Yangian symmetry of scattering amplitudes in $\mathcal{N} = 4$ SYM

James Drummond

LAPTH Annecy

0807.1095 [JMD, Johannes Henn, Gregory Korchemsky, Emery Sokatchev]

0808.2475 [JMD, Johannes Henn]

0902.2987 [JMD, Johannes Henn, Jan Plefka]

Outline

- ✔ Tree-level amplitudes
- ✔ Superconformal and dual superconformal symmetry
- ✔ Superconformal + Dual superconformal $\Longrightarrow$ Yangian symmetry

$\mathcal{N}=4$ **Super-amplitudes**

$\mathcal{N}=4$ SYM is special because it is described by PCT self-conjugate supermultiplet:

Chiral representation:

$$\Phi(\eta) = G^+ + \eta^A \Gamma_A + \frac{1}{2}\eta^A\eta^B S_{AB} + \frac{1}{3!}\eta^A\eta^B\eta^C\epsilon_{ABCD}\bar{\Gamma}^D + \frac{1}{4!}(\eta)^4 G^-$$

$$p^{\alpha\dot{\alpha}} = \lambda^\alpha\tilde{\lambda}^{\dot{\alpha}}, \qquad q^{\alpha A} = \lambda^\alpha\eta^A, \qquad \bar{q}^{\dot{\alpha}}_A = \tilde{\lambda}^{\dot{\alpha}}\frac{\partial}{\partial\eta^A}.$$

Super-amplitudes:

$$\mathcal{A}(\Phi_1\ldots\Phi_n) = (\eta_1)^4(\eta_2)^4\mathcal{A}(G_1^-G_2^-G_3^+\ldots G_4^+) + \ldots$$

$$p^{\alpha\dot{\alpha}} = \sum_i \lambda_i^\alpha\tilde{\lambda}_i^{\dot{\alpha}}, \qquad q^{\alpha A} = \sum_i \lambda_i^\alpha\eta_i^A, \qquad \bar{q}^{\dot{\alpha}}_A = \sum_i \tilde{\lambda}_i^{\dot{\alpha}}\frac{\partial}{\partial\eta_i^A}.$$

Symmetries:

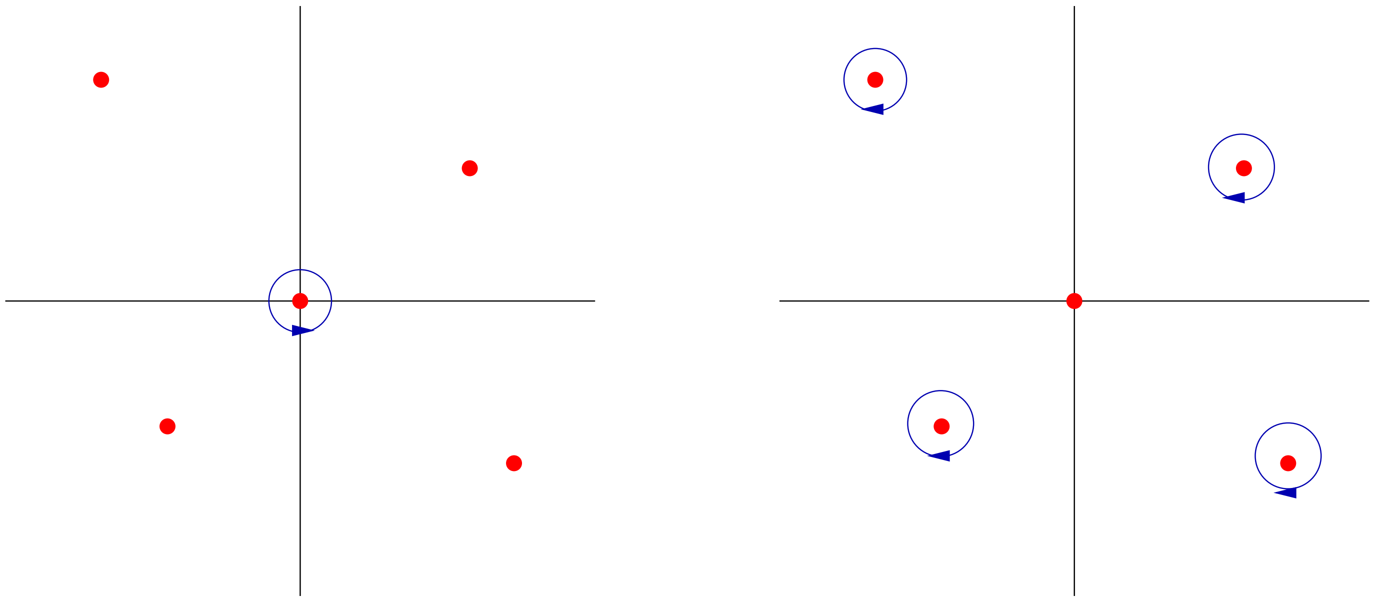
$$h_i\mathcal{A}_n(\lambda_i,\tilde{\lambda}_i,\eta_i) = \mathcal{A}_n(\lambda_i,\tilde{\lambda}_i,\eta_i) \qquad h_i = -\frac{1}{2}\lambda_i^\alpha\frac{\partial}{\partial\lambda_i^\alpha} + \frac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\alpha}}} + \frac{1}{2}\eta_i^A\frac{\partial}{\partial\eta_i^A}$$

