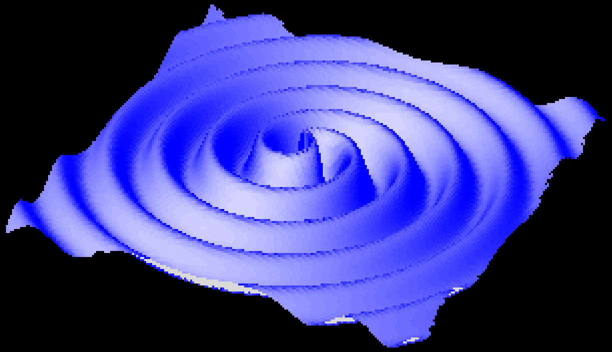
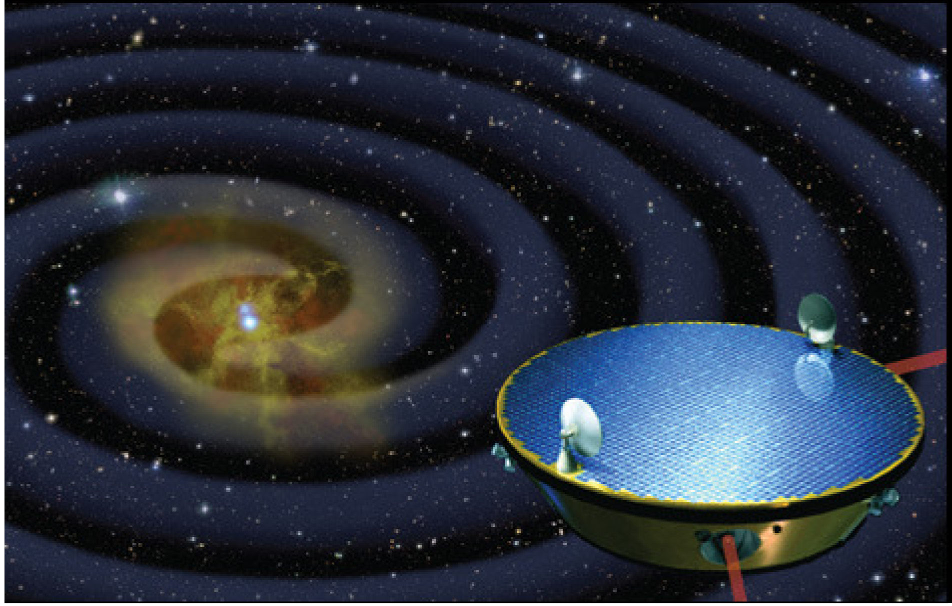
Sources of Gravitational Waves

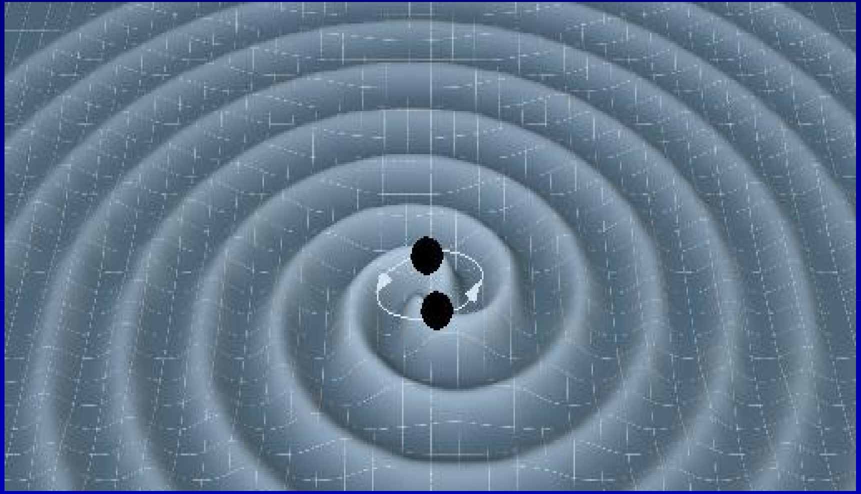


T. Tauris

The last 400 years of astronomy were about
"seeing" a silent movie.
LIGO/VIRGO and LISA hope to deliver the "sound track"



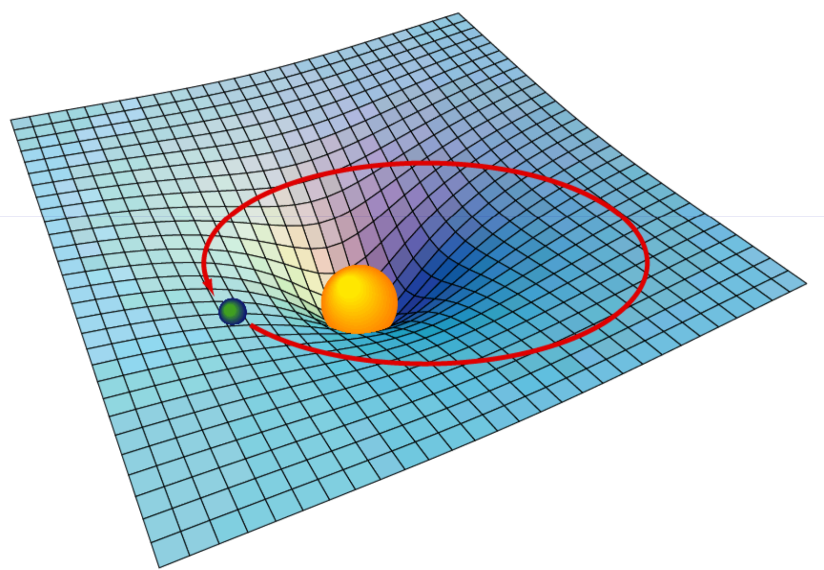
A very brief introduction….



- Concept and emission of GWR
- Detection
  - LIGO/VIRGO
  - LISA
- Astrophysical sources
  - Burst emission (extra galactic)
  - Continuous emission (Galactic)
- Merger time-scale

General relativity in a nutshell

- Imagine space as a stretched rubber sheet
- A mass on the surface will cause a deformation
- Another mass dropped onto the sheet will roll towards that mass



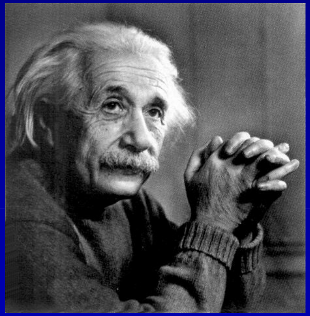
”The curvature of space determines how matter should move
- and the matter determines the curvature of space”

(Einstein’s field equations)

Einstein’s field equations

How does the distribution of mass-energy determine the geometry ?

$G_{\mu\nu} = K\, T_{\mu\nu}$ strain (curvature) = const. x stress (mass, energy)



space-time curvature tensor

stress-energy tensor (source term)

scalar constant ”effectiveness of distorting space-time”

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu} + \Lambda g_{\mu\upsilon}$$

(cosmological constant)

Metric

Riemannian coordinates
(curved space):

$$ds^2 = g_{\mu\upsilon}\, dx^{\mu} dx^{\upsilon}$$

$$g_{\mu\upsilon} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}$$

Minkowski flat space:

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2$$

(special relativity)

Weak field vacuum limit

Consider a small pertubation from a flat space-time:

Let the metric tensor $g_{\mu\upsilon} = \eta_{\mu\upsilon} + h_{\mu\upsilon}$ where $\eta_{\mu\upsilon} = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

and $|h_{\mu\upsilon}| << 1$ is a small pertubation

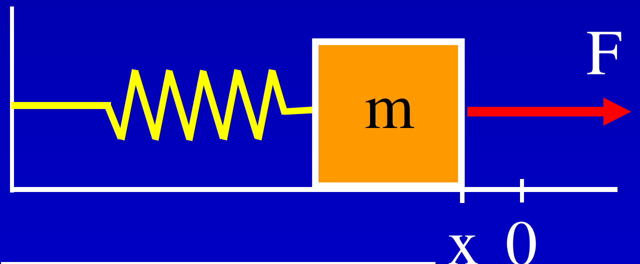
It can be shown that $\nabla^2 h_{\mu\upsilon} - \frac{\partial^2 h_{\mu\upsilon}}{\partial t^2} = 0$ is a solution to Einstein's field eq. (the equation for a plane wave)

