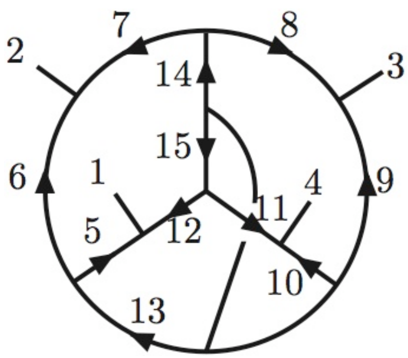
Non-Planar $\mathcal{N}=4$ SYM at Four Loops and Supersum Structures

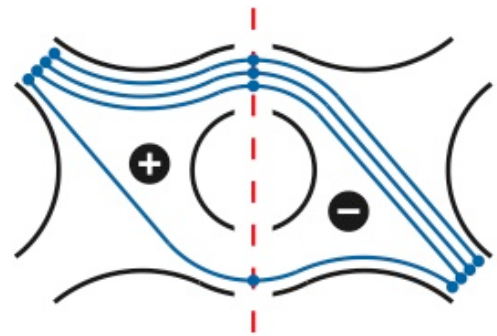


NBIA workshop

Aug 12, 2009

Henrik Johansson

UCLA & IPhT Saclay



0903.5348[hep-th]: Z.Bern, J.J.Carrasco, H.Ita, HJ, R.Roiban

to appear: Z.Bern, J.J.Carrasco, L.Dixon, HJ, R.Roiban

Outline

- Motivation & introduction
- Calculating multi-loop amplitudes in $\mathcal{N} = 4$ SYM
  - Unitarity & maximal cuts
  - Special cuts ↔ heuristic rules
  - Supersum structure in cuts
  - 4-loop non-planar $\mathcal{N}=4$
- 1,2,3,4-loop UV divergences
  - Full color structure of divergences
- Conclusions

Motivation - hidden structures

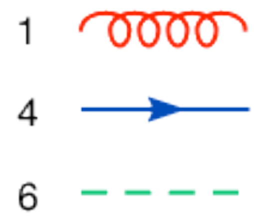
- Maximal SUSY theories are remarkably rich in hidden structures
  - $\mathcal{N} = 4$ SYM – AdS/CFT, dual conformal symmetry (Yangian), integrability, BDS resummation, twistors
  - $\mathcal{N} = 8$ SUGRA – UV finite, $E_{7(7)}$, simplest theory ?
- $\mathcal{N} = 4$ SYM input to $\mathcal{N} = 8$ SUGRA ampl. through KLT & unitarity

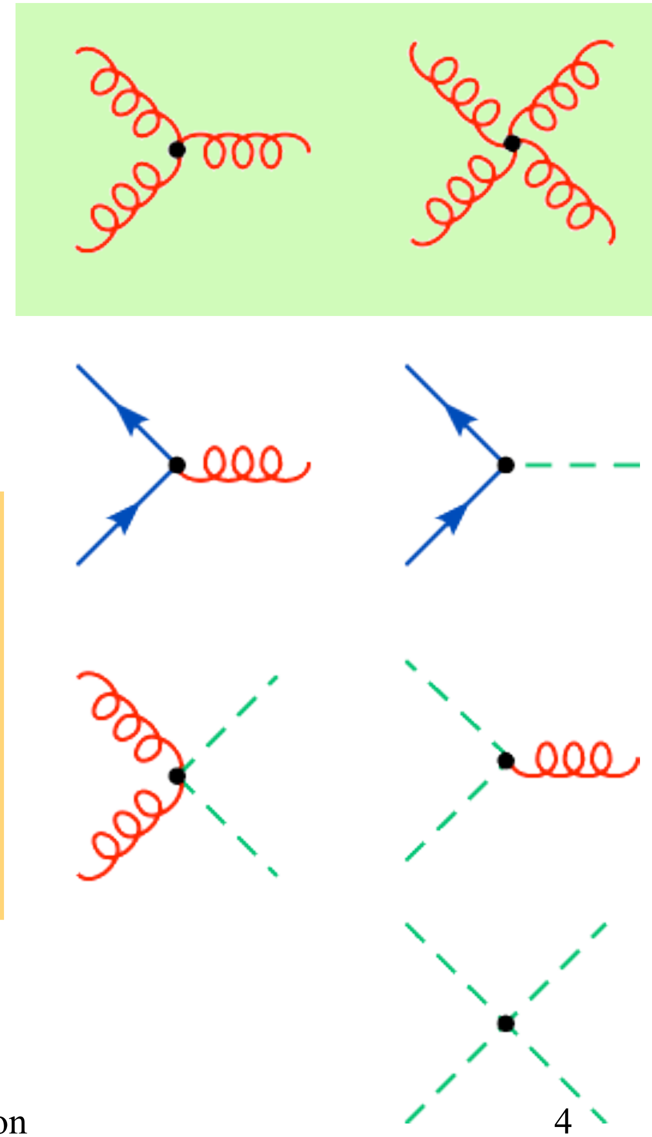
  $\Rightarrow$ talks by Carrasco, Roiban
- Goal: study the less-well-understood non-planar sector of $\mathcal{N} = 4$ SYM

$\mathcal{N} = 4$ super-Yang-Mills

$$\mathcal{L}_{\text{YM}} = -\frac{1}{4g^2} F^a_{\mu\nu} F^{a\,\mu\nu}$$

Maximal SUSY extension of YM





On-shell spectrum:

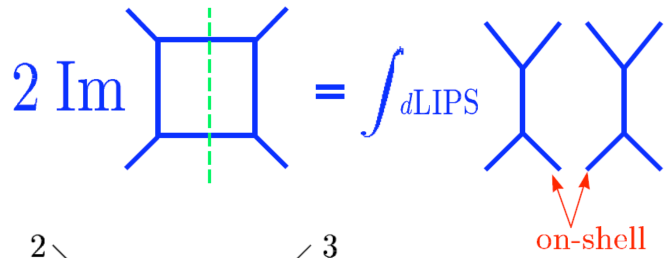
	g^-	f^-	s	f^+	g^+
helicity	-1	$-1/2$	0	$1/2$	1
#	1	4	6	4	1

Particles in adjoint group G, usually $\text{SU}(N_c)$

Unitarity Method

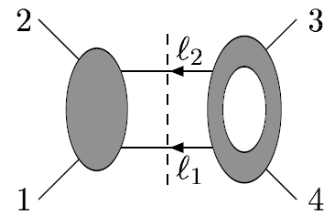
TIME

optical theorem



unitarity method

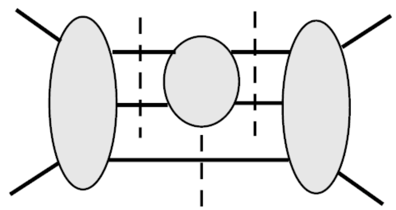
Bern, Dixon, Dunbar and Kosower (1994)



⇒talks by Dunbar, Kosower

generalized unitarity

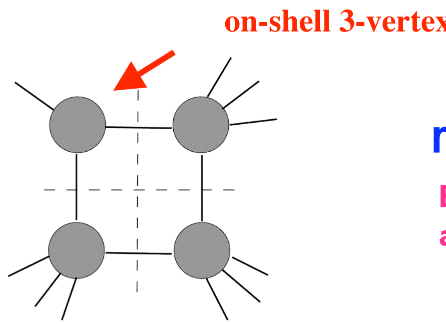
Bern, Dixon and Kosower



quadruple cut
(leading singularity)

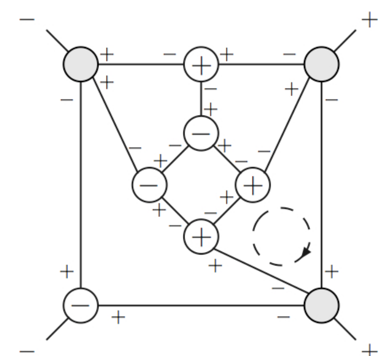
Britto, Cachazo, Feng;
Buchbinder, Cachazo (2004)

Cachazo and Skinner
Cachazo, Spradlin, Volovich
(2008)



maximal cut

Bern, Carrasco, HJ
and Kosower (2007)



Calculation Strategy

Ansatz:

$$A_4^{4-\text{loop}} = g^{10} st A_4^{\text{tree}} \sum_{S_4} \sum_{i=1}^{\#\text{topologies}} c_i \mathcal{I}_i$$

leg perms → S_4; diagrams → $\mathcal{I}_i$; symmetry factor → c_i

Separate color from kinematics:

$$\mathcal{I}_i = C_i I_i \qquad C_i = f^{abc} f^{cde} \cdots f^{xyz}$$

color factor → C_i

Integrals:

$$I_i = \int d^D l_1 d^D l_2 d^D l_3 d^D l_4 \frac{N_i(l_j, k_j)}{l_1^2 l_2^2 l_3^2 l_4^2 l_5^2 l_6^2 l_7^2 l_8^2 l_9^2 l_{10}^2 l_{11}^2 l_{12}^2 l_{13}^2}$$

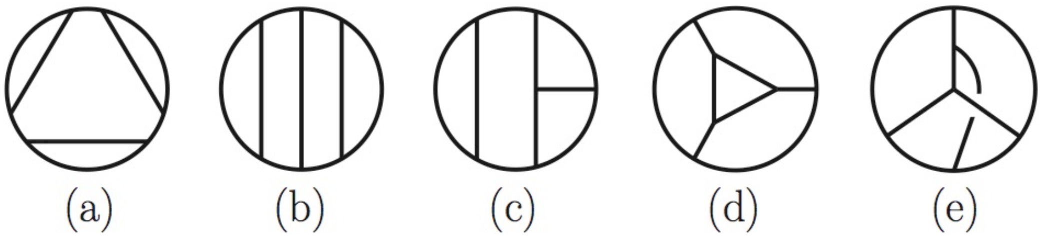
numerator → $N_i(l_j, k_j)$

Find all integral topologies & numerators !