$$p\mathcal{A} = q\mathcal{A} = \bar{q}\mathcal{A} = 0 \implies \mathcal{A}(\Phi_1,\ldots,\Phi_n) = \frac{\delta^4(p)\delta^8(q)}{\langle 12\rangle\ldots\langle n1\rangle}\mathcal{P}(\lambda,\tilde{\lambda},\eta), \qquad \bar{q}\mathcal{P} = 0.$$

$$\mathcal{P} = \mathcal{P}^{\mathrm{MHV}} + \mathcal{P}^{\mathrm{NMHV}} + \ldots + \mathcal{P}^{\overline{\mathrm{MHV}}}.$$

Tree-level BCFW recursion relations

Shift $p_1 \to p_1 - z\lambda_n\tilde{\lambda}_1, \quad p_n \to p_n + z\lambda_n\tilde{\lambda}_1$. Consider $\dfrac{\mathcal{A}(z)}{z}$ in the complex z-plane.



Assuming $\mathcal{A}(z) \longrightarrow 0$ as $z \longrightarrow \infty$ we have

$$\mathcal{A}(0) = -\sum \text{other residues} = \sum_r \sum_{\text{states}} \mathcal{A}_L(z_r)\frac{1}{P_r^2}\mathcal{A}_R(z_r)$$

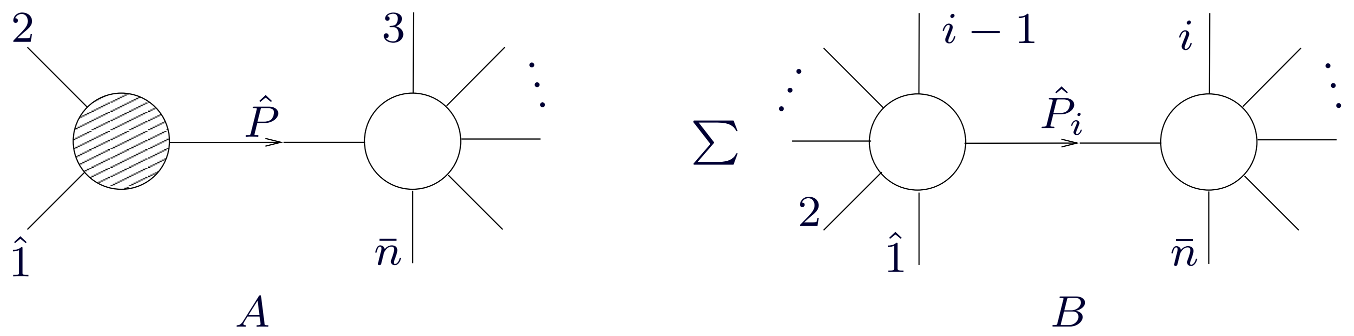
This can be shown to be true for $h_i = h_j = +$

For superamplitudes, one can use $\bar{q}$-supersymmetry to set $\eta_i = \eta_j = 0$. Then $\sum_{\text{states}} \longrightarrow \int d\eta$

[Arkani-Hamed,Cachazo,Kaplan],[Brandhuber,Heslop,Travaglini],[Elvang,Freedman,Kiermaier]