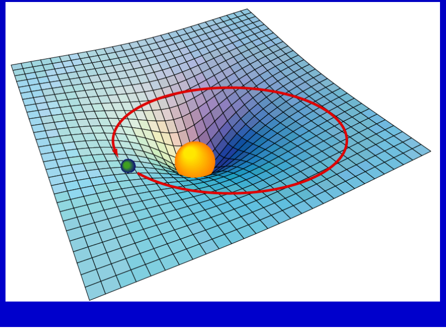
Analogue to Hooke's law:

$F = -kx$

"force" "displacement"

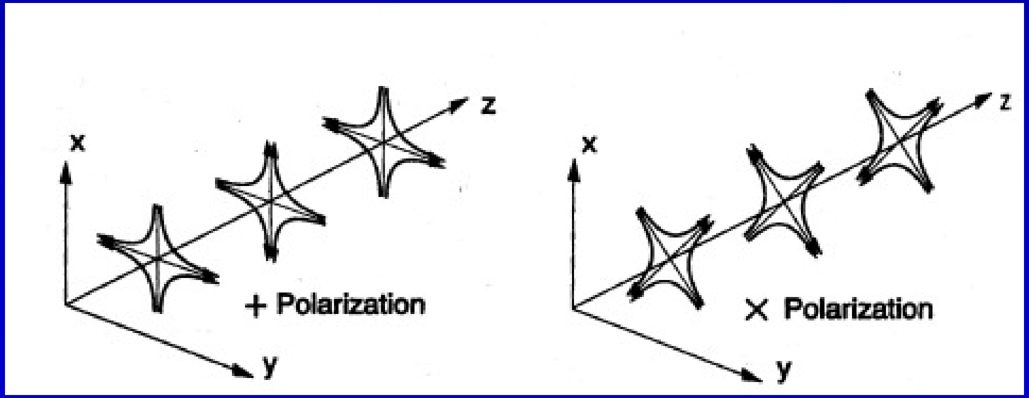
$$T_{\mu\nu} = \frac{c^4}{8\pi G} G_{\mu\upsilon}$$

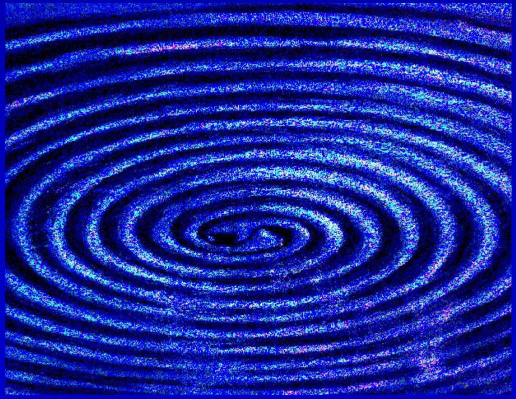




Nature of the gravitational waves

- The emitted waves carry information of the changes in the gravitational field of the source as a result of a change in the distribution of mass, energy and momentum
- Gravitational waves propagate with the speed of light (the graviton has zero rest mass)
- They give rise to fluctuations in the metric where they pass through
- The waves force field is transverse to its propagation direction and has quadrupolar symmetry (i.e. the graviton has S=2)





Gravitational wave emission

A time-varying quadrupole moment* gives rise to emission of gravitational waves with a strain amplitude:

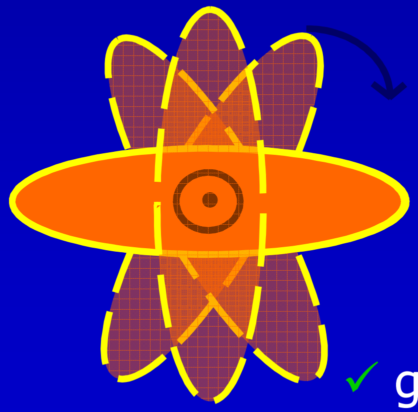
$$h_{\mu\upsilon} \approx \frac{2G}{c^4 d}\ddot{Q}_{\mu\upsilon}$$

quadrupole moment

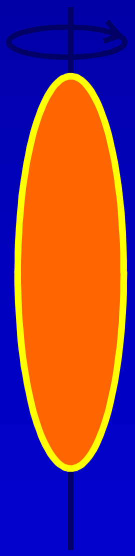
distance to source

(Newtonian/quadrupole approximation)

* an asymmetric distribution of mass with respect to the rotation axis:



✓ gravitational waves



☐ no gravitational waves

The physical meaning of "h"

Remember: $ds^2 = g_{\mu\upsilon}\, dx^\mu dx^\upsilon = (\eta_{\mu\upsilon} + h_{\mu\upsilon})\, dx^\mu dx^\upsilon$

Consider the following geometry:

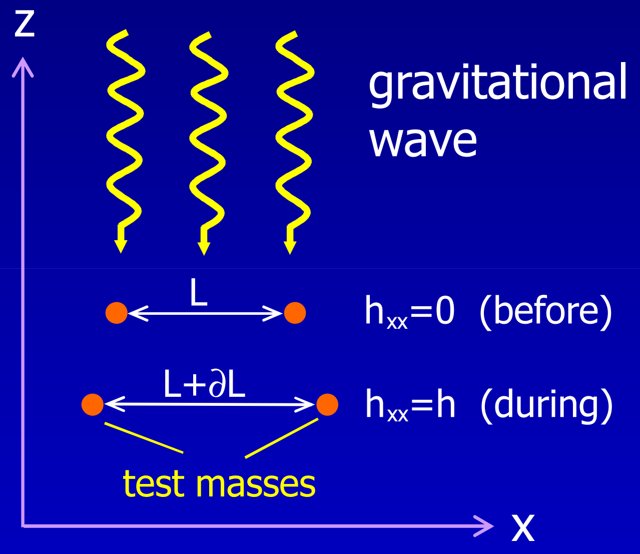
$ds = L + \delta L$ $\quad dx = L$

$dy = dz = dt = 0$

$h_{xx} = h$

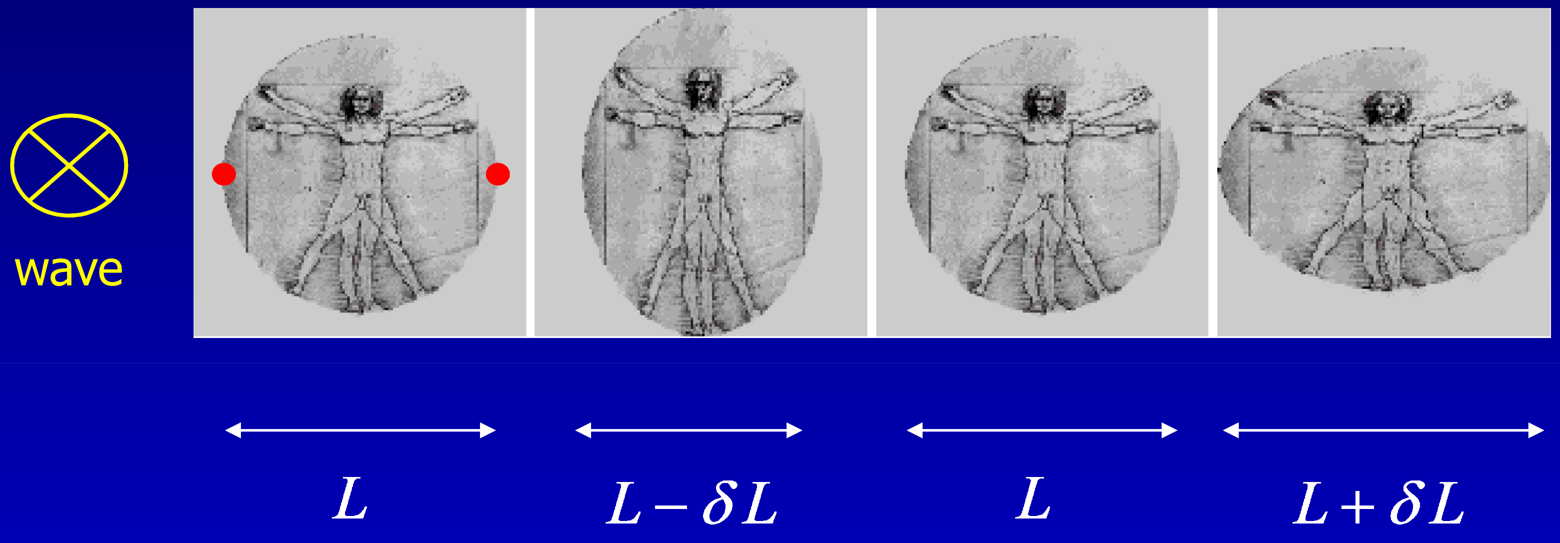
⇕

$$(L+\delta L)^2 = (1+h)\,L^2 \quad \Leftrightarrow \quad \frac{\delta L}{L} \approx \frac{h}{2}$$