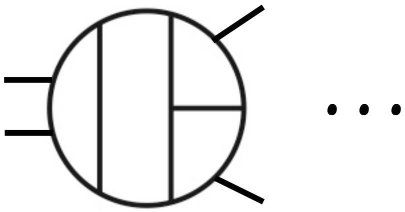
Integral topologies

- We choose to work with *only* trivalent (cubic) diagram topologies
- Contact terms are absorbed into the numerator N

Start with 4-loop trivalent 1PI vacuum graphs

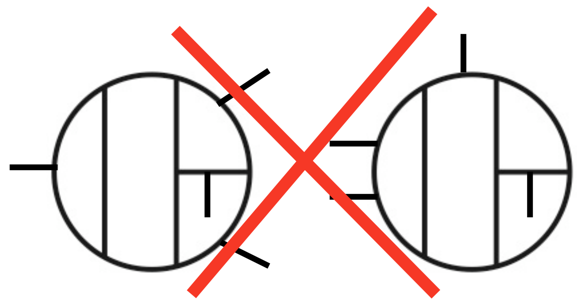


Attach 4 external legs



Remove diagrams with 2, 3-point sub-graphs

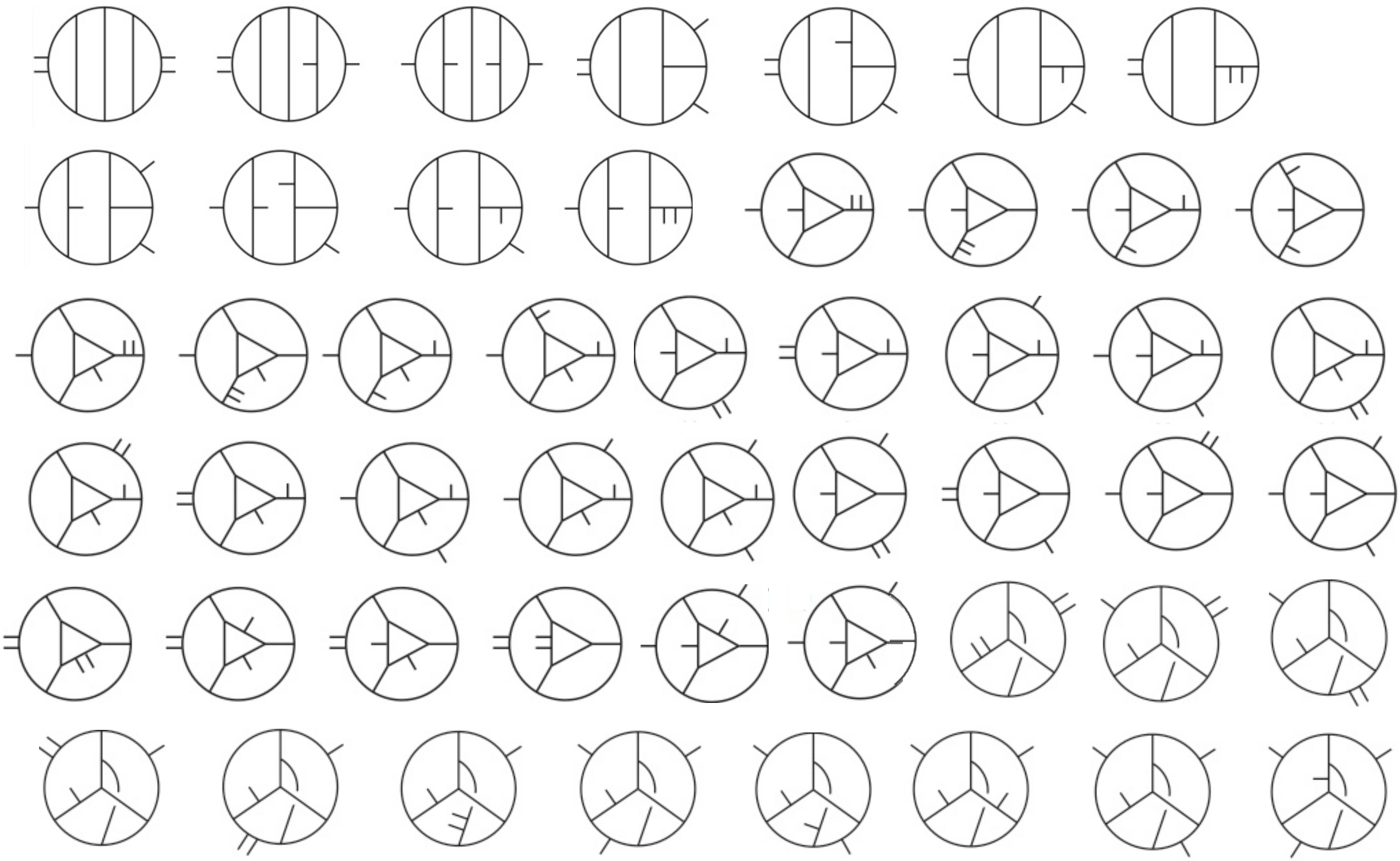
consistent with No-Triangle property (checked by cuts)



....gives 50 diagram topologies or integrals

⇒ talk by Carrasco

50 Integral topologies



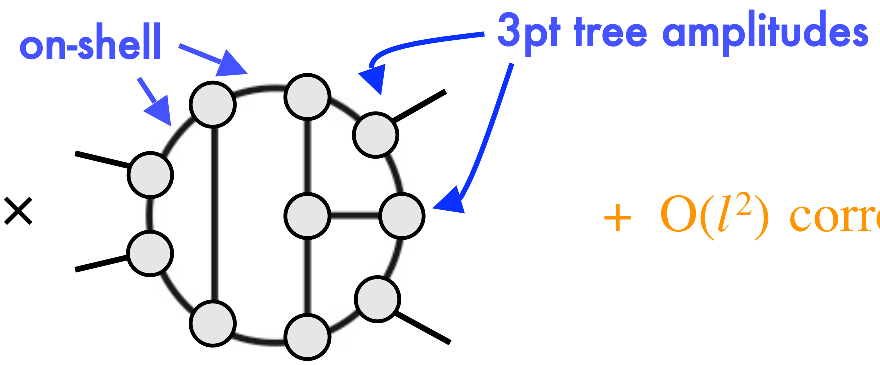
Bern, Carrasco, Dixon, HJ, Roiban [to appear]

Fix Numerators with Maximal Cuts

Bern, Carrasco, HJ and Kosower (2007)

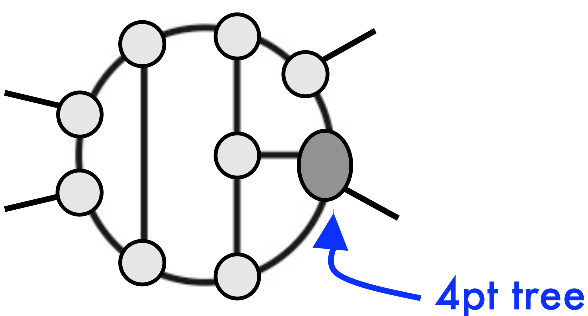
- put maximum number of propagator on-shell → simplifies calculation

$$N_i = \frac{1}{s t A_4^{\rm tree}} \times$$

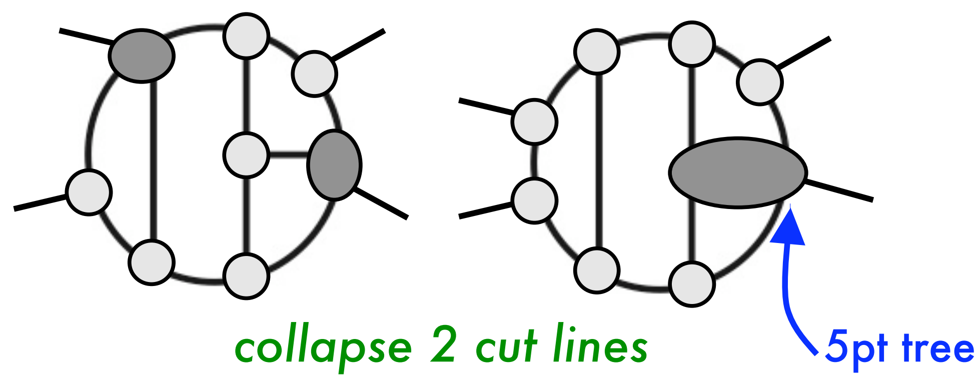


$+ \ O(l^2)$ corrections

- systematically release cut conditions → great control of missing terms



collapse 1 cut line



collapse 2 cut lines

Reconstructs the amplitude piece by piece !

$\mathcal{N}$=4 bag of tricks!

- Rules & assumptions for N_i can used at intermediate steps
- Correctness of amplitude established at the end (complete set of cuts)

Power counting & singlet maximal cuts

Bern, Dixon, Dunbar, Rozowsky, Perelstein; Howe, Stelle

- Power counting $D_c = 4 + \frac{6}{L}$ can constrain numerators
- Worst case 4 loops: 2 inverse propagators $N \sim s l_1^2 l_2^2$
- Such terms fixed by maximal cuts with two collapsed cut lines
- Remarkably all needed maximal cuts have corners in phase space where only gluon states propagate in loops: susy invariant "singlet cuts"

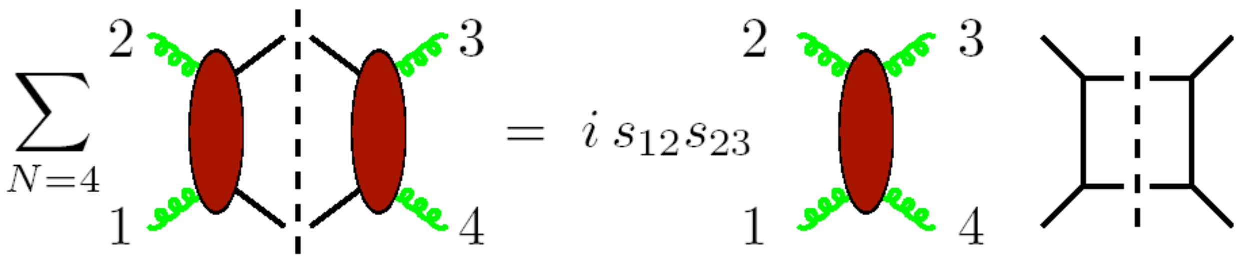
Heuristic rules for numerators ↔ special cuts (that iterate)

