpha \sim 10^{-2}$.

Irreducible Fine Tuning.

Superpotential Corrections

Also corrections to the superpotential:

$$\delta W = \frac{I^n}{M_p^{n-3}} \quad (6)$$

At least the low n terms must be suppressed. This might occur as a result of discrete symmetries.The leading power of I in the superpotential controls the scale of inflation.

$N = 4$, gives $\mu \approx 10^{11}$ GeV and $\kappa \approx 10^{-10}$.

$N = 5$, one obtains $\mu \approx 10^{13}$ GeV, and $\kappa \approx 10^{-5}$

The scale μ grows slowly with N, reaching 10^{14} GeV at $N = 7$ and 10^{15} GeV for $N = 12$.

So hard to understand a high scale of inflation without a rather absurd sort of discrete symmetry. An argument for a low scale of inflation.

But pointing in the opposite direction is κ, which gets smaller rapidly with V_0,

In addition, achieving $n_s < 1$, consistent with Planck, required a balancing of Kahler and superpotential corrections.

Indeed, from the abstract of the Planck theory paper: “the simplest hybrid inflationary models, and monomial potential models of degree $n > 2$ do not provide a good fit to the data."

So, even without the BICEP2 claim, the theoretical arguments for small field over large field inflation are hardly so persuasive. On the one hand, it is necessary to suppress many Kahler potential terms to obtain a large scale of inflation. On the other, even at low scales it is still necessary to have control over Planck scale corrections, and tuning of parameters (at least at the part in 10^{-2} level) is required. One also needs a very small dimensionless parameter, progressively smaller as the scale of inflation becomes smaller.

Generalizing hybrid inflation to large fields: moduli inflation

So, given both the theoretical situation and the recent data, it is clearly interesting to explore the possibility of inflation on (non-compact) moduli spaces with Planck scale fields undergoing variations of order Planck scale or larger. Such moduli spaces are quite familiar from string theory.

First consider another situation where such a small field/large field dichotomy arises.

Small Field and Large Field Solutions to the Strong CP Problem

To *solve* the strong CP problem one must account for an accidental global symmetry which is of extremely high *quality* .

Small field solutions:

Most models designed to obtain a Peccei-Quinn symmetry are constructed with small axion decay constant, $f_a \ll M_p$, with $f_a = \langle \phi \rangle$ Organize the effective field theory in powers of ϕ / M_p.

Axion *Quality*

Require

$$Q_a \equiv \frac{1}{f_a m_a^2} \frac{\partial V}{\partial a} = 10^4 f_a \frac{\partial V}{\partial a} < 10^{-11}.$$

In small field models, if ϕ contains axion field, PQ symmetry $\phi \to e^{i\alpha}\phi$ need to suppress $\frac{\phi^N}{M_p^{N-3}}$ up to very high N. E.g. Z_N, with $N > 11$ or more, depending on f_a.

Not terribly plausible. Not singled out by anthropic or similar considerations. (Carpenter, Festuccia, Ubaldi, M.D.)

Large field solutions of the Strong CP Problem

String theory has long suggested a large field perspective on the axion problem (Witten). Frequently axions; exhibit continuous shift symmetries in some approximation (e.g. perturbatively in the string coupling). Non-perturbatively broken, but usually a discrete shift symmetry left which is exact.

$a \rightarrow a + 2\pi$ is an exact symmetry of the theory

f_a depends on the precise form of the axion kinetic term. The (non-compact) moduli which accompany these axions typically have Planck scale vev's). Calling the full chiral axion superfield $\mathcal{A} = s + ia + \dots$, this periodicity implies that, for large s, in the superpotential the axion appears as $e^{-\mathcal{A}}$. Solving the strong CP problem requires suppressing only a small number of possible terms (Bobkov, Raby; Dine, Festuccia, Wu).

Typically several moduli which must be stabilized. Not a well understood problem (many ideas).

Whatever the mechanism, the axion multiplet is special. If the superpotential plays a significant role in stabilization of the *saxion*, it is difficult to understand why the axion should be light. $e^{-\mathcal{A}}$ would badly break the PQ symmetry if responsible for saxion stabilization.

