

# **Niels Bohr CompSchool on Compact Objects**

**Neutron Star Observables,  
Masses, Radii and Magnetic Fields**

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# **Lecture 1: Surfaces of Neutron Stars**

# Neutron Star Sources and Observables

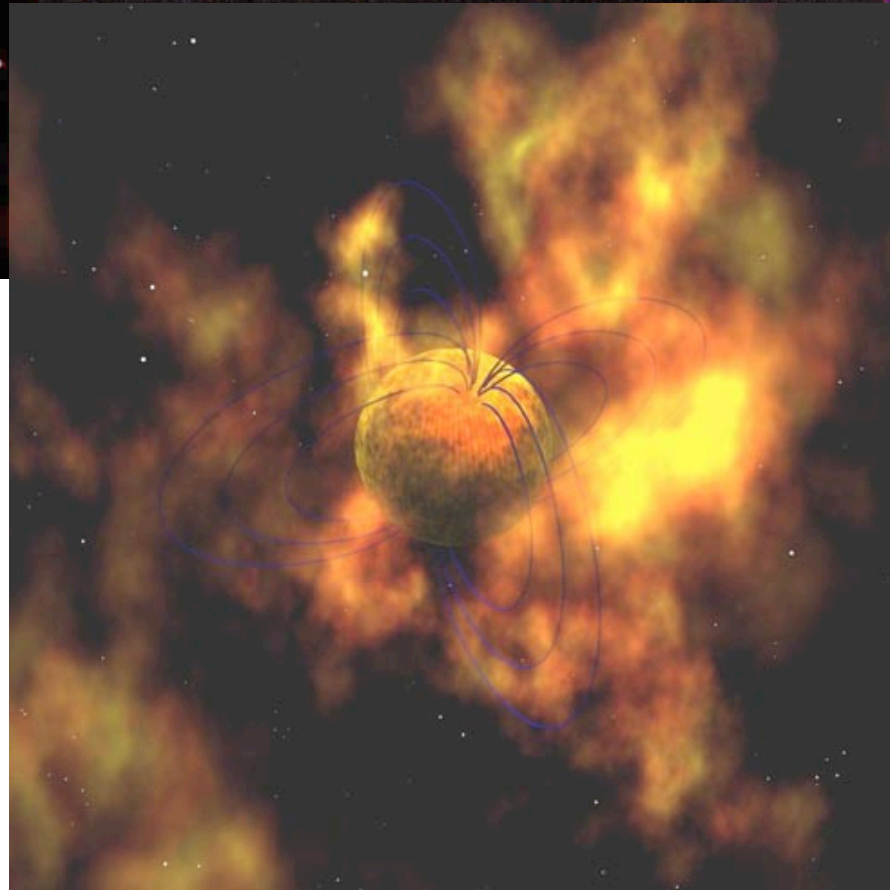
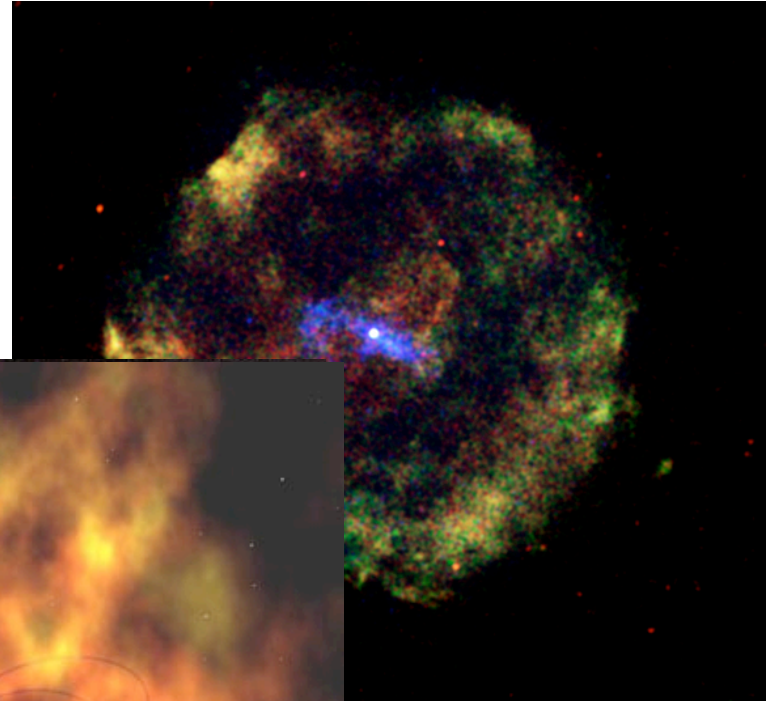
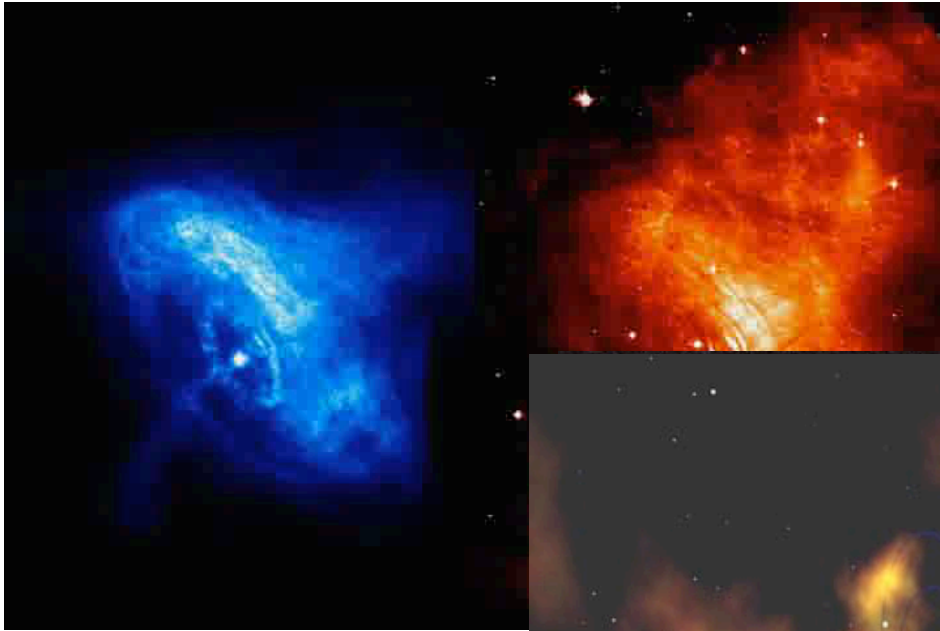
## SOURCES

- Isolated Sources
- Binaries

## PHYSICS GOALS

- M-R relations (equation of state)
- Magnetic fields, energy sources
- Energetic bursts

## Gallery of Young Neutron Stars



# Neutron Star Sources and Observables

## SOURCES

- Isolated Sources
- Binaries

## PHYSICS GOALS

- Neutron star Mass-Radius relations (equation of state)
- Magnetic fields, energy sources
- Energetic bursts
- Particle acceleration mechanisms

Surface + magnetosphere (Lecture 3) determine observables

## **Need a model for the surface emission!**

This is both to determine NS mass and radius but also to understand a wide range of phenomena happening on neutron stars.

# Emission from the Surfaces of Neutron Stars: Isolated NS

## I. Composition of the Surface:

1. How much material is necessary to cover the surface and dominate the emission properties?

Assume zero magnetic field, need material to optical depth  $\tau=1$ .

$$\begin{aligned} m &= \rho V \\ &= \rho 4\pi R_{NS}^2 h \\ &= N_p m_p 4\pi R_{NS}^2 h \end{aligned}$$

$N_e = N_p$  and  $d\tau = N_e \sigma_T dz \implies \tau = N_e \sigma_T z$  (assuming electron density is independent of depth)

$$m = \frac{\tau}{\sigma_T} m_p 4\pi R_{NS}^2$$

For typical values,  $m=10^{-17} M_\odot$  for an unmagnetized neutron star.

2. How long does it take to cover the NS surface with a  $10^{-17} M_\odot$  hydrogen or helium skin by accreting from the ISM?

Using Bondi-Hoyle formalism:

$$\dot{M} = \frac{4\pi(GM)^2 \rho_{ISM}}{v^3}$$

If we take  $v \approx 10^7 \text{ cm/s}$

$$\rho \approx m_p / \text{cm}^3 \approx 1.7 \times 10^{-24} \text{ g/cm}^3$$

$$M \approx 1.5 \times 2 \times 10^{33} \text{ g}$$

$$\dot{M}_{ISM} \approx 7 \times 10^8 \text{ g/s} \approx 10^{-17} M_\odot / \text{yr} \quad \longrightarrow \quad t_{\text{accr}} = 1 \text{ yr.}$$

Assuming magnetic fields do not prevent accretion, very quickly, NS surfaces can be covered by H/He.

### 3. Settling of Heavy Elements (Bildsten, Salpeter, & Wasserman)

Heavy elements settle by ion diffusion, as they are pulled down by gravity and electron current.

How long does it take for them to settle below optical depth  $\sim 1$  (where they no longer affect the spectrum?)

$$t_{\text{settle}} \approx 13 \text{ s} \left( \frac{g}{10^{14}} \right)^{-1} \left( \frac{kT}{1 \text{ keV}} \right)^{-3/2}$$

(T enters because it affects the speed of ions and the inter-particle distances)



## II. Ionization State of the Atmosphere and Magnetic Fields:

1. The ionization state of a gas is given by the Saha equation:

$$\frac{n_H}{n_p n_e} = \frac{V Z_H}{Z_e Z_p}$$

Partition function Z defined for each species:

$$Z_e = \frac{V}{2\pi\lambda_e^3}, \quad \lambda_e = \left(\frac{2\pi\hbar^2}{m_e kT}\right)^{1/2}$$

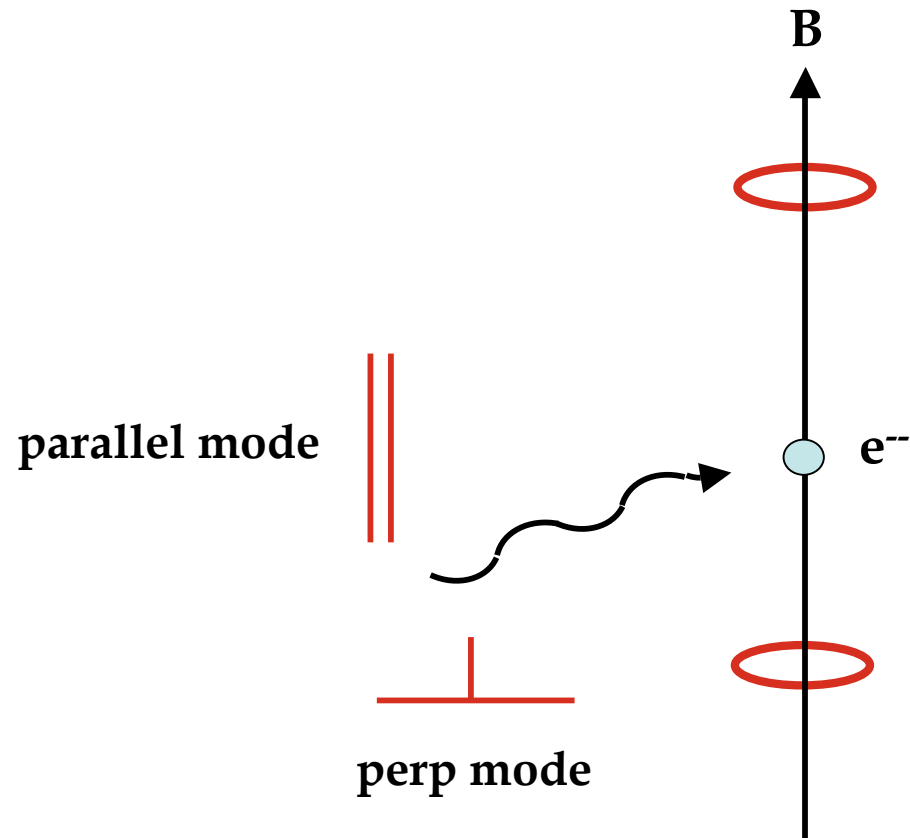
$$Z_p = \frac{V}{2\pi\lambda_p^3} e^{-\chi/kT}$$

When we consider H atoms at  $kT \approx 1\text{keV}$ ,  $\chi \ll kT$  so the atmosphere is completely ionized. For lower temperatures ( $kT_{\text{eff}} \sim 50\text{ eV}$ ), need to consider the presence of neutral atoms.

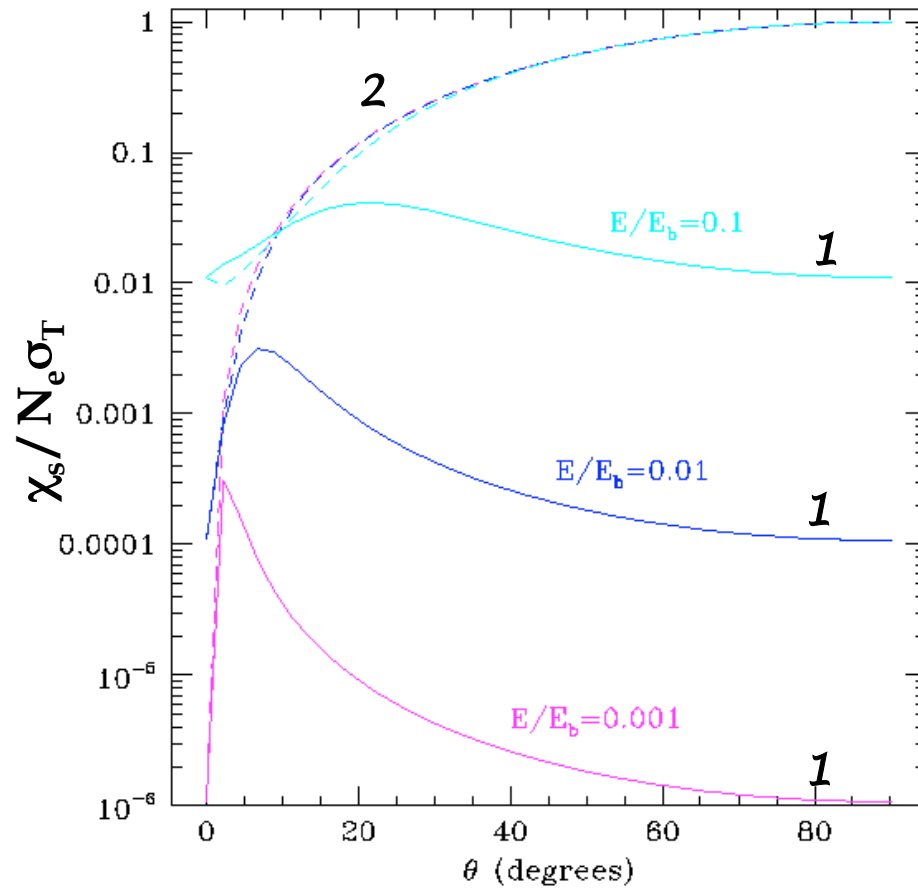
2. Magnetic Fields

At  $B \geq 10^{10}\text{ G}$ , magnetic force is the dominant force,  $\gg$  thermal, Fermi, Coulomb energies.

# Photon-Electron Interaction in Confining Fields



## Magnetic Opacities



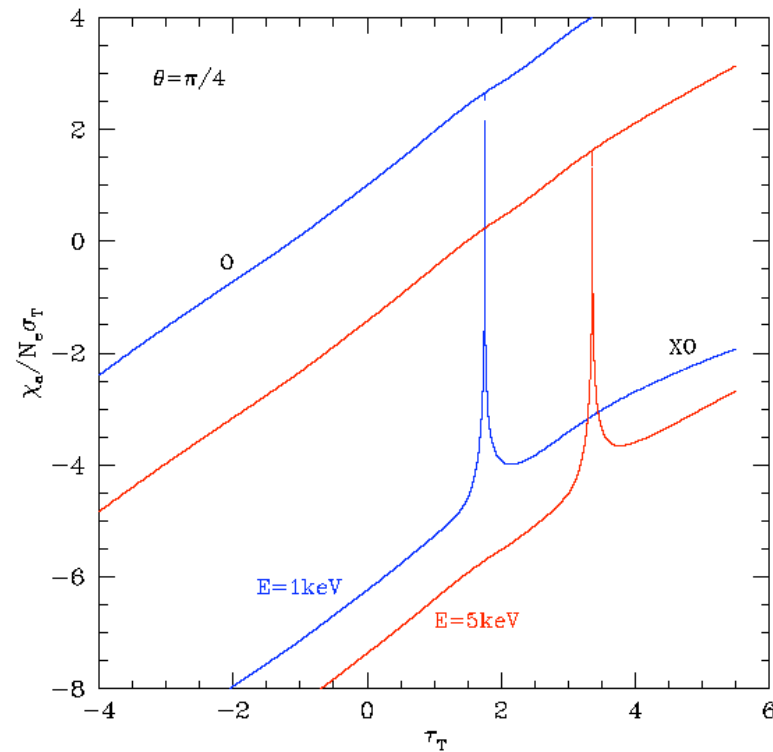
$$\chi_s^1 \approx (E / E_b)^2 \sigma_T$$

$$\chi_s^2 \approx \sigma_T$$

Energy, angle & polarization dependence

→ expect non-radial beaming and deviations from a blackbody spectrum

# Vacuum Polarization Resonance



Vacuum-dominated ← → Plasma-dominated

- at  $B \sim B_{cr}$  virtual  $e^+ e^-$  pairs affect photon transport
- resonance appears at an energy-dependent density
- proton cyclotron absorption features appear at  $\sim \text{keV}$ , and are weak

# Emission from the Surfaces of Neutron Stars: Accreting Case

## I. Composition of the Surface:

A steady supply of heavy elements from accretion as well as thermonuclear bursts

Atmosphere models need to take the contribution of Fe, Si, etc.

## II. Ionization State:

Temperatures reach ~few keV. Magnetic field strengths are very low ( $10^8$ -- $10^9$  G) for LMXBs,  $\sim 10^{11-12}$  G for X-ray pulsars

Light elements are fully ionized. Bound species of heavy elements.

## III. Emission Processes: Compton Scattering

Most important process is non-coherent scattering of photons off of hot electrons  
Bound-bound and bound-free opacities also important for heavy elements

# Compton Scattering

“Compton” scattering is a scattering event between a photon and an electron where there is some energy exchange (unlike Thomson scattering which changes direction but not the energies)

By writing 4-momentum conservation for a photon scattering through angle  $\theta$ , we find

$$\frac{E_f}{E_i} = \frac{1 - \beta_i \cos \alpha_i}{1 - \beta_i \cos \alpha_f + \frac{E_i}{\gamma m c^2} (1 - \cos \theta)}$$

Energy gain                  Recoil term  
from the electron

Typical to expand this expression in orders of  $\beta$ , and average over angles.

To first order, photons don't gain or lose energy due to the motion of the electrons (angles average out to zero)

# Compton Scattering

To second order, we find on average

$$\frac{\Delta E}{E_i} = \frac{1}{3}\beta_i^2 - \frac{E_i}{mc^2}$$

Energy gain  
from K.E. of electron

Energy loss  
from recoil

If electrons are thermal,

$$\beta_i^2 = \frac{3kT}{mc^2}$$

$$\frac{\Delta E}{E_i} = \frac{kT - E_i}{mc^2}$$

If  $E_i < kT$ , photons gain energy

If  $E_i > kT$ , photons lose energy

# Model Atmospheres:

## Hydrostatic balance:

Gravity sustains pressure gradients

$$\frac{dP}{d\tau} = \frac{g\rho}{y_G^2 N_e \sigma_T} \quad (\tau = \int_0^h N_e \sigma_T dz)$$

$y_G$  is the correction to the proper distance in GR

$$y_G = \left(1 - \frac{2GM}{Rc^2}\right)^{1/2}$$

## Equation of State:

Assume ideal gas  $P = 2NkT$



**Equation of Transfer:**

$$y_G \mu \frac{dI_E^i}{d\tau_{es}} = \chi_a^i I^i - \chi_a^i \frac{B_E}{2} + \chi_s^i I^i - \sum_{j=1,2} \int \chi_s^{ij}(\mu, \mu') I'^j d\mu'$$

for  $i = 1, 2$

**Radiative Equilibrium :**

$$H(\tau) = \sigma T_{eff}^4 = \int I(\tau, \mu, E) \mu d\mu dE$$

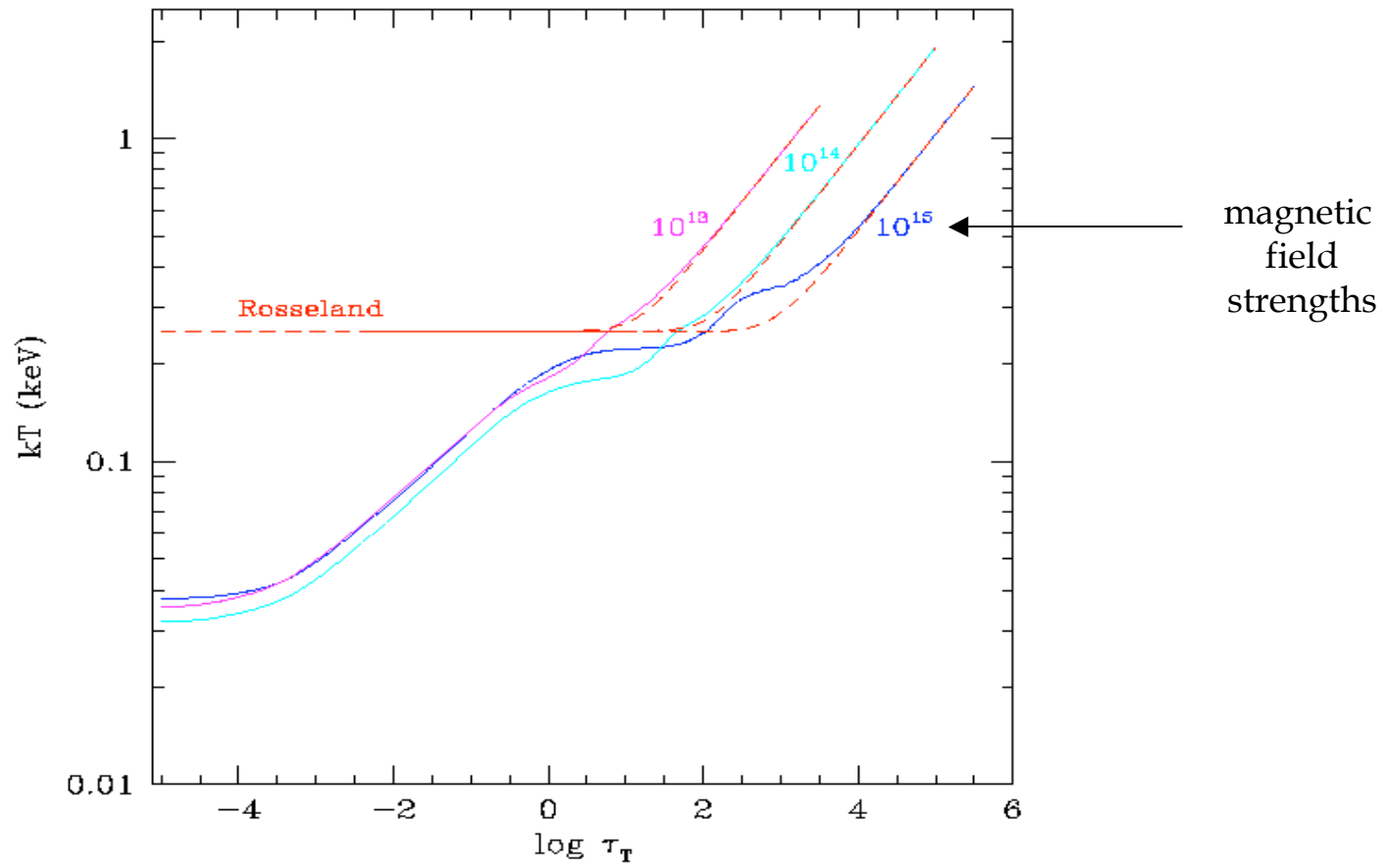
Techniques for solving the Transfer equation (with scattering):

Feautrier Method, Variable Eddington factors, Accelerated Lambda Iteration...

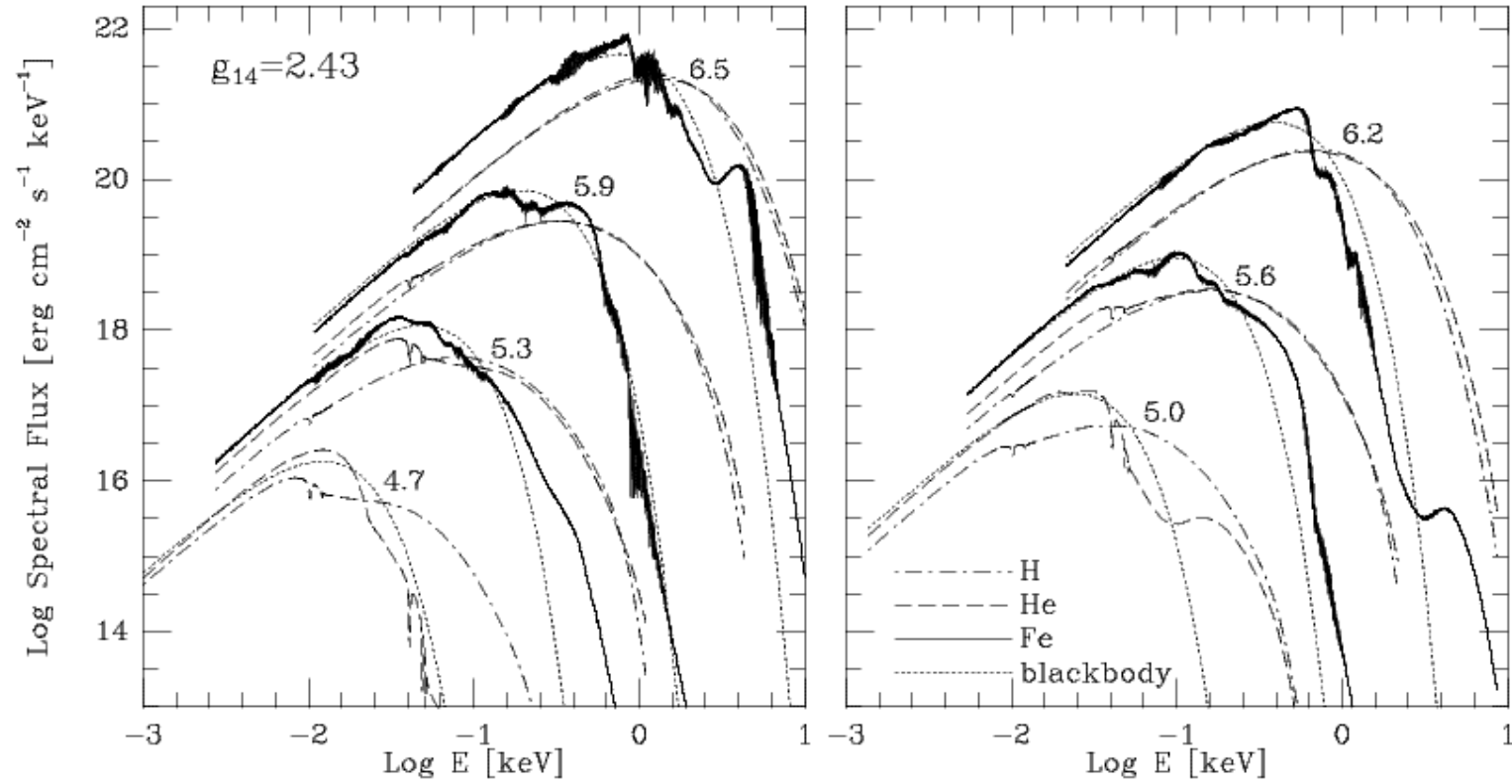
Techniques for achieving Radiative Equilibrium:

Lucy-Unsold Scheme, Complete Linearization...