- rung rule ↔ two-particle cut
- box substitution rule ↔ box cut
- diagram twist rule ↔ (BCJ) Jacobi-like numerator & amplitude relations

valid in D dimensions

Two-particle cut ↔ Rung Rule

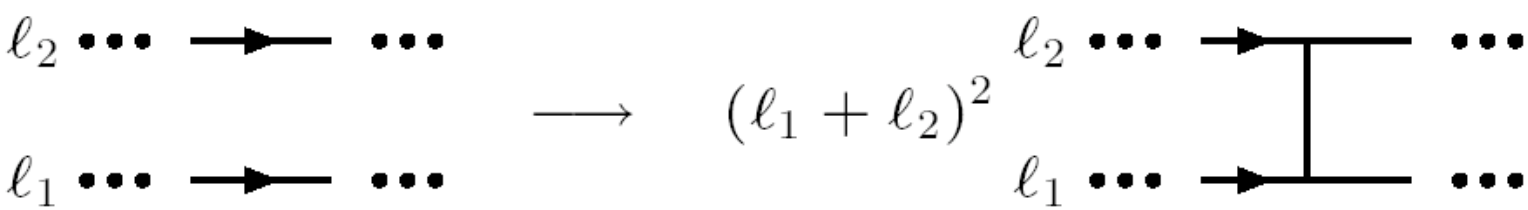
- A simple property of the 2-particle cuts at one loop



Lead to easy rules for computing iterated 2-particle cuts in multiloop ampls

- Inspired "rung rule" for easily finding numerator factors

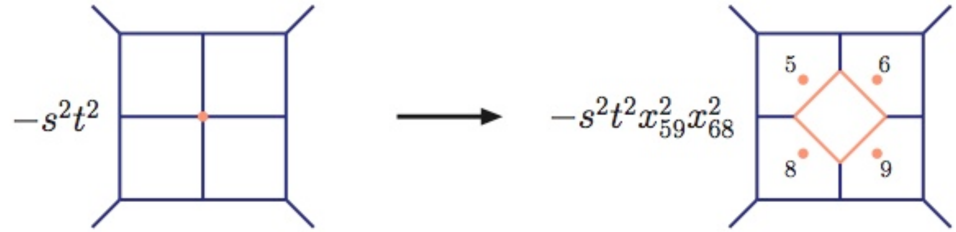
Bern, Rozowsky, Yan (1997)



- rung rule is less useful for non-planar diagrams
- better use the 2-particle cut directly

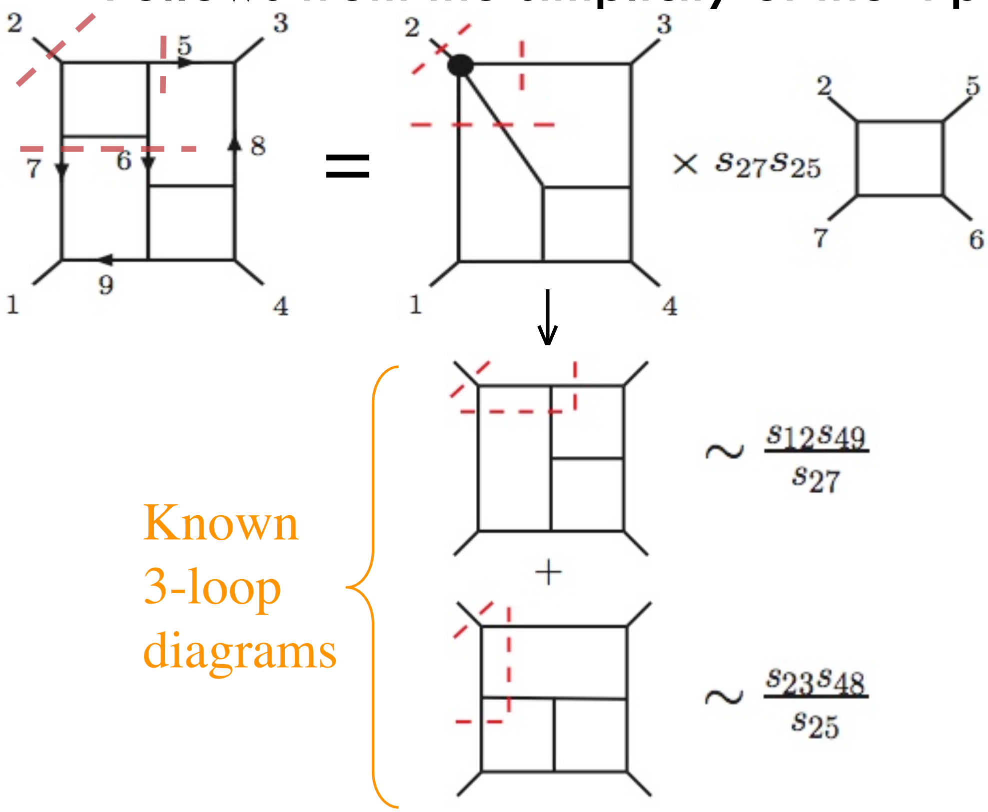
Box substitution rule ⟺ box cut

Observation in 0705.1864 [hep-th] inspired a "box substitution rule"



Bern, Carrasco, HJ and Kosower

- N_i can be obtained easily for diagrams containing a box subdiagram
- Follows from the simplicity of the 4-point $\mathcal{N}$=4 SYM amplitude.



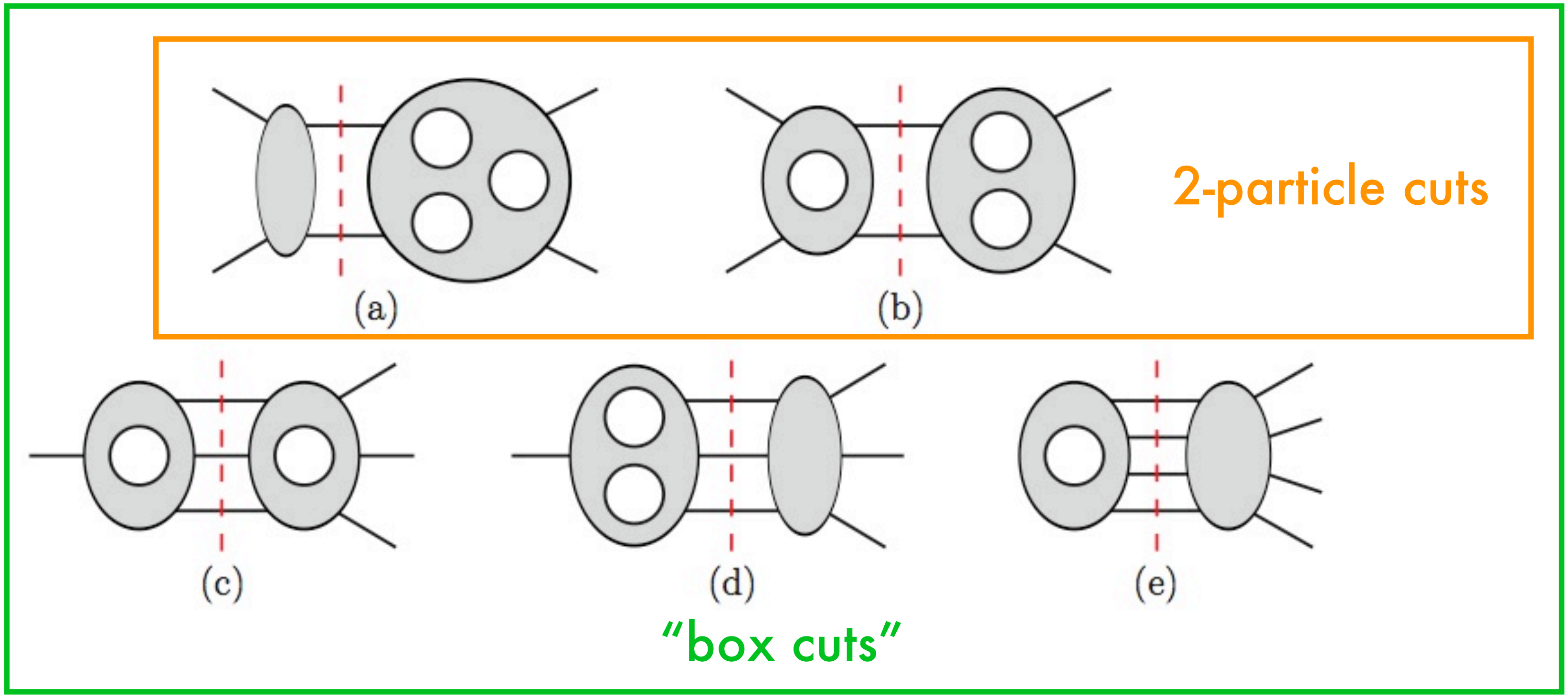
"Box cut":

- Allows back-of-the-envelope calculation of N
- Input comes from known lower loop integrals
- Result is D-dim, but may miss contact terms.

$$N = s_{27}s_{25}\Big(\frac{s_{12}s_{49}}{s_{27}} + \frac{s_{23}s_{48}}{s_{25}}\Big)$$
$$= s_{12}s_{25}s_{49} + s_{23}s_{27}s_{48}$$

Box cut

- A box cut may not involve a box subdiagram
- Isolating any 4-point $\mathcal{N}$=4 SYM *loop-amplitude* will do it

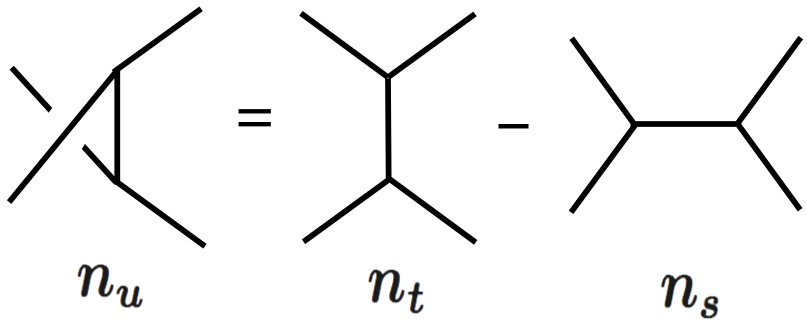


- 44 (out of 50) of the cubic topologies have box subdiagrams or other 4-pt subdiagrams
- But, many contact terms cannot be determined by the box cut