the wave (strain) amplitude is twice the relative length change

The effect on Leonardo da Vinci



$\frac{\delta L}{L} = \frac{1}{2} h \approx 10^{-21}$ for many astrophysical sources

NS-NS collision at 200 Mpc

The value of h for astrophysical sources

$$h_{\mu\upsilon} = \frac{2G}{c^4 d}\ddot{Q}_{\mu\upsilon}$$

order-of-magnitude estimate

$$Q \sim MR^2 \quad \Rightarrow \quad \ddot{Q} \sim \frac{MR^2}{T^2} \sim Mv^2$$

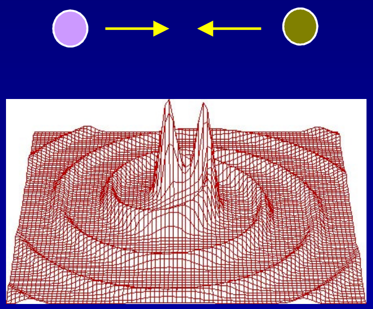
where (M,R,T,v) are characteristic values of the source

$$h \sim \frac{2G}{c^4 d}Mv^2 = \frac{2G}{c^2}\frac{M}{d}\left(\frac{v}{c}\right)^2 \sim 1.0\times10^{-19}\,\frac{M/M_\odot}{d/Mpc}\left(\frac{v}{c}\right)^2$$

h= {

10^{-17} at outskirts of our Milky Way (10 kpc)
10^{-20} at the Virgo cluster of galaxies (15 Mpc)
10^{-21} at 200 Mpc
10^{-22} at the Hubble distance (3 Gpc)

Gravitational wave luminosity



$$h_{\mu\upsilon} = \frac{2G}{c^4 d}\ddot{Q}_{\mu\upsilon}$$

wave amplitude

$$F = \frac{c^3}{32\pi G}\langle \dot{h}_{\mu\upsilon}\,\dot{h}_{\mu\upsilon}\rangle$$

energy flux

$$L = r^2 \int F\, d\Omega$$

luminosity

$$\Rightarrow \quad \boxed{L_{gwr} \equiv \frac{dE}{dt} = \frac{G}{5c^5}\langle \dddot{Q}_{\mu\upsilon}\,\dddot{Q}_{\mu\upsilon}\rangle}$$

Gravitational wave luminosity

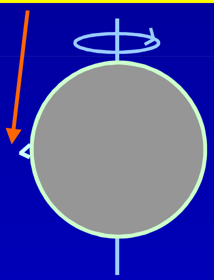
$$L_{gwr} \equiv \frac{dE}{dt} = \frac{G}{5c^5} \langle \dddot{Q}_{\mu\upsilon} \dddot{Q}_{\mu\upsilon} \rangle$$

order-of-magnitude estimate

$$Q \sim MR^2 \quad \Rightarrow \quad \dddot{Q} \sim \frac{MR^2}{T^3} \sim \frac{Mv^3}{R}$$

where (M,R,T,v) are characteristic values of the source

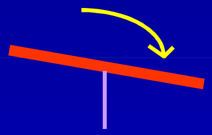
1 mm mountain on a neutron star:



$M=1.4\ M_\odot$
$R=10$ km, $\Delta r=1$ mm
$\Omega=2\pi\nu=1000$ rad/s

$L_{gwr}=10^{36}$ erg/s

steel rod:



M=140.000 kg
R=20 meters
v=300 m/s

$L_{gwr}=10^{-24}$ erg/s

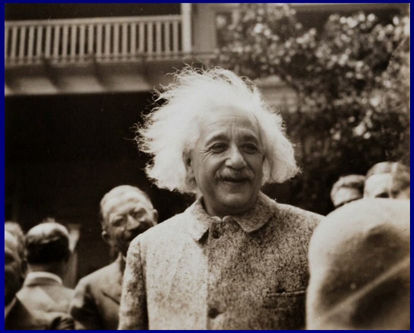
$$I \sim MR^2$$

$$L_{gwr} \equiv \frac{|dE|}{dt} = \frac{32G}{5c^5} I^2 \varepsilon^2 \Omega^6$$

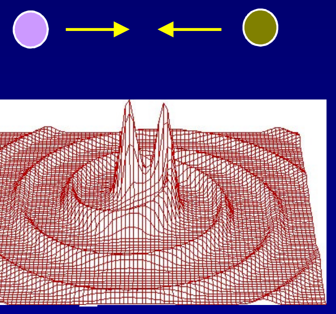
$$\varepsilon = \frac{a-b}{(a+b)/2} \approx \frac{\Delta r}{R}$$

a factor of ~ 10^{60} in difference!!!

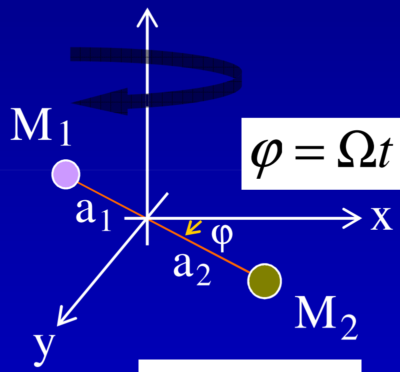
Merging neutron star / black hole binaries



$$L_{gwr} \cong \frac{G}{5c^5} \langle \dddot{Q}_{\mu\upsilon} \dddot{Q}_{\mu\upsilon} \rangle$$



$$Q = \frac{1}{2}\mu a^2 \begin{pmatrix} \cos(2\varphi) + const & \sin(2\varphi) + const \\ \sin(2\varphi) + const & -\cos(2\varphi) + const \end{pmatrix}$$



$$\varphi = \Omega t$$

$$M = M_1 + M_2$$

$$M_1 a_1 = M_2 a_2 = \mu a$$

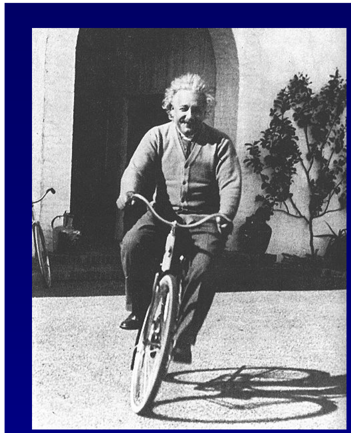
$$\mu = \frac{M_1 M_2}{M_1 + M_2}$$

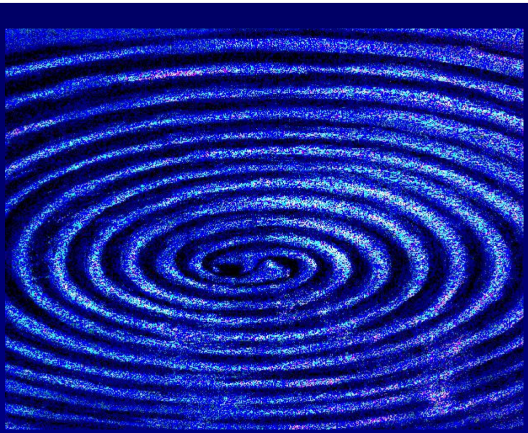
$$\boxed{L_{gwr}(n,e) = \frac{32}{5} \frac{G^4}{c^5} \frac{M^3 \mu^2}{a^5} g(n,e)}$$