# Typical Temperature Profiles:

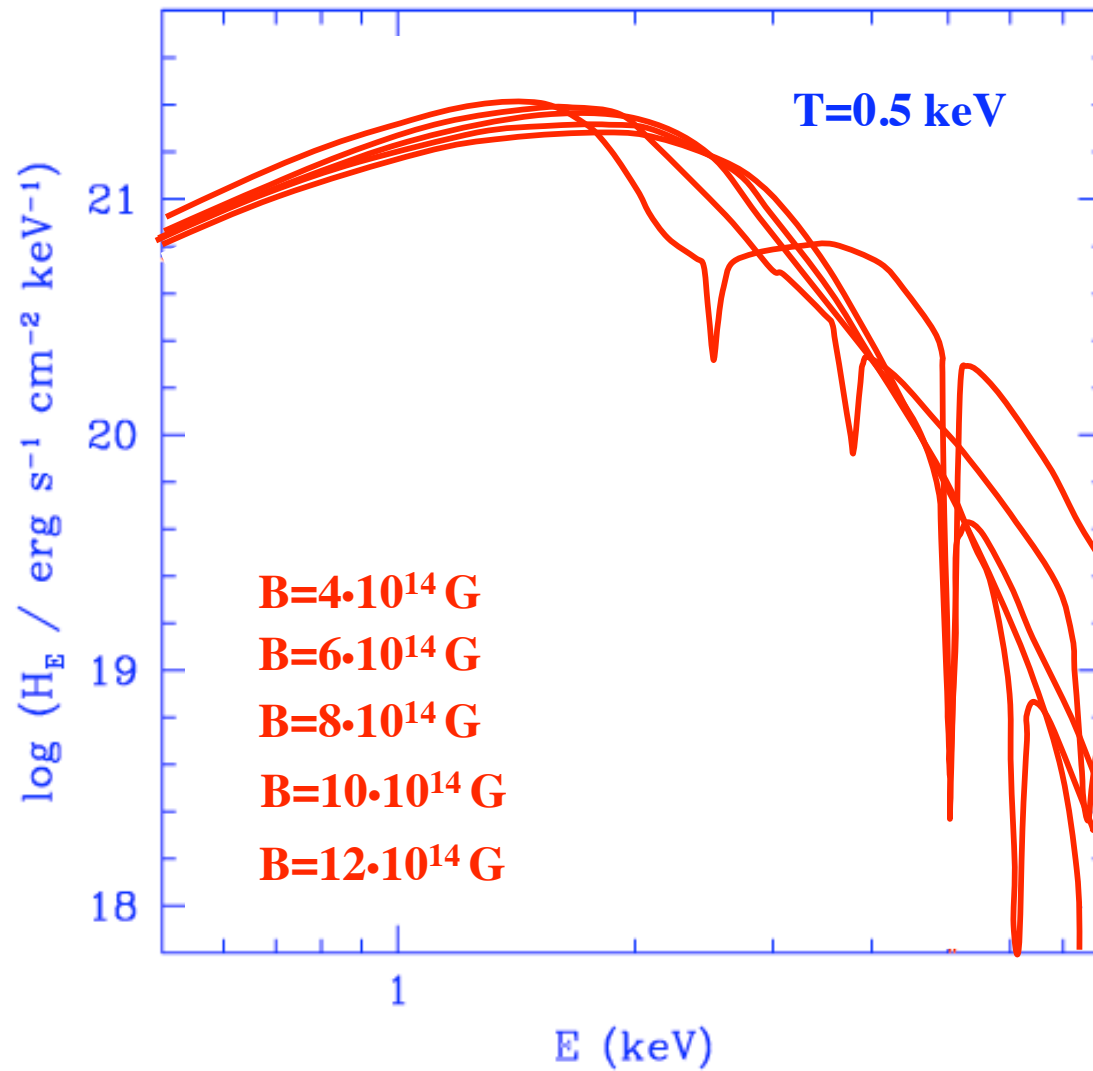


## Typical Spectra (Isolated, Non-Magnetic):

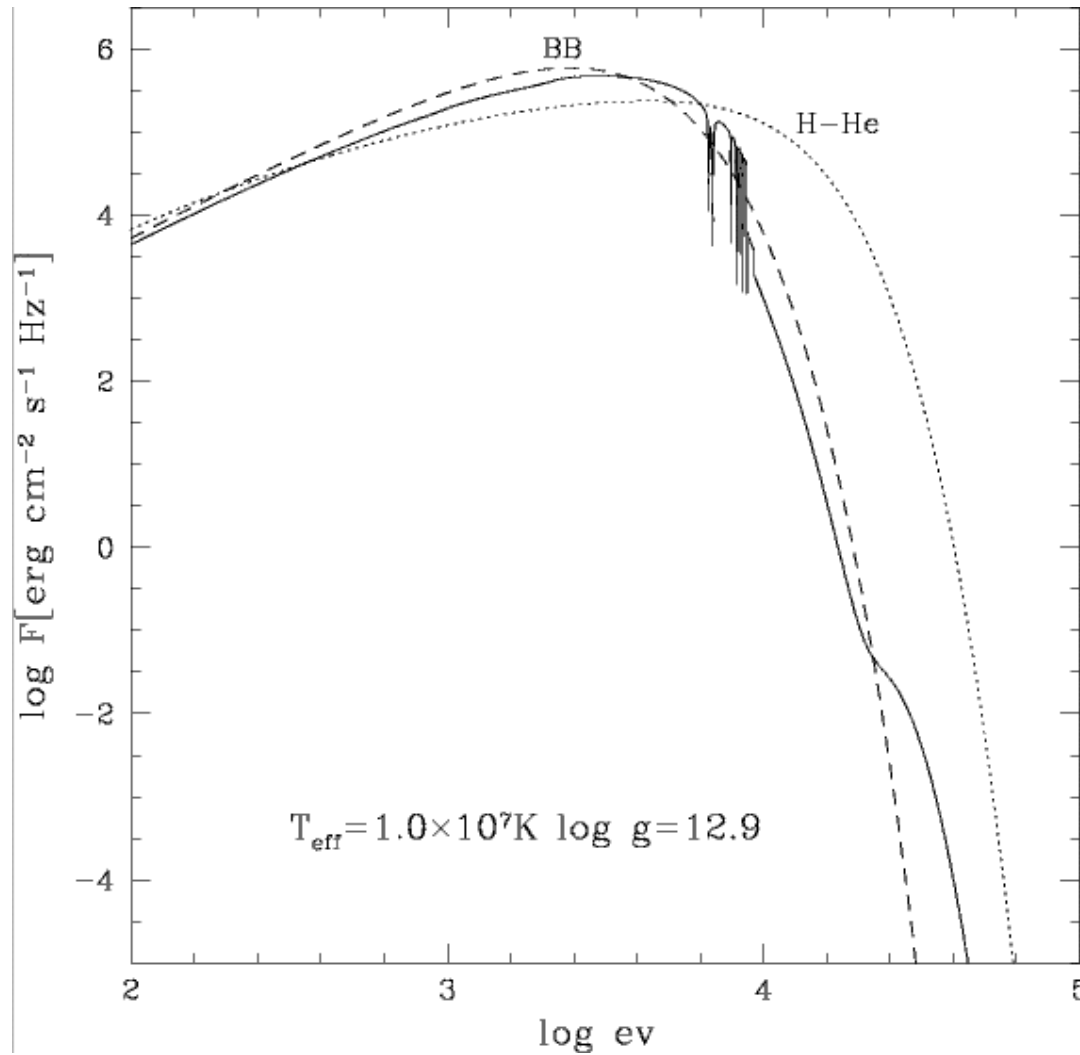


From Zavlin et al. 1996

## Typical Spectra (Isolated, Magnetic):



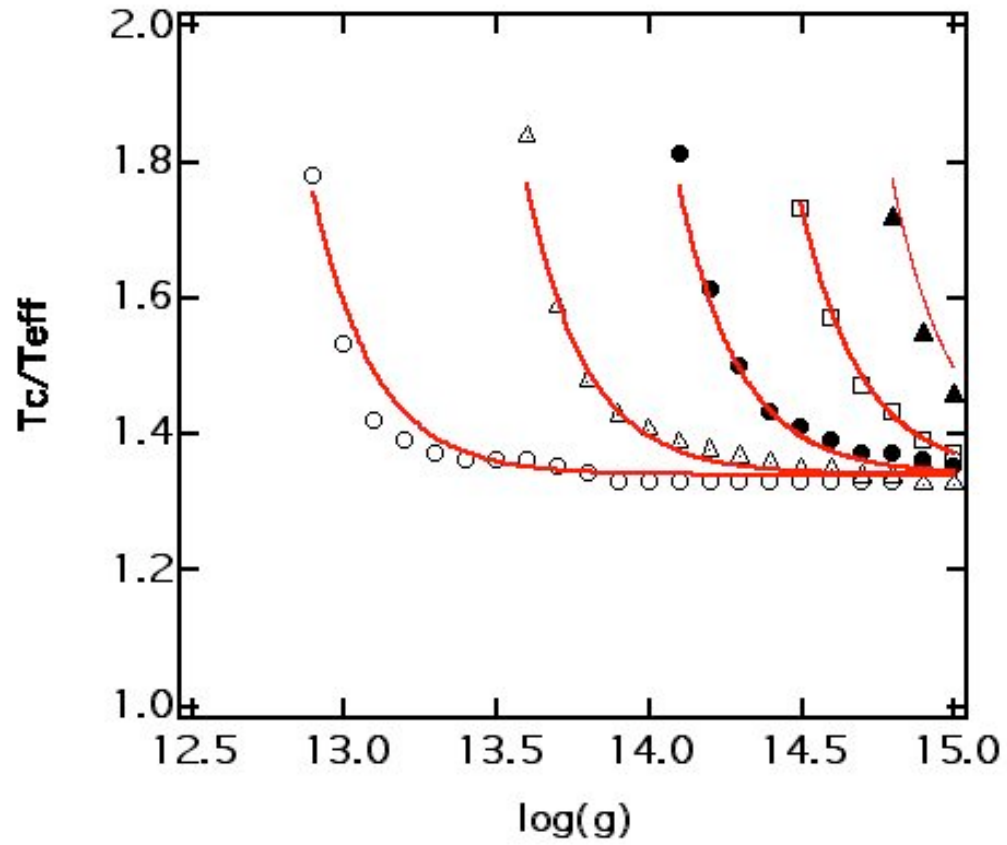
## Typical Spectra (Accreting, Burster):



From Madej et al. 2004, Majczyna et al 2005

- Comptonization produces high-energy “tails” beyond a blackbody
- Heavy elements produce absorption features

# Color Correction Factors



From Madej et al. 2004, Majczyna et al 2005

# Seeing the Surface Light

- We can see the emission from the surface itself in a variety of sources
- Isolated neutron stars (thermal component), millisecond pulsars (accreting and isolated), thermonuclear bursts
- To focus on the surface, it is important to find sources where the magnetospheric emission or the disk emission do not dominate

## Pros and Cons of Surface Emission from Isolated vs. Accreting:

### Isolated:

### Accreting:

#### Pros:



No heavy elements  
--atmospheres simple  
No accretion luminosity

Eddington-limited phenomena

(Redshifted) spectral features  
more likely

Surface emission likely to be uniform

Bright

#### Cons:



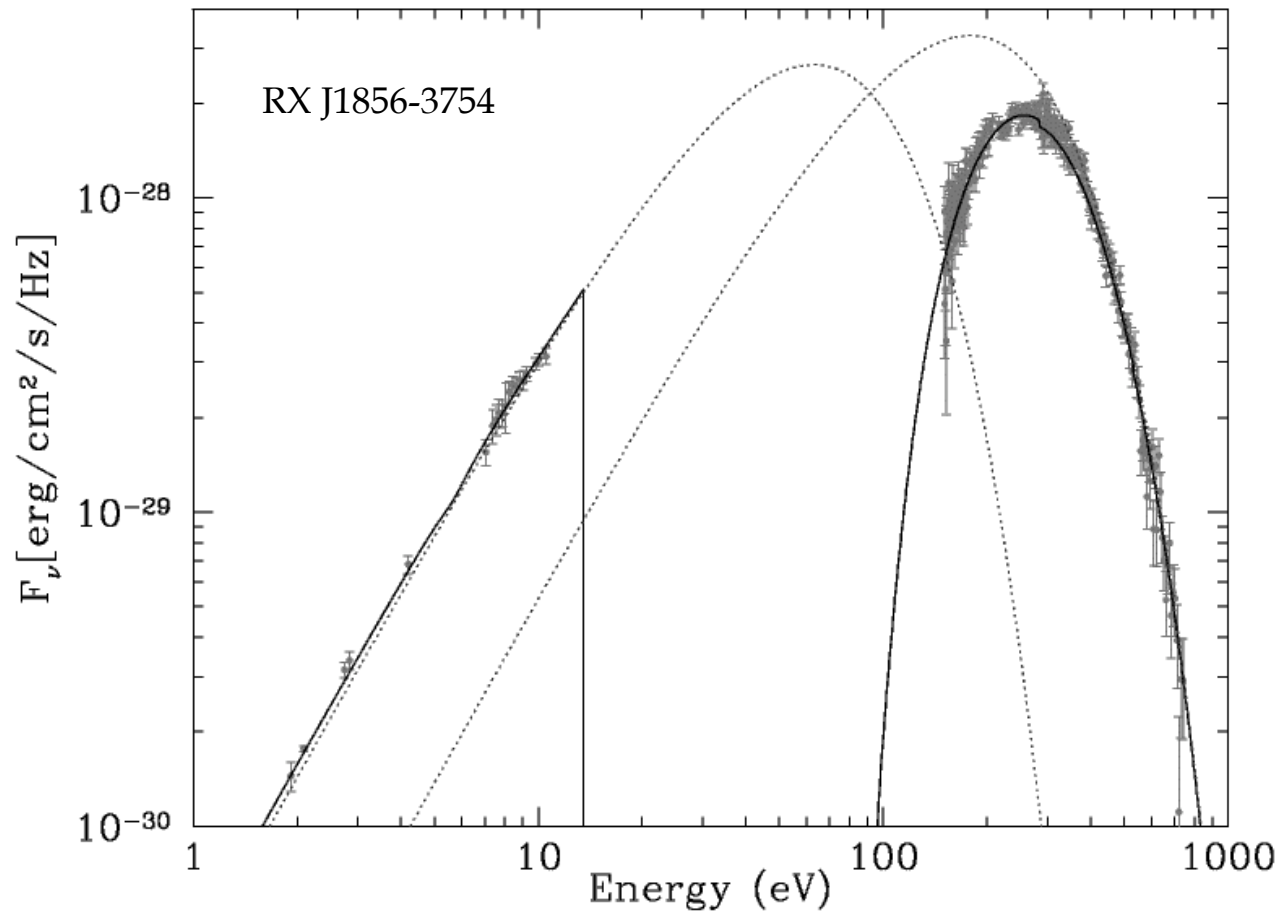
Strong magnetic fields  
--atmospheres complicated  
Non-thermal emission often dominates  
Heavy elements may not be present  
-- redshifted lines unlikely  
Surface emission non-uniform

Heavy elements  
--atmospheres complicated

Accretion luminosity can be high

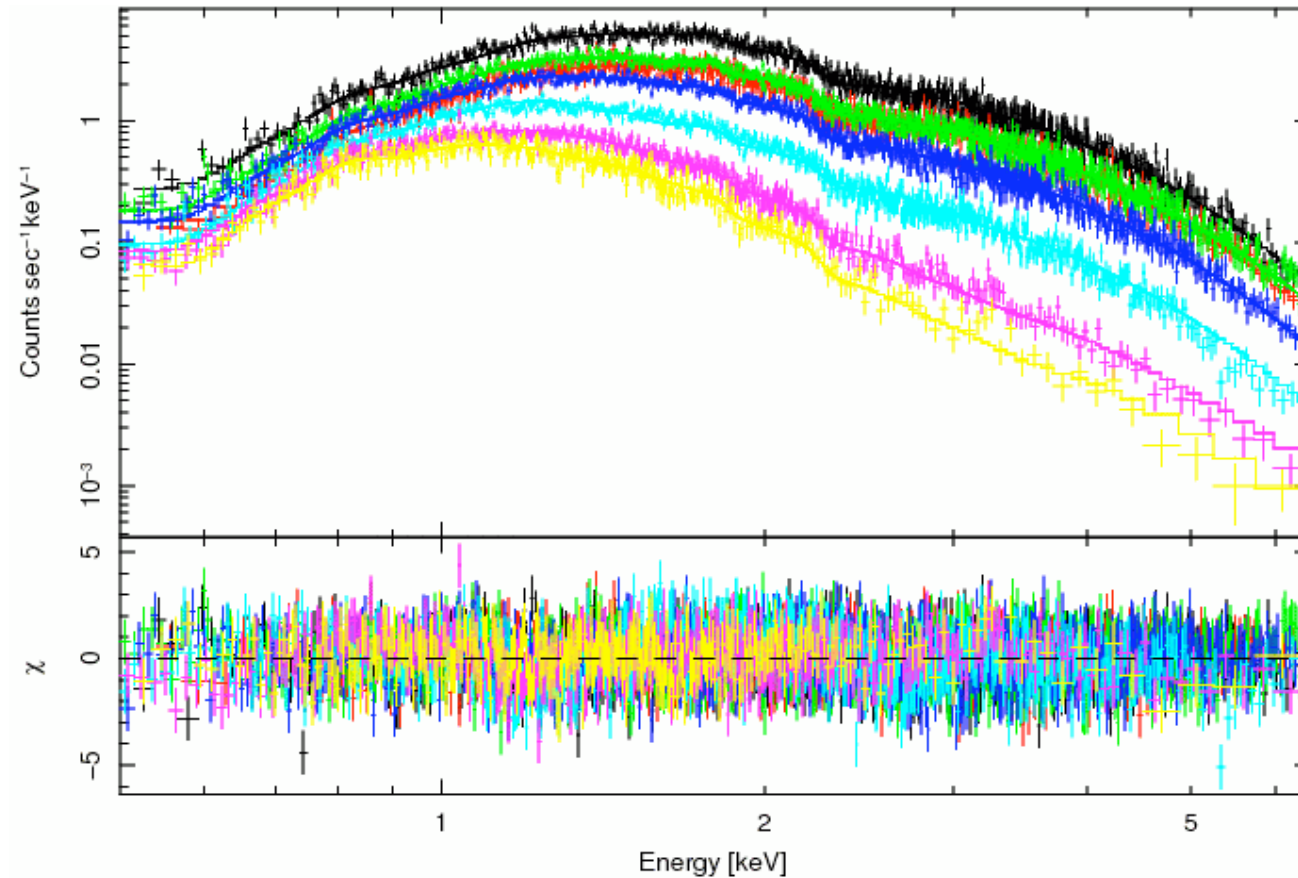


# Spectrum of an Isolated Source

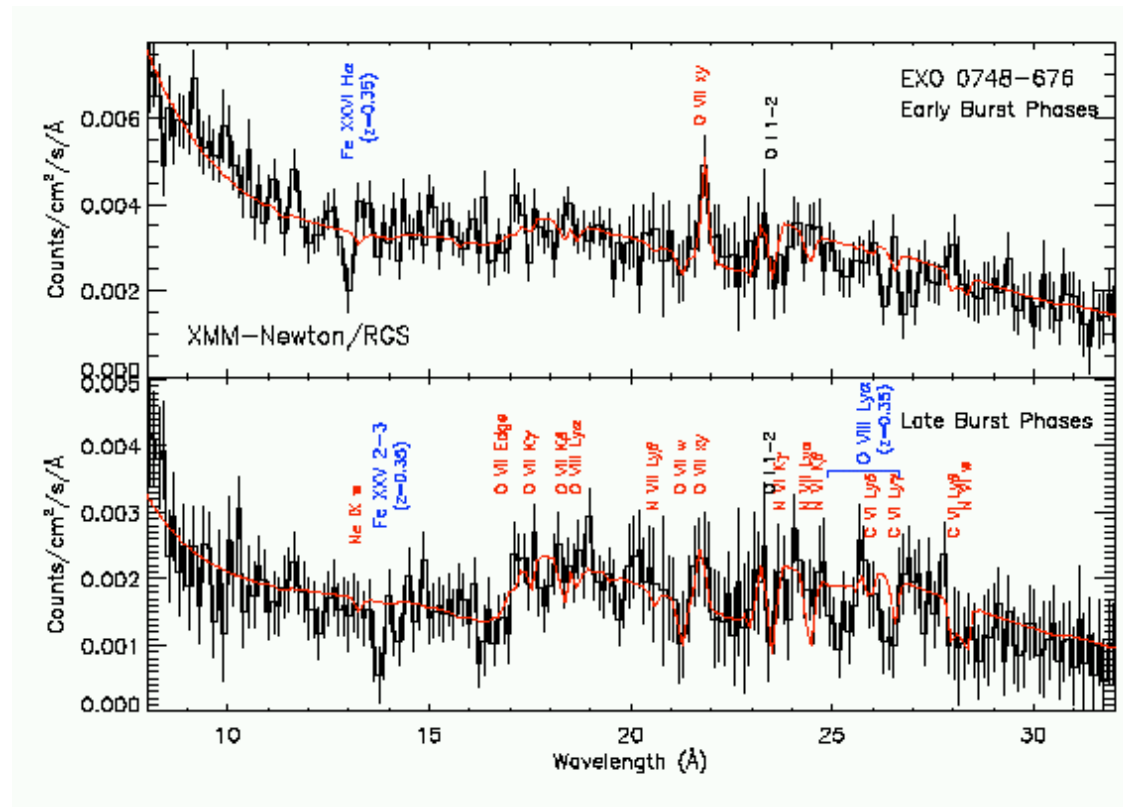


# Spectrum of an Ultramagnetic Source

Seven epochs of XMM data on XTE J1810-197

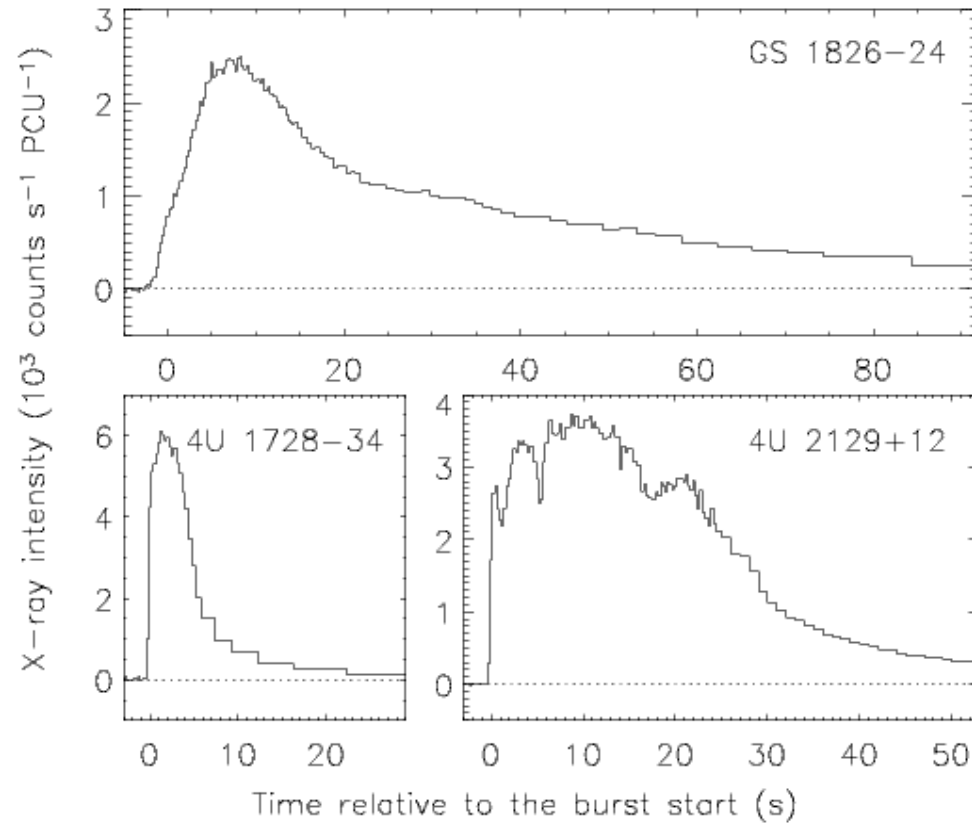


# Atomic Lines in Accreting Sources



Cottam et al. 2003

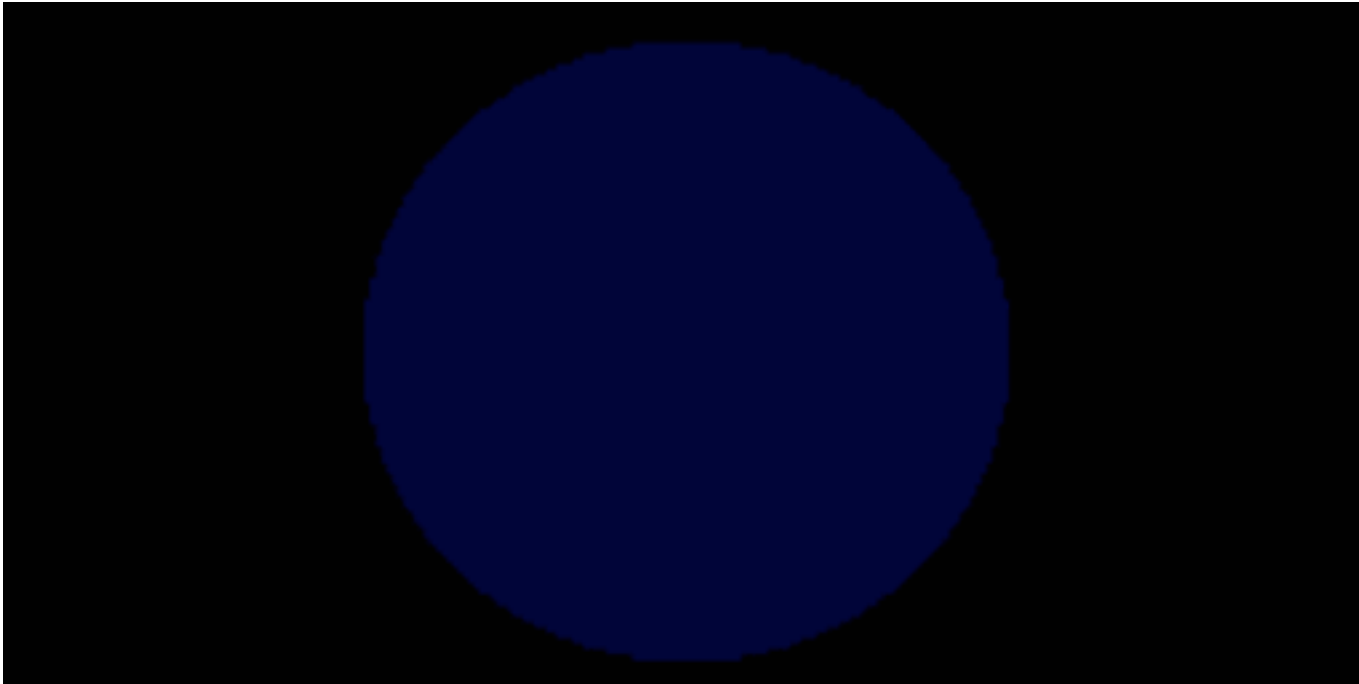
# Thermonuclear Bursts of Low-Mass X-ray Binaries



Sample lightcurves, with different durations and shapes.

Spectra look pretty featureless and are traditionally fit with blackbodies of  $kT \sim \text{few keV}$ .

# Thermonuclear Bursts

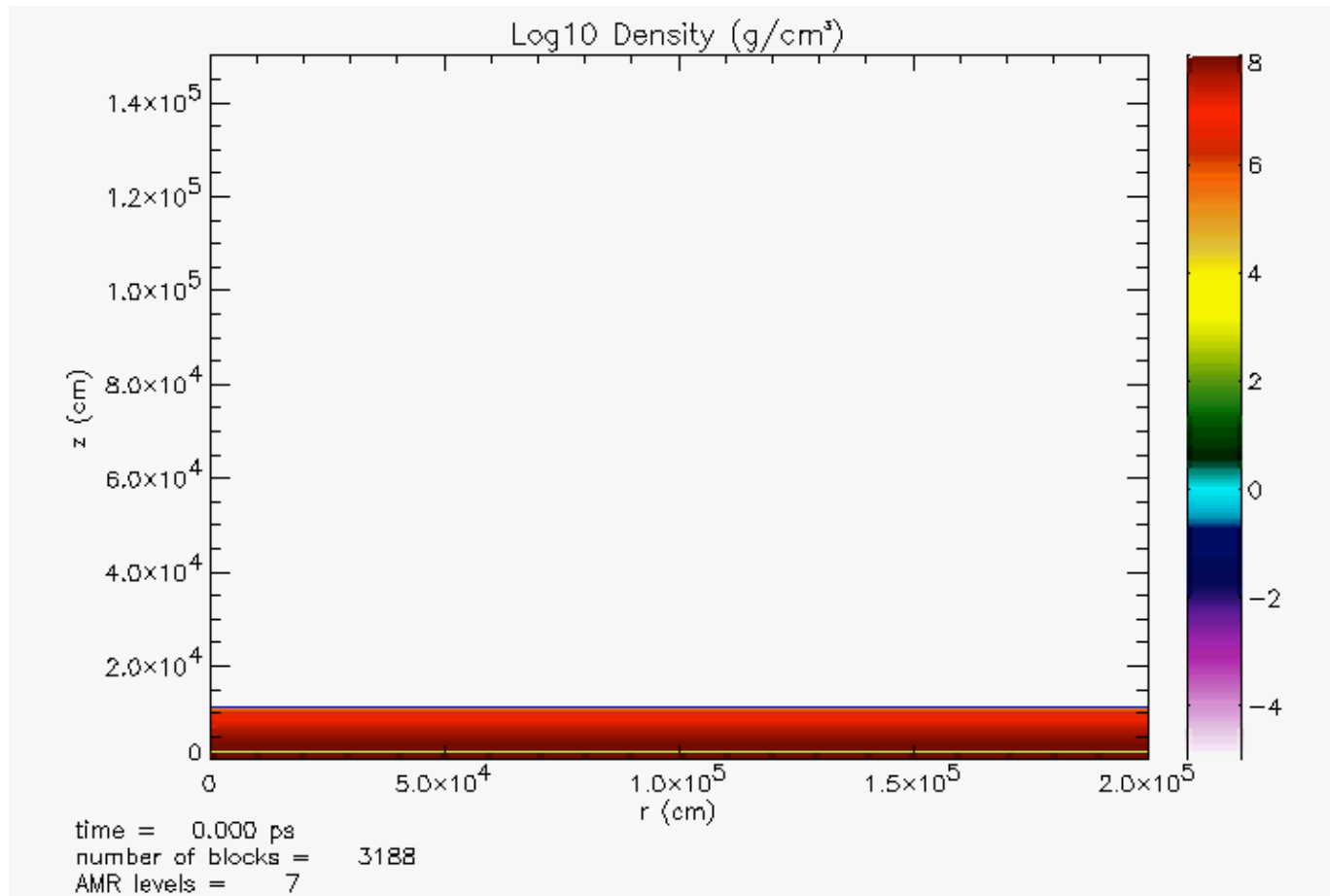


from Spitkovsky et al.