A diagram "twist rule"

We can use the Jacobi-like numerator identity of 0805.3993 [hep-ph]

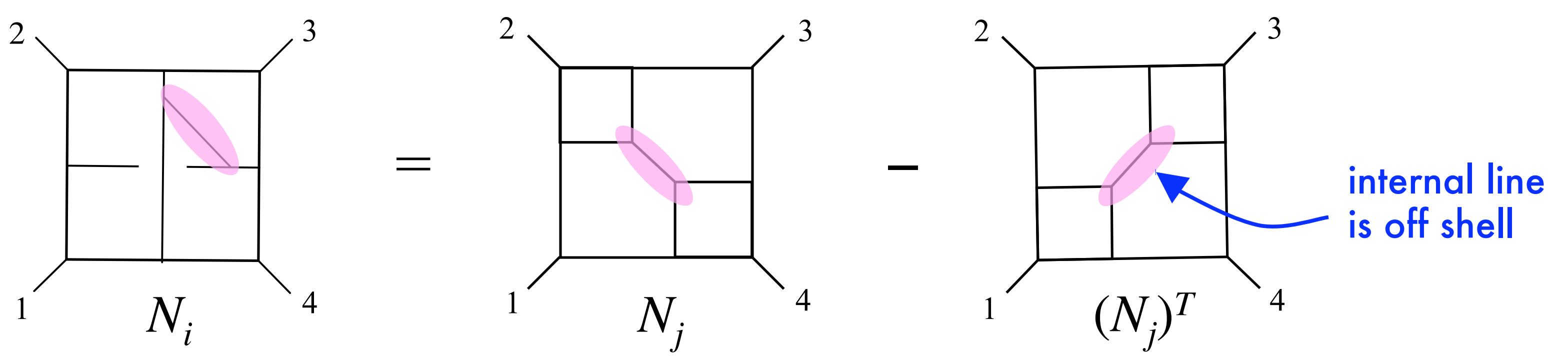
Bern, Carrasco, HJ



$$\mathcal{A}_4^{\text{tree}}(1,2,3,4) = g^2\left(\frac{n_s c_s}{s} + \frac{n_t c_t}{t} + \frac{n_u c_u}{u}\right)$$

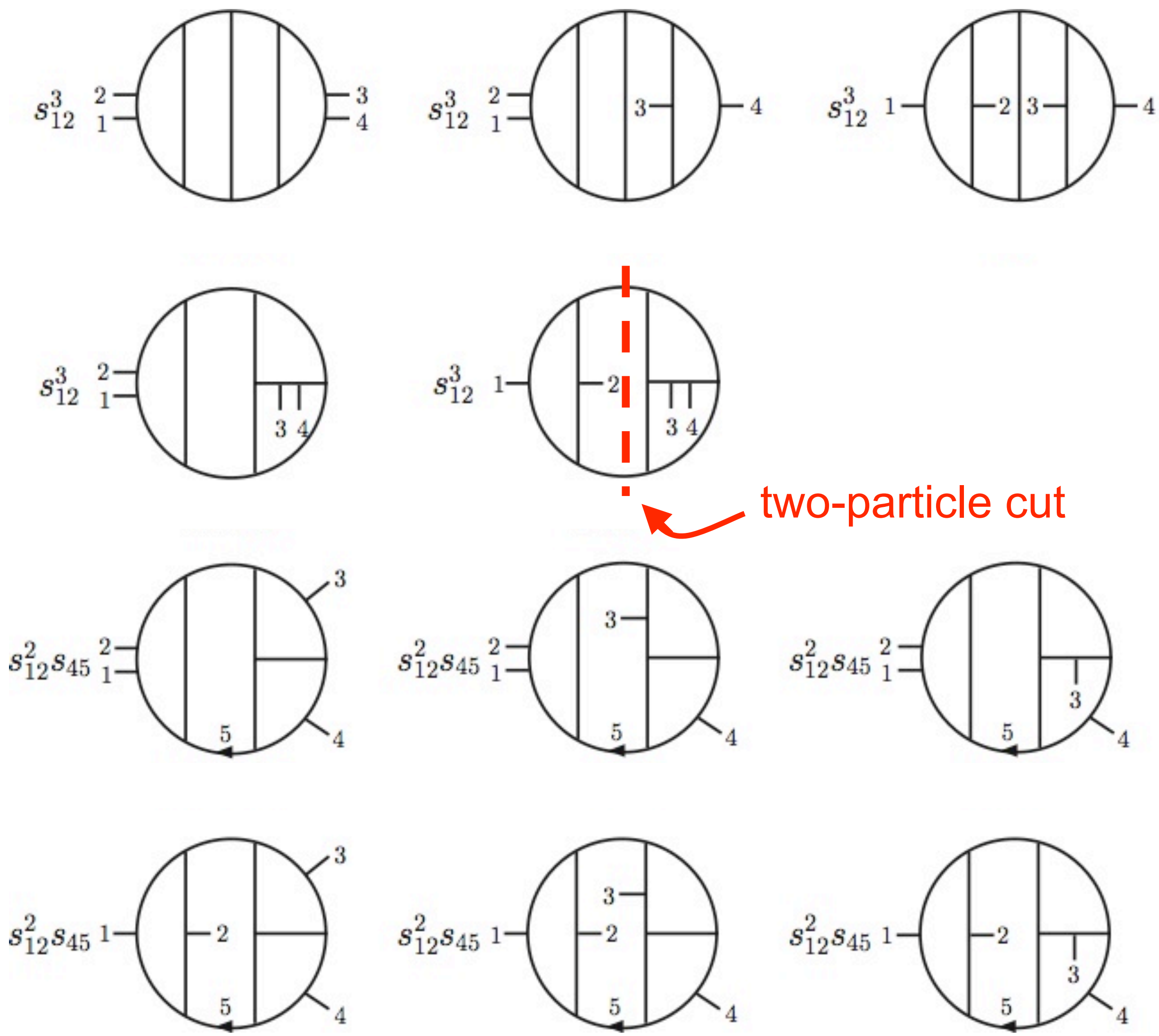
⇒ talk by Bern

Numerators of diagrams entering a cut are not independent



- Possible to relate non-planar topologies to planar ones
- In general, the N_i are constrained by a large linear eqn system

Some results from two-particle cuts



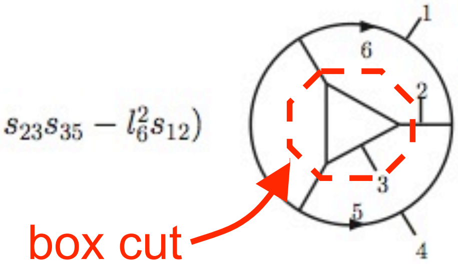
Simple numerator structure follow from the simplicity of the two-particle cut

$$s_{ij} = (k_i + k_j)^2$$

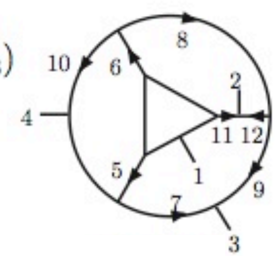
Bern, Carrasco, Dixon, HJ, Roiban, [to appear]

Some Results from Box Cuts

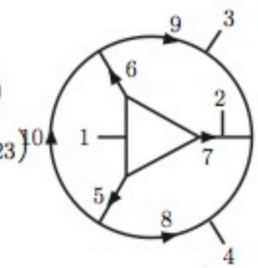
$s_{56}(s_{12}s_{56} - s_{23}s_{35} - l_6^2 s_{12})$



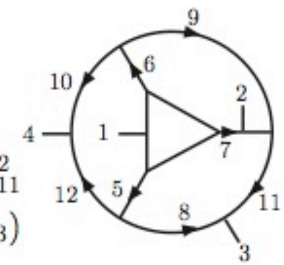
$s_{56}(s_{23}s_{9,10} - s_{12}s_{29} - l_{10}^2 s_{23})$
$+s_{12}s_{15}s_{78}$
$+l_5^2(s_{12}s_{29} - l_{12}^2 s_{23})$



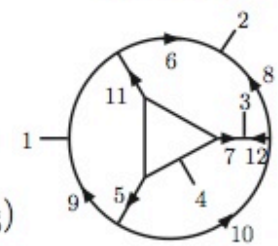
$s_{15}(s_{13}s_{56} - s_{12}s_{78} - l_8^2 s_{13})$
$-s_{16}(s_{23}s_{56} - s_{12}s_{79} - l_9^2 s_{23})$
$+l_7^2 s_{12}\tau_{1,10}$



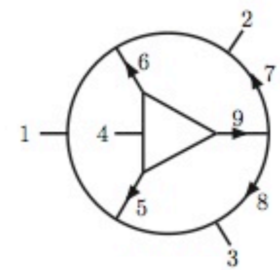
$s_{16}(s_{12}s_{79} + l_{10}^2 s_{23})$
$+s_{15}s_{12}(s_{23} - s_{78} + l_9^2)$
$+s_{23}s_{10,11}(s_{17} + l_6^2) - l_5^2 s_{23} l_{11}^2$
$-l_7^2(s_{12}s_{1,10} - l_{12}^2 s_{12} + l_{10}^2 s_{23})$



$-s_{47}(s_{12}s_{67} + s_{23}s_{89}$
$-s_{12}s_{23} - s_{12}l_{10}^2)$
$+s_{12}s_{45}s_{6,10} + l_{11}^2 l_{12}^2 s_{23}$
$+s_{12}(l_5^2 l_7^2 - l_5^2 s_{6,10} - l_7^2 s_{1\bar{5}})$
$+l_{11}^2 s_{12}(s_{67} - s_{23}) + \frac{1}{2} l_5^2 s_{12}s_{23}$



$s_{46}(s_{12}s_{28} - s_{23}s_{67})$
$+s_{45}(s_{13}s_{37} - s_{23}s_{58})$
$+s_{23}s_{46}s_{45} + l_9^2 s_{12}s_{13}$
$-s_{23}l_5^2 l_6^2$

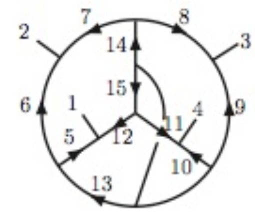


numerator structure more complicated, but still quite modest...

