The Nuclear Equation of State and Neutron Star Structure

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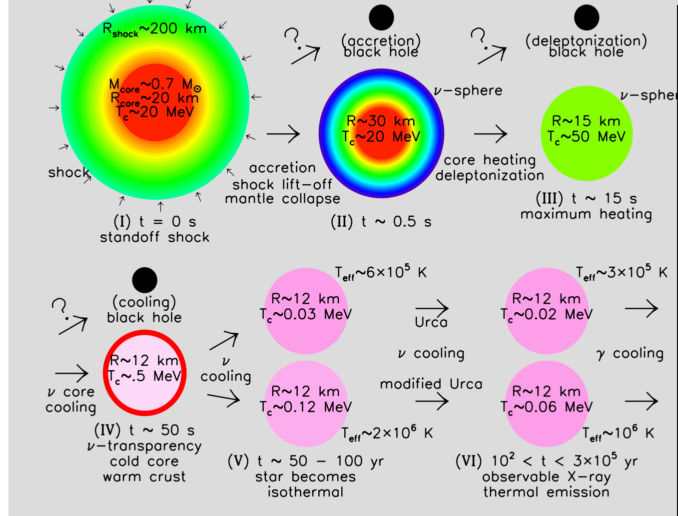
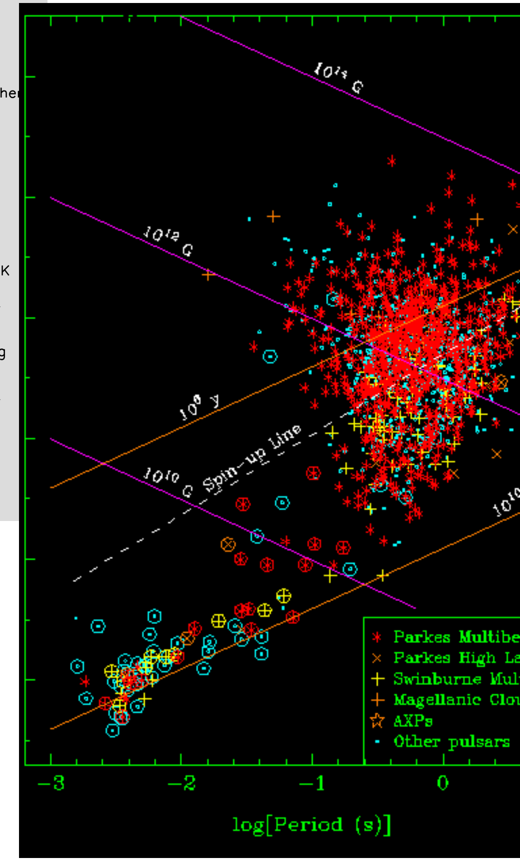
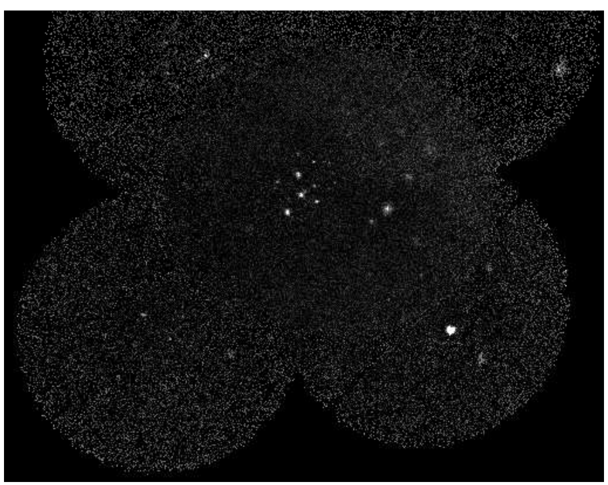
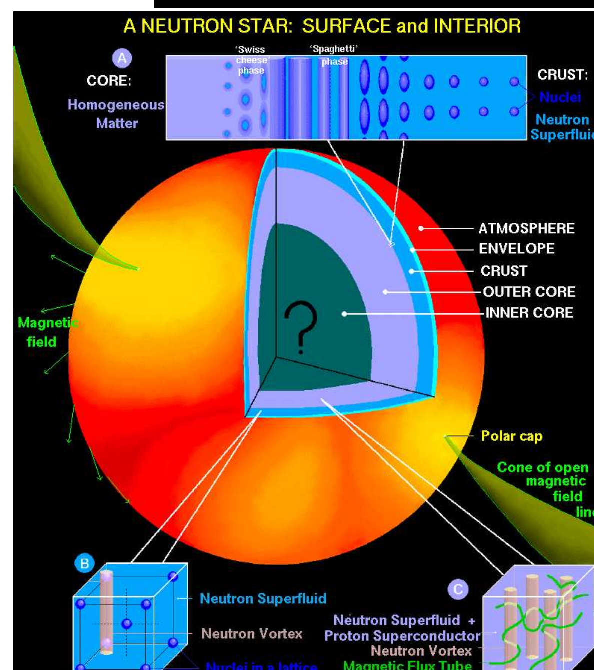
Outline

- Structure
- Dense Matter
  Equation of State
- Formation and Evolution
- Observational Constraints









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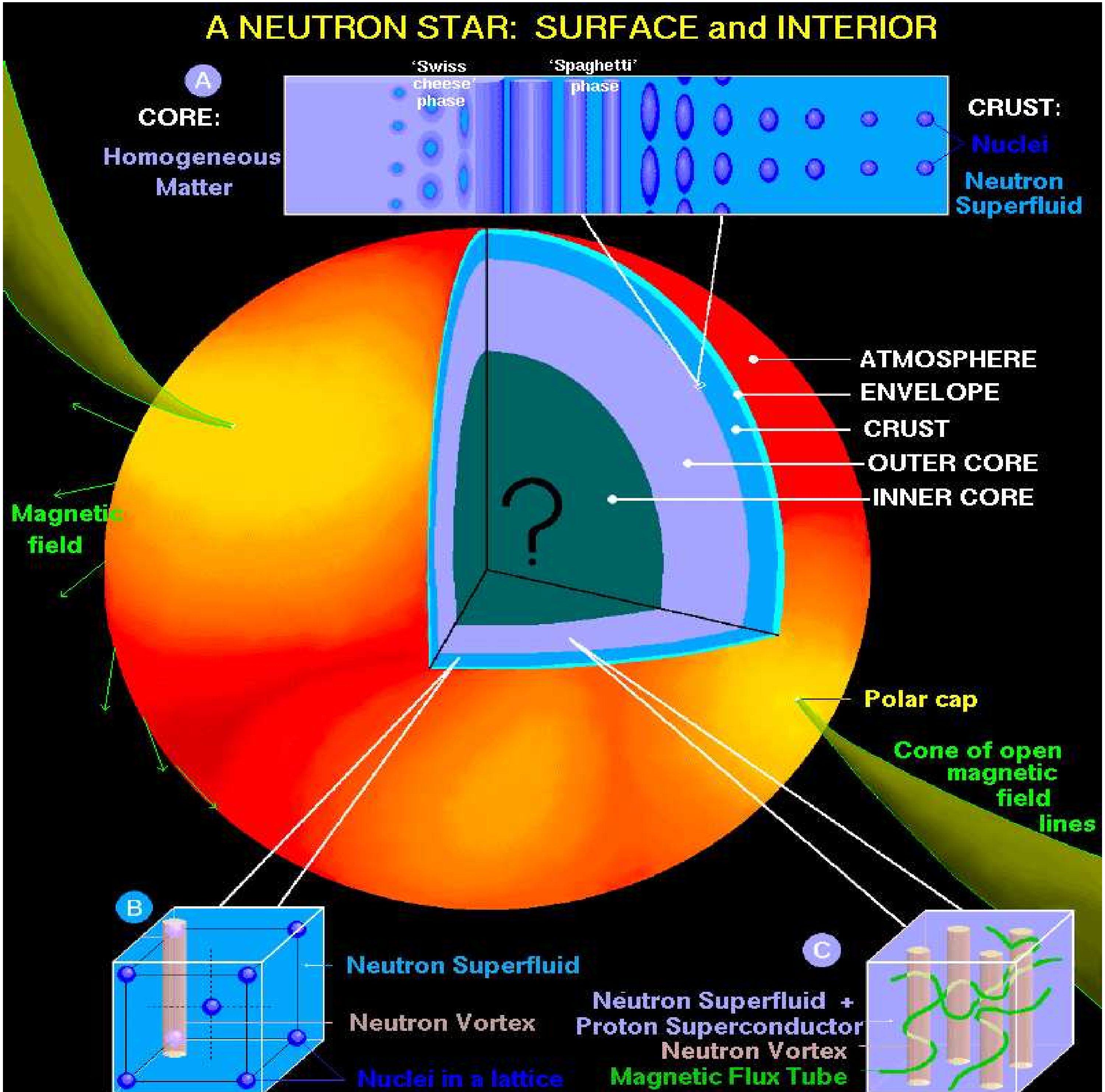


Credit: Dany Page, UNAM

Amazing Facts About Neutron Stars

- Densest objects this side of an event horizon: 10^{15} g cm^{-3}
  Four teaspoons, on the Earth's surface, would weigh as much as the Moon.
- Largest surface gravity: 10^{14} cm s^{-2}
  This is 100 billion times the Earth's gravity.

- Fastest spinning objects known: $\nu = 716$ Hz
  This spin rate was measured for PSR J1748-2446ad, which is located in the globular cluster Terzan 5 located 28,000 light years away. (33 pulsars have been found in this cluster.)
  If the radius is about 15 km, the velocity at the equator is one fourth the speed of light.
- Largest known magnetic field strengths: $B = 10^{15}$ G
  The Sun's field strength is about 1 G.
- Highest temperature superconductor: $T_c = 10$ billion K
  The highest known superconductor on the Earth is mercury thallium barium calcium copper oxide ($Hg_{12}T_{l3}Ba_{30}Ca_{30}Cu_{45}O_{125}$), at 138 K.
- Highest temperature, at birth, anywhere in the Universe since the Big Bang: $T = 700$ billion K
- Fastest measured velocity of a massive object in the Galaxy: 1083 km/s
  This velocity was measured for PSR B1508+55, and is 1/300 the speed of light. It is actually an underestimate, since only the transverse velocity can be measured.
- The only place in the universe except for the Big Bang where neutrinos become *trapped*.

Spherically Symmetric General Relativity

Static metric: $$ds^2 = e^{\lambda(r)}dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right) - e^{\nu(r)}dt^2$$

Einstein's equations:

$$\begin{aligned}
8\pi\epsilon(r) &= \frac{1}{r^2}\left(1 - e^{-\lambda(r)}\right) + e^{-\lambda(r)}\frac{\lambda'(r)}{r}, \\
8\pi p(r) &= -\frac{1}{r^2}\left(1 - e^{-\lambda(r)}\right) + e^{-\lambda(r)}\frac{\nu'(r)}{r}, \\
p'(r) &= -\frac{p(r)+\epsilon(r)}{2}\nu'(r).
\end{aligned}$$

Mass: $$m(r) = 4\pi\int_0^r \epsilon(r')r'^2 dr', \qquad e^{-\lambda(r)} = 1 - 2m(r)/r$$

Boundaries:

$$\begin{aligned}
r = 0 &\qquad m(0) = p'(0) = \epsilon'(0) = 0, \\
r = R &\qquad m(R) = M, \quad p(R) = 0, e^{\nu(R)} = e^{-\lambda(R)} = 1 - 2M/R
\end{aligned}$$

Thermodynamics:

$$\begin{aligned}
d(\ln n) &= \frac{d\epsilon}{\epsilon + p} = -\frac{1}{2}\frac{d\epsilon}{dp}d\nu, \qquad h = \frac{d\epsilon}{dn}, \qquad \epsilon = n(m+e), \qquad p = n^2\frac{de}{dn} \\
mn(r) &= (\epsilon(r) + p(r))e^{(\nu(r)-\nu(R))/2} - n_0 e_0; \qquad p = 0 : n = n_0, e = e_0 \\
N &= \int_0^R 4\pi r^2 e^{\lambda(r)/2} n(r) dr; \qquad \mathrm{BE} = Nm - M
\end{aligned}$$

Neutron Star Structure

Tolman-Oppenheimer-Volkov equations of relativistic hydrostatic equilibrium:

$$\frac{dp}{dr} = -\frac{G}{c^2}\frac{(m+4\pi p r^3)(\epsilon+p)}{r(r-2Gm/c^2)}$$

$$\frac{dm}{dr} = 4\pi\frac{\epsilon}{c^2}r^2$$

p is pressure, ϵ is mass-energy density

Useful analytic solutions exist:

- Uniform density $\epsilon =$ constant
- Tolman VII $\epsilon = \epsilon_c[1-(r/R)^2]$
- Buchdahl $\epsilon = \sqrt{pp_*} - 5p$

Uniform Density Fluid

$$
\begin{aligned}
m(r) &= \frac{4\pi}{3}\epsilon r^3, \qquad \beta \equiv \frac{M}{R} \\
e^{-\lambda(r)} &= 1 - 2\beta(r/R)^2, \\
e^{\nu(r)} &= \left[\frac{3}{2}\sqrt{1-2\beta} - \frac{1}{2}\sqrt{1-2\beta(r/R)^2}\right]^2, \\
p(r) &= \epsilon\left[\frac{\sqrt{1-2\beta(r/R)^2} - \sqrt{1-2\beta}}{3\sqrt{1-2\beta} - \sqrt{1-2\beta(r/R)^2}}\right], \\
\epsilon(r) &= \text{constant}; \qquad n(r) = \text{constant} \\
\frac{\text{BE}}{M} &= \frac{3}{4\beta}\left(\frac{\sin^{-1}\sqrt{2\beta}}{\sqrt{2\beta}} - \sqrt{1-2\beta}\right) \simeq \frac{3\beta}{5} + \frac{9}{14}\beta^2 + \cdots
\end{aligned}
$$

$$p_c < \infty \Longrightarrow \beta < 4/9$$

$$c_s^2 = \infty$$

Tolman VII

$$
\begin{aligned}
\epsilon(r) &= \epsilon_c\left[1-(r/R)^2\right] \equiv \epsilon_c[1-x] \\
e^{-\lambda(r)} &= 1-\beta x(5-3x) \\
e^{\nu(r)} &= (1-5\beta/3)\cos^2\phi, \\
p(r) &= \frac{1}{4\pi R^2}\left[\sqrt{3\beta e^{-\lambda(r)}}\tan\phi(r)-\frac{\beta}{2}(5-3x)\right], \\
n(r) &= \frac{\epsilon(r)+p(r)}{m_b}\,\frac{\cos\phi(r)}{\cos\phi_1} \\
\phi(r) &= \frac{w_1-w(r)}{2}+\phi_1, \qquad \phi_1=\phi(x=1)=\tan^{-1}\sqrt{\frac{\beta}{3(1-2\beta)}}, \\
w(r) &= \ln\left[x-\frac{5}{6}+\sqrt{\frac{e^{-\lambda(r)}}{3\beta}}\right], \qquad w_1=w(x=1)=\ln\left[\frac{1}{6}+\sqrt{\frac{1-2\beta}{3\beta}}\right].
\end{aligned}
$$

$$(P/\epsilon)_c=\frac{2\tan\phi_c}{15}\sqrt{\frac{3}{\beta}}-\frac{1}{3}, \qquad c^2_{s,c}=\tan\phi_c\left(\frac{1}{5}\tan\phi_c+\sqrt{\frac{\beta}{3}}\right)$$

$$\frac{\mathrm{BE}}{M}\simeq\frac{11}{21}\beta+\frac{7187}{18018}\beta^2+\cdots$$

$$p_c<\infty\Longrightarrow\phi_c<\frac{\pi}{2},\beta<0.3862$$

$$c^2_{s,c}<1\Longrightarrow\beta<0.2698$$

Buchdahl

$$\epsilon = \sqrt{p_* p} - 5p$$

$$\begin{aligned}
e^{\nu(r)} &= (1-2\beta)(1-\beta-u(r))(1-\beta+u(r))^{-1}, \\
e^{\lambda(r)} &= (1-2\beta)(1-\beta+u(r))(1-\beta-u(r))^{-1}(1-\beta+\beta\cos Ar')^{-2}, \\
8\pi p(r) &= A^2 u(r)^2 (1-2\beta)(1-\beta+u(r))^{-2}, \\
8\pi\epsilon(r) &= 2A^2 u(r)(1-2\beta)(1-\beta-3u(r)/2)(1-\beta+u(r))^{-2}, \\
m_b n(r) &= \sqrt{p_* p(r)}\left(1-4\sqrt{\frac{p(r)}{p_*}}\right)^{3/2}, \qquad c_s^2(r) = \left(\frac{1}{2}\sqrt{\frac{p_*}{p(r)}}-5\right)^{-1} \\
u(r) &= \frac{\beta}{Ar'}\sin Ar' = (1-\beta)\left(\frac{1}{2}\sqrt{\frac{p_*}{p(r)}}-1\right)^{-1}, \\
r' &= r(1-2\beta)(1-\beta+u(r))^{-1}, \\
A^2 &= 2\pi p_*(1-2\beta)^{-1}, \qquad R = (1-\beta)\sqrt{\frac{\pi}{2p_*(1-2\beta)}}.
\end{aligned}$$

$$p_c = \frac{p_*}{4}\beta^2, \qquad \epsilon_c = \frac{p_*}{2}\beta(1-\frac{5}{2}\beta), \qquad n_c m_b = \frac{p_*}{2}\beta(1-2\beta)^{3/2}$$

$$\frac{\mathrm{BE}}{M} = (1-\frac{3}{2}\beta)(1-2\beta)^{-1/2}(1-\beta)^{-1} \simeq \frac{\beta}{2}+\frac{\beta^2}{2}+\cdots$$

$$c_{s,c}^2 < 1 \Longrightarrow \beta < 1/6$$

Tolman IV Variation: A Self-Bound Star

Finite, large surface energy density, *e.g.*, strange quark star

Lake's solution

$$
\begin{aligned}
e^{\nu(r)} &= \frac{[1-\frac{5}{2}\beta(1-\frac{1}{5}x)]^2}{(1-2\beta)}, \\
e^{\lambda(r)} &= \frac{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{2/3}}{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{2/3}-2(1-\beta)^{2/3}\beta x}, \\
4\pi p R^2 &= \frac{\beta}{1-\frac{5}{2}\beta(1-\frac{1}{5}x)}\left[1-(1-\beta)^{2/3}\frac{(1-\frac{5}{2}\beta(1-x))}{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{2/3}}\right], \\
4\pi\epsilon R^2 &= 3(1-\beta)^{2/3}\beta\frac{1-\frac{5}{2}\beta(1-\frac{1}{3}x)}{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{5/3}}, \\
m &= \frac{(1-\beta)^{\frac{2}{3}}Mx^{3/2}}{(1-\frac{5}{2}\beta(1-\frac{3}{5}x))^{2/3}} \\
c_s^2 &= \frac{(2-5\beta+3\beta x)}{5(2-5\beta+\beta x)^3}\left[\frac{(2-5\beta+3\beta x)^{5/3}}{2^{2/3}(1-\beta)^{2/3}}+(2-5\beta)^2-5\beta^2 x\right], \\
\frac{\epsilon_{surf}}{\epsilon_c} &= \left(1-\frac{5}{3}\beta\right)\left(1-\frac{5}{2}\beta\right)^{2/3}(1-\beta)^{-5/3}.
\end{aligned}
$$

$$0.30 < c_{s,c}^2 < 0.44, \qquad 0.265 < \frac{\epsilon_{surf}}{\epsilon_c} < 1$$

Mass-Radius Diagram and Theoretical Constraints

GR:

$R > 2GM/c^2$

$P < \infty$:

$R > (9/4)GM/c^2$

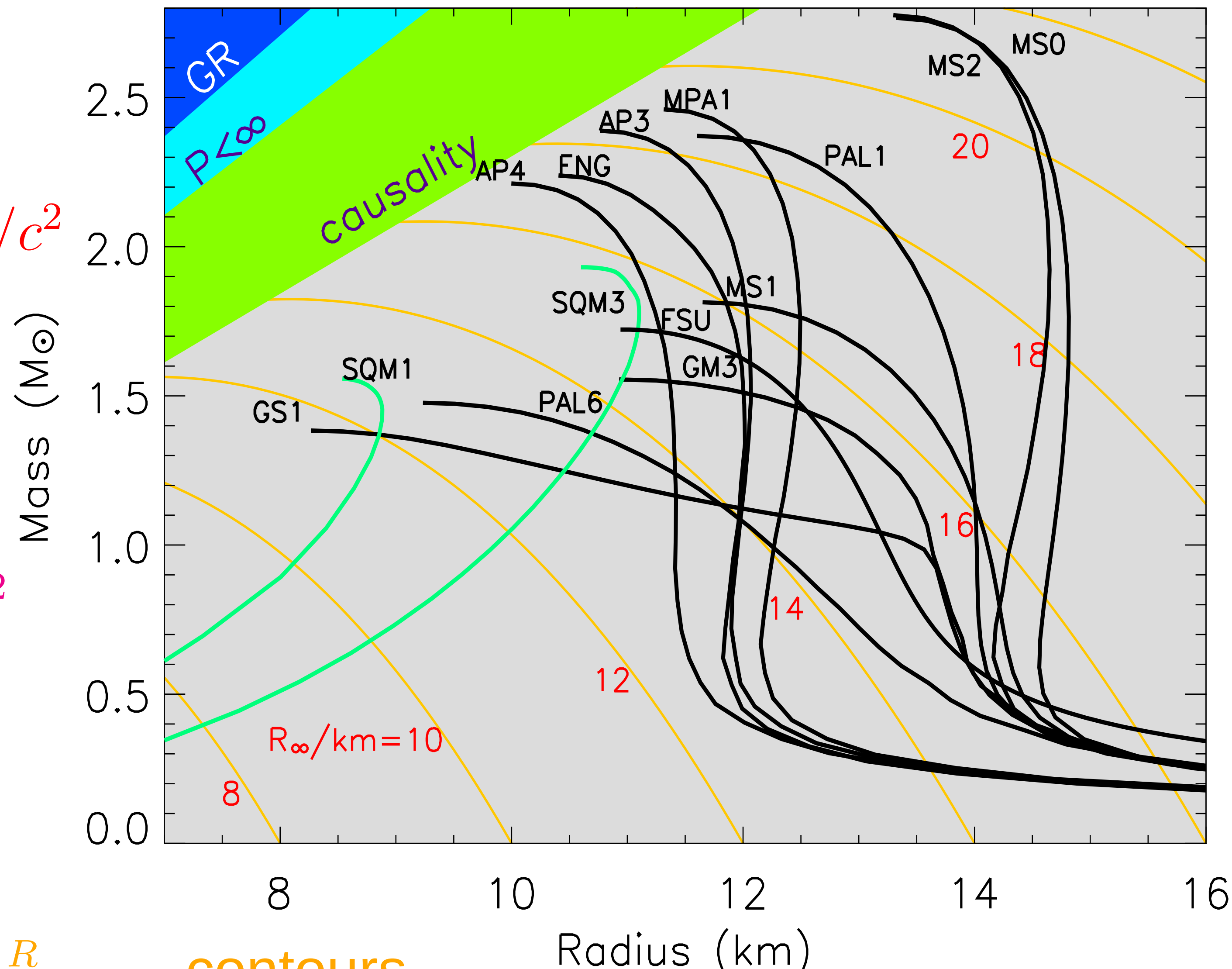
causality:

$R \gtrsim 2.9GM/c^2$

— normal NS

— SQS

— $R_\infty = \frac{R}{\sqrt{1-2GM/Rc^2}}$ contours



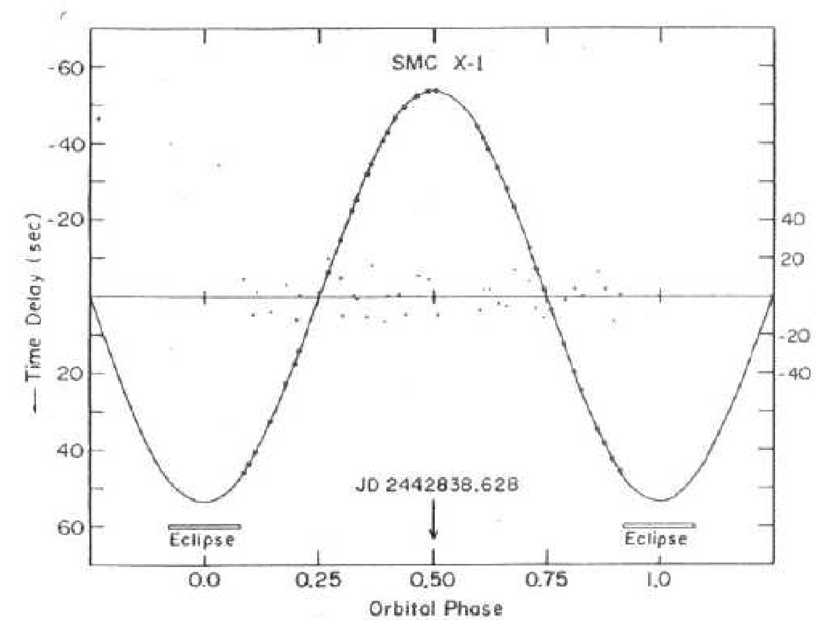
Mass Measurements In X-Ray Binaries

Mass function

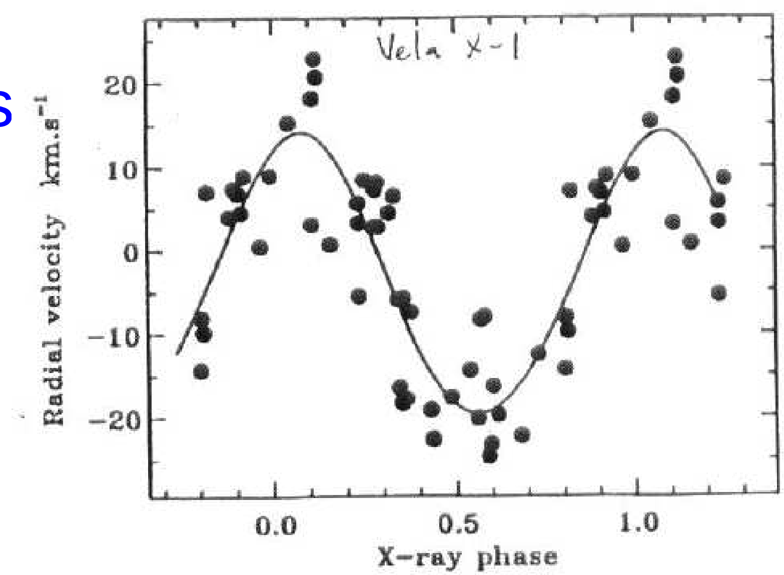
$$f(M_1) = \frac{P(v_2 \sin i)^3}{2\pi G} = \frac{(M_1 \sin i)^3}{(M_1+M_2)^2} > M_1$$

$$f(M_2) = \frac{P(v_1 \sin i)^3}{2\pi G} = \frac{(M_2 \sin i)^3}{(M_1+M_2)^2} > M_2$$

X-ray timing



Optical spectroscopy



In an X-ray binary, $v_{optical}$ has the largest uncertainties. In some cases $\sin i \sim 1$ if eclipses are observed. If eclipses are not observed, limits to i can be made based on the estimated radius of the optical star.

