Analytical models of the initial mass function

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Some simple considerations…

Volume of the sphere in k-space

Volume of the sphere in real space

$$\begin{cases} dN(k) \propto k^2 dk \\ M \propto R^3 \propto k^{-3} \Rightarrow dM \propto k^{-4} dk \end{cases}$$

$$\Rightarrow dN(k) \propto k^6 dM$$

$$\Rightarrow \frac{dN}{dM} \propto M^{-2}$$

M^{-2} is the most natural mass spectrum, one can infer from elementary geometrical considerations.

Salpeter: -2.3
CO clumps: -1.7

=> A robust theory must predict mass spectra with sufficient accuracy…

Different types of models

Theories based on pure gravity
-recursive fragmentation (Larson 1972, Elmegreen & Mathieu 1983, Field et al. 2006)
=> tends to produce lognormal distribution

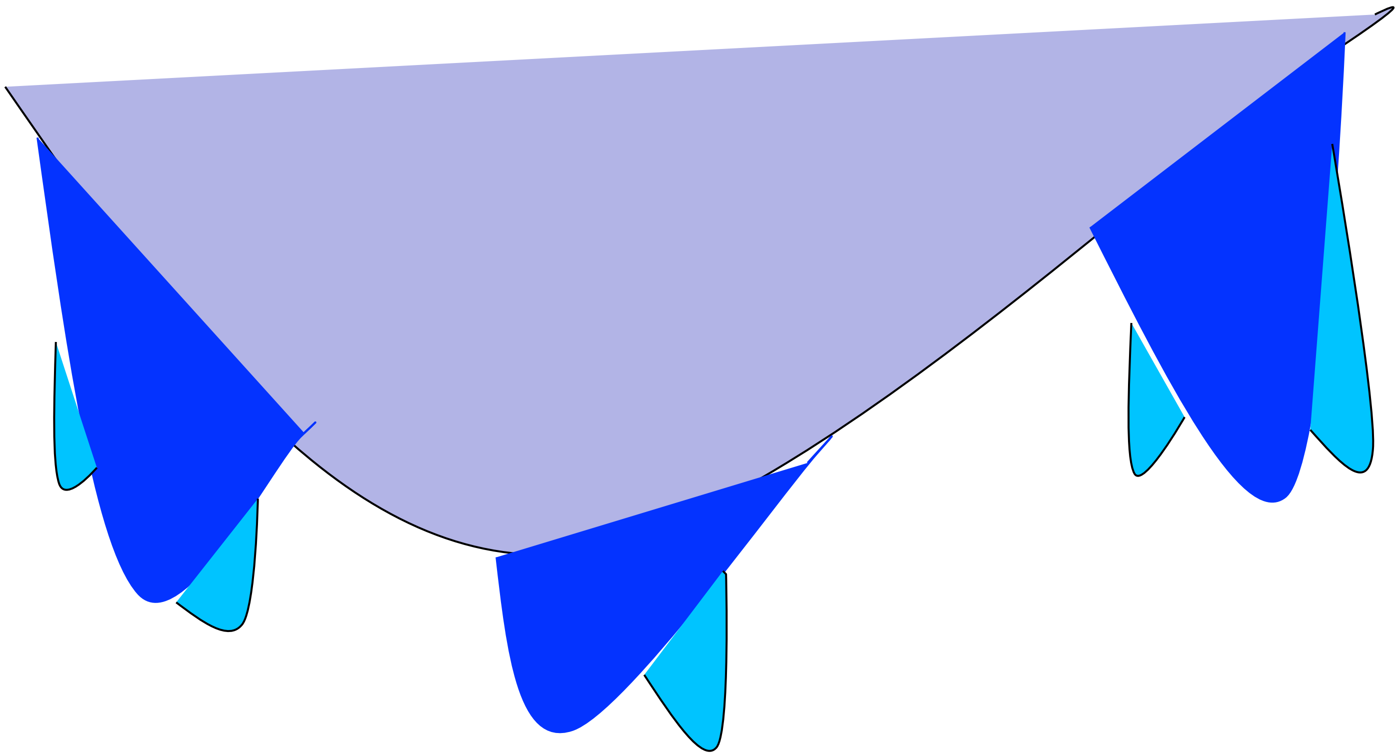
Theories based on stochastic processes
-e.g. use the central limit theorem (Adams & Fatuzzo 1996, Elmegreen 2001)

Theories based on accretion (Initial Jeans mass is unimportant)
-competitive accretion (Zinnecker 1982, Larson 1992, Bonnell et al. 2001, Bate et al. 2003)
-"stopped" accretion (Adams & Fatuzzo 1996, Basu & Jones 2004, Bate & Bonnell 2005, Myers 2008, Maschberger 2013)

Theories based on "core" formations (central role played by the initial Jeans mass)
-gravity+MHD shocks (Padoan & Nordlund 1997, 2002, Padoan et al. 2007)
-gravity+turbulent support/dispersion (Hennebelle & Chabrier 2008, 2009, 2013, Hopkins 2013abc)

=> PPVI review by Offner et al. 2014

A hierarchy of wells:



Direct mapping between the wells and the stars ?
Exchange between the wells ?
Likely both ! But how much ?

1-Theories based on accretion

2- “Reservoir”/Core based theory

3- Sensibility to physics and cloud conditions

4- Self-regulation of initial conditions / cluster formation

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Theories based on accretion

Competitive accretion (Zinnecker 1982, Bonnell et al. 2001**)**

Main idea is that stars compete for the gas and the big ones are more hungry…

The accretion rate depends on the local density, the relative velocity and the accretion radius:

$$\dot{M}_* = \pi \rho V_{rel} R_{acc}^2$$

Gas dominated Potential:
R position in the cloud

$$\frac{dN}{dM} \propto M^{-3/2}$$

Assume $\rho_{gas} \propto R^{-2}, n_* \propto R^{-2}$: singular isothermal sphere (Shu 1977)

$$R_{acc} \approx R_{tidal} \approx \left(\frac{M_*}{M_{enc}}\right)^{1/3} R, \quad M_{enc} = 4\pi \int \rho R^2 dR \propto R, \quad V_{rel} \approx \sqrt{GM_{enc}/R} \approx cst$$

$$\Rightarrow \dot{M}_* \propto M_*^{2/3} R^{-2/3} \Rightarrow M_*^{1/3} \propto R^{-2/3} \Rightarrow M_* \propto R^{-2} \Rightarrow dR \propto M_*^{-3/2} dM_*$$

$$dN_* = 4\pi n_* R^2 dR \Rightarrow dN_* \propto M_*^{-3/2} dM_*$$

Different density profils lead to different mass spectrum

Stellar dominated Potential: $\frac{dN}{dM} \propto M^{-2}$

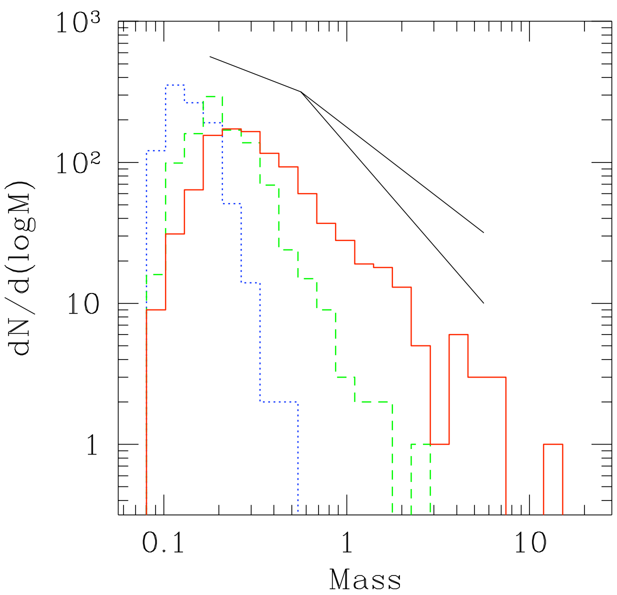
Assume $\rho_{gas} \propto R^{-3/2}$: typical after rarefaction wave propagates away (Shu 1977)

$$\dot{M}_* = \pi \rho V_{rel} R_{acc}^2$$

$$R_{acc} \approx R_{BH} \approx \frac{GM_*}{V_{rel}^2}, \qquad V_{rel} \approx \sqrt{GM_{cluster}/R} \approx R^{-1/2}$$

$\Rightarrow \dot{M}_* \propto M_*^2$ (accretion independent on the position in the cluster)

$\Rightarrow dN \propto M_*^{-2} dM_*$ (under reasonable assumptions…)



Mass spectrum from Bonnell et al. (2001)

1000 stars initially of mass 0.1 Ms, 10% of the total Mass

The mass spectrum develops and lead to a Salpeter type Slope

1-Theories based on accretion

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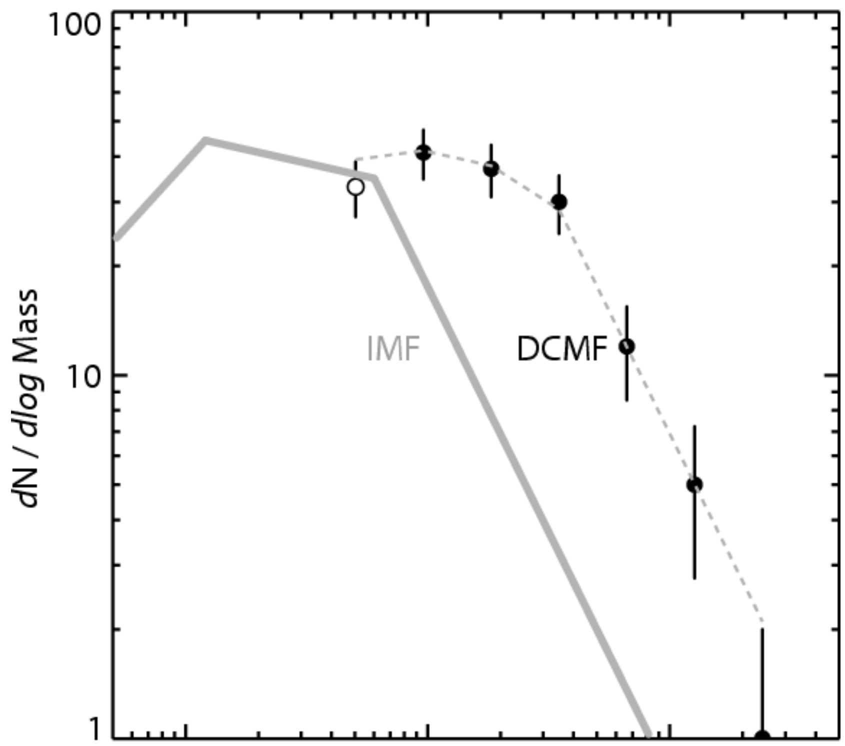
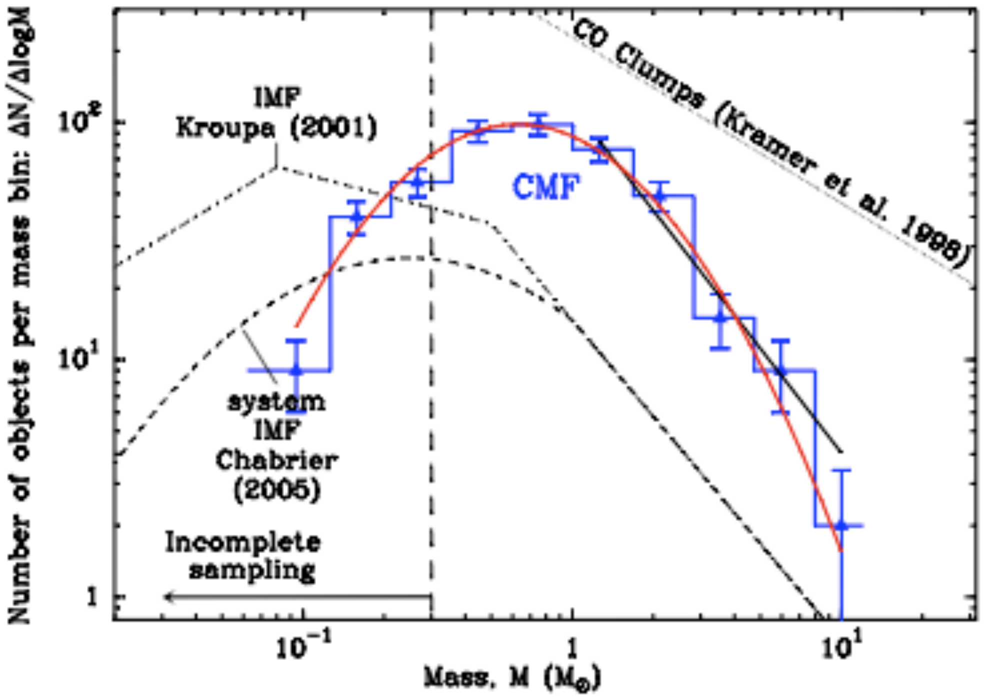
Theories based on cores - Central role played by the Jeans mass