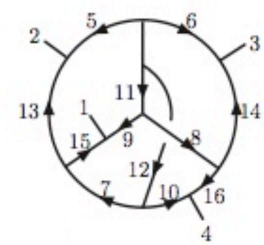
The most complicated numerators

Related to planar contributions through the "twist rule"

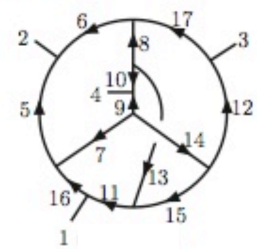
$$
\begin{aligned}
&s_{12}s_{28}s_{4,12} - s_{12}s_{37}s_{1,11} - s_{23}s_{16}s_{3,10} \\
&+s_{23}s_{25}s_{49} + \tfrac{1}{2}s_{12}s_{23}(s_{13,15} - s_{13,14}) \\
&+s_{12}(l_6^2 l_{10}^2 - l_5^2 l_9^2) + s_{23}(l_7^2 l_{11}^2 - l_8^2 l_{12}^2)
\end{aligned}
$$



$$
\begin{aligned}
&s_{12}(s_{2,10}s_{39} - s_{47}s_{18} + s_{2,10}s_{59} + s_{39}s_{6,10} + s_{23}s_{6,11}) - s_{23}s_{57}s_{68} - s_{13}s_{59}s_{6,10} \\
&+l_6^2(s_{12}s_{35} + s_{12}s_{4,\overline{12}} - s_{23}s_{59}) + l_5^2(s_{12}s_{26} + s_{12}s_{1,\overline{11}} - s_{23}s_{6,10}) \\
&+l_9^2(s_{12}s_{12,\overline{13}} - s_{13}s_{10,11}) + l_{10}^2(s_{12}s_{11,\overline{14}} - s_{13}s_{9,12}) \\
&-l_{13}^2 s_{12}s_{11,\overline{14}} - l_{14}^2 s_{12}s_{12,\overline{13}} + (s_{13} - 2s_{12})l_9^2 l_{10}^2 \\
&+s_{23}(l_5^2 l_6^2 - l_7^2 l_8^2 + l_6^2 l_7^2 + l_5^2 l_8^2) + s_{12}l_{13}^2 l_{14}^2 + s_{12}l_5^2 l_6^2 \\
&+s_{12}(-l_5^2 l_8^2 + l_5^2 l_9^2 - l_5^2 l_{11}^2 - l_5^2 l_{15}^2 - l_9^2 l_{15}^2) \\
&+s_{12}(-l_6^2 l_7^2 + l_6^2 l_{10}^2 - l_6^2 l_{12}^2 - l_6^2 l_{16}^2 - l_{10}^2 l_{16}^2) \\
&+s_{23}(l_9^2 l_{12}^2 + l_{10}^2 l_{11}^2 - l_7^2 l_9^2 - l_8^2 l_{10}^2) + s_{13}(l_9^2 l_{11}^2 + l_{10}^2 l_{12}^2)
\end{aligned}
$$



$$
\begin{aligned}
&s_{12}(s_{47}s_{5,12} - s_{19}s_{36} - s_{48}s_{36}) + s_{23}(s_{48}s_{6,11} - s_{15}s_{3,10} - s_{15}s_{47}) - s_{12}s_{23}s_{11,12} \\
&+l_5^2(s_{23}s_{7,12} - s_{23}s_{4,15} - s_{13}s_{10,11}) + l_6^2(s_{12}s_{8,11} - s_{12}s_{4,\overline{15}} - s_{13}s_{9,12}) \\
&+l_9^2(s_{23}s_{3,15} - s_{12}s_{3\overline{8}} + s_{23}s_{6,10}) + l_{10}^2(s_{12}s_{1,\overline{15}} - s_{23}s_{1\overline{7}} + s_{12}s_{59}) \\
&+l_{13}^2(s_{12}s_{23} + s_{12}s_{38} - s_{23}s_{6,11}) + l_{14}^2(s_{23}s_{12} + s_{23}s_{17} - s_{12}s_{5,12}) \\
&+l_{11}^2 s_{23}(s_{4,12} - s_{6,10}) + l_{12}^2 s_{12}(s_{4,11} - s_{59}) \\
&+s_{13}(l_7^2 l_8^2 + l_5^2 l_8^2 + l_6^2 l_7^2 + l_{11}^2 l_{12}^2 + l_{10}^2 l_{16}^2 + l_9^2 l_{17}^2 - l_9^2 l_{12}^2 - l_{10}^2 l_{11}^2) \\
&+s_{12}(-l_5^2 l_{10}^2 + l_6^2(l_{14}^2 + l_{13}^2 - l_{10}^2) + l_{12}^2(l_9^2 - l_5^2 - l_7^2 + l_{14}^2) + l_8^2(l_9^2 + l_{16}^2)) \\
&+s_{23}(-l_6^2 l_9^2 + l_5^2(l_{13}^2 + l_{14}^2 - l_9^2) + l_{11}^2(l_{10}^2 - l_6^2 - l_8^2 + l_{13}^2) + l_7^2(l_{10}^2 + l_{17}^2)) \\
&+s_{12}(l_{12}^2 l_{13}^2 - l_8^2 l_{13}^2 - l_{10}^2 l_{13}^2 - l_{10}^2 l_{14}^2 - l_{13}^2 l_{17}^2) + s_{23}(l_{11}^2 l_{14}^2 - l_7^2 l_{14}^2 - l_9^2 l_{14}^2 - l_9^2 l_{13}^2 - l_{14}^2 l_{16}^2)
\end{aligned}
$$

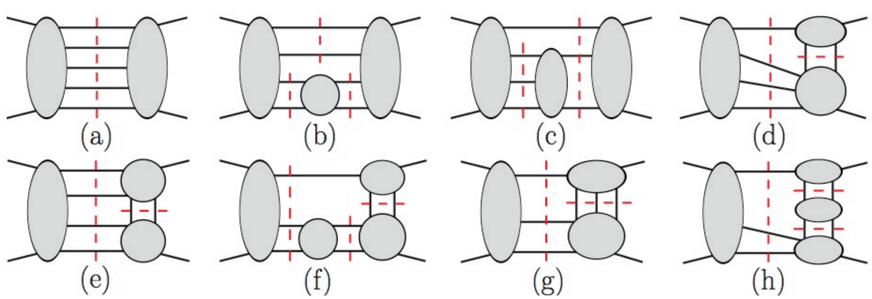


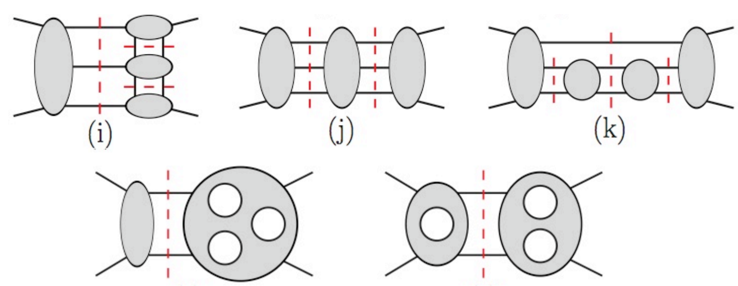
Contact-term corrections determined by maximal cuts

Proof of amplitude

- To do:

- Find missing contact terms not given by heuristic rules
  - ☑ automate (singlet) maximal cuts
- Check correctness of cuts in $D=4$ using a complete set of cuts
  - ☑ automate general cuts, and
  - ☑ include full $\mathcal{N}=4$ supersum   ⇒ talk by Roiban
- Check correctness of amplitude using D-dimensional cuts
  - ☐ automate D-dim cuts and superspace
  - ☑ partial checks done: two-particle cuts, box cuts

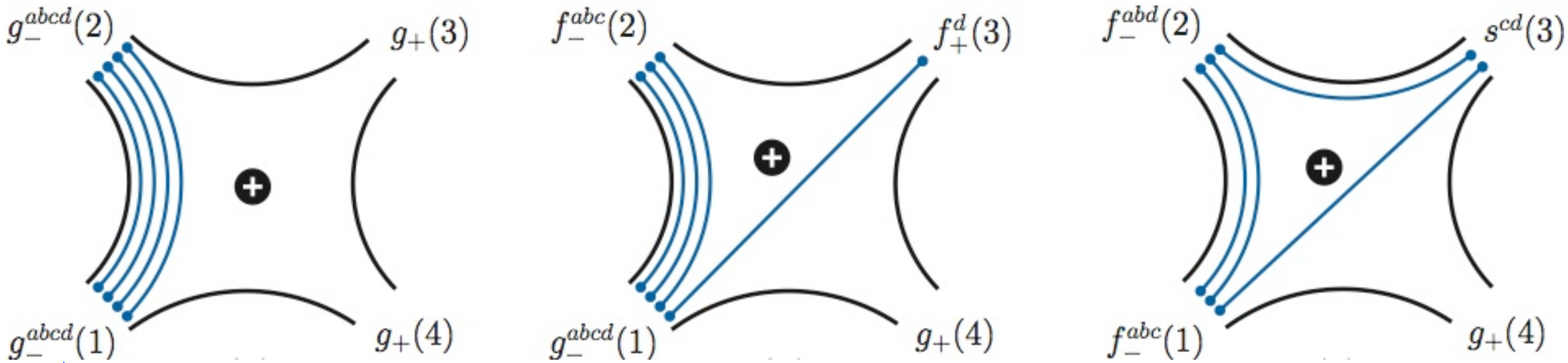




Note: for 1,2,3 loops, 4-dim cuts capture the full 4pt amplitude

Supersum Structure

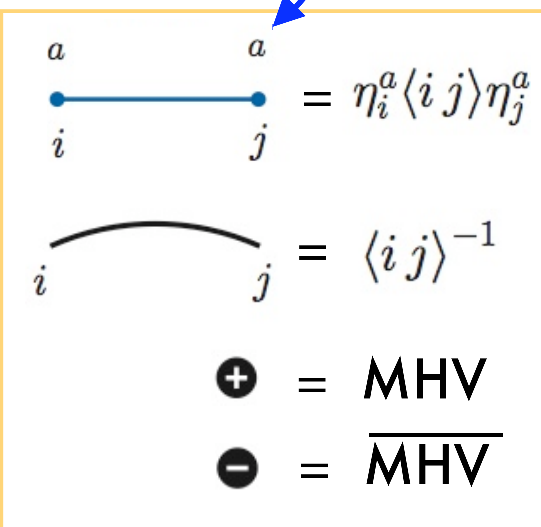
Convenient to use "index diagrams" to visualize the supersum structure

