Mathematical Aspects of General Relativity

Report of Contributions

https://indico.nbi.ku.dk/e/7
One-parameter families of conformally related asymptotically flat, static vacuum data

*Saturday, 12 April 2008 11:35 (45 minutes)*

We give a complete description of the asymptotically flat, conformally non-flat, static vacuum data which admit conformal mappings onto other such data which extend smoothly to space-like infinity. These data form a 3-parameter family which decomposes into 1-parameter families of data which are conformal to each other.

**Presenter:** FRIEDRICH, Helmut (AEI Golm)
Ricci flow as an algorithm to construct black hole metrics

Friday, 11 April 2008 10:20 (45 minutes)

Ricci flow potentially provides a tool to allow explicit numerical construction of black hole metrics of interest in physics. Whilst in 4 dimensions stationary black holes of interest are known analytically, I will discuss how black holes in theories with extra dimensions (such as string theory) are generally not known analytically. Then a numerical construction of the metric is likely the only way to examine them. I will show that black holes (of interest) are actually unstable fixed points of Ricci flow and I will discuss how to use the flow in practice for this application.

Presenter: WISEMAN, Tobias (Imperial College)
Gluing construction for asymptotically hyperbolic manifolds with constant scalar curvature

Thursday, 10 April 2008 14:00 (1 hour)

We will see that the Corvino gluing technique can be adapted to the asymptotically hyperbolic context. More precisely, if a Riemannian metric with constant negative scalar curvature is asymptotic to a hyperbolic model at infinity, it can be glued on an “annulus” with a Schwarzschild-AdS slice to a new constant scalar curvature metric. This produce initial data for the vacuum Einstein equation that are exactly Schwarzschild-AdS slice outside of a compact set. This is joint work with Piotr Chrusciel.

Presenter: DELAY, Erwann (Avignon)
Open issues in the classification and characterization of higher-dimensional black holes

*Friday, 11 April 2008 14:15 (45 minutes)*

After a review of our current knowledge about higher-dimensional black holes, I will discuss some problems that arise due to novel types of black holes that do not arise in four dimensions. In particular, the absence of uniqueness, and the existence of horizons with the geometry of a product space seem to require new ideas and tools in order to achieve a better characterization and classification of black holes.

**Presenter:** EMPARAN, Roberto (Barcelona)
Rigidly Rotating Bodies in General Relativity

Thursday, 17 April 2008 11:00 (1 hour)

The only rigorous result known so far on the existence of isolated bodies in GR in rigid rotation are the ones by Heilig of 1995 on perfect fluids. We outline a method to solve the stationary Einstein equations with source a body in rigid rotation consisting of elastic matter. This is work in progress by R.B., B.G.Schmidt, and L.Andersson.

Presenter: BEIG, Robert (Vienna)
We consider stationary and axisymmetric solutions in General Relativity, primarily in five dimensions. We motivate the introduction of the Rod-structure for any given solution and give examples of Rod-structures for various five-dimensional exact solutions of General Relativity. We consider the questions of uniqueness and existence of a solution given the Rod-structure. Finally we review briefly the uniqueness theorem of Hollands and Yazadjiev which proves that five-dimensional stationary and axisymmetric solutions are unique given the Rod-structure, and the asymptotic charges.

**Presenter:** HARMARK, Troels (Niels Bohr Institute)
I will discuss recent results on existence and regularity of
marginal surfaces, blowup of Jang’s equation, and regularity of the trapped
region. This is joint work with Jan Metzger.

Presenter: ANDERSSON, Lars (AEI Golm)
Classical Effective Field Theory and Non-Relativistic Gravitation

Saturday, 12 April 2008 10:20 (45 minutes)

I shall discuss an improvement to the (Classical) Effective Field Theory approach to the non-relativistic or Post-Newtonian approximation of General Relativity. The “potential metric field” is decomposed through a temporal Kaluza-Klein ansatz into three NRG-fields: a scalar identified with the Newtonian potential, a 3-vector corresponding to the gravito-magnetic vector potential and a 3-tensor. The derivation of the Einstein-Infeld-Hoffmann Lagrangian simplifies such that each term corresponds to a single Feynman diagram providing a clear physical interpretation. Spin interactions are associated with the gravito-magnetic field. Leading correction diagrams corresponding to the 3PN correction to the spin-spin interaction and the 2.5PN correction to the spin-orbit interaction will be presented.

