

Initial stellar populations in star clusters and in galaxies and "how to make a galaxy"

*The early life of stellar clusters,
Copenhagen,
6h of November 2014*

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the IMF

$dN = \xi(m) dm$, the number of stars in $m, m + dm$

... as derived from **detailed star-count analyses**
(Kroupa Tout & Gilmore 1991; 1992, 1993; K. et al 2013)

The Canonical IMF

(m is in units of M_{\odot})

$$\xi_{\text{BD}}(m) = \frac{k}{3} \left(\frac{m}{0.07}\right)^{-0.3 \pm 0.4}, \quad 0.01 < m \lesssim 0.15,$$

$$\xi_{\text{star}}(m) = k \begin{cases} \left(\frac{m}{0.07}\right)^{-1.3 \pm 0.3}, & 0.07 < m \leq 0.5, \\ \left[\left(\frac{0.5}{0.07}\right)^{-1.3 \pm 0.3}\right] \left(\frac{m}{0.5}\right)^{-2.3 \pm 0.36}, & 0.5 < m \leq 150. \end{cases} \quad (4.55)$$

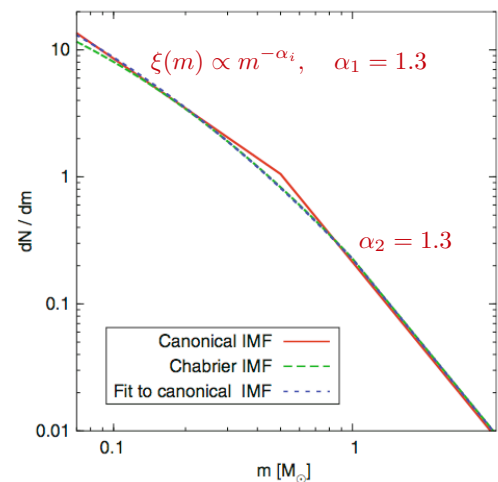
The Log-normal Canonical IMF

(m is in units of M_{\odot})

$$\xi_{\text{BD}}(m) = k k_{\text{BD}} \left(\frac{m}{0.07}\right)^{-0.3 \pm 0.4}, \quad 0.01 < m \lesssim 0.15,$$

$$\xi_{\text{star}}(m) = k \begin{cases} \frac{1}{m} \exp\left[-\frac{(m - m_*)^2}{2\sigma_m^2}\right], & 0.07 < m \leq 1.0, \\ A \left(\frac{m}{1.0}\right)^{-2.3 \pm 0.36}, & 1.0 < m \leq 150. \end{cases} \quad (4.56)$$

$$\sigma_{\ln m} = 0.75, \quad A = 0.244, \quad k_{\text{BD}} = 4.46 \quad (\text{not Chabrier!})$$



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The IMF as a scale-invariant probability density distribution function

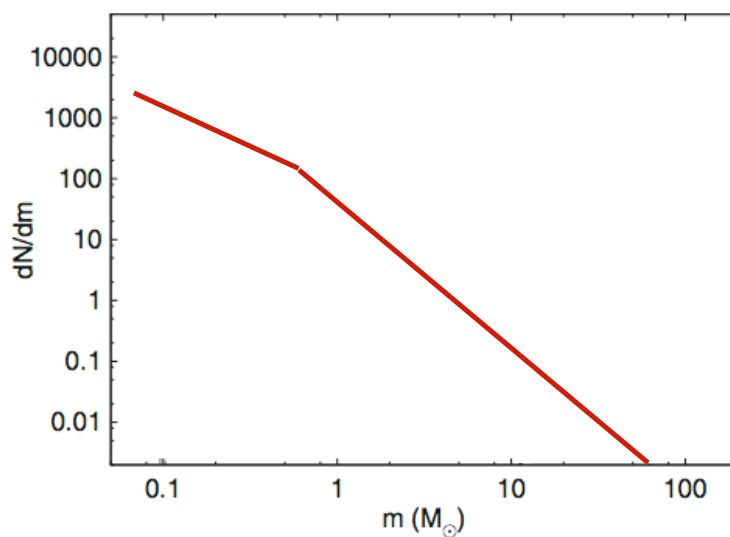
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The IMF as a scale-invariant *probability density distribution function*

⇒ stochastically sampled stellar populations when discretised :



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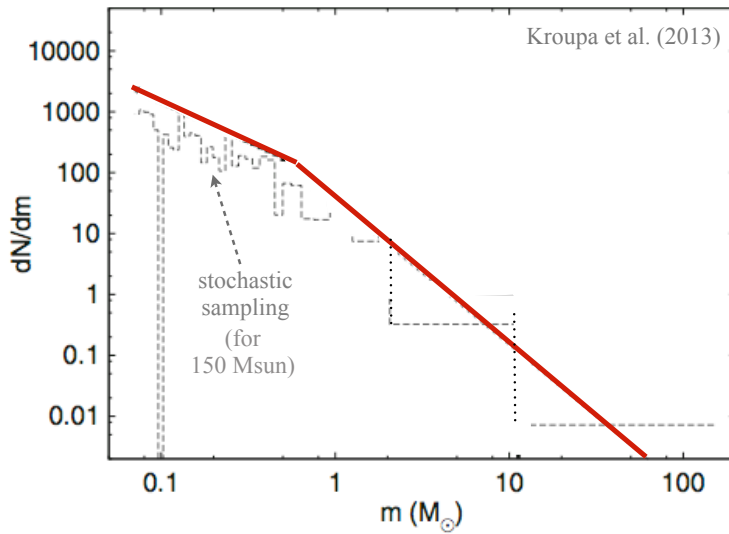
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The IMF as a scale-invariant *probability density distribution function*

==> stochastically sampled stellar populations when discretised :



==> stochastic variations in the shape of the IMF from case to case.

Elmegreen - many papers
Bastian et al. 2010

or *fractions of stars* in low-mass galaxies and low-mass star clusters when analytical description ==> this is *unphysical* but often employed

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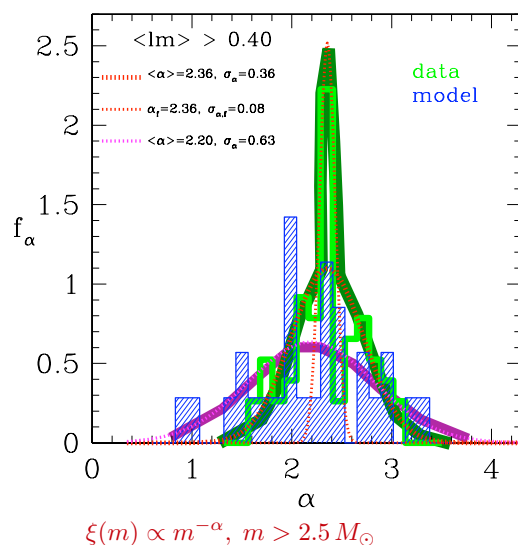
Tests : is the IMF a scale-invariant pdf ?

Is the standard assumption (invariant probabilistic IMF) consistent with observations ?

1. upper mass of stars is limited (to about 150 Msun)
==> i.e. \exists physical constraints on the IMF

(Weidner & Kroupa 2004; Figer 2005;
Oey & Clarke 2005; Koen 2006;
Maiz Appellaniz et al. 2007)

2. observed variations of IMF shapes much smaller than in theory



1. No asymmetries and sharp Salpeter/Massey peak.

2. Model worse than data !?

... but model has no measurement uncertainties

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==> Real young populations are
not stochastic ensembles from an IMF.