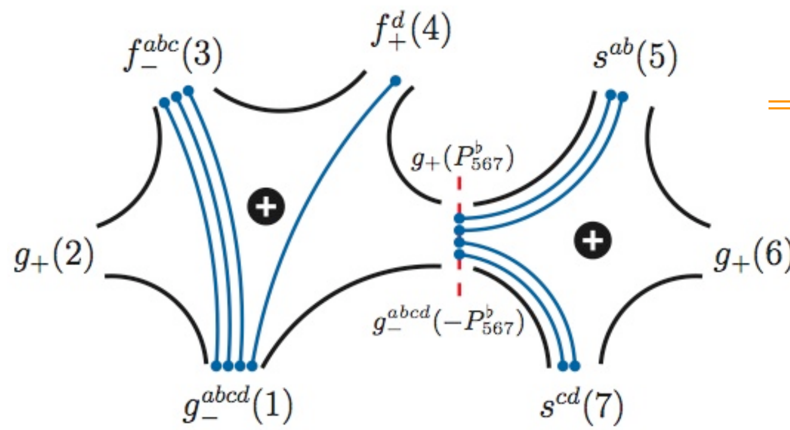


↑ 4pt MHV amplitudes in a graphical notation

R-charge index

Bern, Carrasco, Ita, HJ, Roiban

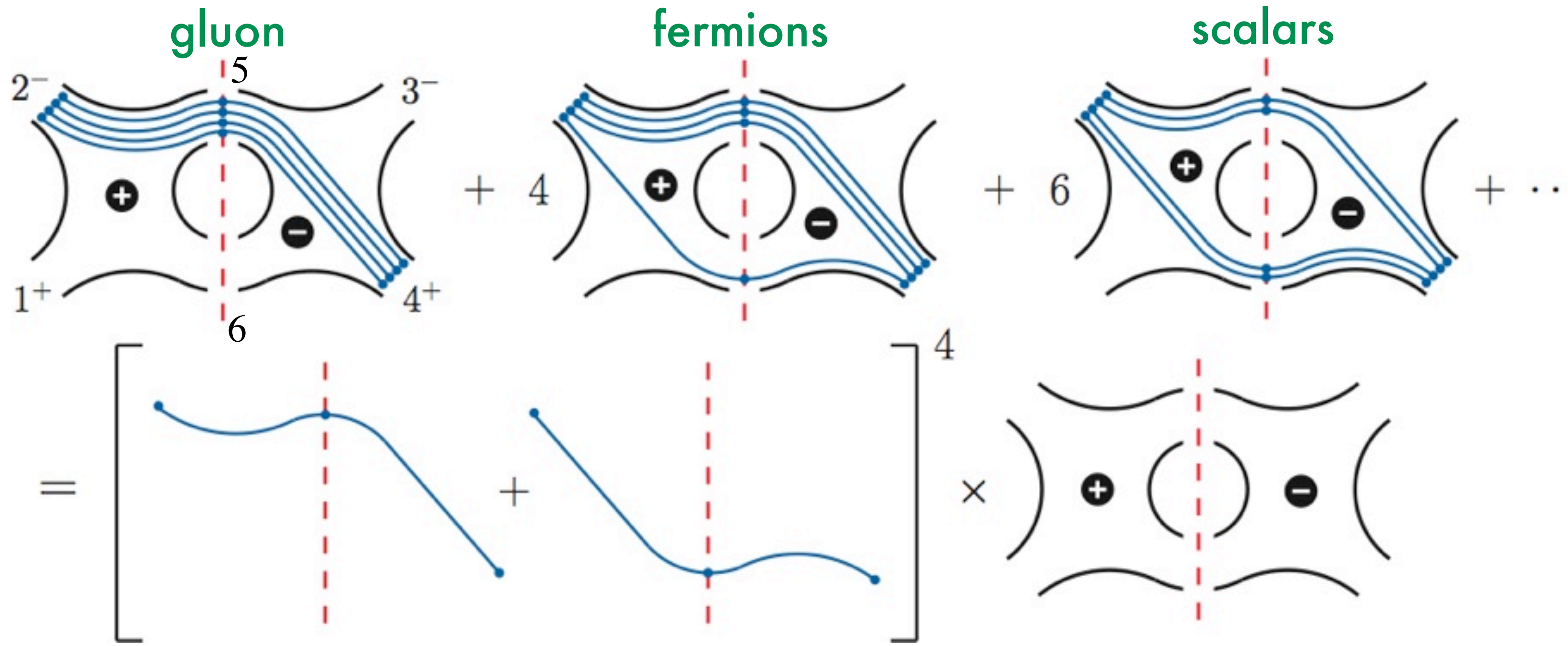




⇒ talk by Roiban

For non-MHV trees → CSW expansion

Tracking the *R* charge index



Helicity dependent part of cut

$$\langle 2|5|4]^4 \;+\; 4\,\langle 2|5|4]^3\,\langle 2|6|4] \;+\; 6\,\langle 2|5|4]^2\,\langle 2|6|4]^2 + \ldots$$

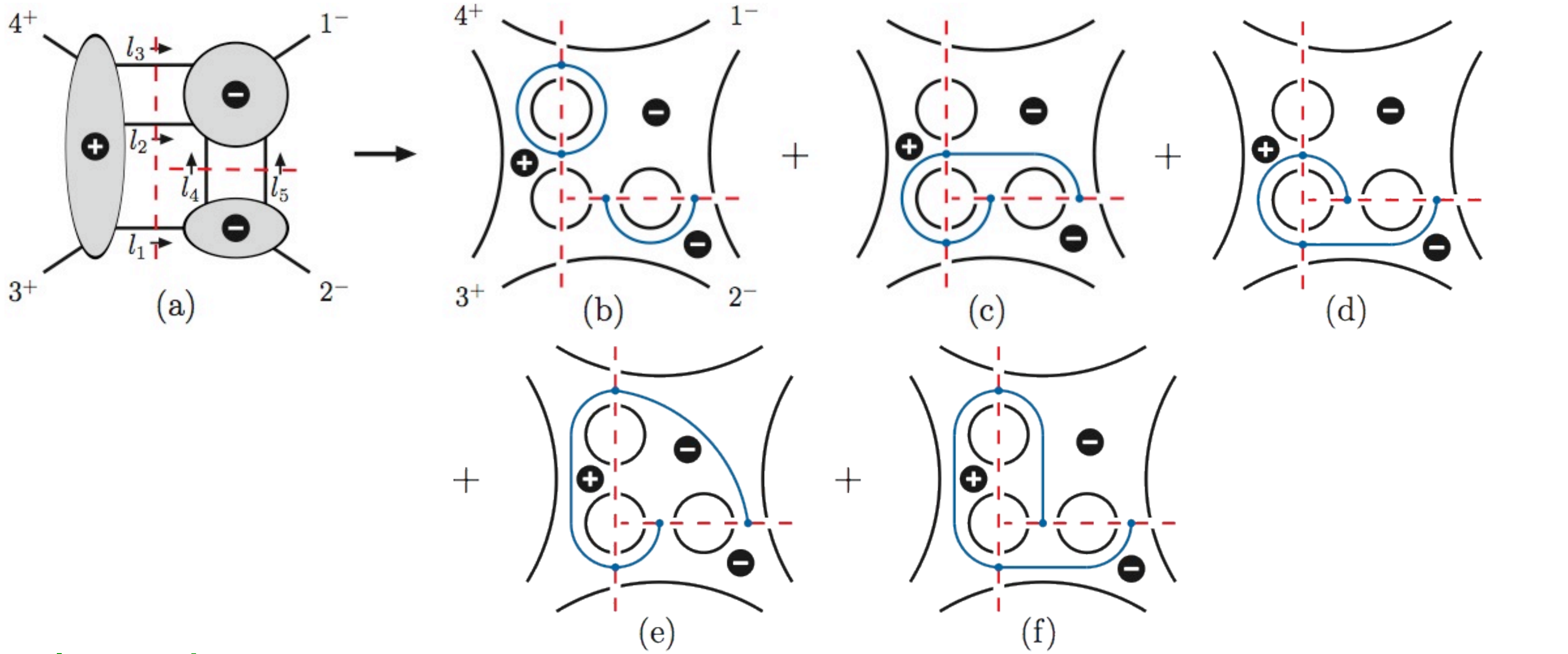
$$= \Big[\,\langle 2|5|4] \;+\; \langle 2|6|4]\,\Big]^4$$

General structure of cuts $(A+B+C+\ldots)^{\mathcal{N}}\,, \qquad \mathcal{N}=4$

"single R index contribution" ↑

⇒ talk by Roiban

Supersum Example



"single *R* index contribution"

$$l_2 \cdot l_3\,[l_4\, l_5] + [l_4|l_1 l_2|l_5] + [l_4|l_2 l_1|l_5] + [l_4|l_1 l_3|l_5] + [l_4|l_3 l_1|l_5] = s\,[l_4\, l_5]$$

$$\text{cut} = s^4\,[l_4\, l_5]^4 \frac{1}{\langle 3\,4\rangle\,\langle 4\,l_3\rangle\,\langle l_3\,l_2\rangle\,\langle l_2\,l_1\rangle\,\langle l_1\,3\rangle}\,\frac{1}{[1\,l_5]\,[l_5\,l_4]\,[l_4\,l_2]\,[l_2\,l_3]\,[l_3\,1]}\,\frac{1}{[2\,l_1]\,[l_1\,l_4]\,[l_4\,l_5]\,[l_5\,2]}$$