Pulsar Mass Measurements

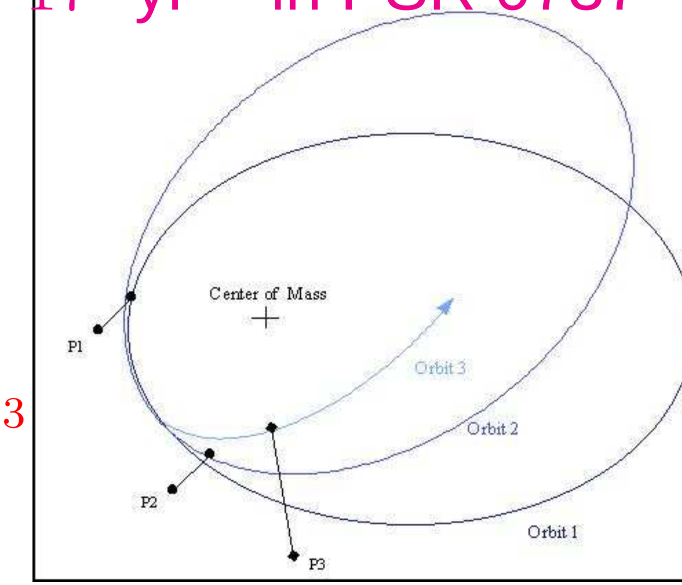
Mass function for pulsar precisely obtained.

It is also possible in some cases to obtain the rate of periastron advance and the Einstein gravitational redshift + time dilation term:

$\dot{\omega} = 3(2\pi/P)^{5/3}(GM/c^2)^{2/3}/(1-e^2)$

$\gamma = (P/2\pi)^{1/3} e M_2 (2M_2 + M_1)(G/M^2c^2)^{2/3}$

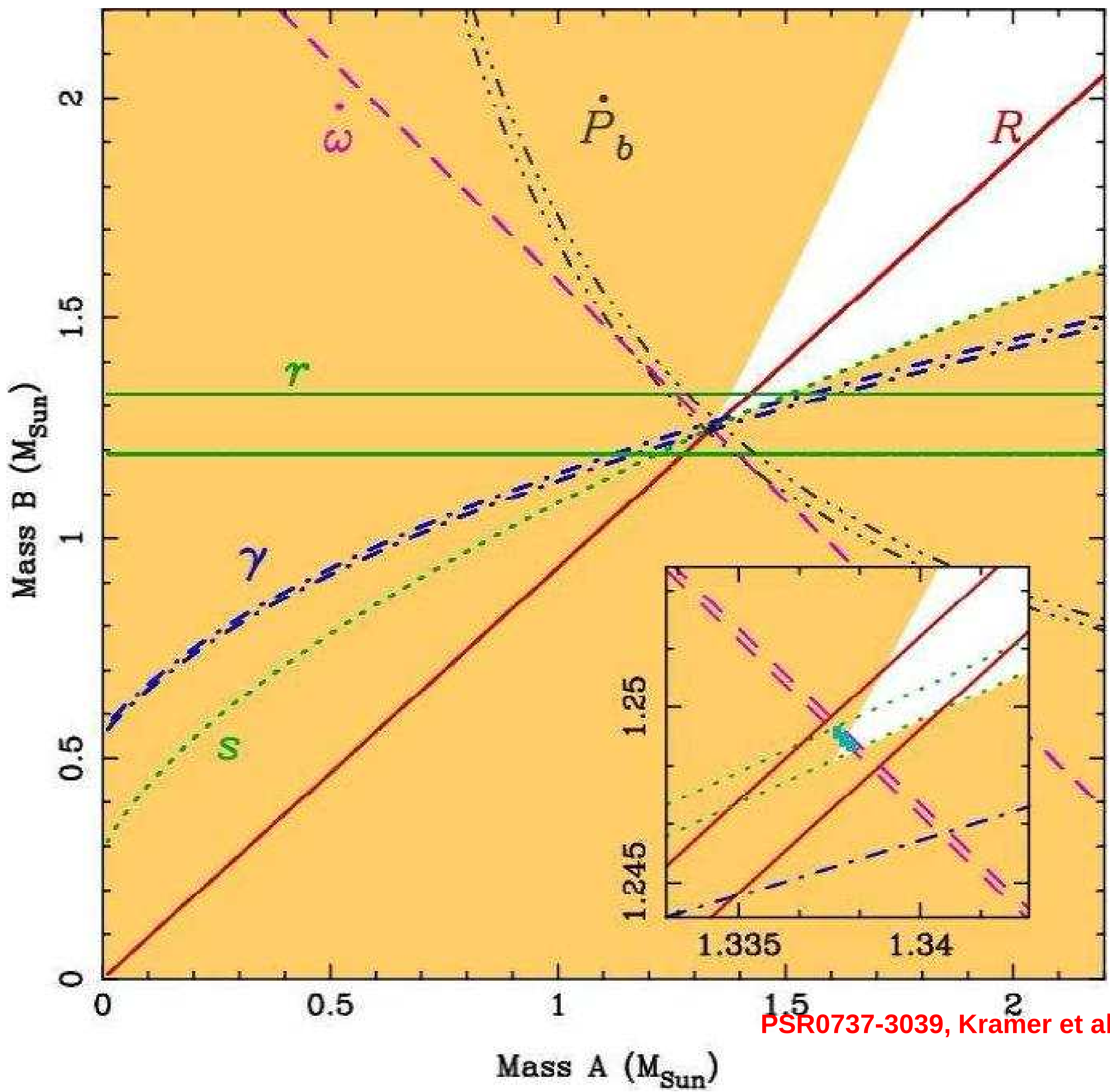
17° yr^{-1} in PSR 0737



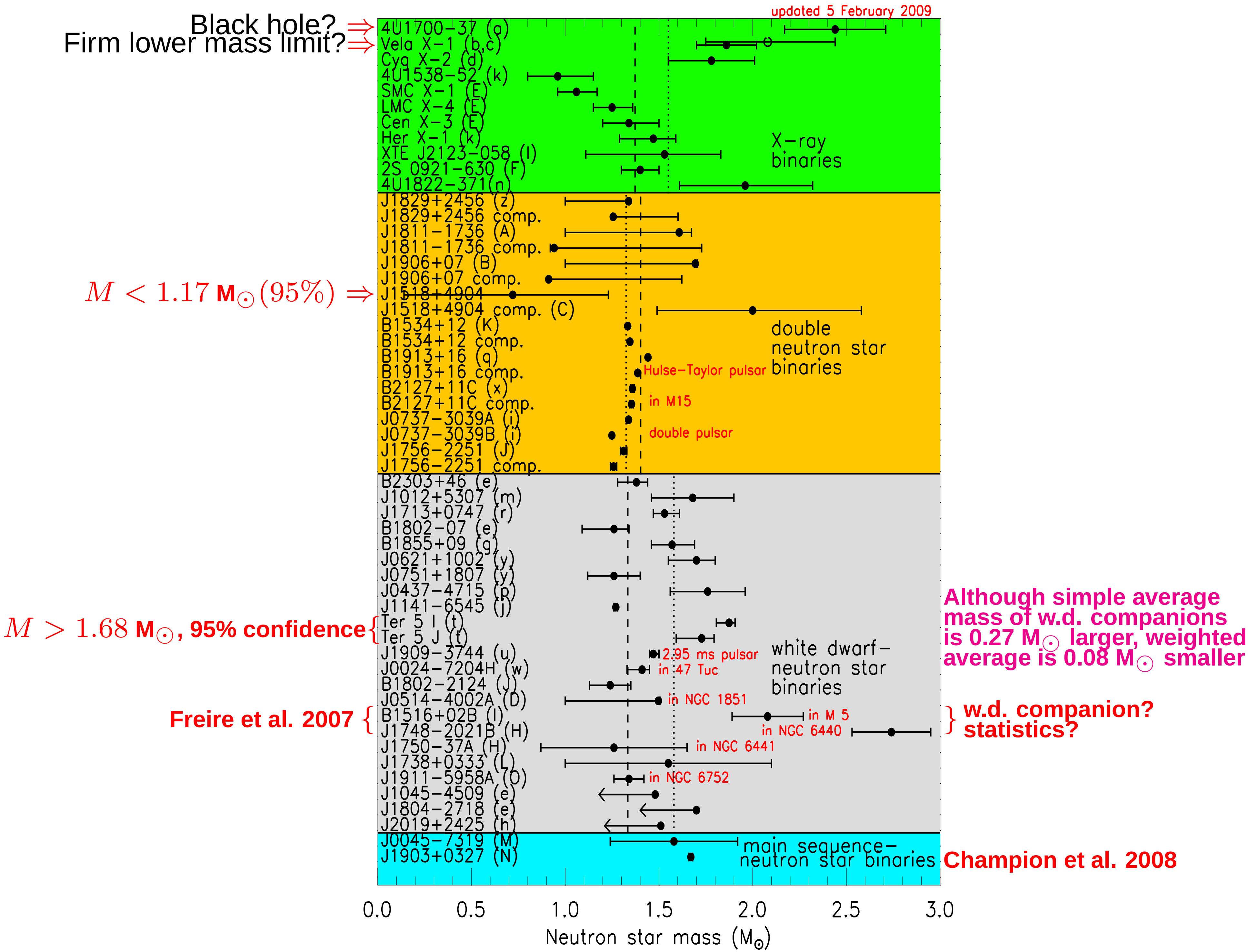
Gravitational radiation leads to orbit decay:

$\dot{P} = -\frac{192\pi}{5c^5}\left(\frac{2\pi G}{P}\right)^{5/3}(1-e^2)^{-7/2}\left(1+\frac{73}{24}e^2+\frac{37}{96}e^4\right)\frac{M_1M_2}{M^{1/2}}$

In some cases, can constrain Shapiro time delay, r is magnitude and $s = \sin i$ is shape parameter.



PSR0737-3039, Kramer et al. (2007)


updated 5 February 2009
Black hole?
Firm lower mass limit?
4U1700−37 (a)
Vela X−1 (b,c)
Cyg X−2 (d)
4U1538−52 (k)
SMC X−1 (E)
LMC X−4 (E)
Cen X−3 (E)
Her X−1 (k)
XTE J2123−058 (l)
2S 0921−630 (F)
4U1822−371 (n)
X−ray binaries
J1829+2456 (z)
J1829+2456 comp.
J1811−1736 (A)
J1811−1736 comp.
J1906+07 (B)
J1906+07 comp.
M < 1.17 M⊙ (95%)
J1518+4904
J1518+4904 comp. (C)
B1534+12 (K)
B1534+12 comp.
B1913+16 (q)
B1913+16 comp.
Hulse−Taylor pulsar
B2127+11C (x)
B2127+11C comp.
in M15
J0737−3039A (i)
J0737−3039B (i)
double pulsar
J1756−2251 (J)
J1756−2251 comp.
double neutron star binaries
B2303+46 (e)
J1012+5307 (m)
J1713+0747 (r)
B1802−07 (e)
B1855+09 (g)
J0621+1002 (y)
J0751+1807 (y)
J0437−4715 (p)
J1141−6545 (j)
M > 1.68 M⊙, 95% confidence
Ter 5 I (t)
Ter 5 J (t)
J1909−3744 (u)
2.95 ms pulsar
J0024−7204H (w)
in 47 Tuc
B1802−2124 (J)
J0514−4002A (D)
in NGC 1851
Freire et al. 2007
B1516+02B (I)
in M 5
J1748−2021B (H)
in NGC 6440
J1750−37A (H)
in NGC 6441
J1738+0333 (L)
J1911−5958A (O)
in NGC 6752
J1045−4509 (e)
J1804−2718 (e)
J2019+2425 (h)
white dwarf−neutron star binaries
Although simple average mass of w.d. companions is 0.27 M⊙ larger, weighted average is 0.08 M⊙ smaller
w.d. companion? statistics?
J0045−7319 (M)
J1903+0327 (N)
main sequence−neutron star binaries
Champion et al. 2008
0.0
0.5
1.0
1.5
2.0
2.5
3.0
Neutron star mass (M⊙)

Roche Model for Maximal Rotation

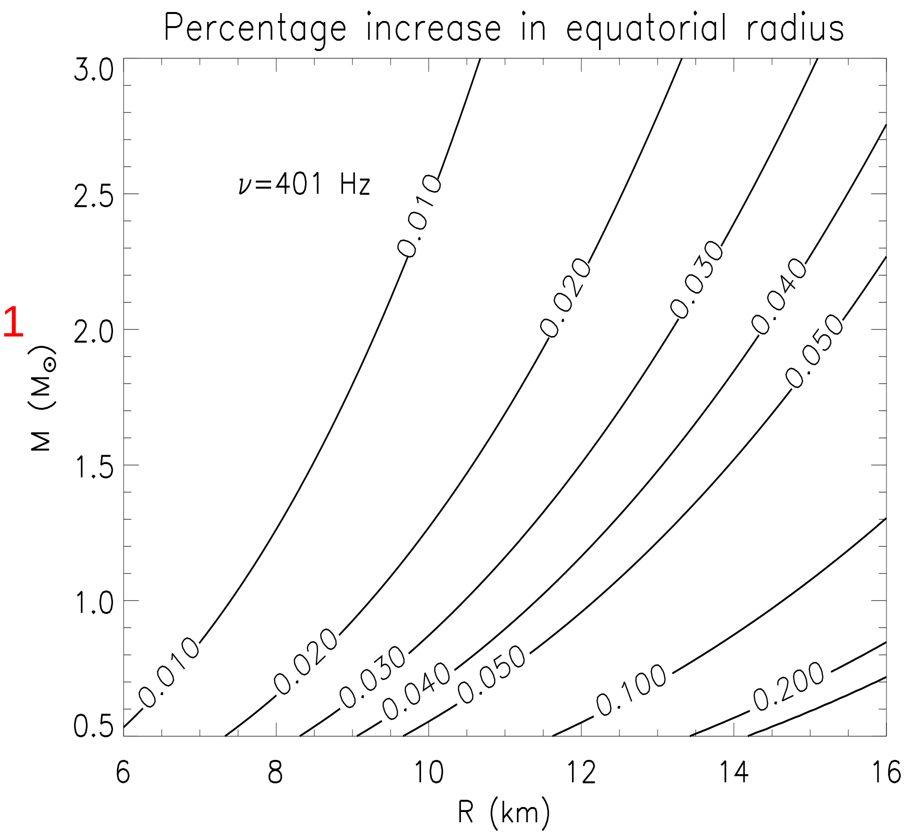
(*c.f.*, Shapiro & Teukolsky 1983)

$\rho^{-1}\nabla P = \nabla h = -\nabla(\Phi_G + \Phi_c), \qquad \Phi_G \simeq -GM/r, \qquad \Phi_c = -\frac{1}{2}\Omega^2 r^2 \sin^2\theta$

Bernoulli integral: $H = h + \Phi_G + \Phi_c = -GM/R_p$

Enthalpy $h = \int_0^p \rho^{-1} dp = \mu_n(\rho) - \mu_n(0)$ in beta equilibrium

Numerical calculations show R_p is nearly constant for arbitrary rotation



Evaluate at equator: $\frac{\Omega^2 R_{eq}^3}{2GM} = \frac{R_{eq}}{R} - 1$

Mass-shedding limit $\Omega^2_{shed} = \frac{GM}{R_{eq}^3} : \frac{R_{eq}}{R} = \frac{3}{2}$

GR: Cook, Shapiro & Teukolsky (1994): 1.43–1.51

$\Omega_{shed} = \left(\frac{2}{3}\right)^{3/2}\sqrt{\frac{GM}{R^3}}$

$P_{shed} = 1.0\ (R/10\ \mathrm{km})^{3/2}(\mathrm{M}_\odot/M)^{1/2}$ ms

GR: Lattimer & Prakash (2005): $0.96 \pm 3\%$

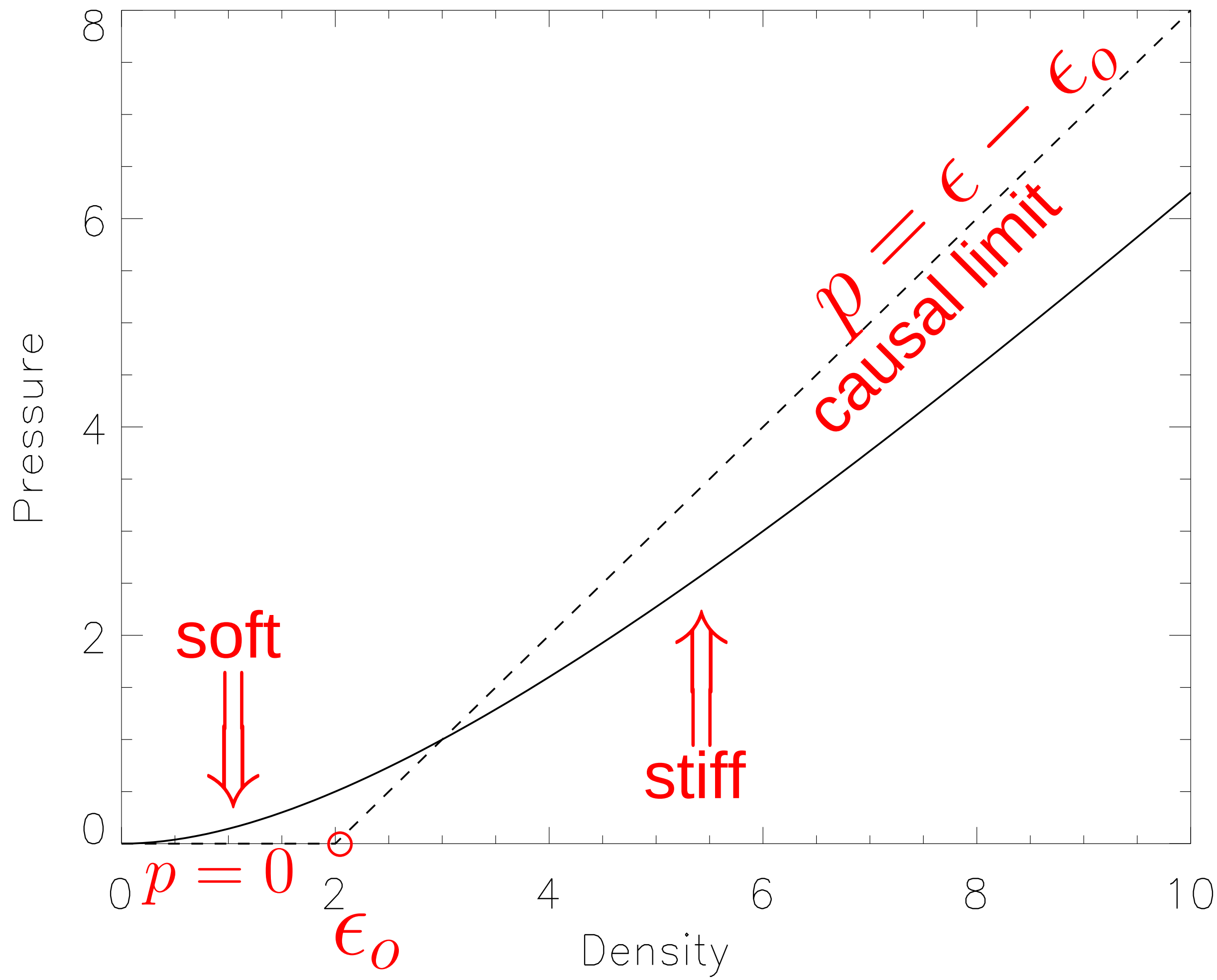
Shape: $\frac{\Omega^2 R^3 \sin^2\theta}{2GM} = \frac{R}{R_p} - 1$

$\frac{R_p}{R(\theta)} = \frac{1}{3} + \frac{2}{3}\cos[\frac{1}{3}\cos^{-1}(1 - 2(\frac{\Omega\sin\theta}{\Omega_{shed}})^2)]$

Limit: $\frac{R_p}{R(\theta)} = \frac{1}{3} + \frac{2}{3}\cos[\frac{1}{3}\cos^{-1}(1 - 2\sin^2\theta)] = \frac{\sin(\theta)}{3\sin(\theta/3)}$.

Extreme Properties of Neutron Stars

- The most compact configurations occur when the low-density equation of state is "soft" and the high-density equation of state is "stiff".