**Burst proceeding by deflagration**

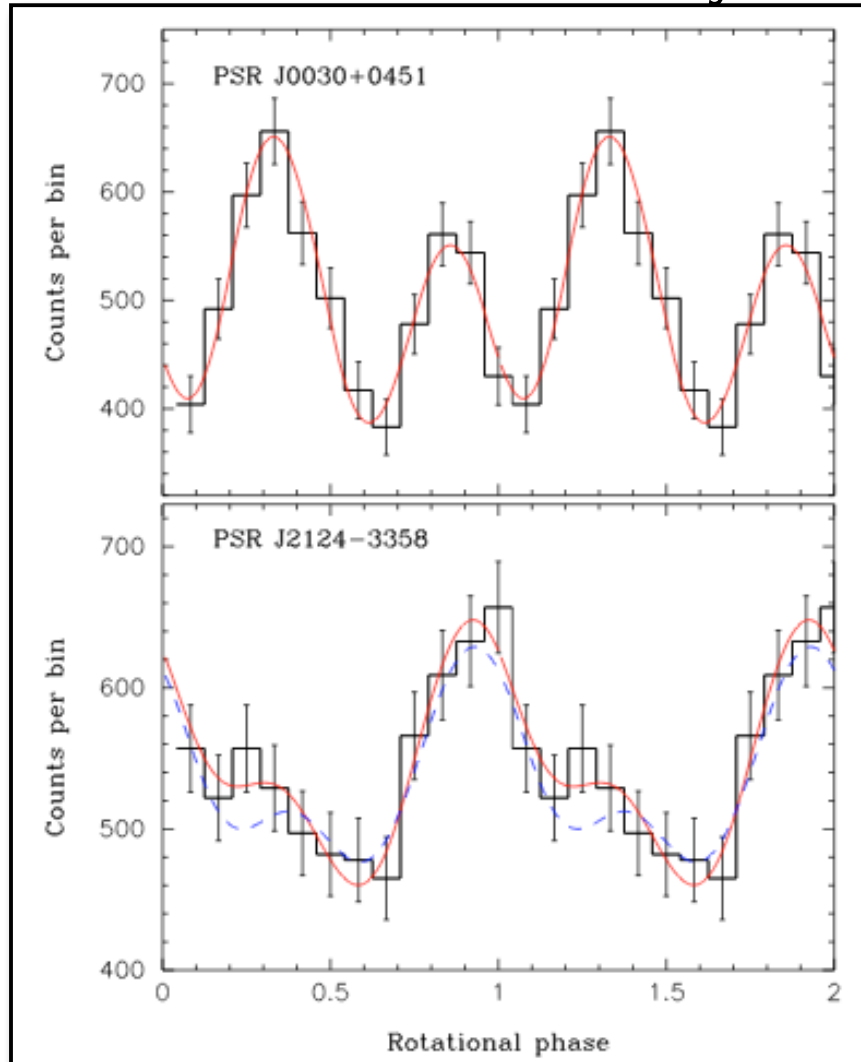
**Bursts propagate and engulf the neutron star at  $t \ll 1$  s.**

# Thermonuclear Bursts



from Zingale et al.

# Isolated Millisecond X-ray Pulsars



Assuming  $M=1.4$

$R > 9.4, 7.8$  km

for different  
sources

Bogdanov, Grindlay, & Rybicki 2008

Accreting ms pulsar profiles: Poutanen et al. 2004

**Question: Are we seeing the whole NS surface?**

X-ray Pulsars: No, by definition

Isolated thermal emitters: Perhaps, sometimes



# Thermonuclear Bursts

Theoretical reasons to think that the emission is uniform and reproducible

Magnetic fields of bursters are dynamically unimportant

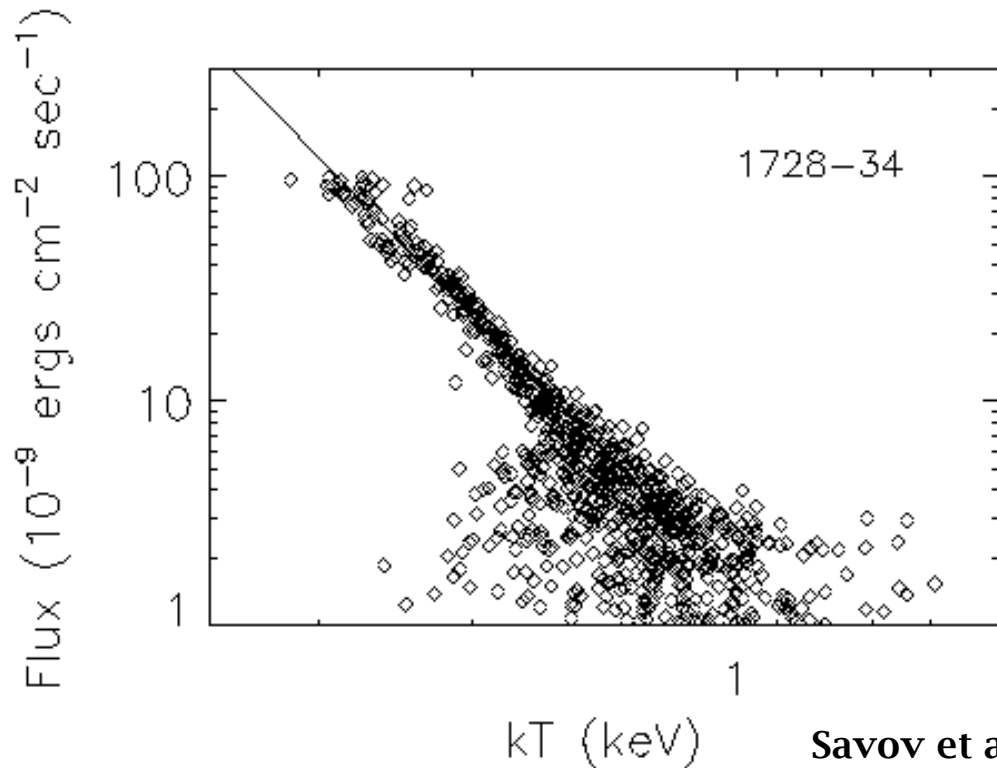
(for EXO 0748: Loeb 2003)

--> fuel spreads over the entire star

Bursts propagate rapidly and burn the entire fuel

# Constant Emitting Area in Bursts

Constant inferred radius from  $\frac{F_{\text{cool}}}{\sigma T_c^4}$



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## Compact Objects

**Neutron Star Observables, Masses, Radii and Magnetic Fields**

**Lecture 2: Neutron Star Observables to Interiors**

# Neutron Star Structure and Equation of State

Structure of a (non-rotating) star in Newtonian gravity:

$$\frac{dM(r)}{dr} = 4\pi r^2 \rho(r) \quad \leftarrow \quad M(r) = \int_0^r 4\pi r^2 \rho(r) dr$$

(enclosed mass)

$$\frac{dP(r)}{dr} = -\frac{GM(r)}{r^2} \rho(r)$$

Need a third equation relating  $P(r)$  and  $\rho(r)$  (called the equation of state --EOS)

$$P = P(\rho)$$

Solve for the three unknowns  $M$ ,  $P$ ,  $\rho$

## Equations in General Relativity:

$$\left. \begin{aligned} \frac{dM(r)}{dr} &= 4\pi r^2 \rho(r) \\ \frac{dP(r)}{dr} &= -\frac{G\left[M(r) + 4\pi r^3 P(r)\right]}{r^2\left[1 - \frac{2GM(r)}{rc^2}\right]} \left(\rho(r) + \frac{P}{c^2}\right) \end{aligned} \right\} \text{Tolman-Oppenheimer-Volkoff Equations}$$

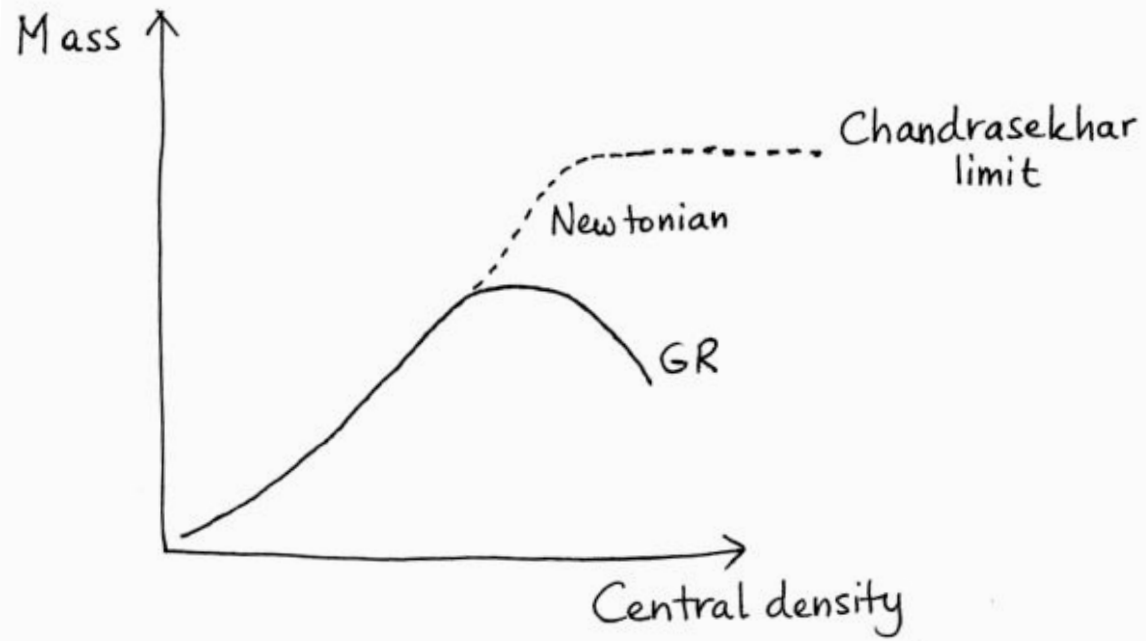
Two important differences between Newtonian and GR equations:

1. Because of the term  $[1-2GM(r)/c^2]$  in the denominator, any part of the star with  $r < 2GM/c^2$  will collapse into a black hole
2. Gravity  $\neq$  mass density  
Gravity = mass density + pressure (because pressure always involves some form of energy)

**Unlike Newtonian gravity, you cannot increase pressure indefinitely to support an arbitrarily large mass**



**Neutron stars have a maximum allowed mass**



# Equation of State of Neutron Star Matter

For *degenerate, ideal, cold Fermi* gas:

$$P \sim \begin{cases} \rho^{5/3} & \text{(non-relativistic neutrons)} \\ \rho^{4/3} & \text{(relativistic neutrons)} \end{cases}$$

Solving Tolman-Oppenheimer-Volkoff equations with this EOS, we get:

$$R \sim M^{-1/3} \quad \longrightarrow \quad \text{As } M \text{ increases, } R \text{ decreases}$$

--- **Maximum Neutron Star mass obtained in this way is  $0.7 M_{\odot}$**

(there would be no neutron stars in nature)

--- **There are lots of reasons why NS matter is *non-ideal***

(so that pressure is not provided only by degenerate neutrons)

**Some additional effects we need to take into account :**

(some of them reduce pressure and thus *soften* the equation of state,  
others increase pressure and *harden* the equation of state)

## I. $\beta$ -stability



In every neutron star,  $\beta$ -equilibrium implies the presence of ~1-10% fraction of protons, and therefore electrons to ensure charge neutrality.

## II. The Strong Force

The force between neutrons and protons (as well as within themselves) has a strong repulsive core. At very high densities, this interaction provides an additional source of pressure. The shape of the potential when many particles are present is very difficult to calculate from first principles, and two approaches have been followed:

- a) The potential energy for the interaction between 2-, 3-, 4-, .. particles is parametrized and the parameter values are obtained by fitting nucleon-nucleon scattering data.
- b) A mean-field Lagrangian is written for the interaction between many nucleons and its parameters are obtained empirically from comparison to the binding energies of normal nucleons.

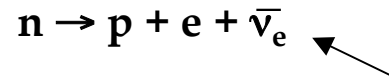
## III. Isospin Symmetry

The Pauli exclusion principle makes it energetically favorable for a system of nucleons to have approximately equal number of protons and neutrons. In neutron stars, there is a significant difference between the neutron and proton fraction and this costs energy. This interaction energy is usually added to the theory using empirical formulae that reproduce the (A,Z) relation of stable nuclei.



#### IV. Presence of Bosons, Hyperons, Condensates

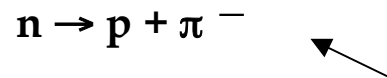
As we saw, neutrons can decay via the  $\beta$ -decay



yielding a relation between the chemical potentials of n, p, and e:

$$\mu_n - \mu_p = \mu_e$$

And they can also decay through a different channel

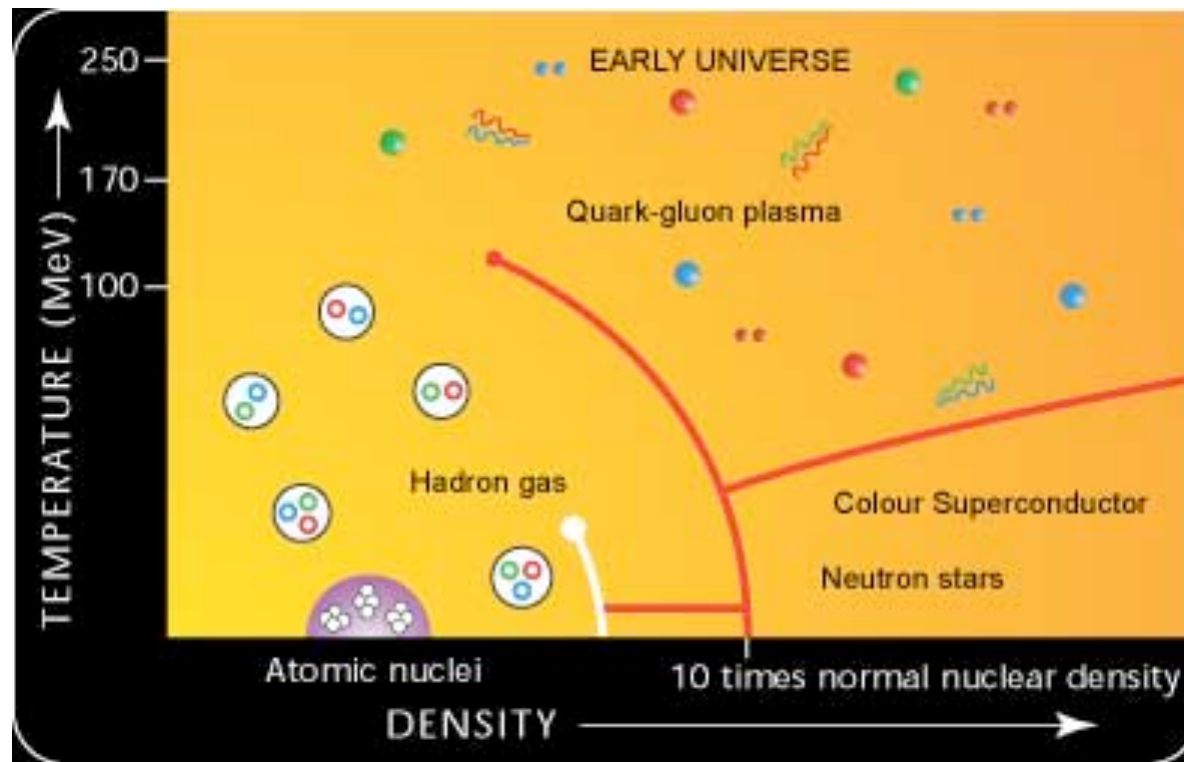


when the Fermi energy of neutrons exceeds the pion rest mass

$$E_{F,n} \approx m_\pi c^2 \approx 140 \text{ MeV}$$

Because pions are bosons and thus follow Bose-Einstein statistics ==> can condense to the ground state. This releases some of the pressure that would result from adding additional baryons and softens the equation of state. The overall effect of a condensate is to produce a “kink” in the M-R relation:

## V. Quark Matter or Strange Matter

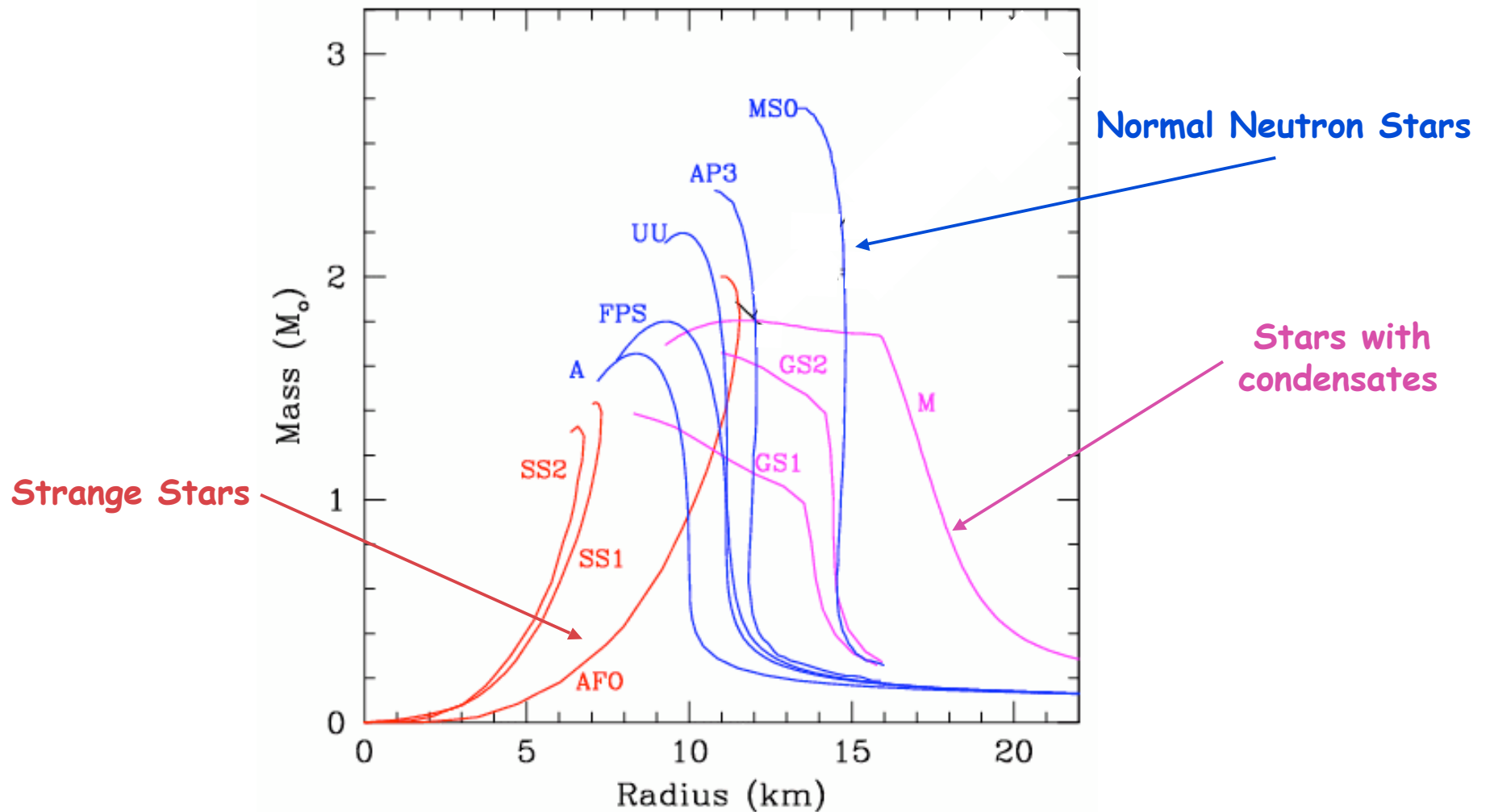


Exceeding a certain density, matter may preferentially be in the form of free (unconfined) quarks. In addition, because the strange quark mass is close to u and d quarks, the “soup” may contain u, d, and s.

Quark/hybrid stars: typically refer to a NS whose cores contain a mixed phase of confined and deconfined matter. These stars are bound by gravity.

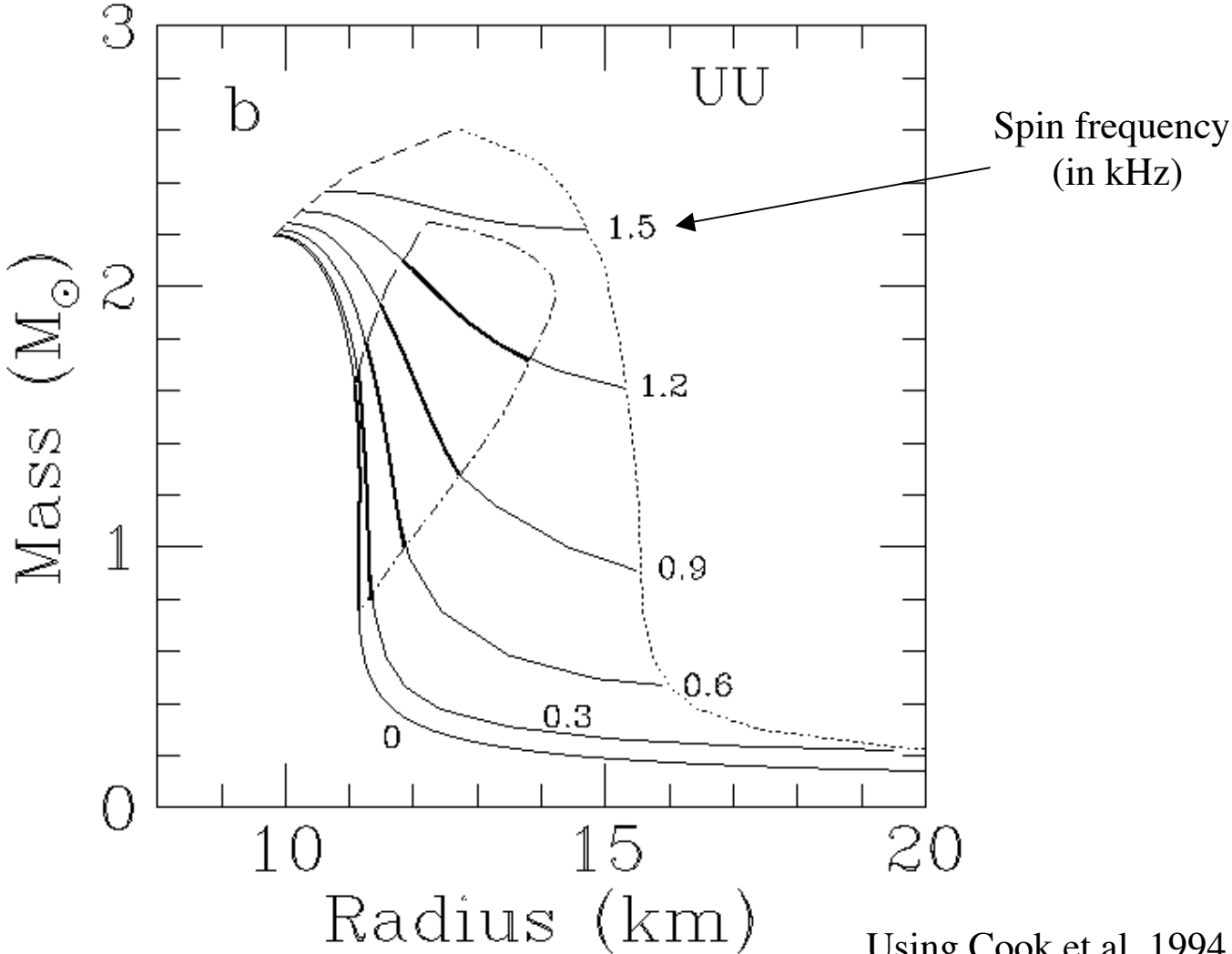
Strange stars: refer to stars that have only unconfined matter, in the form of u, d, and s quarks. These stars are not bound by gravity but are rather one giant nucleus.

# Mass-Radius Relation for Neutron Stars



- We will discuss how accurate M-R measurements are needed to determine the correct EOS. However, even the detection of a massive ( $\sim 2M_{\odot}$ ) neutron star alone can rule out the possibility of boson condensates, the presence of hyperons, etc, all of which have softer EOS and lower maximum masses.

# Effects of Stellar Rotation on Neutron Star Structure

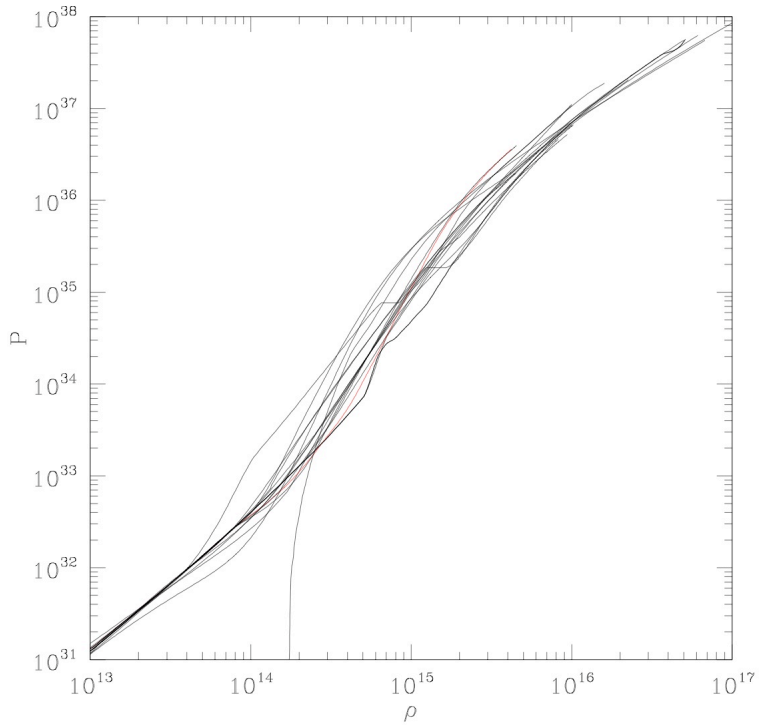
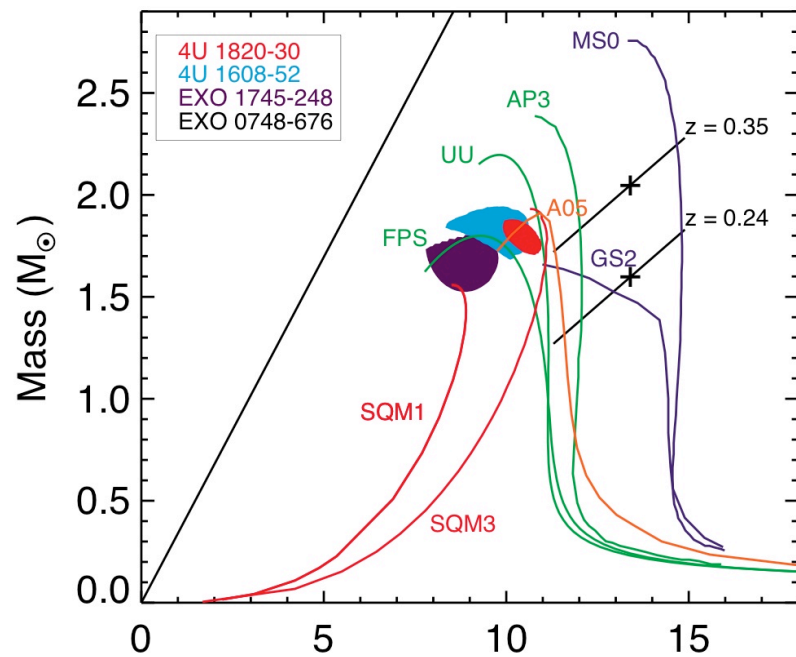


Using Cook et al. 1994

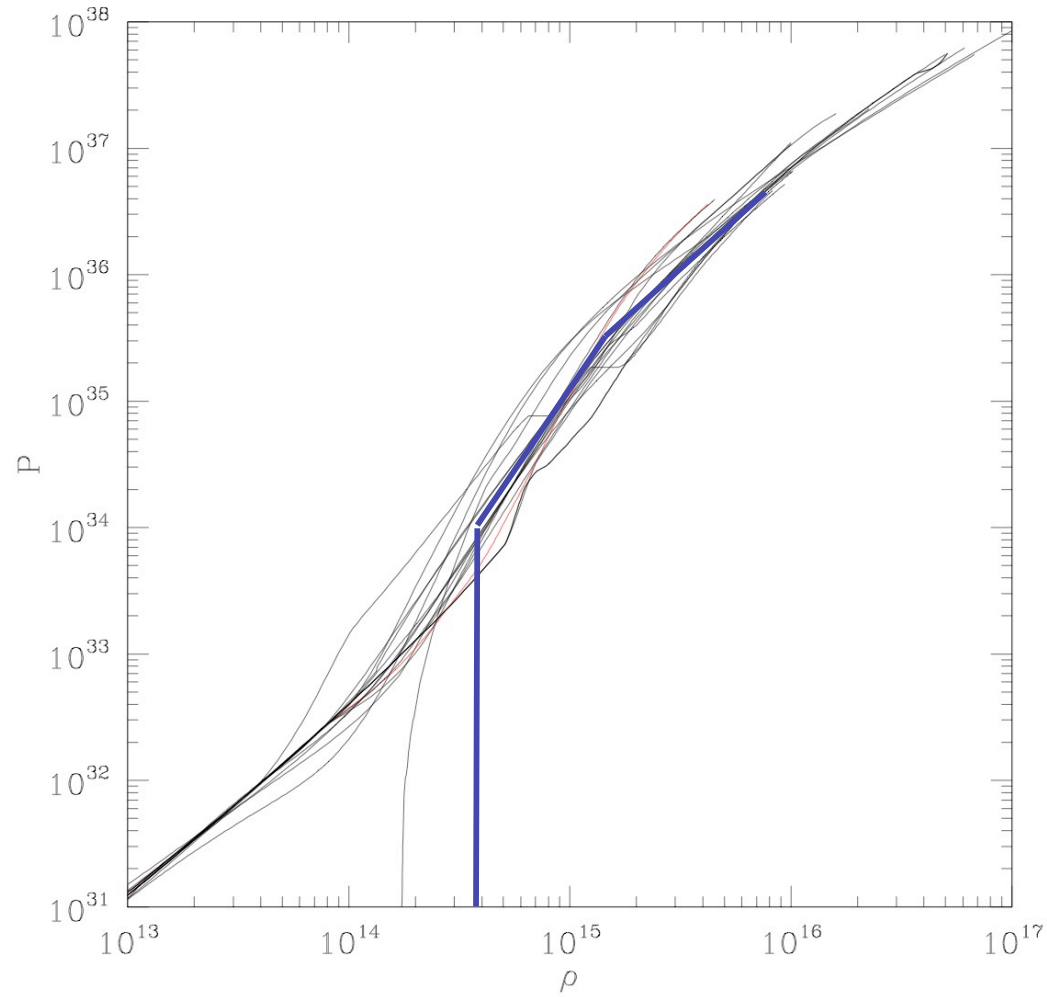
# Effects of Magnetic Field on Neutron Star Structure

Magnetic fields start affecting NS equation of state and structure when  $B \geq 10^{17}$  G. by contributing to the pressure. For most neutron stars, the effect is negligible.