Solving recursion relations

[JMD, Henn]



$$\int \frac{d^4\eta}{P^2} \mathcal{A}_3^{\overline{\mathrm{MHV}}}(\hat{1}, 2, \hat{P}) \mathcal{A}_{n-1}^{\mathrm{MHV}}(-\hat{P}, 3, \ldots, \hat{n}) = \mathcal{A}_n^{\mathrm{MHV}} = \frac{\delta^{(4)}(p)\,\delta^{(8)}(q)}{\langle 1\,2\rangle\langle 2\,3\rangle \ldots \langle n\,1\rangle}$$

$$\mathcal{A}_n^{\mathrm{NMHV}} = \frac{\delta^{(4)}(p)\,\delta^{(8)}(q)}{\langle 1\,2\rangle\langle 2\,3\rangle \ldots \langle n\,1\rangle} \sum_{2\leq s<t\leq n-1} R_{n;st}$$

$$R_{r;s,t} = \frac{\langle s\ s-1\rangle\langle t\ t-1\rangle \delta^4(\langle r|x_{rs}x_{st}|\theta_{tr}\rangle + \langle r|x_{rt}x_{ts}|\theta_{sr}\rangle)}{x_{st}^2 \langle r|x_{rs}x_{st}|t\rangle\langle r|x_{rs}x_{st}|t-1\rangle\langle r|x_{rt}x_{ts}|s\rangle\langle r|x_{rt}x_{ts}|s-1\rangle}$$

Here

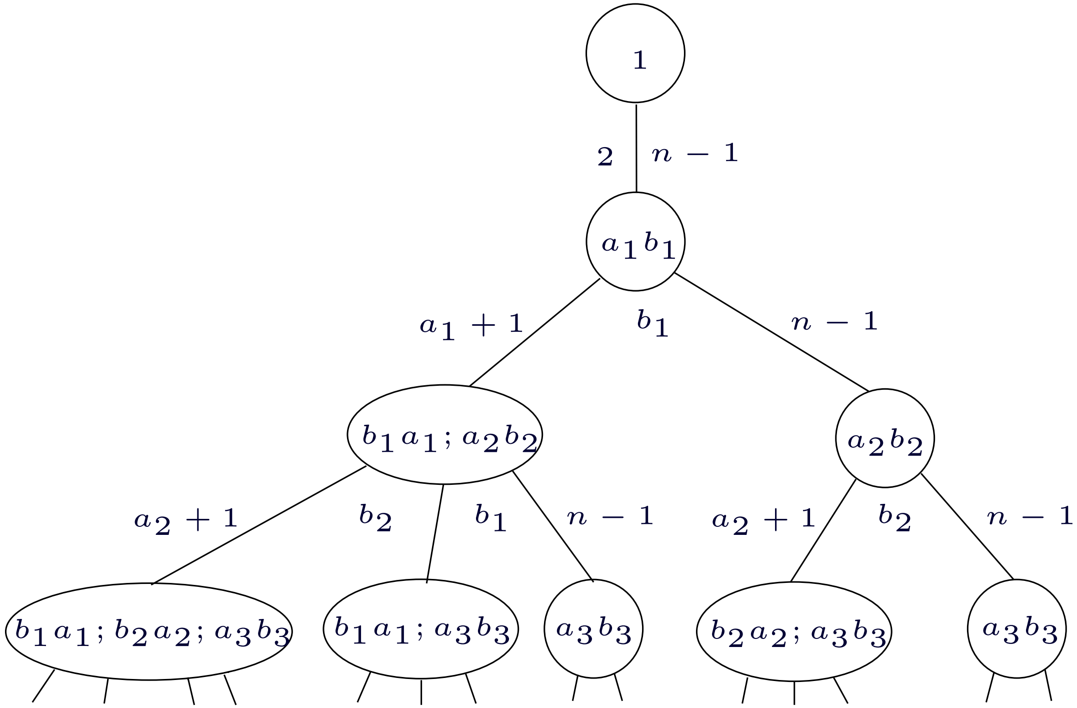
$$x_{st}^{\alpha\dot{\alpha}} = p_s^{\alpha\dot{\alpha}} + \ldots + p_{t-1}^{\alpha\dot{\alpha}}, \qquad \theta_{st}^{\alpha A} = \lambda_s^\alpha \eta_s^A + \ldots + \lambda_{t-1}^\alpha \eta_{t-1}^A.$$

All tree-level amplitudes

The supersymmetric BCFW recursion relation admits a closed-form solution :

$$\mathcal{A} = \mathcal{A}^{\mathrm{MHV}} \mathcal{P}$$

$$\mathcal{P} = \sum \text{vertical paths in this picture} = 1 + \sum_{a_1, b_1} R_{n;a_1 b_1} + \dots$$



Similar structure for $N = 8$ gravity [JMD,Spradlin,Volovich,Wen].