(Motte et al. 1998, Testi & Sargent 1998, Alves et al. 2007, Johnstone et al. 2002, Enoch et al. 2008, Simpson et al. 2008)



Alves et al. 2007



Konyves, André et al. 2010

Core as density fluctuations created by turbulence and selected by gravity

Principles of Press-Schecter analysis

Used in cosmology to predict the mass spectrum of DM haloes: =>*very successful*

-consider a spectrum of density fluctuations (Gaussian in the cosmological case) characterized by its powerspectrum and smooth it at scale R

-setup a criterion to decide which perturbations have to be considered (collapse time should be smaller than the age of the universe)

-sum over the corresponding fluctuations

In the case of Molecular clouds

(Padoan et al. 1997, 2002, H & Chabrier 2008, 2009, 2011, 2013, Hosking 2011, 2012abc)

-assume that the density PDF is log-normal (e.g. Vazquez-Semadeni 1994, Padoan et al. 1997, Federrath et al. 2011, 2014)

-the power-spectrum of log ρ is close to Kolmogorov

-consider self-gravitating structures

Use Virial theorem to estimate when a piece of fluid is going to collapse:

$$2(E_{therm} + E_{kin}) + E_{mag} \le -E_{pot}$$

$$E_{therm} \propto kT,$$

$$E_{kin} \propto V_{rms}^{\;2} \propto R^{2\eta}, \eta \approx 0.4 - 0.5$$

Define 2 equivalent criteria for the piece of fluid to collapse:

$$M > M_R^c = a_J^{2/3}\left(\frac{C_s^2}{G}R + \frac{V_0^2}{3G}\left(\frac{R}{1pc}\right)^{2\eta} R\right)$$

$$\delta > \delta_R^c = \ln\left(a_J^{2/3} \frac{C_s^2 + \frac{V_0^2}{3G}\left(\frac{R}{1pc}\right)^{2\eta}}{G\overline{\rho}R^2} \right)$$

Then sum over all density fluctuations:

$$M_{tot}(R) = \overline{\rho}L_i^{\;3}\int_{\delta_R^c}^{\infty}\exp(\delta)\exp\left(-\frac{\left(\delta + \frac{\sigma^2}{2}\right)^2}{2\sigma^2}\right)d\delta = L_i^{\;3}\int_0^{M_R^c}\frac{dN}{dM'}M'P(M,M')dM'$$

Important terms:

$$\frac{dN}{dM} \propto \frac{1}{R^6}\left(\frac{M}{R^3}\right)^{-\frac{3}{2}-\frac{1}{2\sigma^2}\ln\left(M/R^3\right)}$$

: combination of lognormal functions (dominant at small and large masses) and powerlaws

$$M = R\left(1+\mathcal{M}_*^2 R^{2\eta}\right)$$

$$\mathcal{M}_* \propto \frac{V_0}{C_s}\frac{\lambda_J}{1pc}$$

: Mach number at the Jeans length (due to turbulent *support/dispersion*)
Transition is expected around 1

The theory is controlled by the global Mach number and the Mach number at the Jeans length of the gas *before it forms core*.

At large masses:
-in the absence of turbulent support (identical to Padoan et al. 1997)

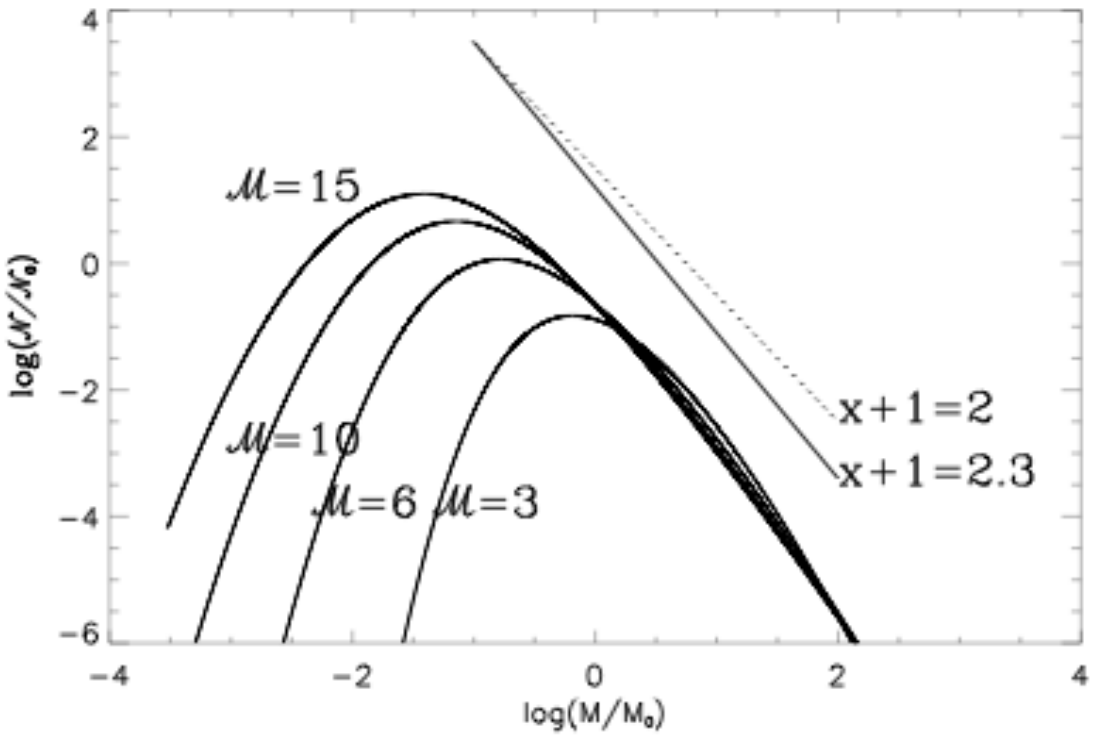
-whereas when turbulent support is significant

$$\mathrm{M}_* \approx 0, \frac{dN}{d\log M} \propto M^{-2}$$

$$\mathrm{M}_* \geq 1, \frac{dN}{d\log M} \propto M^{-\frac{n+1}{2n-4}}$$

For n=11/3, the exponent is 1.4 and for n=4, it is 1.25

Influence of the global Mach number on the CMF

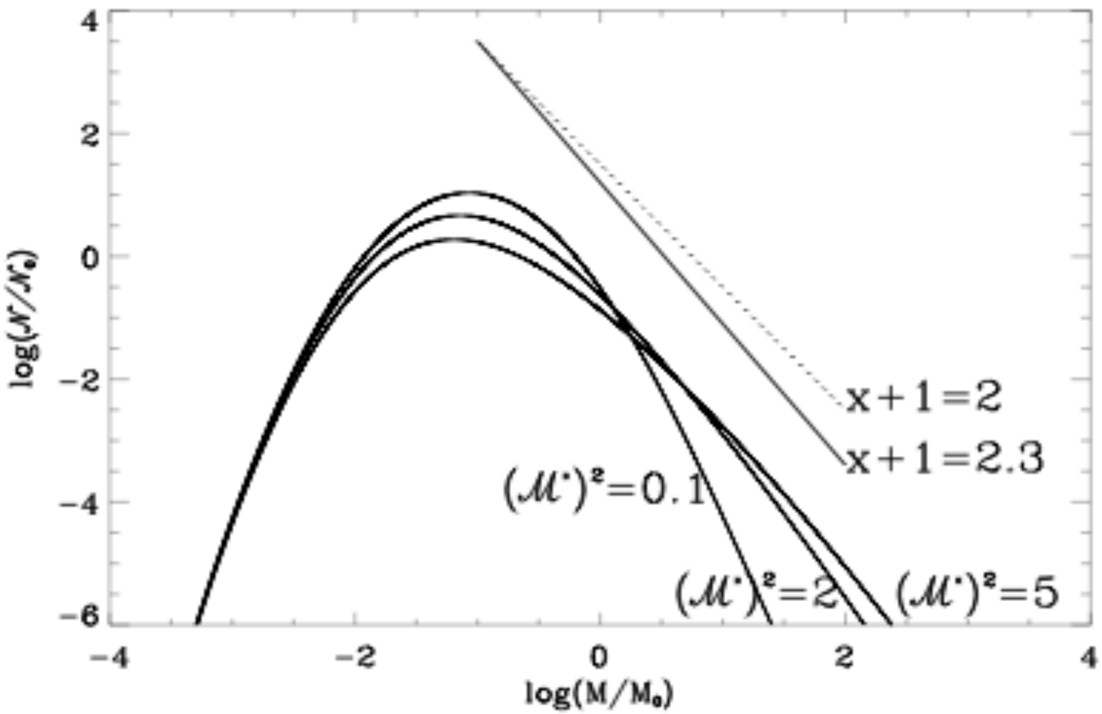


The larger the Mach number, the larger the number of small cores (brown dwarfs) (Padoan & Nordlund 2004)

The large mass core number does not vary with the global Mach number M.