Fourier decomposition factor
(harmonic number, eccentricity)

$$a = -\frac{GM\mu}{2E_{orb}} \quad \Rightarrow \quad \dot{a} = \frac{GM\mu}{2E_{orb}^2} \dot{E}_{orb} \quad \wedge \quad |\dot{E}_{orb}| = L_{gwr}$$

orbital decay!





$$\dot{a} = -\frac{GM\mu}{2E_{orb}^2} L_{gwr}$$

$$\frac{1}{a}\frac{da}{dt} = -\frac{1}{E}\frac{dE}{dt}\bigg|_{e=0} f(e)$$

$$L_{gwr} \cong \frac{32}{5}\frac{G^4}{c^5}\frac{M^3\mu^2}{a^5}\frac{\overbrace{1+(73/24)e^2+(37/96)e^4}}{\left(1-e^2\right)^{7/2}}$$

$$\dot{a} \cong \frac{64}{5}\frac{G^3}{c^5}\frac{M^2\mu}{a^3}\frac{1+(73/24)e^2+(37/96)e^4}{\left(1-e^2\right)^{7/2}}$$

$\Downarrow$

$$\boxed{\tau(a_0,e_0) \cong \frac{12}{19}\frac{C_0{}^4}{\beta}\int_0^{e_0}\frac{e^{29/19}[1+(121/304)e^2]^{1181/2299}}{\left(1-e^2\right)^{3/2}}\,de}$$

Merging timescale

$$\beta = \frac{64}{5}\frac{G^3}{c^5}M^2\mu$$

determine C_0 from initial condition: $a=a_0$, $e=e_0$

$$a(e) = \frac{C_0 e^{12/19}}{(1-e^2)}[1+(121/304)e^2]^{870/2299}$$

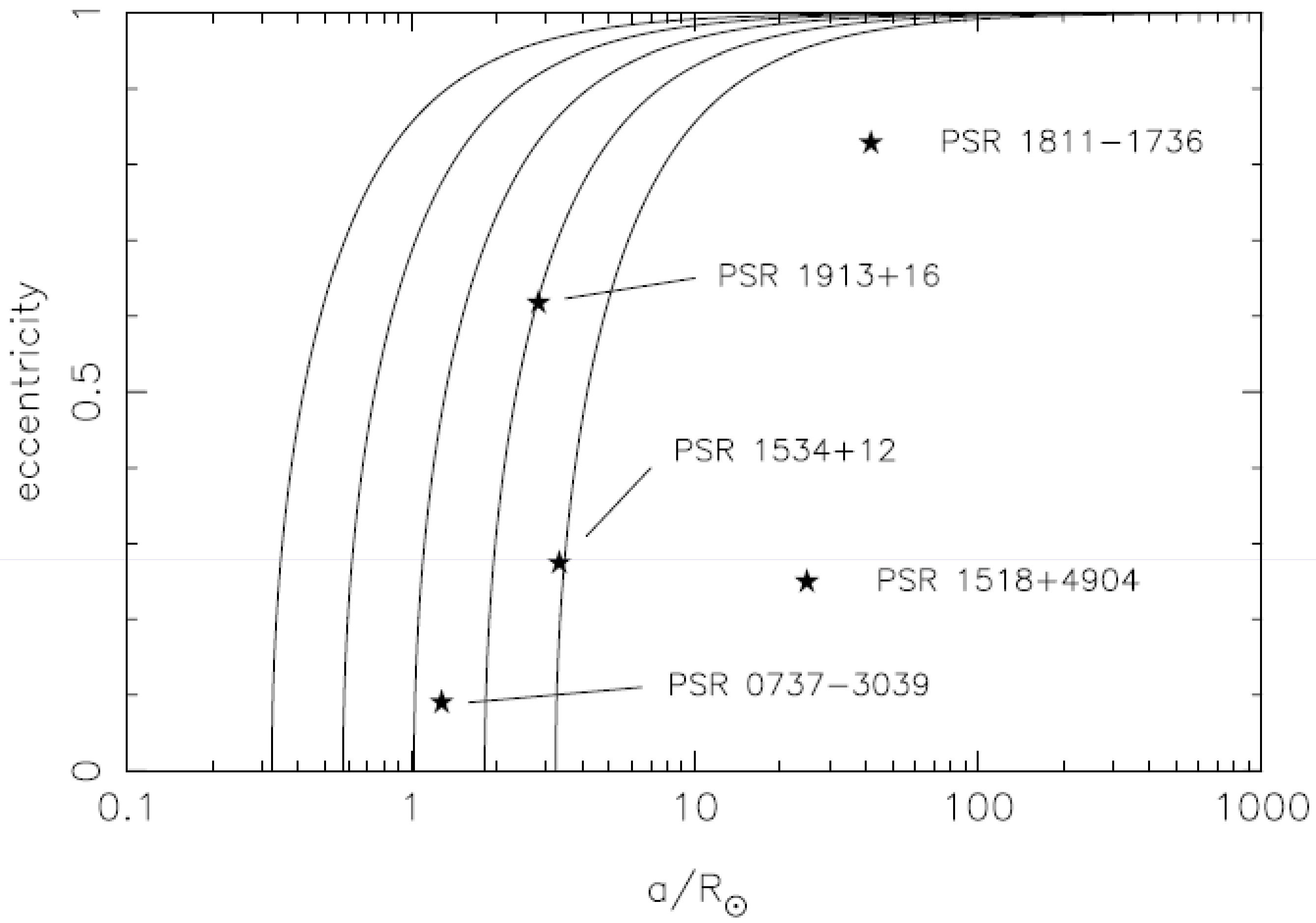
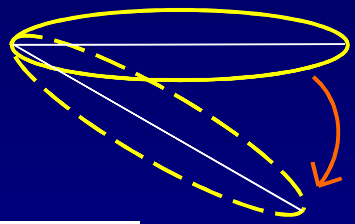
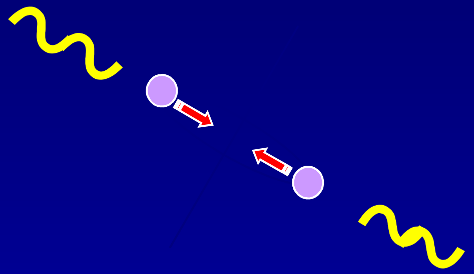


Fig. 16.16. Isochrones for the merging time of double neutron star binaries, as calculated by the authors. The curves correspond to values of (from left to right): 3×10^5 yr, 3 Myr, 30 Myr, 300 Myr and 3 Gyr, respectively. The five detected Galactic double neutron star systems are indicated with ⋆.