ϵ_0 is the only EOS parameter

The TOV solutions scale with ϵ_0

Maximally Compact Equation of State

Koranda, Stergioulas & Friedman (1997)

$$p(\epsilon) = 0, \qquad \epsilon \leq \epsilon_o$$
$$p(\epsilon) = \epsilon - \epsilon_o, \qquad \epsilon \geq \epsilon_o$$

This EOS has a parameter ϵ_o, which corresponds to the surface energy density. The structure equations then contain only this one parameter, and can be rendered into dimensionless form using

$$y = m\epsilon_o^{1/2}, \qquad x = r\epsilon_o^{1/2}, \qquad q = p\epsilon_o^{-1}.$$

$$\frac{dy}{dx} = 4\pi x^2(1+q)$$
$$\frac{dq}{dx} = -\frac{(y+4\pi q x^3)(1+2q)}{x(x-2y)}$$

The solution with the maximum central pressure and mass and the minimum radius:

$$q_{max} = 2.026, \qquad y_{max} = 0.0851, \qquad x_{min}/y_{max} = 2.825$$
$$p_{max} = 307\left(\frac{\epsilon_o}{\epsilon_s}\right) \text{ MeV fm}^{-3}, \quad M_{max} = 4.1\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{M}_\odot, \quad R_{min} = 2.825\frac{GM_{max}}{c^2}.$$

Moreover, the scaling extends to the axially-symmetric case, yielding

$$P_{min} \propto \left(\frac{M_{max}}{R_{min}^3}\right)^{1/2} \propto \epsilon_o^{-1/2}, \quad P_{min} = 0.82\left(\frac{\epsilon_s}{\epsilon_o}\right)^{1/2} \text{ms} = 0.76\left(\frac{R}{10\text{ km}}\right)^{3/2}\left(\frac{\text{M}_\odot}{M}\right)^{1/2} \text{ms}$$

Maximum Possible Density in Stars

The scaling from the maximally compact EOS yields

$$\epsilon_{c,max} = 3.026 \left(\frac{4.1\ \mathrm{M}_\odot}{M_{max}}\right)^2 \epsilon_s \simeq 13.7 \times 10^{15} \left(\frac{\mathrm{M}_\odot}{M_{max}}\right)^2 \mathrm{\ g\ cm}^{-3}.$$

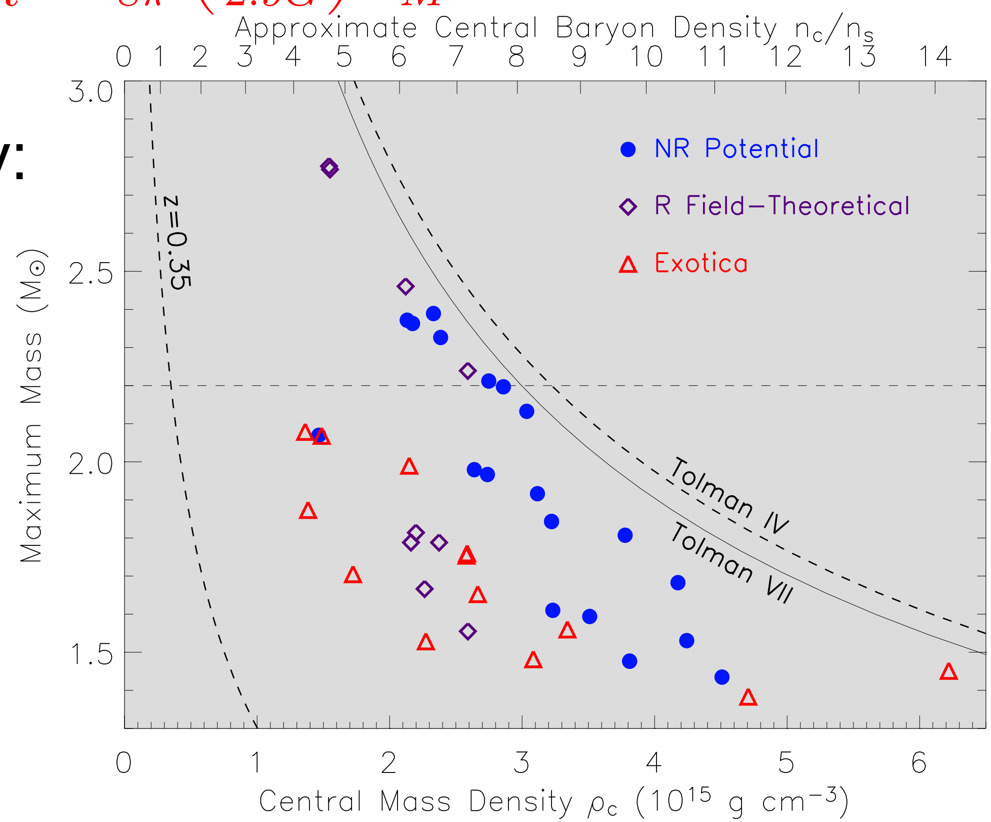
A virtually identical result arises from combining the maximum compactness constraint ($R_{min} \simeq 2.9GM/c^2$) with the Tolman VII relation

$$\epsilon_{c,VII} = \frac{15}{8\pi}\frac{M}{R^3} = \frac{15}{8\pi}\left(\frac{c^2}{2.9G}\right)^3 \frac{1}{M^2}$$

Maximum possible density:

$$2.2\ \mathrm{M}_\odot \Rightarrow$$

$$\epsilon_{max} < 2.8 \times 10^{15} \mathrm{g\ cm}^{-3}$$



Maximum Mass, Minimum Period

Theoretical limits from GR and causality

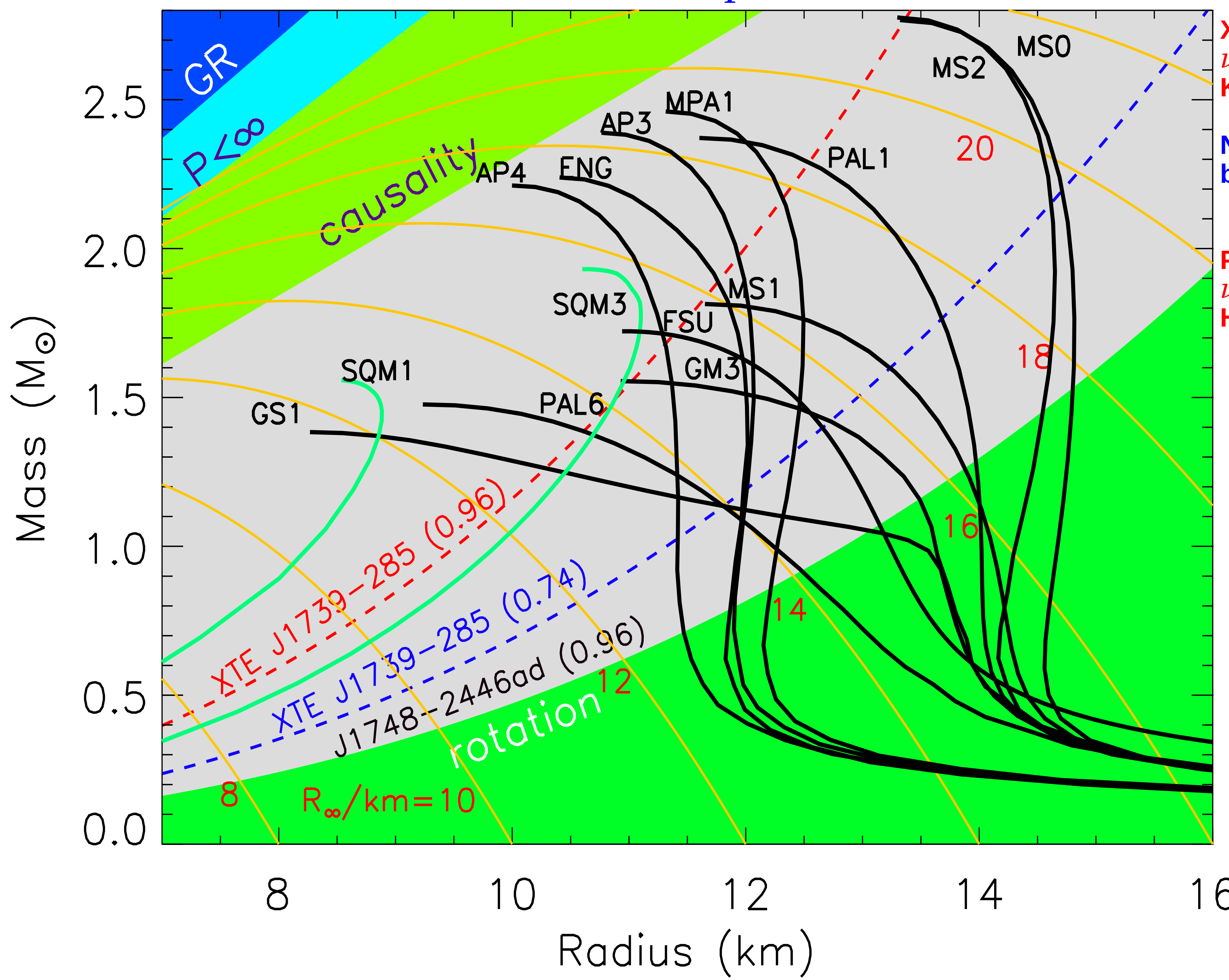
- $M_{max} = 4.2(\epsilon_s/\epsilon_f)^{1/2}\ \mathrm{M}_\odot$ **Rhoades & Ruffini (1974), Hartle (1978)**
- $R_{min} = 2.9GM/c^2 = 4.3(M/\mathrm{M}_\odot)$ km

  Lindblom (1984), Glendenning (1992), Koranda, Stergioulas & Friedman (1997)
- $\epsilon_c < 4.5 \times 10^{15}(\mathrm{M}_\odot/M_{largest})^2$ g cm^{-3} **Lattimer & Prakash (2005)**
- $P_{min} \simeq 0.74(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

  Koranda, Stergioulas & Friedman (1997)
- $P_{min} \simeq 0.96 \pm 0.03(\mathrm{M}_\odot/M_{sph})^{1/2}(R_{sph}/10\ \mathrm{km})^{3/2}$ ms

  (empirical) **Lattimer & Prakash (2004)**
- $\epsilon_c > 0.91 \times 10^{15}(1\ \mathrm{ms}/P_{min})^2$ g cm^{-3} (empirical)
- $cJ/GM^2 \lesssim 0.5$ (empirical, neutron star)

Constraints from Pulsar Spins

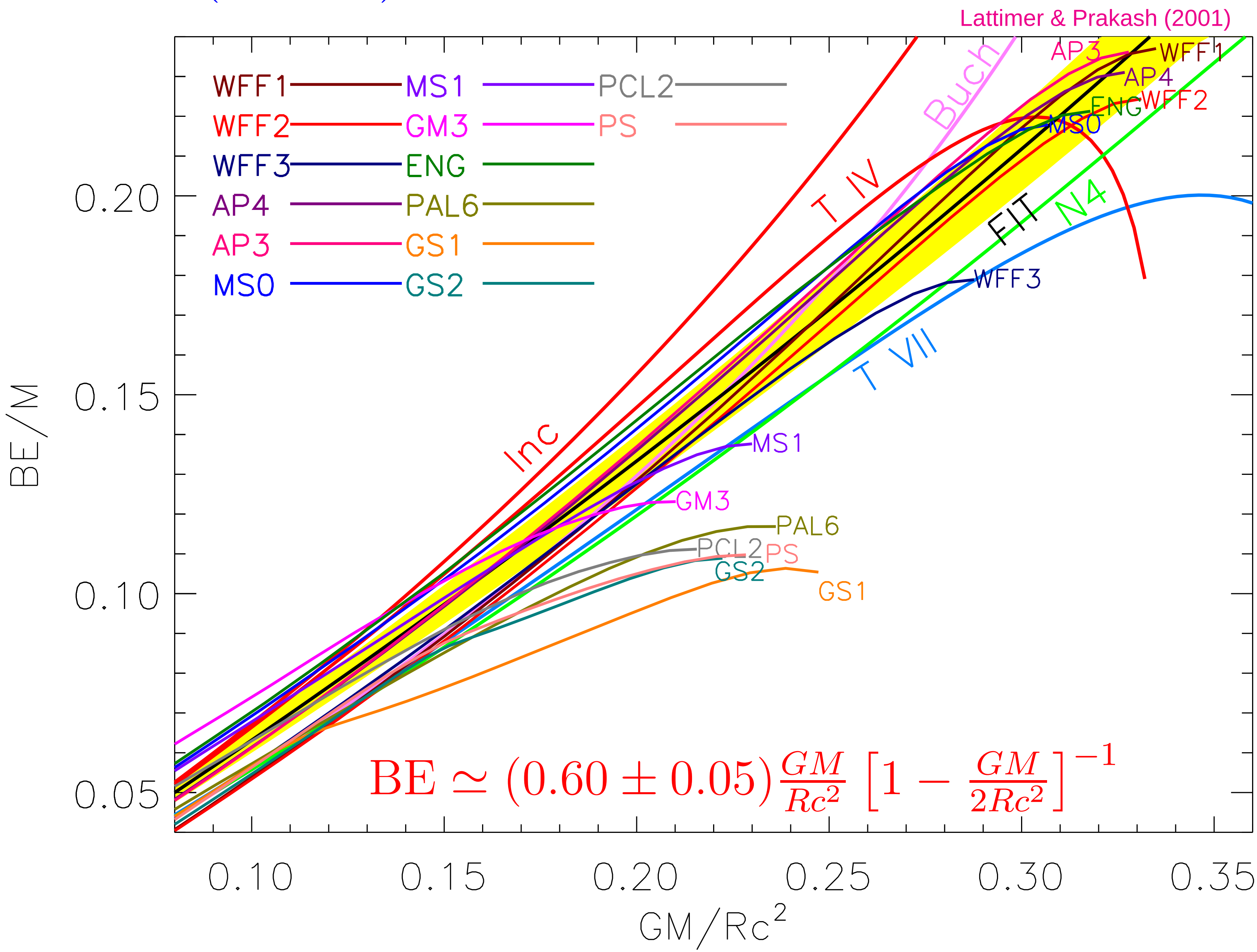


XTE J1739-285
$\nu = 1122$ **Hz**
Kaaret et al. 2006

Not confirmed to be a rotation rate

PSR J1748-2446ad
$\nu = 716$ **Hz**
Hessels et al. 2006

$\mathrm{BE}(M, R)$



Lattimer & Prakash (2001)
WFF1
WFF2
WFF3
AP4
AP3
MS0
MS1
GM3
ENG
PAL6
GS1
GS2
PCL2
PS
Inc
T IV
Buch
FIT
N4
T VII
BE/M
GM/Rc^2
0.05
0.10
0.15
0.20
0.25
0.30
0.35
$\mathrm{BE} \simeq (0.60 \pm 0.05)\frac{GM}{Rc^2}\left[1 - \frac{GM}{2Rc^2}\right]^{-1}$

Moment of Inertia

$$
\begin{aligned}
I &= \frac{8\pi}{3c^4}\int_0^R r^4\left[\epsilon(r)+p(r)\right]e^{(\lambda(r)-\nu(r))/2}\omega(r)dr \\
&= -\frac{2c^2}{3G}\int_0^R r^3\omega(r)\frac{dj(r)}{dr}dr,
\end{aligned}
$$

where

$$
\begin{aligned}
j(r) &= e^{-(\lambda(r)+\nu(r))/2}; \\
\frac{d}{dr}\left[r^4 j(r)\frac{d\omega(r)}{dr}\right] &= -4r^3\omega(r)\frac{dj(r)}{dr}; \\
j(R) &= 1, \qquad \omega(R) = 1-\frac{2GI}{R^3c^2}, \qquad \frac{d\omega(0)}{dr} = 0.
\end{aligned}
$$

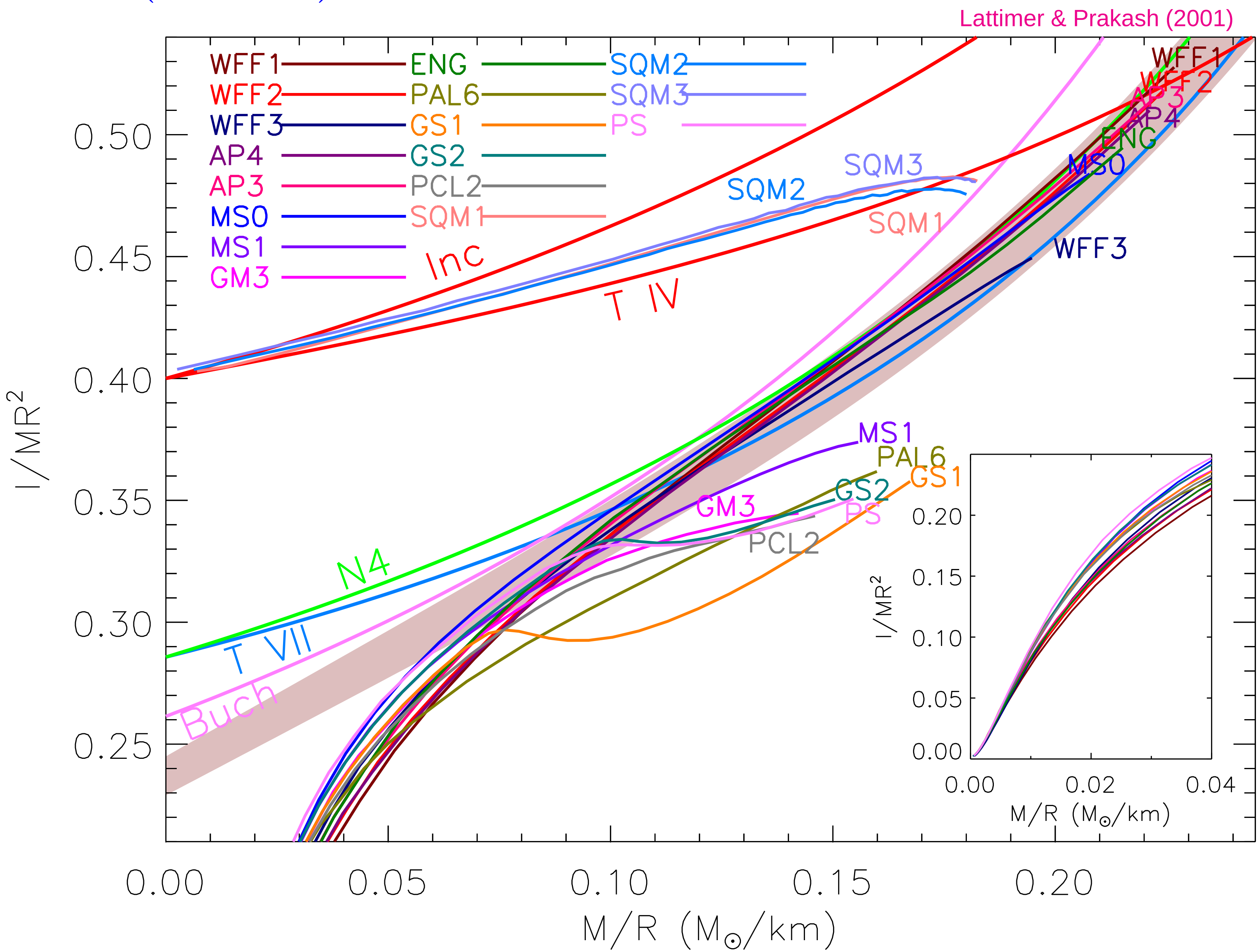
Combining these:

$$
I = \frac{c^2}{6G}R^4\frac{d\omega(R)}{dr}
$$

With $\phi = d\ln\omega/d\ln r$, $\phi(0) = 0$,

$$
\begin{aligned}
\frac{d\phi}{dr} &= -\frac{\phi}{r}(3+\phi)-(4+\phi)\frac{d\ln j}{dr}, \\
I &= \frac{\phi_R c^2}{6G}R^3\omega_R = \frac{\phi_R}{6}\left(\frac{R^3c^2}{G}-2I\right) = \frac{R^3\phi_R c^2}{G(6+2\phi_R)}.
\end{aligned}
$$

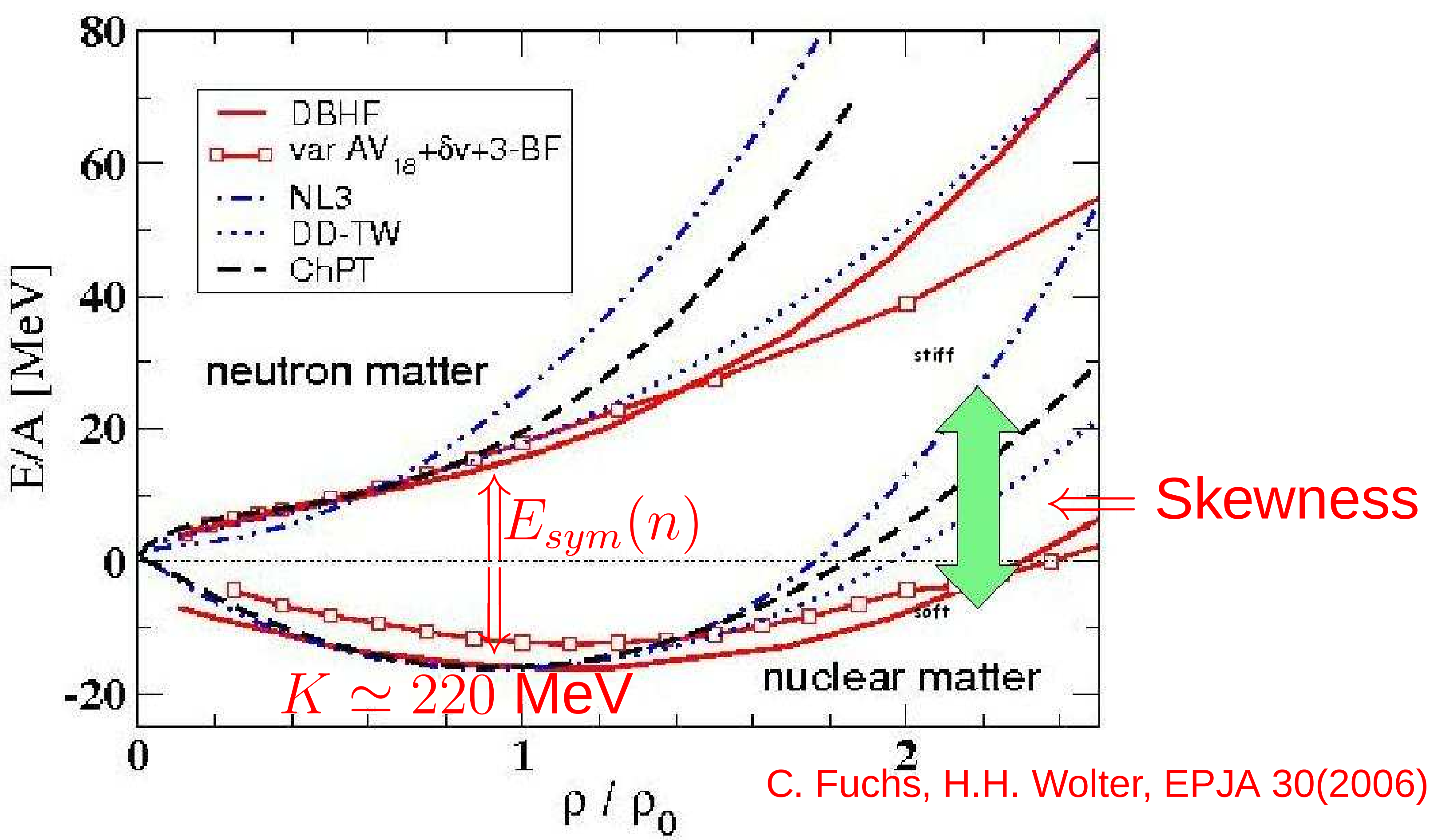
$I(M,R)$



$$I \simeq (0.237 \pm 0.008) MR^2 \left[1 + 4.2\frac{M \text{ km}}{R \text{ M}_\odot} + 90\left(\frac{M \text{ km}}{R \text{ M}_\odot}\right)^4\right]$$

The Uncertain Nuclear Force

The density dependence of $E_{sym}(n)$ is crucial but poorly constrained. Although the second density derivative, the incompressibility K, for symmetric matter is known well, the third density derivative, the skewness, is not.



