

## Core-Collapse Supernovae

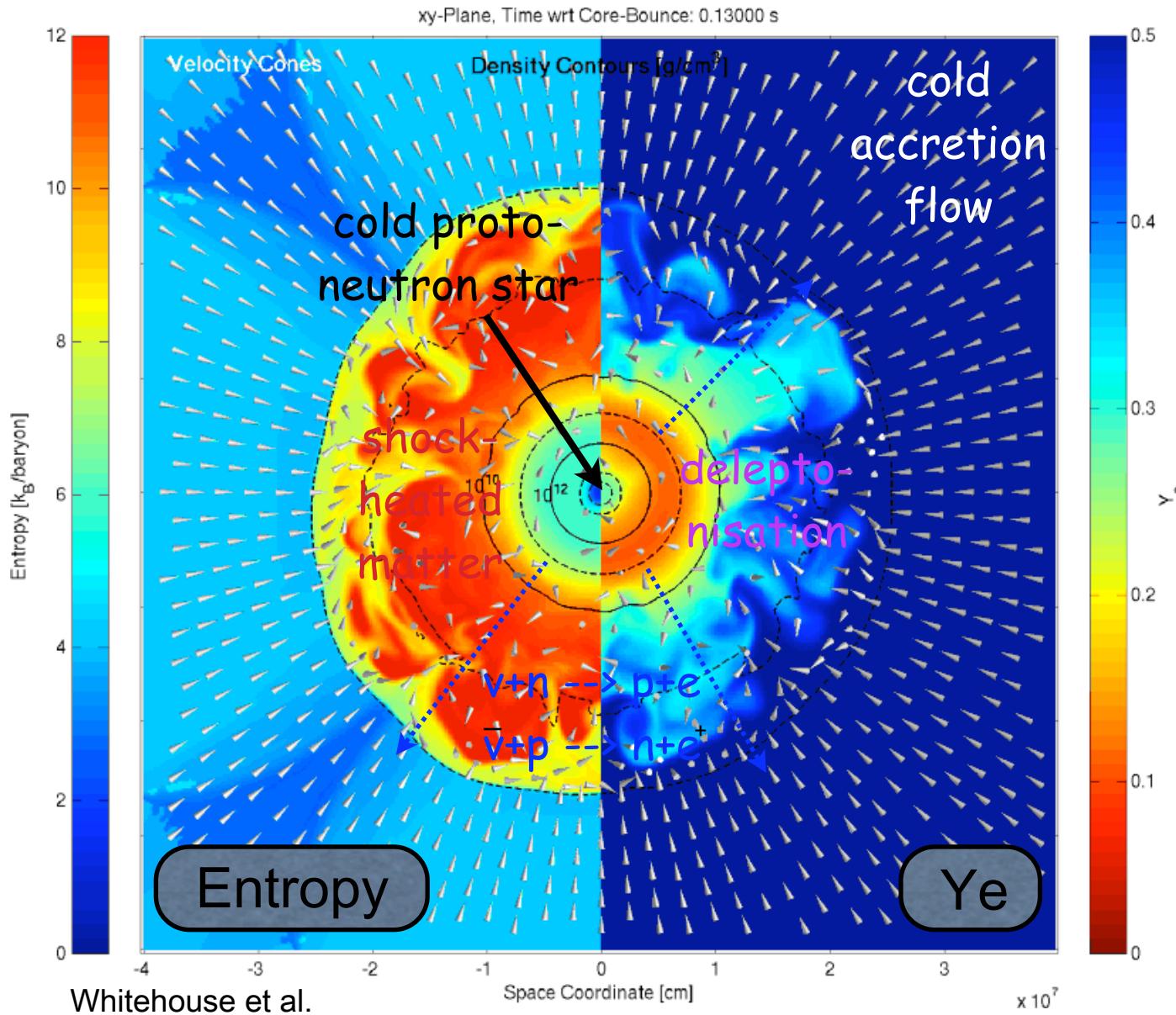
M. Liebendörfer  
University of Basel

- Collapse phase: Dynamics &  $\nu$ -interactions
- Postbounce phase:  $\nu$ -transport & explosion mechanisms
- Models: Approximations & prediction of observables

Large cancellation effects in the total energy budget:

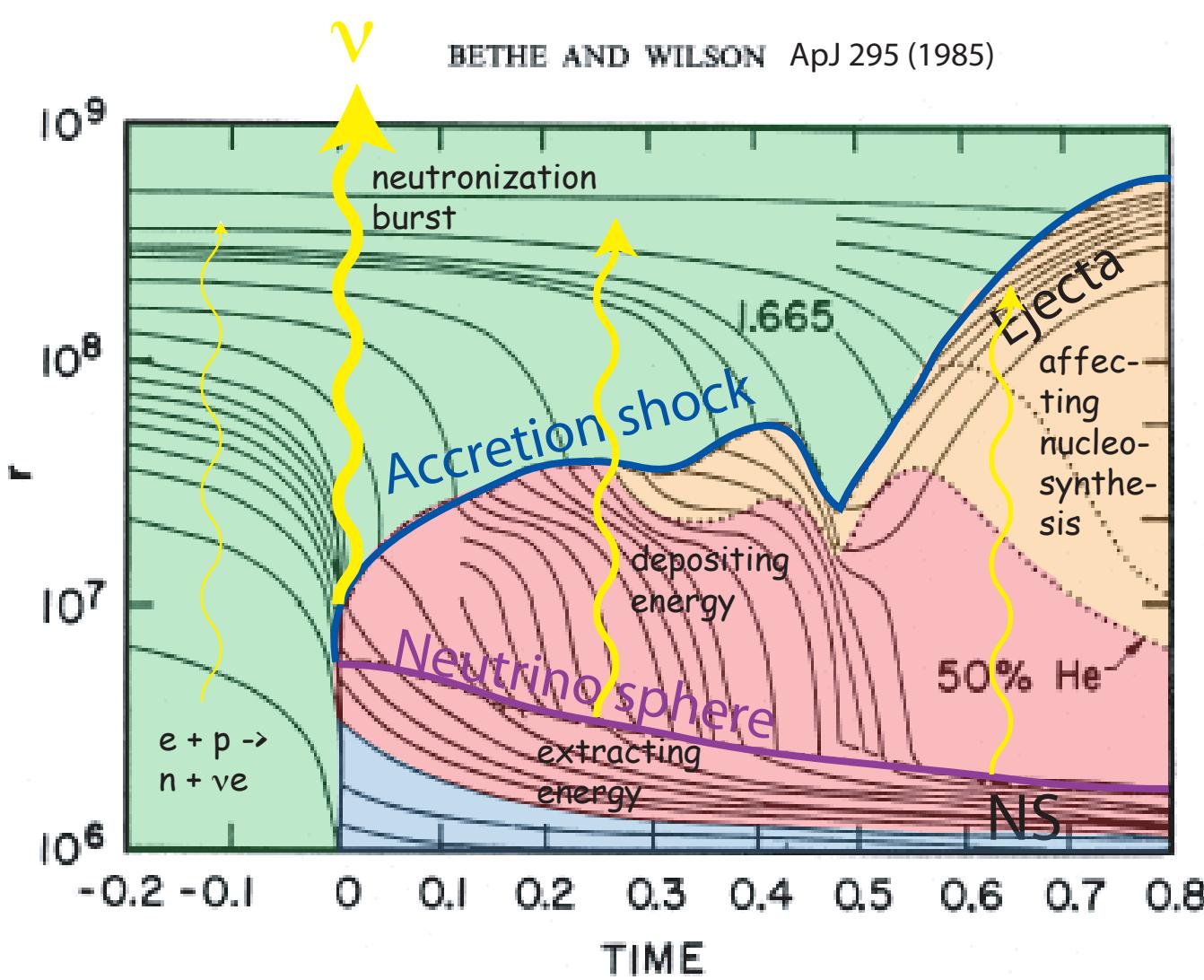
- Huge energy in!
- Huge energy out!
- The rest makes the supernova!
- Leading order contributions from many fields of physics possible...