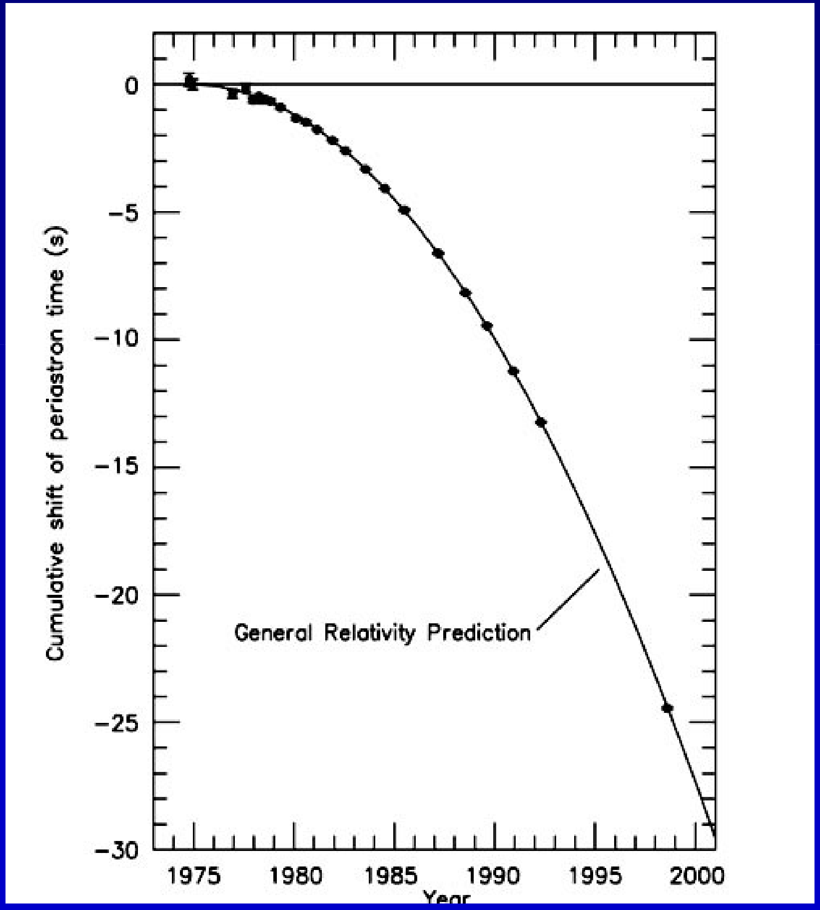
The Hulse-Taylor pulsar

(PSR B1913+16)



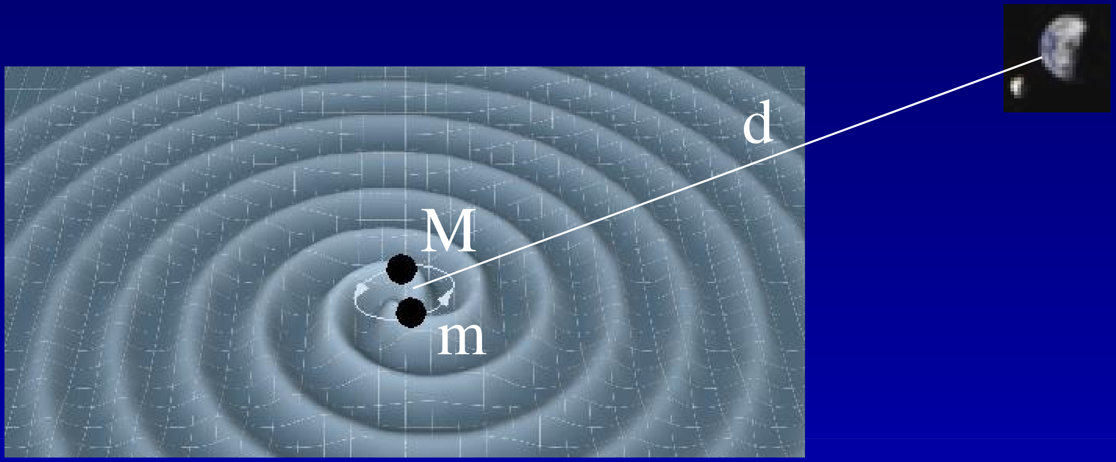


Gravitational waves do exist!



Gravitational wave detection (cont.)

$$\boxed{f_{\mathrm{gwr}} = 2\, f_{\mathrm{orb}} \quad \text{(ecc.=0)}}$$



strain amplitude:

$$h(n,e) = \left(\tfrac{1}{2}[h^2_{+,\mathrm{max}} + h^2_{\times,\mathrm{max}}]\right)^{1/2} = \left[\frac{16\pi G}{c^3\omega^2_{\mathrm{gwr}}}\,\frac{L_{\mathrm{gwr}}(n,e)}{4\pi d^2}\right]^{1/2}$$

$$= 1.0\times10^{-21}\,\frac{\sqrt{g(n,e)}}{n}\,\left(\frac{Mm\,(M+m)^{-1/3}}{\mathrm{M}_{\odot}^{5/3}}\right)\left(\frac{P_{\mathrm{orb}}}{1\,\mathrm{hr}}\right)^{-2/3}\left(\frac{d}{1\,\mathrm{kpc}}\right)^{-1}$$

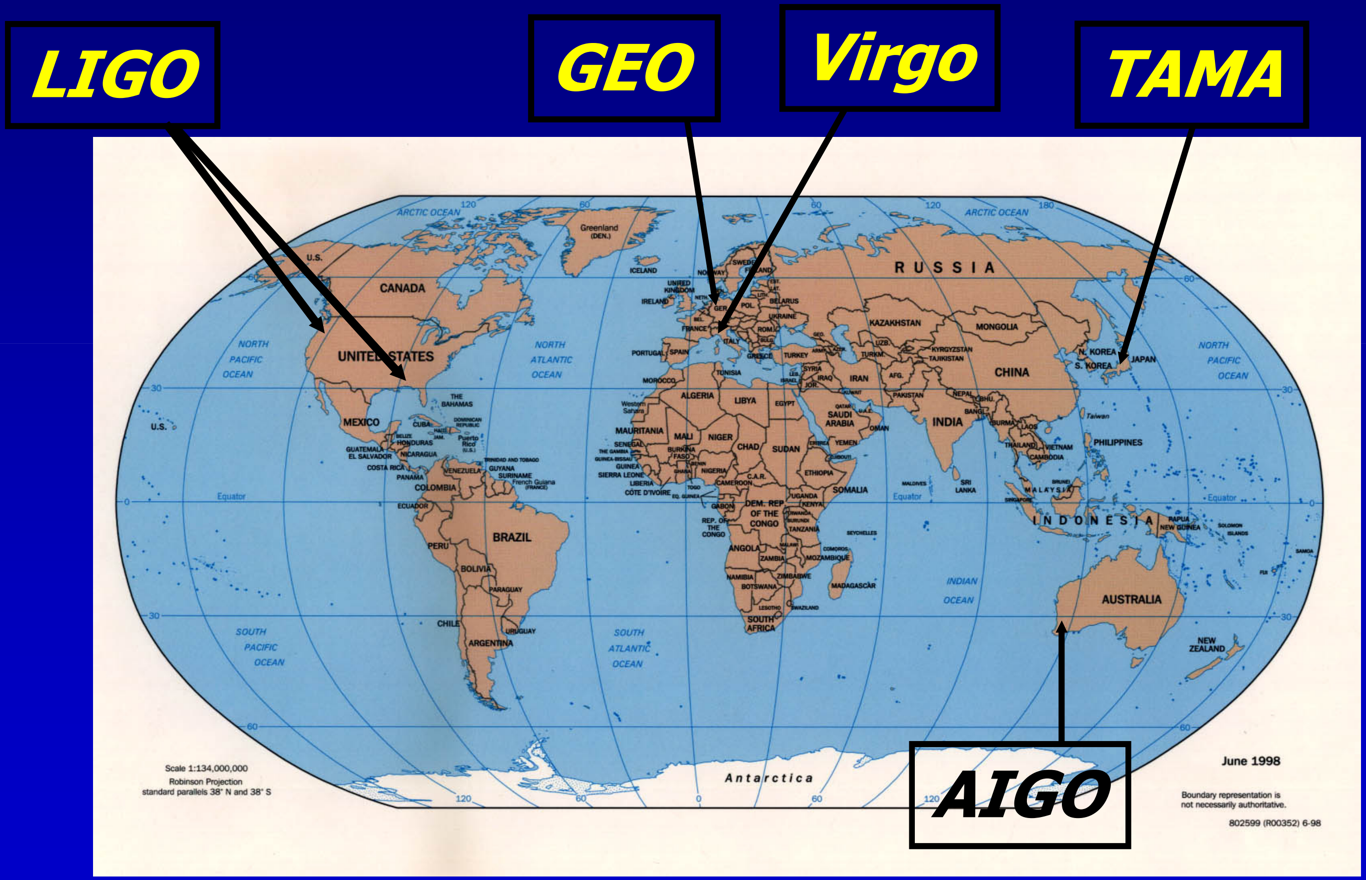
massive

tight

nearby

scale factor

Gravitational wave observatories



LIGO

GEO

Virgo

TAMA

AIGO

LIGO Laser Interferometer Gravitational wave Observatory

wave amplitude: h ~ 10^{-21}

accuracy to achieve: ΔL ~ 10^{-16} cm

iameter of the nucleus of an atom!)