# Reconstructing the Neutron Star Equation of State from Astrophysical Observations



# Parametrizing $P(r)$

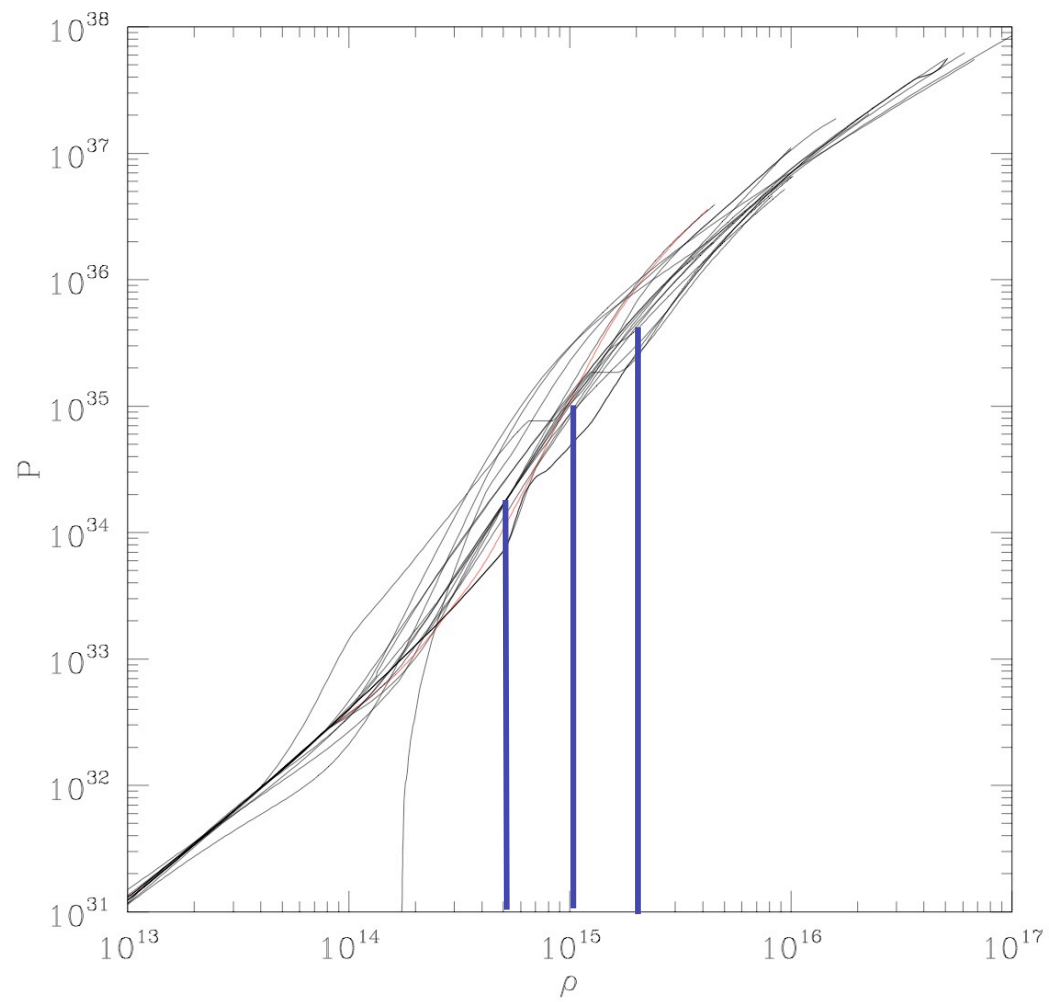


Lattimer & Prakash 2001

Read et al. 2009

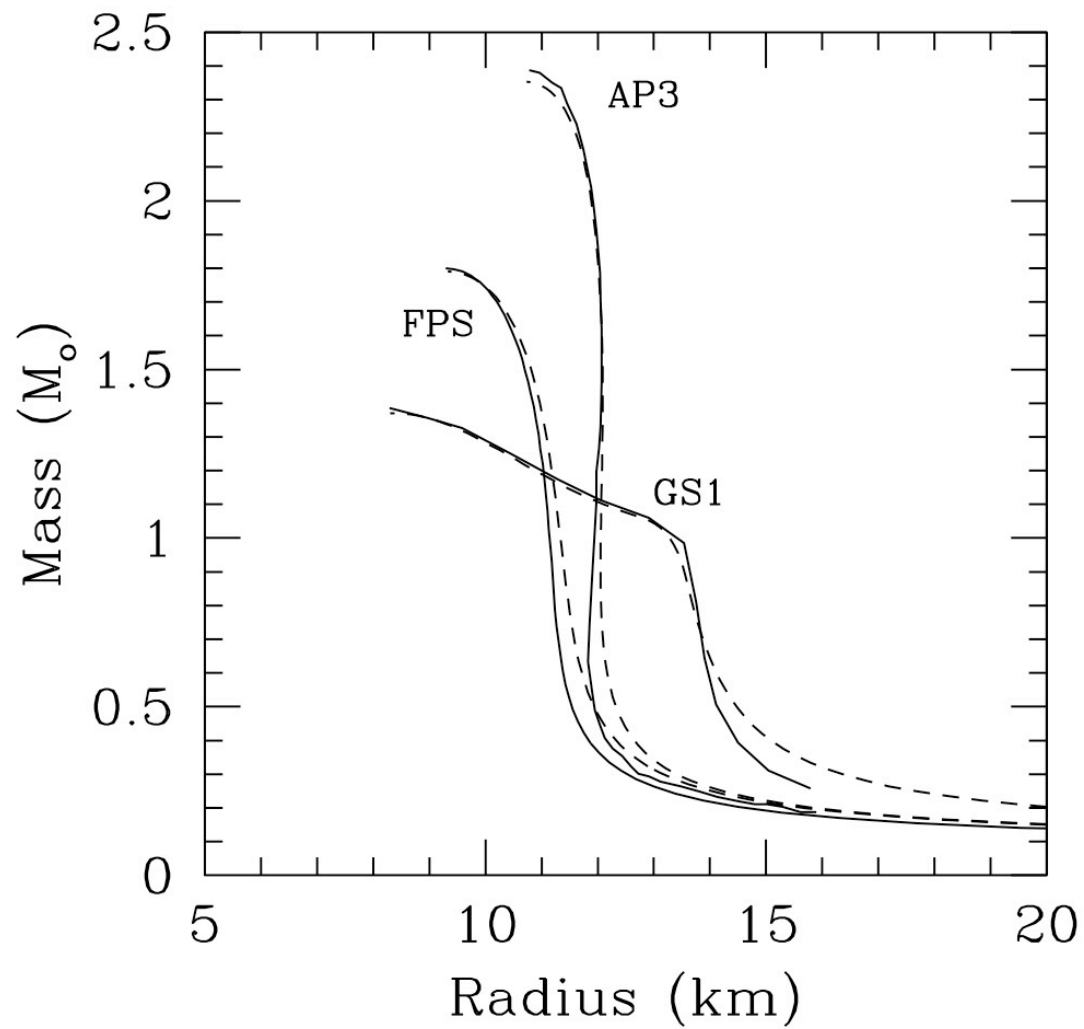
Ozel & Psaltis 2009

# Parametrizing $P(r)$

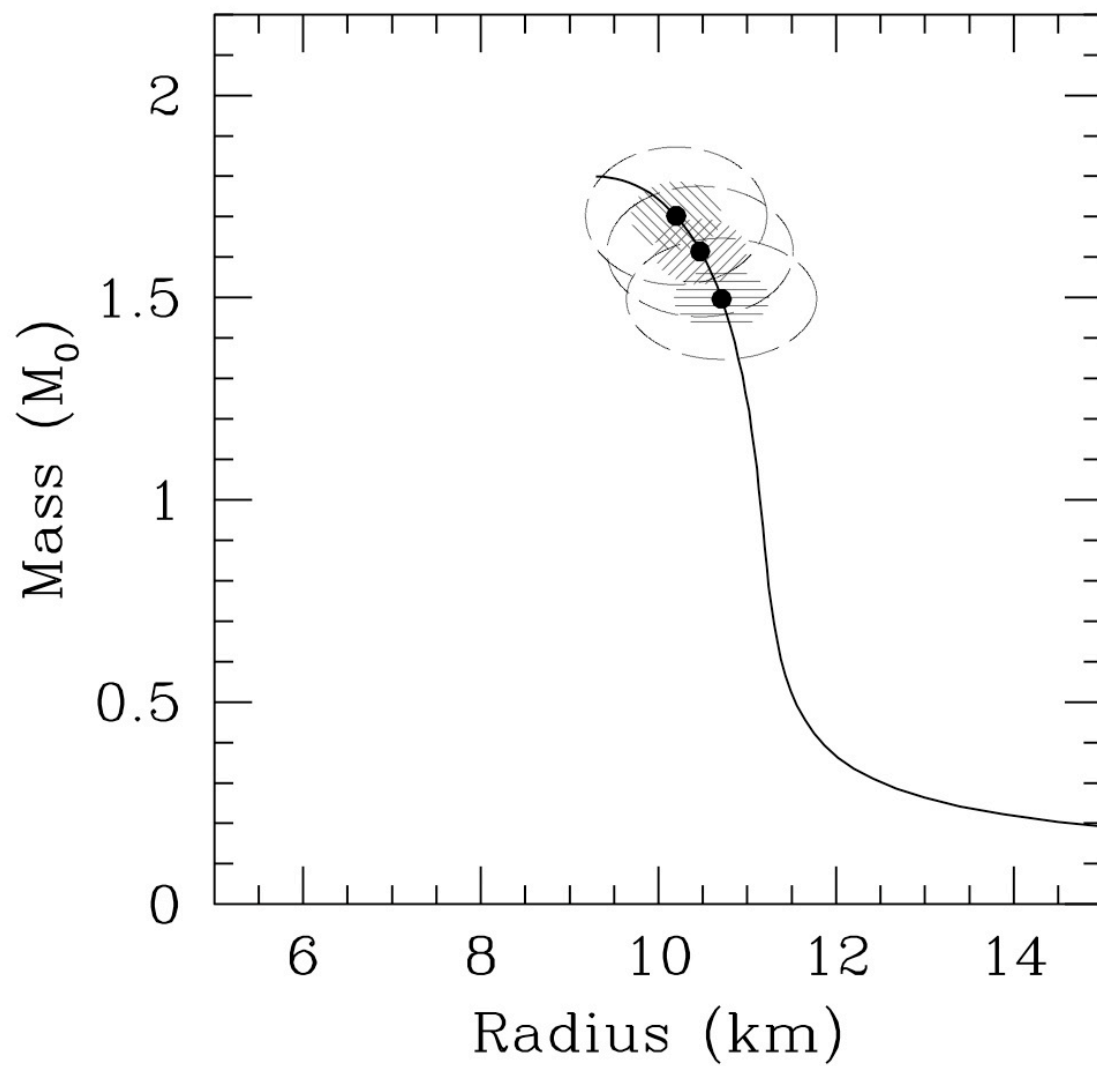




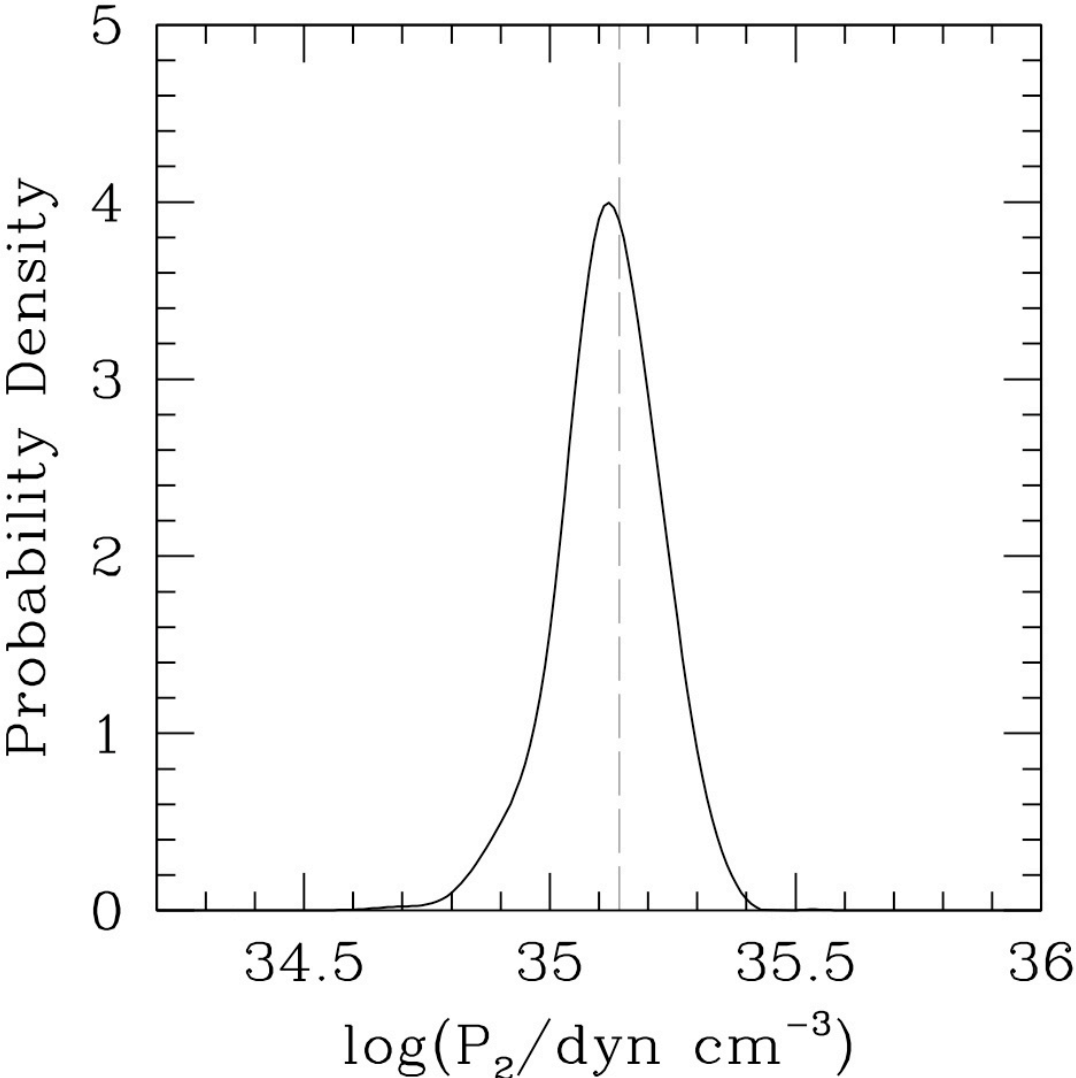
# Parametrized EOS



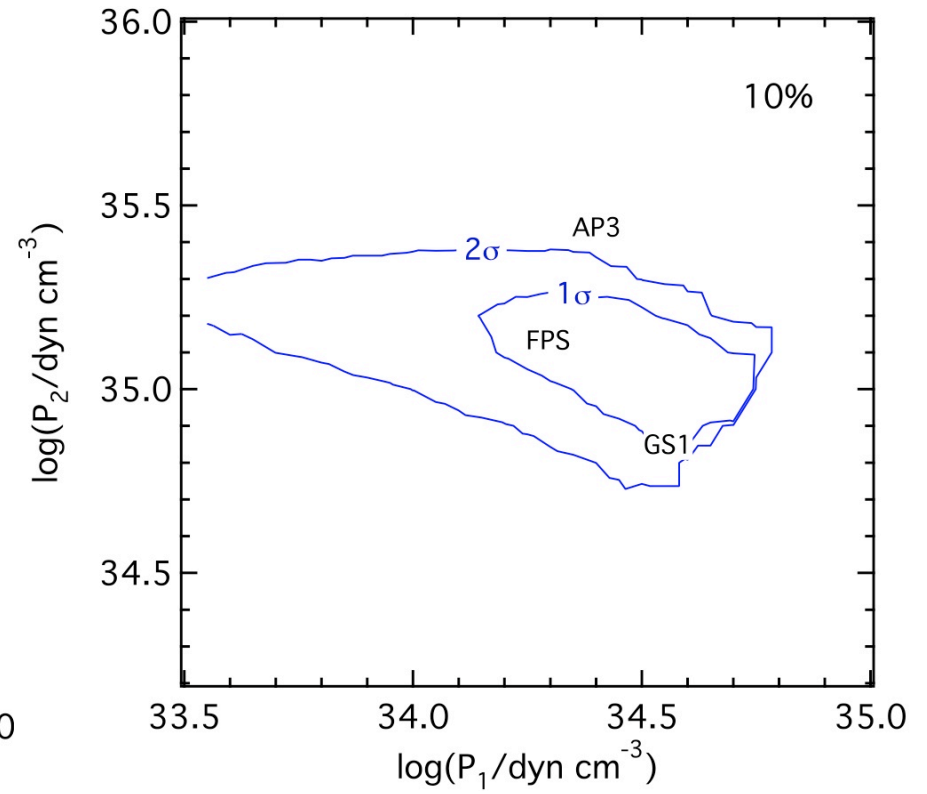
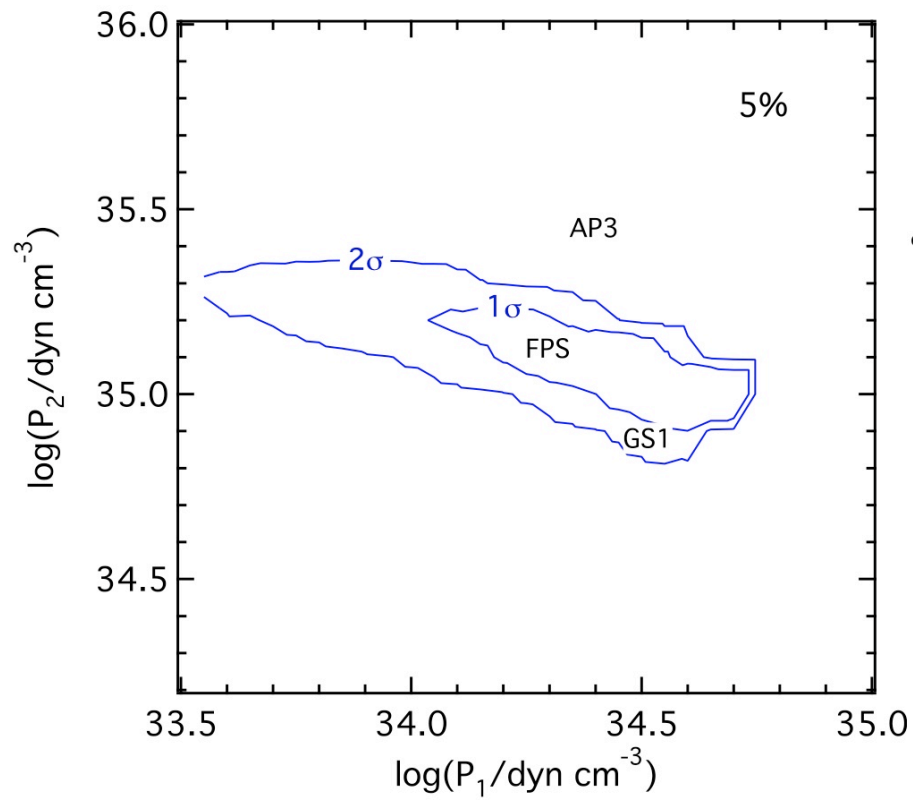
## Simulated Data



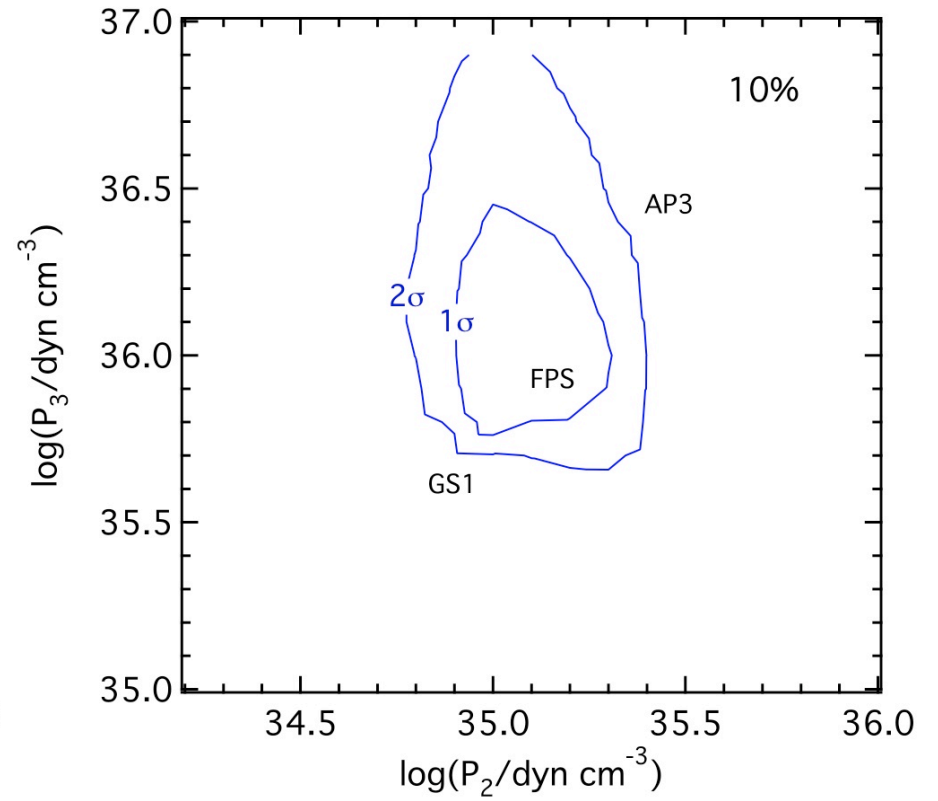
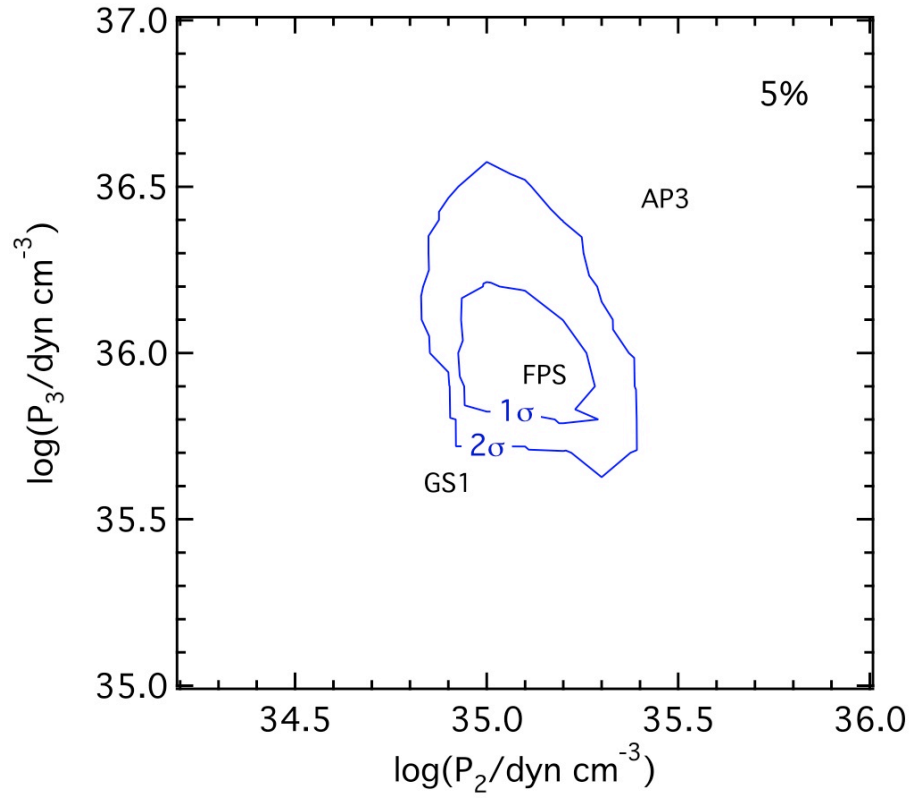
# How Well Can we Measure the Pressure?



# Measured Pressures



# Measured Pressures



## **Methods of Determining NS Mass and/or Radius**

### **More promising methods (entirely in my opinion):**

- Thermal Emission from Neutron Star Surface
- Eddington-limited Phenomena
- Spectral Features

### **Other methods I will discuss at the end:**

- Dynamical mass measurements (very important but mass only)
- Neutron star cooling (provides --fairly uncertain-- limits)
- Quasi Periodic Oscillations
- Glitches (provides limits)
- Maximum spin measurements

## Observables I: Determine M and/or R

Radius for a thermally emitting object from continuum spectra:

$$R^2 = \frac{F D^2}{\sigma T^4}$$

## Observables II: Determine M and/or R

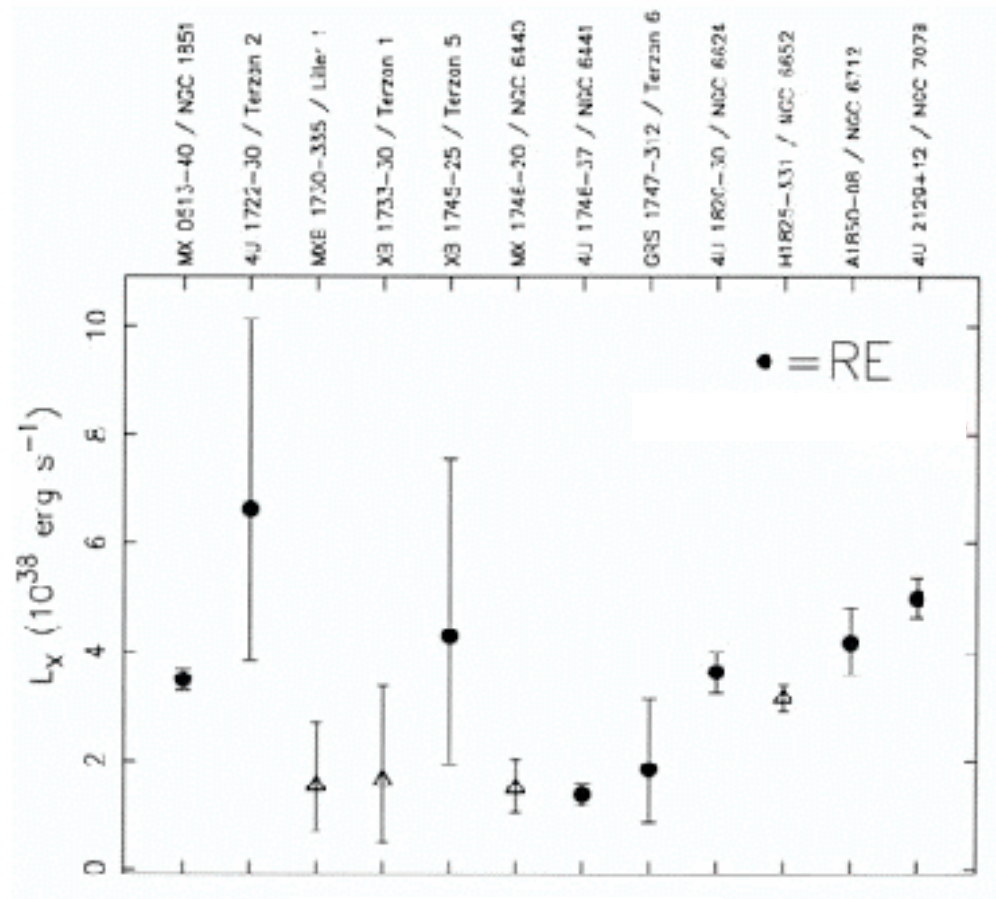
Mass from the Eddington limit:

$$L_{\text{Edd}} = \frac{4 \pi G c M}{\sigma (1+X)}$$

At the Eddington Limit, radiation pressure provides support against gravity



## Observables II: Determine M and/or R



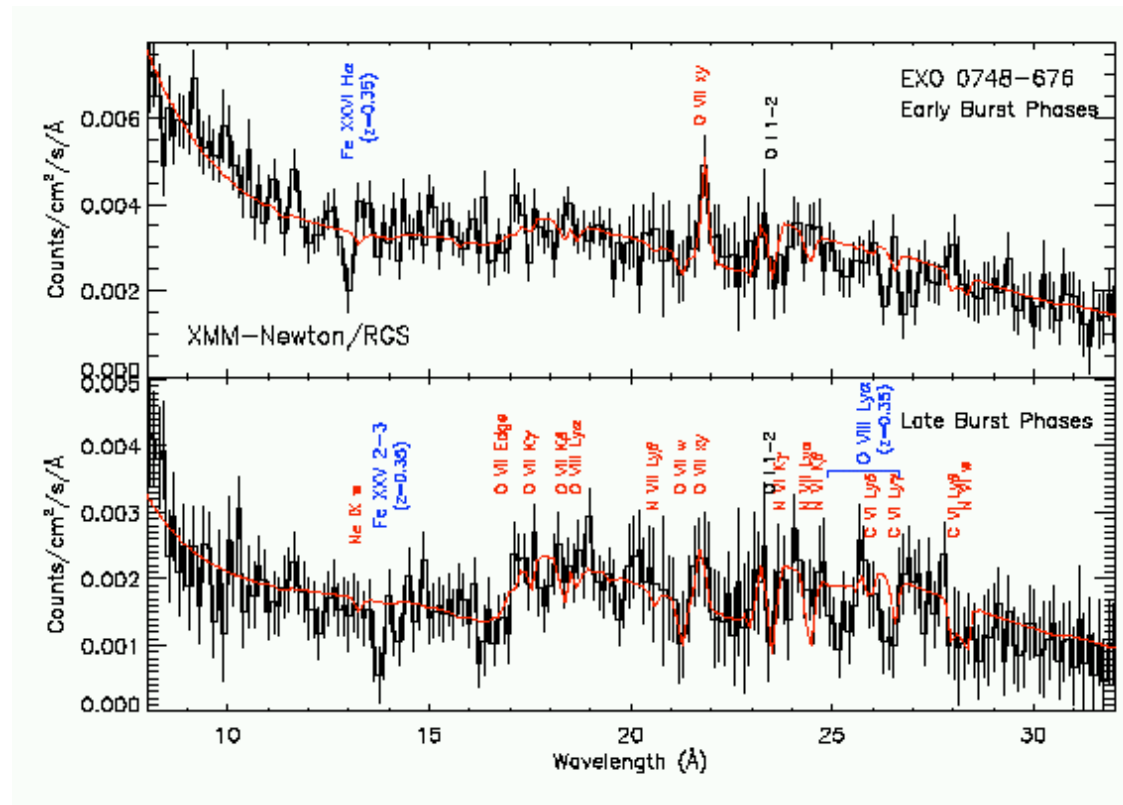
Globular Cluster Burster

Kuulkers et al. 2003

# Observables III: Determine M and/or R

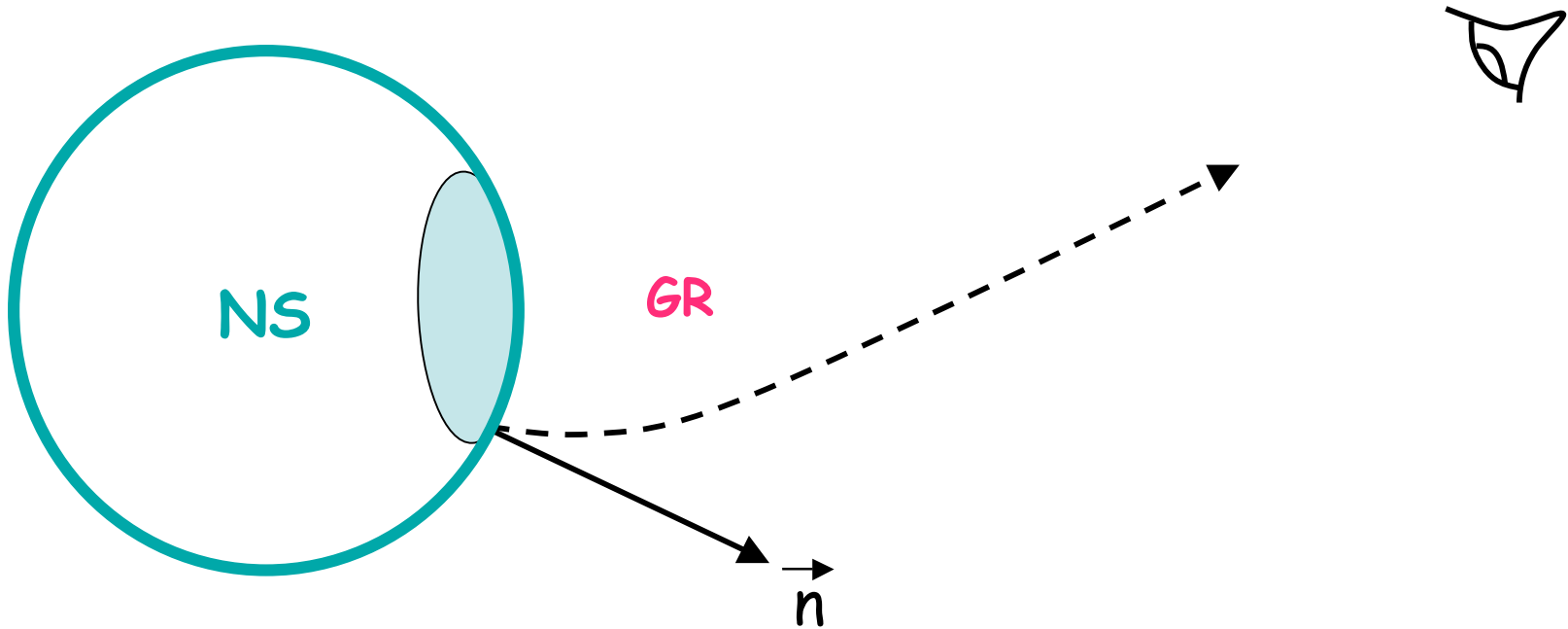
M/R from spectral lines:

$$E = E_0 \left(1 - \frac{2M}{R}\right)$$

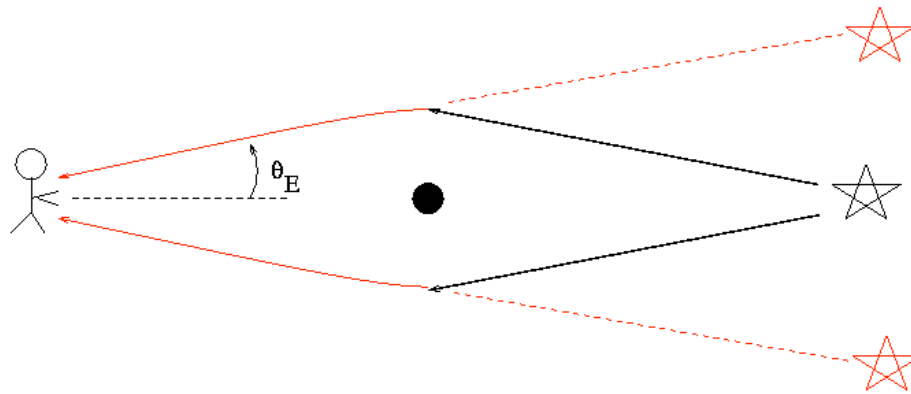


Cottam et al. 2003

In reality, Mass and Radius are always coupled because neutron stars lens their own surface radiation due to their strong gravity



# Gravitational Lensing

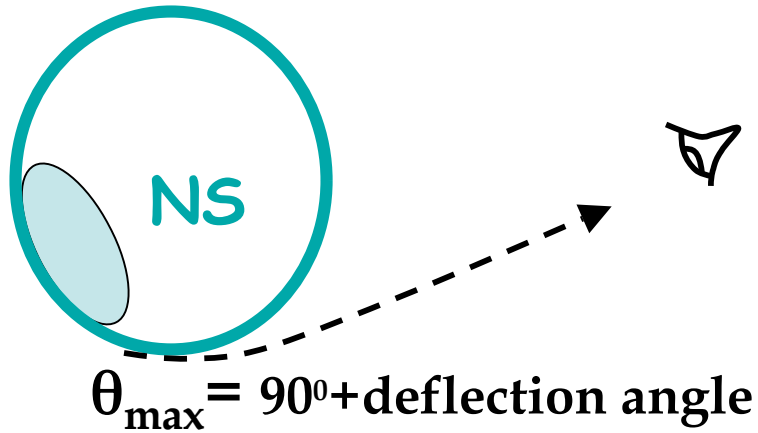


$$\vartheta = \frac{4GM}{c^2 b}$$

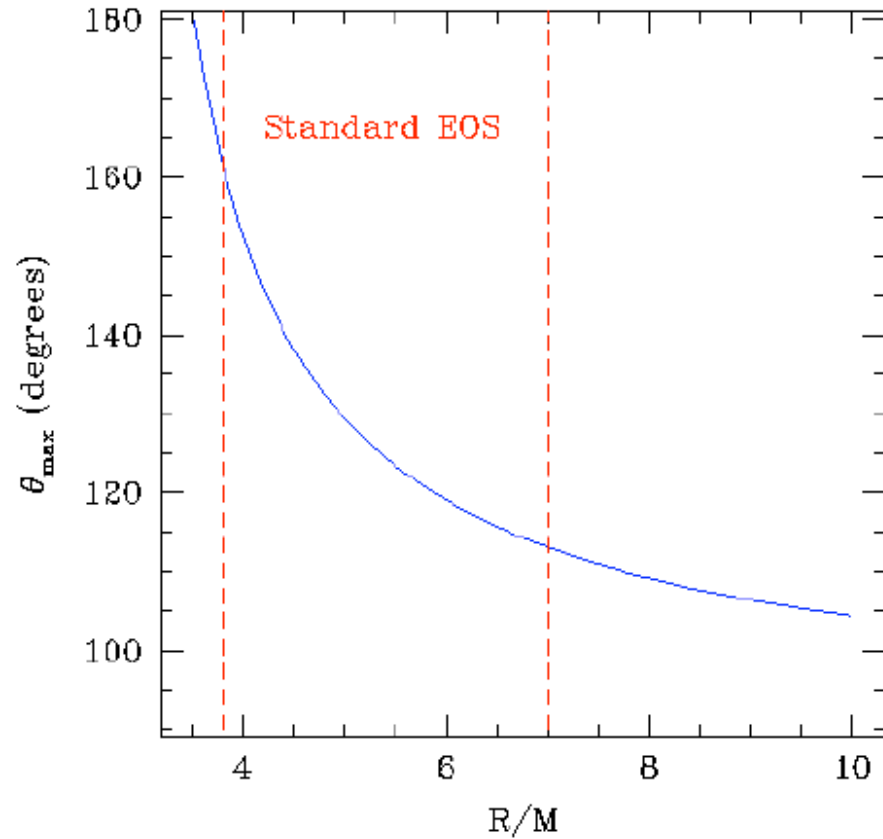
$\theta \rightarrow$  deflection angle

$b \rightarrow$  impact parameter

# Gravitational Self-Lensing



**A perfect ring of radiation:  
→  $R/M = 3.52$**



# Self-Lensing

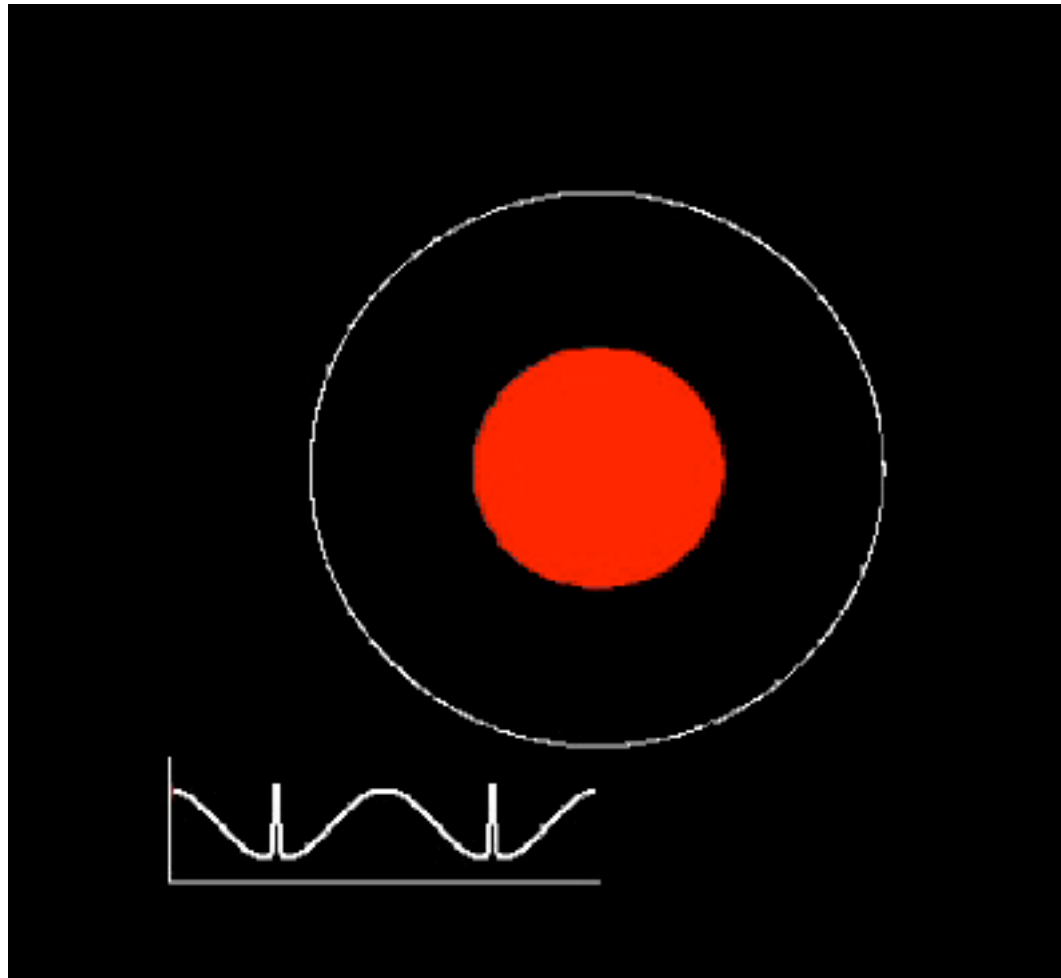
The Schwarzschild metric:

$$ds^2 = dt^2 \left(1 - \frac{2M}{R}\right) - dr^2 \left(1 - \frac{2M}{R}\right)^{-1} - f(\vartheta, \phi)$$

Photons with impact parameters  $b < b_{\max}$  can reach the observer:

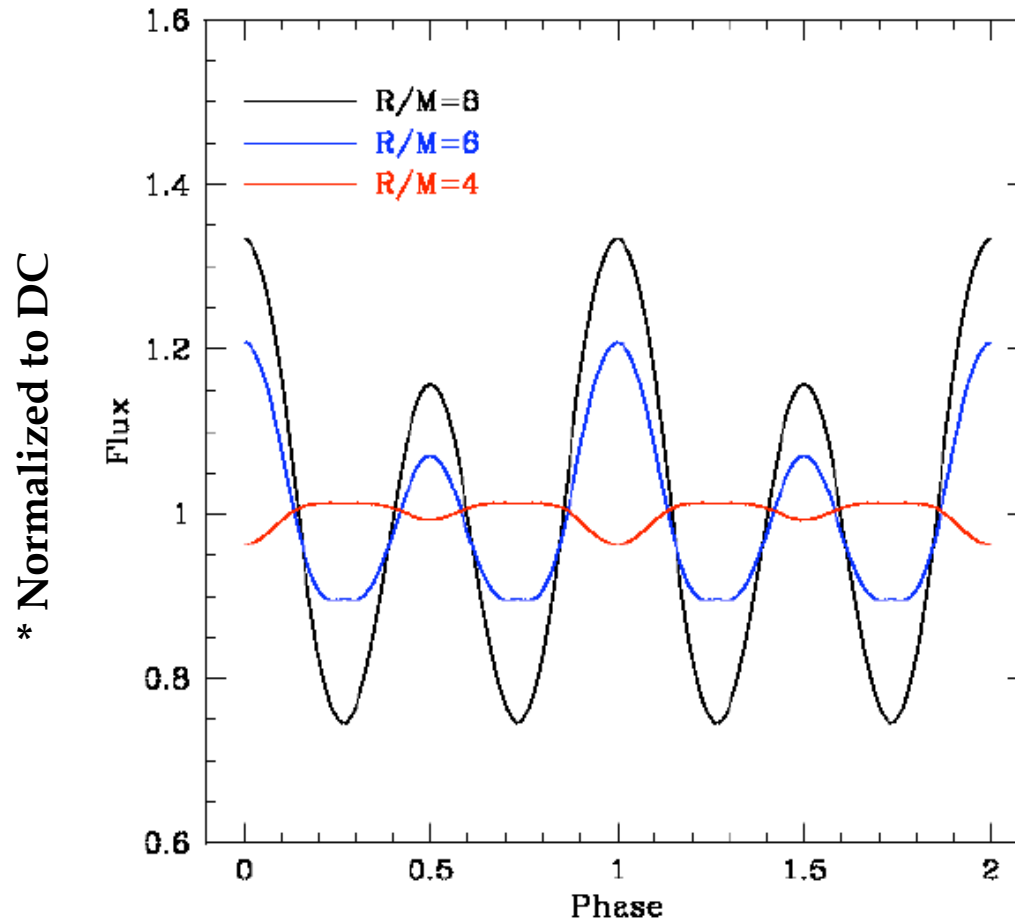
$$b_{\max} = R \left(1 - 2 \frac{M}{R}\right)^{-1/2}$$

## General Relativistic Effects



Lensing of a hot spot on the neutron star surface

# Pulse Amplitudes



Two antipodal hot spots at a 45 degree angle from the rotation axis

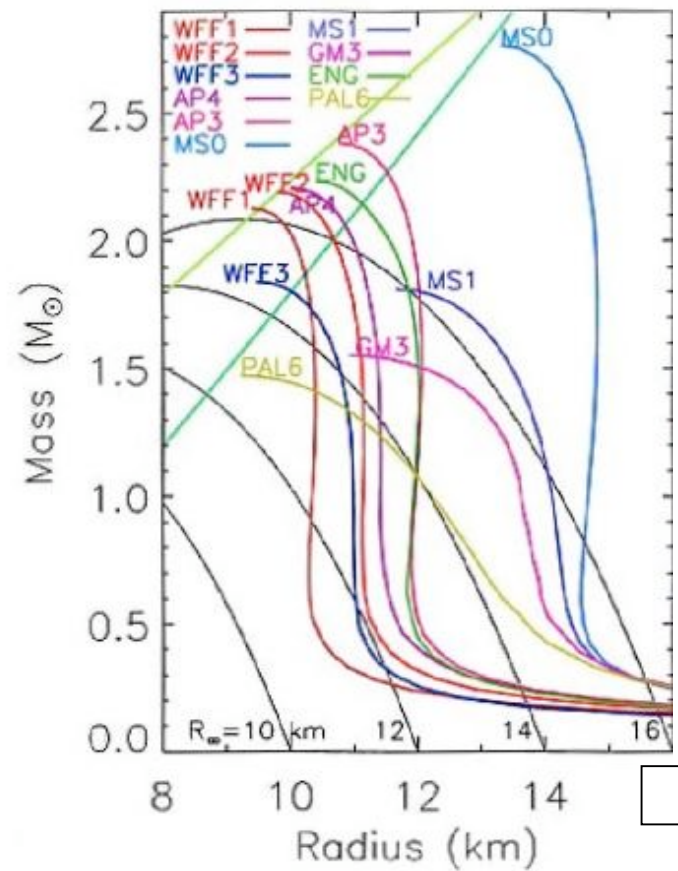
**Note: The pulse amplitudes and shapes make Observable # IV**



# Apparent Radius of a Neutron Star

$$b_{\max} = R \left(1 - 2 \frac{M}{R}\right)^{-1/2}$$

Because of lensing, the apparent radius of neutron stars changes



Lattimer & Prakash 2001

## GR Modifications

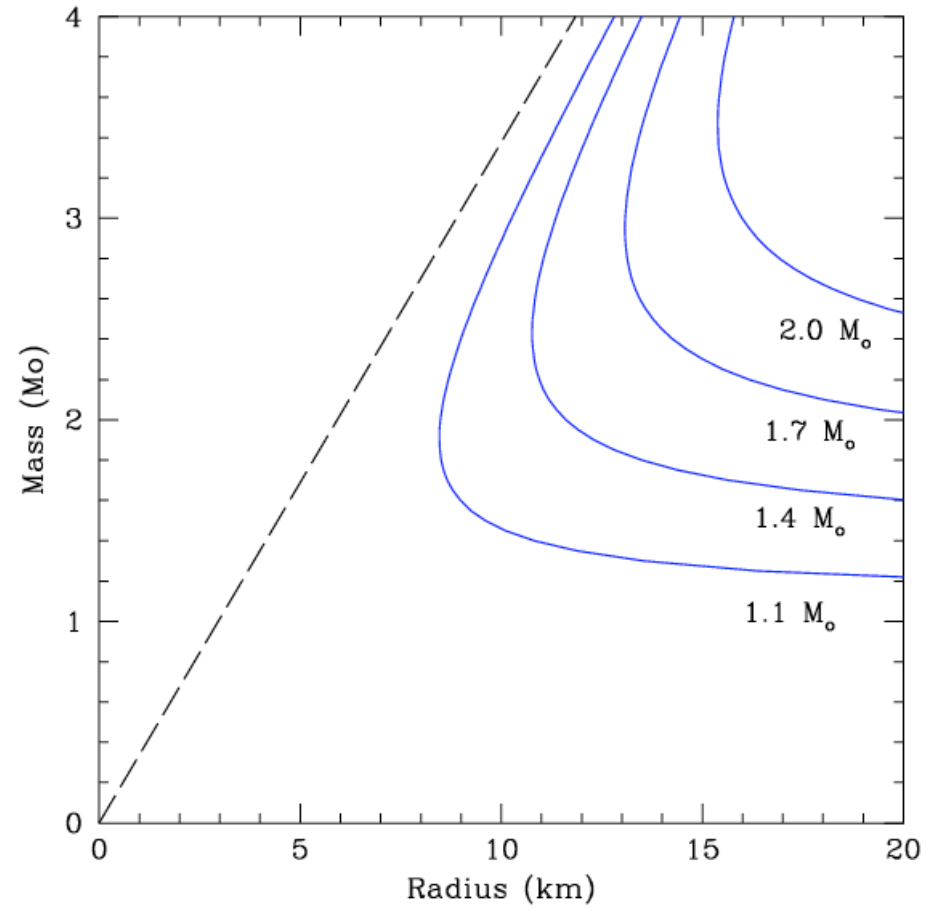
The correct expressions (lowest order)

$$R^2 = \frac{F D^2}{\sigma T^4} \left(1 - \frac{2M}{R}\right)^{-1}$$

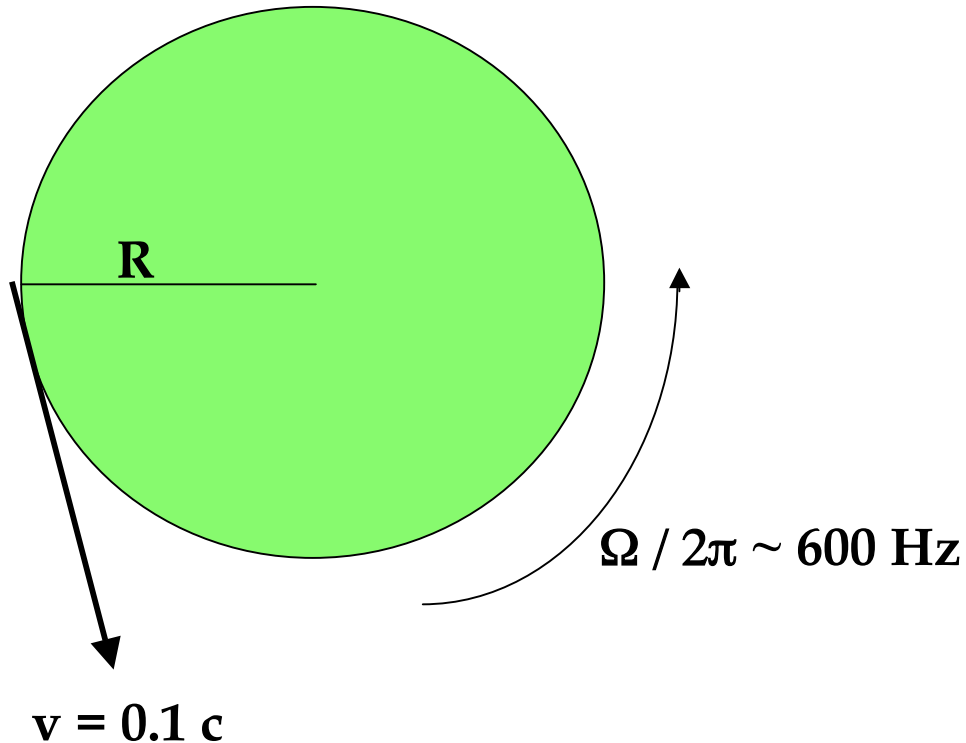
$$L_{\text{Edd}} = \frac{4 \pi G c M}{\sigma (1+X)} \left(1 - \frac{2M}{R}\right)^{1/2}$$

# Effects of GR

## Modifications to the Eddington limit



# What if the NS is rotating rapidly?



$$E_{\infty} = E_0 \gamma (1 + \Omega R/c)$$

**Doppler Boosts**

$$\delta t = \pi/\Omega \sim \pi R/c$$

**Time delays**

Other effects:

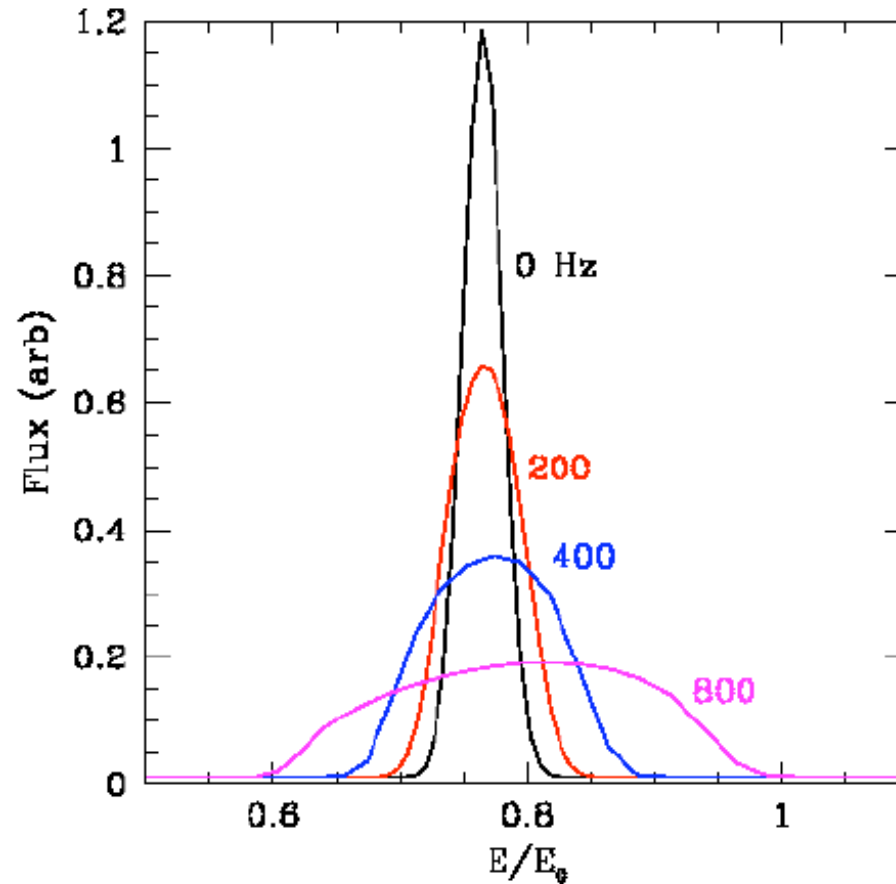
Frame dragging

Oblateness

Equation of State

(Stergioulas, Morsink, Cook)

# Effect of Rotation on Line Widths



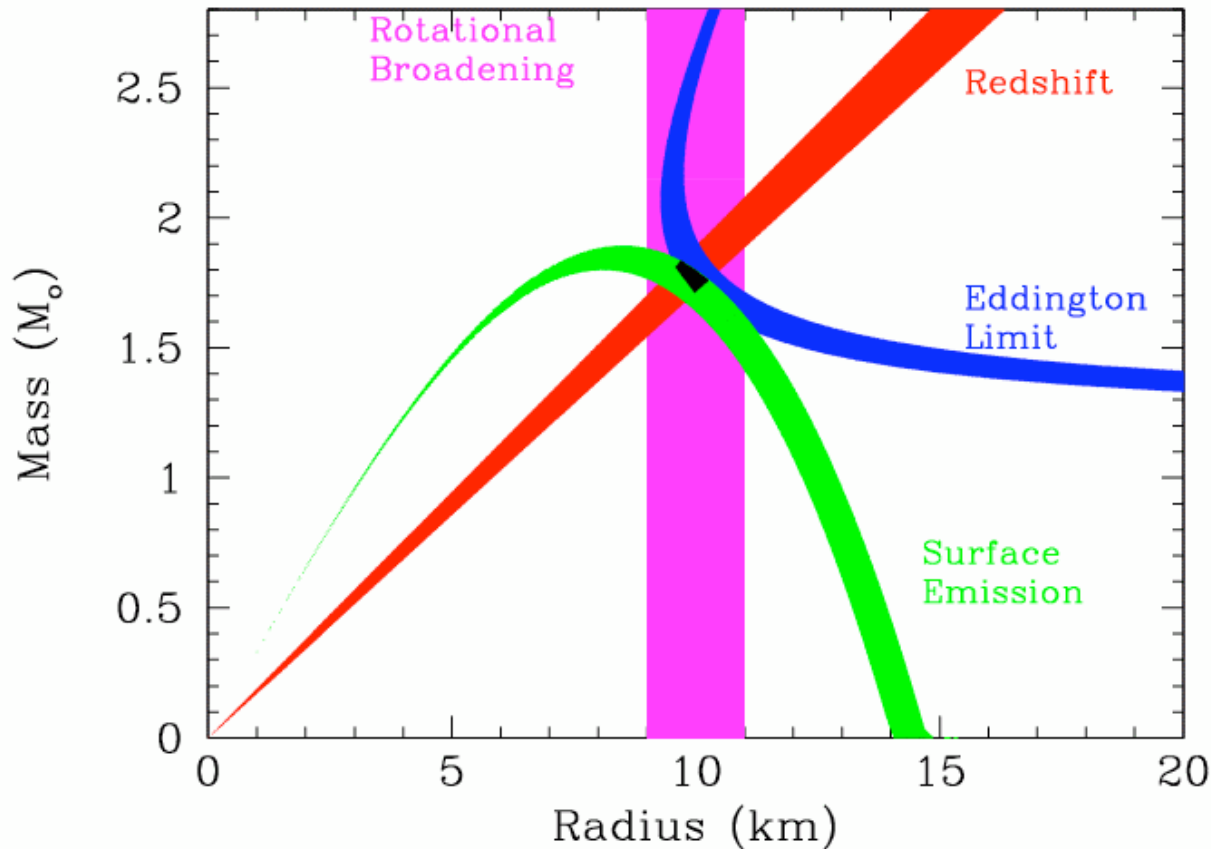
Özel & Psaltis 03

May affect the inferred redshift and detectability **BUT**

$$E/E_0 \rightarrow M/R \quad \text{FWHM} \rightarrow R$$

**Observable # V**

# Determining Mass and Radius



1. The methods have different M-R dependences: they are complementary!
  2. Surface emission gives a maximum NS mass!!
  3. Eddington limit gives a minimum radius!!
- ➔ gravity effects can be undone