# The Supernova Problem



- How does the collapse of single stars lead to explosions that outshine a galaxy?
- Which new physics is observable in the extreme conditions of matter during the explosion?
- Does the nucleo-synthesis of heavy elements explain the abundances on Earth, the Sun and distant stars?

# Delayed explosion: 4 phases



collapse phase || postbounce accretion phase | explosion phase  
bounce

Ensemble of nuclei

Cool bulk nuclear matter

Hot dissociated matter

Freeze-out of nuclei

(-->movie)

# Discussed Explosion Mechanisms

Energy scales:

- Gravitational  
 $\sim 3E+53$  erg
- Explosion  
 $\sim 1E+51$  erg

# Discussed Explosion Mechanisms

## 1) Prompt explosion mechanism, $E_{\text{bounce}}$

(e.g. Baron et al. 1985)

Energy scales:

- Gravitational  
 $\sim 3 \times 10^{53}$  erg
- Explosion  
 $\sim 10^{51}$  erg

# Discussed Explosion Mechanisms

## 1) Prompt explosion mechanism, $E(\text{bounce})$

(e.g. Baron et al. 1985)

## 2) Neutrino-driven explosion mechanism, $E(\text{therm.})$

(Colgate 1966, Arnett, Bruenn, Burrows, Mezzacappa ... Marek & Janka 2009)

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## 3) Magneto-rotational explosion mechanism, $E(\text{rot.})$

(Bisnovatyi-Kogan 1976, Leblanc & Wilson 1979, ...)

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(Burrows et al. 2006)

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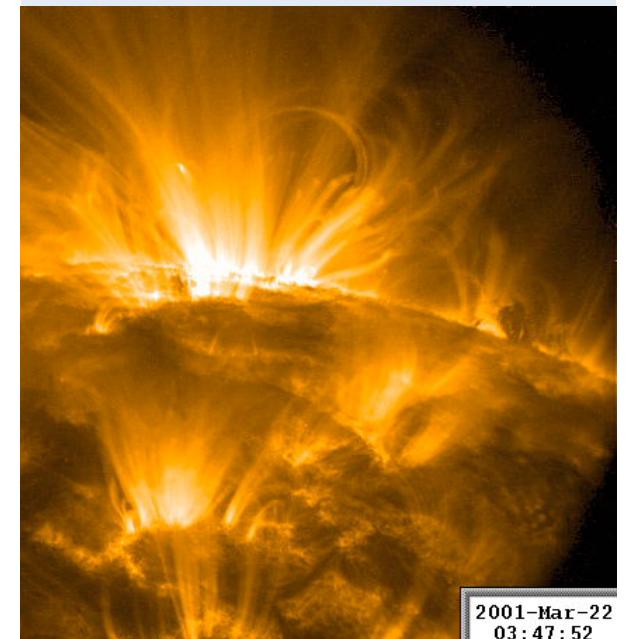
(Burrows et al. 2006)

## 5) Magneto-sonic/viscous expl. mech., $E(\text{buoyancy})$

(Akiyama et al. 2003, Thompson et al. 2003, Socrates et al. 2005)

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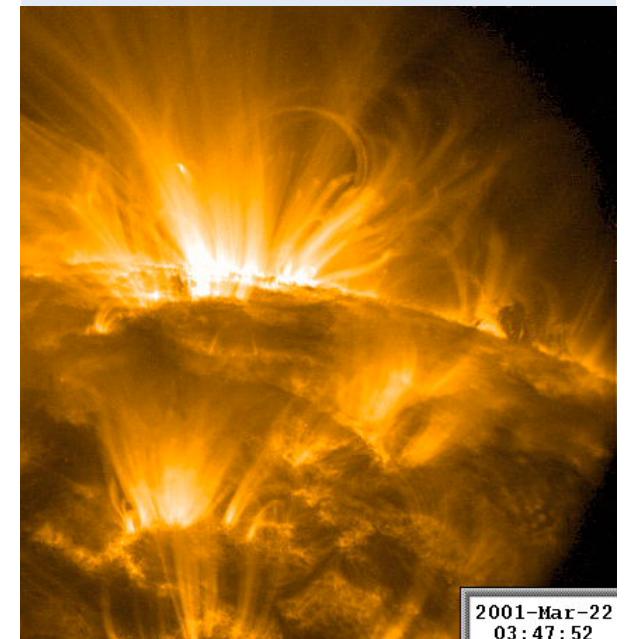
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## 6) Phase transition induced expl. mech., E(compact)

(Migdal et al. 1971, ... Sagert et al. 2009)

Energy scales:

- Gravitational  
~ $3E+53$  erg
- Explosion  
~ $1E+51$  erg



# Input <-> Model <-> Observation

Physics

Modelling challenges

Observations

Reaction network

- uncertainties
- stiff partial diff'eqs.

Magneto-hydrodynamics

- resolution
- time scales

Gravity

- NR: elliptic equations
- GR: metric/horizons

Radiative transfer

- dimensionality
- non-locality

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Observations

Lightcurve  
Polarisation  
Spectra/abundances  
Event rates  
Galactic Evolution  
NS kick & spin  
neutrino signal  
Grav. wave signal  
GRB's

Transient searches

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Transient searches

Quality must match!