Influence of the Mach number at the Jeans scale, M_*

Low M_*: only thermal support

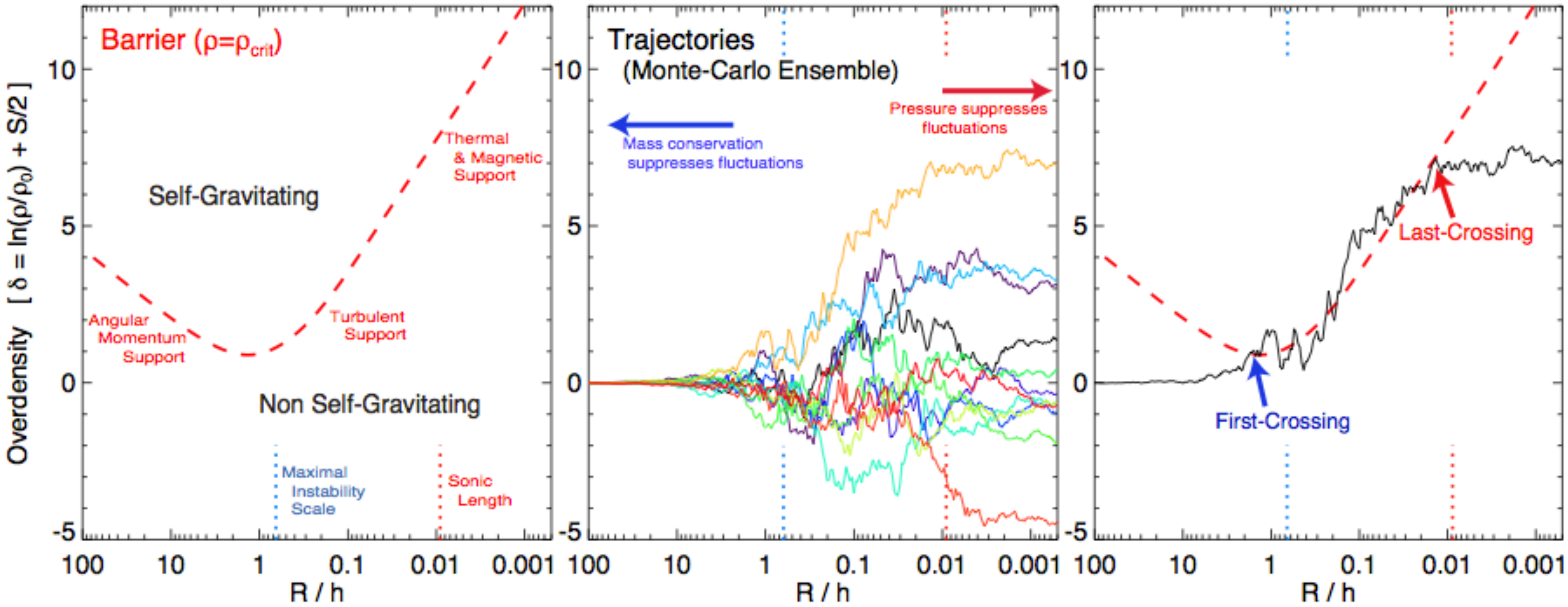


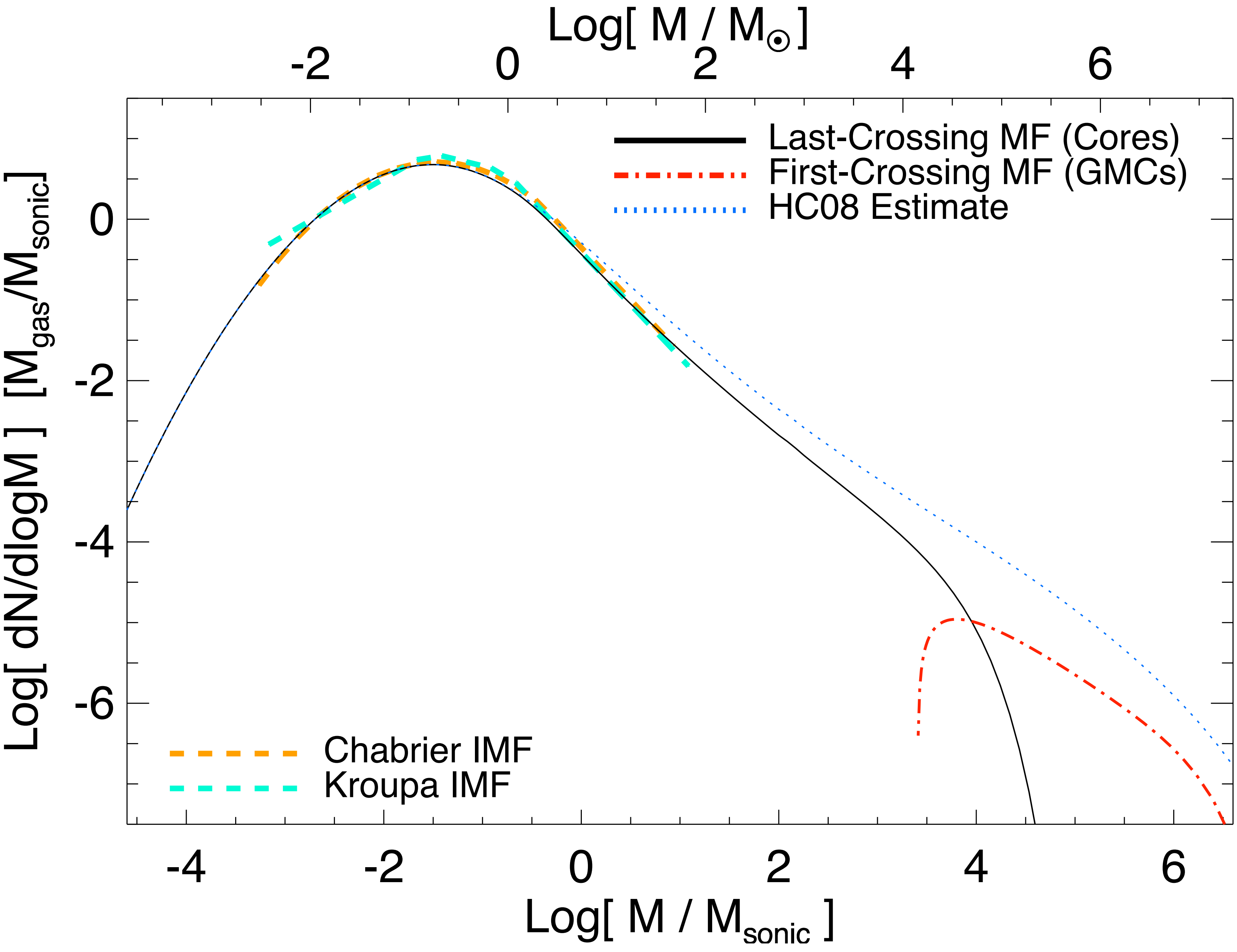
Behaviour at small masses: unchanged
(the thermal support is dominant)

At large masses, stiff power spectrum for low M_* and powerspectrum compatible with Salpeter for M_* around or greater than 1

The excursion set theory

Hopkins 2012ab, 2013ab



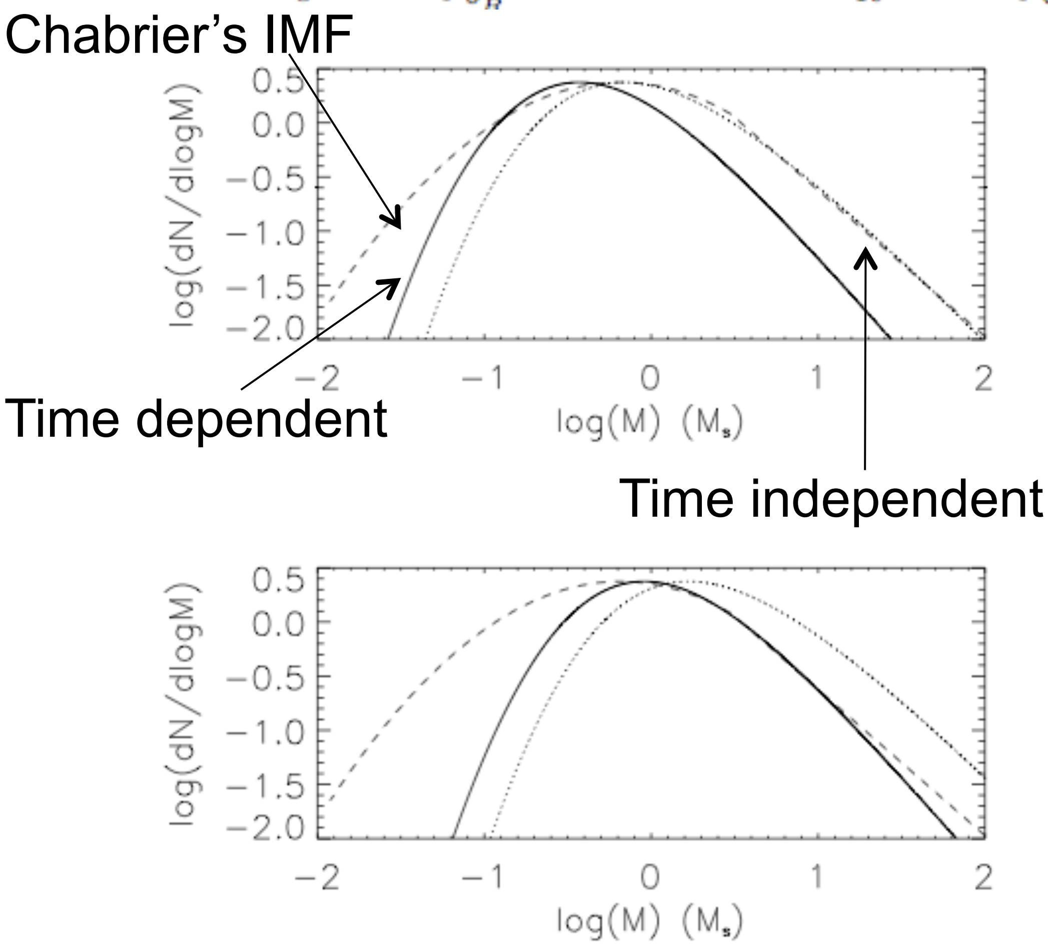


Log[M / M$_\odot$]
-2
0
2
4
6
Last-Crossing MF (Cores)
First-Crossing MF (GMCs)
HC08 Estimate
Chabrier IMF
Kroupa IMF
Log[dN/dlogM] [M$_{gas}$/M$_{sonic}$]
0
-2
-4
-6
-4
-2
0
2
4
6
Log[M / M$_{sonic}$]

Time dependence issue

Small scales rejuvenate faster than large scales
Mass spectrum should be weighted by the time needed to produce the next “generation” of fluctuations

$$\frac{M_{\rm tot}(R)}{V_c} = \int_{\delta_R^c}^{\infty} \bar{\rho}\exp(\delta)\mathcal{P}_R(\delta)(\frac{\tau_0}{\tau_R})d\delta = \int_0^{M_R^c} M'\,\mathcal{N}(M')\,P(R,M')\,dM'.$$



Most important effects:

-shift the peak toward smaller masses

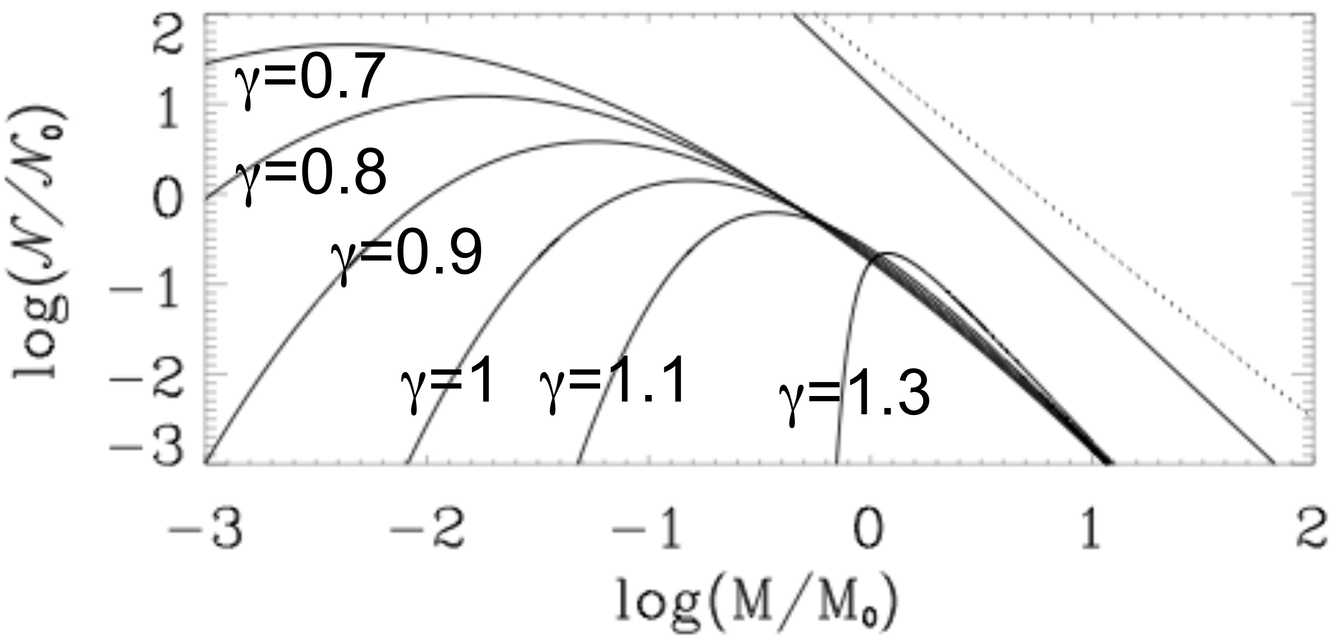
-Makes the slope at high masses a little stiffer