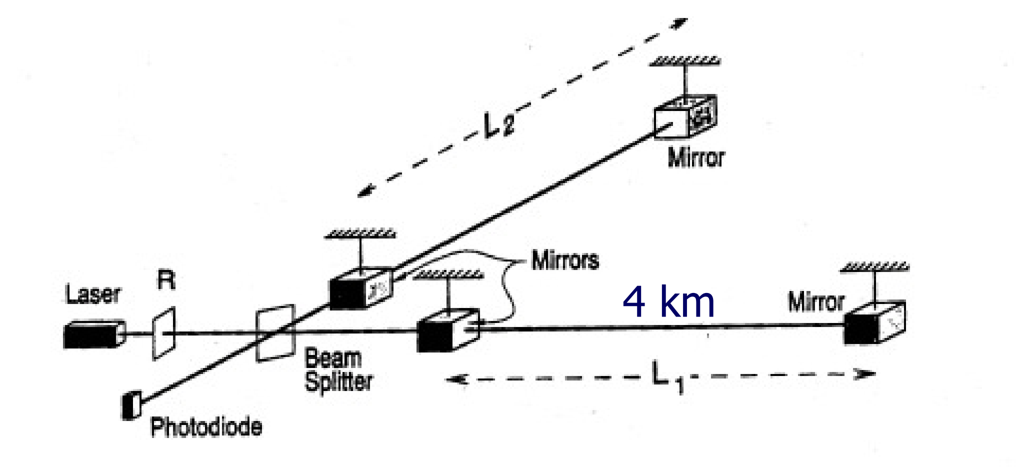
arm length: L=ΔL/h ~ 4 km

LIGO Observations began in 2004

VIRGO Observations began in 2007

Laser interferometer

- how to achieve $\Delta L \sim 10^{-16}$ cm ?



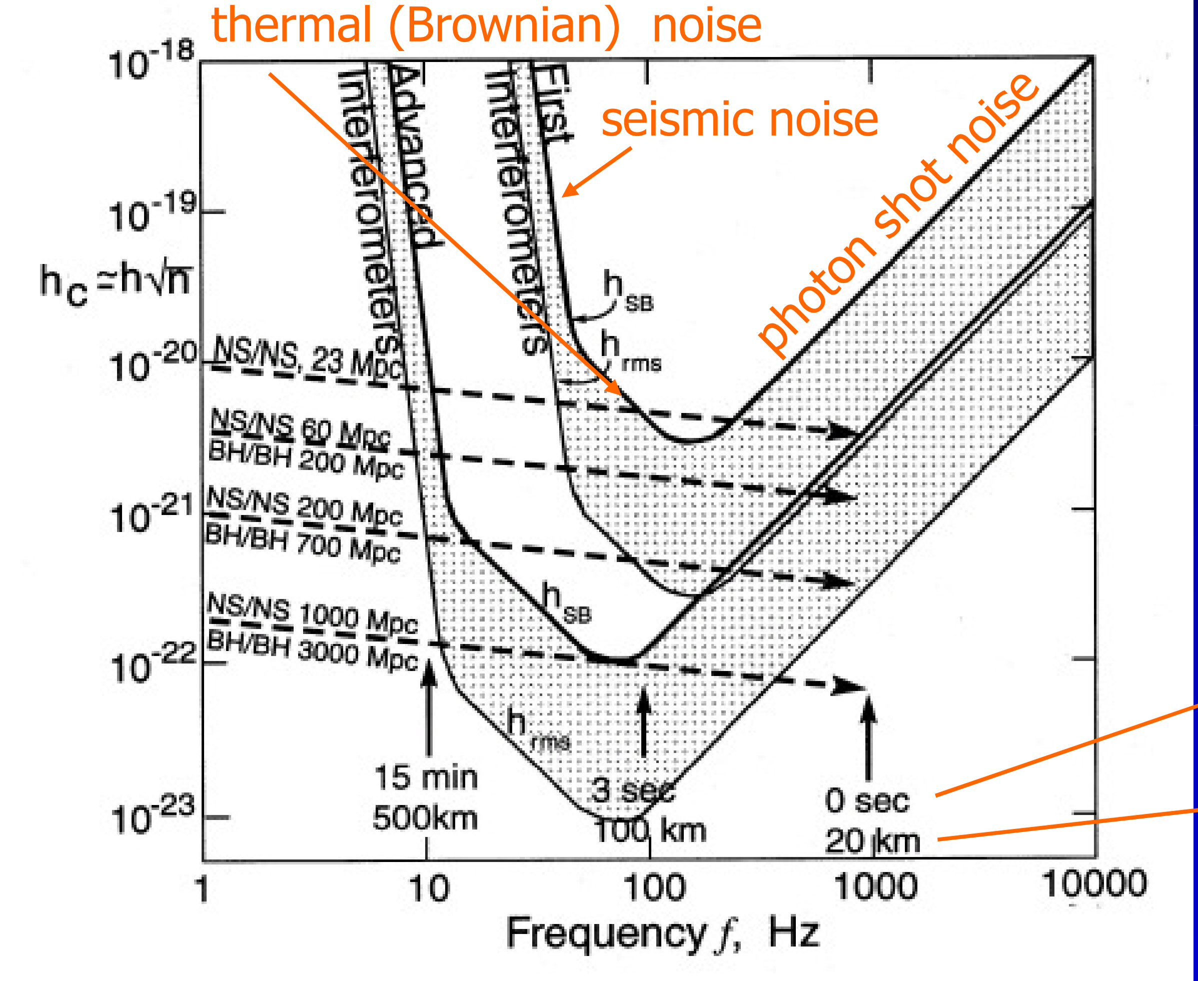
Sensitivity noise: photon, thermal and seismic

5 cm wide laser beam shining on 10^{17} atoms

Light is reflected 100 times |←→|

$$\Delta I_{pd} \propto \Delta\Phi \propto \Delta L \propto h(t)$$

Sensitivity of LIGO/VIRGO



<u>angular resolution</u>:
~ 1 sq. deg. on sky
(3 detectors LIGO/VIRGO)

merger time

separation

Expected Detected Merger Rates of Compact Binaries

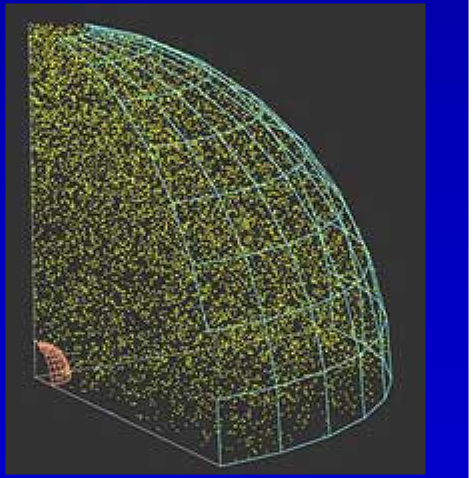
- Initial LIGO
  - NS-NS 1/3000 yrs – 1/3

I am pessimistic !!

until 2015 (advanced LIGO)

- Advanced LIGO
  - NS-NS 1/yr – 2/day
  - BH-BH 2/month – 10/day

Advanced LIGO will equal the 1-yr integrated observation time of initial LIGO in roughly 3 hours