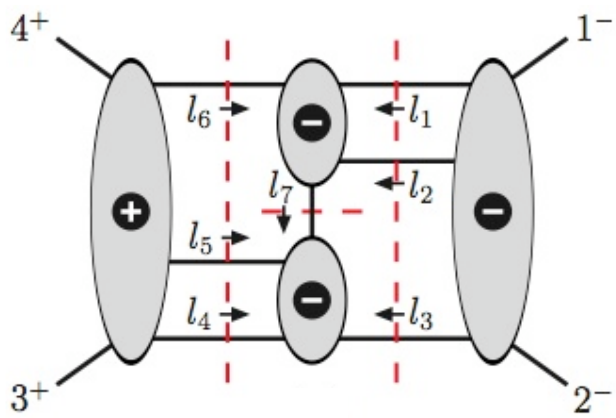
$5^4 = 625$ contributions from individual states re-summed

Automate Supersums

- After the supersum structure is understood, automating calculations is straightforward
- Exploit similarity of N=4 SYM and pure Yang Mills (QCD)

Bern, Carrasco, Ita, HJ, Roiban

⇒ talk by Roiban



QCD: $A^4+B^4+C^4+D^4+E^4+F^4+G^4+H^4$

N=4 SYM: $\left[A+B+C+D+E+F+G+H\right]^4$

Algorithm ⇒ Find the (few) QCD contributions and map to N=4 SYM

$$A=\langle l_4\, l_5\rangle\,[l_4\, l_5]\,[l_2\, l_7]\,[l_1\, l_3]\,,\quad B=\langle l_4\, l_5\rangle\,[l_4\, l_5]\,[l_7\, l_1]\,[l_2\, l_3]\,,\quad C=\langle l_4\, l_6\rangle\,[l_4\, l_7]\,[l_2\, l_6]\,[l_1\, l_3]\,,$$

$$D=\langle l_4\, l_6\rangle\,[l_4\, l_7]\,[l_6\, l_1]\,[l_2\, l_3]\,,\quad E=\langle l_5\, l_6\rangle\,[l_5\, l_7]\,[l_2\, l_6]\,[l_1\, l_3]\,,\quad F=\langle l_5\, l_6\rangle\,[l_5\, l_7]\,[l_6\, l_1]\,[l_2\, l_3]\,,$$

$$G=\langle l_4\, l_6\rangle\,[l_2\, l_1]\,[l_3\, l_4]\,[l_6\, l_7]\,,\quad H=\langle l_5\, l_6\rangle\,[l_2\, l_1]\,[l_3\, l_5]\,[l_6\, l_7]\,.$$

$$\left[A+B+C+D+E+F+G+H\right]^4=\left[s\,[l_1\, l_2]\,[l_7\, l_3]\right]^4$$

$8^4 = 4096$ contributions from individual states re-summed

UV properties

UV properties

- N=4 SYM UV properties are interesting due to recent studies of potential counterterms   Bossard, Howe, Stelle, 0901.4661   ⇒talk by Howe

- Planar amplitudes have established divergences in critical dimensions:

$$D_c = 8 \qquad\qquad L = 1$$
$$D_c = 4 + 6/L \qquad\qquad L = 2, 3, 4$$

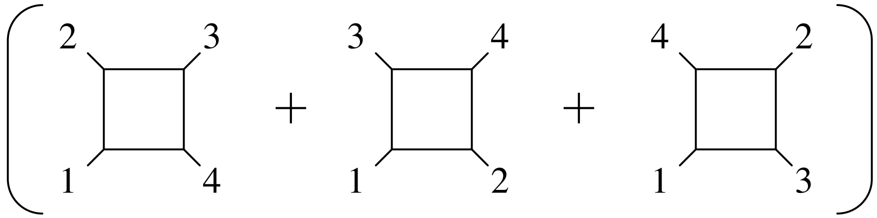
- We wish to determine the full color dependence of the UV divergences

- In gauge group $SU(N_c)$, using color structures:

$$\begin{aligned}\mathrm{Tr}_{ijkl} &\equiv \mathrm{Tr}(T^{a_i}T^{a_j}T^{a_k}T^{a_l}) \\ \mathrm{Tr}_{ij} &\equiv \mathrm{Tr}(T^{a_i}T^{a_j}) = \delta^{a_i a_j}\end{aligned}$$

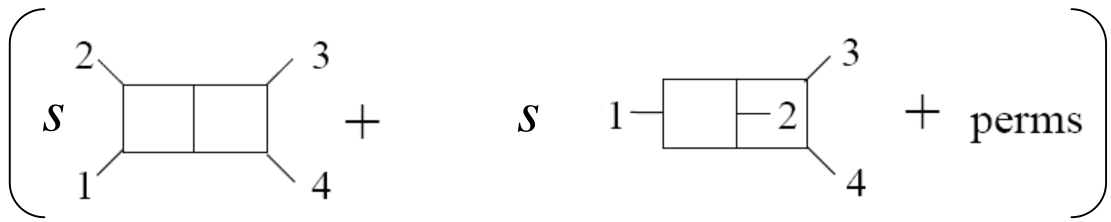
1,2-Loop Amplitudes

1-loop: $\mathcal{K}$



Green, Schwarz, Brink (1982)

2-loop: $\mathcal{K}$



Bern, Dixon, Dunbar, Perelstein and Rozowsky (1998)

$s = (k_1 + k_2)^2$

Helicity containing prefactor: $\mathcal{K} = st A_4^{\text{tree}}$

Color factors: dress each diagram with $f^{abc} = \text{Tr}(T^a [T^b, T^c])$

1,2-Loop UV Divergences

1 loop: $D_c = 8 - 2\varepsilon$

$$\mathcal{A}_4^{(1)}(1,2,3,4)|_{\text{pole}} = -\frac{g^4\,\mathcal{K}}{6\,(4\pi)^4\,\epsilon}\Big[N_c\,(\,\text{Tr}_{1324} + \text{Tr}_{1423} + \text{Tr}_{1243} + \text{Tr}_{1342} + \text{Tr}_{1234} + \text{Tr}_{1432}) + 6\,(\,\text{Tr}_{12}\,\text{Tr}_{34} + \text{Tr}_{14}\,\text{Tr}_{23} + \text{Tr}_{13}\,\text{Tr}_{24})\Big]$$

Counterterms ~ $\text{Tr}\,F^4$ $\quad$ $\text{Tr}\,F^2\;\;\text{Tr}\,F^2$

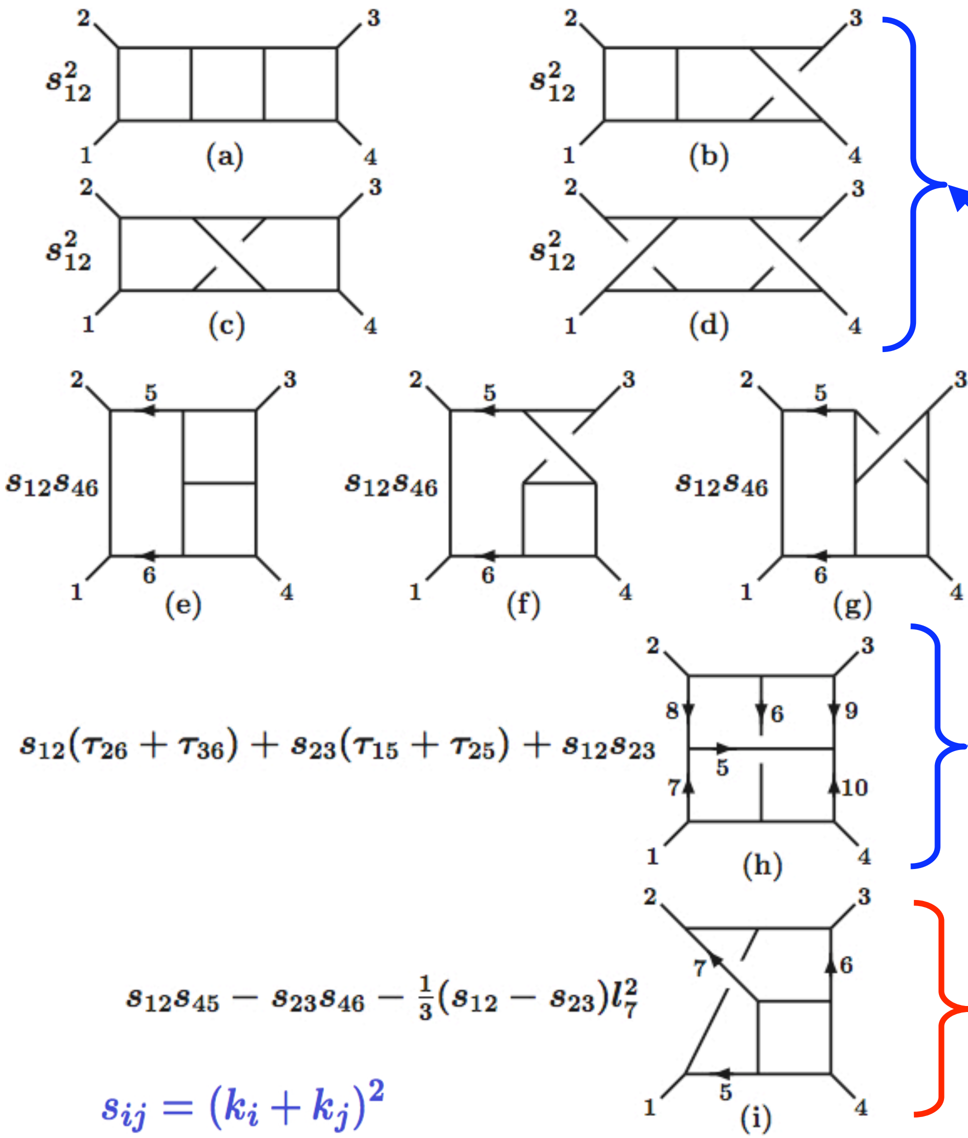
2 loops: $D_c = 7 - 2\varepsilon$

$$\mathcal{A}_4^{(2)}(1,2,3,4)|_{\text{pole}} = \frac{g^6\,\pi\,\mathcal{K}}{20\,(4\pi)^7\,\epsilon}\Big[(N_c^2 + 20)(s_{12}\,(\,\text{Tr}_{1324} + \text{Tr}_{1423}) + s_{23}\,(\,\text{Tr}_{1243} + \text{Tr}_{1342}) + s_{13}\,(\,\text{Tr}_{1234} + \text{Tr}_{1432})) - 20N_c\,(s_{12}\,\text{Tr}_{12}\,\text{Tr}_{34} + s_{23}\,\text{Tr}_{14}\,\text{Tr}_{23} + s_{13}\,\text{Tr}_{13}\,\text{Tr}_{24})\Big]$$