Note: for reasonable Mach numbers, CMF too narrow

Influence of the equation of state

Influence of γ on the CMF/IMF

$$C_s^2 \propto \rho^{\gamma-1}$$

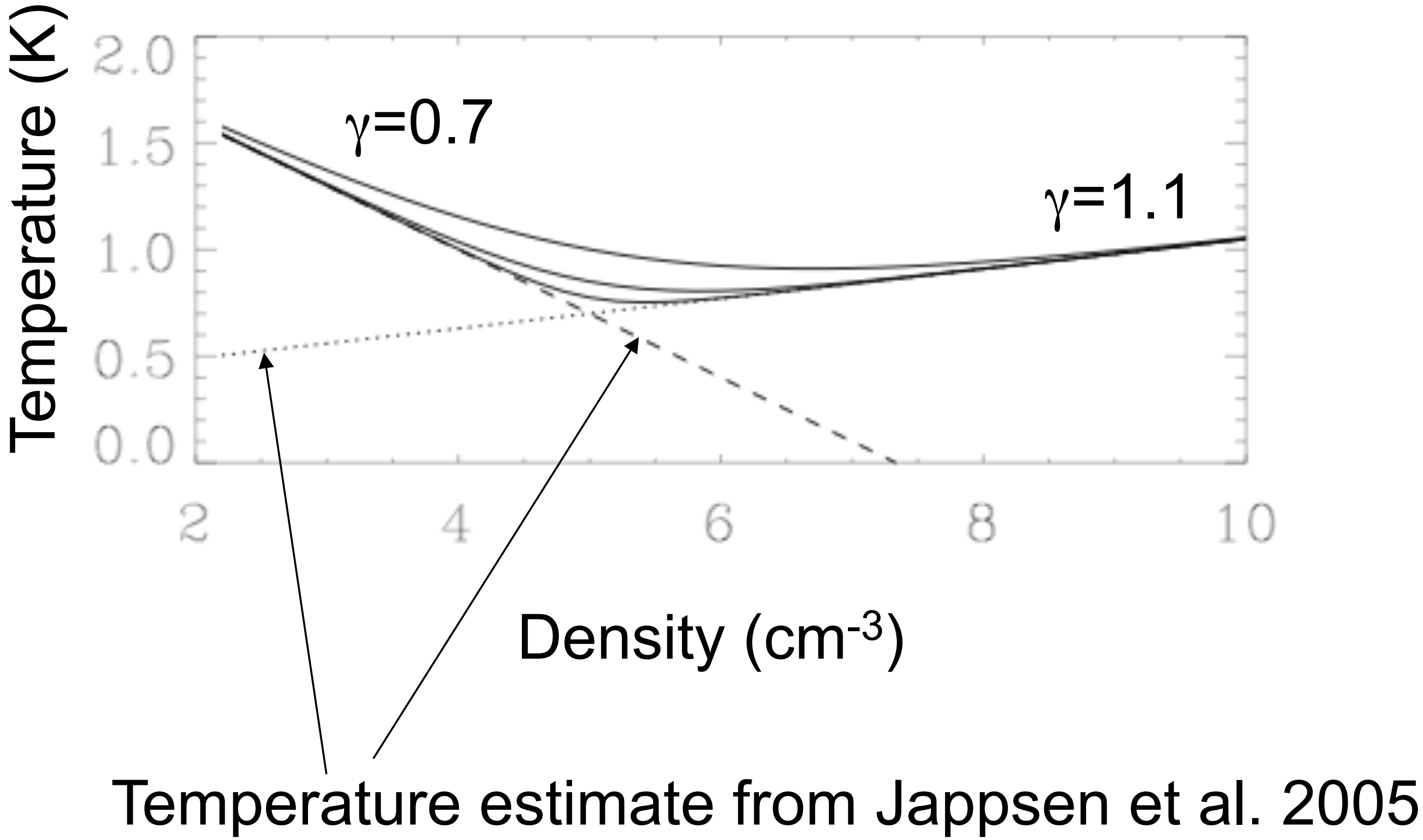


Strong influence of the equation of state on the low mass regime

H & Chabrier 2009

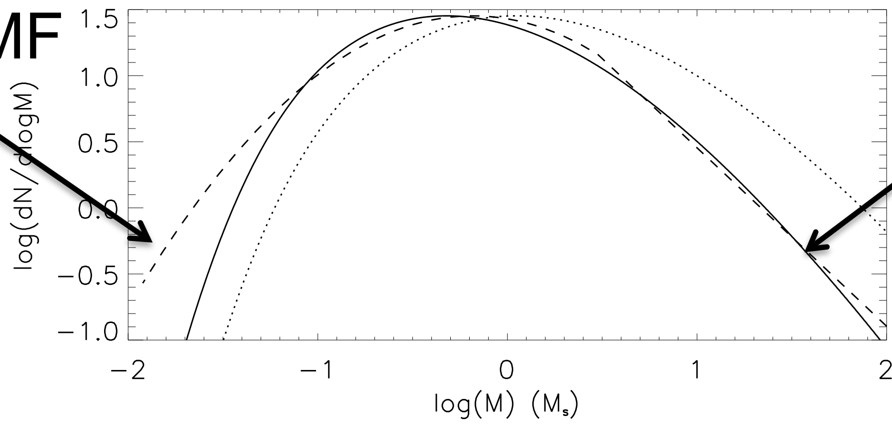
Beyond isothermality

Cold gas in molecular clouds is not isothermal



Comparison with Chabrier IMF

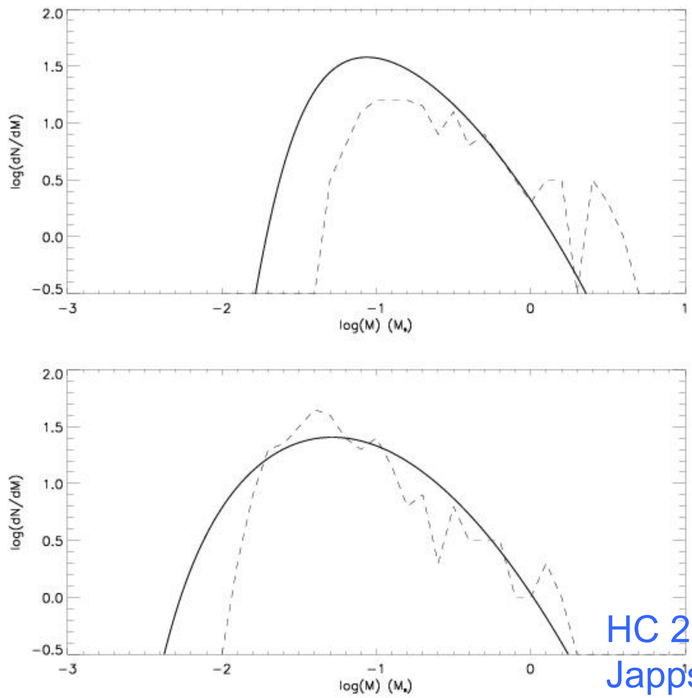
Chabrier's IMF



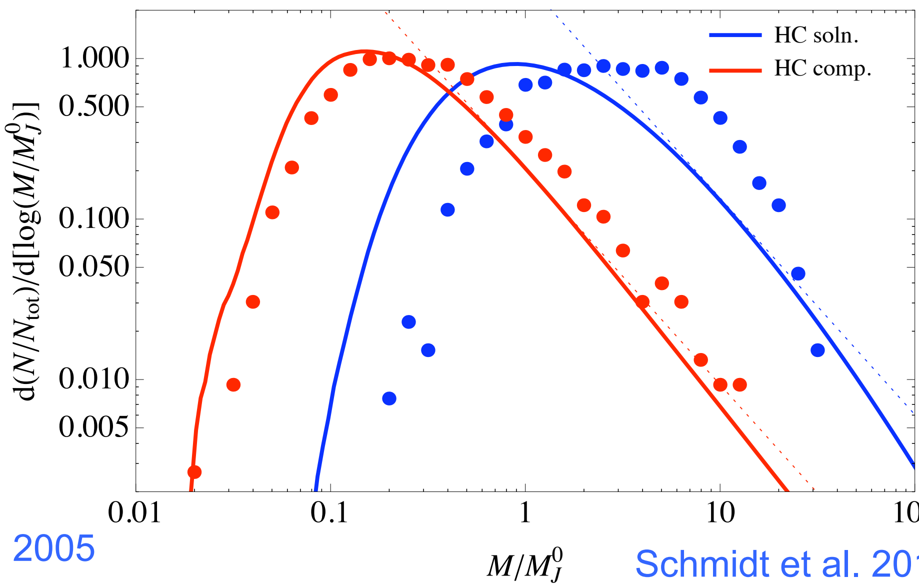
Analytical prediction

HC 2008, 2013

Comparison with high resolution numerical simulations



HC 2009
Jappsen et al. 2005



Schmidt et al. 2010

1-Theories based on accretion

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Reasonable success of CMF based theories to explain the high mass part of the IMF.

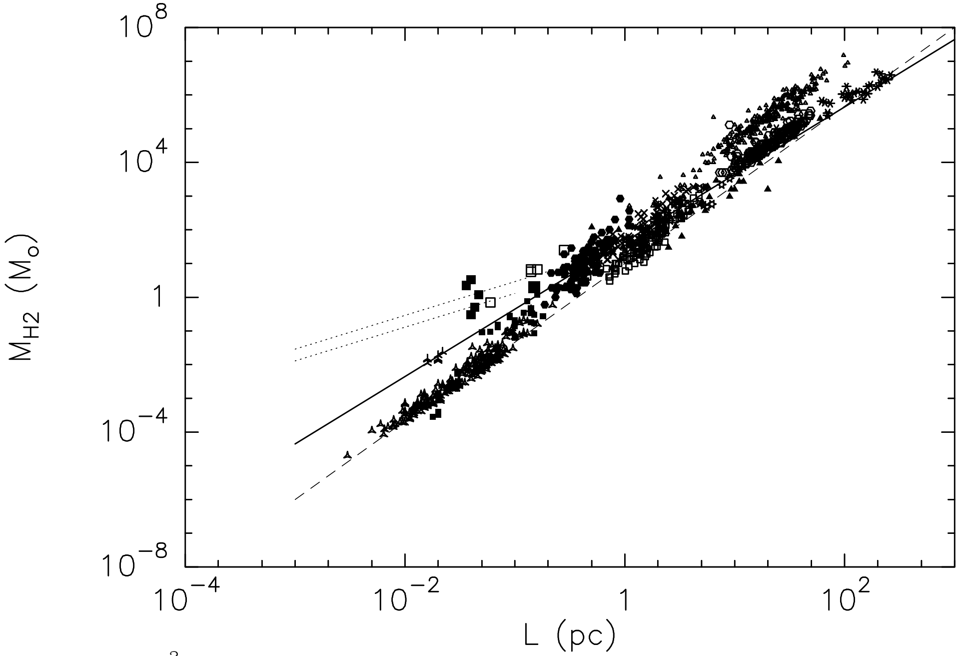
Indeed, the *shape* does not depend too strongly on physical parameters (within a “reasonable range”).

However, the peak does.
A good approximation (isothermal case) is given by:

$$M_{peak} \approx \frac{M_{Jeans}}{1 + b^2 \mathcal{M}^2}$$

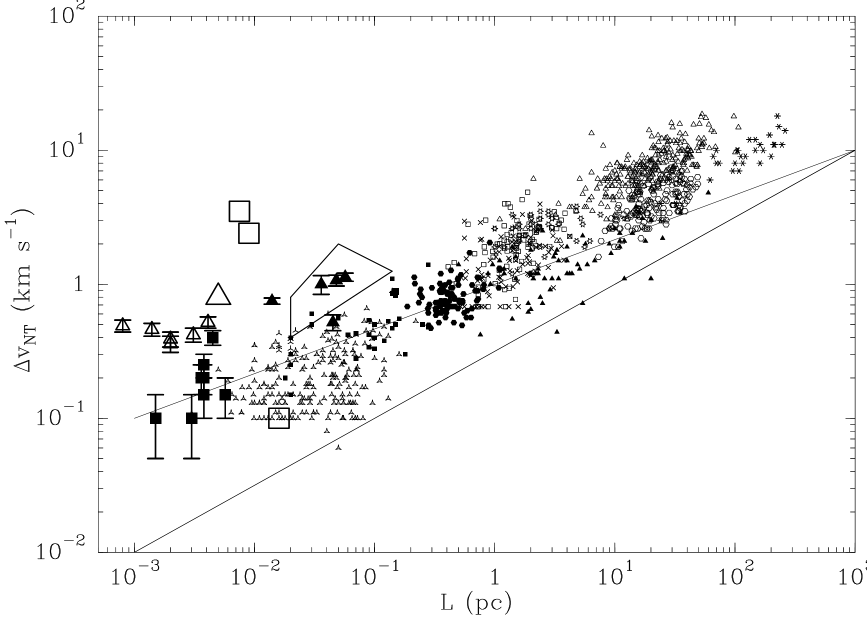
Source of non-universality, at least :
-density variations
-Mach number variations
-temperature variations
(-magnetic field variation)