Superconformal symmetry

Since $\mathcal{N}=4$ SYM is a superconformal theory so we expect an action of the superconformal algebra on amplitudes [Witten].

$$p^{\dot\alpha\alpha} = \sum_i \tilde\lambda_i^{\dot\alpha}\lambda_i^{\alpha}, \qquad k_{\alpha\dot\alpha} = \sum_i \partial_{i\alpha}\partial_{i\dot\alpha},$$

$$\overline{m}_{\dot\alpha\dot\beta} = \sum_i \tilde\lambda_{i(\dot\alpha}\partial_{i\dot\beta)}, \qquad m_{\alpha\beta} = \sum_i \lambda_{i(\alpha}\partial_{i\beta)},$$

$$d = \sum_i [\tfrac{1}{2}\lambda_i^{\alpha}\partial_{i\alpha} + \tfrac{1}{2}\tilde\lambda_i^{\dot\alpha}\partial_{i\dot\alpha} + 1], \qquad r^A{}_B = \sum_i [-\eta_i^A\partial_{iB} + \tfrac{1}{4}\eta_i^C\partial_{iC}],$$

$$q^{\alpha A} = \sum_i \lambda_i^{\alpha}\eta_i^A, \qquad \bar q_A^{\dot\alpha} = \sum_i \tilde\lambda_i^{\dot\alpha}\partial_{iA},$$

$$s_{\alpha A} = \sum_i \partial_{i\alpha}\partial_{iA}, \qquad \bar s_{\dot\alpha}^A = \sum_i \eta_i^A\partial_{i\dot\alpha}.$$

$$c = \sum_i [1 + \tfrac{1}{2}\lambda_i^{\alpha}\partial_{i\alpha} - \tfrac{1}{2}\tilde\lambda^{\dot\alpha}\partial_{i\dot\alpha} - \tfrac{1}{2}\eta^A\partial_{iA}]$$

$$\partial_{i\alpha} = \frac{\partial}{\partial\lambda_i^{\alpha}}, \qquad \partial_{i\dot\alpha} = \frac{\partial}{\partial\tilde\lambda_i^{\dot\alpha}}, \qquad \partial_{iA} = \frac{\partial}{\partial\eta_i^A}.$$

Some operators are zeroth order, some first order and some second order.
In twistor space they all become first order.

Dual conformal symmetry

Dual coordinates $x_i^\mu - x_{i+1}^\mu = p_i^\mu$.

Dual conformal symmetry: $K^\mu = \sum_i \left[x_i^\mu x_i \cdot \partial_i - \frac{1}{2} x_i^2 \partial_i^\mu\right]$

It is the symmetry of a Wilson loop in dual space with cusps at the points x_i.
(WL/MHV amplitude duality - see talks by Gregory Korchemsky and Paul Heslop)

We need the action of the dual conformal generators on the spinors $\lambda, \tilde{\lambda}$.

The momenta satisfy two constraints:

$$\sum p_i^{\alpha\dot\alpha} = 0 \implies p_i^{\alpha\dot\alpha} = x_i^{\alpha\dot\alpha} - x_{i+1}^{\alpha\dot\alpha}$$
$$p_i^2 = 0 \implies p_i^{\alpha\dot\alpha} = \lambda_i^\alpha \tilde{\lambda}_i^{\dot\alpha}$$

Together these imply the constraints: $x_i - x_{i+1} - \lambda_i \tilde{\lambda}_i = 0$

Extend dual conformal generators so that they commute with the constraints up to constraints:

$$K_{\alpha\dot\alpha} = \sum_i [x_{i\alpha}{}^{\dot\beta} x_{i\dot\alpha}{}^{\beta} \partial_{i\beta\dot\beta} + x_{i\dot\alpha}{}^{\beta} \lambda_{i\alpha} \partial_{i\beta} + x_{i+1\,\alpha}{}^{\dot\beta} \tilde{\lambda}_{i\dot\alpha} \partial_{i\dot\beta}]$$

Dual superconformal symmetry

[JMD,Henn,Korchemsky,Sokatchev]

Momentum conservation $\delta^4(p)$ suggests the introduction of the dual x_i:

$$x_i^{\alpha\dot{\alpha}} - x_{i+1}^{\alpha\dot{\alpha}} - \lambda_i^{\alpha}\tilde{\lambda}_i^{\dot{\alpha}} = 0.$$

Supersymmetry $\delta^8(q)$ suggests the introduction of dual θ_i:

$$\theta_i^{\alpha A} - \theta_{i+1}^{\alpha A} - \lambda_i^{\alpha}\eta_i^{A} = 0.$$

Now we can extend dual conformal symmetry to dual superconformal symmetry by extending the standard chiral representation so that all generators commute with the constraints up to constraints.