3. observed systematic change of galaxy-wide IMFs with SFR

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ROYAL ASTRONOMICAL SOCIETY



Mon. Not. R. Astron. Soc. **415**, 1647–1662 (2011)

doi:10.1111/j.1365-2966.2011.18800.x

Galaxy and Mass Assembly (GAMA): the star formation rate dependence of the stellar initial mass function

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ABSTRACT

The stellar initial mass function (IMF) describes the distribution in stellar masses produced from a burst of star formation. For more than 50 yr, the implicit assumption underpinning most areas of research involving the IMF has been that it is universal, regardless of time and environment. We measure the high-mass IMF slope for a sample of low-to-moderate redshift galaxies from the Galaxy and Mass Assembly survey. The large range in luminosities and galaxy masses of the sample permits the exploration of underlying IMF dependencies. A strong IMF–star formation rate dependency is discovered, which shows that highly star-forming galaxies form proportionally more massive stars (they have IMFs with flatter power-law slopes) than galaxies with low star formation rates. This has a significant impact on a wide variety of galaxy evolution studies, all of which rely on assumptions about the slope of the IMF. Our result is supported by, and provides an explanation for, the results of numerous recent explorations suggesting a variation of or evolution in the IMF.

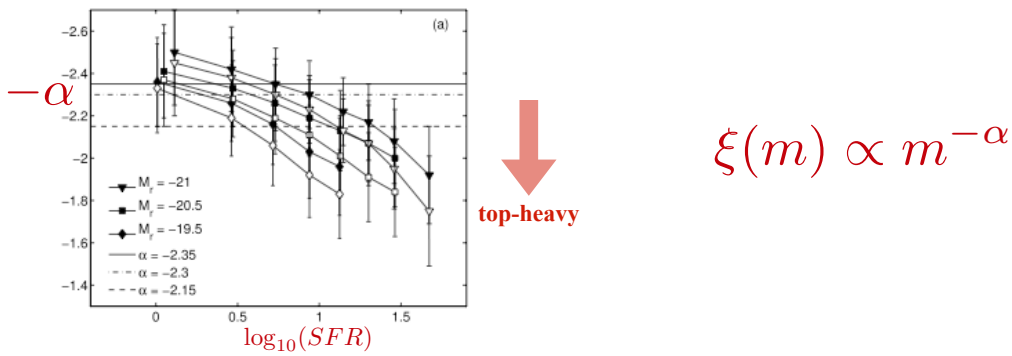


Figure 13. The best-fitting IMF slope for each of the (a) SFR (Fig. 6), (b) specific SFR (Fig. 11) and (c) SFR surface density (Fig. 12) sub-groups of the three volume-limited samples used in this study. The filled symbols denote results when using the Calzetti (2001)/Cardelli et al. (1989) dust corrections and open symbols represent the Fischera & Dopita (2005) dust corrections. The solid horizontal line indicates a Salpeter slope, the dot-dashed line indicates a Kroupa (2001) high-mass slope of $\alpha = -2.3$ and the dashed line dictates the Baldry & Glazebrook (2003) IMF slope.

Very comparable results by

Hoversten E. A., Glazebrook K., 2008, *ApJ*, 675, 163

Meurer G. R. et al., 2009, *ApJ*, 695, 765

Lee, J. C. et al., 2009, *ApJ*, 706, 599

4. Previously noted evidence:

Observational evidence for top-heavy IMF :

In star bursting regions: luminosity too large for amount of gas mass to sustain the burst unless IMF top-heavy.

Stellar counts: cosmic SFH implies too many low-mass stars locally unless IMF was top heavy at larger z when SFRs were higher.

The **high rate of type II supernovae** in the starburst galaxy Arp 220 suggests a top-heavy IMF in that system.

Since recently also deduced locally
(e.g. in globular clusters)...

(Elmegreen 2005; Eisenhauer 2011; Dabringhausen et al. 2012;
Marks et al. 2012; Kroupa et al. 2013)

5. M33 - radial distribution of young star clusters

stochastic ?

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The galactocentric radius dependent upper mass limit of young star clusters: stochastic star formation ruled out

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ABSTRACT

It is widely accepted that the distribution function of the masses of young star clusters is universal and can be purely interpreted as a probability density distribution function with a constant upper mass limit. As a result of this picture the masses of the most massive objects are exclusively determined by the size of the sample. Here we show, with very high confidence, that the masses of the most massive young star clusters in M33 decrease with increasing galactocentric radius in contradiction to the expectations from a model of a randomly sampled constant cluster mass function with a constant upper mass limit. Pure stochastic star formation is thereby ruled out. We use this example to elucidate how naive analysis of data can lead to unphysical conclusions.

Key words: stars: formation – galaxies: individual: M33 – galaxies: star clusters: general.

5 CONCLUSION

We have found that the formation of very massive star clusters is increasingly suppressed with increasing galactocentric radius in M33. We have ruled out with extremely high significance that this is the result of a size-of-sample effect, where a constant and environment independent ICMF is populated entirely randomly and environmental effects can be neglected.

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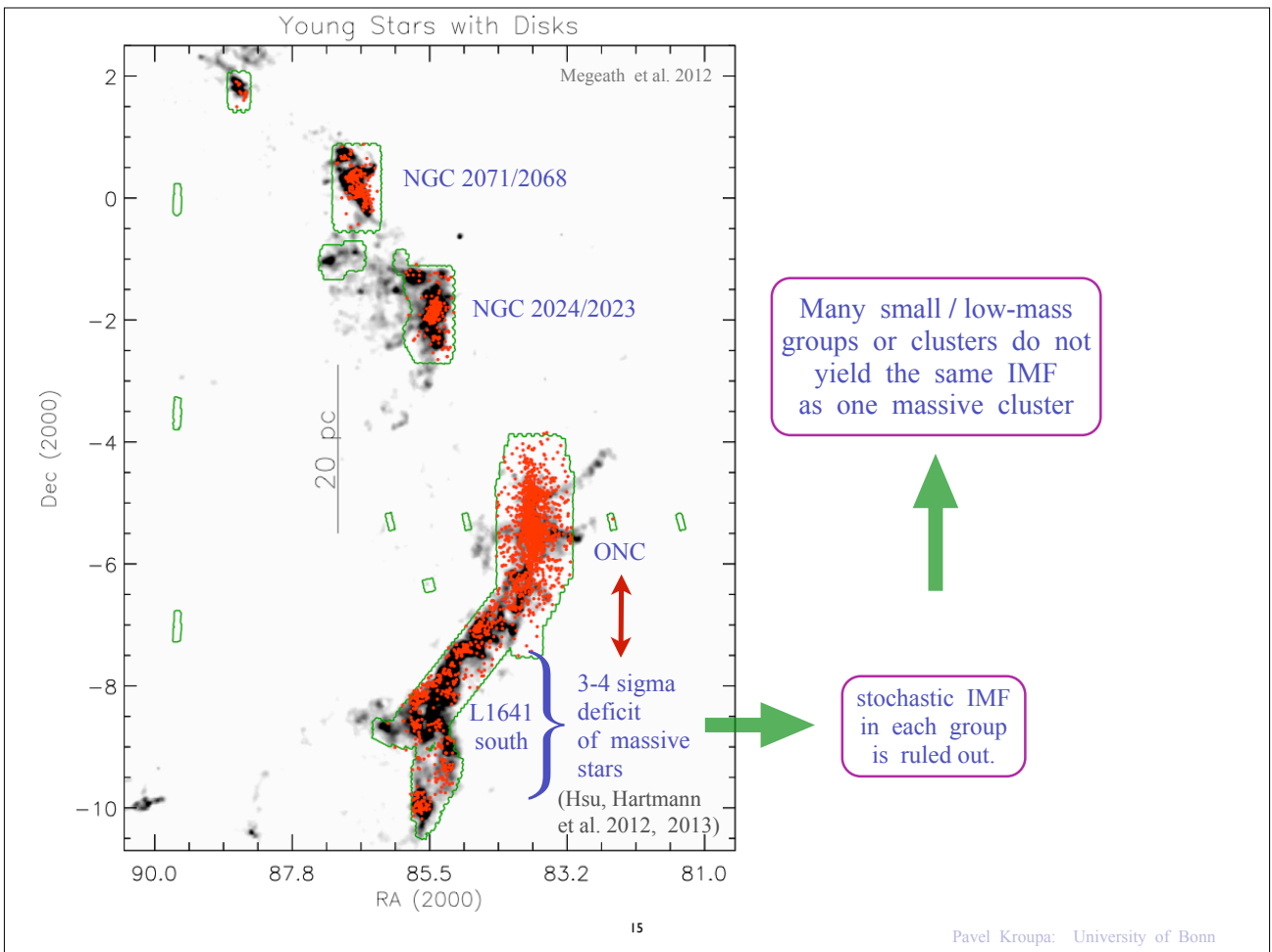
1. + 2. + 3. + 4. + 5.