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Schematic Energy Density

n: number density; x: proton fraction; T: temperature

$n_s \simeq 0.16 \pm 0.01$ fm^{-3}: nuclear saturation density

$B \simeq -16 \pm 1$ MeV: saturation binding energy

$K \simeq 220 \pm 15$ MeV: incompressibility parameter

$S_v \simeq 30 \pm 6$ MeV: bulk symmetry parameter

$a \simeq 0.065 \pm 0.010$ MeV^{-1}: bulk level density parameter

$$\begin{aligned}
\epsilon(n,x,T) &= n\left[B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)^2 + S_v\frac{n}{n_s}(1-2x)^2 + a\left(\frac{n_s}{n}\right)^{2/3}T^2\right] \\
P &= n^2\frac{\partial(\epsilon/n)}{\partial n} = \frac{n^2}{n_s}\left[\frac{K}{9}\left(\frac{n}{n_s}-1\right) + S_v(1-2x)^2\right] + \frac{2an}{3}\left(\frac{n_s}{n}\right)^{2/3}T^2 \\
\mu_n &= \frac{\partial\epsilon}{\partial n} - \frac{x}{n}\frac{\partial\epsilon}{\partial x} \\
&= B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)\left(1-3\frac{n}{n_s}\right) + 2S_v\frac{n}{n_s}(1-2x) - \frac{a}{3}\left(\frac{n_s}{n}\right)^{2/3}T^2 \\
\hat{\mu} &= -\frac{1}{n}\frac{\partial\epsilon}{\partial x} = \mu_n - \mu_p = 4S_v\frac{n}{n_s}(1-2x) \\
s &= \frac{1}{n}\frac{\partial\epsilon}{\partial T} = 2a\left(\frac{n_s}{n}\right)^{2/3}T
\end{aligned}$$

Phase Coexistence

Schematic energy density

$$
\begin{aligned}
\epsilon &= n\left[B+\frac{K}{18}\left(1-\frac{n}{n_s}\right)^2+S_v\frac{n}{n_s}(1-2x)^2+a\left(\frac{n_s}{n}\right)^{2/3}T^2\right]\\
P &= \frac{n^2}{n_s}\left[\frac{K}{9}\left(\frac{n}{n_s}-1\right)+S_v(1-2x)^2\right]+\frac{2an}{3}\left(\frac{n_s}{n}\right)^{2/3}T^2\\
\mu_n &= B+\frac{K}{18}\left(1-\frac{n}{n_s}\right)\left(1-3\frac{n}{n_s}\right)+2S_v\frac{n}{n_s}(1-2x)-\frac{a}{3}\left(\frac{n_s}{n}\right)^{2/3}T^2\\
\hat{\mu} &= \mu_n-\mu_p=4S_v\frac{n}{n_s}(1-2x)\,, \qquad s=2a\left(\frac{n_s}{n}\right)^{2/3}T
\end{aligned}
$$

Free Energy Minimization With Two Phases

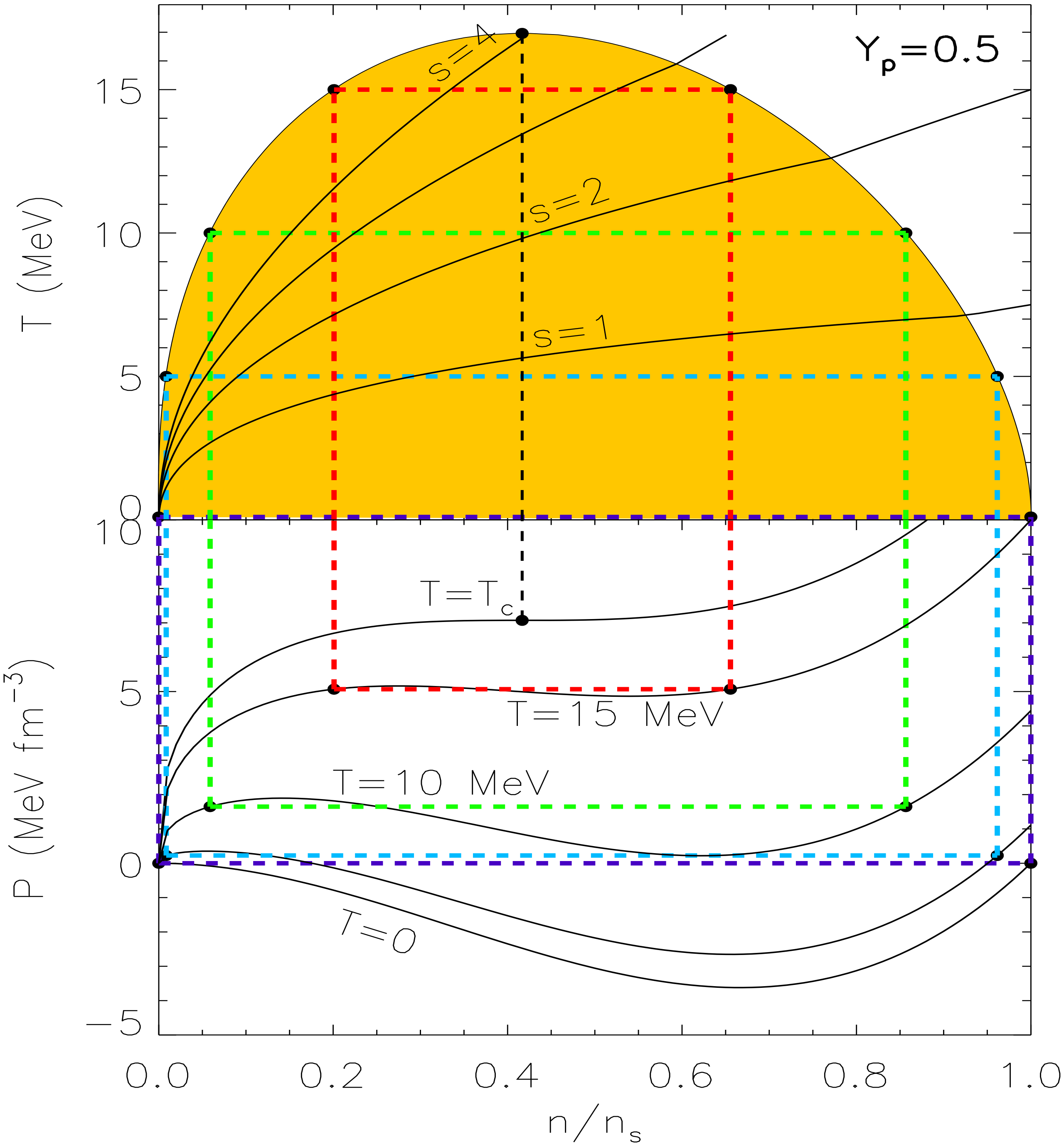
$$F=\epsilon-nTs=uF_I+(1-u)F_{II}, \qquad n=un_I+(1-u)n_{II}, \qquad nY_e=ux_In_I+(1-u)x_{II}n_{II}$$

$$\frac{\partial F}{\partial n_I}=0\,, \quad \frac{\partial F}{\partial x_I}=0\,, \quad \frac{\partial F}{\partial u}=0 \quad \Longrightarrow \mu_{nI}=\mu_{nII}\,, \qquad \mu_{pI}=\mu_{pII}\,, \qquad P_I=P_{II}$$

Critical Point ($Y_e=0.5$)

$$\left(\frac{\partial P}{\partial n}\right)_T=\left(\frac{\partial^2 P}{\partial n^2}\right)_T=0$$

$$n_c=\frac{5}{12}n_s\,, \qquad T_c=\left(\frac{5}{12}\right)^{1/3}\left(\frac{5K}{32a}\right)^{1/2}, \qquad s_c=\left(\frac{12}{5}\right)^{1/3}\left(\frac{5Ka}{8}\right)^{1/2}$$



$Y_p=0.5$
s=4
s=2
s=1
T (MeV)
$T=T_c$
T=15 MeV
T=10 MeV
T=0
P (MeV fm^{-3})
n/n_s

The Pressure of Neutron Star Matter

Expansion of cold nucleonic matter energy near n_s and isospin symmetry $x = 1/2$:

$$\begin{aligned} E(n,x) &\simeq E(n,1/2) + E_{sym}(n)(1-2x)^2 + \frac{3\hbar c}{4}x(3\pi^2 nx)^{1/3}\,, \\ P(n,x) &\simeq n^2\left[\frac{dE(n,1/2)}{dn} + \frac{dE_{sym}}{dn}(1-2x)^2\right] + \frac{\hbar c}{4}nx(3\pi^2 nx)^{1/3}\,, \\ \mu_e &= \hbar c(3\pi^2 nx)^{1/3}\,, \qquad E(n,1/2) \simeq -B + \frac{K}{18}\left(1-\frac{n}{n_s}\right)^2. \end{aligned}$$

Beta Equilibrium:

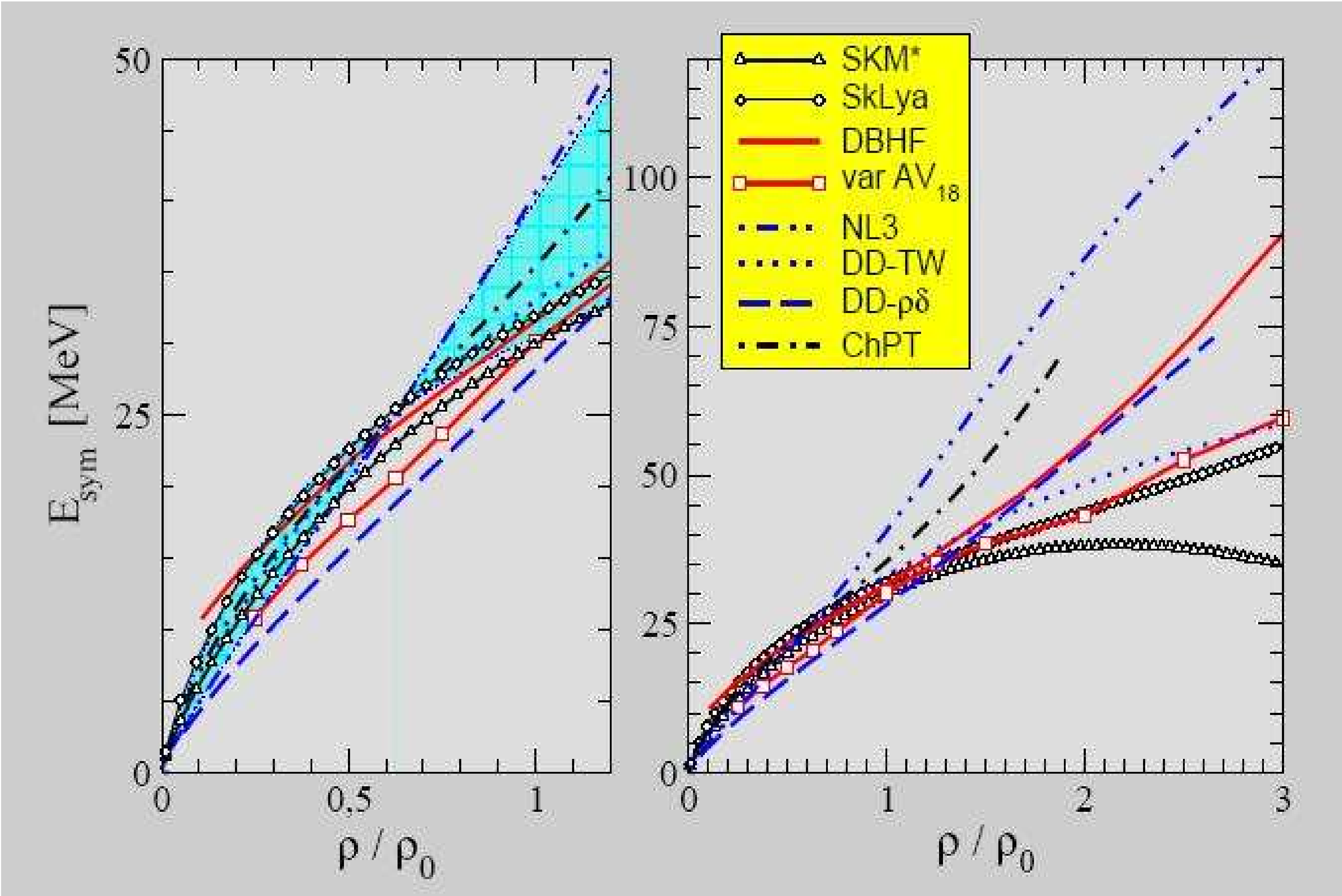
$$\left(\frac{\partial E}{\partial x}\right)_n = \mu_p - \mu_n + \mu_e = 0\,.$$

$$\begin{aligned} x_\beta &\simeq (3\pi^2 n)^{-1}\left(\frac{4E_{sym}}{\hbar c}\right)^3\,, \\ P_\beta &= \frac{Kn^2}{9n_0}\left(\frac{n}{n_s}-1\right) + n^2(1-2x_\beta)^2\frac{dE_{sym}}{dn} + E_{sym}nx_\beta(1-2x_\beta) \end{aligned}$$

$E_{sym}(n_s) \equiv S_v \simeq 30$ MeV, $\hbar c \simeq 200$ MeV/fm, $\qquad n \to n_s \Longrightarrow$

$$x_\beta \to 0.04\,, \qquad P_\beta \to n_s^2 \left.\frac{dE_{sym}}{dn}\right|_{n_s}.$$

The Uncertain $E_{sym}(n)$



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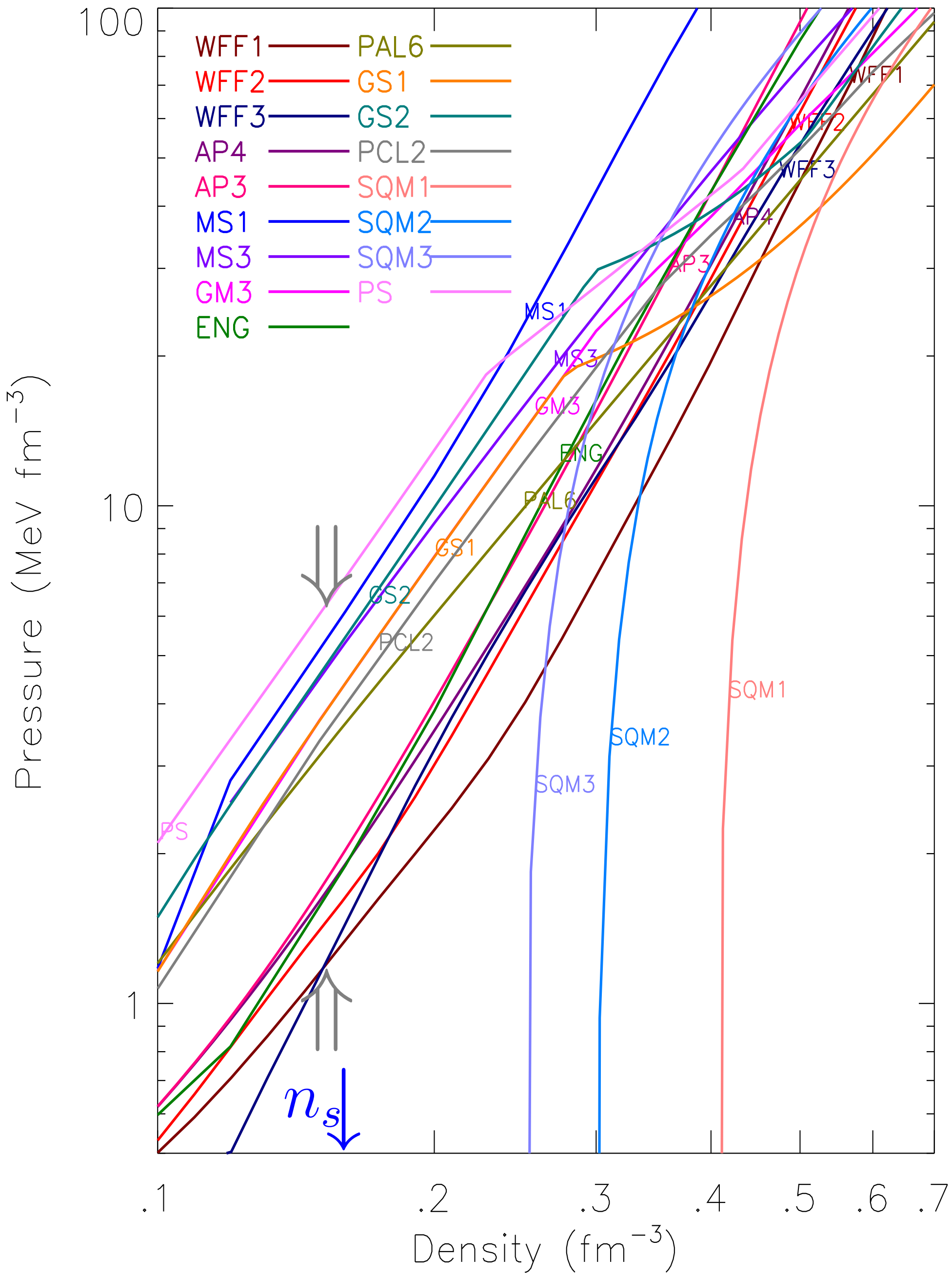
Neutron Star Matter Pressure

$$p \simeq K n^\gamma$$

$$\gamma = d \ln p / d \ln\ n \sim 2$$

Wide variation:

$$1.2 < \frac{p(n_s)}{\mathrm{MeV\ fm^{-3}}} < 7$$



Polytropes

Polytropic Equation of State: $p = Kn^\gamma$
n is number density, γ is polytropic exponent.
Hydrostatic Equilibrium in Newtonian Gravity:

$$\frac{dp(r)}{dr} = -\frac{Gm(r)n(r)}{r^2}, \qquad \frac{dm(r)}{dr} = 4\pi n r^2$$

Dimensional analysis:

$$M \propto n_c R^3, \quad p \propto \frac{M^2}{R^4}, \quad R \propto K^{1/(3\gamma-4)} M^{(\gamma-2)/(3\gamma-4)}$$

When $\gamma \sim 2$: $$R \propto K^{1/2} M^0 \propto p_f^{1/2} n_f^{-1} M^0$$

General Relativistic analysis using Buchdahl's solution:

$$R = (1-\beta)\sqrt{\frac{\pi}{2p_*(1-2\beta)}}, \quad \left.\frac{d\ln R}{d\ln p}\right|_{n,M} = \frac{1}{2}\frac{(1-\beta)(2-\beta)}{(1-3\beta+3\beta^2)}\frac{1-10\sqrt{p/p_*}}{1+2\sqrt{p/p_*}}.$$

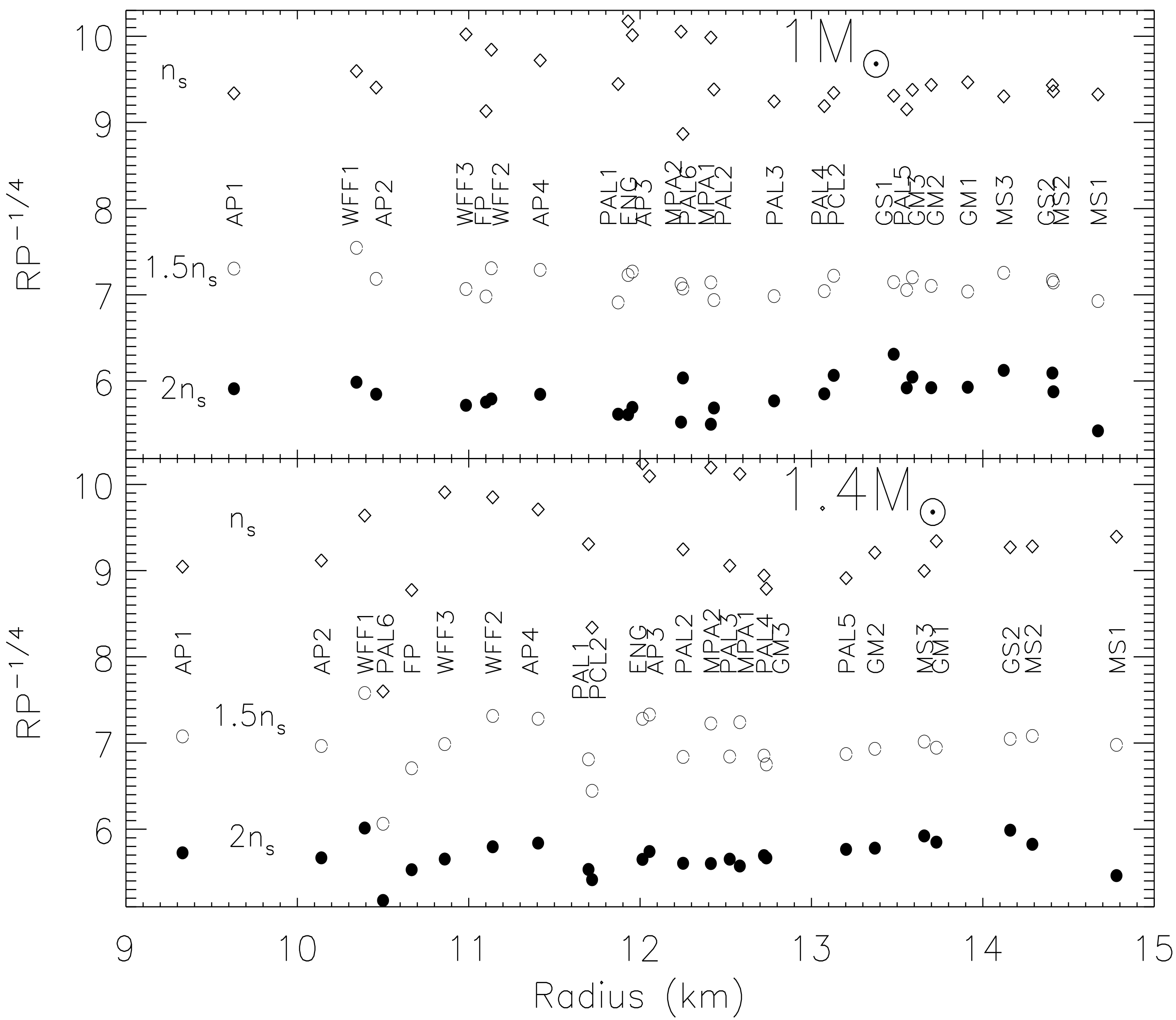
For $M = 1.4\mathrm{M}_\odot$, $R = 14$ km, $n = 1.5n_s$, $\epsilon = 1.5m_b n_s \simeq 3\times10^{-4}$ km^{-2}:

$$\beta = 0.148, \quad p_* = 0.00826, \quad p/p_* = 0.00221.$$

$$\left.\frac{d\ln R}{d\ln p}\right|_{n,M} \simeq 0.234$$

The Radius – Pressure Correlation

$R \propto p^{1/4}$



Lattimer & Prakash (2001)

Nuclear Symmetry Energy

The density dependence of $E_{sym}(n)$ is crucial. Some information is available from nuclei (for $n < n_s$). Heavy ion collisions have potential for constraining it for $n > n_s$. It is common to expand $E_{sym}(n)$ as

$$E_{sym}(n) \simeq J + \frac{L}{3}\left(\frac{n}{n_s} - 1\right) + \frac{K_{sym}}{18}\left(\frac{n}{n_s} - 1\right)^2 + \cdots$$

$$J = E_{sym}(n_s), \qquad L = 3n_s\left(\frac{\partial E_{sym}}{\partial n}\right)_{n_s}, \qquad K_{sym} = 9n_s^2\left(\frac{\partial^2 E_{sym}}{\partial n^2}\right)_{n_s}$$

Almost no information is available for K_{sym}.

What information can constrain $E_{sym}(n)$ from the laboratory?