# Solving the Boltzmann equation



$$\frac{\partial F}{\alpha c \partial t} + \frac{\partial (4\pi r^2 \alpha \rho \mu F)}{\alpha \partial m} .$$

$$= \frac{j}{\rho} - \tilde{\chi}F$$

$$\frac{\partial Y_e}{\partial t} = -\frac{2\pi m_B}{h^3 c^2} \int E^2 dE d\mu \left( \frac{j}{\rho} - \tilde{\chi}F \right) \quad \frac{\partial e}{\partial t} = \dots \quad \frac{\partial u}{\partial t} = \dots$$

(Mezzacappa & Bruenn 1993, Liebendörfer 2000, Liebendörfer et al. 2004)

Evolution of specific neutrino distr. function:

$$F(t, m, \mu, E) = f(t, r, \mu, E) / \rho$$

=> 3D implicit problem

Comoving metric:

$$ds^2 = -\alpha^2 dt^2 + \left( \frac{1}{\Gamma} \frac{\partial r}{\partial a} \right)^2 + r^2 (d\vartheta^2 + \sin^2 \vartheta d\varphi^2)$$

Stress-energy tensor:

$$\begin{aligned} T^{tt} &= \rho (1 + e + J) \\ T^{ta} = T^{at} &= \rho H \\ T^{aa} &= p + \rho K \\ T^{\vartheta\vartheta} = T^{\varphi\varphi} &= p + \frac{1}{2} \rho (J - K) \end{aligned}$$

# Solving the Boltzmann equation



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$$\begin{aligned}
 &= \frac{j}{\rho} - \tilde{\chi}F + \frac{1}{h^3 c^4} E^2 \int d\mu' R_{is}(\mu, \mu', E) F(\mu', E) \\
 &- \frac{1}{h^3 c^4} E^2 F \int d\mu' R_{is}(\mu, \mu', E) \\
 &+ \frac{1}{h^3 c^4} \left[ \frac{1}{\rho} - F(\mu, E) \right] \int E'^2 dE' d\mu' \tilde{R}_{nes}^{in}(\mu, \mu', E, E') F(\mu', E) \\
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$$\frac{\partial F}{\alpha c \partial t} + \frac{\partial (4\pi r^2 \alpha \rho \mu F)}{\alpha \partial m} + \Gamma \left( \frac{1}{r} - \frac{\partial \alpha}{\alpha \partial r} \right) \frac{\partial [(1 - \mu^2) F]}{\partial \mu}$$

$$\begin{aligned}
 &+ \left[ -\mu \Gamma \frac{\partial \alpha}{\alpha \partial r} \right] \frac{1}{E^2} \frac{\partial (E^3 F)}{\partial E} \\
 &= \frac{j}{\rho} - \tilde{\chi} F + \frac{1}{h^3 c^4} E^2 \int d\mu' R_{is}(\mu, \mu', E) F(\mu', E) \\
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 & + \left( \frac{\partial \ln \rho}{\alpha c \partial t} + \frac{3u}{rc} \right) \frac{\partial [\mu (1 - \mu^2) F]}{\partial \mu} \\
 & + \left[ \mu^2 \left( \frac{\partial \ln \rho}{\alpha c \partial t} + \frac{3u}{rc} \right) - \frac{1}{rc} u - \mu \Gamma \frac{\partial \alpha}{\alpha \partial r} \right] \frac{1}{E^2} \frac{\partial (E^3 F)}{\partial E} \\
 & = \frac{j}{\rho} - \tilde{\chi} F + \frac{1}{h^3 c^4} E^2 \int d\mu' R_{is}(\mu, \mu', E) F(\mu', E) \\
 & - \frac{1}{h^3 c^4} E^2 F \int d\mu' R_{is}(\mu, \mu', E) \\
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# Finite differencing of time evolution



Forward differencing:  
evaluate slope with  
current state vector

Backw. differencing:  
evaluate slope with  
future state

Let's see...

# Explicit finite differencing



Forward differencing:  
evaluate slope with  
current state vector

- simple
- accurate for small time steps
- limited by characteristic time scale

Go faster...

# Explicit finite differencing



Forward differencing:  
evaluate slope with  
current state vector

- simple
- inaccurate for large time steps
- even catastrophic!