Real young populations
are *not stochastic ensembles*
from invariant distr. functions

~~the IMF / star cluster MF an invariant probability density distribution function~~

The
IMF
as a
constrained
probability density
distribution function



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↓

the IMF as a **constrained probability distribution function (pdf)**

==> But it is not a pure pdf in individual clusters :
(The Orion case suggests that the most massive star may depend on cluster mass.)

Consider : in each *correlated star-formation event*
(CSFE ≡ embedded cluster)
there is *one most massive star* :

$$1 = \int_{m_{\max}}^{m_{\max*}} \xi(m) dm$$

$$M_{\text{ecl}} = \int_{m_1}^{m_{\max}} m \xi(m) dm$$

→ $m_{\max} = \text{fn}(M_{\text{ecl}})$
an m_{\max} -- M_{ecl} relation

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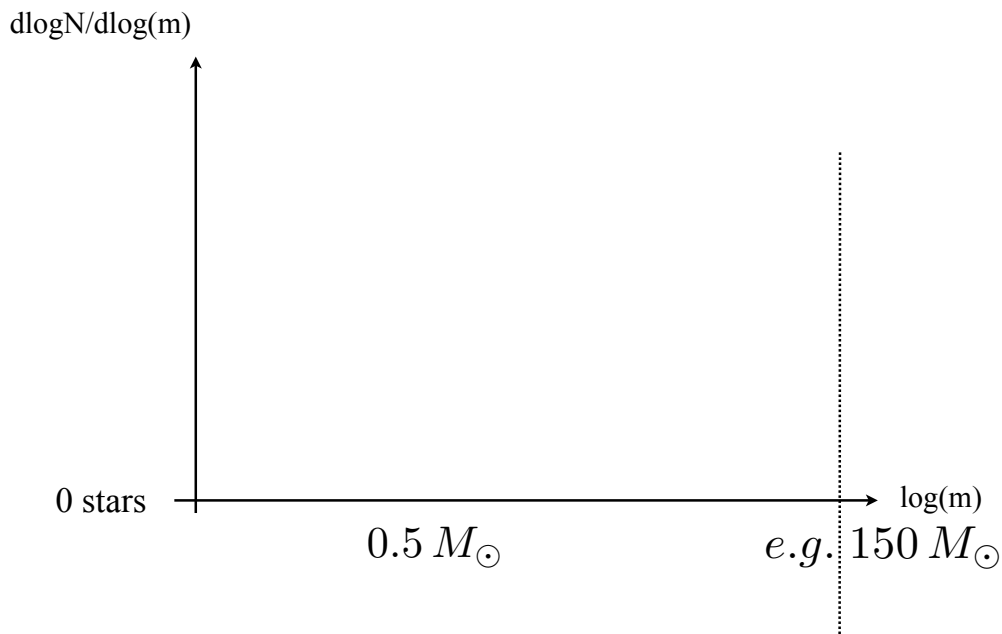
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... So :

Variation of the canonical / standard / universal IMF ?



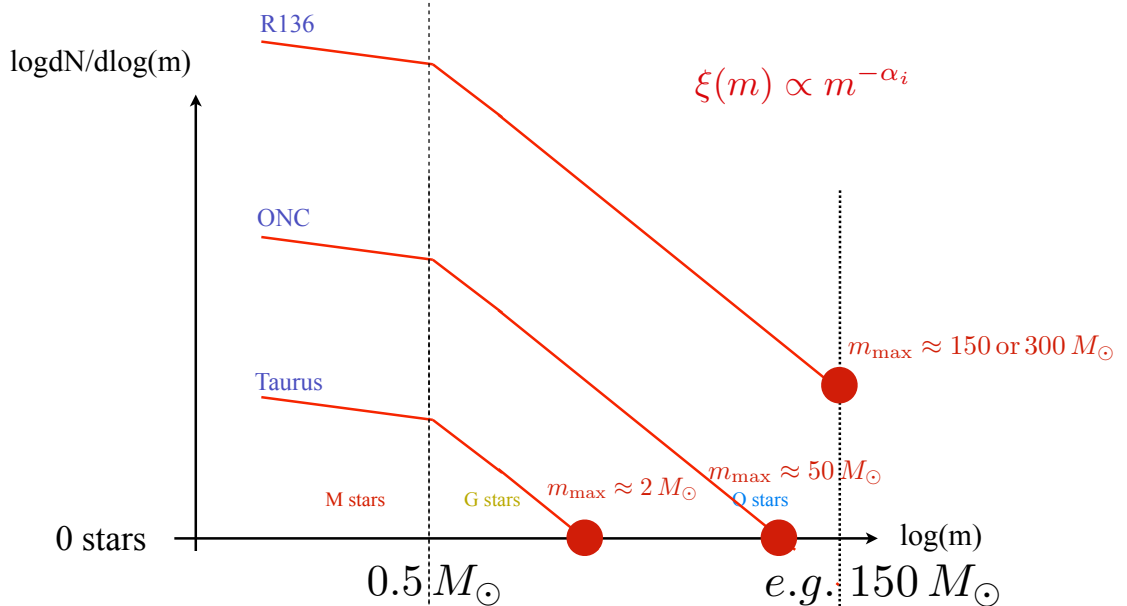
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... So :