All generators of dual superconformal symmetry

$$P_{\alpha\dot{\alpha}} = \sum_i \partial_{i\alpha\dot{\alpha}},$$

$$Q_{\alpha A} = \sum_i \partial_{i\alpha A},$$

$$\overline{Q}^A_{\dot{\alpha}} = \sum_i [\theta_i^{\alpha A}\partial_{i\alpha\dot{\alpha}} + \eta_i^A \partial_{i\dot{\alpha}}],$$

$$D = \sum_i [-x_i^{\dot{\alpha}\alpha}\partial_{i\alpha\dot{\alpha}} - \tfrac{1}{2}\theta_i^{\alpha A}\partial_{i\alpha A} - \tfrac{1}{2}\lambda_i^\alpha \partial_{i\alpha} - \tfrac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\partial_{i\dot{\alpha}}],$$

$$C = \sum_i [-\tfrac{1}{2}\lambda_i^\alpha \partial_{i\alpha} + \tfrac{1}{2}\tilde{\lambda}_i^{\dot{\alpha}}\partial_{i\dot{\alpha}} + \tfrac{1}{2}\eta_i^A \partial_{iA}] = \sum_i h_i,$$

$$S^A_\alpha = \sum_i [-\theta^B_{i\alpha}\theta_i^{\beta A}\partial_{i\beta B} + x_{i\alpha}{}^{\dot{\beta}}\theta_i^{\beta A}\partial_{\beta\dot{\beta}} + \lambda_{i\alpha}\theta_i^{\gamma A}\partial_{i\gamma} + x_{i+1\,\alpha}{}^{\dot{\beta}}\eta_i^A \partial_{i\dot{\beta}} - \theta^B_{i+1\,\alpha}\eta_i^A \partial_{iB}],$$

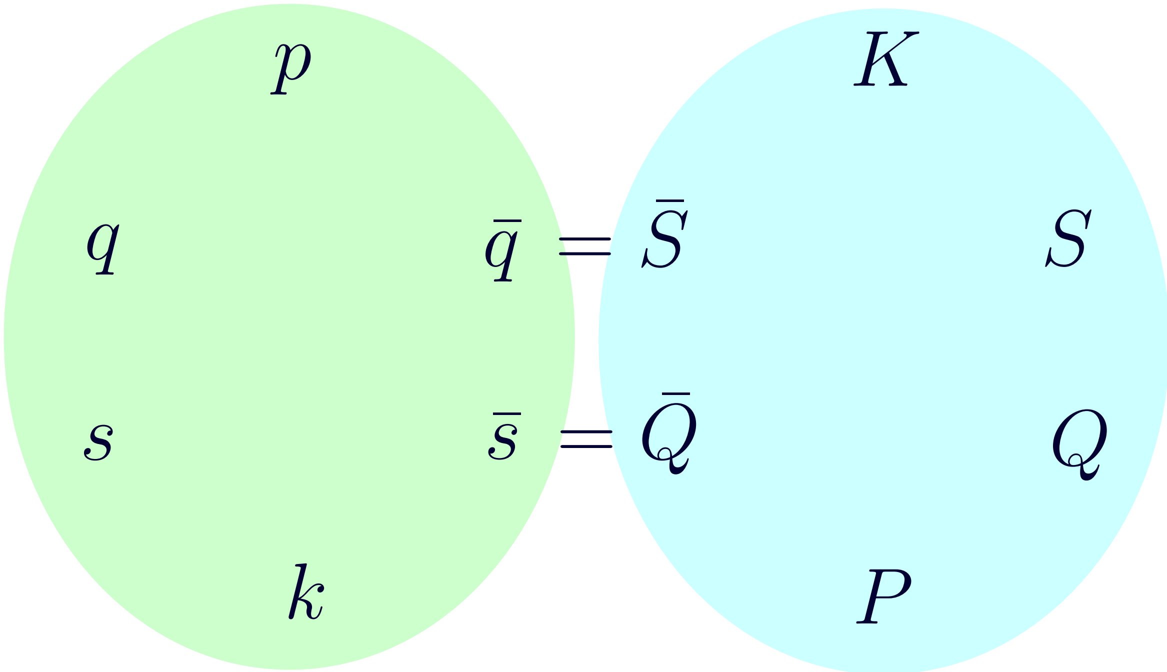
$$\overline{S}_{\dot{\alpha}A} = \sum_i [x_{i\dot{\alpha}}{}^{\beta}\partial_{i\beta A} + \tilde{\lambda}_{i\dot{\alpha}}\partial_{iA}],$$