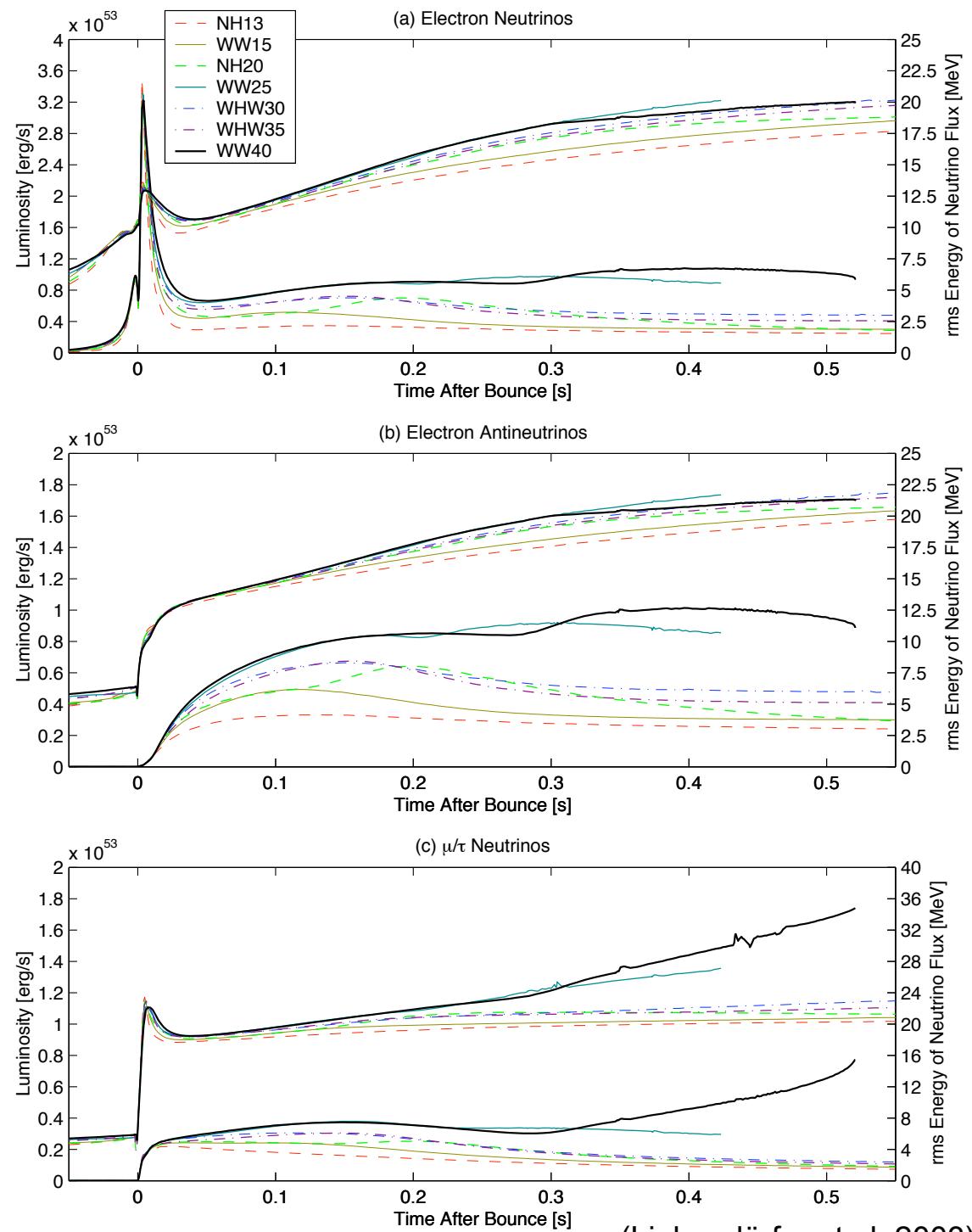
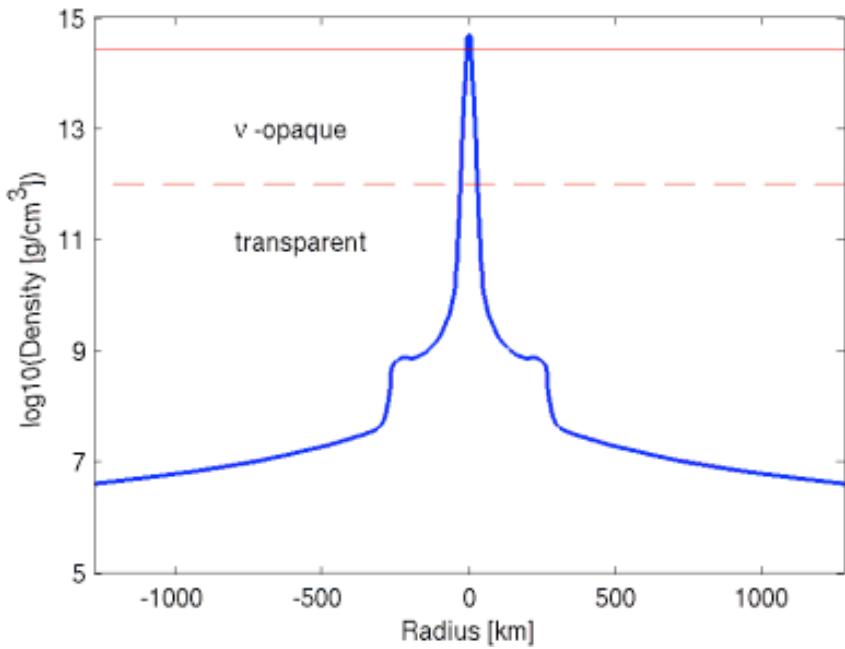
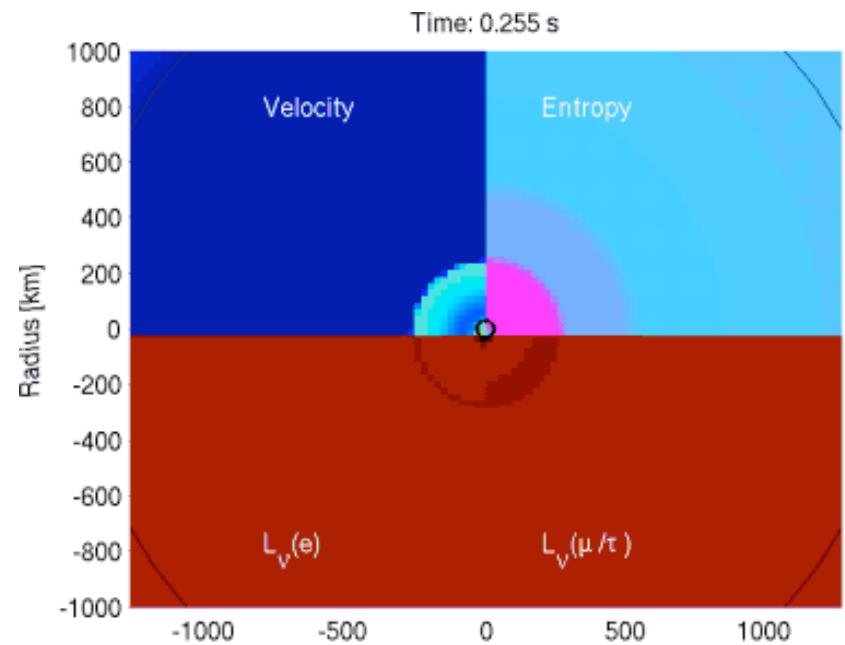
Think...

# Implicit finite differencing



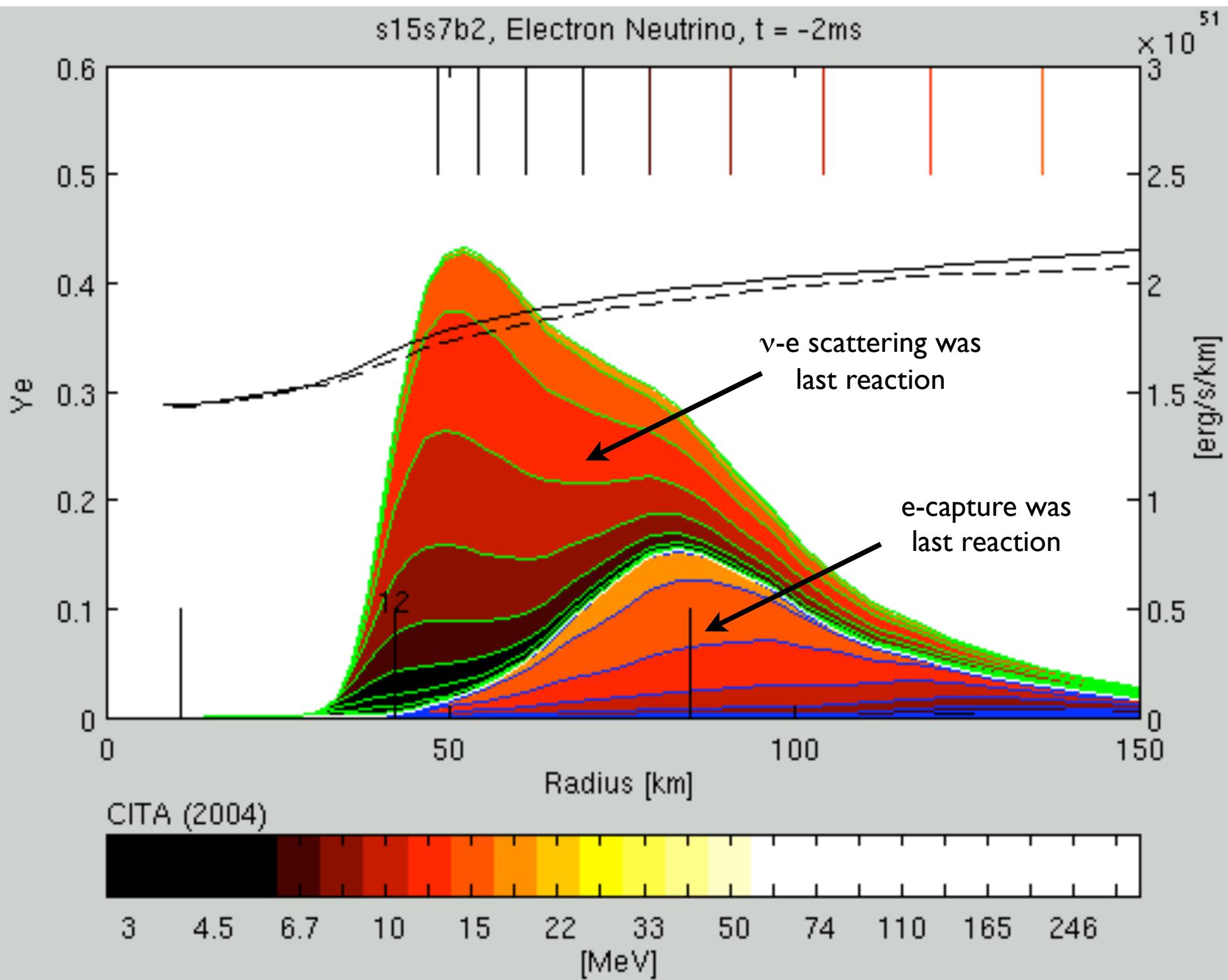
Backw. differencing:  
evaluate slope with  
future state vector