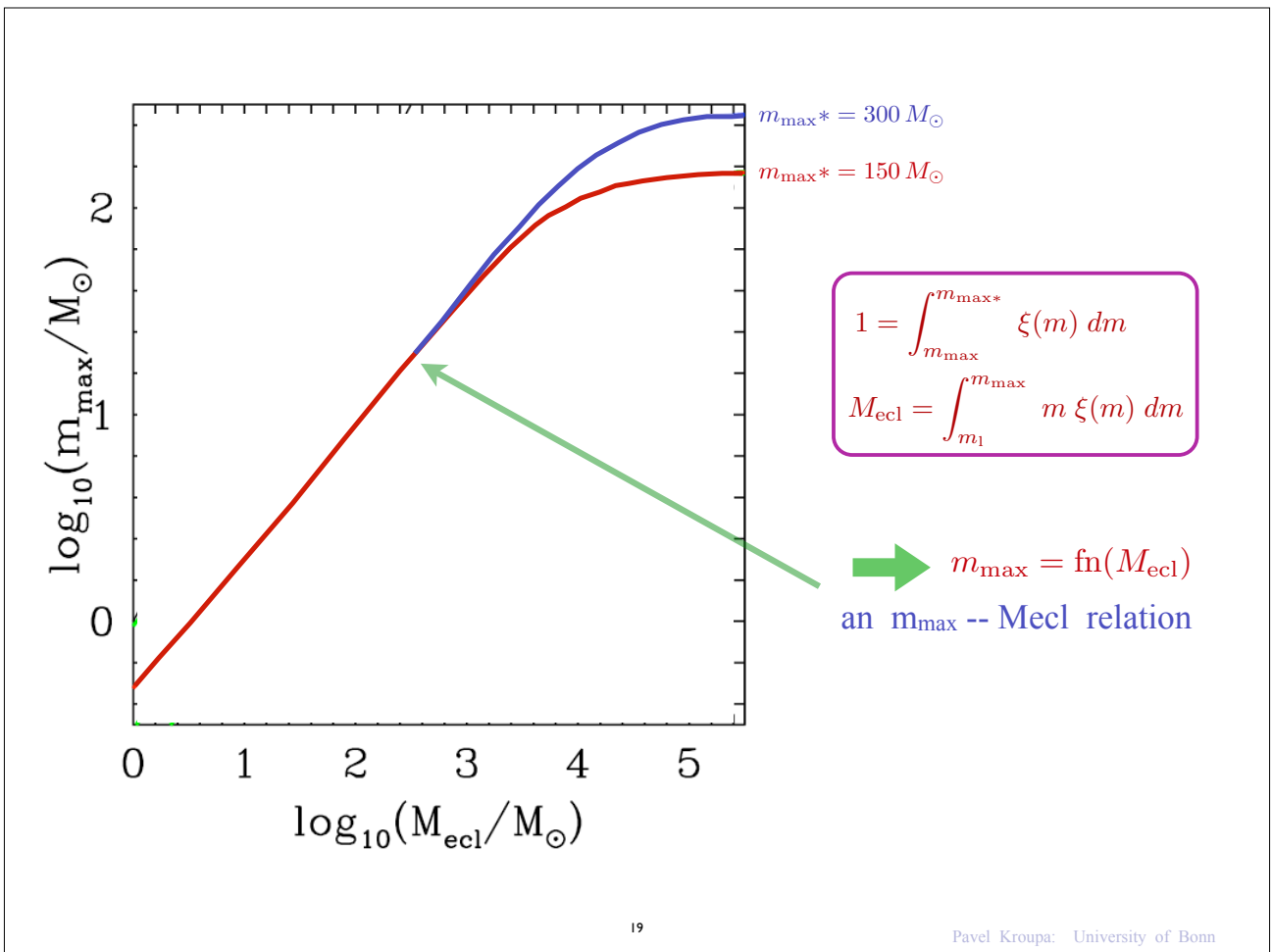
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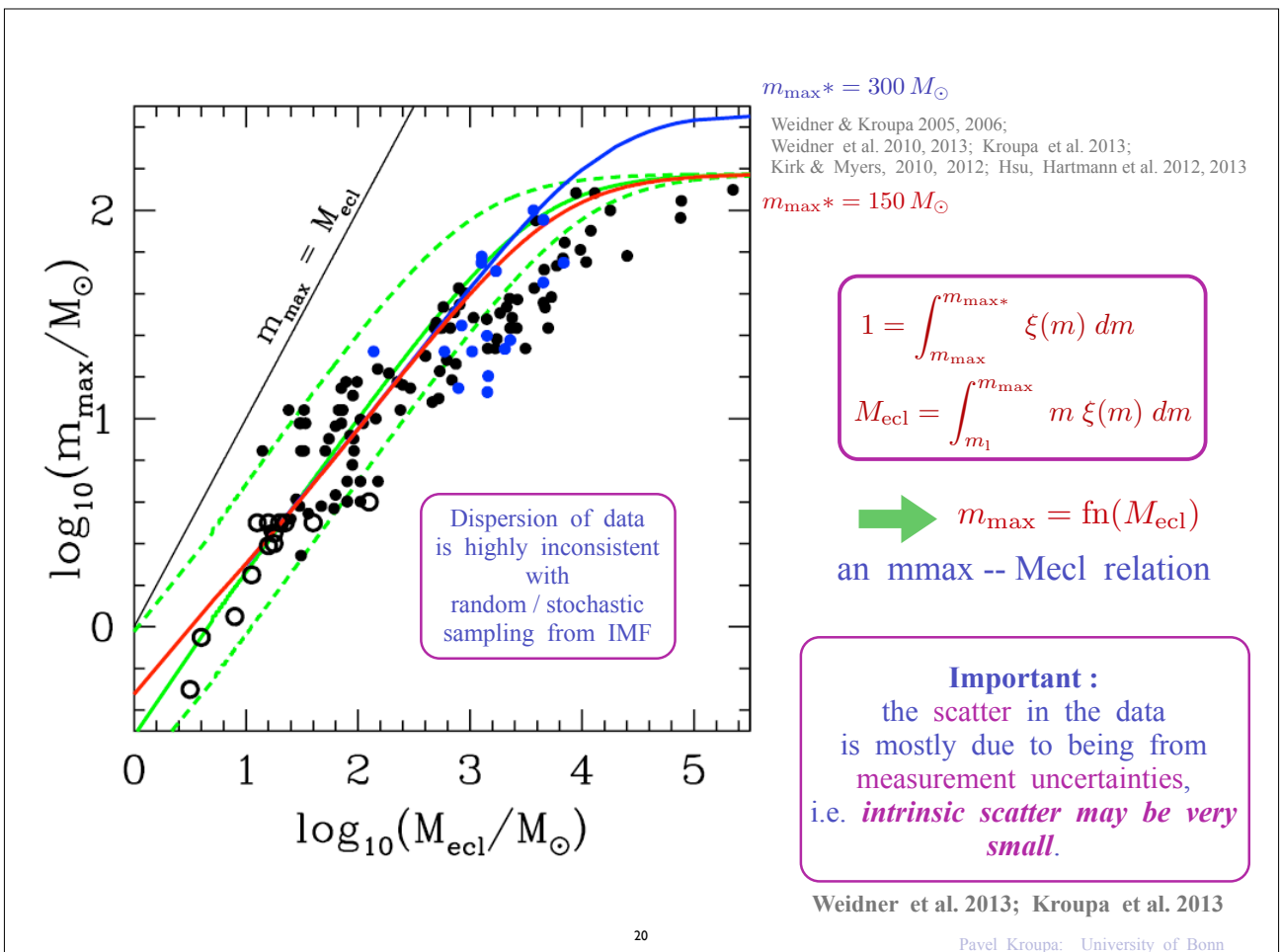
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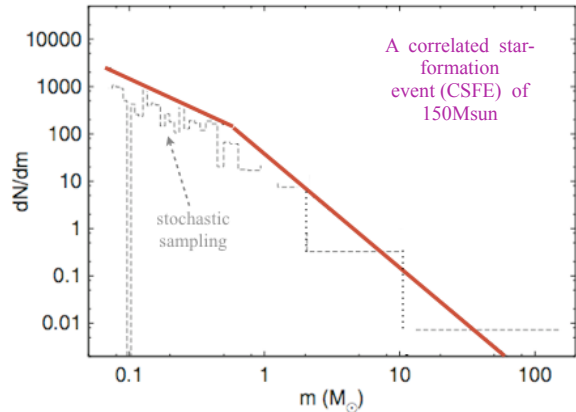
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A RESULT :
 the scatter in the data
 is mostly due to being from
 measurement uncertainties,
 i.e. *intrinsic scatter may be very
 small.*



The IMF can only be a probability distribution
 function,
 if *sampling from it is close to optimal :*

Remember :
 expect large dispersion in
 measured IMF shapes

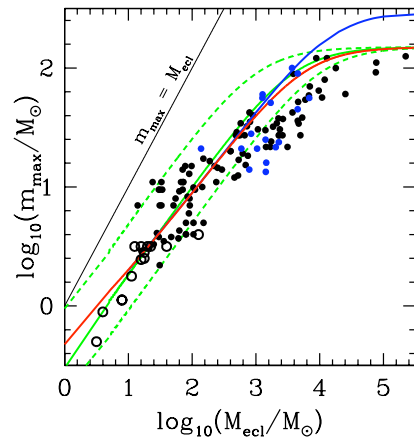


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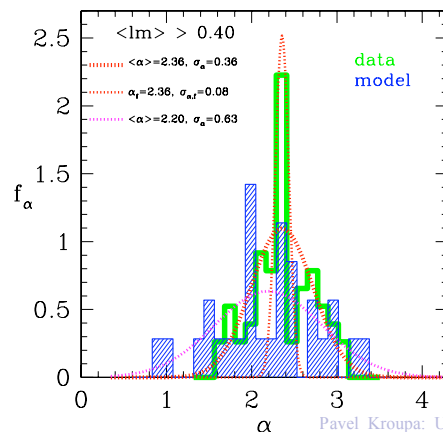
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Nature in actuality appears to
 be closer to optimal sampling :

Tight $m_{\max}(M_{\text{ecl}})$ relation :



Small dispersion of α_3 values :



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~~the IMF as a **constrained probability density distribution**~~

the IMF as an **optimally sampled density distribution**

What is an optimally sampled distribution function ?