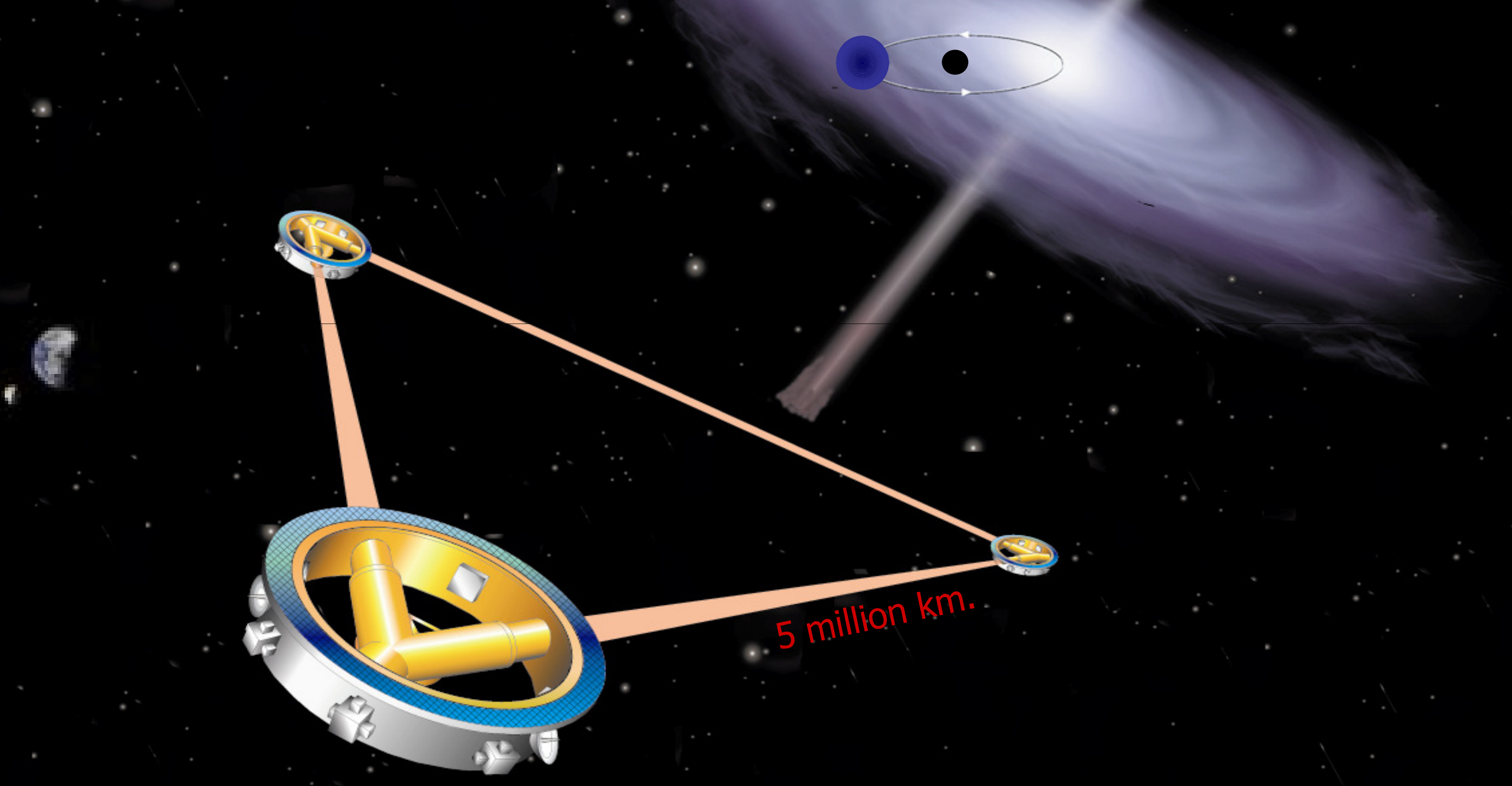


Laser Interferometer Space Antenna

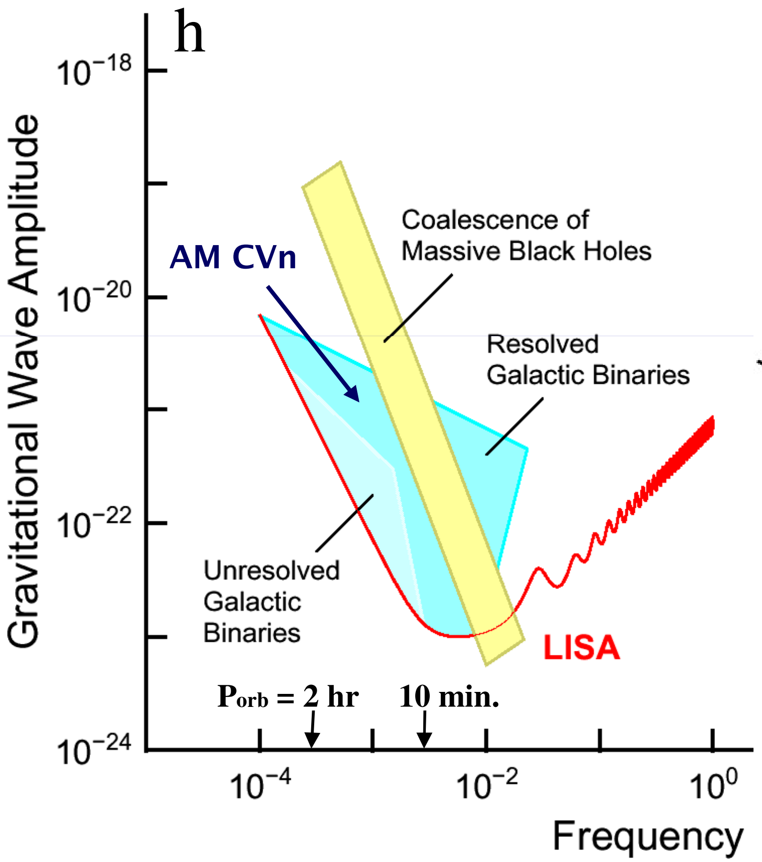
LISA

launch ~2018

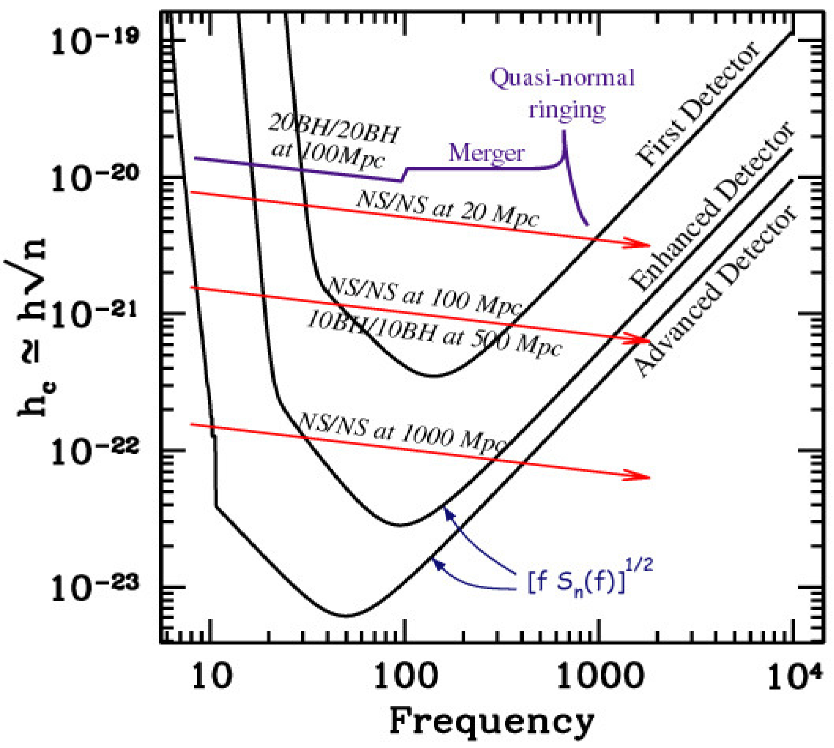
While one can only measure LISA's absolute armlength (distance between the test masses) to within 10 meters, one will be able to measure any changes in the armlengths much more accurately — down to 10 picometers (about 1/10th the size of an atom)!