- Long time steps possible
- Follows ‘average’ evolution
- nonlinear system
- computationally expensive!

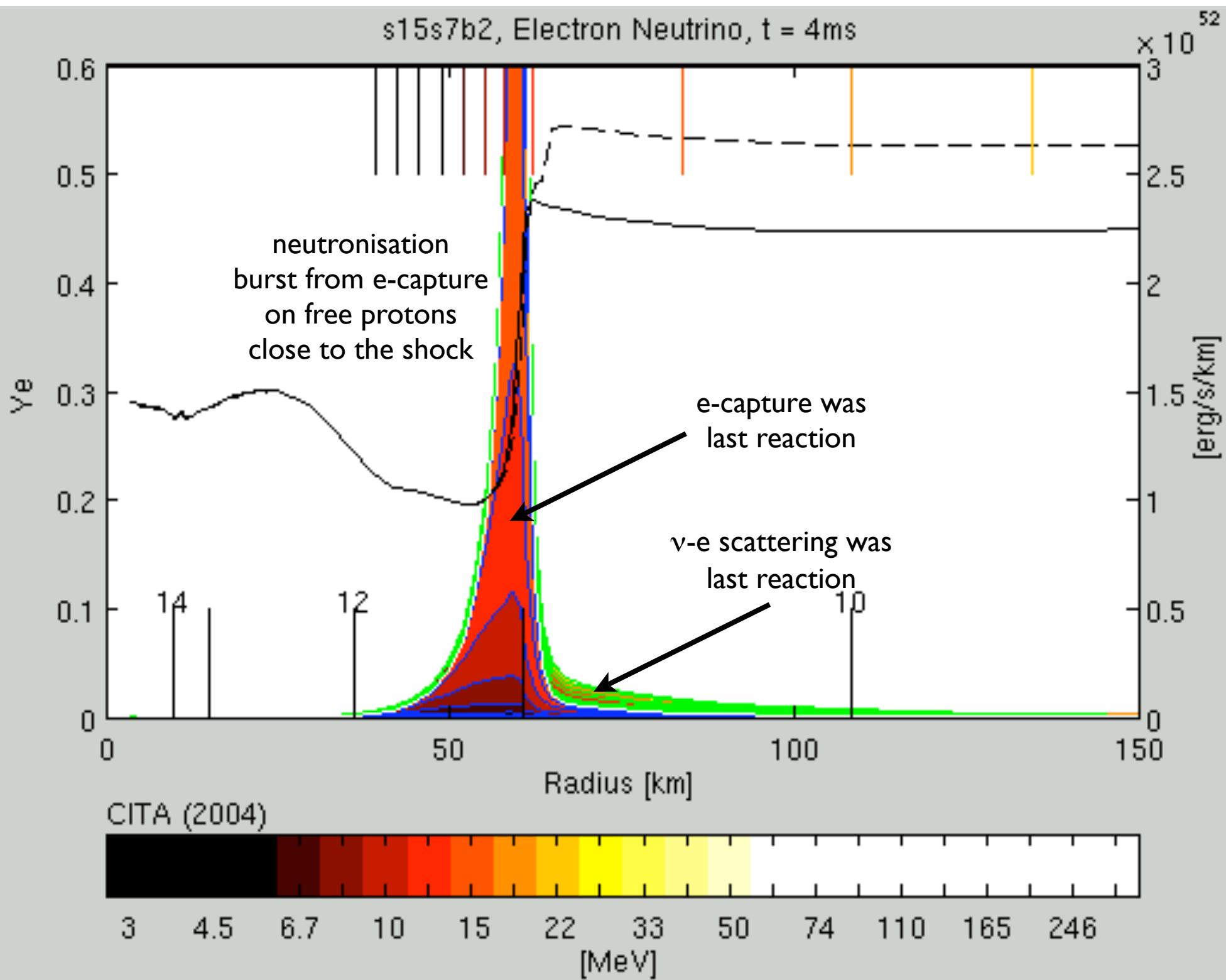


(Liebendörfer et al. 2003)