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The
IMF
as an
optimally sampled
probability density
distribution function

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Discussions with
Douglas Heggie
in Heidelberg
in about 2011

The
IMF
as an
optimally
sampled
probability density
distribution function

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What is an optimally sampled distribution function ?

Given the *mass reservoir* in stellar mass M_{ecl} ,
starting with the most massive star,
select the next most massive such that
 M_{ecl} is distributed over the distribution function
without Poisson scatter.

Kroupa et al. 2013

The above ansatz can be extended to a discretized optimal distribution of stellar masses:
Given the mass, M_{ecl} , of the population, the following sequence of individual stellar masses
yields a distribution function which exactly follows $\xi(m)$,

$$m_{i+1} = \int_{m_{i+1}}^{m_i} m \xi(m) dm, \quad m_L \leq m_{i+1} < m_i, \quad m_1 \equiv m_{\text{max}}. \quad (4.9)$$

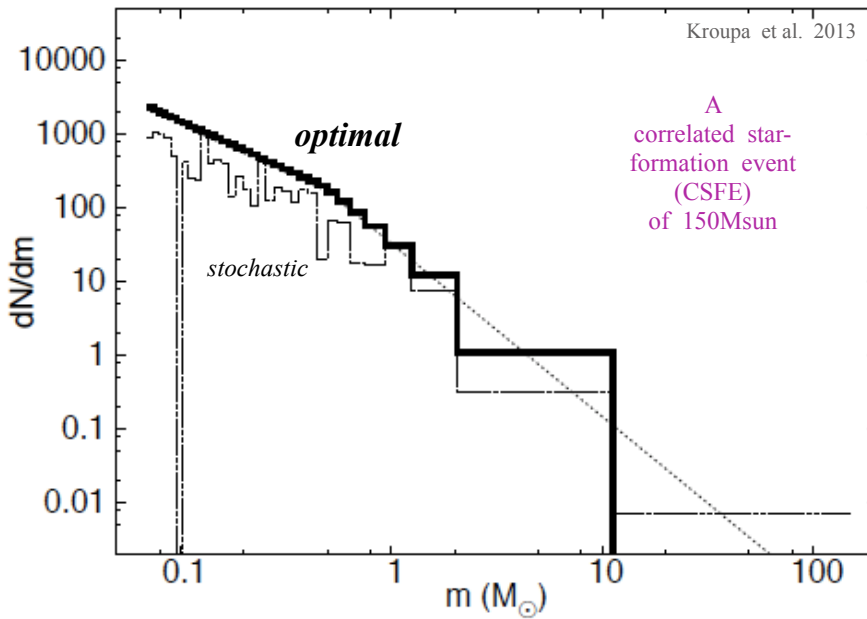
The normalization and the most massive star in the sequence are set by the following two equations:

$$1 = \int_{m_{\text{max}}}^{m_{\text{max}^*}} \xi(m) dm, \quad (4.10)$$

with

$$M_{\text{ecl}}(m_{\text{max}}) - m_{\text{max}} = \int_{m_L}^{m_{\text{max}}} m \xi(m) dm \quad (4.11)$$

as the closing condition. These two equations need to be solved iteratively. An excellent approx-



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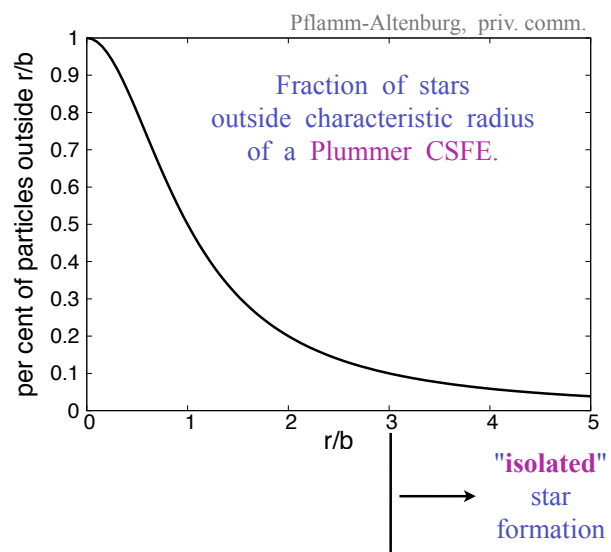
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Correlated star formation events (CSFEs)

The stellar population of an individual CSFE of stellar mass M_{ecl} is now specified.

Assuming star formation takes place in CSFEs, the stellar population from an ensemble of CSFEs can be computed, if the *distribution of CSFEs* is known.



(dynamical activity leads to rapid dispersal of additional stars which may appear to have formed "in isolation")

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Correlated star formation events CSFEs (embedded clusters)

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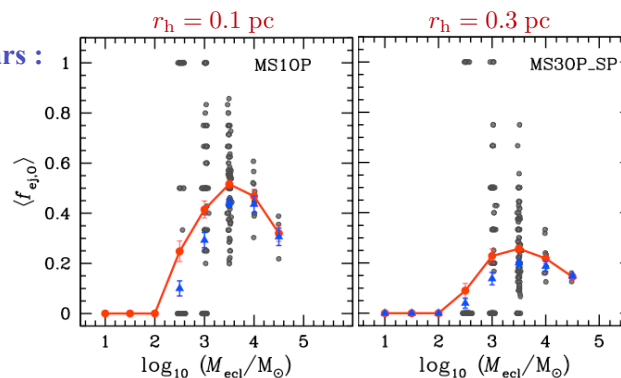
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What do we know about Correlated Star Formation Events (CSFEs) ?

Stellar-mass distribution : $\xi_{\text{ECMF}}(M_{\text{ecl}}) = k M_{\text{ecl}}^{-\beta}$; $\beta \approx 2$ (Lada & Lada 2003)

Radius - mass relation : $\frac{r_h}{\text{pc}} = 0.1^{+0.07}_{-0.04} \times \left(\frac{M_{\text{ecl}}}{M_{\odot}}\right)^{0.13 \pm 0.04}$ (Marks & Kroupa 2012)
c.f. simulation results from **Matthew Bate**
& observational results for Carina SF region by **Mauricio Tapia**

Efficient ejections of O stars :
(within 3Myr)



(Oh, Kroupa
& Pflamm-
Altenburg 2014)