Variability of the density and Mach number



$$\rho = d_0 \times 1000\, cm^{-3} \left(\frac{L}{1pc}\right)^{-0.7}$$

$$V = V_0 \times 0.8\, km\, s^{-1} \left(\frac{L}{1pc}\right)^{0.4-0.5}$$



Larson 1981
Falgarone et al. 2009
H & Falgarone 2012

Let us combine Larsons relations with the peak dependence:

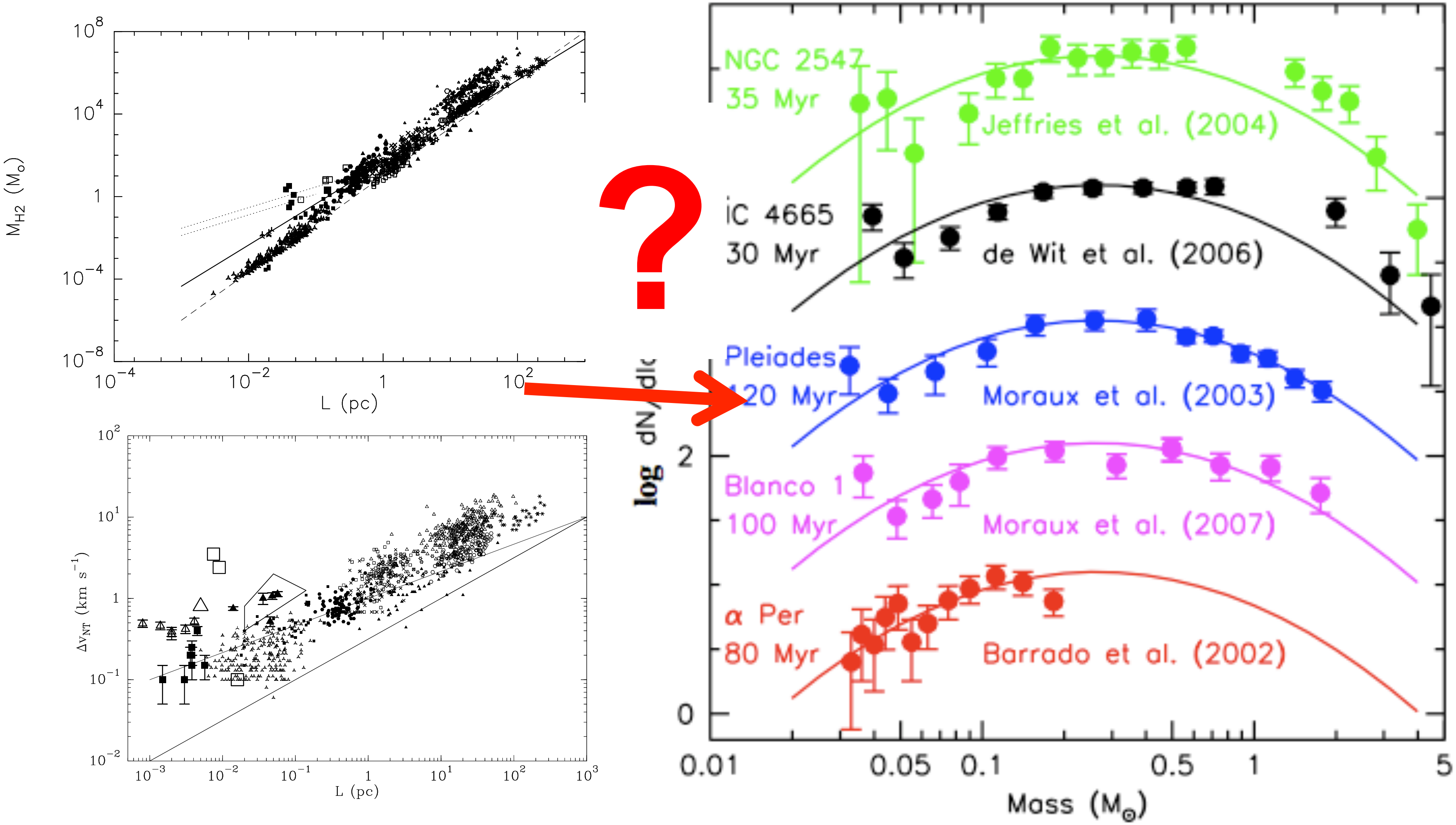
$$\rho = d_0 \times 1000\, cm^{-3} \left(\frac{L}{1pc} \right)^{-0.7}$$

$$V = V_0 \times 0.8\, km\, s^{-1} \left(\frac{L}{1pc} \right)^{0.4-0.5}$$

$$M_{peak} \approx \frac{M_{Jeans}}{1 + b^2 \mathcal{M}^2}$$

$$M_{peak} \propto \frac{C_s^5}{V^2 \sqrt{\rho}} \propto \frac{C_s^5}{V_0^{\;2} \sqrt{d_0} L^{\approx 0.5}} \propto \frac{C_s^5}{V_0^{\;2} \sqrt{d_0}} \frac{1}{M^{\approx 0.2}}$$

Weak dependence on the mass but stiff dependence on C_s, V_0 and d_0. All are varying by a fair amount.



Jeffries 2011
Chabrier 2003

1-Theories based on accretion

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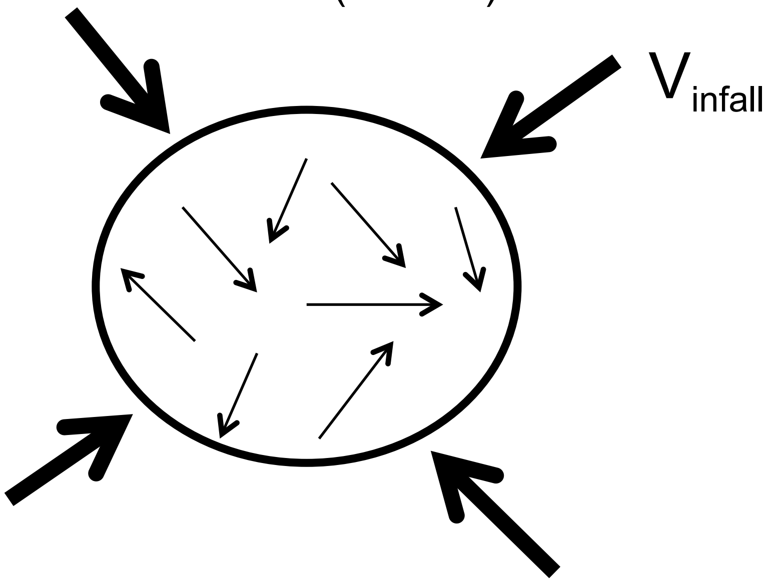
Is there a self-regulated process that set (or help setting) the IMF ?

-Radiative Feedback
(Bate 2009, Offner et al. 2009, Krumholz 2011)

-Initial conditions
Stars form only in special places… Most stars form in clusters (Lada & Lada 2003).

Self-regulated initial conditions ? A model for protocluster

(H 2012)



Assume:

1) Virial equilibrium between turbulence and gravity

$$\dot{M}^2 \frac{dR_*^2}{dM_*} + M_* \sigma_*^2 - 3 P_{\rm ram} V_* - \frac{3}{5} \frac{G M_*^2}{R_*} = 0.$$

2) Equilibrium between turbulent energy dissipation and accretion driven turbulence

(Klessen & Hennebelle 2010, Goldbaum et al. 2011)

$$\frac{M_* \sigma_*^2}{2 \tau_{\rm cct}} \simeq \dot{E}_{\rm ext} + \dot{E}_{\rm int},$$

Which accretion ?

3) Use Larsons relations to construct an accretion rate.

$$
\begin{aligned}
\dot{M} &\simeq \frac{M_c}{\tau_c} \simeq \frac{M_c}{2R_c/(\sigma_{rms}/\sqrt{3})}, \\
&\simeq M_c^{\frac{2-\eta_d+\eta}{3-\eta_d}} \frac{\sigma_0}{2\sqrt{3}} \left(\frac{4\pi}{3} n_0 m_p\right)^{\frac{1-\eta}{3-\eta_d}} (1\,\mathrm{pc})^{\frac{\eta_d-3\eta}{3-\eta_d}}, \\
&= \dot{m}_4 \left(\frac{M_c}{10^4 M_\odot}\right)^{\eta_{acc}}, \\
&\simeq 9\times10^{-4} M_\odot\,\mathrm{yr}^{-1} \left(\frac{M_c}{10^4 M_\odot}\right)^{\frac{2-\eta_d+\eta}{3-\eta_d}} \\
&\quad \times \frac{\sigma_0}{0.8\,\mathrm{km\,s^{-1}}} \left(\frac{n_0}{1000\,\mathrm{cm^{-3}}}\right)^{\frac{1-\eta}{3-\eta_d}},
\end{aligned}
$$

We get

$$\dot{M} = \dot{m}_4 \left(\frac{\alpha_{*,c}}{10^4 M_\odot}\right)^{\eta_{acc}} M_*^{\eta_{acc}}$$

η_{acc}~0.75

Assumptions 1), 2) and 3) allows to estimate the mass-size relation of the proto-cluster:

$$
\begin{aligned}
M_* &= M_*^0 \left(\frac{R_*}{1\mathrm{pc}}\right)^2, \\
n_* &\simeq 4600\,\mathrm{cm}^{-3} \frac{M_*^0}{10^3 M_\odot} \left(\frac{R_*}{1\mathrm{pc}}\right)^{-1}, \\
\Sigma_{*,c} &= 2.8\times10^{22}\mathrm{cm}^{-2} \frac{M_*^0}{10^3 M_\odot},
\end{aligned}
$$

Mechanical equilibrium

$$\sigma \propto \sqrt{\frac{M}{R}}$$

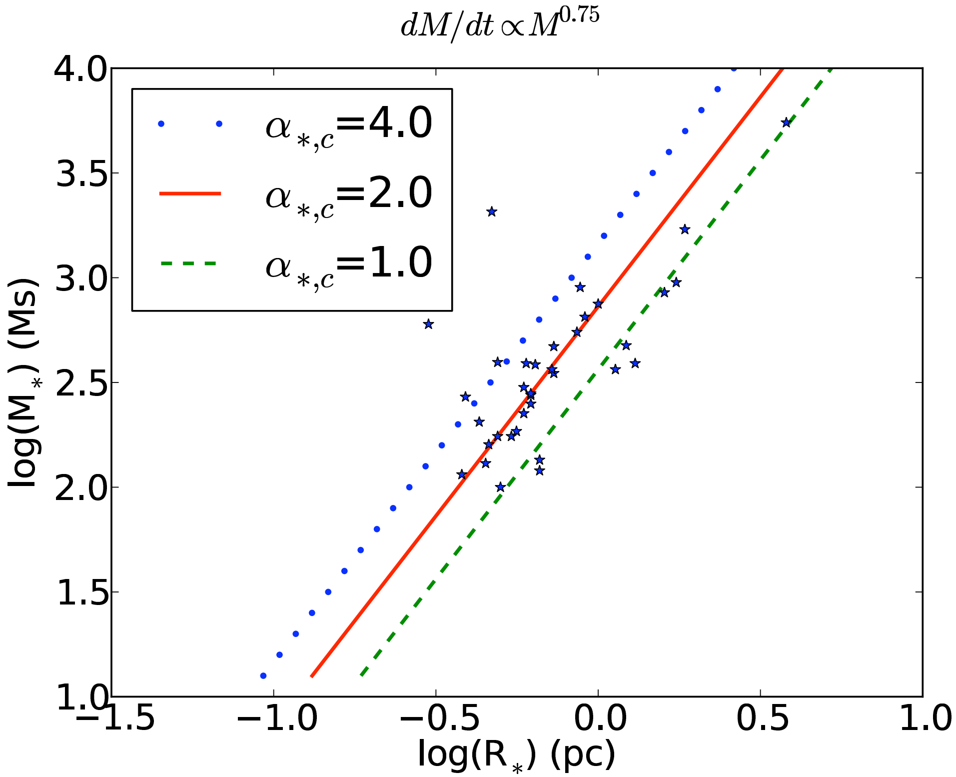
Energy balance : dissipation=injection

$$\frac{M\sigma^3}{R} \propto \sigma^2 M^{0.75}$$

$$\Rightarrow \frac{M}{R}\sqrt{\frac{M}{R}} \propto M^{0.75}$$

$$\Rightarrow M \propto R^2$$

Comparison between the model and data of embedded clusters from Lada & Lada (2003):