$$K_{\alpha\dot{\alpha}} = \sum_i [x_{i\alpha}{}^{\dot{\beta}} x_{i\dot{\alpha}}{}^{\beta}\partial_{i\beta\dot{\beta}} + x_{i\dot{\alpha}}{}^{\beta}\theta^B_{i\alpha}\partial_{i\beta B} + x_{i\dot{\alpha}}{}^{\beta}\lambda_{i\alpha}\partial_{i\beta} + x_{i+1\,\alpha}{}^{\dot{\beta}}\tilde{\lambda}_{i\dot{\alpha}}\partial_{i\dot{\beta}} + \tilde{\lambda}_{i\dot{\alpha}}\theta^B_{i+1\,\alpha}\partial_{iB}].$$

$$\partial_{i\alpha\dot{\alpha}} = \frac{\partial}{\partial x_i^{\alpha\dot{\alpha}}}, \qquad \partial_{i\alpha A} = \frac{\partial}{\partial \theta_i^{\alpha A}}, \qquad \partial_{i\alpha} = \frac{\partial}{\partial \lambda_i^\alpha}, \qquad \partial_{i\dot{\alpha}} = \frac{\partial}{\partial \tilde{\lambda}_i^{\dot{\alpha}}}, \qquad \partial_{iA} = \frac{\partial}{\partial \eta_i^A}.$$

Conventional and dual superconformal symmetries

The generators of conventional and dual superconformal symmetry are not all independent:



Similar picture found by [Berkovits,Maldacena],[Beisert,Ricci,Tseytlin,Wolf] by combining bosonic T-duality with a fermionic one in the AdS sigma model.

How tree amplitudes behave under the symmetries

Invariance under the superconformal algebra.

$$J_a \mathcal{A}_n = 0$$

Covariance under the dual superconformal algebra:

$$\{P_{\alpha\dot\alpha}, Q_{\alpha A}, \bar{Q}^A_{\dot\alpha} = \bar{s}^A_{\dot\alpha}, \bar{S}_{\dot\alpha A} = \bar{q}_{\dot\alpha A}\}\mathcal{A}_n = 0$$

$$K^{\alpha\dot\alpha}\mathcal{A}_n = -\textstyle\sum_{i=1}^n x_i^{\alpha\dot\alpha}\mathcal{A}_n \,, \qquad\qquad S^A_\alpha \mathcal{A}_n = -\textstyle\sum_{i=1}^n \theta^A_{i\alpha}\mathcal{A}_n$$

[JMD,Henn,Korchemsky,Sokatchev],[Brandhuber,Heslop,Travaglini],[JMD,Henn].

Note: $\bar{s}\mathcal{A} = 0 \implies s\mathcal{A} = 0 \implies k\mathcal{A} = 0$

$$\mathcal{A}_{\rm tree} = \frac{\delta^4(p)\delta^8(q)}{\langle 12\rangle \ldots \langle n1\rangle}\mathcal{P} = \frac{\delta^4(p)\delta^8(q)}{\langle 12\rangle \ldots \langle n1\rangle}\big(1 + \sum_{s,t} R_{r;s,t} + \ldots\big)$$

Here $R_{r;s,t}$ is a dual superconformal invariant (best to use x, θ, λ variables):

$$R_{r;s,t} = \frac{\langle s\ s-1\rangle\langle t\ t-1\rangle\delta^4(\langle r|x_{rs}x_{st}|\theta_{tr}\rangle + \langle r|x_{rt}x_{ts}|\theta_{sr}\rangle)}{x_{st}^2\langle r|x_{rs}x_{st}|t\rangle\langle r|x_{rs}x_{st}|t-1\rangle\langle r|x_{rt}x_{ts}|s\rangle\langle r|x_{rt}x_{ts}|s-1\rangle}$$

Commuting the two algebras

What algebraic structure combines both superconformal and dual superconformal algebras? [JMD,Henn,Plefka]

We want to commute charges coming from both algebras.

First we must reformulate dual superconformal symmetry as an invariance.

Subtract the weight terms:

$$\tilde{K}^{\alpha\dot{\alpha}} = K^{\alpha\dot{\alpha}} + \textstyle\sum_{i=1}^{n} x_i^{\alpha\dot{\alpha}} \qquad \text{and} \qquad \tilde{S}_{\alpha}^{A} = S_{\alpha}^{A} + \textstyle\sum_{i=1}^{n} \theta_{i\alpha}^{A}$$

So that: $\tilde{K}\mathcal{A} = 0$ and $\tilde{S}\mathcal{A} = 0$.

We want to remove all x and θ dependence.