Nuclear Mass Formula

Bethe-Weizsäcker (neglecting pairing and shell effects)

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2.$$

Myers & Swiatecki introduced the surface asymmetry term:

$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2 - S_s (N-Z)^2/A^{4/3}.$$

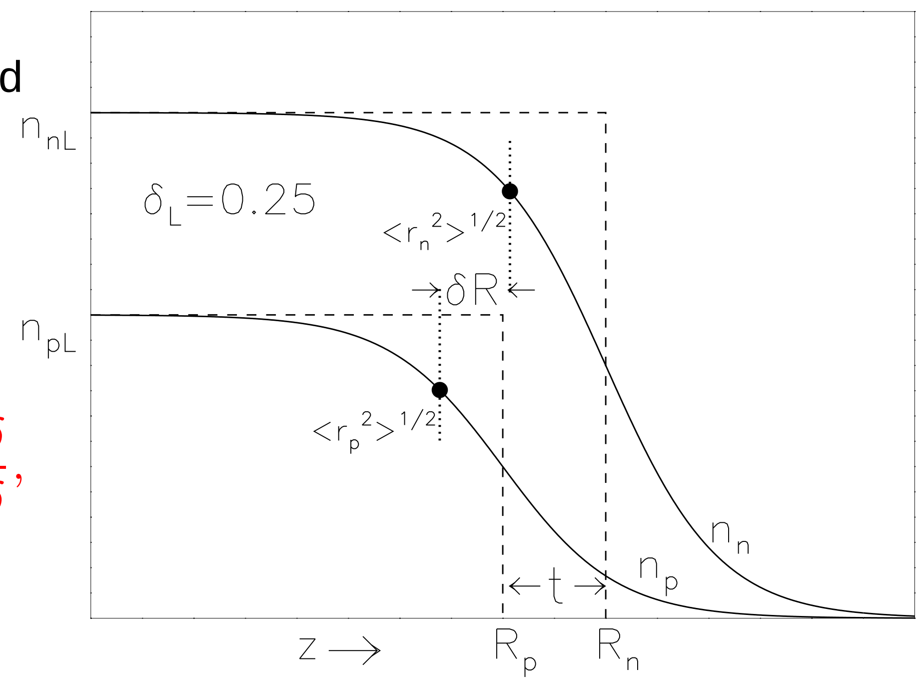
Droplet extension: consider the neutron/proton asymmetry of the nuclear surface.

$$E(A,Z) = (-a_v + S_v \delta^2)(A - N_s) + a_s A^{2/3} + a_C Z^2/A^{1/3} + \mu_n N_s.$$

N_s is the number of excess neutrons associated with the surface, $I = (N-Z)/(N+Z)$, $\delta = 1 - 2x = (A - N_s - 2Z)/(A - N_s)$ is the asymmetry of the nuclear bulk fluid, and μ_n is the neutron chemical potential. From thermodynamics,

$$N_s = -\frac{\partial a_s A^{2/3}}{\partial \mu_n} = \frac{S_s}{S_v}\frac{\delta}{1-\delta} = A\frac{I-\delta}{1-\delta},$$

$$\delta = I\left(1 + \frac{S_s}{S_v A^{1/3}}\right)^{-1},$$



$$E(A,Z) = -a_v A + a_s A^{2/3} + a_C Z^2/A^{1/3} + S_v A I^2 \left(1 + \frac{S_s}{S_v A^{1/3}}\right)^{-1}.$$

Nuclear Structure Considerations

Information about E_{sym} can be extracted from nuclear binding energies and models for nuclei. For example, consider the schematic liquid droplet model (Myers & Swiatecki):

$$E(A,Z) \simeq -a_v A + a_s A^{2/3} + \frac{S_v}{1+(S_s/S_v)A^{-1/3}} A + a_c Z^2 A^{-1/3}$$

Fitting binding energies results in a strong correlation between S_v and S_s, but not definite values.

Blue: $\Delta E < 0.01$ MeV/b

Green: $\Delta E < 0.02$ MeV/b

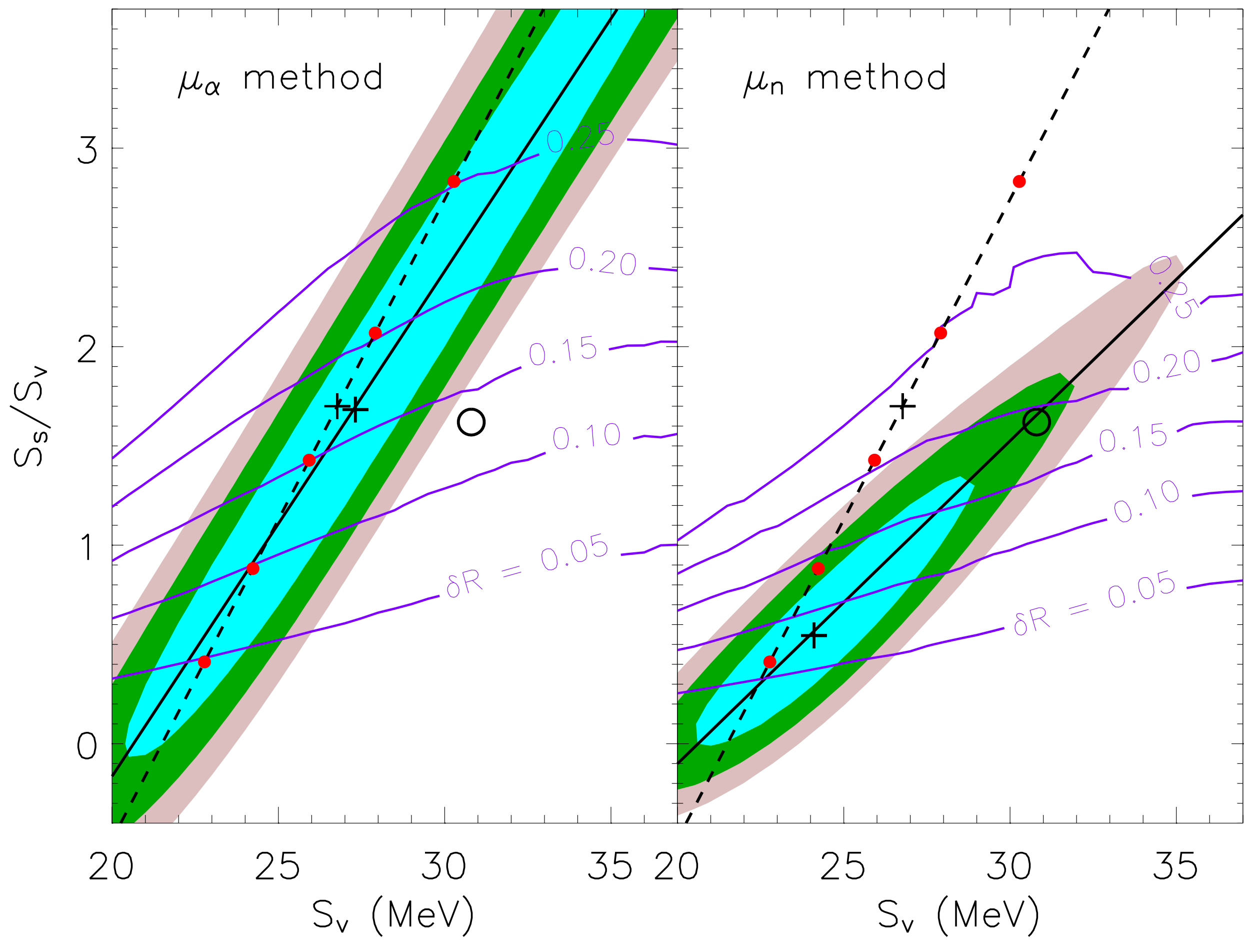
Gray: $\Delta E < 0.03$ MeV/b

Circle: Moeller et al. (1995)

Crosses: Best fits

Dashed: Danielewicz (2004)

Solid: Steiner et al. (2005)



Schematic Dependence

Nuclear Hamiltonian:

$$H = H_B + \frac{Q}{2} n'^2 , \quad H_B \simeq n \left[-B + \frac{K}{18} \left(1 - \frac{n}{n_s} \right)^2 \right] + E_{sym} (1 - 2x)^2$$

Lagrangian minimization of energy with respect to n (symmetric matter):

$$H_B - \mu_0 n = \frac{Q}{2} n'^2 = \frac{K}{18} n \left(1 - \frac{n}{n_s} \right)^2 , \qquad \mu_0 = -a_v$$

Liquid Droplet surface parameters: $a_s = 4\pi r_0^2 \sigma_0, \qquad S_s = 4\pi r_0^2 \sigma_\delta$

$$\sigma_0 = \int_{-\infty}^{+\infty} [H - \mu_0 n] dz = \int_0^{n_s} (H_B - \mu_0 n) \frac{dn}{n'} = \frac{4}{45} \sqrt{Q K n_s^3}$$

$$t_{90-10} = \int_{0.1 n_s}^{0.9 n_s} \frac{dn}{n'} = 3 \sqrt{\frac{Q n_s}{K}} \int_{0.1}^{0.9} \frac{du}{\sqrt{u}(1-u)} \simeq 9 \sqrt{\frac{Q n_s}{K}}$$

$$\sigma_\delta = S_v \sqrt{\frac{Q}{2}} \int_0^{n_s} n \left(\frac{S_v}{E_{sym}} - 1 \right) (H_B - \mu_0 n)^{-1/2} dn$$

$$= \frac{S_v t_{90-10} n_s}{3} \int_0^1 \frac{\sqrt{u}}{1-u} \left(\frac{S_v}{E_{sym}} - 1 \right) du$$

$$E_{sym} \simeq S_v \left(\frac{n}{n_s} \right)^p \Longrightarrow \int \rightarrow 0.28 \ (p = \frac{1}{2}) , \ 0.93 \ (p = \frac{2}{3}) , \ 2.0 \ (p = 1)$$

$$E_{sym} \simeq S_v + \frac{L}{3} \left(\frac{n}{n_s} \right) \Longrightarrow \int \rightarrow 2 - \sqrt{\frac{3S_v}{L} - 1} \tan^{-1} \sqrt{\left(1 + \frac{S_v}{3L} \right)^{-1}} \simeq 1 + \frac{L}{3S_v}$$

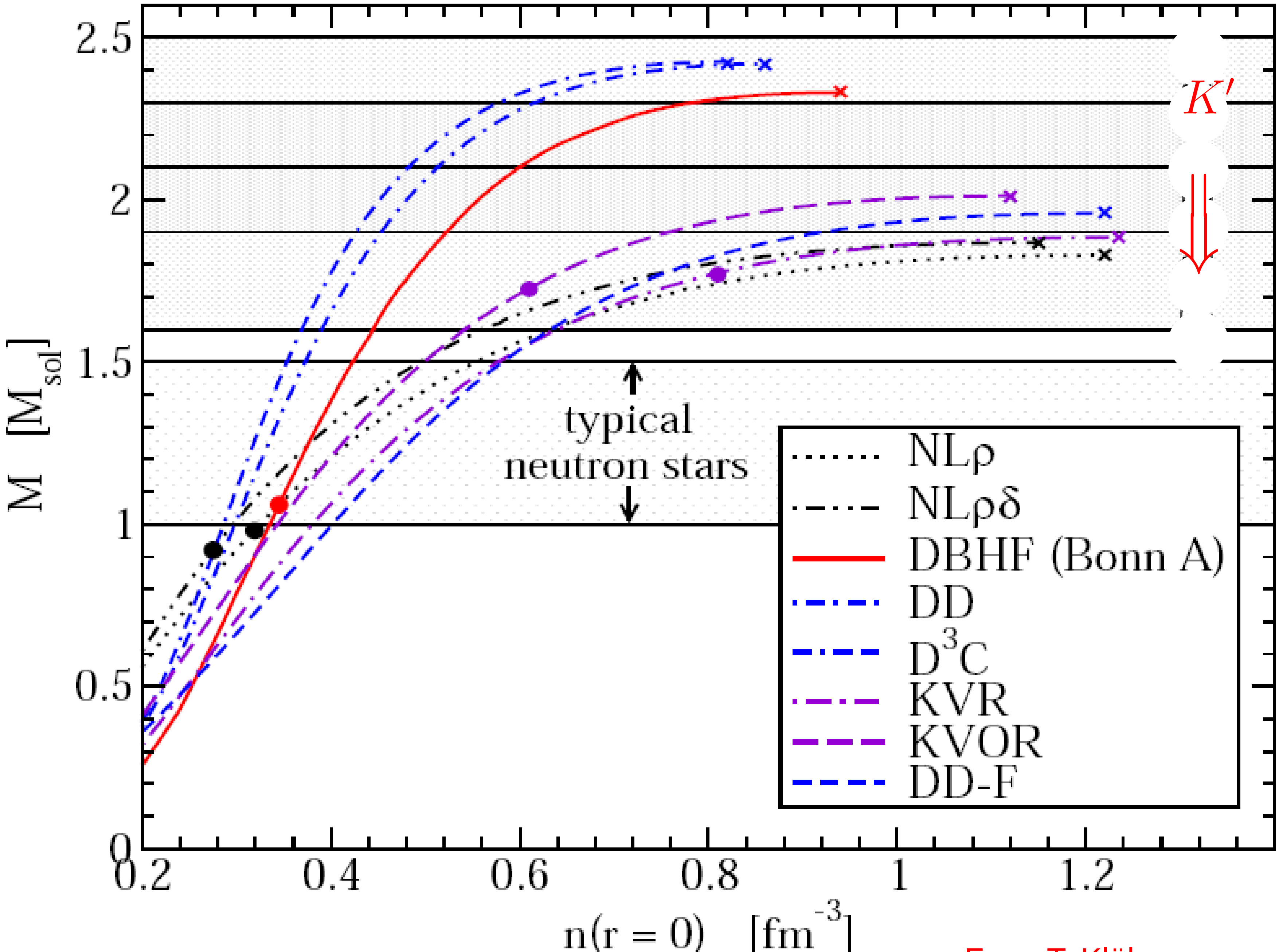
Schematic Dependence

$$\frac{S_s}{S_v} \simeq \frac{t_{90-10}}{r_0} \int \simeq 2.05 \int \Longrightarrow 0.57 \qquad 1.91 \qquad 4.1\,.$$

For Pb^{208}:

$$\delta R = \sqrt{\frac{3}{5}}(R_n - R_p) \simeq \frac{t_{90-10}}{6} \frac{A-2Z}{Z(1-Z/A)} \int \Longrightarrow 0.05 \qquad 0.16 \qquad 0.35$$

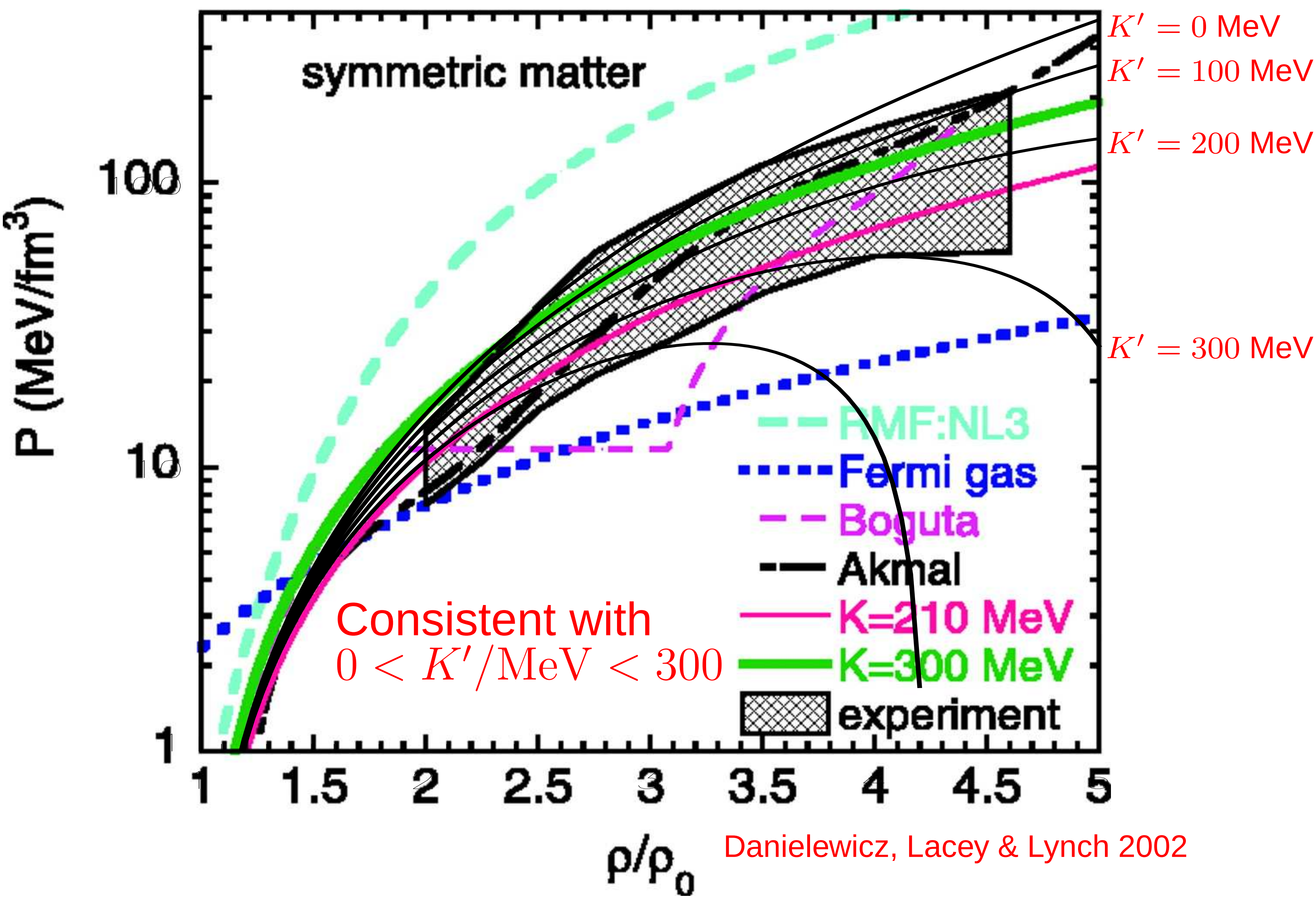
PREX experiment (E06002) at Jefferson Lab to measure the neutron radius of lead to about 1% accuracy (current accuracy is about 5%) using the parity violating asymmetry in elastic scattering due to the weak neutral interaction. Requires corrections for Coulomb distortions (Horowitz).



2.5
2
1.5
1
0.5
0
0.2
0.4
0.6
0.8
1
1.2
M [M_sol]
n(r = 0) [fm^-3]
K'
typical
neutron stars
NLρ
NLρδ
DBHF (Bonn A)
DD
D^3C
KVR
KVOR
DD-F

From T. Klähn

Flow Constraint From Heavy Ions



Danielewicz, Lacey & Lynch 2002

Nuclei in Dense Matter

Liquid Droplet Model, Simplified

$$F = u(F_I + f_{LD}/V_N) + (1-u)F_{II}\,, \qquad f_{LD} = f_S + f_C + f_T$$

$$\begin{aligned}
f_C &= \frac{3}{5}\frac{Z^2e^2}{R_N}\left(1 - \frac{3}{2}u^{1/3} + \frac{u}{2}\right) = \frac{3}{5}\frac{Z^2e^2}{R_N}D(u) \\
f_T &= T\ln\left(\frac{u}{n_Q V_N A^{3/2}}\right) - T = \mu_T - T\,, \qquad n_Q = \left(\frac{mT}{2\pi\hbar^2}\right)^{3/2} \\
f_S &= 4\pi R_N^2 \sigma(\mu_s)
\end{aligned}$$

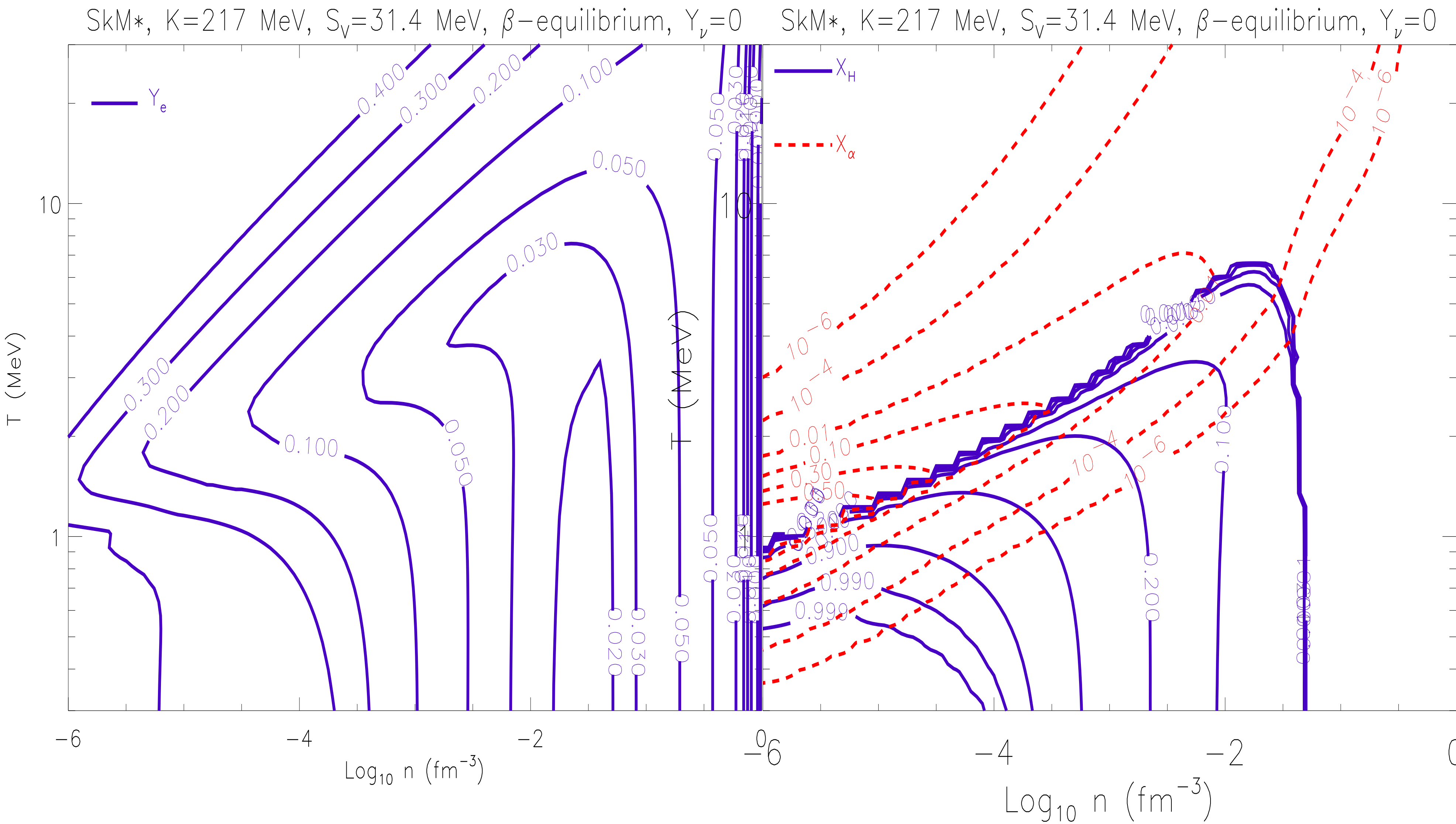
$$n = un_I + (1-u)n_{II}\,, \qquad nY_e = un_I x_I + (1-u)n_{II}x_{II} + u\frac{N_s}{V_N}$$