Presenter: KOL, Barak (Jerusalem)
U(1) symmetric gravitationnal collapse in Iwasawa variable

Friday, 11 April 2008 09:30 (45 minutes)

Presenter: CHOQUET-BRUHAT, Yvonne (Paris Jussieu)
Inverse scattering in gravity

Saturday, 12 April 2008 12:25 (45 minutes)

Presenter: ELVANG, Henriette (MIT)
When can one extend the conformal metric through a space-time singularity?

Wednesday, 9 April 2008 14:00 (1 hour)

One knows, for example by proving well-posedness for an initial value problem with data at the singularity, that there exist many cosmological solutions of the Einstein equations with an initial curvature singularity but for which the conformal metric can be extended through the singularity. Here we consider a converse, a local extension problem for the conformal structure: given an incomplete causal curve terminating at a curvature singularity, when can one extend the conformal structure to a set containing a neighbourhood of a final segment of the curve? We obtain necessary and sufficient conditions based on boundedness of tractor curvature components. (Based on work with Christian Luebbe: arXiv:0710.5552, arXiv:0710.5723.)

Presenter: TOD, Poul (Oxford)
In this talk we will discuss a geometric inequality which is in the same spirit as the Positive Mass Theorem and the Penrose Inequality for black holes. Whereas the cases of equality of these first two theorems are respectively Minkowski space (which can be thought of as Schwarzschild with zero mass) and the Schwarzschild spacetime with positive mass, the case of equality for the inequality we will discuss is the Schwarzschild spacetime with negative mass.

Physically speaking, when positive amounts of energy are concentrated as much as possible, black holes result. However, when negative amounts of energy are "concentrated" as much as possible, it is in fact possible to form point singularities in each spacelike slice (which form a timelike curve of singularities in the spacetime).

As usual we will focus on maximal, spacelike slices of spacetimes as a first step. The assumption of nonnegative energy density on these slices implies that these Riemannian 3-manifolds have nonnegative scalar curvature. However, we will allow these 3-manifolds to have singularities which contribute negatively to the total mass. The standard example is the negative Schwarzschild metric on \( \mathbb{R}^4 \) minus a ball of radius \( m/2 \), \((1 - m/2r)^4 \delta_{ij}\). This metric (which has total mass \(-m\)) has zero scalar curvature everywhere but has a singularity at \( r = m/2 \). We will propose a definition for the mass of a singularity, and prove a sharp lower bound on the ADM mass in terms of the masses of the singularities in the 3-manifold, modulo an interesting geometric conjecture.

**Presenter:** BRAY, Hubert (Duke)
Future stability of the Einstein-non-linear scalar field system, power law expansion

Monday, 14 April 2008 14:00 (1 hour)

In the case of Einstein’s equations coupled to a non-linear scalar field with a suitable exponential potential, there are solutions for which the expansion is accelerated and of power law type. In the talk I will discuss the future global non-linear stability of such models. The results generalize those of Mark Heinzle and Alan Rendall obtained using different methods.

Presenter: RINGSTROM, Hans (Stockholm (KTH))
An Introduction to the Geometry of Black Holes

In this lecture we will study the Schwarzschild spacetime, which represents a nonrotating black hole in vacuum, from a variety of perspectives. After considering the more intuitive coordinate chart representations of Schwarzschild, we will then focus on Kruskal coordinates which is a global coordinate chart on the whole spacetime. From this introductory material, we will then transition into a discussion about what the correct, or most geometric, statement of the Penrose Conjecture for black holes should be. Time permitting, we'll prove the Penrose Conjecture in a very special case, discuss white holes as compared to black holes, and define a new notion of horizon, called a generalized apparent horizon, which may be an important notion useful for proving the Penrose Conjecture.

Presenter: BRAY, Hubert (Duke)
Quantum Field Theory in Curved Spacetime

Tuesday, 8 April 2008 15:15 (1 hour)

Presenter: WALD, Robert (Chicago)
The interface between mathematics and astrophysics in the study of cosmic acceleration.

Monday, 7 April 2008 11:15 (1 hour)

For about the last ten years cosmic acceleration has been a subject of wide interest in cosmology. By now there are a number of interesting mathematical results in this area. A closer examination reveals that while the mathematical theorems are often of greater generality than what is considered in the astrophysical literature there are topics of astrophysical interest which fail to be addressed at all by the mathematical developments up to now. This talk will discuss possibilities of improving the interface between the two subjects in this context, concentrating on the case of the massive scalar field as a source for the Einstein equations. Other aspects of the question will be illuminated by consideration of a modification of the Einstein equations given by Cardassian models, following work of Nikolaus Berndt.