Counterterms ~ $\partial^2\,\text{Tr}F^4$ $\quad$ $\partial^2[\,\text{Tr}F^2]^2$

As expected, both single and double trace terms appears

The 3-loop amplitude



$$\mathcal{K}\sum_{S_4}\Big[C^{(\mathrm{a})}I^{(\mathrm{a})}+C^{(\mathrm{b})}I^{(\mathrm{b})}+\tfrac{1}{2}C^{(\mathrm{c})}I^{(\mathrm{c})}+\tfrac{1}{4}C^{(\mathrm{d})}I^{(\mathrm{d})}+2C^{(\mathrm{e})}I^{(\mathrm{e})}+2C^{(\mathrm{f})}I^{(\mathrm{f})}+4C^{(\mathrm{g})}I^{(\mathrm{g})}+\tfrac{1}{2}C^{(\mathrm{h})}I^{(\mathrm{h})}+2C^{(\mathrm{i})}I^{(\mathrm{i})}\Big]$$

Finite in $D = 6$ (a), (b), (c), (d), (h)

Divergent in $D = 6$ (e), (f), (g), (i)

Bern, Carrasco, Dixon, HJ, Kosower, Roiban,
Bern, Carrasco, Dixon, HJ, Roiban

$s_{ij}=(k_i+k_j)^2$
$\tau_{ij}=2k_i\cdot l_j$

3-Loop UV Divergence

Bern, Carrasco, Dixon, HJ, Roiban [to appear]

3 loops: $D_c = 6 - 2\varepsilon$

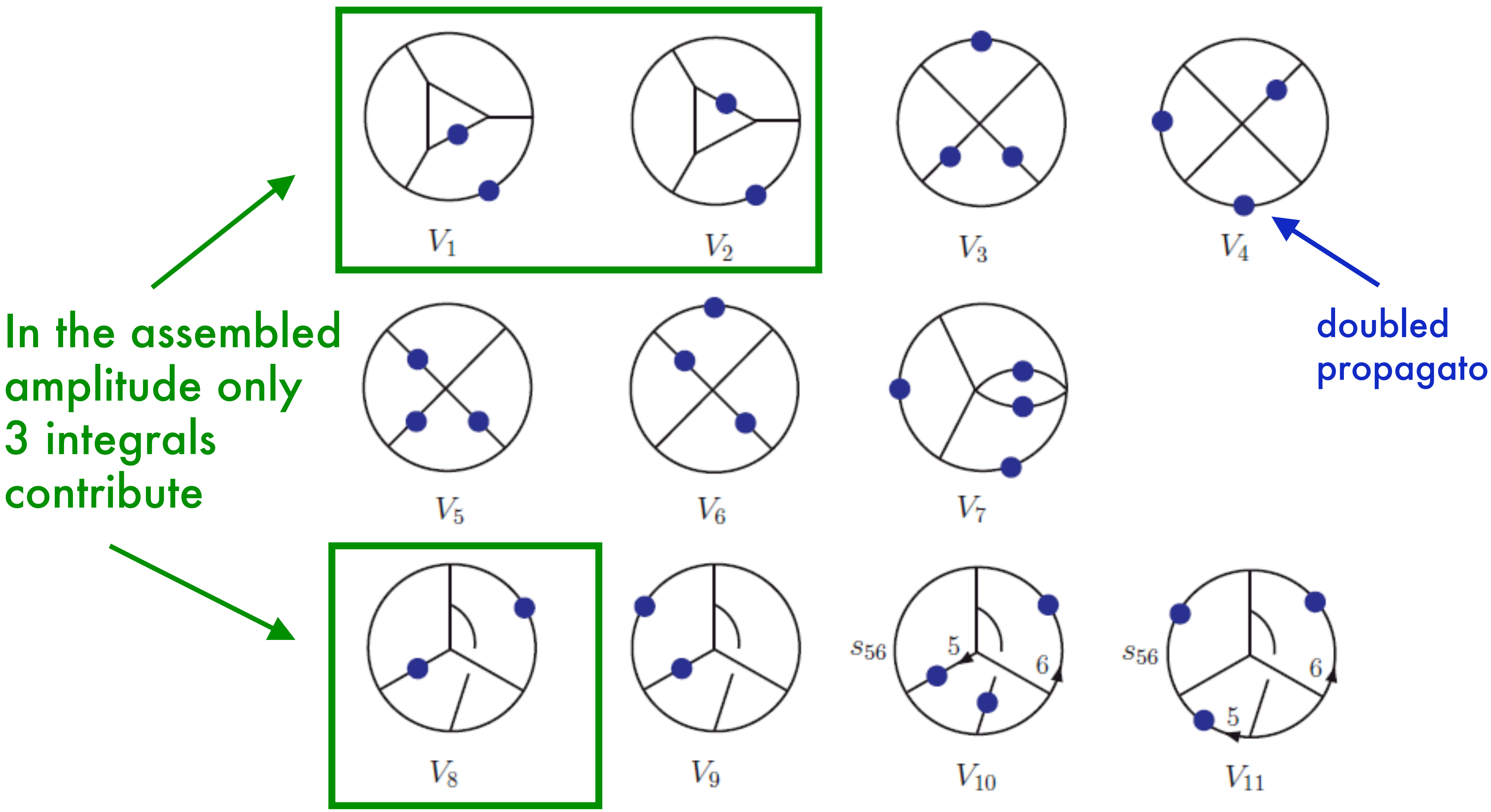
$$\mathcal{A}_4^{(3)}(1,2,3,4)|_{\text{pole}} = -\frac{g^8\,\mathcal{K}}{3\,(4\pi)^9\,\epsilon}(N_c^3 + 36\,\zeta(3)\,N_c)\,\Big[s_{12}\,(\text{Tr}_{1324} + \text{Tr}_{1423}) + s_{23}\,(\text{Tr}_{1243} + \text{Tr}_{1342}) + s_{13}\,(\text{Tr}_{1234} + \text{Tr}_{1432})\Big]$$

Counterterms ~ $\partial^2\,\text{Tr}F^4$ ~~$\partial^2[\,\text{Tr}F^2]^2$~~

Remakably the double-trace contributions are finite in D=6

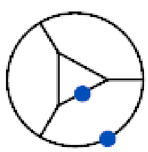
4-loop UV vacuum integrals

(before cancellations between different topologies)

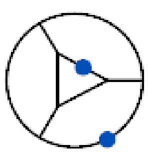


UV integral evaluation

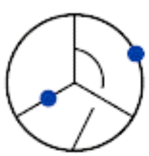
- Vacuum integrals factorize into product of 1-loop integral with UV pole and a finite 3-loop propagator (2pt) integral
- Finite 3-loop integrals reduce to master integrals using integration by parts (IBP), a la MINCER. Chetyrkin, Tkachov (1981)
- Most nontrivial integral is nonplanar master integral, for which we only have numerical results (obtained using Gegenbauer polynomial x-space technique) Chetyrkin, Tkachov (1981); Bekavac, hep-ph/0505174



$$V_1 = \frac{1}{(4\pi)^{11}\,\epsilon}\left[\frac{512}{5}\,\Gamma^4(\tfrac{3}{4}) - \frac{2048}{105}\,\Gamma^3(\tfrac{3}{4})\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{4})\right] \;+\; \mathcal{O}(1)$$



$$V_2 = \frac{1}{(4\pi)^{11}\,\epsilon}\left[-\frac{4352}{105}\,\Gamma^4(\tfrac{3}{4}) + \frac{832}{105}\,\Gamma^3(\tfrac{3}{4})\Gamma(\tfrac{1}{2})\Gamma(\tfrac{1}{4})\right] \;+\; \mathcal{O}(1)$$



$$V_8 = \frac{1}{(4\pi)^{11}}\frac{4}{21}\frac{1}{\Gamma(\frac{3}{4})}\frac{V_8^{\rm fin}}{\epsilon} \qquad V_8^{\rm fin} \;=\; 1.428452926283(3)$$

Evaluated in critical dimension $D_c = 4 + 6/4 = 11/2$

4-Loop UV Divergence

Combining UV poles of integrals with color factors

$$\mathcal{A}_4^{(4)}(1,2,3,4)|_{\text{pole}} = -6\, g^{10}\, \mathcal{K}\, N_c^2 \Big[N_c^2\, V_1 + 12\, (V_1 + 2\, V_2 + V_8) \Big] \times \Big[s_{12}\, (\,\text{Tr}_{1324} + \,\text{Tr}_{1423}) + s_{23}\, (\,\text{Tr}_{1243} + \,\text{Tr}_{1342}) + s_{13}\, (\,\text{Tr}_{1234} + \,\text{Tr}_{1432}) \Big]$$

Counterterms ~ $\partial^2\, \text{Tr} F^4$

Bern, Carrasco, Dixon, HJ, Roiban [to appear]

- Again the double-trace contributions are finite in $D_c = 11/2$
- Also $(N_c)^0$ term is finite
- Absence of double-trace and $(N_c)^0$ terms at 3 *and* 4 loops calls out for explanation.
- Related to better UV behavior of colorless theories?

$\Rightarrow$ talk by Vanhove

Conclusions

- Full color 4-point 4-loop amplitude has been computed in $\mathcal{N}$=4 super-Yang-Mills theory
- Tools: rung rule, box cut, twist rules, maximal cuts, and generalized cuts with full susy multiplet
- "Index diagrams" introduced to clarify supersum structure in cuts, paving the way for automated calculations
- $L = 4$ UV divergence have been extracted, and compared with results for $L = 1,2,3$
- Double-trace and $(N_c)^0$ terms drop out after $L = 2$
- Future studies of the IR information is possible ... once technology is developed for doing non-planar 4-point integrals (even numerically) in $D = 4 - 2\varepsilon$ at $L = 3,4$