100 % initial
binarity

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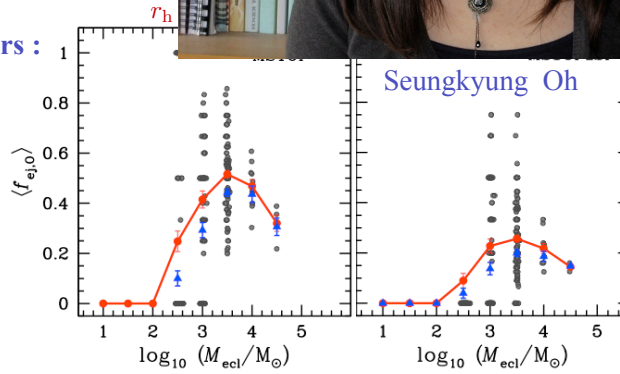
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Kroupa
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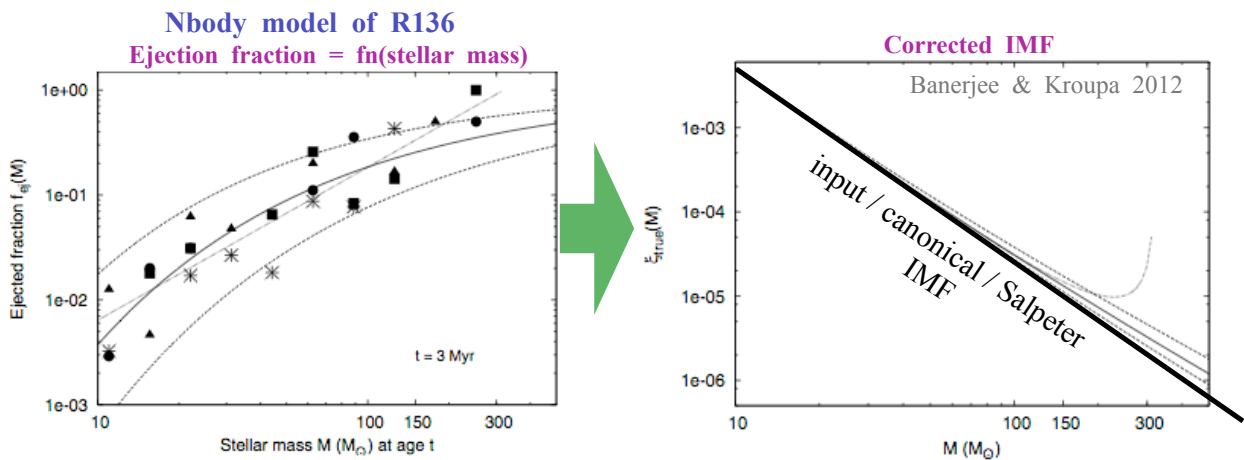
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What do we know about Correlated Star Formation Events (CSFEs) ?

IMF becomes top heavy with increasing density :



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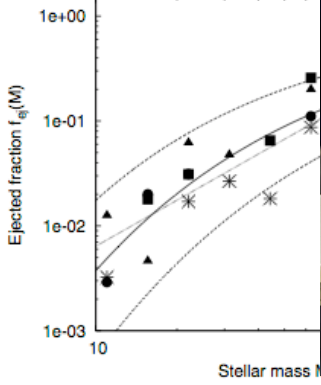
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*What do we know about
Correlated Star Formation Events (CSFEs) ?*

IMF becomes top heavy

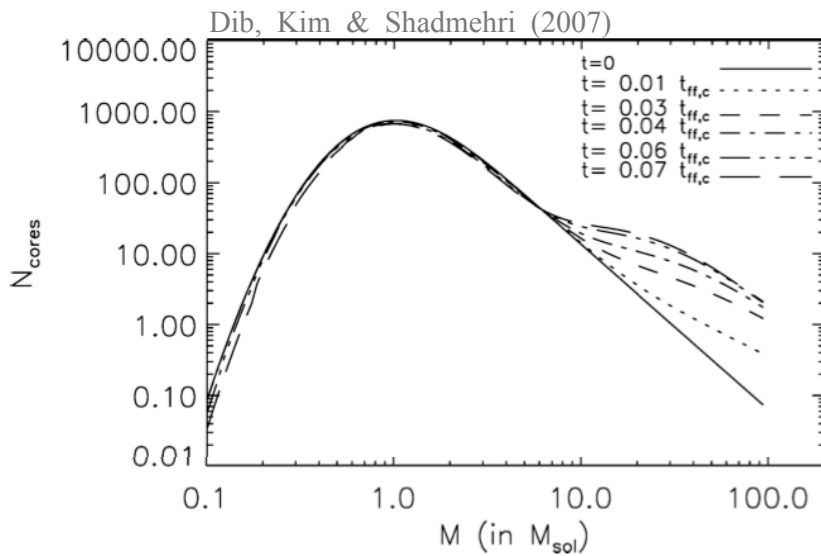
Nbody model
Ejection fraction



Sambaran Banerjee + Sverre's code

*What do we know about
Correlated Star Formation Events (CSFEs) ?*

IMF becomes top heavy with increasing density :



Models of coalescing and collapsing cloud cores in a dense proto cluster.

What do we know about Correlated Star Formation Events (CSFEs) ?

Globular clusters : deficit of low-mass stars increases with decreasing concentration

- disagrees with dynamical evolution
- correlate energy needed to expell residual gas with number of OB stars required.