In perturbative string models the Kahler potential is often a function of $\mathcal{A} + \mathcal{A}^\dagger$. There is no guarantee that would-be corrections to K which stabilize $\mathcal{A}$ do not violate this symmetry substantially, but will take as a hypothesis.

E.g. as a model, suppose there is some other modulus, $T = t + ib$, appearing in the superpotential as e^{-T}, where e^{-T} might set the scale for supersymmetry breaking.

$$W(T) = Ae^{-T/b} + W_0 \tag{7}$$

with small W_0, leading to

$$T \approx b \log(W_0). \tag{8}$$

The potential for s would arise from terms in the supergravity potential:

$$V_s = e^K \left| \frac{\partial K}{\partial \mathcal{A}} W \right|^2 g^{\mathcal{A}\,\mathcal{A}^*} + \dots \tag{9}$$

For suitable $K(\mathcal{A}, \mathcal{A}^*)$, V might exhibit a minimum as a function of s. If s is, say, twice t at the minimum, $e^{-\mathcal{A}}$ is severely suppressed, as is the potential for the (QCD) axion, a.

A Remark on Distances in the Modulus Geometry

Typical metrics for non-compact moduli fall off as powers of the field for large field. Defining s to be dimension one,

$$g_{\mathcal{A},\mathcal{A}^*} = C^2 M_p^2 / s^2 \tag{10}$$

for some constant, C. So large s is far away (a distance of order $a\, M_p \log(s/M_p)$ away) in field space. If, for example, the smallness of $e^{-(s+ia)}$ is to account for an axion mass small enough to solve the strong CP problem, we might require $s \sim 110\ M_p$, corresponding to a distance of order $8 M_p$ from $s = M_p$ if $C = \sqrt{3}$. (Compare BICEP2)

Non-Compact Moduli as Inflatons

So the strong CP problem points to Planck scale regions of field space as the arena for phenomenology.

Ingredients for moduli as setting for inflation:

1. In the present epoch, one or more moduli responsible for hierarchical supersymmetry breaking.
2. In the present epoch, a modulus whose superpotential is highly suppressed, whose compact component is the QCD axion. This is not necessary for inflation, but is the essence of a modular (large field) solution to the strong CP problem.
3. At an earlier epoch, a stationary point in the effective action with higher scale supersymmetry breaking and a positive cosmological constant. The BICEP2 result would imply a suppression by 10^8 relative to M_p^4. Setting this aside, we might contemplate significantly lower scales.
4. At an earlier epoch, a field with a particularly flat potential which is a candidate for slow roll.

Fields need not play the same role in the inflationary era that they do now. The Peccei-Quinn symmetry might be badly broken during inflation. Then the axion will be heavy during this period and isocurvature fluctuations may not be an issue.

In such a case the initial axion misalignment angle, θ_0, would be fixed rather than being a random variable.

We know that the scale of inflation is well below M_p^4. So plausible that even during inflation moduli have large vev's, $e^{-\mathcal{A}},\ e^{-T} \ll 1$, though much larger than at present.

E.g. suppose a pair of moduli, $\mathcal{A}, T$ responsible for supersymmetry breaking, and an additional field, I, which will play the role of the inflaton. During inflation,

$$H_I \sim W \sim e^{-t} \tag{11}$$

For typical Kahler potentials, the curvature of the t and i potentials will be of order H_I (for i, this is the usual "η problem"). We will exhibit a model with lower curvature below.

A successful model requires a complicated interplay between effects due to the Kahler potential and superpotential (“hilltop inflation" and variants)..

- The *potential* must possess local, supersymmetry breaking minima in $\mathcal{A}$ and T, one of higher, one of lower, energy. The former is the setting for the inflationary phase; the latter for the current, nearly Minkowski, universe.
- In the inflationary domain, the potential for I, must be very flat over some range.
- In the inflationary domain, the imaginary parts of $\mathcal{A}$ and T should be comparable in mass to H_I (or slightly larger) , if the system is to avoid difficulties with isocurvature fluctuations. This would arise if $e^{-s} \approx e^{-t}$.
- In the present universe, the imaginary part of $\mathcal{A}$ should be quite light and that of I much lighter.