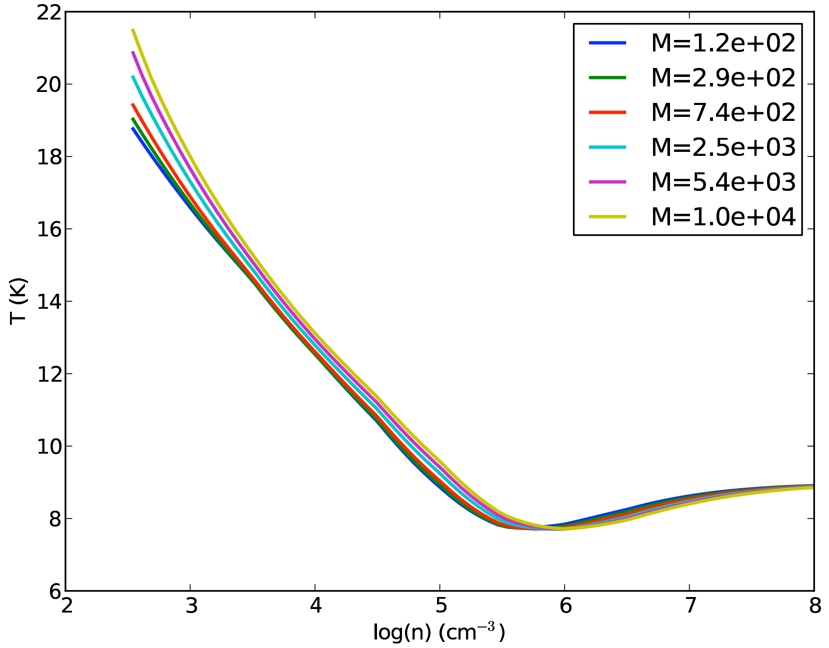
Global trend is reproduced. Apart from a few points, the dispersion is compatible with an accretion rate varying by about a factor 2.

Before, we can proceed with the IMF, we need the temperature.

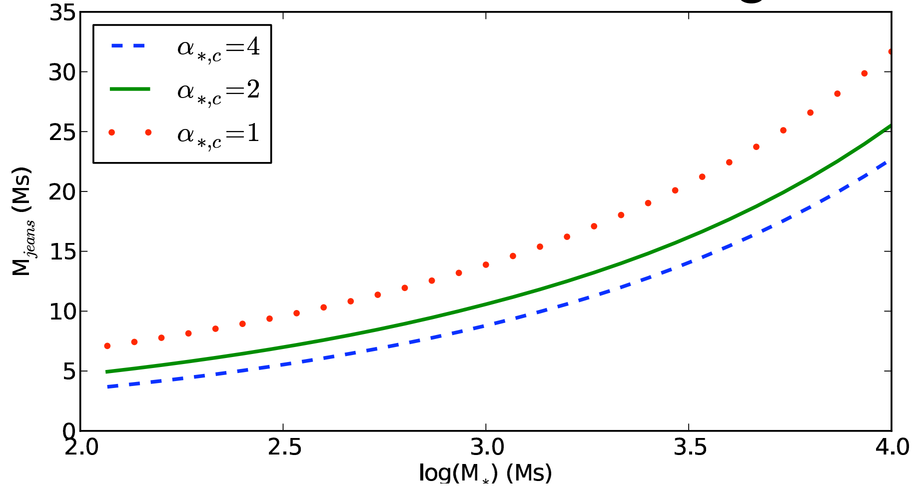
We compute thermal balance:

turbulent dissipation + cosmic ray heating = molecular + dust cooling

Molecular: Neumann et al. 95
Dust temperature: Zucconi et al. 2001

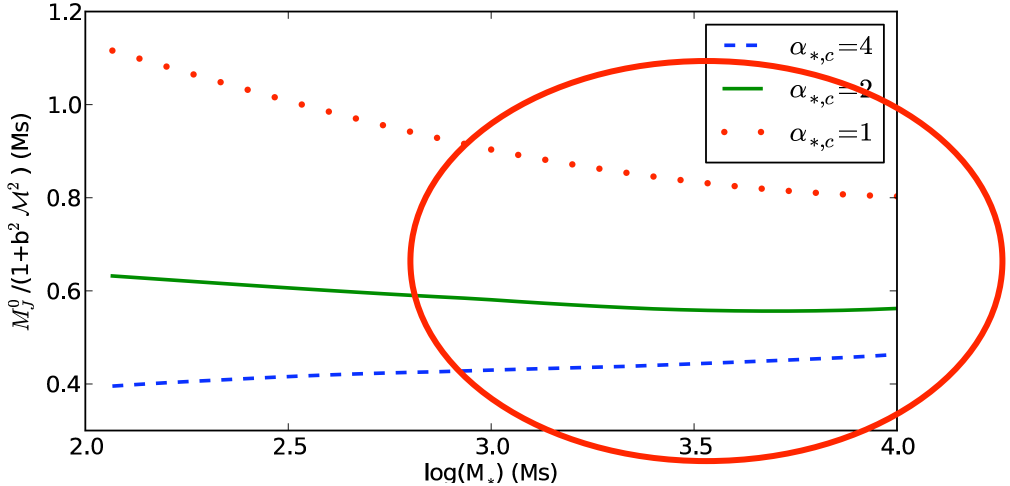


Jeans mass as a function of gas cluster mass



=>change by a factor of 5-6 when M varies over 2 orders of magnitude

$M_{jeans}/(1+b^2 Mach^2)$ as a function of gas cluster mass

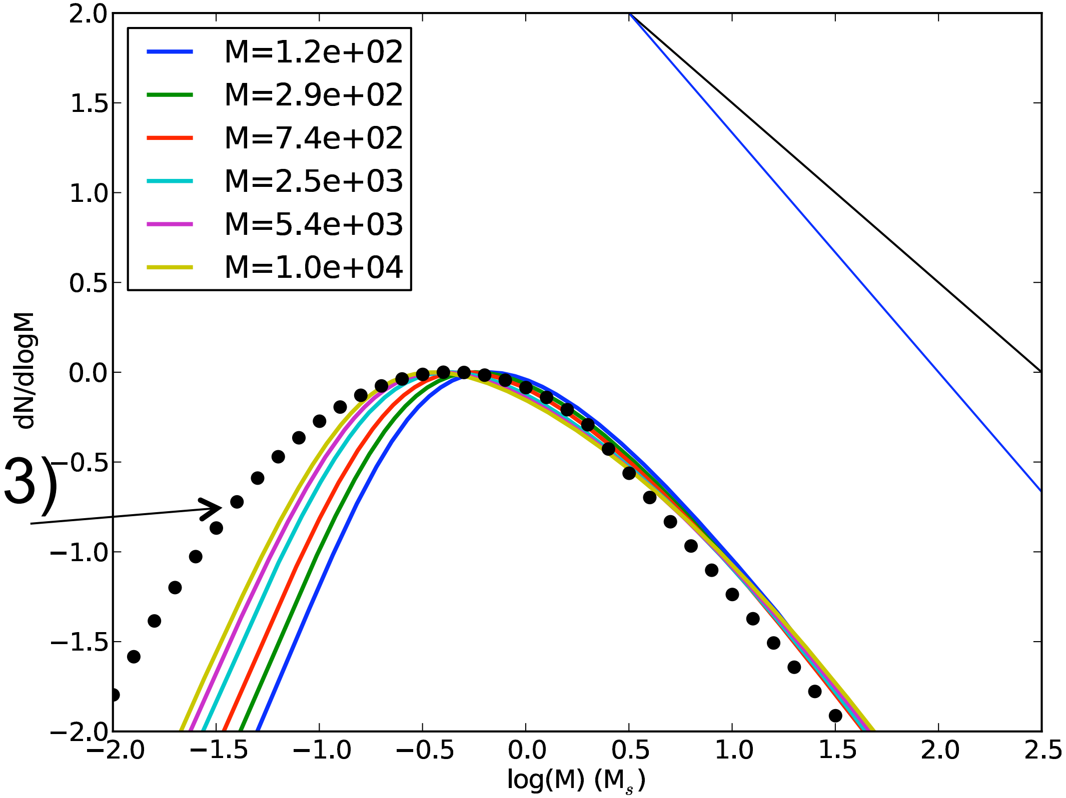


Can be understood analytically. Linked in particular to the cooling function.

=>change by a factor of 2-3 when M varies over 2 orders of magnitude and accretion by a factor of about 4

Apply HC (time-dependent model) to the proto-cluster model for various masses and “standard” accretion rate.

No free parameters



Shifted (factor 3)
Chabrier IMF

⇒The peak position is well reproduced

⇒Almost no dependence on the CMF when changing M ($>10^3 M_s$)

Conclusion

The high mass part of the IMF appears to be robust because it is due to the combination of two generic processes: gravity and turbulence

The low mass end (the peak) is much sensitive to initial conditions and thermal physics

Constancy of the IMF within clusters is surprising and difficult to explain. Self-regulation of feedback or/and initial conditions (possibly both…) are interesting possibilities.

Which mechanism is at play in gravo-turbulent simulations ?

Competitive accretion or core formation ?

Smith et al. have run SPH simulations with gravity and sink particles

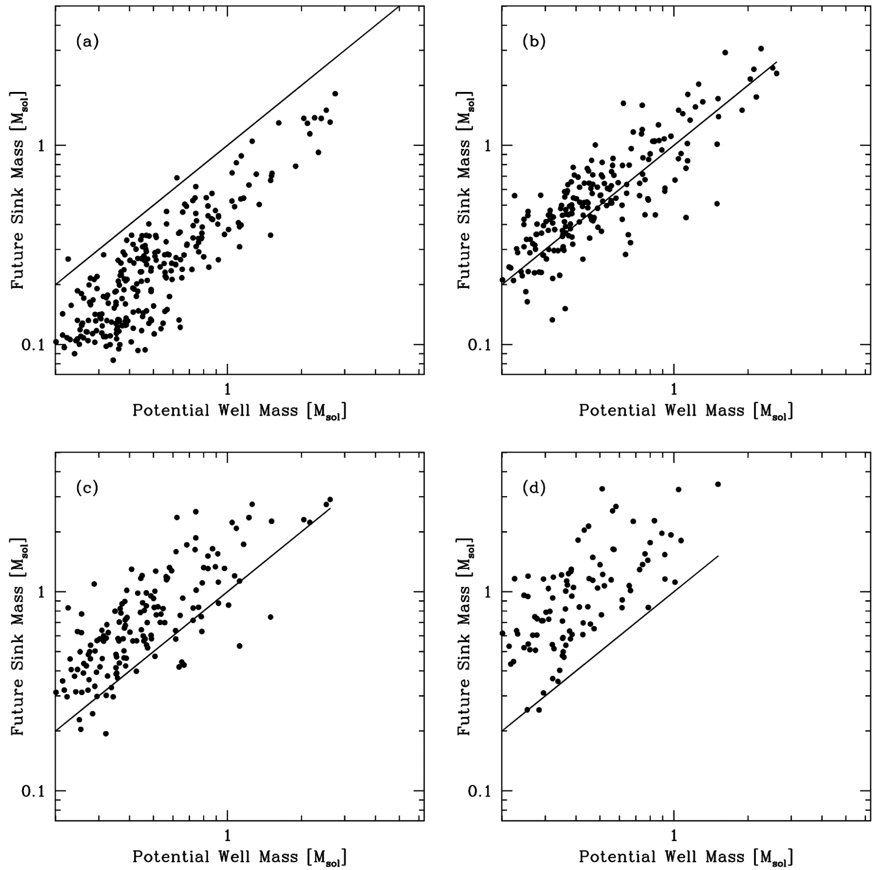
They identify cores and look at the correlation between the core masses and the sink masses.

The correlation is very good initially (few freefall times) and becomes progressively less good.