Marks et al. 2012

UCDs : higher dynamical M/L ratios

- cannot be exotic dark matter => top-heavy IMF

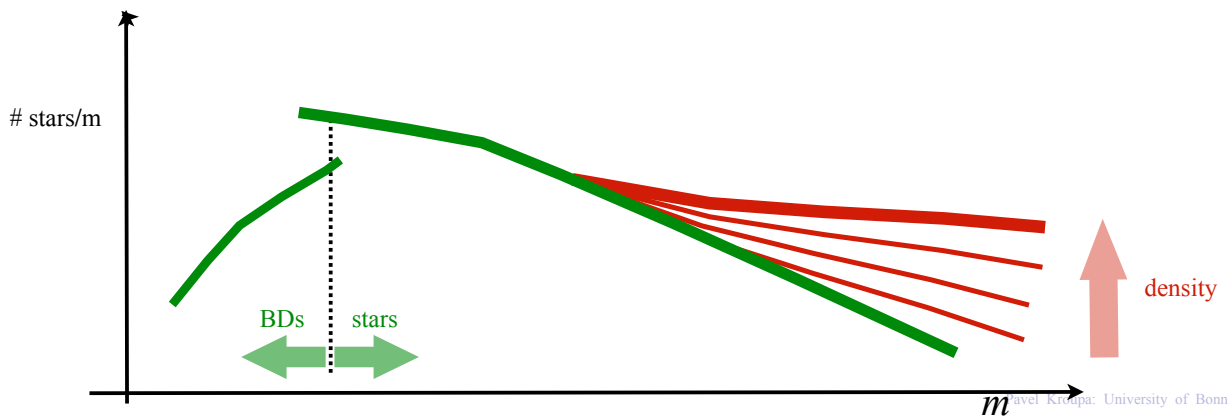
Dabringhausen et al. 2009

UCDs : larger fraction of X-ray sources than expected

- no explanation other than many remnants => top-heavy IMF

Dabringhausen et al. 2012

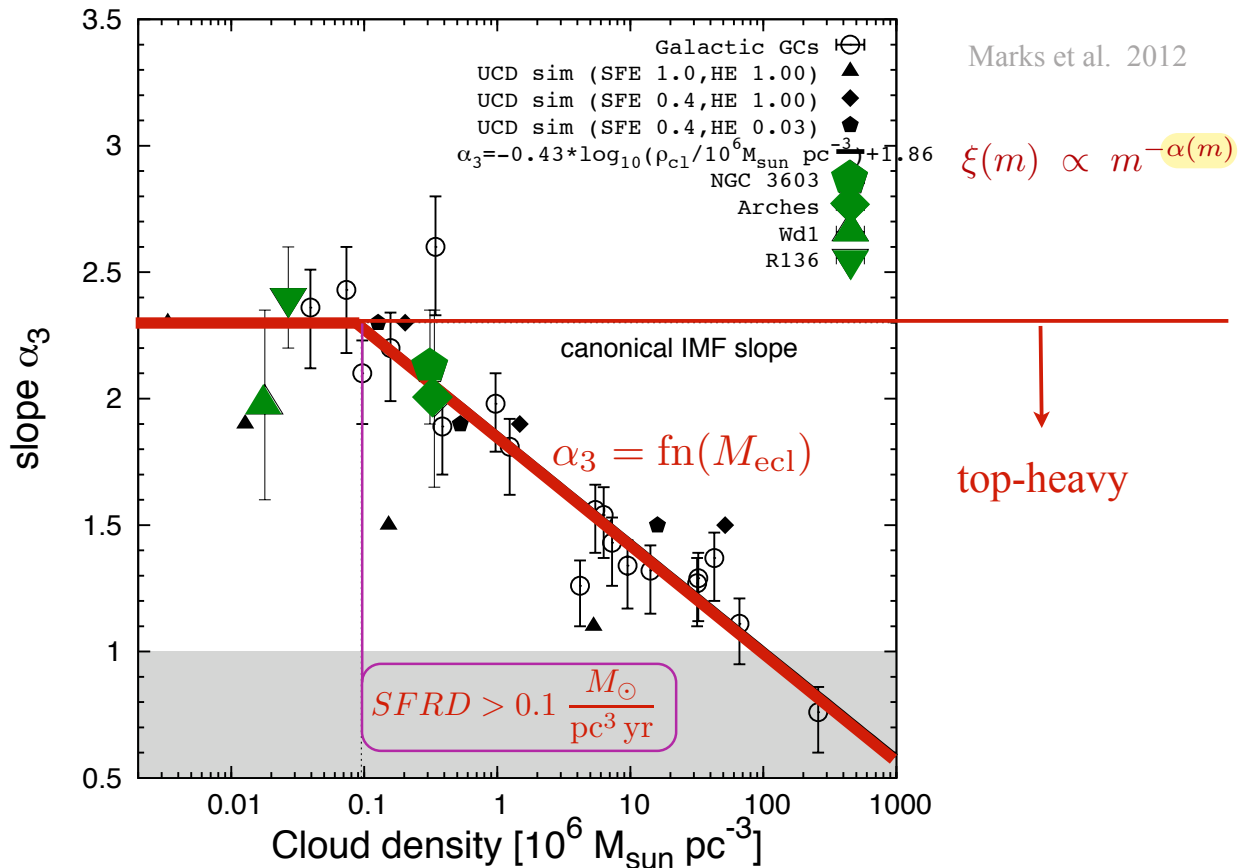
What this implies :



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Top-heavy IMF in extreme-density environments :



Marks et al. 2012

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Top-heavy IMF in extreme-density environments :

THE STELLAR IMF DEPENDENCE ON DENSITY AND METALLICITY: Resolved stellar populations show an invariant IMF (Eq. 55), but for $SFRD \gtrsim 0.1 M_{\odot}/(\text{yr pc}^3)$ the IMF becomes top-heavy, as inferred from deep observations of GCs. The dependence of α_3 on cluster-forming cloud density, ρ , (stars plus gas) and metallicity, $[\text{Fe}/\text{H}]$, can be parametrised as

$$\begin{aligned} \alpha_3 &= \alpha_2, & m > 1 M_{\odot} \quad \wedge \quad x < -0.89 \\ \alpha_3 &= -0.41 \times x + 1.94, & m > 1 M_{\odot} \quad \wedge \quad x \geq -0.89 \\ x &= -0.14 [\text{Fe}/\text{H}] + 0.99 \log_{10} (\rho / (10^6 M_{\odot} \text{pc}^{-3})) . \end{aligned} \tag{65}$$

Marks et al. 2012
Kroupa et al. 2013 (arXiv:1112.3340)
Recchi & Kroupa 2014 (arXiv:1411.0318)

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Thus there is good and independent quantifiable evidence
for the IMF
becoming top-heavy
with increasing density and decreasing metallicity.

Note : extraction of this evidence requires understanding
the data and of the dynamical evolution of the CSFEs.

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From CSFEs to galaxies

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Correlated star formation events building up a galaxy

The total mass in stars formed in a galaxy over time δt is $M_{\text{tot}} = SFR \times \delta t$

But
$$M_{\text{tot}} = \int_{M_{\text{ecl},\text{min}}}^{M_{\text{ecl},\text{max}}} \xi_{\text{ecl}}(M_{\text{ecl}}) M_{\text{ecl}} dM_{\text{ecl}}$$

For $M_{\text{ecl},\text{min}} = 5 M_{\odot}$ and with
$$1 = \int_{M_{\text{ecl},\text{max}}}^{M_{\text{ecl},\text{max}^*}} \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}$$

where $M_{\text{ecl},\text{max}^*} \approx 10^7 M_{\odot}$

Thus $M_{\text{ecl},\text{max}} = \text{fn}(SFR)$

What is δt ?

The galaxy-wide time-scale of transforming the ISM via molecular clouds into a new stellar population (Egusa et al. 2004; 2009).

Disappearance of large molecular clouds around young star clusters (Leisawitz 1989).



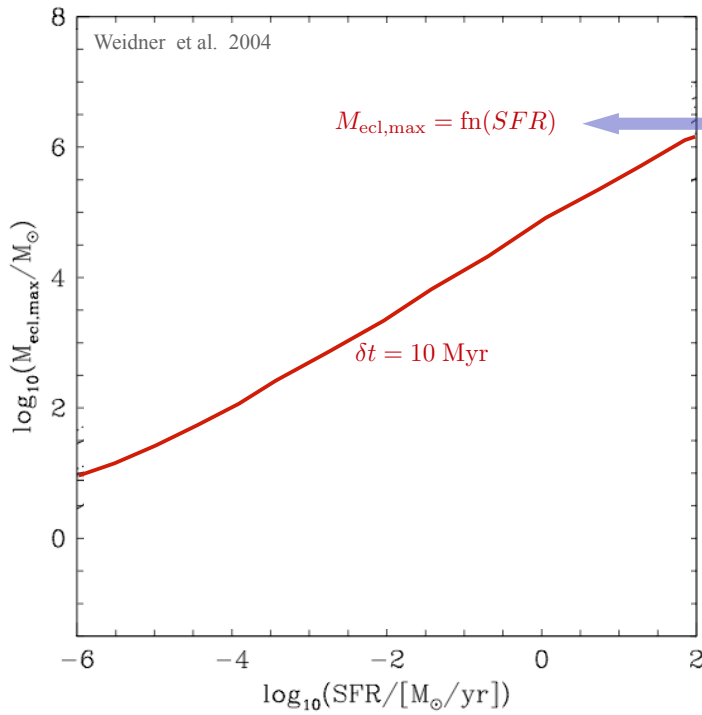
$$\delta t \approx 10 \text{ Myr}$$

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$$M_{\text{tot}} = \text{SFR} \times \delta t$$

$$M_{\text{tot}} = \int_{M_{\text{ecl},\text{min}}}^{M_{\text{ecl},\text{max}}} \xi_{\text{ecl}}(M_{\text{ecl}}) M_{\text{ecl}} dM_{\text{ecl}}$$

$$1 = \int_{M_{\text{ecl},\text{max}}}^{M_{\text{ecl},\text{max}^*}} \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}$$

$$\delta t = 10 \text{ Myr}$$

$$M_{\text{ecl},\text{min}} = 5 M_{\odot}$$

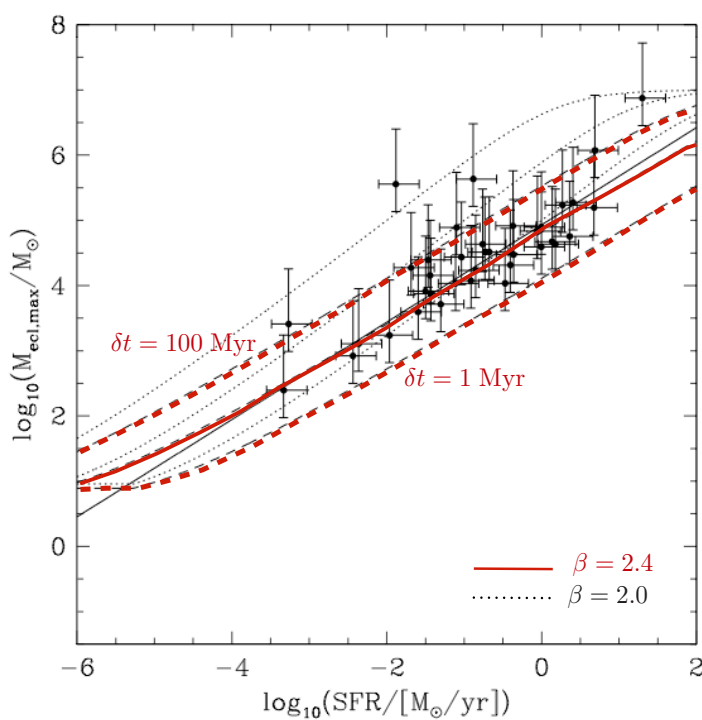
$$M_{\text{ecl},\text{max}^*} \approx 10^7 M_{\odot}$$