## A Unique Solution for Neutron Star M and R

Observable	Dependence on NS Properties
$F_{\text{Edd}}$	$\frac{1}{4\pi D^2} \frac{4\pi G M c}{\kappa_{\text{es}}} \left(1 - \frac{2GM}{c^2 R}\right)^{1/2}$
$z$	$\left(1 - \frac{2GM}{Rc^2}\right)^{-1/2} - 1$
$F_{\text{cool}}/\sigma T_c^4$	$f_\infty^2 \frac{R^2}{D^2} \left(1 - \frac{2GM}{Rc^2}\right)^{-1}$

NS Property	Dependence on Observables
M	$\frac{f_\infty^4 c^4}{4G\kappa_{\text{es}}} \left(\frac{F_{\text{cool}}}{\sigma T_c^4}\right) \frac{[1-(1+z)^{-2}]^2}{(1+z)^3} F_{\text{Edd}}^{-1}$
R	$\frac{f_\infty^4 c^2}{2\kappa_{\text{es}}} \left(\frac{F_{\text{cool}}}{\sigma T_c^4}\right) \frac{1-(1+z)^{-2}}{(1+z)^3} F_{\text{Edd}}^{-1}$
D	$\frac{f_\infty^2 c^2}{2\kappa_{\text{es}}} \left(\frac{F_{\text{cool}}}{\sigma T_c^4}\right)^{1/2} \frac{1-(1+z)^{-2}}{(1+z)^3} F_{\text{Edd}}^{-1}$

**M and R not affected by source inclination because they involve flux ratios**

## **Applying the Methods to Sources:**

For isolated sources: Can use surface emission from cooling to get area contours  
(and possibly a redshift)

For accreting sources: Can possibly apply all these methods, especially if there  
is Eddington limited phenomena



## Good Isolated Candidates

- Nearby neutron stars with no (or very low) pulsations
- No observed non-thermal emission (as in a radio pulsar)
- (Unidentified) spectral absorption features have been observed in some

# Thermonuclear Bursts and Eddington-limited Phenomena

Theoretical reasons to think that the emission is uniform and reproducible

Magnetic fields of bursters (in particular 0748-676) are dynamically unimportant

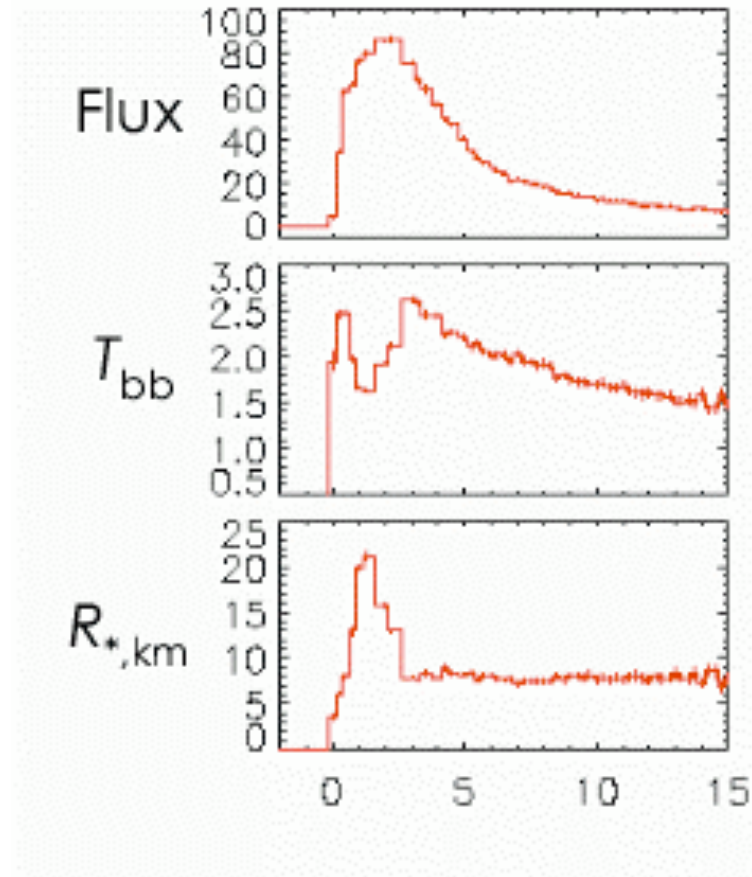
(for EXO 0748: Loeb 2003)

--> fuel spreads over the entire star

Emission from neutron stars during thermonuclear bursts are likely to be uniform and reproducible

# Thermonuclear Bursts and Eddington-limited Phenomena

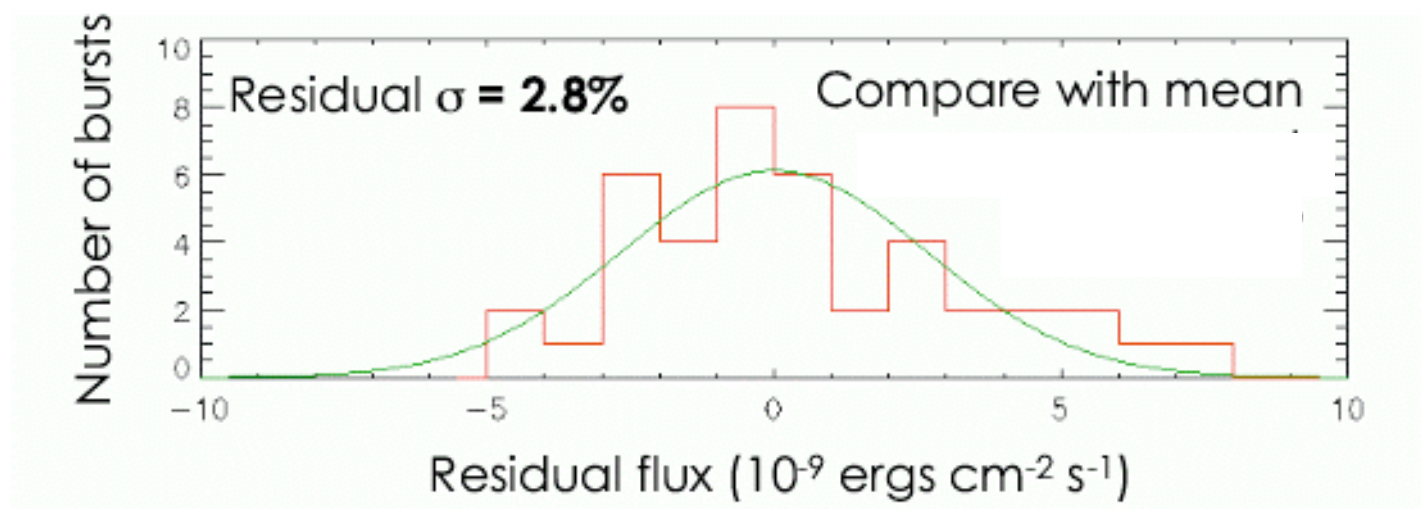
An Eddington-limited (i.e., a radius-expansion) Burst



A flat-topped flux, a temperature dip, a rise in the inferred radius

# Thermonuclear Bursts and Eddington-limited Phenomena

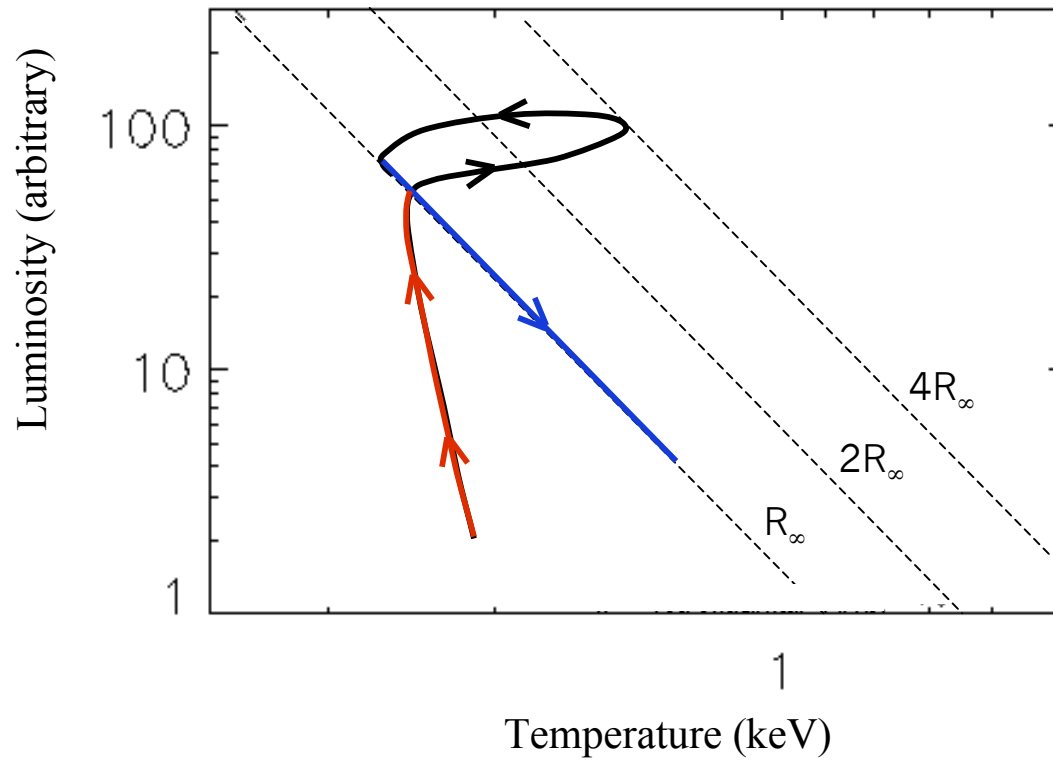
The peak luminosity is constant to 2.8% for 70 bursts of 4U 1728-34



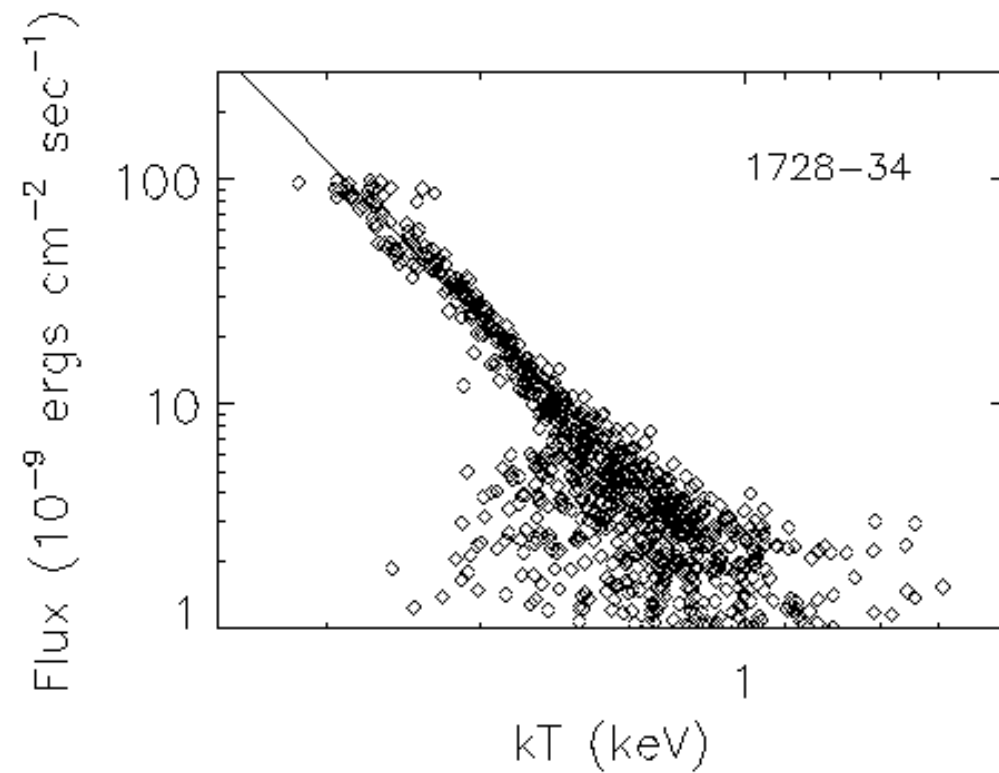
Galloway et al. 2003

# Measuring the Eddington Limit: The Touchdown Flux

An “H-R” diagram for a burst

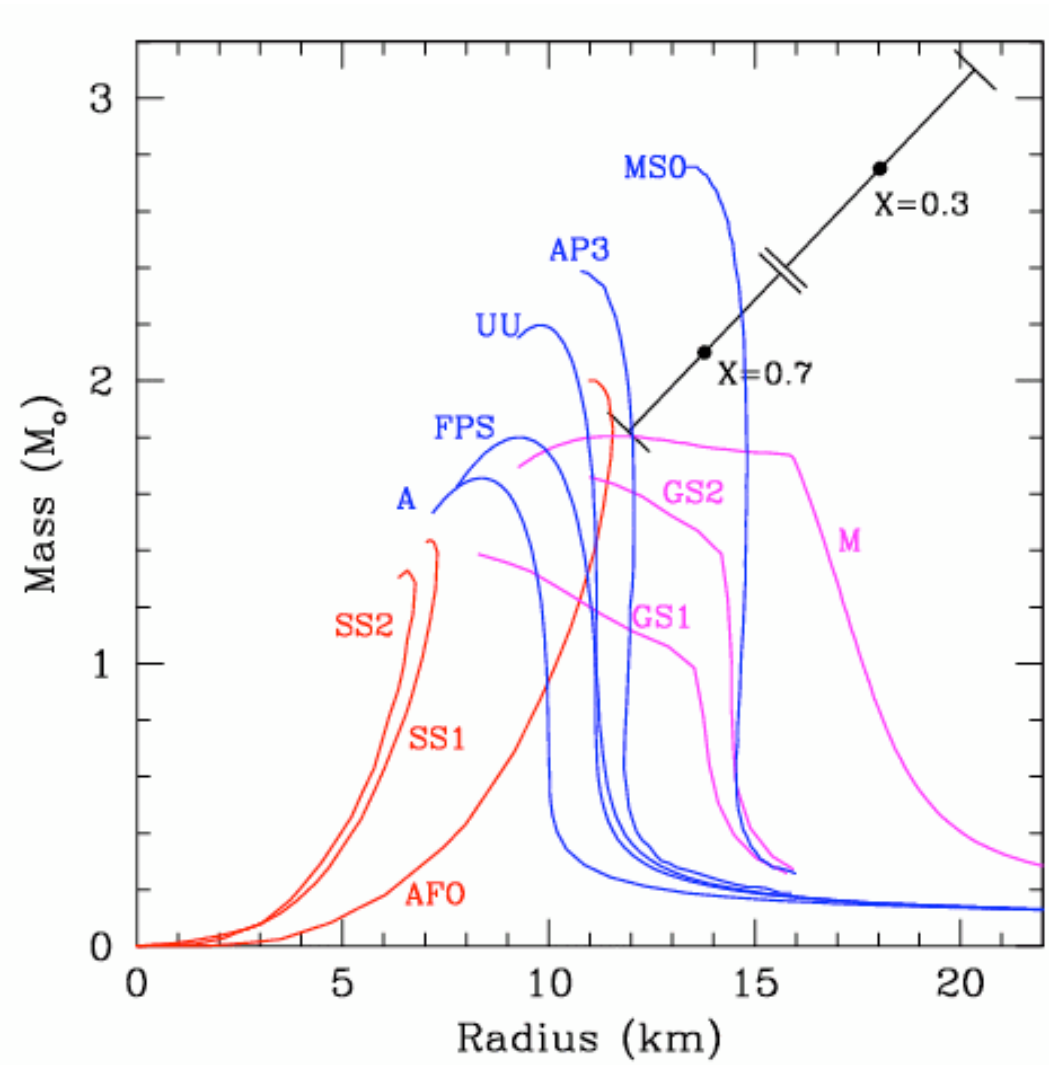


## Constant Radii Imply Emission from Whole Surface



Savov et al. 2001

# Mass and Radius of EXO 0748-676



**M-R limits:**

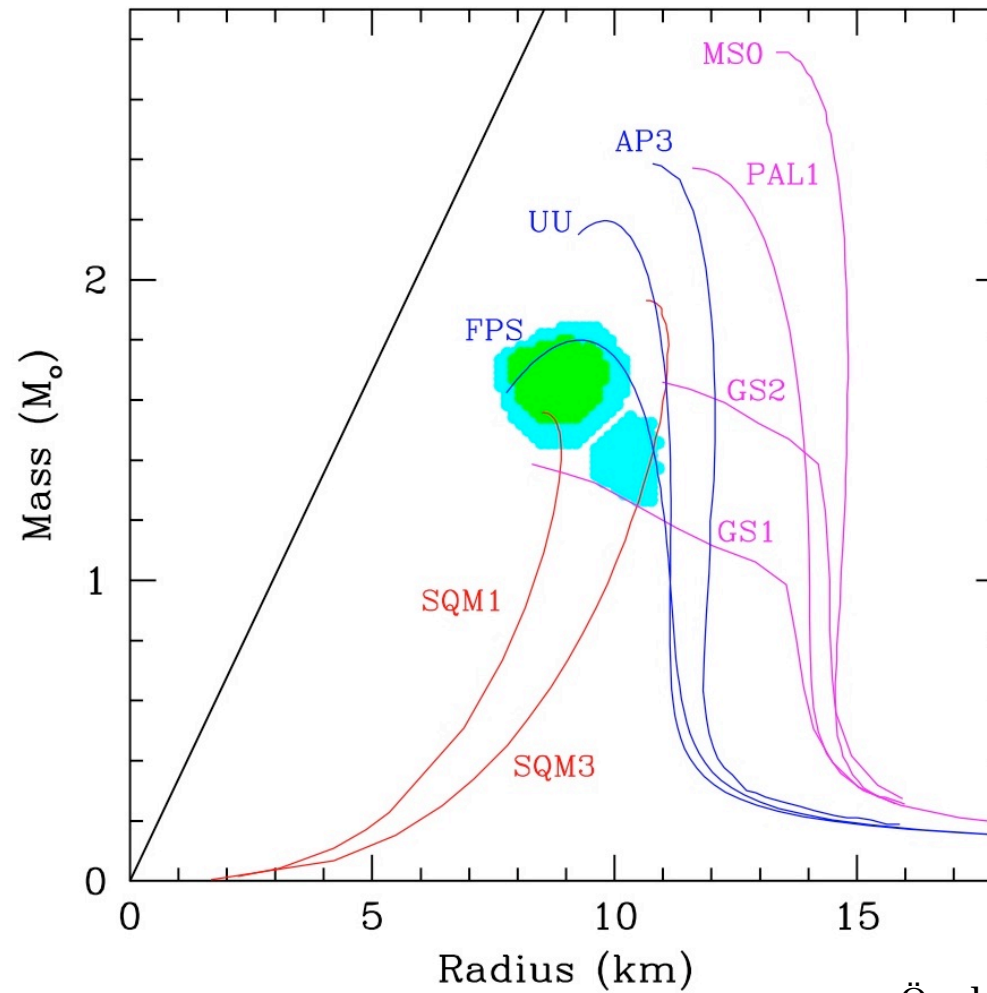
$$M = 2.10 \pm 0.28 M_{\odot}$$

$$R = 13.8 \pm 1.8 \text{ km}$$

Özel 2006

# Measurements Using Distances to Sources

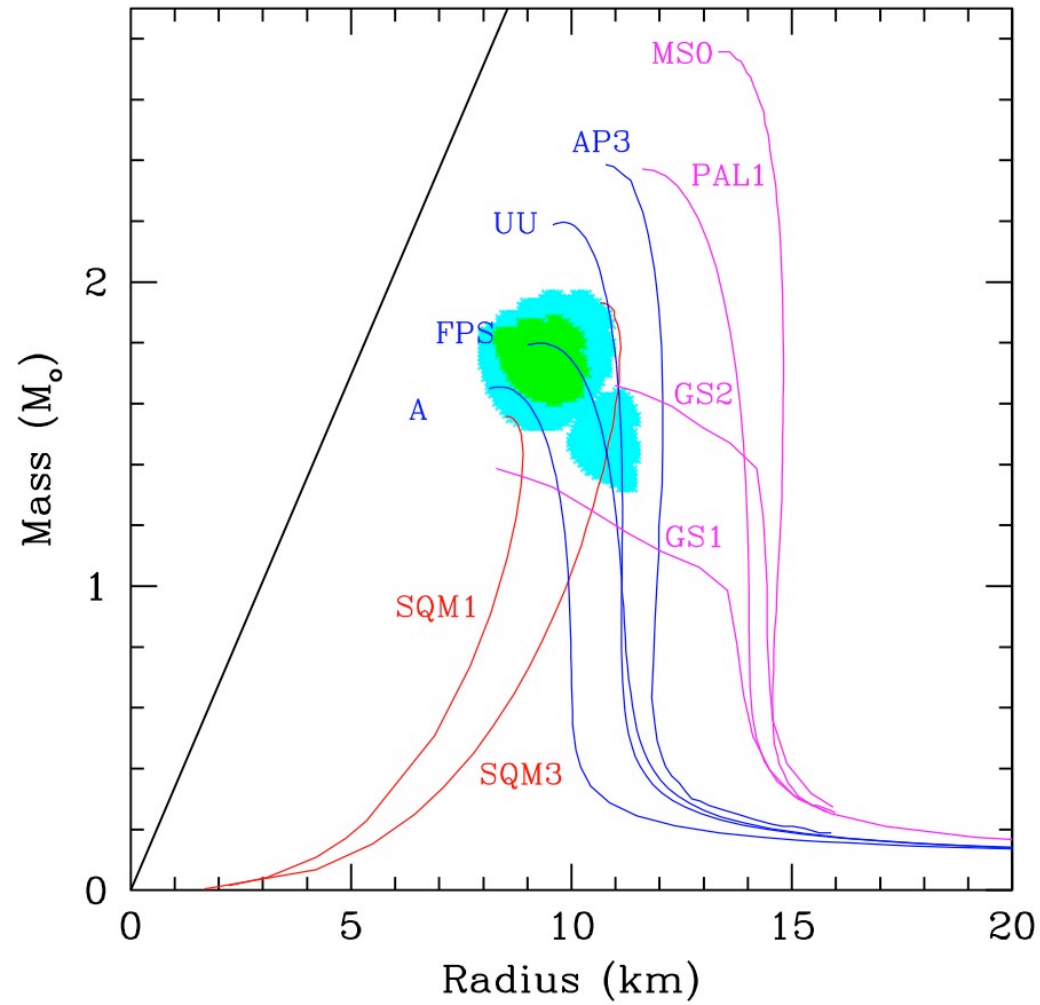
**EXO 1745-248** in Globular Cluster Terzan 5 ( $D = 6.5$  kpc from HST NICMOS)



Özel et al. 2008

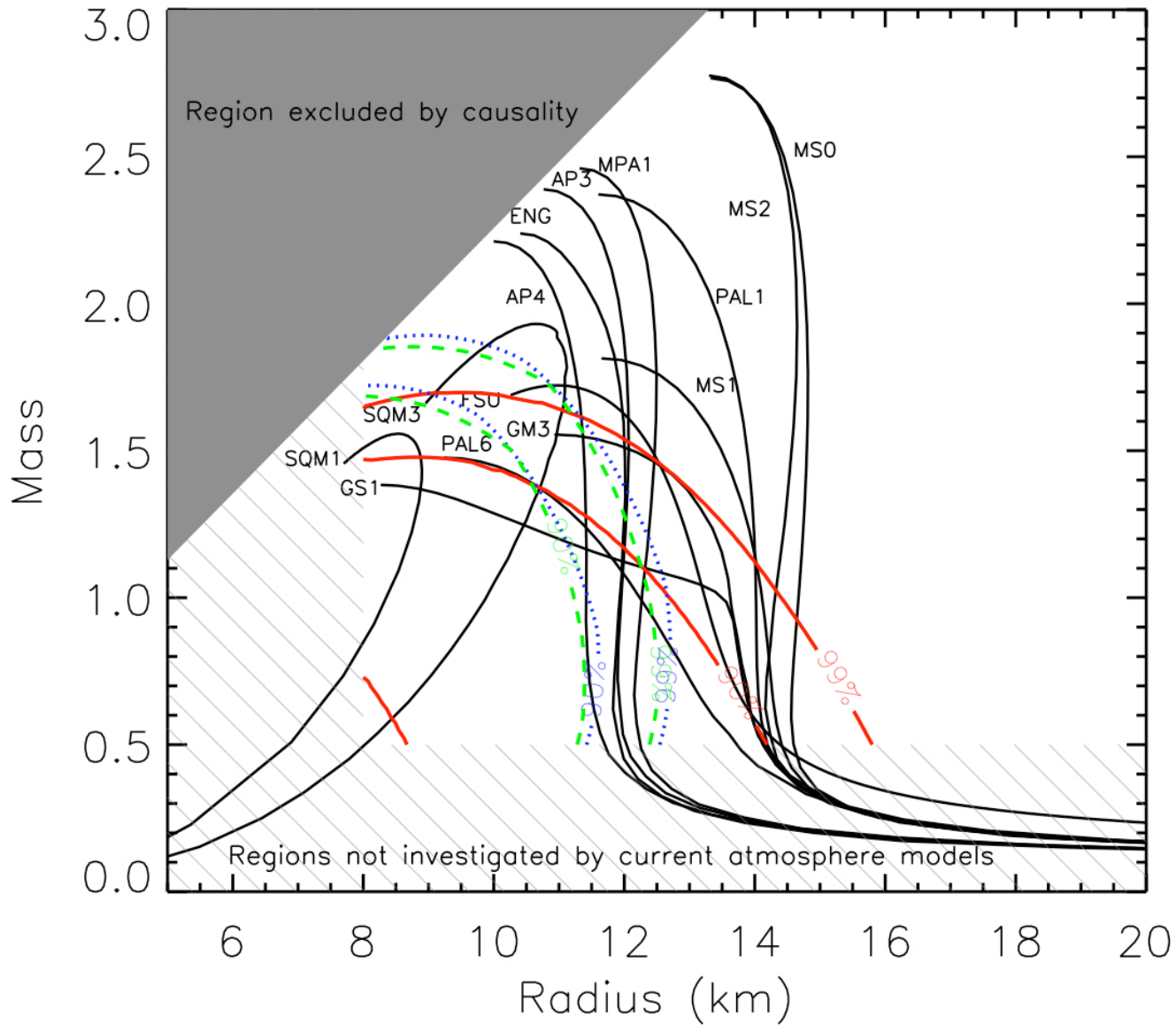


# The Mass and Radius of 4U 1608-52

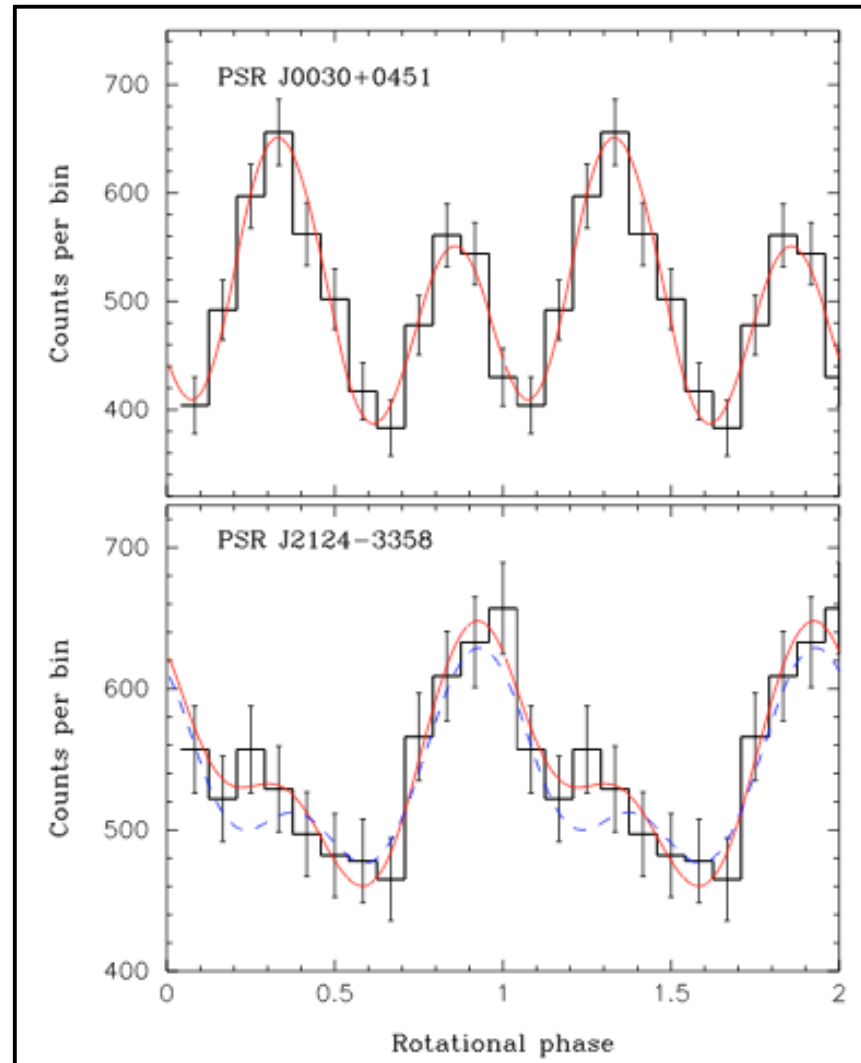


Guver et al. 2009

# Neutron Star in Globular Cluster M 13



## Isolated Millisecond Pulsar Pulse Profiles (in X-rays)



Assuming  $M=1.4$

$R > 9.4, 7.8$  km

for different  
sources

Bogdanov, Grindlay, & Rybicki 2008

Accreting ms pulsar profiles: Poutanen et al. 2004

## Methods of Determining NS Mass and/or Radius

- **Dynamical mass measurements (very important but mass only)**
- Neutron star cooling (provides --fairly uncertain-- limits)
- Quasi Periodic Oscillations
- Glitches (provides limits)
- Maximum spin measurements

## Dynamical Mass Measurements

Use the general relativistic decay of a binary orbit containing a NS

$$(\dot{P}_b)_{GR} = f(m_1, m_2, \sin(i))$$

The observed binary period derivative can be expressed in terms of the binary mass function.