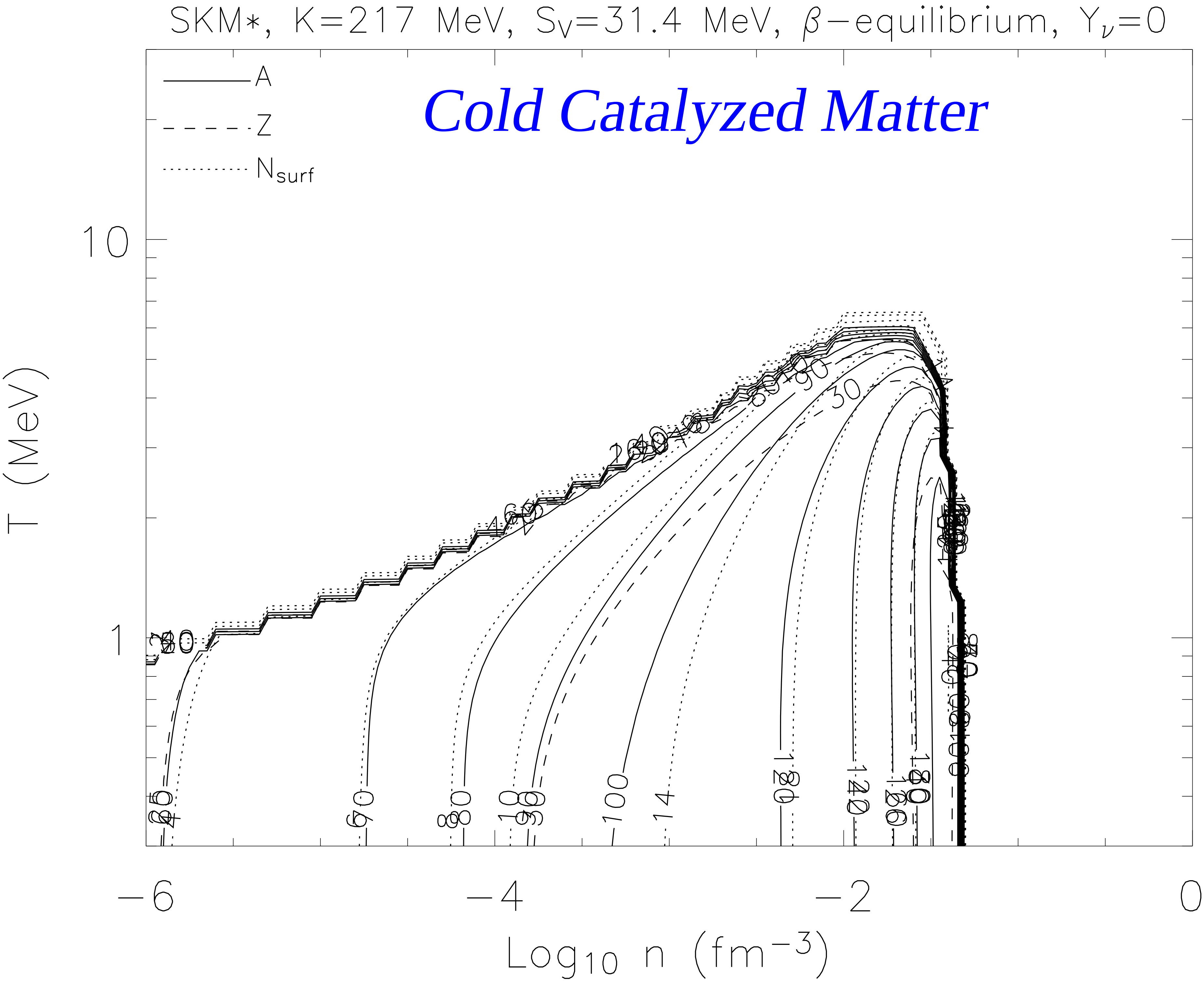
Free Energy Minimization

$$\frac{\partial F}{\partial z_i} = 0\,, \qquad z_i = (n_I, x_I, R_N, u, \nu_s, \mu_s)$$

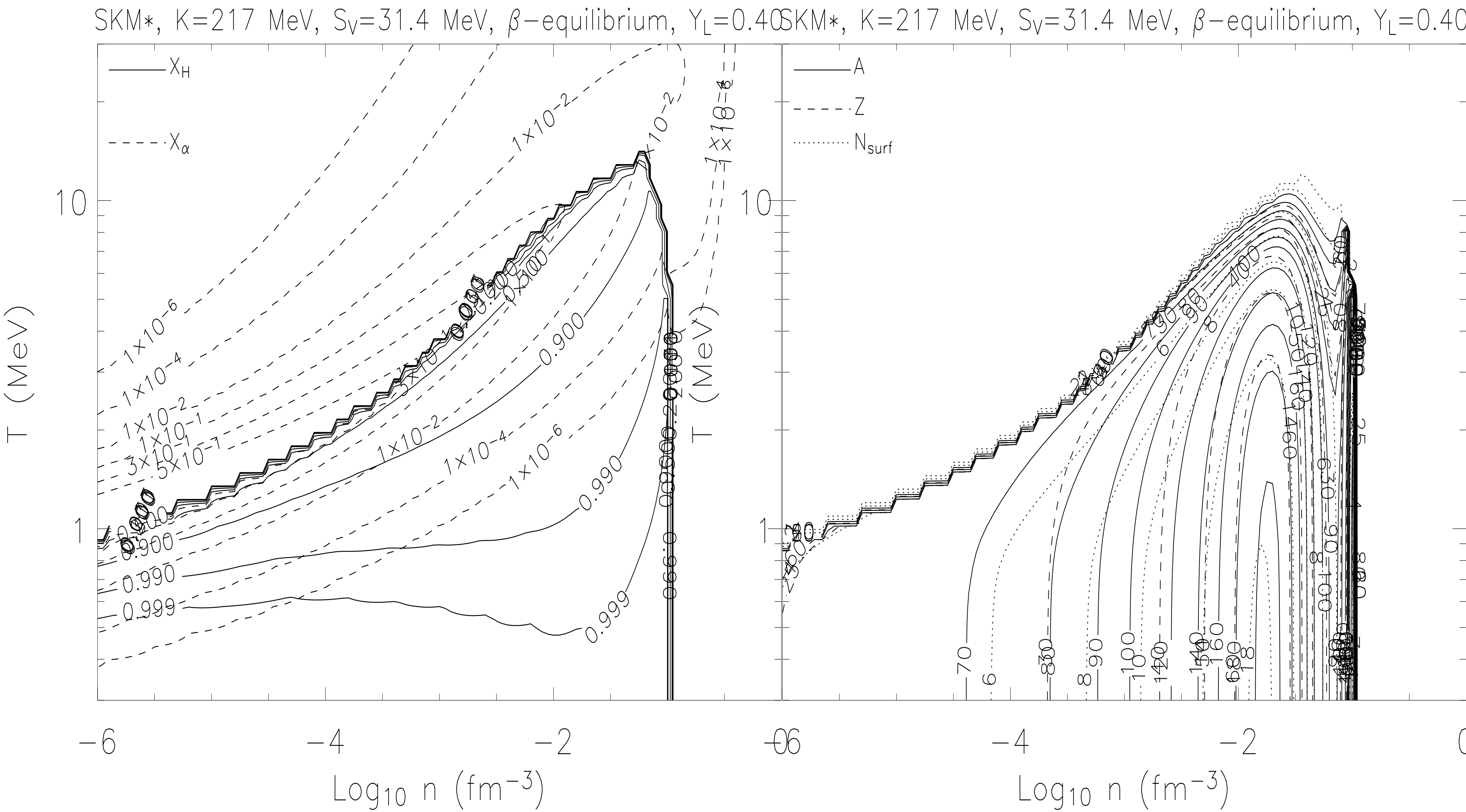
$$\begin{aligned}
\mu_{n,II} &= \mu_{n,I} + \frac{\mu_T}{A}\,, \qquad \hat\mu_{II} = \hat\mu_I - \frac{3\sigma}{R_N n_I x_i} = -\mu_s\,, \qquad N_s = -4\pi R_N^2\frac{\partial\sigma}{\partial\mu_s} \\
P_{II} &= P_I + \frac{3\sigma}{2R_N}\left(1 + \frac{uD'}{D}\right)\,, \qquad R_N = \left(\frac{15\sigma}{8\pi n_I^2 x_I^2 e^2 D}\right)^{1/3}
\end{aligned}$$

Cold Catalyzed Matter

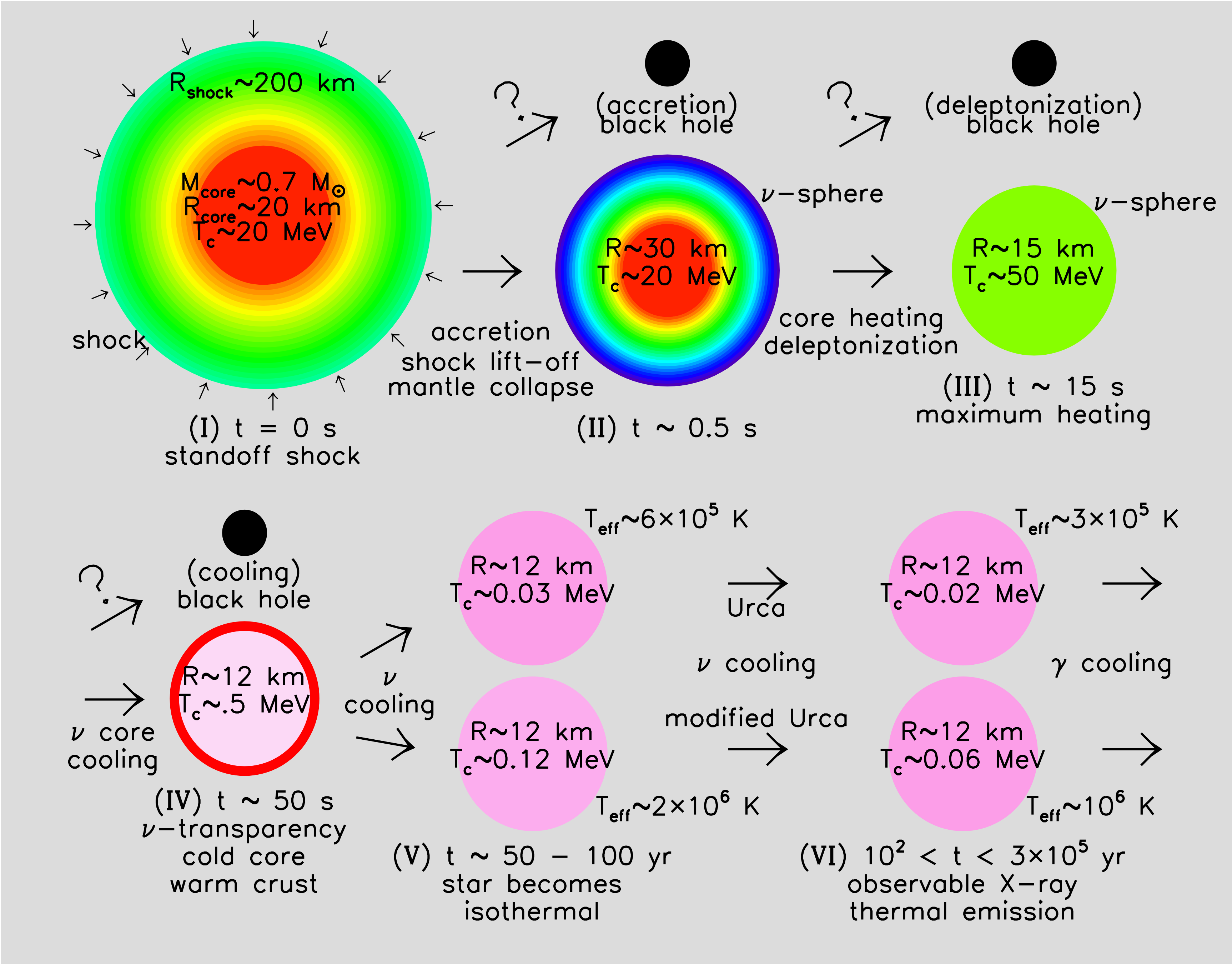



SKM*, K=217 MeV, S_V=31.4 MeV, β−equilibrium, Y_ν=0
A
Z
N_surf
Cold Catalyzed Matter
T (MeV)
10
1
−6
−4
−2
0
Log_10 n (fm^−3)

Supernova Matter



Proto-Neutron Stars



Proto-Neutron Star Evolution

$$n\frac{dY_L}{dt} = n\frac{dY_e}{dt} + \frac{dY_\nu}{dt} = \frac{1}{r^2}\frac{\partial}{\partial r} r^2 F_\nu$$

$$nT\frac{ds}{dt} = -\frac{1}{4\pi r^2}\frac{\partial L_\nu}{\partial r} - n\sum_{n,p,e,\nu}\mu_i\frac{dY_i}{dt}.$$

In the diffusion approximation, fluxes are driven by density gradients:

$$F_\nu = -\int_0^\infty \frac{c}{3}\left(\lambda_\nu \frac{\partial n_\nu(E_\nu)}{\partial r} - \lambda_{\bar\nu}\frac{\partial n_{\bar\nu}(E_\nu)}{\partial r}\right) dE_\nu,$$

$$L_\nu = -\int_0^\infty 4\pi r^2 \sum_i \frac{c\lambda_E^i}{3}\frac{\partial \epsilon_i(E_\nu)}{\partial r} dE_\nu.$$

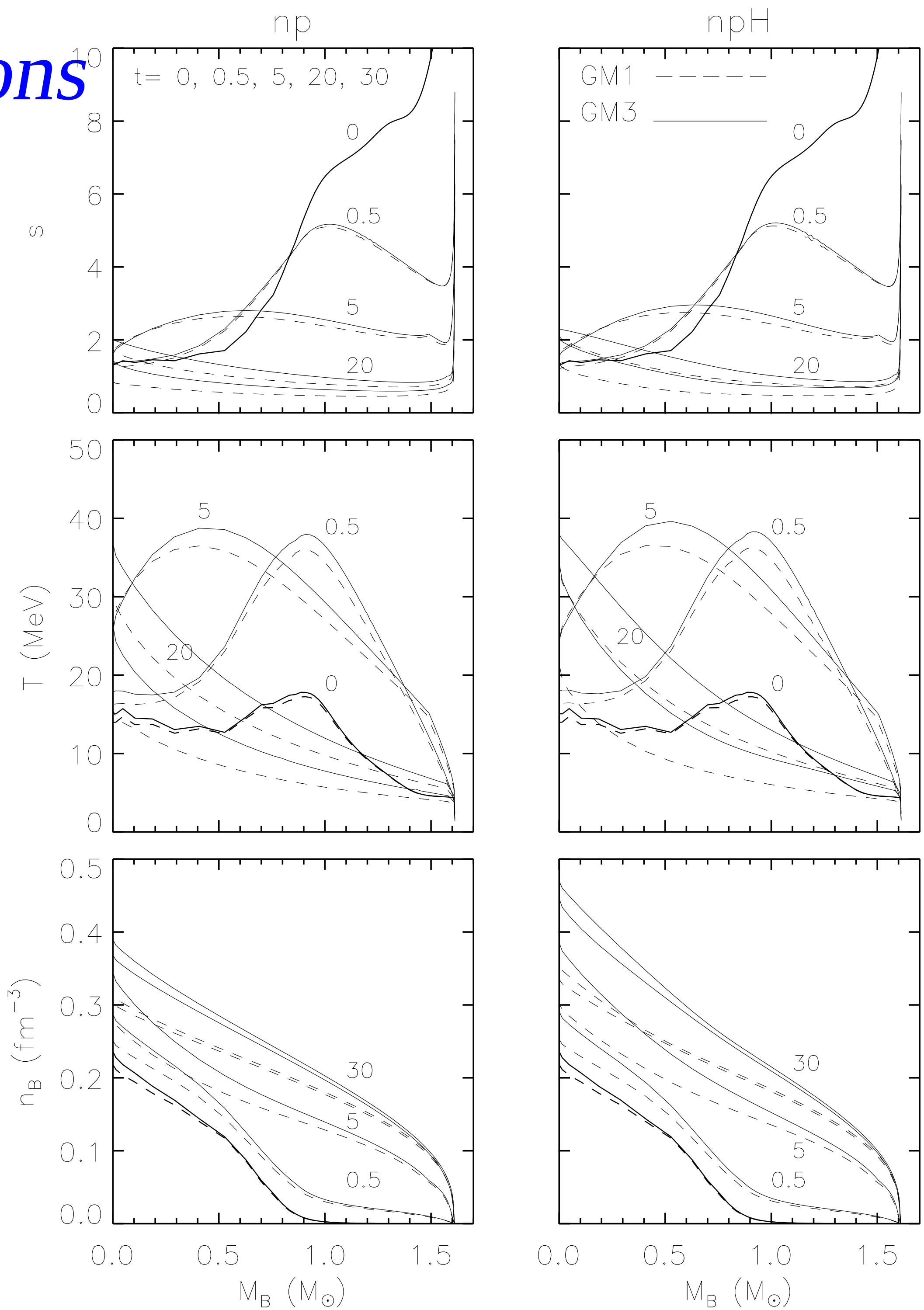
λ_ν and λ_E^i's are mean free paths for number and energy transport, respectively. $n_\nu(E_\nu)$ and $\epsilon_i(E_\nu)$ are the number and energy density of species $i = e, \mu$ at neutrino energy E_ν.
There are two main sources of opacity:

1. ν-nucleon absorption. Affects only $e-$types.
2. Neutrino-electron scattering. Inelastic scattering affects all types of neutrinos.

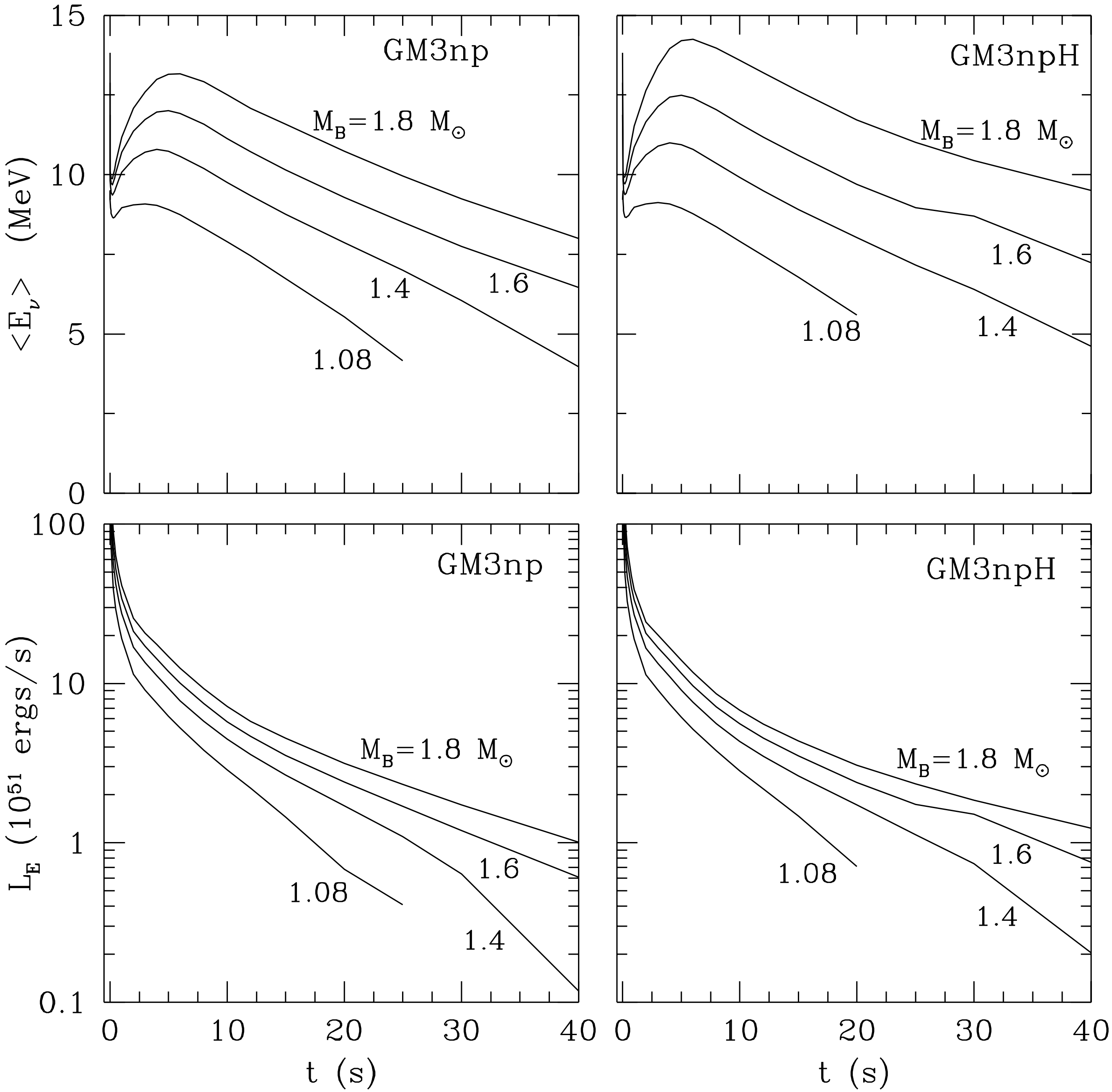
Mean free paths for these processes are approximately:

1. $\lambda_\nu \simeq \lambda_{\bar\nu} \simeq 5$ cm, $\lambda_\nu \propto E_\nu^{-2}$;
2. $\lambda_E^i \simeq 100$ cm, $\lambda_E^i \propto T^{-1}E_\nu^{-2}$.