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$$M_{\text{tot}} = \text{SFR} \times \delta t$$

$$M_{\text{tot}} = \int_{M_{\text{ecl},\text{min}}}^{M_{\text{ecl},\text{max}}} \xi_{\text{ecl}}(M_{\text{ecl}}) M_{\text{ecl}} dM_{\text{ecl}}$$

$$1 = \int_{M_{\text{ecl},\text{max}}}^{M_{\text{ecl},\text{max}^*}} \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}$$

$$\delta t = 10 \text{ Myr}$$

$$M_{\text{ecl},\text{min}} = 5 M_{\odot}$$

$$M_{\text{ecl},\text{max}^*} \approx 10^7 M_{\odot}$$

Note:
the true spread
is smaller because of the
substantial measurement uncertainties

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Composite Stellar Populations

The Integrated Galactic IMF now follows from

$$\xi_{\text{IGIMF}}(m, t) = \int_{M_{\text{ecl}, \text{min}}}^{M_{\text{ecl}, \text{max}}(SFR(t))} \xi(m \leq m_{\text{max}}(M_{\text{ecl}})) \xi_{\text{ecl}}(M_{\text{ecl}}) dM_{\text{ecl}}$$

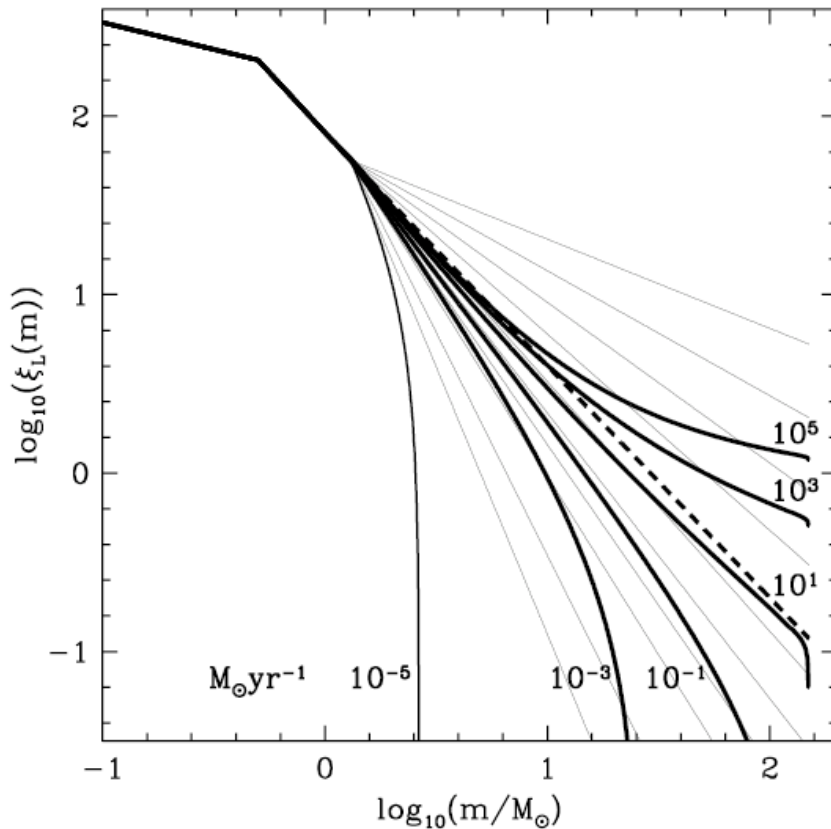
Kroupa & Weidner (2003); Weidner & Kroupa (2005, 2006)
Vanbeveren (1982)



adding-up all IMFs
in all SCFEs!
The LEGO principle

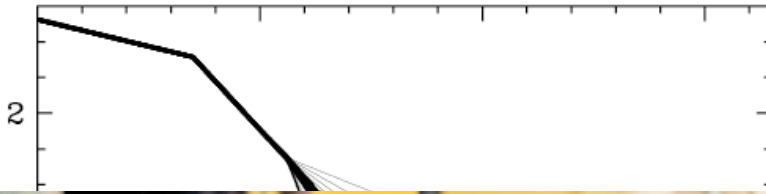
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Weidner et al. 2013; Kroupa et al. 2013



The IGIMF for galaxies with different SFRs

Weidner et al. 2013; Kroupa et al. 2013



The IGIMF for galaxies with different SFRs



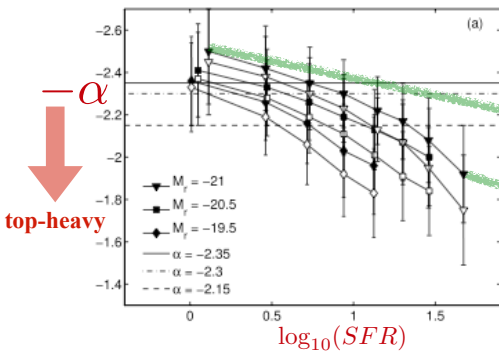
Jan Pflamm-Altenburg & Carsten Weidner

Pavel Kroupa: University of Bonn

Donnerstag, 6. November 14

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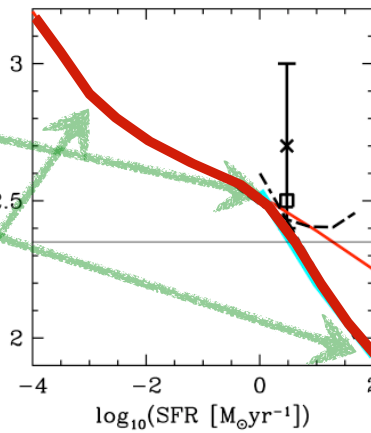
1660 M. L. P. Gunawardhana et al. 2011



Very comparable / consistent results by

- Hoversten E. A., Glazebrook K., 2008, ApJ, 675, 163
- Meurer G. R. et al., 2009, ApJ, 695, 765
- Lee, J. C. et al., 2009, ApJ, 706, 599

IGIMF theory



Weidner et al. 2013

top-light

↑

top-heavy

↓

E galaxies formed with top-heavy IMFs

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Conclusions

- The *stellar IMF* is found to be *surprisingly invariant* up to a critical star-formation rate density (SFRD) on a <pc scale
- Above this critical SFRD [GCs + UCDs] ==> evidence for top-heavy IMF in starbursts
- IMF : *not* a probabilistic / stochastic distribution function
- IMF : appears to be closer to an *optimally* sampled distribution function (i.e. **star-formation is feedback regulated**)
- **Superposition principle** : Sum over all CSFEs (=embedded clusters) : very powerful approach to quantify properties of freshly hatched stellar populations in galaxies :

IGIMF, binary properties, thick disks

- Pioneering application of this concept in self-consistent full-scale hydrodynamic simulations of galaxies by *Ploeckinger et al. (2014a,2014b)*