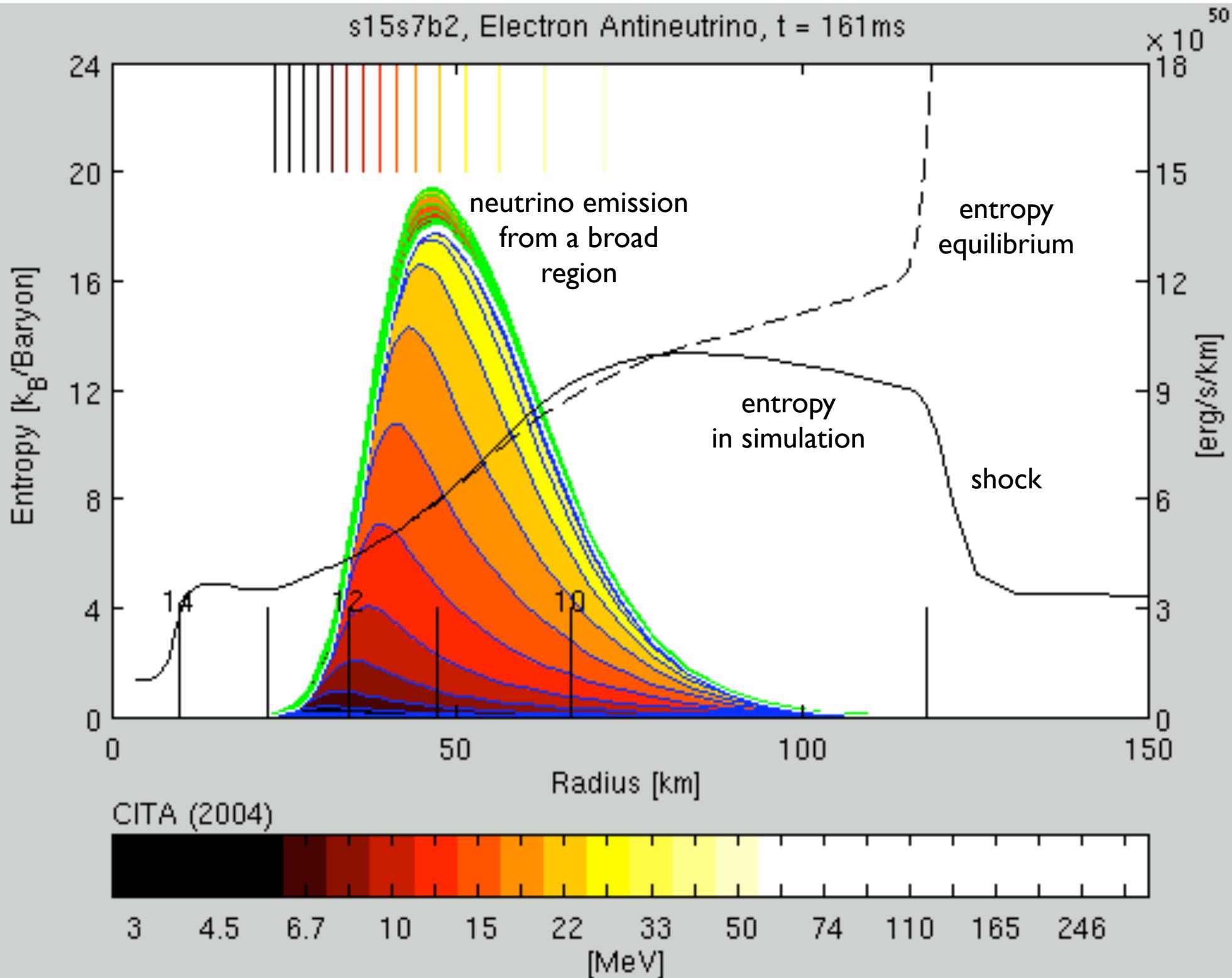
s15s7b2, Electron Neutrino,  $t = -2\text{ms}$

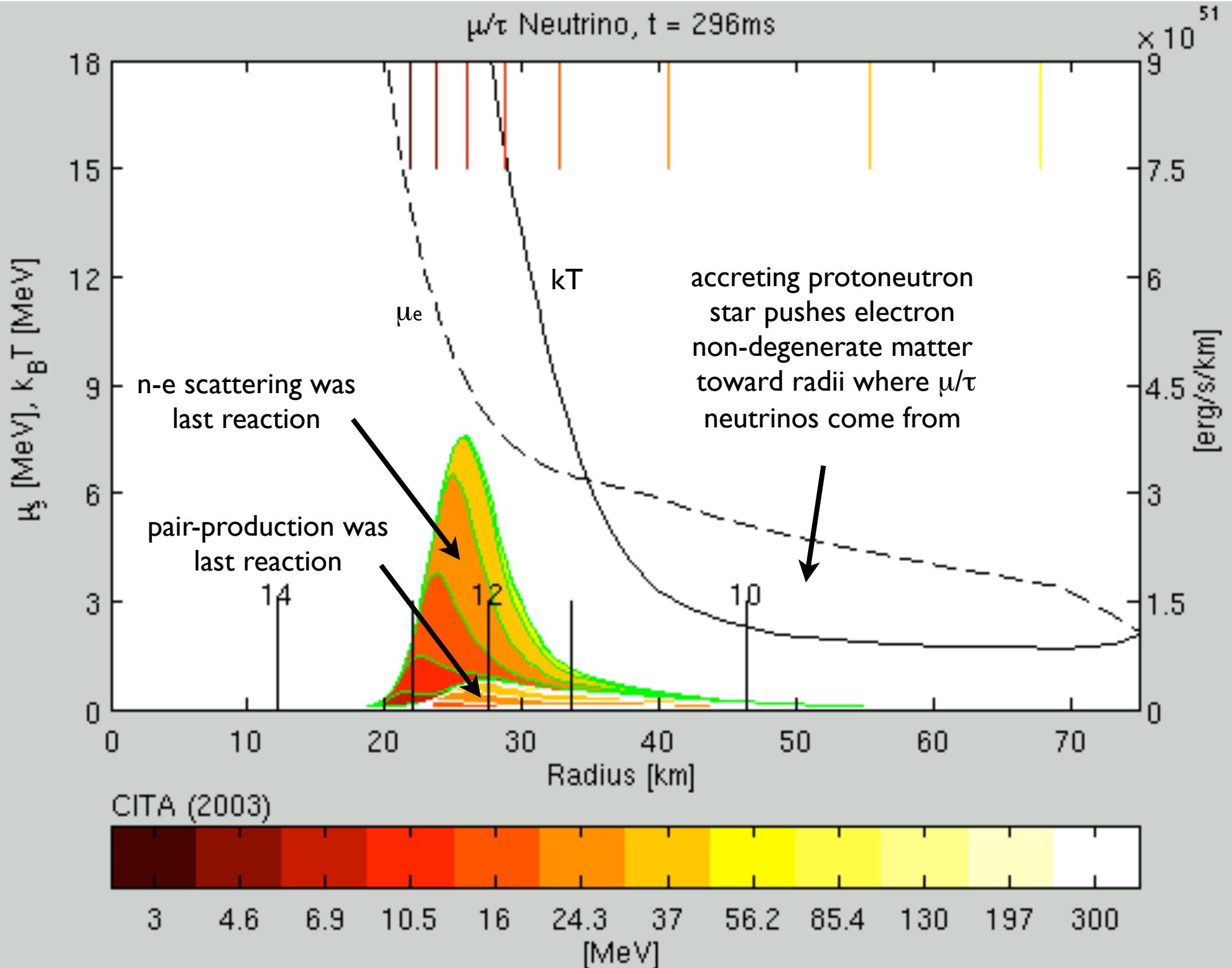


s15s7b2, Electron Neutrino,  $t = 4\text{ms}$



s15s7b2, Electron Antineutrino,  $t = 161\text{ms}$





# Comparison among independent groups

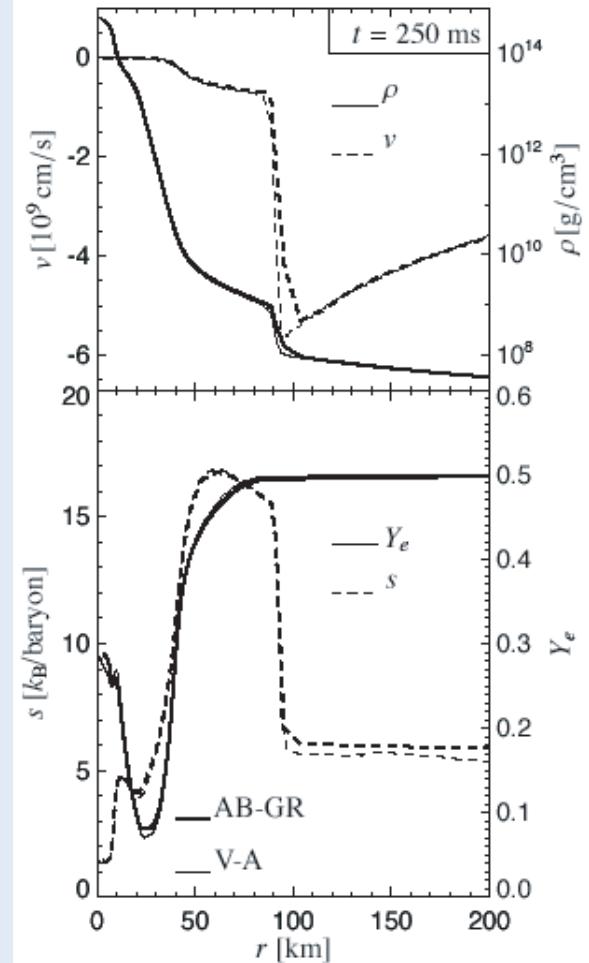
(datafiles.tar.gz in ApJ electronic edition)