Use $P_{\alpha\dot{\alpha}}$ and $Q_{\alpha A}$ to set $x_1 = 0$ and $\theta_1 = 0$. Eliminate all other x_i and θ_i in favour of $\lambda_i, \tilde{\lambda}_i, \eta_i$.

$$S'^{A}_{\alpha} = -\sum_{i=1}^{n}\left[\sum_{j=1}^{i-1}\lambda_j^{\gamma}\eta_j^{A}\lambda_{i\alpha}\frac{\partial}{\partial\lambda_i^{\gamma}} + \sum_{j=1}^{i}\lambda_{j\alpha}\tilde{\lambda}_j^{\dot{\beta}}\eta_i^{A}\frac{\partial}{\partial\tilde{\lambda}_i^{\dot{\beta}}} - \sum_{j=1}^{i}\lambda_{j\alpha}\eta_j^{B}\eta_i^{A}\frac{\partial}{\partial\eta_i^{B}} + \sum_{j=1}^{i-1}\lambda_{j\alpha}\eta_j^{A}\right]$$

Now on the same footing as the ordinary superconformal generators.

Yangians

Consider a Lie (super)algebra g: $[J_a, J_b] = f_{ab}{}^c J_c$

Can introduce some 'level one' generators

$$[J_a, J_b^{(1)}] = f_{ab}{}^c J_c^{(1)}$$

The Jacobi identity can be 'quantised' (Drinfel'd):

$$[J_a^{(1)}, [J_b^{(1)}, J_c]] + \mathrm{cyc}(a,b,c) = h f_{ar}{}^l f_{bs}{}^m f_{ct}{}^n f^{rst} \{J_l, J_m, J_n\}$$

Then J and $J^{(1)}$ generate the Yangian $Y(g)$.

On a chain the generators J can be given by sums of single site generators

$$J_a = \sum_i J_{ia}$$

Then $J_a^{(1)}$ can take the bilocal form [Dolan,Nappi,Witten]

$$J_a^{(1)} = f_a{}^{cb} \sum_{i<j} J_{ib} J_{jc}$$

if the representation $\mathcal{R}$ of J_i satisfies the condition that the adjoint appears only once in $\mathcal{R} \otimes \bar{\mathcal{R}}$.

From dual conformal symmetry to the Yangian

We want to identify two bilocal Yangian generators $J_a^{(1)}$ with the symmetries K' and S'

Inspecting the dimensions and Lorentz and $su(4)$ labels suggests the identification

$$p^{(1)}_{\alpha\dot{\alpha}} \sim K'_{\alpha\dot{\alpha}}, \qquad\qquad q^{(1)}{}_{\alpha}^{A} \sim S'^{A}_{\alpha}$$

Indeed we can add terms to S' which annihilate the amplitudes on their own

$$\Delta S_\alpha^A = \tfrac{1}{2}\left[-q_\gamma^A m_\alpha^\gamma + q_\alpha^A \tfrac{1}{2} d_\lambda + n q_\alpha^A + p_\alpha^{\dot{\beta}} \bar{s}^A_{\dot{\beta}} + q_\alpha^B r_B^A - q_\alpha^A \tfrac{1}{4} d_\eta + q_\alpha^A\right]$$

and we arrive at the bilocal formula

$$q_\alpha^{(1)A} := \textstyle\sum_{i>j}\left[m_{i\alpha}^{\gamma} q_{j\gamma}^{A} - \tfrac{1}{2}(d_i + c_i) q_{j\alpha}^{A} + p_{i\alpha}^{\dot{\beta}} \bar{s}^{A}_{j\dot{\beta}} + q_{i\alpha}^{B} r_{jB}^{A} - (i \leftrightarrow j)\right].$$

The remaining generators in the level one multiplet come by acting with level zero generators.

The generator $p^{(1)}$ so obtained coincides with K' after similarly adding terms which annihilate the amplitude.

Cyclicity

The bilocal representation of Yangians is not normally consistent with the cyclicity of a closed chain.

Here this problem is avoided by a remarkable mechanism.

Consider

$$J_a^{(1)} = f_a{}^{cb} \sum_{1 \leq i < j \leq n} J_{ib} J_{jc}$$

$$\tilde{J}_a^{(1)} = f_a{}^{cb} \sum_{2 \leq i < j \leq n+1} J_{ib} J_{jc}$$

Then cyclicity implies $J_a^{(1)} - \tilde{J}_a^{(1)}$ should annihilate the amplitude.