- There are additional constraints from the requirement that inflation ends. For some value of Re I, the inflationary minimum for T and $\mathcal{A}$ must be destabilized (presumably due to Kahler potential couplings of I to $\mathcal{A}$ and T). At this point, the system must transit to another local minimum of the potential, with nearly vanishing cosmological constant.
- The process of transiting from the inflationary region of the moduli space to the present day one is subject to serious constraints. Even assuming that there is a path from the inflationary regime to the present one, the system is subject to the well-known concerns about moduli in the early universe. If they are sufficiently massive (as might be expected given current constraints on supersymmetric particles), they may reheat the universe to nucleosynthesis temperatures, avoiding the standard cosmological moduli problem. T and $\mathcal{A}$ are vulnerable to the moduli overshoot problem, for which various solutions have been proposed.

The nature of Models of Moduli Inflation

A strong coupling problem. Assumption is that there are light degrees of freedom, described, over a suitable range of field space (and for a period in the history of the universe) by an approximately supersymmetric effective field theory. Superpotential constrained by assumptions of periodicity, small exponential factors. Kahler potential essentially an arbitrary function.

Questions are whether there exist (classes of) W and K consistent with phenomenological requirements, and how generic (tuned) they may be.

Outputs/predictions might arise through connections to low energy physics, genericity of predictions. This is not different than other models of inflation (e.g. earlier remarks about parameters of low scale models).

Inflationary Models: Large r

We consider, first, the case where r is in the range of claimed by BICEP2. A simple model consistent with the data on r and n_s as well as the fluctuation spectrum:

$$K = -\mathcal{N}^2 \log(I + I^*). \tag{12}$$

With

$$I = e^{\phi/\mathcal{N}} \tag{13}$$

the kinetic term for ϕ is simply $|\partial\phi|^2$.

$$V(\phi) = e^{-\mathcal{N}\phi} V_0, \tag{14}$$

V_0 being the minimum of the S, T potential.

The slow roll parameters are:

$$\epsilon = \frac{1}{2}\mathcal{N}^2; \ \eta = \mathcal{N}^2 = 2\epsilon. \tag{15}$$

Note

$$n - 1 = -2\epsilon. \tag{16}$$

If $r = 0.2 = 16\epsilon$,

$$n_s - 1 = 0.025 \tag{17}$$

on the high end of the range favored by the Planck measurement.

This model [originally proposed by Lucchin and Matarreseis] is discussed in the Planck theory paper which rules it out based on their measurement of r. Suitable modifications will be discussed below. A model with similar features (with cosh rather than exponential potential) has been discussed recently by Nojiri et al.

Connection to Chaotic Inflation

Chaotic inflation has, for decades, provided a simple model for slow roll inflation, and its prediction of transplanckian field motion and observable gravitational radiation now seems to be validated. As we look at the moduli inflation model of the previous section (and more generally moduli models of large field inflation), we see, in fact, a realization of the ideas of chaotic inflation. Again, the potential behaves as

$$V \sim H_I^2 M_p^2 e^{\mathcal{N}\phi} \tag{18}$$

We have seen that the exponent changes, during inflation, by a factor of about $3/2$. So we can make a crude approximation, expanding the exponent and keeping only a few terms. If we focus on each monomial in the expansion, the coefficient of ϕ^p, in Planck units, is:

$$\lambda^p = \frac{10^{-8}\mathcal{N}^p}{p!}. \tag{19}$$

where N is the humber of e-folds.

We can compare this with the required coefficients of chaotic inflation driven by a monomial potential, ϕ^p. In this case,

$$\lambda_p = \frac{3 \times 10^{-7}}{(2Np)^{\frac{p}{2}-1}} \tag{20}$$

These coefficients are not so different. For example, for $p = 1$, the moduli coefficient is about 2×10^{-9}, while for the chaotic case it is about four times smaller; the discrepancy is about a factor of two larger for $p = 2$. So we see that these numbers, which would one hardly expect to be identical, are in a similar ballpark.