Presenter: RENDALL, Alan (AEI Golm)
Asymptotic Stability of the five-dimensional Schwarzschild metric under biaxial perturbations

Wednesday, 9 April 2008 15:15 (1 hour)

Presenter: HOLZEGEL, Gustav (Cambridge)
Regularity conditions at spatial infinity revisited

H. Friedrich has shown that if one considers a time symmetric initial data set for the Einstein vacuum equations admitting an analytic compactification at infinity, then necessary conditions for the solutions to the transport system implied by the conformal Einstein equations at the cylinder at spatial infinity to extend smoothly to the critical sets where null infinity touches spatial infinity is that the Cotton-Bach tensor of the conformal metric, and its trace-free symmetrised higher order derivatives vanish at spatial infinity.

In this talk the generalisation of this regularity condition to data with non-vanishing second fundamental forms is examined. It is discussed how these regularity conditions can be phrased in terms of the vanishing at infinity of a pair of tensors and their higher order symmetrised derivatives. It is shown that these “generalised regularity conditions” are only a restriction on the freely specifiable data. The relation of these “generalised regularity conditions” to stationary data is considered. Finally, it is also discussed how these regularity conditions can be used to construct purely radiative data at “past null infinity”.

Presenter: VALIENTE, Juan A (Queen Mary, London)
Stability of marginally trapped surfaces, and applications to black holes

Friday, 11 April 2008 11:35 (45 minutes)

A basic result in the theory of black holes is Hawking’s theorem on the topology of black holes, which asserts that cross sections of the event horizon in (3+1)-dimensional asymptotically flat stationary black hole spacetimes obeying the dominant energy condition are topologically 2-spheres. Recent interest and developments in the study of higher dimensional black holes has drawn attention to the question of what are the allowable black hole topologies in higher dimensions. We have addressed this question in two recent papers, the first with Rick Schoen, resulting in a natural generalization of Hawking’s theorem to higher dimensions. In this talk we discuss these works and some further related developments. The results we describe are based on properties of marginally outer trapped surfaces, which are natural spacetime analogues of minimal surfaces, and which form the focus of our talk.

Presenter: GALLOWAY, Greg (Miami)
The non-linear stability problem for black hole spacetimes

Tuesday, 15 April 2008 13:45 (1 hour)

Presenter: DAFERMOS, Mihalis (Cambridge)
The non-linear stability problem for black hole spacetimes

Wednesday, 16 April 2008 15:15 (1 hour)

Presenter: DAIFEROMOS, Mihalis (Cambridge)
The Motion of ‘Point Particles’

Thursday, 10 April 2008 15:15 (1 hour)

Presenter: WALD, Robert (Chicago)
On the Penrose Conjecture for Arbitrary Slices of a Spacetime

Tuesday, 8 April 2008 11:30 (1 hour)

The proofs of the Riemannian Penrose Conjecture by Huisken-Ilmanen in 1997 (for one black hole) and by the speaker in 1999 (for any number of black holes) describe the geometric relationships between the total mass of a slice of a spacetime and the size and number of black holes in the slice, in the special case that the slice has zero second fundamental form in the spacetime. However, Penrose’s original 1973 conjecture concerns any asymptotically flat, space-like slice of a spacetime and, consequently, is still open in its most general form. In this talk, the speaker will describe a joint effort with Marcus Khuri to reduce the general case of the Penrose Conjecture to the known case using a generalization of Jang’s equation (used to prove the general case of the positive mass theorem) and a new geometric identity, which we are calling the generalized Schoen-Yau identity, which is designed to recognize arbitrary space-like slices of static spacetimes (like the Schwarzschild spacetime, which is the case of equality of the Penrose Conjecture), and hence is ideally suited for our purposes. We will then discuss three different systems of p.d.e.s whose solutions, when they exist, imply the Penrose Conjecture.

Presenter: BRAY, Hubert (Duke)
Rate of change of widths under flows

Saturday, 12 April 2008 09:30 (45 minutes)

Presenter: COLDING, Tobias
A uniqueness theorem for the Kerr metric

Wednesday, 16 April 2008 14:00 (1 hour)

Presenter: CHRUSCIEL, Piotr