Observed frequencies: LISA & LIGO



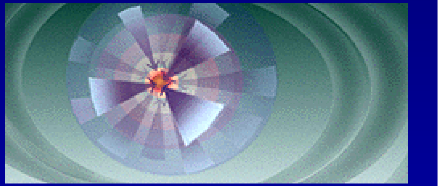
Sensitivity of LIGO to coalescing binaries



Astrophysical sources

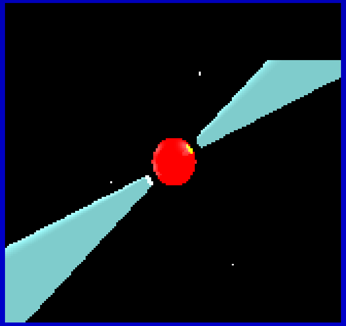
Single (one-time) burst events: extra galactic

- LISA ❖ Massive black hole mergers
- LIGO ❖ Supernova core collapse (non axisymm.)
- LIGO ❖ Colliding neutron star + black hole binaries



Persistent sources (continuous emission): Galactic

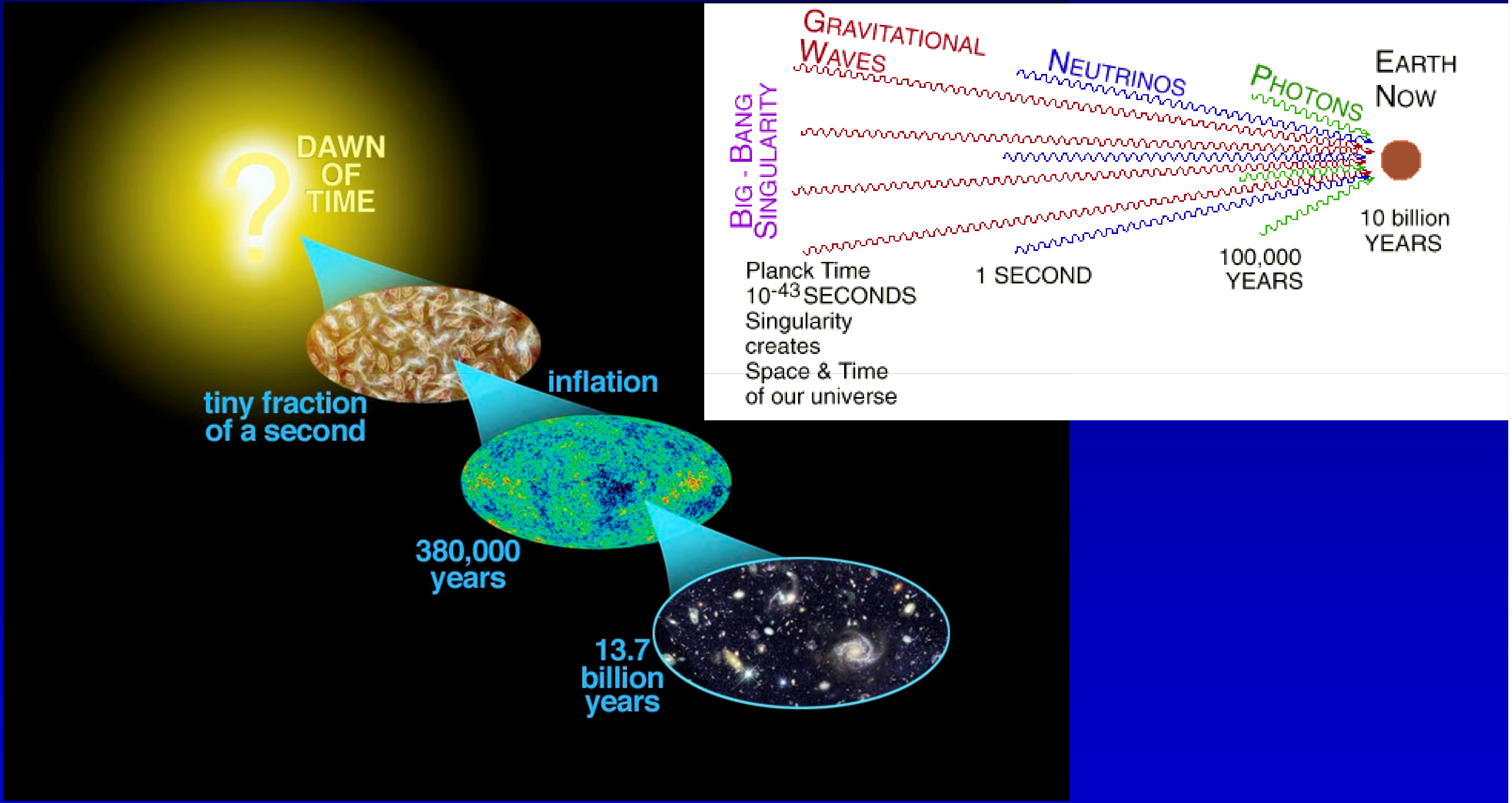
- LISA ❖ Galactic resolved compact binaries (WD, NS, BH)
- LIGO ❖ Pulsars or accreting NS (non axisymm.)



- ❖ Big Bang ?

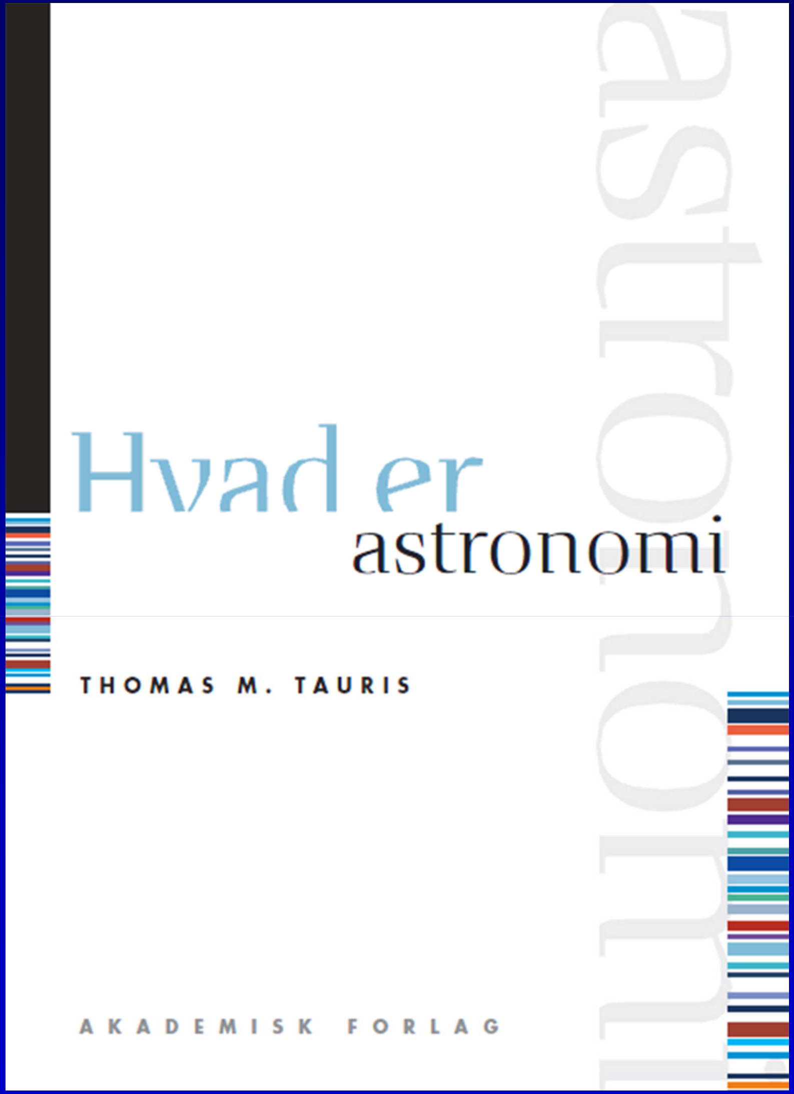
`Murmurs' from the Big Bang

signals from the early Universe



Summary of lecture

- ✓ Brief intro. to GTR and concept of gravitational waves
- ✓ Wave amplitude and physical deformation
- ✓ Gravitational wave luminosity
- ✓ Detection of gravitational waves
  - LIGO: high frequencies (10 Hz - 10 kHz)
  - LISA: low frequencies (0.1 mHz – 0.1 Hz)
- ✓ Sources of gravitational waves
  - Burst sources: extra galactic
  - Continuous sources: Galactic
- ✓ A new epoch of astronomy!



Hvad er astronomi

THOMAS M. TAURIS

AKADEMISK FORLAG