So moduli inflation provides a rationale for the effective field theories of chaotic inflation. The typical potential is not a monomial, but one has motion on a non-compact field space, over distances of several M_p, with a scale, in Planck units, roughly that expected for chaotic inflation. The structure is enforced by supersymmetry and discrete shift symmetries.

Moduli Inflation: Small r

It might yet turn out that r is small. For moduli inflation, fields would be of order M_p, but their excursions would be small.

In small r models, $\epsilon \ll \eta$.

Expect the superpotential, W_I and F_Z are roughly constant during inflation. Then

$$V = e^K \left[\left| \frac{\partial K}{\partial I} W_I \right|^2 g^{II^*} + V_0. \right]. \qquad (21)$$

The requirements that V produce the desired values of ϵ and η yield constraints on K. In general, smaller ϵ implies greater tuning of the Kahler potential. The Planck results for n_s, in addition, require that η be negative. Consider a class of Kahler potentials:

$$K(I, I^*) = -\mathcal{N}^2 \log(I + I^*) + \frac{A}{I + I^*} + \frac{B}{(I + I^*)^2}. \tag{22}$$

For a range of parameters, one can have suitable n_s with small ϵ. As an example,

$$\mathcal{N} = 0.026, \; A = -0.0003, \; B = 0.0001 \tag{23}$$

yields

$$\epsilon = 3.55 \times 10^{-4}, \; \eta = -1.90 \times 10^{-2}, \; n_s = .960 \tag{24}$$

This corresponds to an energy scale somewhat below that implied by the central BICEP2 value of r, about

$$V_I = (6.3 \times 10^{15})^4. \tag{25}$$

With $\frac{\delta\rho}{\rho}$ fixed at its observed value, ϵ is proportional to V. As a result, decreasing V to $(10^{14})^4\ \mathrm{GeV}^4$, say, would decrease ϵ by more than 10^5, with a corresponding increase in tuning.

Making ϵ smaller by a factor c corresponds to a tuning of order $\sqrt{c}$.

A realization of a well-known argument for higher scale inflation.

Summary of Our Observations

Explaining inflation from an underlying microscopic theory is an extremely challenging problem, quite possibly inaccessible to our current theoretical technologies. As we have reviewed, even in so-called small field inflation, it requires control over Planck scale phenomena. Within string theory, this requires understanding of supersymmetry breaking (whether large or small) and fixing of moduli in the present universe as well as at much earlier times. It requires an understanding of cosmological singularities, and almost certainly of something like a landscape.

Helpful, then, to understand what might be generic features of the underlying microscopic theory. As we reviewed, the essence of hybrid inflation is motion on a (non-compact) pseudomoduli space. In string theory, at least at the classical level, such moduli spaces are ubiquitous, and the features of these moduli suggest a picture for inflation in which the (canonical) fields have Planck scale motions.

We have stressed a parallel between small/large field inflation and small/large field solutions to the strong CP problem. The existence of moduli in string models is strongly suggestive of the large field solutions to both problems. The proposal we have put forward here is similar to the large field solutions of the strong CP problem.

Several moduli likely play a role in inflation, achieving the needed degree of supersymmetry breaking and slow roll. We have noted that small r is more tuned than large r, giving some weight to the former possibility. We have noted the contrast with small field inflation, where extreme tuning to achieve low scale inflation is replaced by the requirement of an extremely small dimensionless coupling.

Returning to the strong CP problem, any would-be Peccei-Quinn symmetry is an accident, and the accident which holds in the current configuration of the universe need not hold during inflation; this would resolve the axion isocurvature problem. It would imply that θ_0 is not a random variable.

The inflationary paradigm is highly successful; the question is whether we can provide some compelling microscopic framework and whether it is testable. In the present proposal, one does not attempt (at least for now) a detailed microscopic understanding, but considers a class of theories. Within those considered here:

1. Higher scales of inflation are preferred
2. High scale axions (even an *axiverse*).

In a more detailed picture, one might hope to connect some lower energy phenomenon, such as supersymmetry breaking, with inflation.