Comparison of spherically symmetric simulations:  
Oak Ridge/Basel group and Garching group

Liebendörfer, Rampp, Janka, Mezzacappa, ApJ 620 (2005)

Explosions only in exceptional cases!

excellent agreement:



(Marek et al., A&A 2006)

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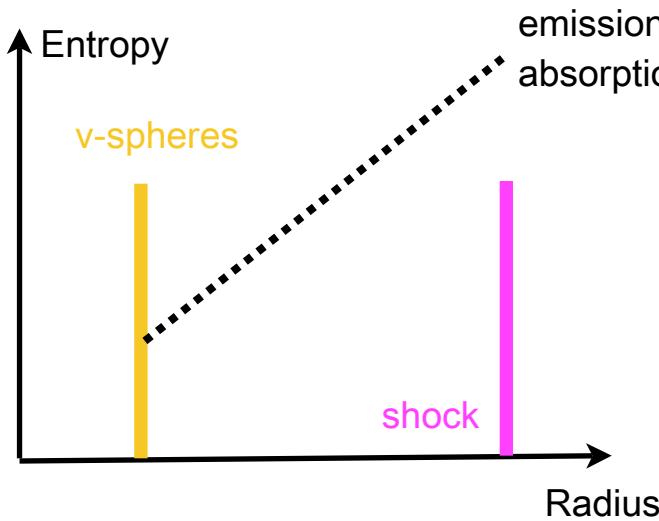
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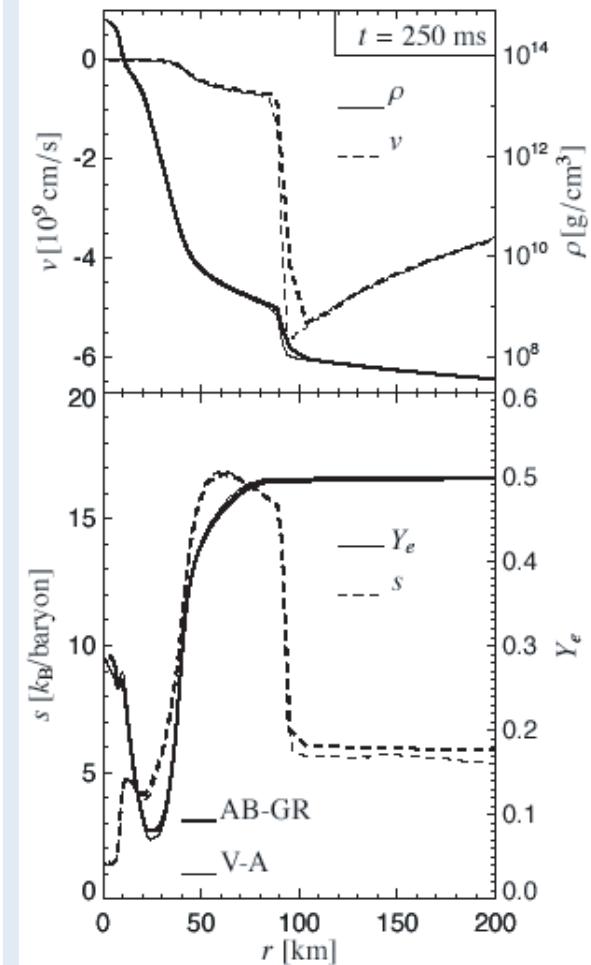
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## Explosions only in exceptional cases!