One finds the following term which, in general, does not annihilate the amplitude

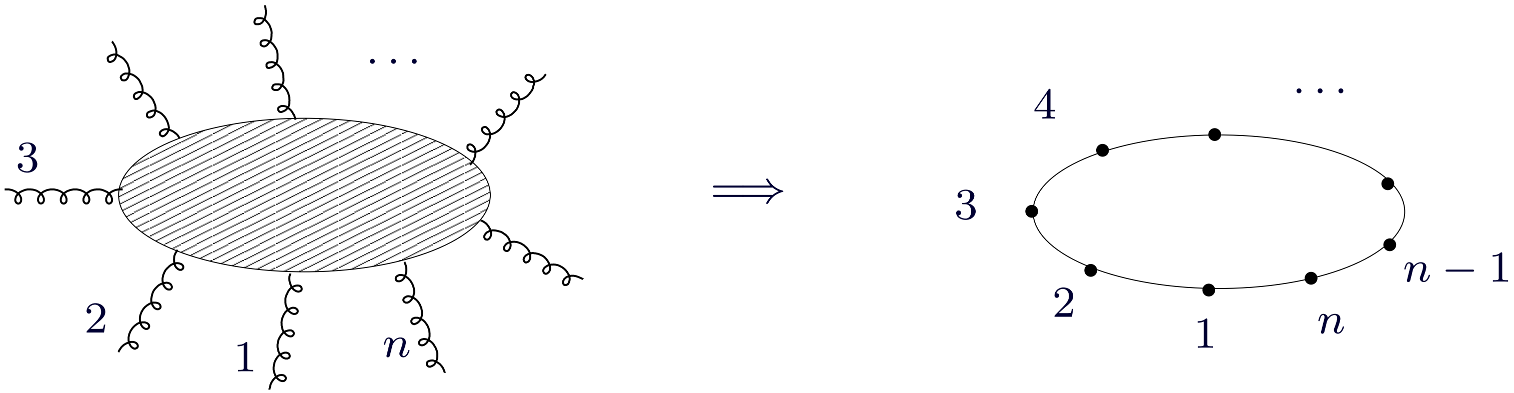
$$f_a{}^{cb} f_{bc}{}^{d} J_{1d}$$

But for certain superalgebras this vanishes identically (those with vanishing Killing form):

$$psl(n|n), osp(2n+2|2n), D(2,1;\alpha), P(n), Q(n)$$

Amplitudes and spin chains

As far as the algebraic representations are concerned, amplitudes are identical to local operators.



Fields in a single trace operator can be written as (e.g.) $\Phi_{AB} = c^{\dagger}_{A} c^{\dagger}_{B} |0\rangle, \quad \Psi_{\alpha A} = a^{\dagger}_{\alpha} c^{\dagger}_{A} |0\rangle$.

The free generators get deformed by coupling-dependent corrections [Beisert].

They involve operators which can increase or decrease the number of sites.

Yangian symmetry implies there are extra charges which commute with the spin chain Hamiltonian (anomalous dilatation generator) [Dolan,Nappi,Witten].

Suggests that all the extremely powerful techniques (Bethe Ansatz etc.) applied to the spectral problem should have a version for amplitudes.

Classical corrections

In fact tree-level amplitudes are invariant under the free representation only up to collinear contact contributions [Bargheer,Beisert,Galleas,Loebbert,McLoughlin],[Korchemsky,Sokatchev].

$$\bar{s}^{A}_{\dot{\alpha}} \mathcal{A}_n = \sum \delta(\langle i\ i+1 \rangle)\delta([i\ i+1])\chi^{A}_{\dot{\alpha}} \mathcal{A}_{n-1}$$

Possible to absorb the anomalous variation into the generator such that

$$\bar{s}^{\mathrm{cl}\,A}_{\ \ \dot{\alpha}} \mathcal{A} = (\bar{s}^{\mathrm{free}\,A}_{\ \ \ \ \dot{\alpha}} + \bar{s}'^{A}_{\ \dot{\alpha}})\mathcal{A} = 0.$$

Then Yangian generators have schematically the same form

$$J_a^{(1)\mathrm{cl}} = f_a{}^{cb} \sum_{1 \le i < j \le n} J^{\mathrm{cl}}_{ib} J^{\mathrm{cl}}_{jc}.$$

Then if tree-level amplitudes are fixed by collinear limits one can say equivalently they are fixed by the classical generators.

Should we expect something similar for loop corrections?

Summary and Outlook

All tree-level amplitudes are explicitly known

They are invariant under ordinary superconformal and covariant under dual superconformal symmetries

Ordinary superconformal + dual superconformal $\longrightarrow$ Yangian symmetry.

The central problem we face is how the ordinary superconformal symmetry ($s, \bar{s}, k$) is realised at loop level.

INFRARED DIVERGENCES!

Could the full planar amplitude be fixed by symmetry?

Is there a generalisation of the Wilson loop/ MHV amplitude duality to all amplitudes?

Recent formula of [Arkani-Hamed,Cachazo,Cheung,Kaplan] - integral over Grassmannian. Is it invariant under the Yangian symmetry?