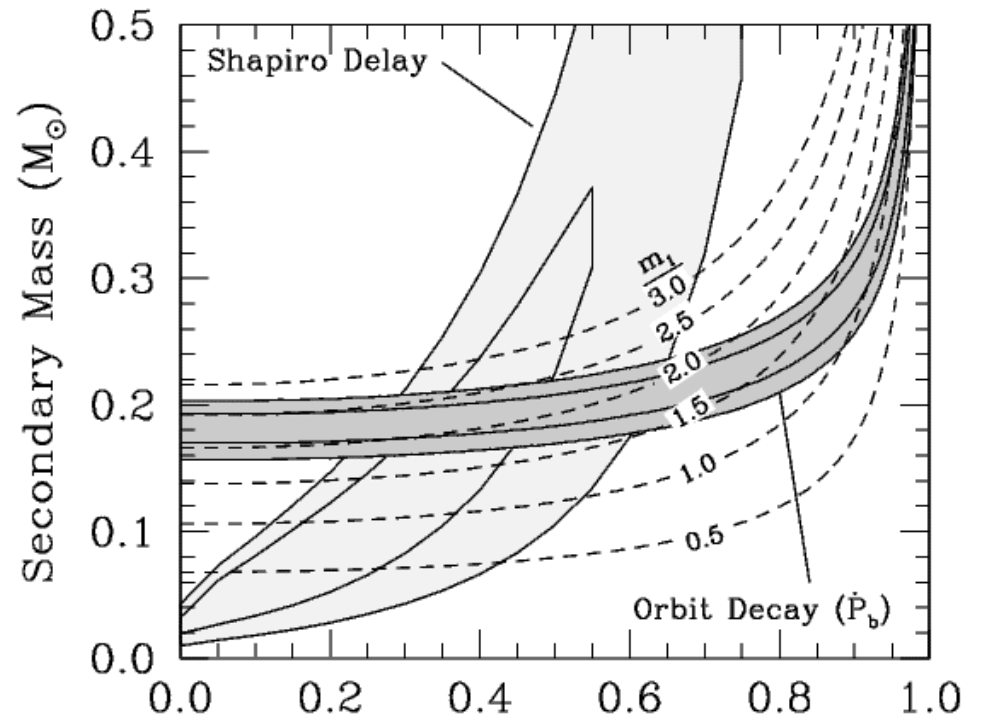
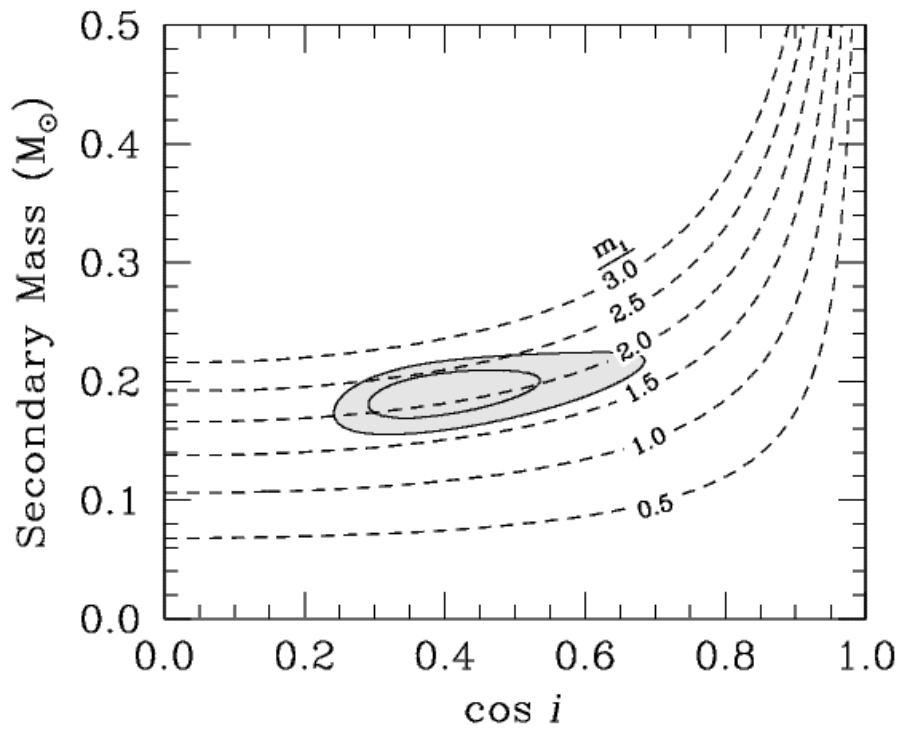
Need a short binary period, preferably a fast pulsar, a long baseline to get accurate timing parameters.

Also use Shapiro delay,

$$\Delta t = f(m_2, \sin(i))$$

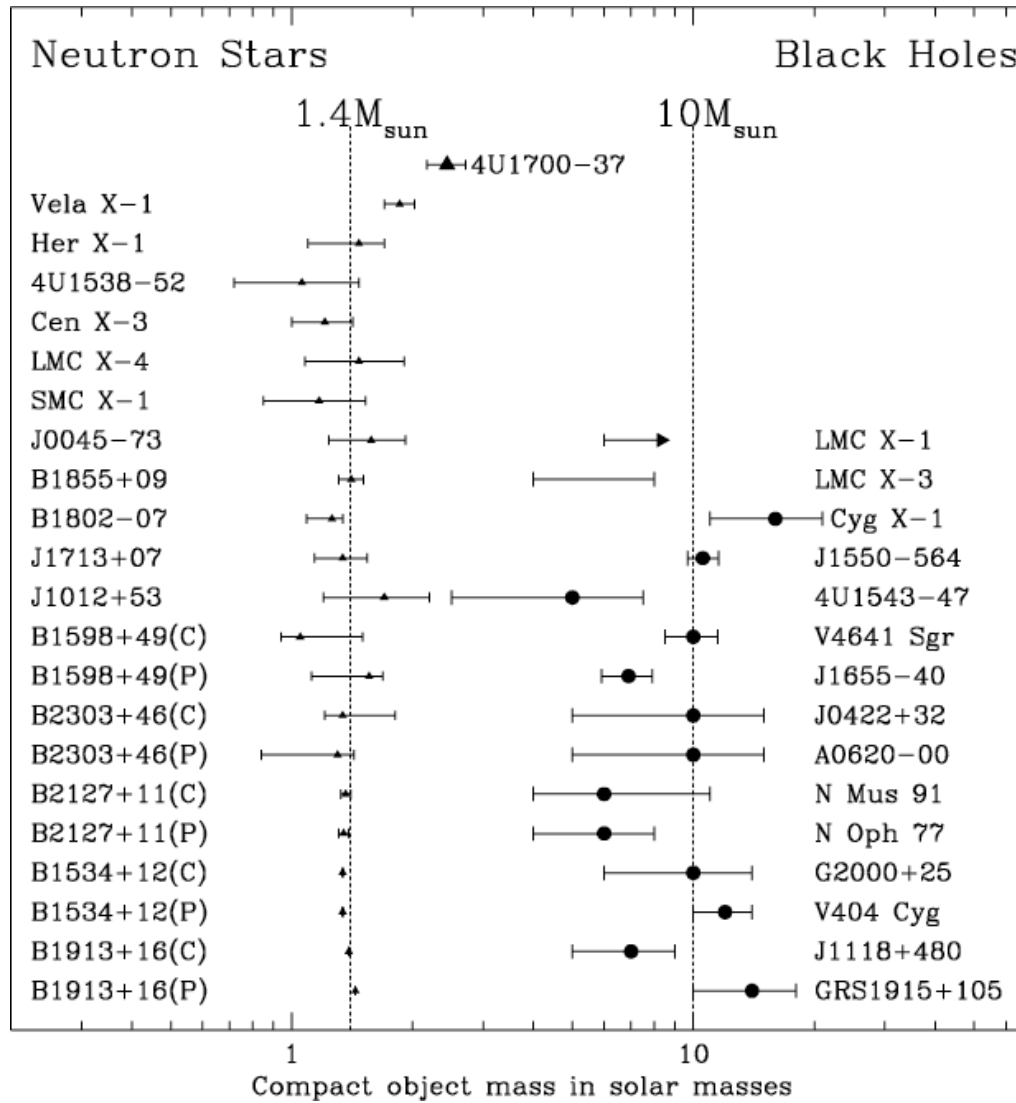
(For black holes, measurements are more approximate and rely on the binary mass function)

## Limits on PSR J0751+1807



from Nice et al. 05

$$M = 2.1 M_{\odot}$$



## Methods of Determining NS Mass and/or Radius

- Dynamical mass measurements (very important but mass only)
- **Neutron star cooling (provides --fairly uncertain-- limits)**
- Quasi Periodic Oscillations
- Glitches (provides limits)
- Maximum spin measurements



## Neutron Star Cooling

Why is cooling sensitive to the neutron star interior?

The interior of a proto-neutron star loses energy at a rapid rate by neutrino emission.

Within  $\sim 10$  to 100 years, the thermal evolution time of the crust, heat transported by electron conduction into the interior, where it is radiated away by neutrinos, creates an isothermal core.

The star continuously emits photons, dominantly in X-rays, with an effective temperature  $T_{\text{eff}}$  that tracks the interior temperature.

The energy loss from photons is swamped by neutrino emission from the interior until the star becomes about  $3 \times 10^5$  years old.

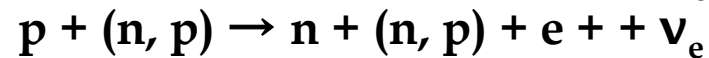
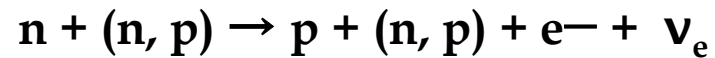
The overall time that a neutron star will remain visible to terrestrial observers is not yet known, but there are two possibilities: the standard and enhanced cooling scenarios. The dominant neutrino cooling reactions are of a general type, known as Urca processes, in which thermally excited particles alternately undergo  $\beta^-$  and inverse- $\beta$  decays. Each reaction produces a neutrino or antineutrino, and thermal energy is thus continuously lost.

## Neutron Star Cooling

The most efficient Urca process is the direct Urca process.

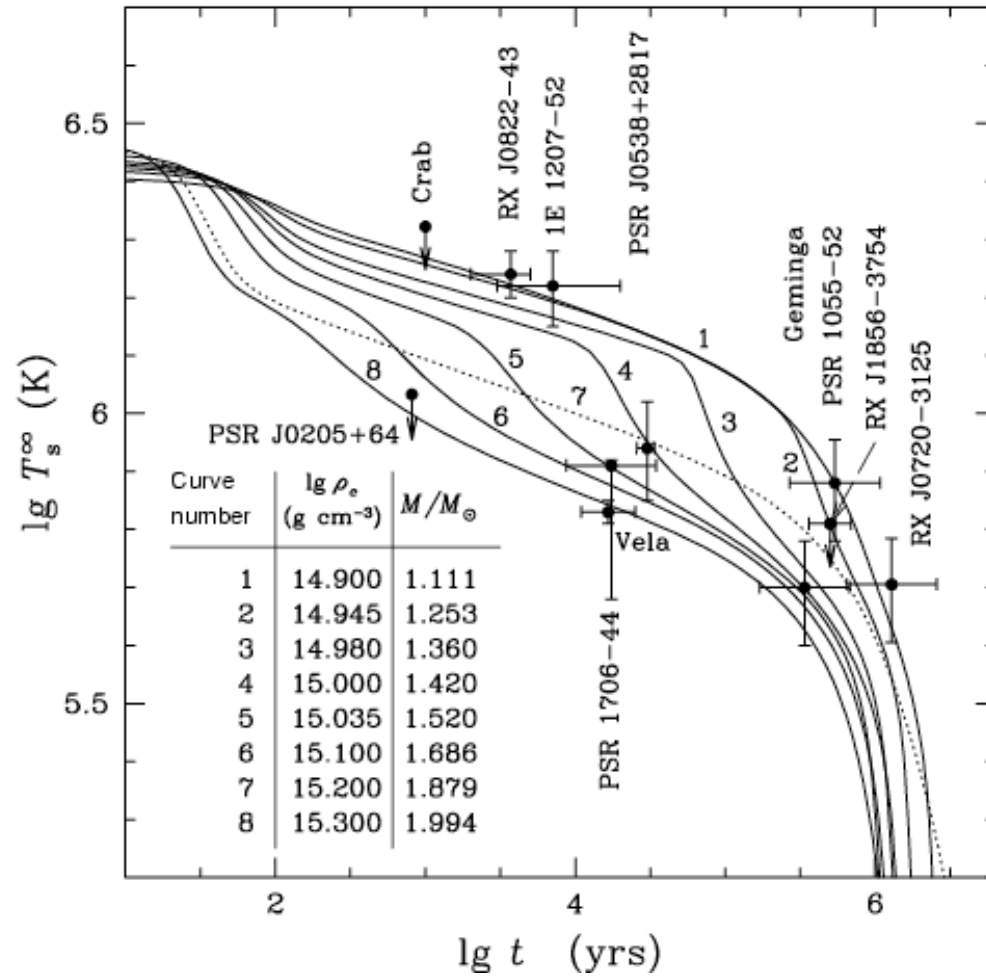
This process is only permitted if energy and momentum can be simultaneously conserved. This requires that the proton to neutron ratio exceeds 1/8, or the proton fraction  $x \geq 1/9$ .

If the direct process is not possible, neutrino cooling must occur by the modified Urca process



Which of these processes take place, and where in the interior, depend sensitively on the composition of the interior.

# Neutron Star Cooling

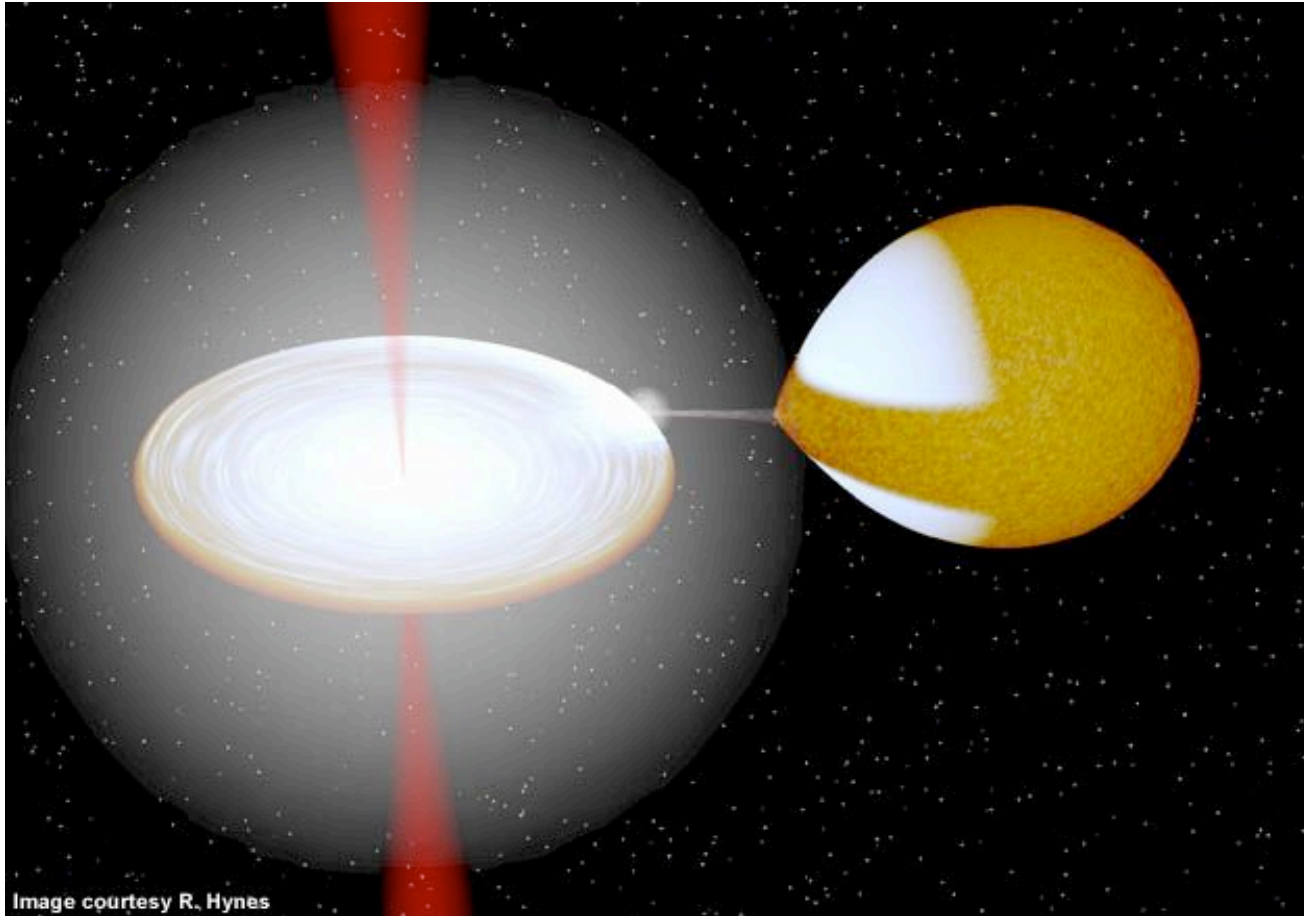


Caveats: Very difficult to determine ages and distances  
Magnetic fields change cooling rates significantly

## Methods of Determining NS Mass and/or Radius

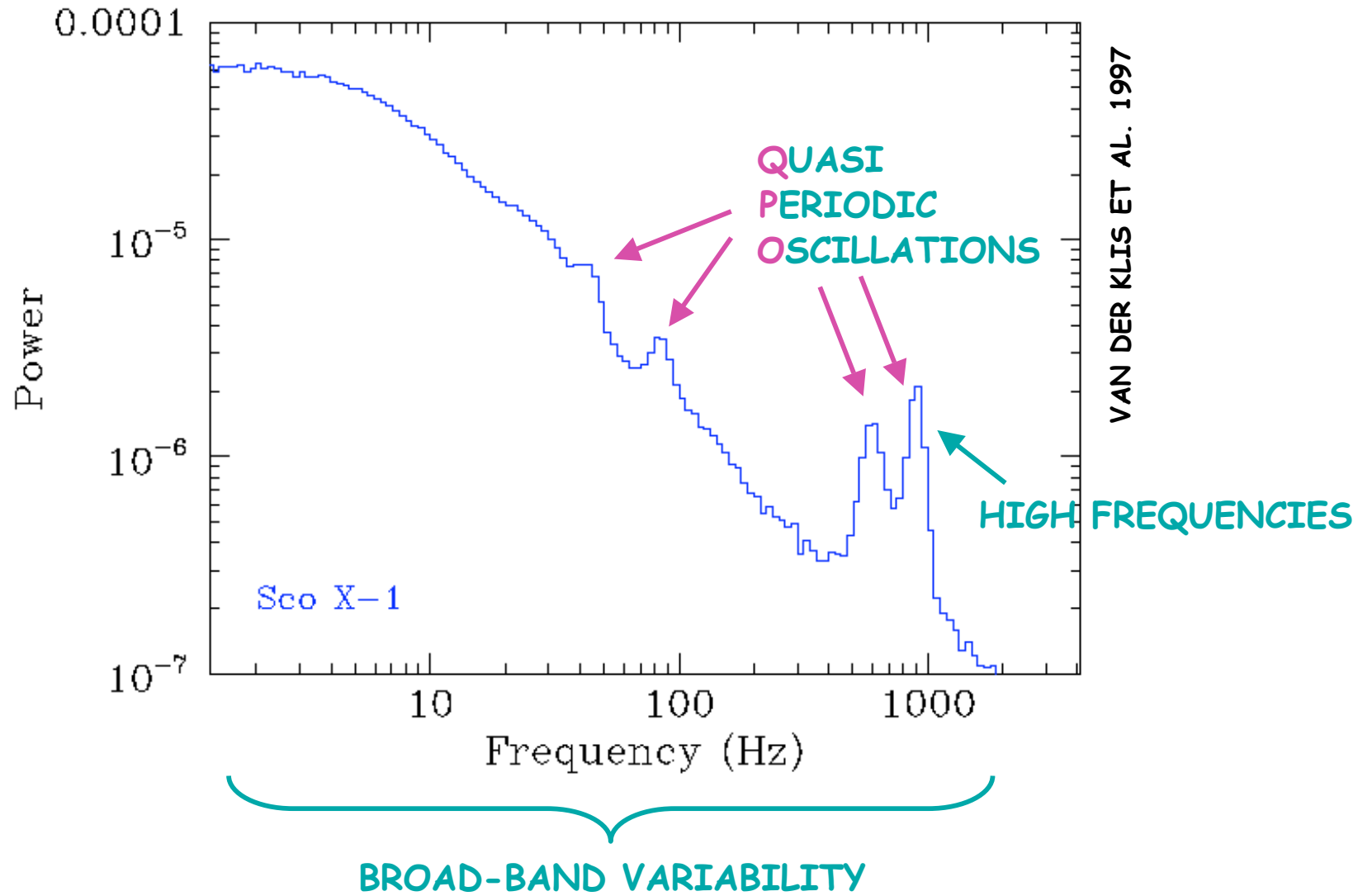
- Dynamical mass measurements (very important but mass only)
- Neutron star cooling (provides --fairly uncertain-- limits)
- **Quasi Periodic Oscillations**
- Glitches (provides limits)
- Maximum spin measurements

## Quasi-periodic Oscillations

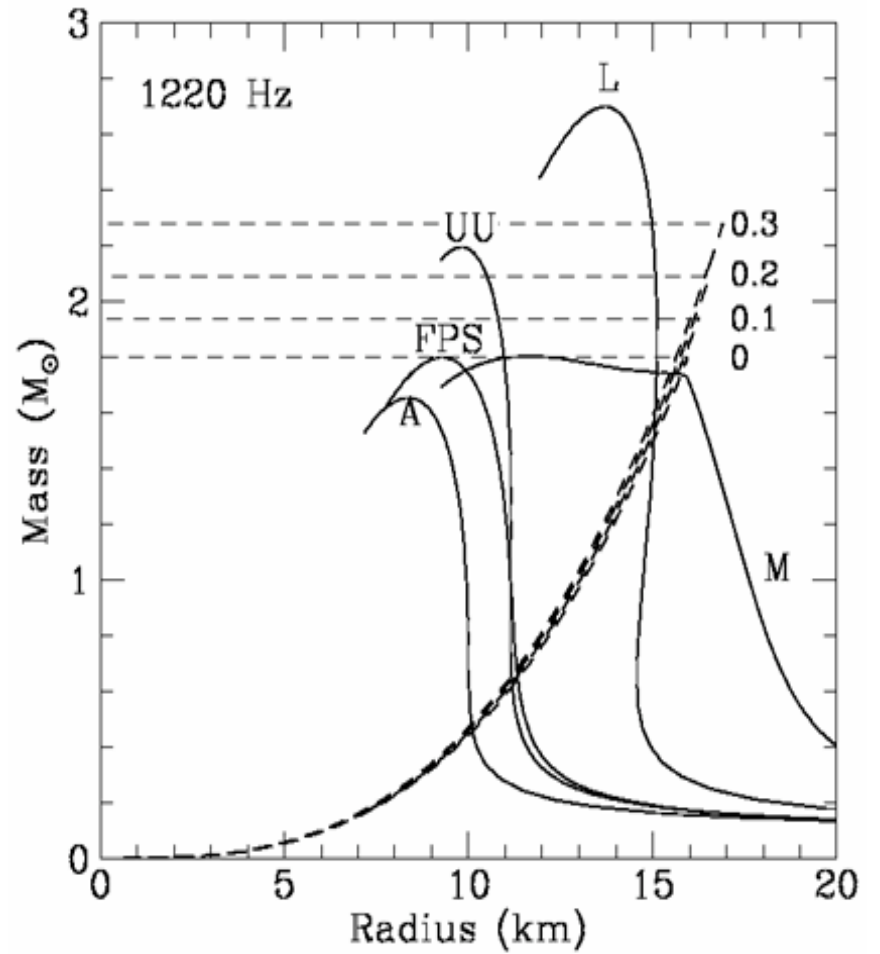
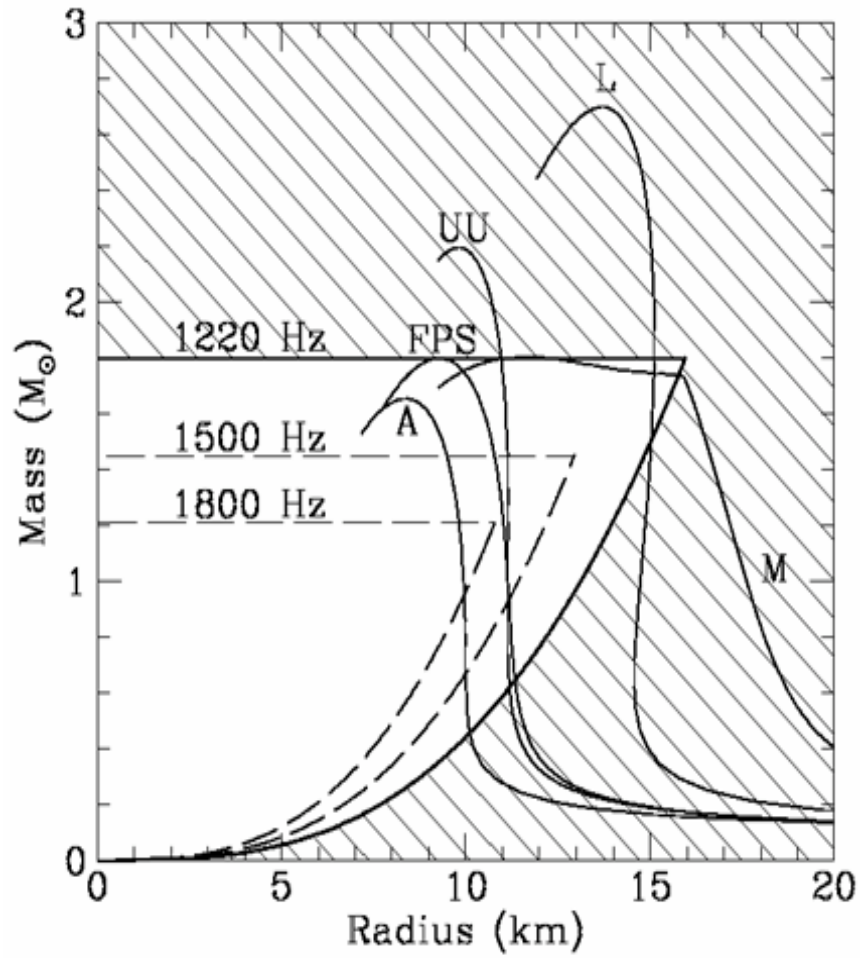


Accretion flows are very variable, with timescales ranging from 1ms to 100 days!

# Power Spectra of Variability:



# Quasi-periodic Oscillations



from Miller, Lamb, & Psaltis 1998

## Methods of Determining NS Mass and/or Radius

- Dynamical mass measurements (very important but mass only)
- Neutron star cooling (provides --fairly uncertain-- limits)
- Quasi Periodic Oscillations
- Glitches (provides limits)
- **Maximum spin measurements**



## Limits from Maximum Neutron Star Spin

The mass-shedding limit for a rigid Newtonian sphere is the Keplerian rate:

$$P_{\min}^N = 2\pi \left( \frac{R^3}{GM} \right)^{1/2} = 0.545 \left( \frac{M_{\odot}}{M} \right)^{1/2} \left( \frac{R}{10km} \right)^{3/2} ms$$

Fully relativistic calculations yield a similar result:

$$P_{\min} = 0.83 \left( \frac{M_{\odot}}{M} \right)^{1/2} \left( \frac{R}{10km} \right)^{3/2} ms$$

for the maximum mass, minimum radius configuration.

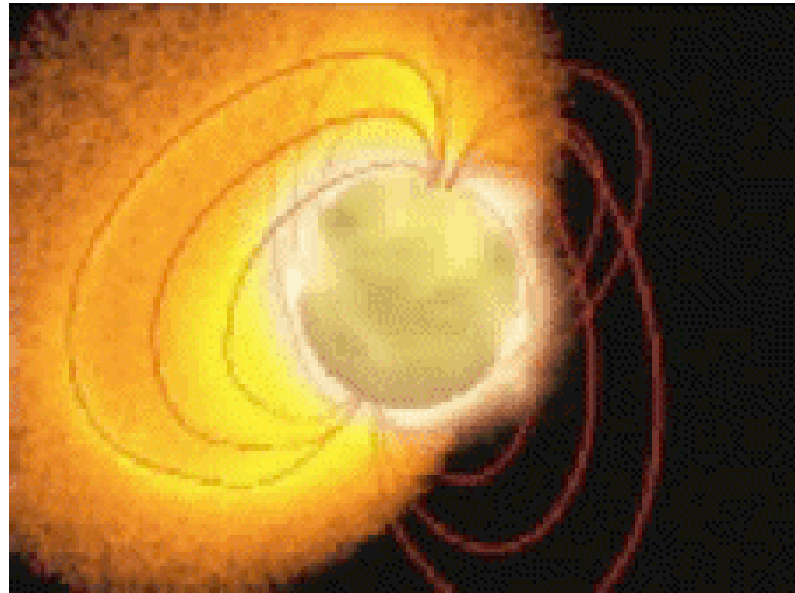
Depending on the actual values of M and R in each equation of state, the obtainable maximum spin frequency changes.