Problem 1: Exposure  
to heating limited by  
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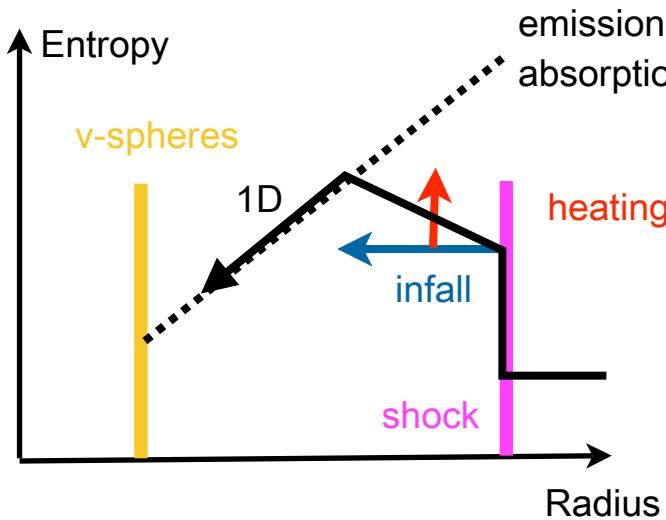
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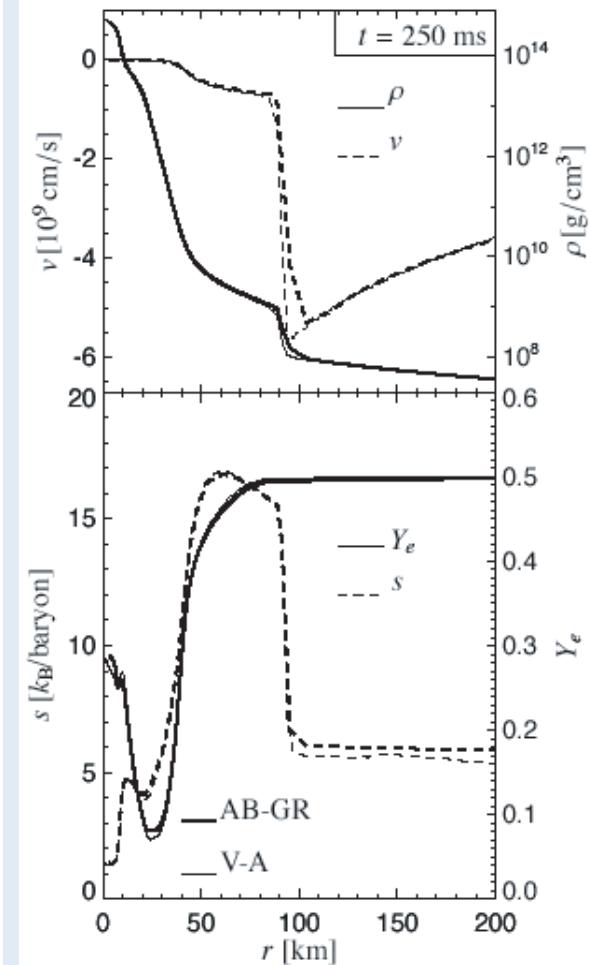
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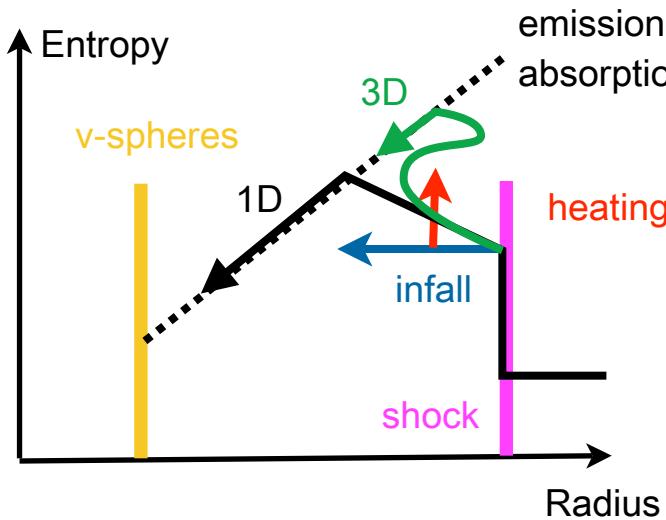
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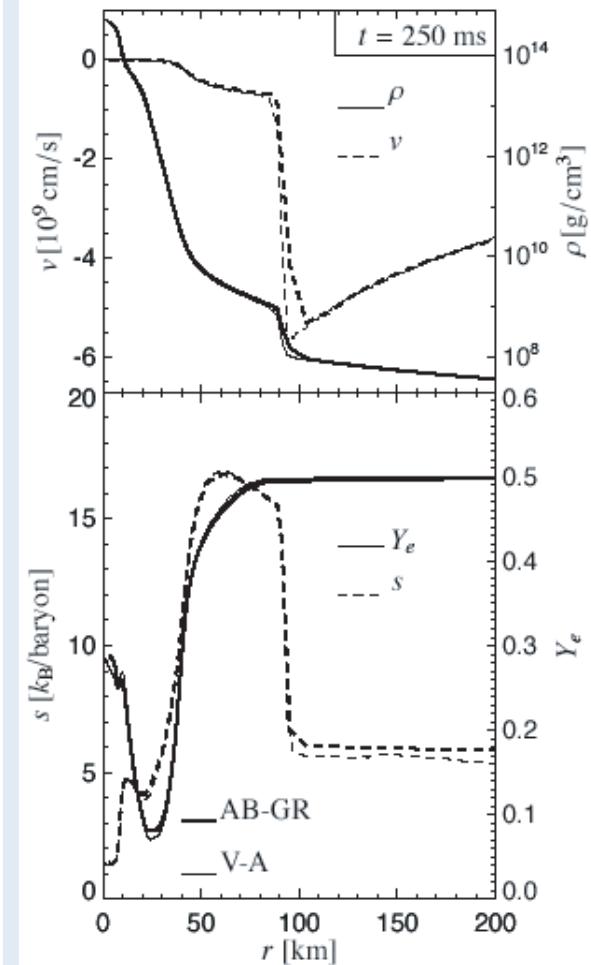
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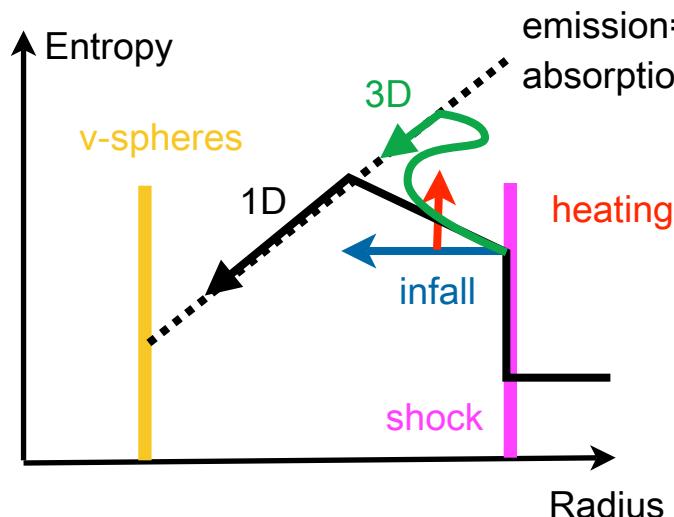
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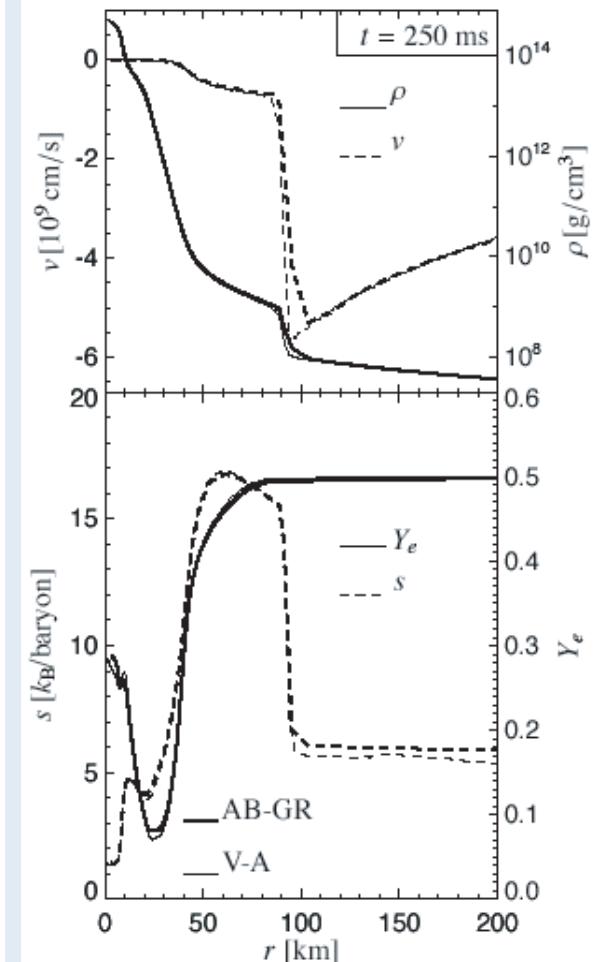
## Explosions only in exceptional cases!

Problem 1: Exposure  
to heating limited by  
high infall velocity



Problem 2: Accretion (-luminosity)  
shuts off after onset of explosion

excellent agreement:



(Marek et al., A&A 2006)