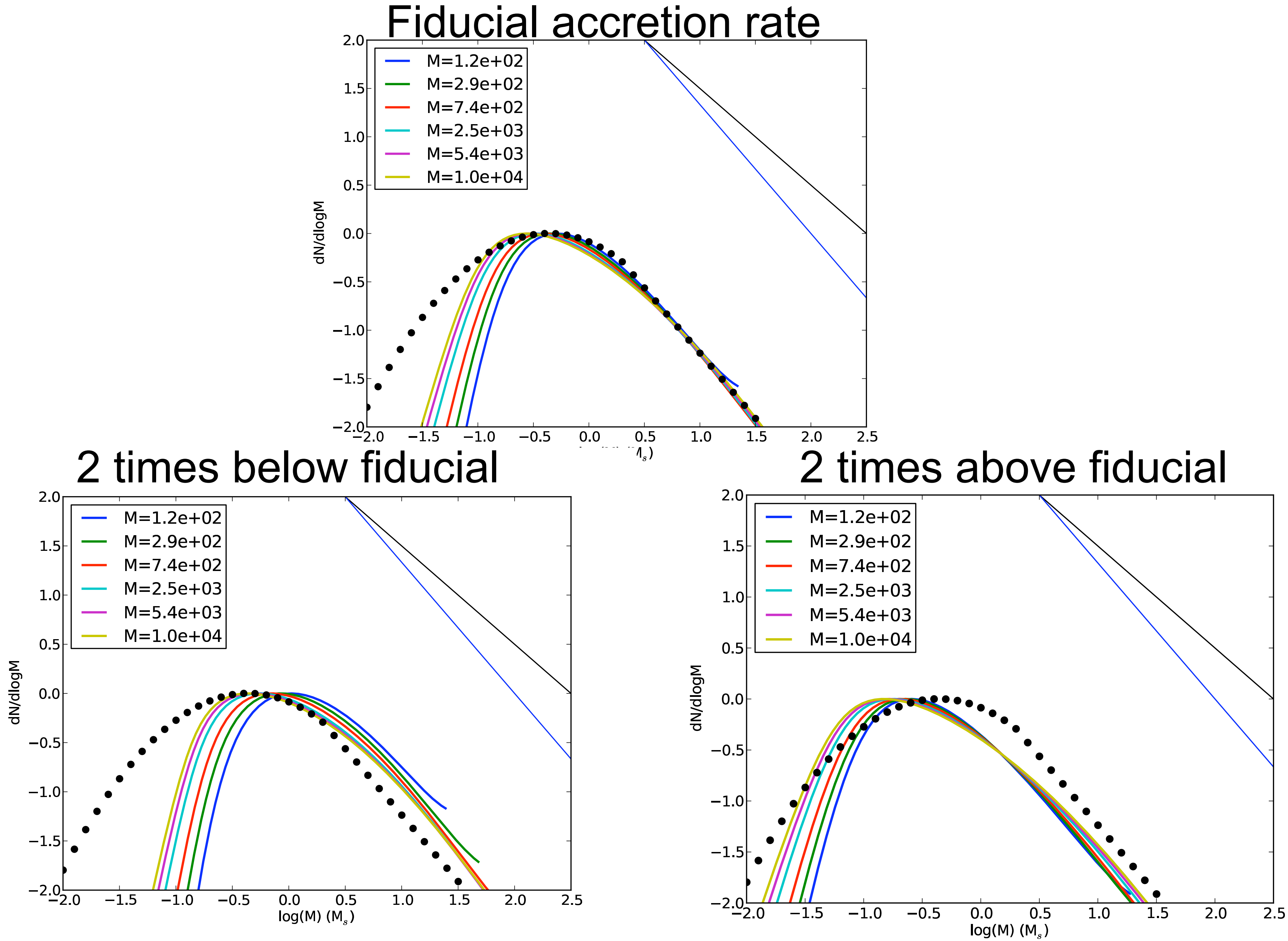
=>*This is compatible with the core mass function being able to produce a reasonable IMF.*

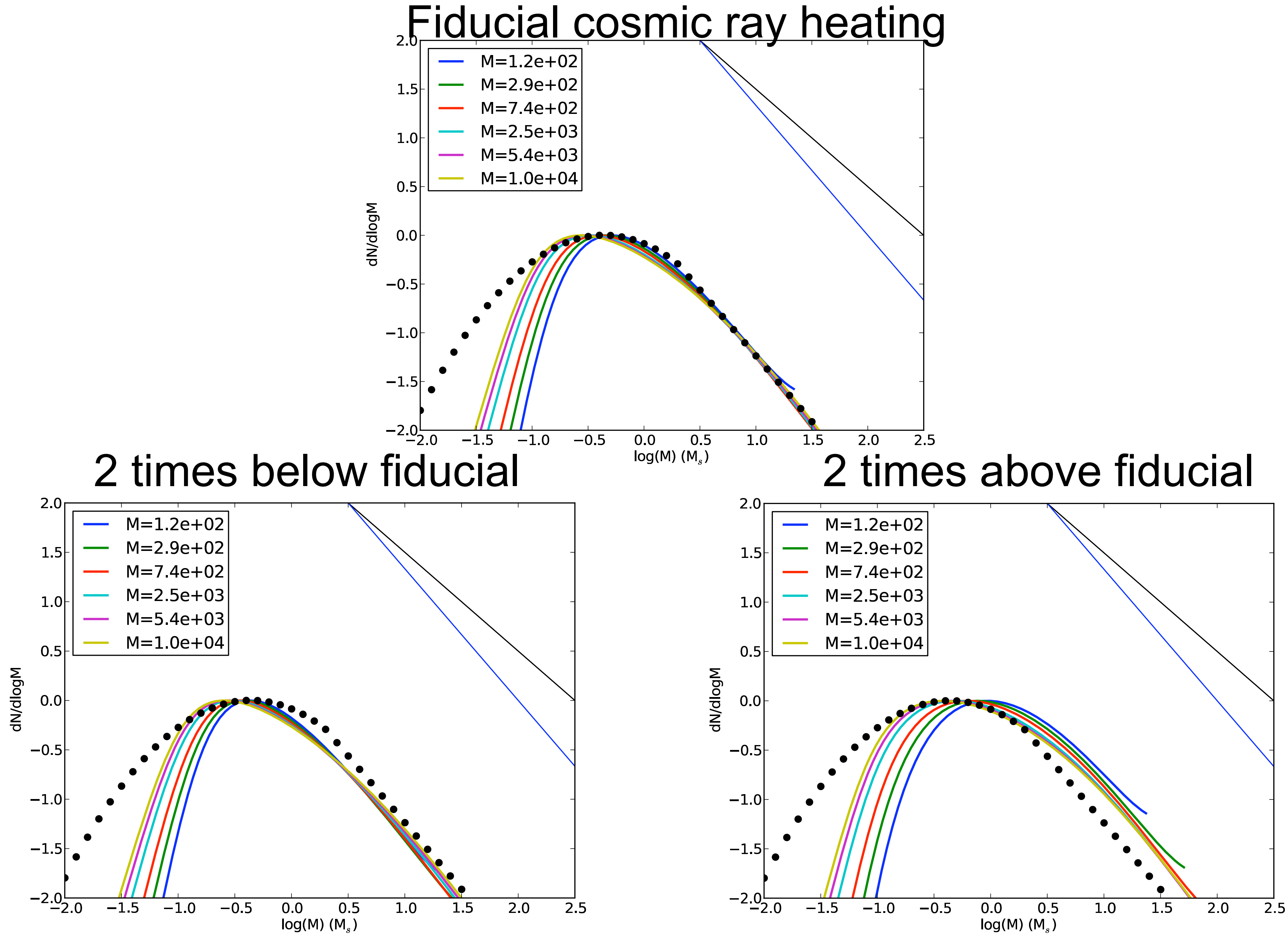
=>*The question is whether the observed IMF is nevertheless not the consequence of the initial CMF produced.*

Until how many freefall times are the cores accreting ?



Smith et al. 2008


Fiducial accretion rate
2 times below fiducial
2 times above fiducial
dN/dlogM
log(M) (M$_s$)
M=1.2e+02
M=2.9e+02
M=7.4e+02
M=2.5e+03
M=5.4e+03
M=1.0e+04



Fiducial cosmic ray heating
2 times below fiducial
2 times above fiducial
M=1.2e+02
M=2.9e+02
M=7.4e+02
M=2.5e+03
M=5.4e+03
M=1.0e+04
dN/dlogM
log(M) (M$_s$)

Larsons relations and eos

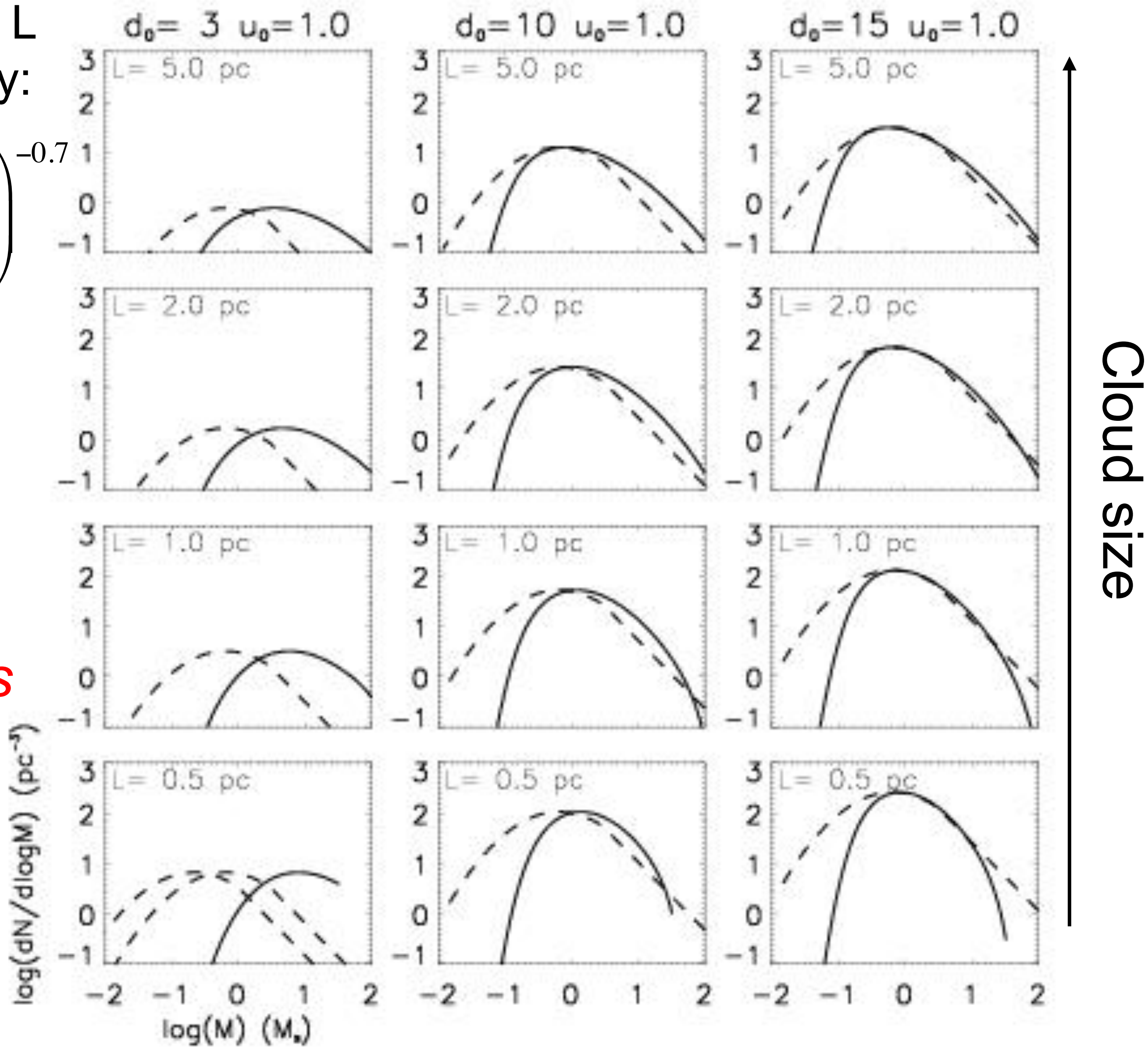
d_0=3 d_0=10 d_0=15

Consider clouds of size L with density and velocity:

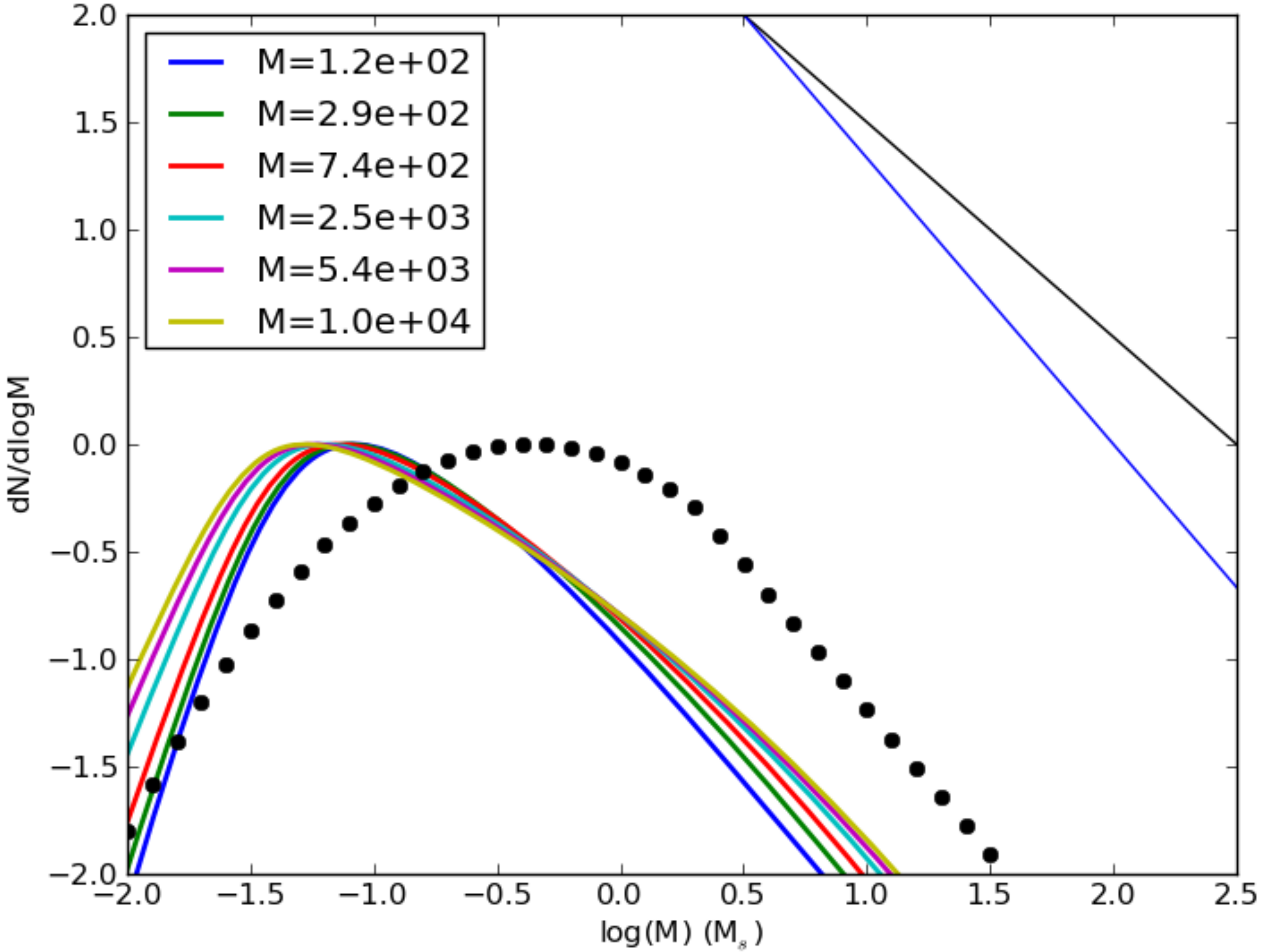
$$\rho = d_0 \times 1000\,cm^{-3}\left(\frac{L}{1pc}\right)^{-0.7}$$

$$V = 0.8\,km\,s^{-1}\left(\frac{L}{1pc}\right)^{0.5}$$

CMF also depends on cloud size

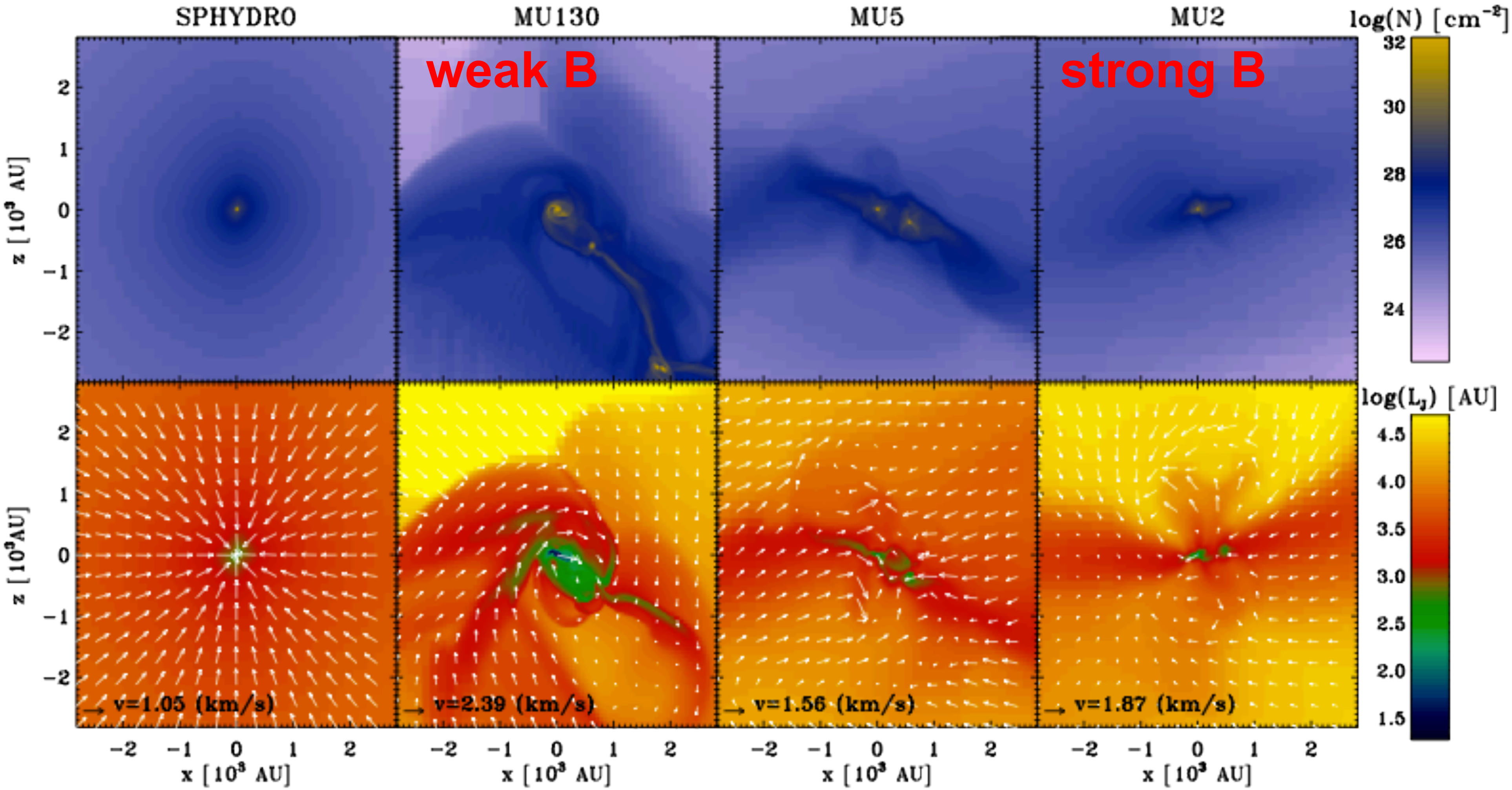


Ten times above fiducial



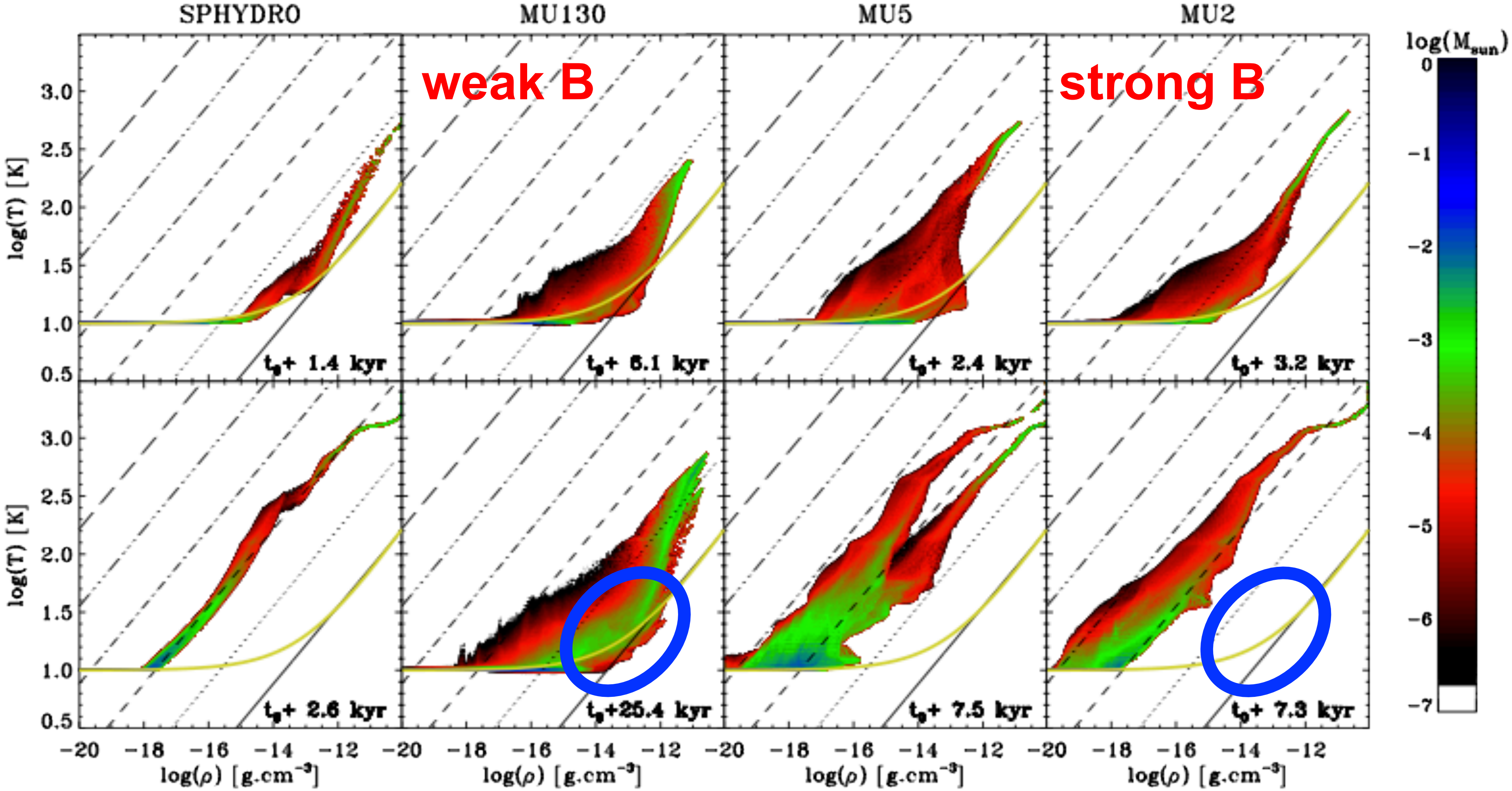
100 M☉ turbulent dense core collapse

Eturb/Egrav=20% initially



Commerçon, Hennebelle & Henning, ApJL 2011

100 M⊙ turbulent dense core collapse



Commerçon, Hennebelle & Henning, ApJL 2011