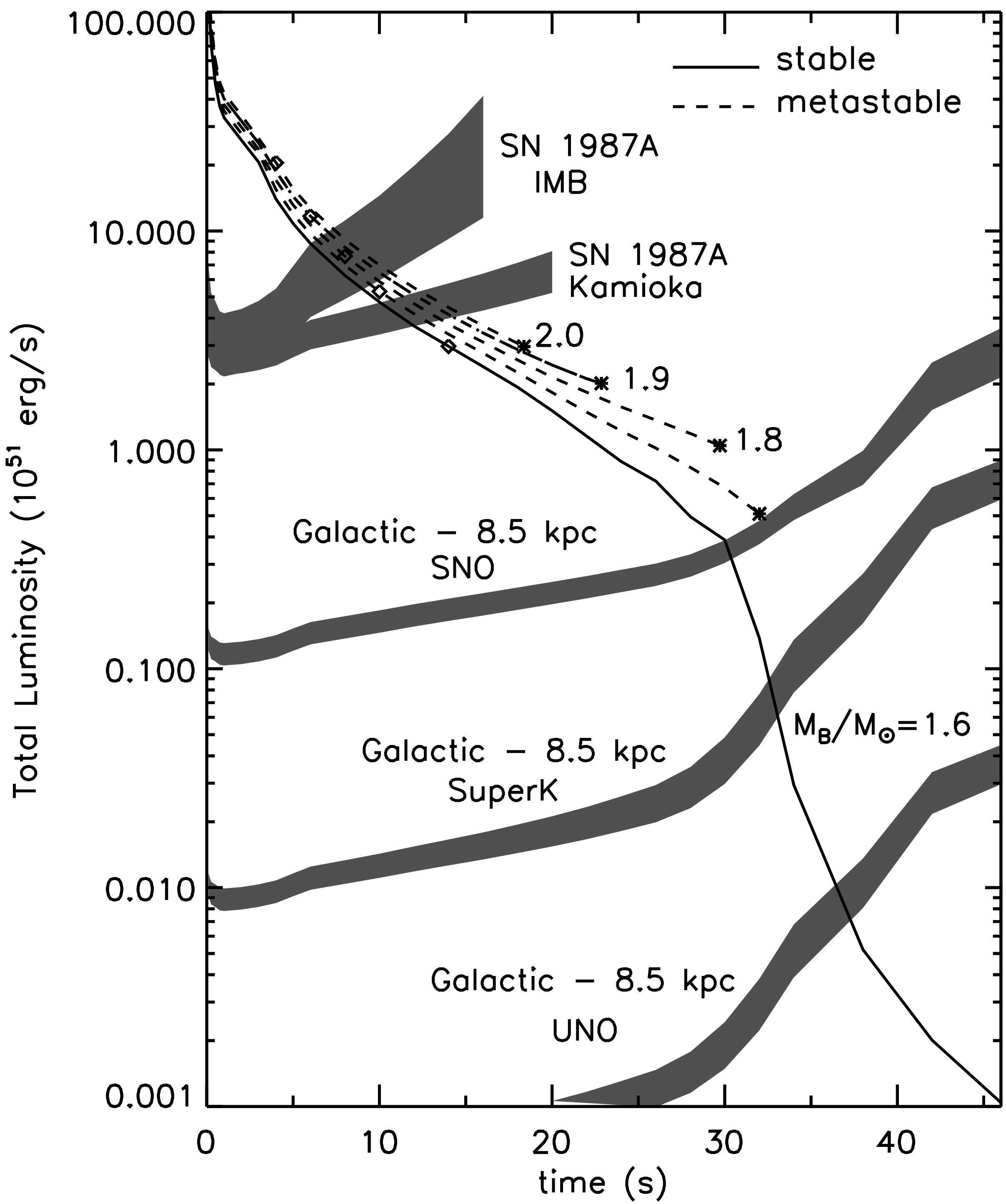
Model Simulations



Model Simulations

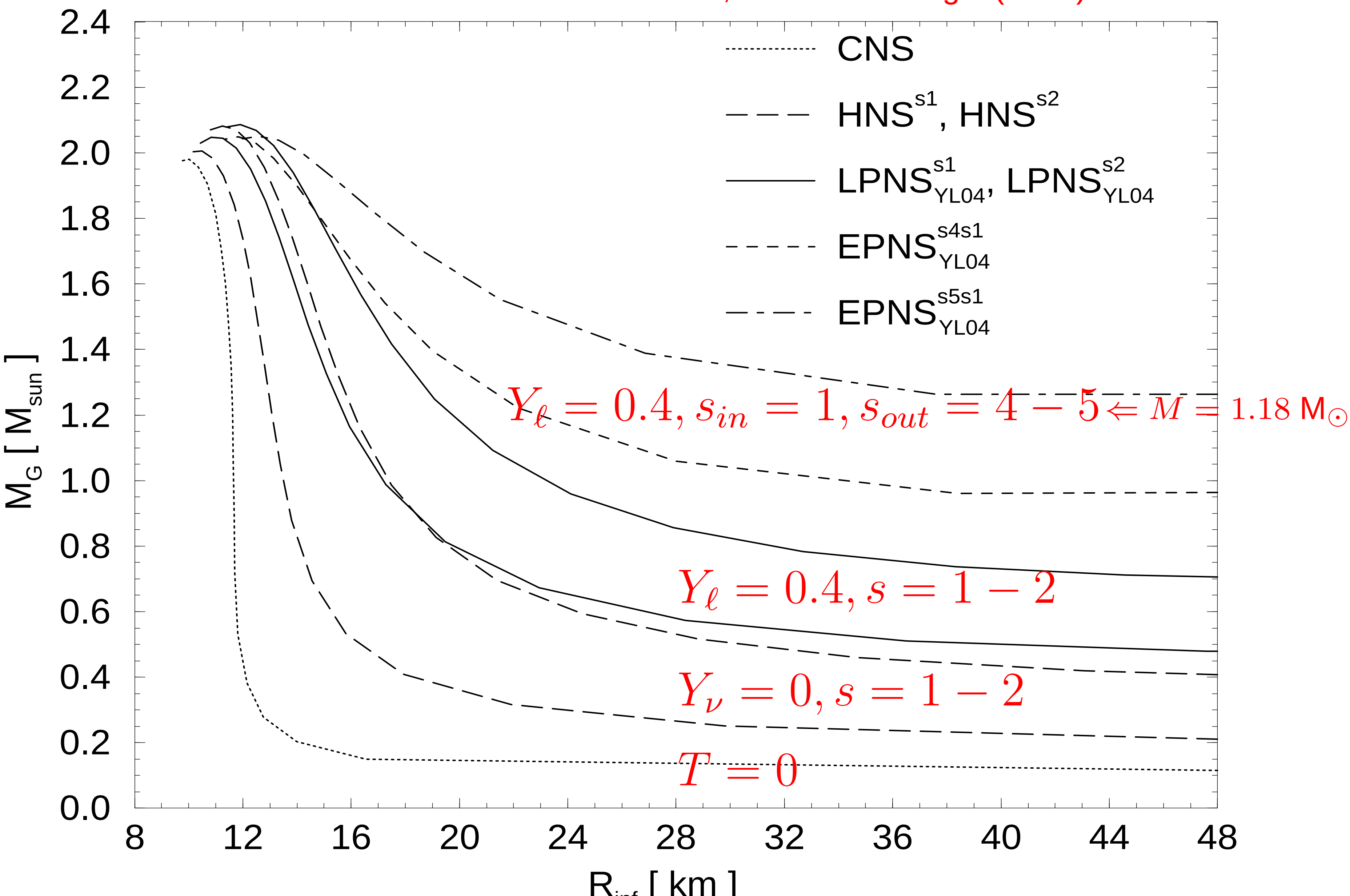


Model Signal



Effective Minimum Masses

Strobel, Schaab & Weigel (1999)

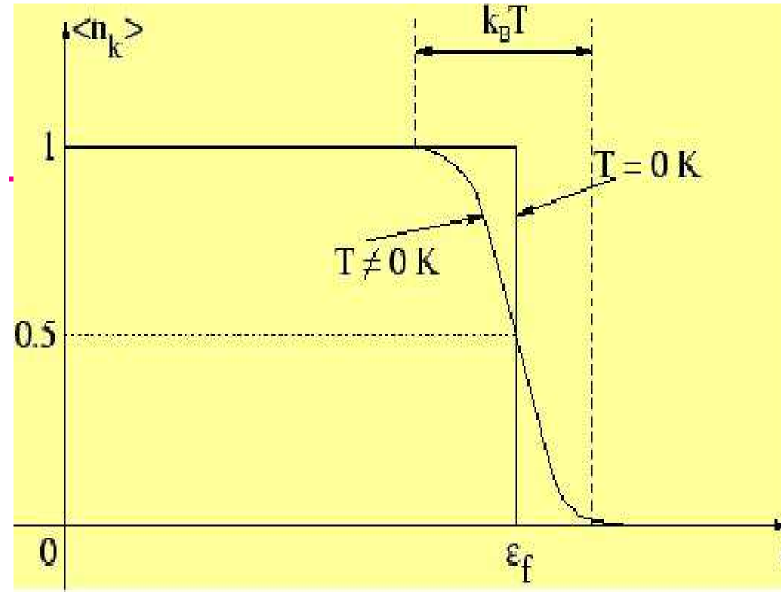


Neutron Star Cooling

Gamow & Schönberg proposed the direct Urca process: nucleons at the top of the Fermi sea beta decay. c

$$n \rightarrow p + e^- + \nu_e \,, \qquad p \rightarrow n + e^+ + \bar{\nu}_e$$

Energy conservation guaranteed by beta equilibrium
$\mu_n - \mu_p = \mu_e$
Momentum conservation requires $|k_{Fn}| \leq |k_{Fp}| + |k_{Fe}|$.
Charge neutrality requires $k_{Fp} = k_{Fe}$,
therefore $|k_{Fp}| \geq 2|k_{Fn}|$.
Degeneracy implies $n_i \propto k_{Fi}^3$, thus $x \geq x_{DU} = 1/9$.
With muons $(n > 2n_s), x_{DU} = \frac{2}{2+(1+2^{1/3})^3} \simeq 0.148$
If $x < x_{DU}$, bystander nucleons needed:
modified Urca process is then dominant.



$$(n,p) + n \rightarrow (n,p) + p + e^- + \nu_e, \qquad (n,p) + p \rightarrow (n,p) + n + e^+ + \bar{\nu}_e$$

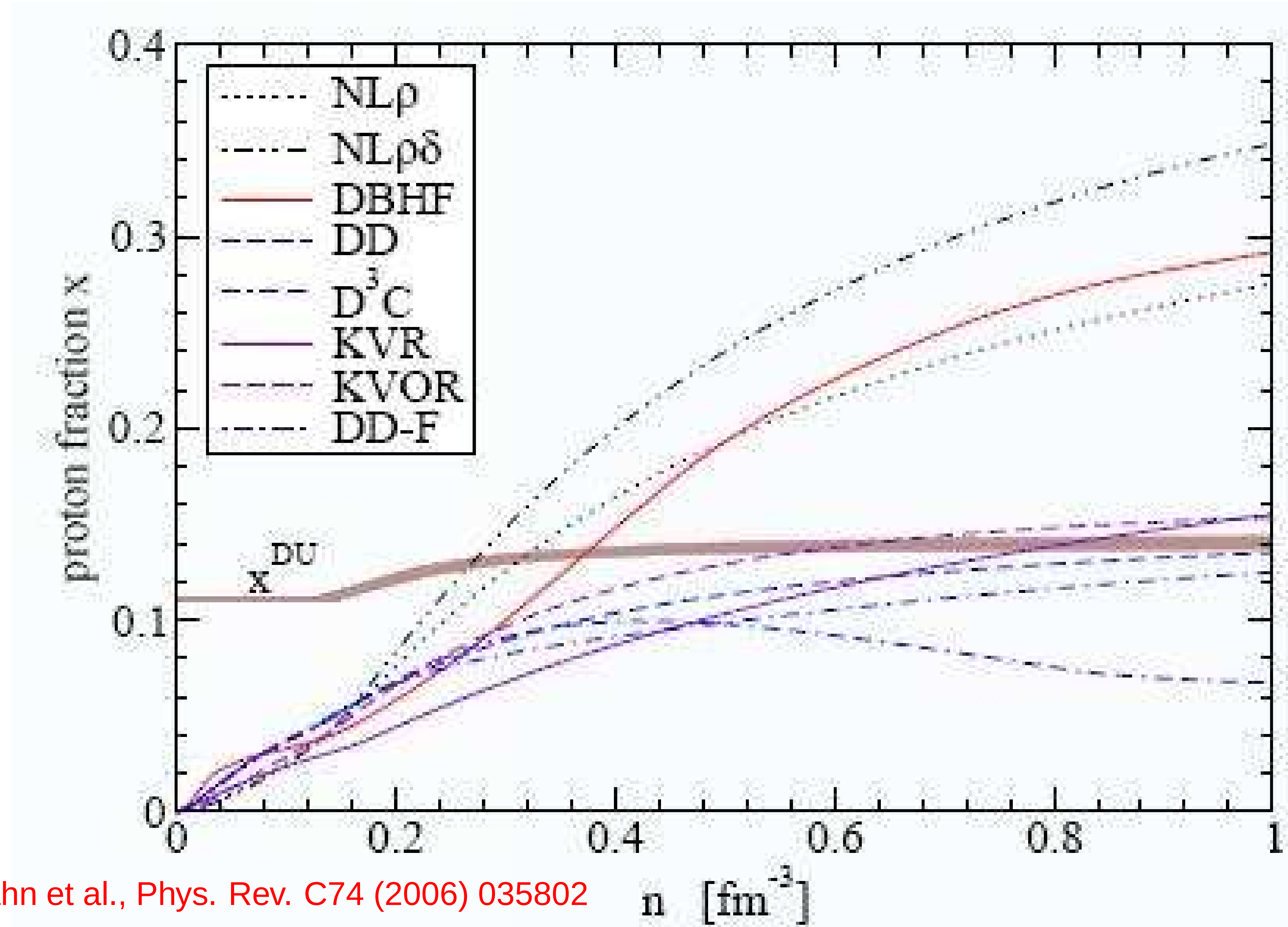
Neutrino emissivities:

$$\dot{\epsilon}_{MURCA} \simeq \left(\frac{T}{\mu_n}\right)^2 \dot{\epsilon}_{DURCA} \,.$$

Beta equilibrium composition:

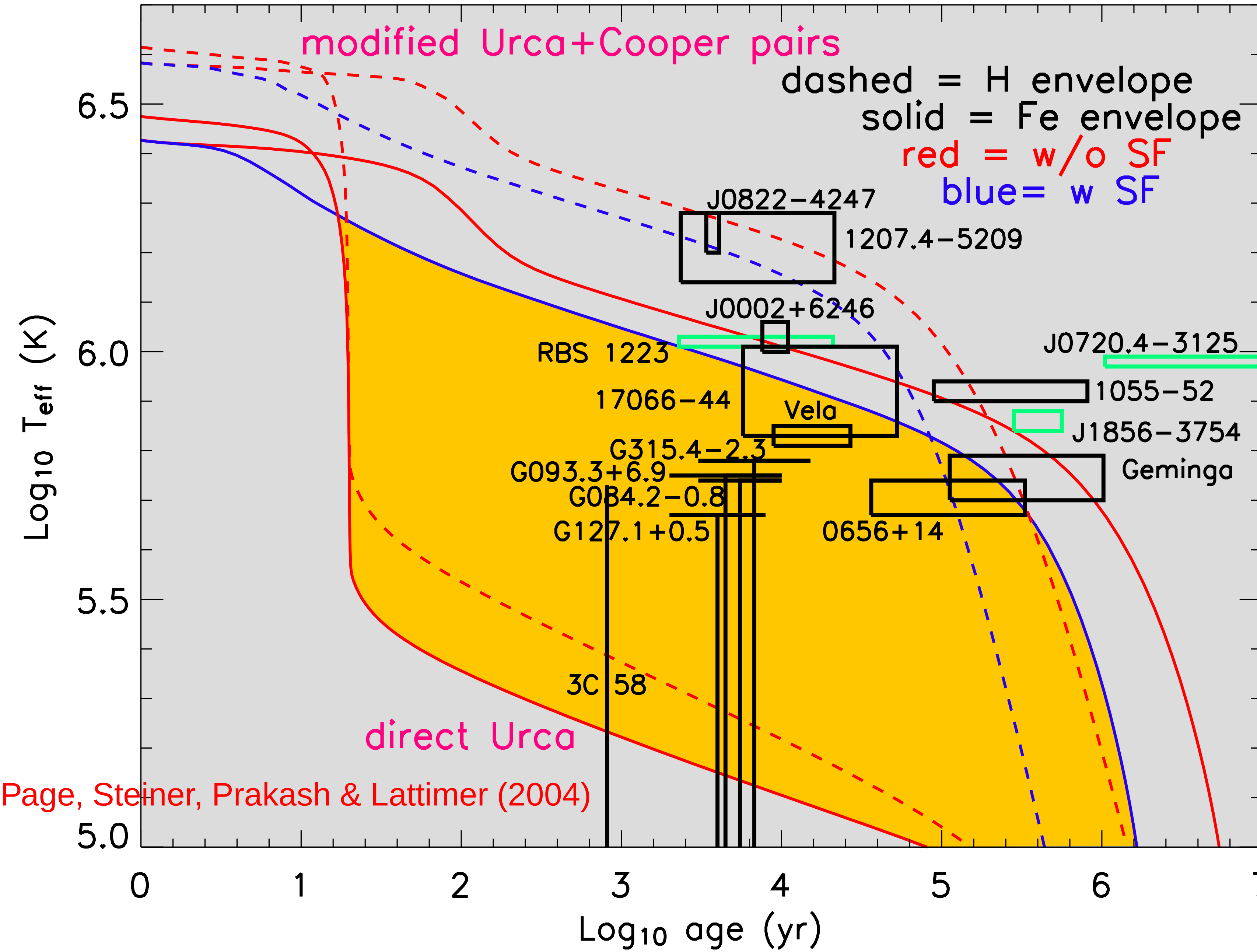
$$x_\beta \simeq (3\pi^2 n)^{-1} \left(\frac{4E_{sym}}{\hbar c}\right)^3 \simeq 0.04 \left(\frac{n}{n_s}\right)^{0.5-2} \,.$$

Direct Urca Threshold



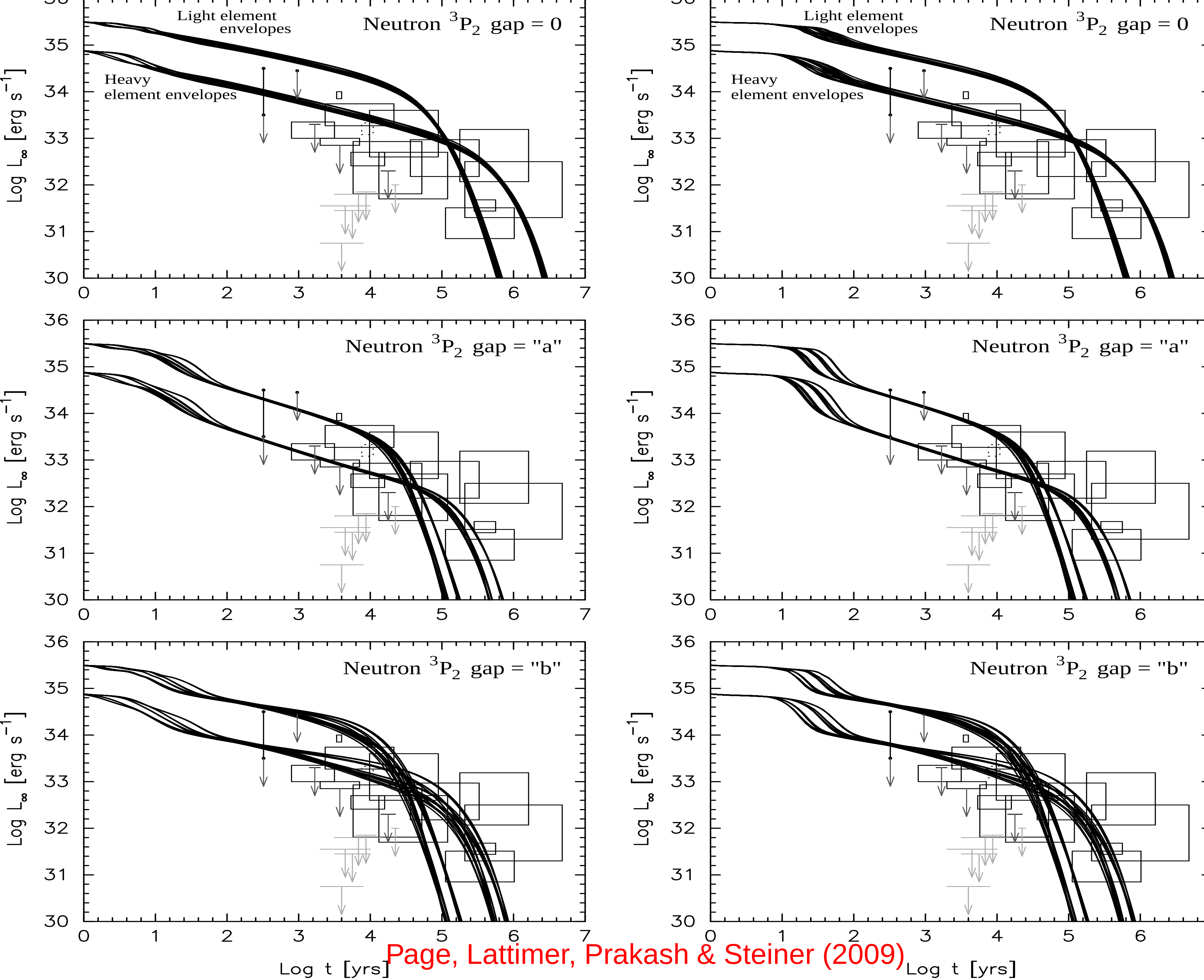
Klähn et al., Phys. Rev. C74 (2006) 035802

Neutron Star Cooling



Page, Steiner, Prakash & Lattimer (2004)

Minimal Cooling Paradigm



Page, Lattimer, Prakash & Steiner (2009)

Minimal Cooling Paradigm

- If some observations are inconsistent with the MCP, then according to Sherlock Holmes, rapid cooling must occur for these exceptions.
- All sources are consistent with the MCP only IF
  - tight conditions are placed on the magnitude and density dependence of the neutron ^{3}P$_2$ gap, AND
  - some neutron stars have heavy Z envelopes and others have light Z envelopes, AND
  - ALL core-collapse supernova remnants with no observable thermal emission contain black holes.
- Highly suggestive that rapid cooling occurs in some neutron stars (of higher masses?)
- A possible constraint on $E_{sym}(n)$, *i.e.*, it's not supersoft?

Possible Kinds of Observations

- Maximum and Minimum Mass (binary pulsars)
- Minimum Rotational Period*
- Radiation Radii or Redshifts from X-ray Thermal Emission*
- Crustal Cooling Timescale from X-ray Transients*
- X-ray Bursts from Accreting Neutron Stars*
- Seismology from Giant Flares in SGR's*
- Neutron Star Thermal Evolution (URCA or not)*
- Moments of Inertia from Spin-Orbit Coupling*
- Neutrinos from Proto-Neutron Stars (Binding Energies, Neutrino Opacities, Radii)*
- Pulse Shape Modulations*
- Gravitational Radiation from Neutron Star Mergers* (Masses, Radii from tidal Love numbers)

* Significant dependence on symmetry energy

Potentially Observable Quantities

- Apparent angular diameter from flux and temperature measurements
  $\beta \equiv GM/Rc^2$

$$\frac{R_\infty}{D} = \frac{R}{D}\frac{1}{\sqrt{1-2\beta}} = \sqrt{\frac{F_\infty}{\sigma}}\frac{1}{f_\infty^2 T_\infty^2}$$

- Redshift

$$z = (1-2\beta)^{-1/2} - 1$$

- Eddington flux

$$F_{EDD} = \frac{GMc}{\kappa c^2 D^2}(1-2\beta)^{1/2}$$

- Crust thickness

$$\frac{m_b c^2}{2}\ln\mathcal{H} \equiv h_t = \int_0^{p_t}\frac{dp}{n} = \mu_{n,t} - \mu_{n,t}(p=0)$$

$$\frac{\Delta}{R} \equiv \frac{R-R_t}{R} \simeq \frac{(\mathcal{H}-1)(1-2\beta)}{\mathcal{H}+2\beta-1} \simeq (\mathcal{H}-1)\left(\frac{1}{2\beta}-1\right).$$

- Moment of Inertia

$$I \simeq (0.237\pm0.008)MR^2(1+2.84\beta+18.9\beta^4)\ \mathrm{M}_\odot\ \mathrm{km}^2$$