# Niels Bohr CompSchool

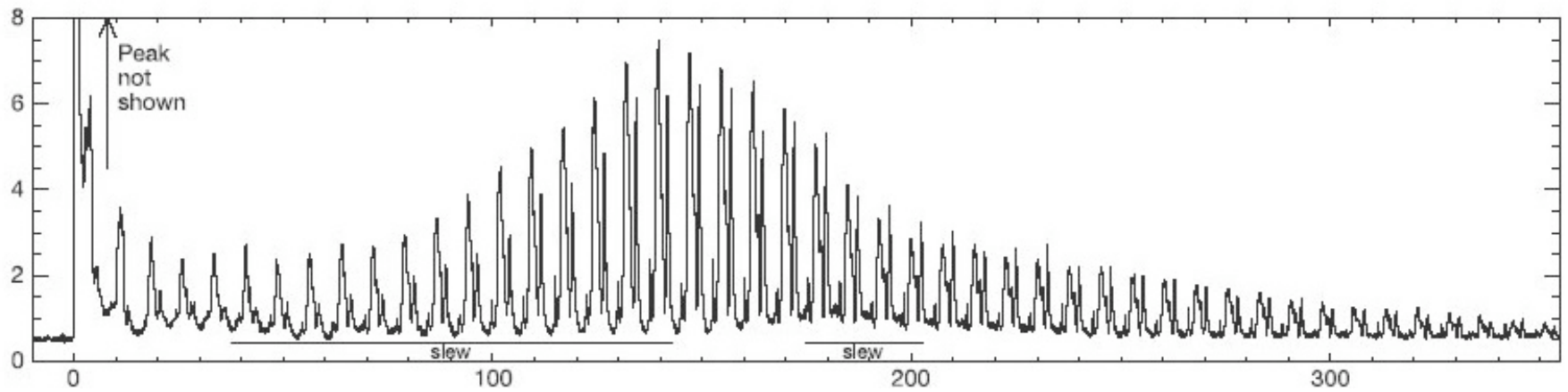
## Compact Objects

**Neutron Star Observables, Masses, Radii and Magnetic Fields**

**Lecture 3: Magnetic Neutron Stars**



## 27 December 2004 burst of SGR 1806



**Why are they “Magnetars”?**

**Dipole spindown argument:**

$$B = 2 \cdot 10^{14} \left( \frac{P}{6s} \frac{\dot{P}}{10^{-11}} \right)^{1/2} G$$

**No concrete evidence.**

## Questions:

- Magnetic field strength
- Magnetic field geometry
- Energy source (for quiescent emission and bursts)

# Magnetospheres

- Accreting sources

Some accreting sources have virtually no magnetospheres (low-mass X-ray binaries)

In others (high-mass X-ray binaries), the magnetosphere interacts with the accretion disk, channelling the flow and causing pulsations

- Radio pulsars

- Magnetars

# Magnetospheres

- Accreting sources

- Radio pulsars

Emission is completely dominated by the magnetosphere

Thought to be synchrotron and curvature radiation from a

Goldreich-Julian density of particles

- Magnetars



## Processes in Magnetar Magnetospheres

Thompson, Lyutikov & Kulkarni '02, Lyutikov & Gavriil '06,  
Guver, Ozel & Lyutikov '06, Fernandez & Thompson '06

- large scale currents in the magnetosphere of a magnetar can result in particle densities  $\gg$  Goldreich & Julian
- mildly relativistic charges Compton upscatter atmospheric photons

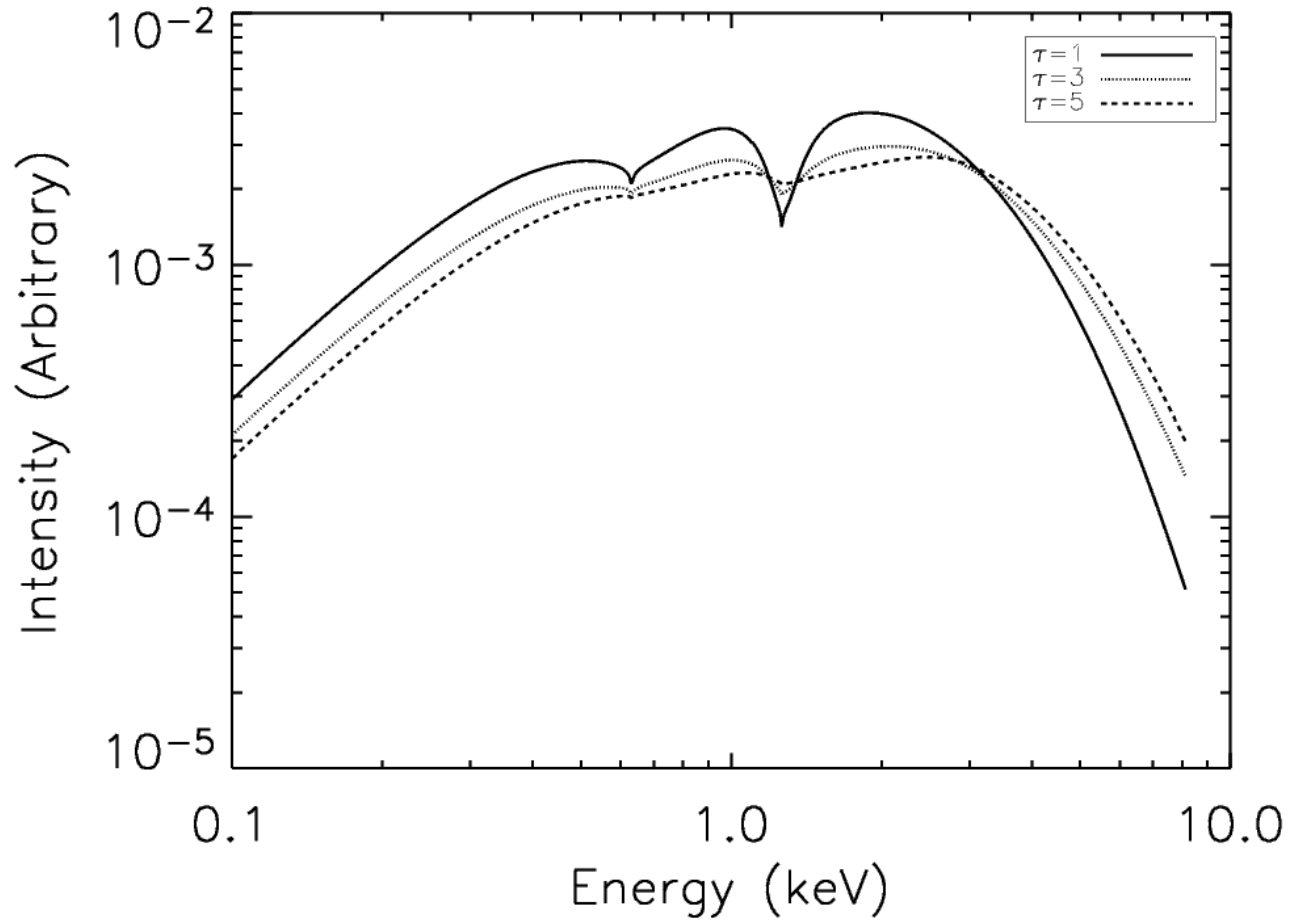
$$\tau = \int \sigma N_e dz$$

- resonant layers appear at  $r \approx r_{NS} \left( \frac{heB_{NS}}{Emc} \right)^{1/3}$  for dipole fields
- solve radiative transfer using two-stream approximation for thermal electrons

Atmos.+Magnetosph.+GR = Surface thermal Emission and Magnetospheric Scattering Model

# Spectra

## Atmosphere + Magnetosphere



Guver, Ozel, & Lyutikov 07

## Anomalous X-ray Pulsars and Soft Gamma-ray Repeaters

- X-ray bright pulsars,  $L_x \sim 10^{33-35} \text{ erg s}^{-1}$
- some are in SNRs
- some show radio, optical, and IR emission
  
- soft spectra ( $kT \sim 0.5 \text{ keV}$ )
- power-law like tails
- no features
  
- 6-12 s periods
- large period derivatives
- large Pulsed Fractions (PF)
  
- powerful, recurrent, soft gamma-ray, hard X-ray bursts

## **AXP 4U 0142+61**

**A (mostly) stable, bright AXP**

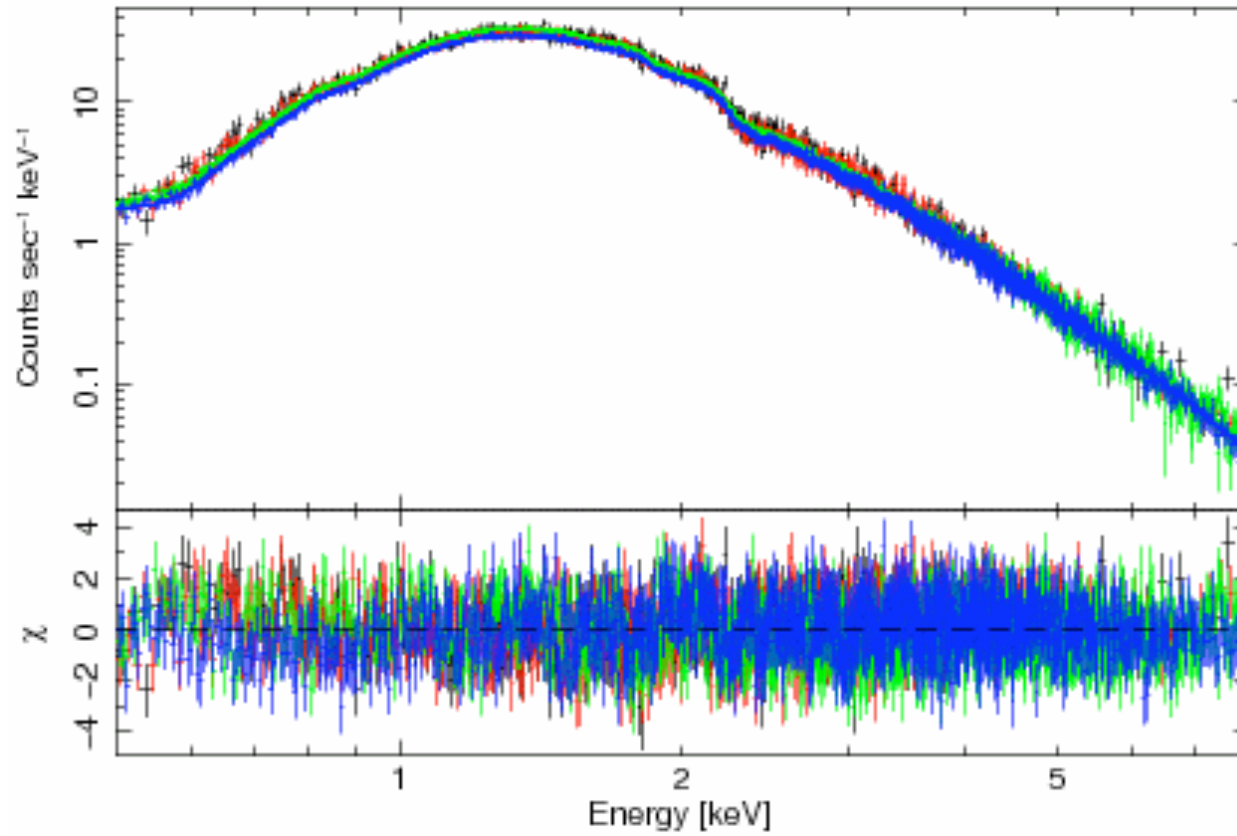
(See Kaspi, Gavriil & Dib '06, Dib et al. '06 for recent bursts)

**Dominant hard X-ray spectrum detected with INTEGRAL in 20-230 keV**

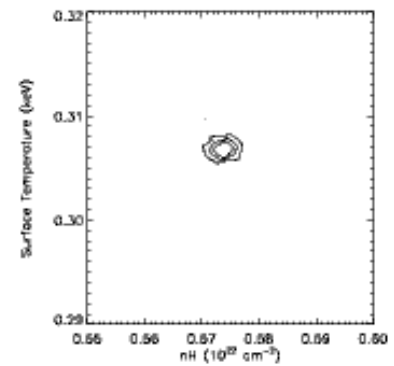
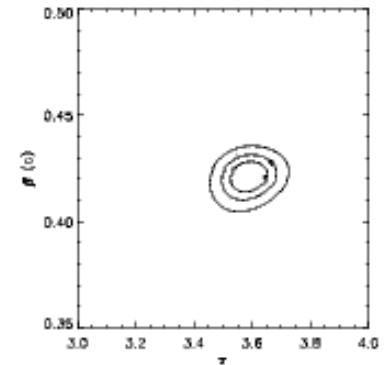
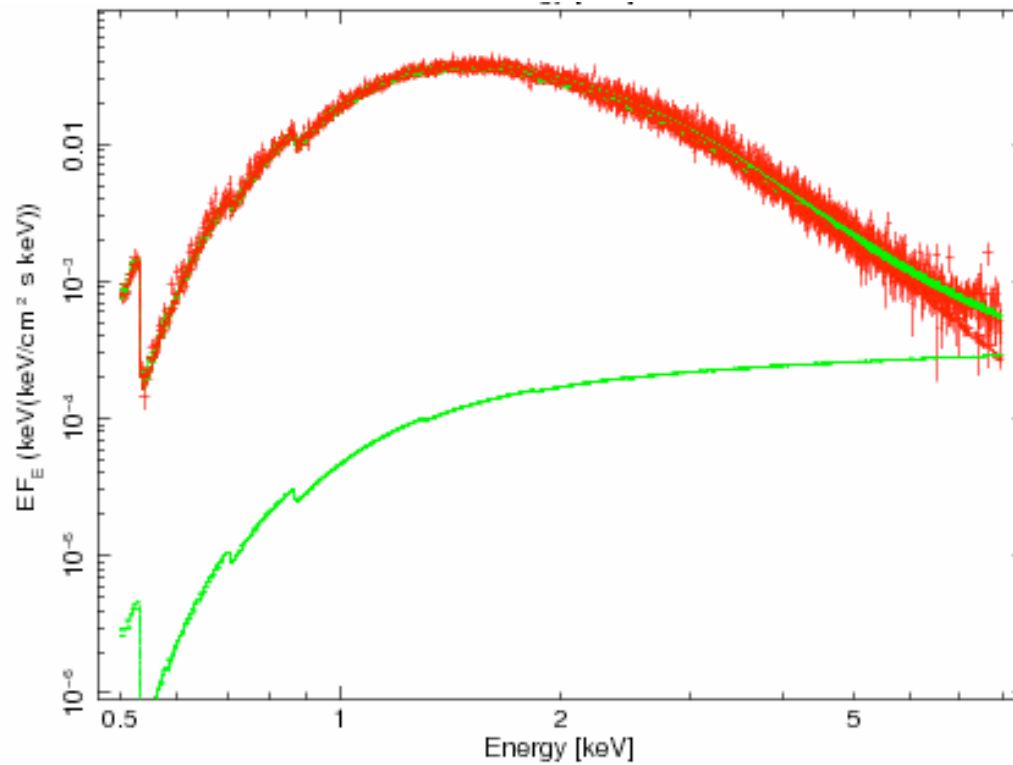
(Kuiper et al. '06, den Hartog et al. '07)

**Many epochs of XMM+Chandra data**

# AXP 4U 0142+61



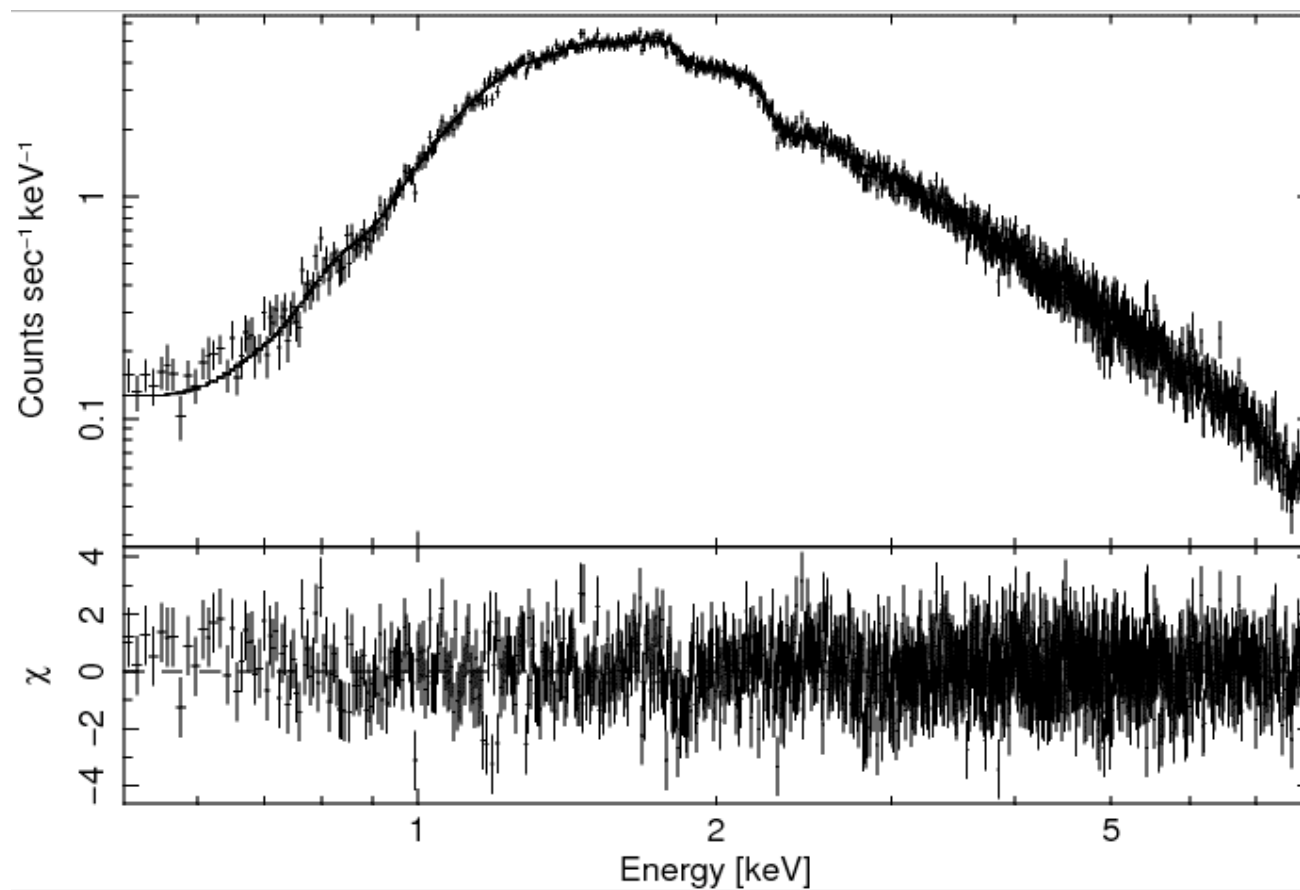
# AXP 4U 0142+61



$$B_{\text{surf}} = (4.6 \pm 0.14) \times 10^{14} \text{ G}$$

$$B_{\text{spindown}} = 1.3 \times 10^{14} \text{ G (Gavriil \& Kaspi 02)}$$

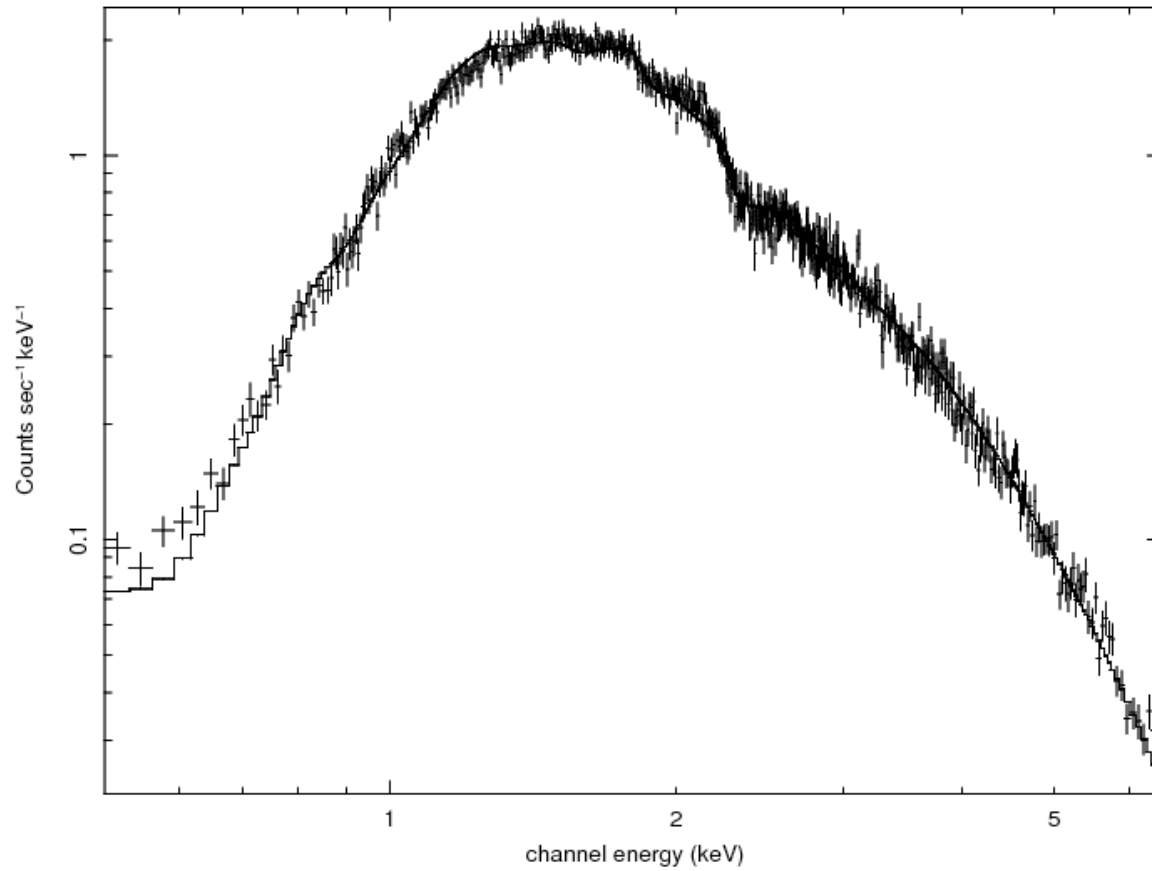
# 1RXS J 1708-40



$$B_{\text{surf}} = (3.95 \pm 0.17) \times 10^{14} \text{ G}$$

$$B_{\text{spindown}} = 4.6 \times 10^{14} \text{ G}$$

# 1E 1048.1-5937



$$B_{\text{surf}} = 2.48 \times 10^{14} \text{ G}$$

$$B_{\text{spindown}} = (2.4 - 4) \times 10^{14} \text{ G (Gavriil \& Kaspi 02)}$$

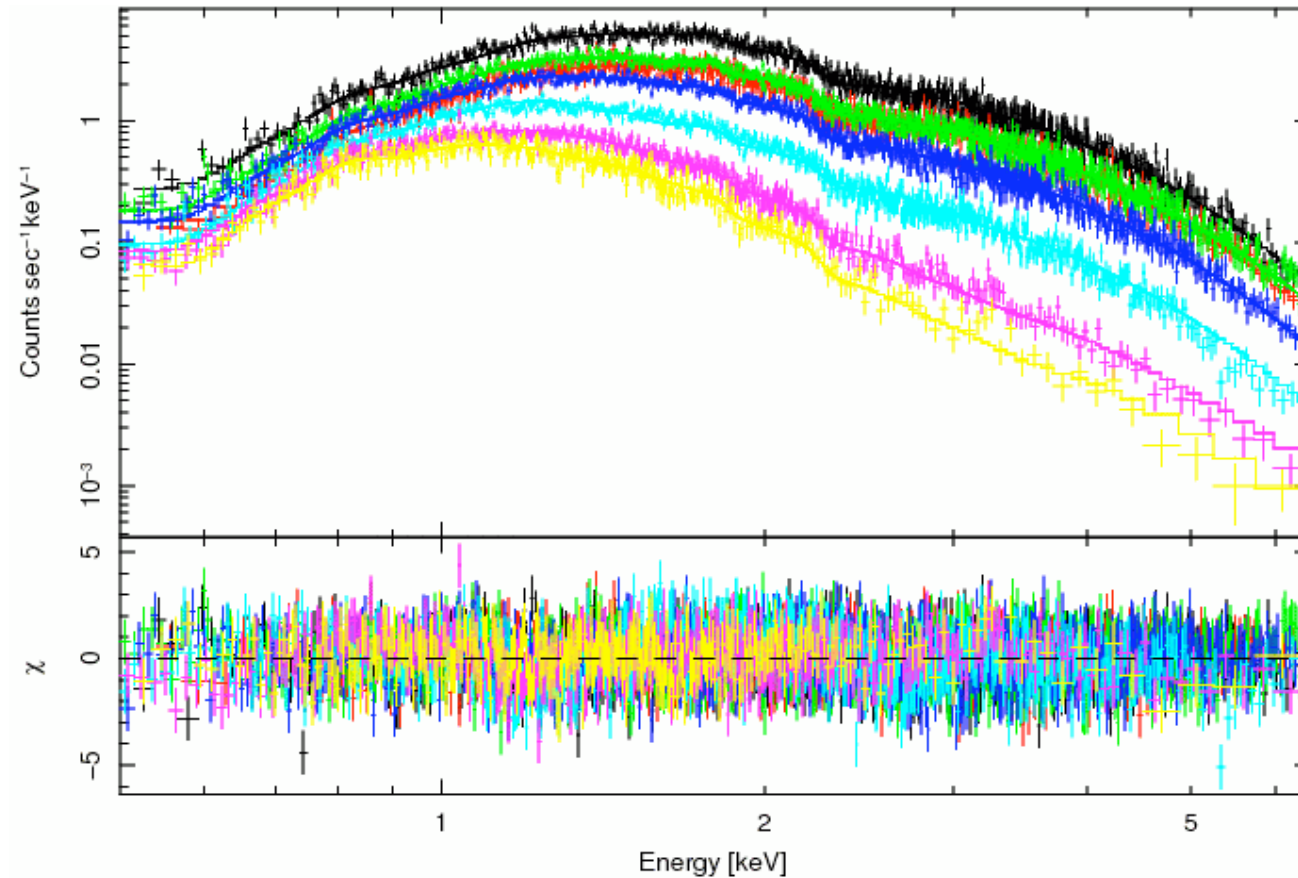


## XTE J1810-197: A Highly Variable (Transient) AXP

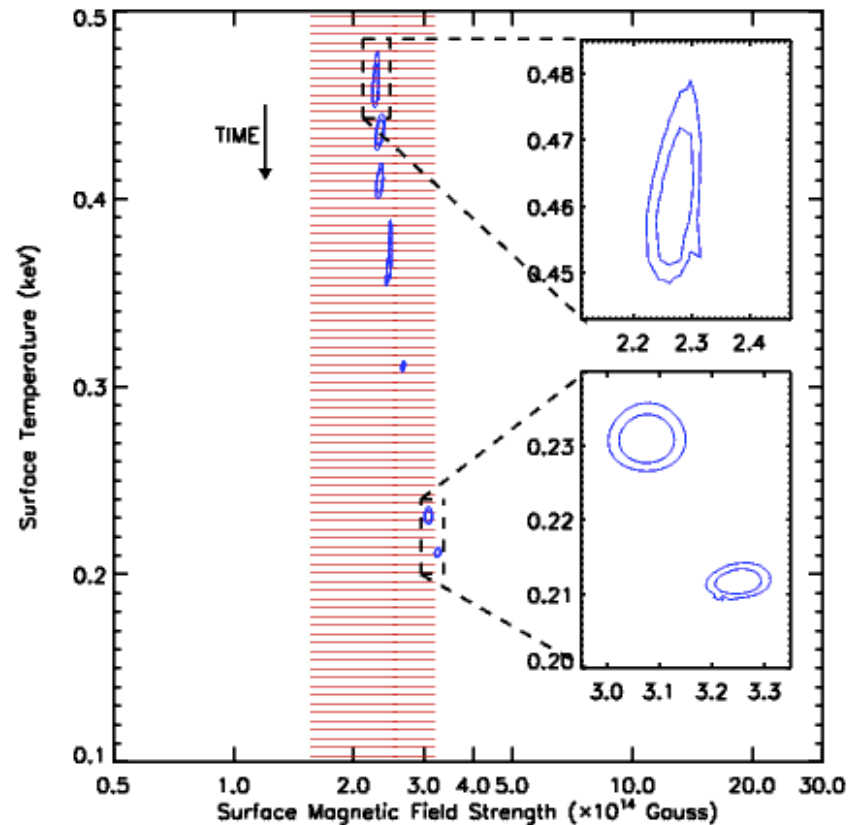
- **Discovered in 2003 when it went into outburst** (Ibrahim et al. 03)
- **Source flux has declined ~100-fold since** (Gotthelf & Halpern 04, 05, 06)
- **Significant spectral evolution during decay**
- **B (spindown)  $\sim 2.5 \times 10^{14}$  G**

# Spectral Analysis

## Fits to seven epochs of XMM data on XTE J1810-197



## Temperature Evolution and Magnetic Field of XTE J1810-197



**Magnetic field remains nearly constant; is equal to spindown field!**

**Temperature declines steadily and dramatically**

**No changes in magnetospheric parameters during these observations**

# Summary

- Modeling the surfaces and magnetospheres of neutron stars allow us to make sense out of many types of sources
- In turn, we have begun measuring NS masses and radii with reasonable accuracy
- We can address magnetic field strengths, geometries, and burst mechanisms of isolated sources