- Crustal Moment of Inertia

$$\frac{\Delta I}{I} \simeq \frac{8\pi}{3}\frac{R^6 p_t}{IMc^2}$$

- Binding Energy

$$\mathrm{B.E.} \simeq (0.60\pm0.05)\frac{\beta}{1-\beta/2}$$

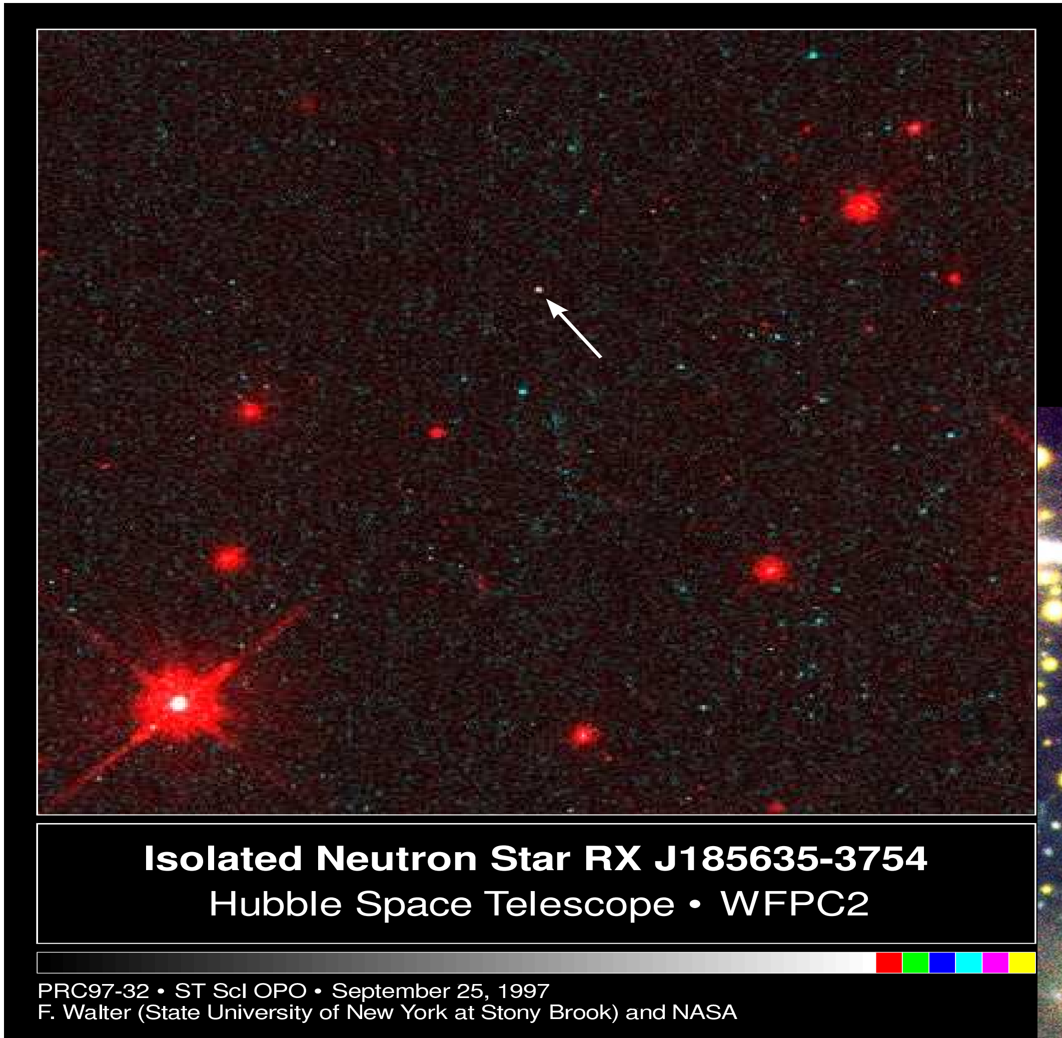
Radiation Radius

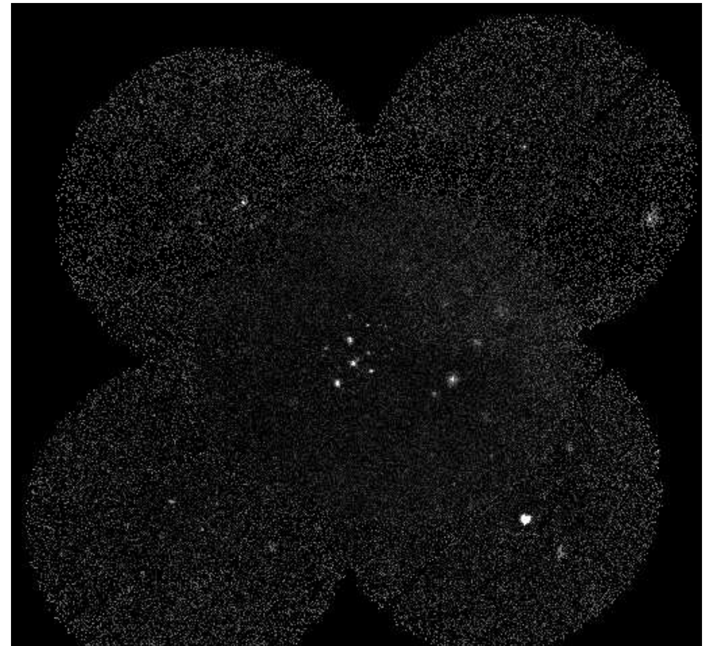
- Combination of flux and temperature measurements yields apparent angular diameter (pseudo-BB):

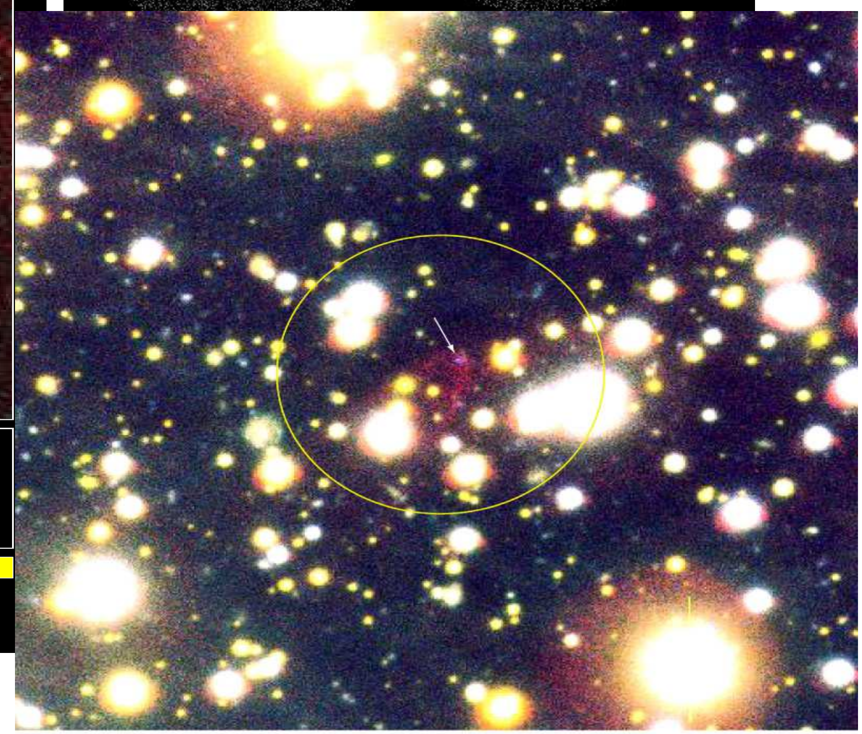
$$\frac{R_\infty}{d} = \frac{R}{d}\frac{1}{\sqrt{1-2GM/Rc^2}}$$

- Observational uncertainties include distance, interstellar H absorption (hard UV and X-rays), atmospheric composition
- Best chances for accurate radii are from
  - Nearby isolated neutron stars (parallax measurable)
  - Quiescent X-ray binaries in globular clusters (reliable distances, low B H-atmospheres)
  - X-ray pulsars in systems of known distance
    - CXOU J010043.1-721134 in the SMC: $R_\infty \geq 10.8$ km (Esposito & Mereghetti 2008)

RX J1856-3754



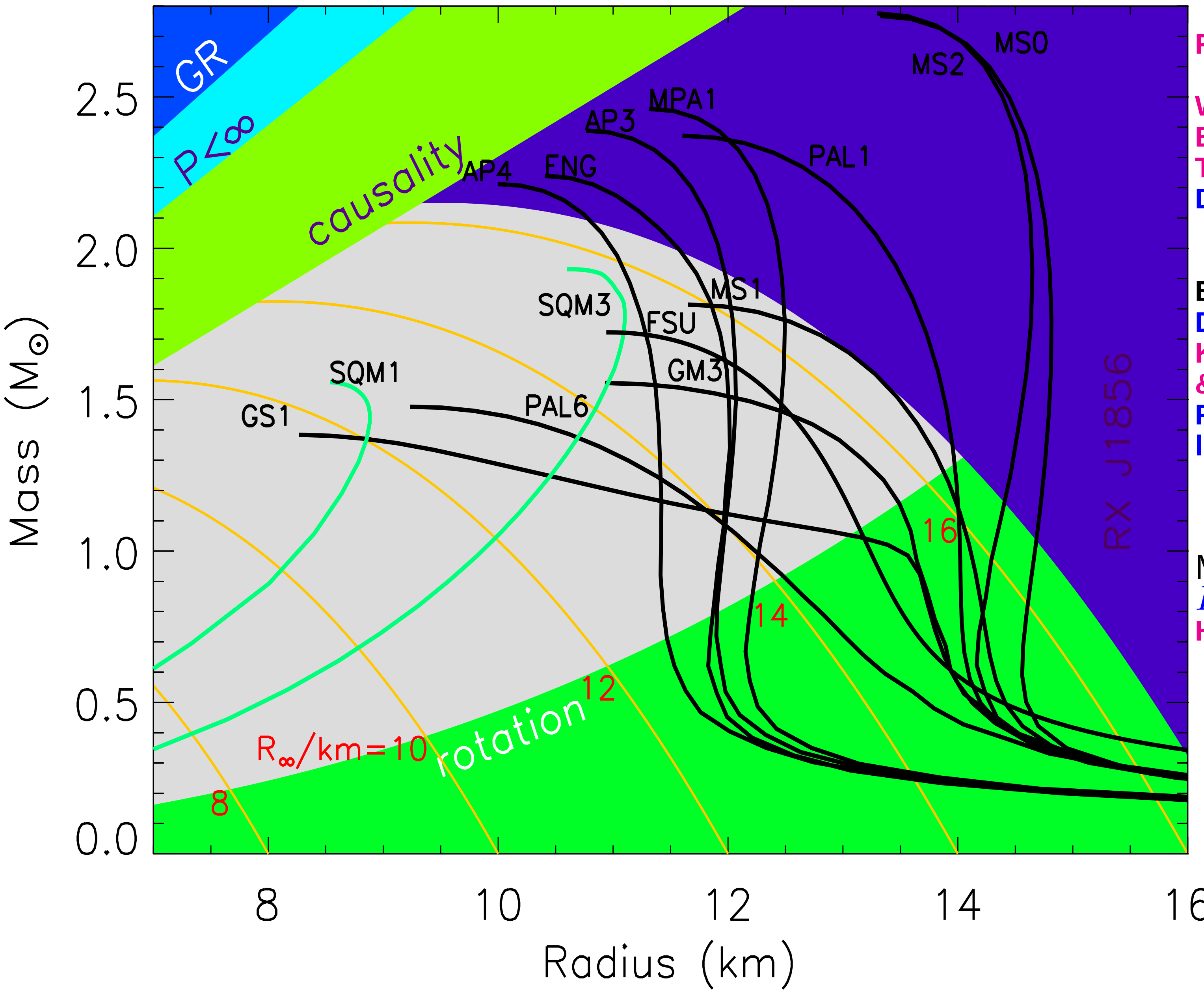




A Bowshock Nebula Near the Neutron Star RX J1856.5-3754 (Detail)
(VLT KUEYEN + FORS2)

ESO PR Photo 23b/00 (11 September 2000) ©European Southern Observatory

Radiation Radius: Nearby Neutron Star



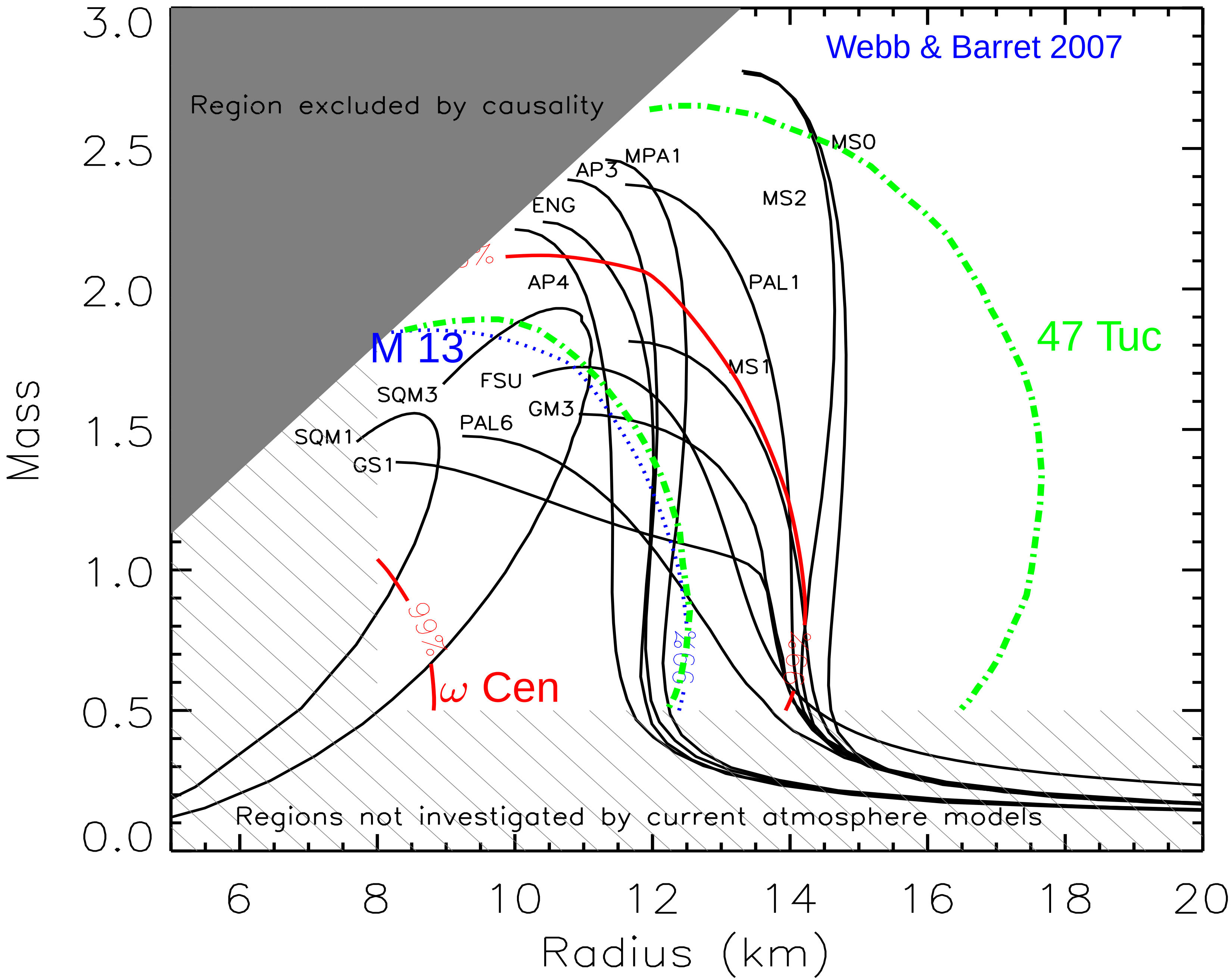
RX J1856-3754:

Walter & Lattimer 2002
Braje & Romani 2002
Truemper 2005
D=120 pc

BUT
D=140 pc
Kaplan, van Kerkwijk
& Anderson 2002
Raises R_∞
limit to 19.5 km

Magnetic H atmospher
$R_\infty \approx 17$ **km**
Ho et al. 2007

Radiation Radius: Globular Cluster Sources



The Neutron Star Crust

Hydrostatic equilibrium in the crust:

$$\frac{dp(r)}{m_b n} = \frac{d\mu}{m_b} = -\frac{GM}{r^2 - 2GMr/c^2} dr.$$

$$\frac{\mu_t - \mu_0}{m_b c^2} = \frac{1}{2} \ln \left[\frac{r_t(R - 2GM/c^2)}{R(r_t - 2GM/c^2)} \right].$$

Defining $\ln \mathcal{H} = 2(\mu_t - \mu_0)/m_b c^2$,

$$\frac{\Delta}{R} \equiv \frac{R - R_t}{R} = \frac{(\mathcal{H} - 1)(1 - 2\beta)}{\mathcal{H} + 2\beta - 1} \simeq (\mathcal{H} - 1)(1 - 2\beta) \frac{R^2}{2M}.$$

Crustal moment of inertia

$$\Delta I = \frac{8\pi}{3c^4} \int_{R-\Delta}^{R} r^4 (\epsilon + p) e^{-\lambda} j \omega dr \simeq \frac{8\pi\omega(R)}{3Mc^2} \int_{p(R-\Delta)}^{0} r^6 dp$$

Approximately, $\int_{p(R-\Delta)}^{0} r^6 dp \simeq R^6 p_t e^{-4.8\Delta/R}$. $p_t < 0.65$ MeV fm^{-3}.

$$\frac{\Delta I}{I} \simeq \frac{8\pi R^4 p_t}{3M^2 c^2} \left(\frac{MR^2}{I} - 2\beta \right) e^{-4.8\Delta/R}$$

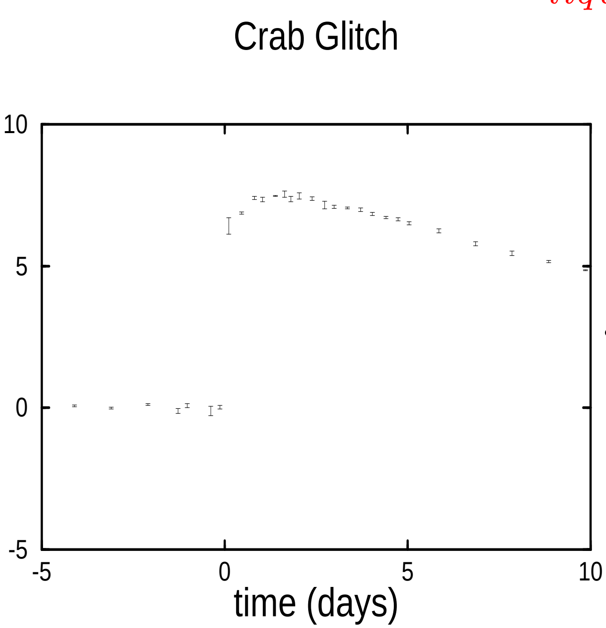
Pulsar Glitches

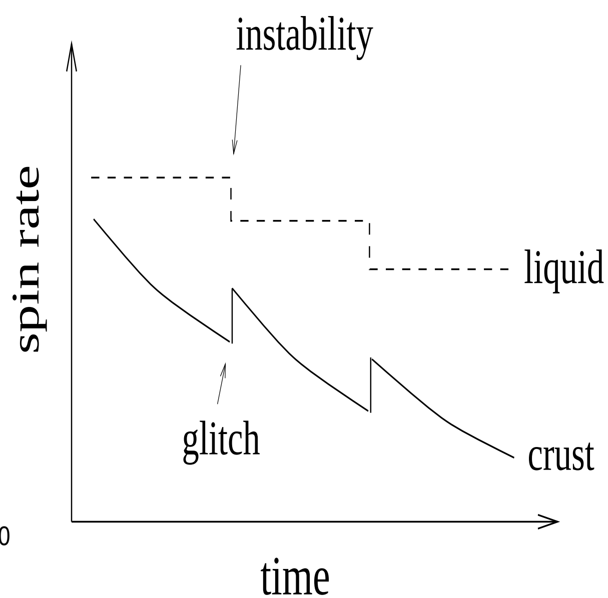
Pulsars occasionally undergo glitches, when the spin rate "hiccups".
Each glitch changes the angular momentum of the star by $\Delta J = I_{liquid}\Delta\Omega$.
The glitches are stochastic, but total angular momentum transfer is regular.

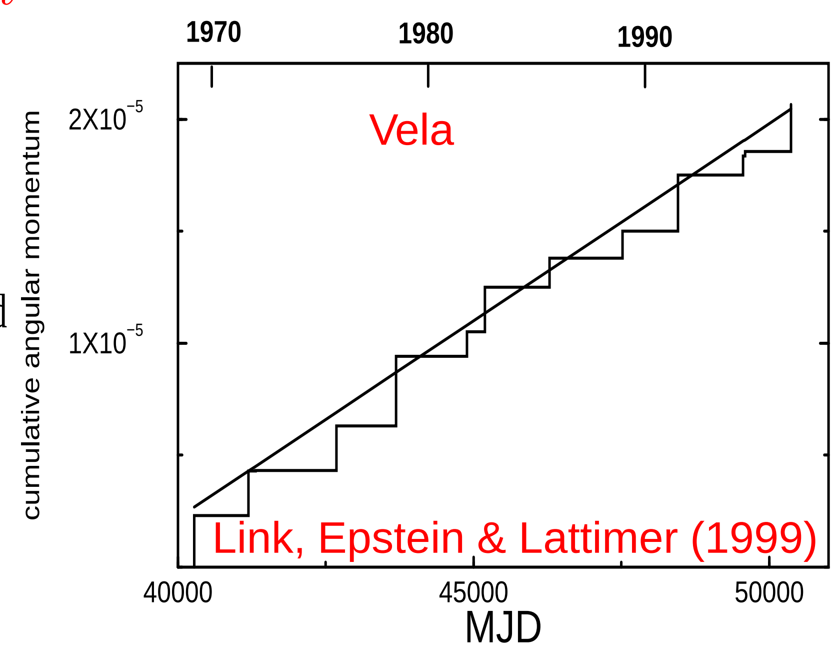
$$J(t) = I_{liquid}\bar{\Omega}\sum\frac{\Delta\Omega}{\Omega}, \qquad \dot{J} = I_{liquid}\bar{\Omega}\frac{d(\Delta\Omega/\Omega)}{dt}.$$

A leading model is that as the crust slows due to pulsar's dipole radiation, the interior acquires an excess differential rotation. The acquired excess is limited: $\dot{J} \leq \dot{\Omega} I_{crust}$.

$$\frac{I_{crust}}{I_{liquid}} = \frac{I_{crust}}{I - I_{crust}} \geq \frac{\bar{\Omega}}{\dot{\Omega}}\frac{d\sum(\Delta\Omega/\Omega)}{dt} \simeq 0.014.$$







Moment of Inertia

- Spin-orbit coupling of same magnitude as post-post-Newtonian effects (Barker & O'Connell 1975, Damour & Schaeffer 1988)
- Precession alters inclination angle and periastron advance
- More EOS sensitive than R: $I \propto MR^2$
- Requires extremely relativistic system to extract
- Double pulsar PSR J0737-3037 is a marginal candidate
- Even more relativistic systems should be found, based on dimness and nearness of PSR J0737-3037

Moments of Inertia and Precession

- Spin-orbit coupling: $\dot{\vec{S}}_A = -\dot{\vec{L}} = \frac{G(4M_A+3M_B)}{2M_A a^3 c^2} \vec{L} \times \vec{S}_A$

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$$P_p = \frac{2(M_A+M_B)ac^2}{GM_B(4M_A+3M_B)}P(1-e^2) \simeq 74.9 \text{ years}$$

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$$\delta_i = \frac{|\vec{S_A}|}{|\vec{L}|} \sin\theta \simeq \frac{I_A(M_A + M_B)}{a^2 M_A M_B} \frac{P}{P_A} \sin\theta \simeq (3.6 - 7.2) \sin\theta \times 10^{-5}$$

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$$\frac{2\pi I_A}{P_A}\left(\frac{2+3M_B/M_A}{3M_A^2+3M_B^2+2M_AM_B}\right)\sqrt{\frac{M_A+M_B}{Ga}}\cos\theta \simeq (2.2-4.3)\times 10^{-4}\cos\theta$$

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- Moment of Inertia – Mass – Radius

$$I \simeq (0.237\pm0.008)MR^2\left[1+4.2\frac{M\text{ km}}{R\text{ M}_\odot}+90\left(\frac{M\text{ km}}{R\text{ M}_\odot}\right)^4\right]$$

Comparison of Binary Pulsars

References	PSR 0707-3039 a, b, c	PSR 1913+16 d	PSR 1534+12 e, f
a/c (s)	2.93	6.38	7.62
P (h)	2.45	7.75	10.1
e	0.088	0.617	0.274
M_A ($M_\odot$)	1.337 ± 0.005	1.4414 ± 0.0002	1.333 ± 0.001
M_B ($M_\odot$)	1.250 ± 0.005	1.3867 ± 0.0002	1.345 ± 0.001
T_{GW} (M yr)	85	245	2250
i	$87.9 \pm 0.6^\circ$	47.2°	77.2°
P_A (ms)	22.7	59	37.9
θ_A	$13^\circ \pm 10^\circ$	21.1°	$25.0^\circ \pm 3.8^\circ$
ϕ_A	$246^\circ \pm 5^\circ$	9.7°	$290^\circ \pm 20^\circ$
P_{pA} (yr)	74.9	297.2	700
$\delta t_a/I_{A,80}$ (μs)	0.7 ± 0.6	11.2	7.9 ± 1.1
$A_{pA}/(A_{1PN}I_{A,80})$	$3.4^{+0.2}_{-0.1} \times 10^{-5}$	1.0×10^{-5}	$1.15^{+0.04}_{-0.03} \times 10^{-5}$
A_{2PN}/A_{1PN}	5.2×10^{-5}	4.7×10^{-5}	2.3×10^{-5}

a: Lyne et al. (2004); b: Solution 1, Jenet & Ransom (2004); c: Coles et al. (2004)
d: Weisberg & Taylor (2002, 2004); e: Stairs et al. (2002, 2004); f: Bogdanov et al. (2002)

EOS Constraint