QCD Scattering Amplitudes from Wilson Loops

Talk by Poul Olesen
NBI, August 12, 2009

Work done with
Yuri Makeenko, ITEP

PRL 102 (2009) 07602 and PRD 80 (2009) 026002[arXiv:0903.4114]

August 10, 2009

Overview

- *Goal:* We want to find the scattering amplitude when the Wilson loop is dominated by the non-perturbative minimal area behavior.

- *Procedure:*

- There exists a connection between the Wilson loop and the large N (or quenched) QCD scattering amplitudes.

- Lattice Gauge Theory gives this behavior of the quenched Wilson loop $W[C]$ quite precisely at large and not so large distances for various N's.

- The relation Amplitude/Wilson loop is expressed in terms of Feynman path integrals.

- The leading behavior of $W[C]$ obtained on the lattice is just $W \sim \exp(-\mathrm{Minimal\ Area})$, with Lüscher term correction.

- Since the amplitude is a path integral over W, to do a simple calculation we need the minimal area expressed as a path integral.

- Jesse Douglas (1929-1930) showed how to compute the minimal area from a variational principle involving *only* the boundary curve C.

- Inserting Douglas' path integral for $W[C]$ we find that the spectrum is of the Regge type $\mathrm{Mass}^2 \propto$ integer.

- If the $\#$ of external particles is *very large*
we find that the Veneziano amplitude in the Koba-Nielsen form
is valid in QCD for large energies/momentum transfers.
We need incoming energy $>>$ momentum transfers.

The Wilson Loop/Scattering Amplitude Relation

[Wilson 1974, Makeenko and Migdal 1981]

External momenta $p_i = q_{i-1} - q_i$, $i = 1, ..., M$,

$$G(x_1, ..., x_M) =< \bar{q}(x_1)q(x_1)...\bar{q}(x_M)q(x_M) >$$

In momentum space:

$$G(p_1, ..., p_M) \propto \int_0^\infty d\tau \tau^{M-1} e^{-m\tau} \prod_{i=1}^{M-1} \int_0^{\phi_{i+1}} d\phi_i \int Dk(\phi)\ \mathcal{F}$$

with $\mathcal{F}$ given by

$$\int_{Z(0)=Z(2\pi)=0} DZ(\phi)\mathrm{tr}\ \mathrm{P}e^{i\int_0^{2\pi} d\phi[(k(\phi)+p(\phi))\dot{Z}(\phi)-\tau\gamma(\phi)k(\phi)/2\pi]} W[Z(\phi)]$$

Explanation of symbols.

- $p(\phi) = q_i, \quad \phi_i < \phi < \phi_{i+1}$ and $\dot{p}(\phi) = -\sum p_i \delta(\phi - \phi_i)$.

- $k(\phi)$ is virtual momentum, related to the covariant derivative in QCD.

- The ϕ's are related to the Schwinger proper time variables,
  and $Z(\phi_i) = x_i$ is the Fourier conjugate to the external momentum p_i.

- $W[Z(\phi)]$ goes through the points x_i. These are integrated
  because the Schwinger angles ϕ_i are.

- No integration over $Z(0) = Z(2\pi)$.
  Would give an infinite volume factor because of translational invariance.

Douglas and the minimal area (Plateau's problem)

[Douglas 1930]. The minimal area bounded by a curve $Z(\phi)$ is given by the minimum of

$$-\frac{1}{4\pi}\int_0^{2\pi} d\theta \int_0^{2\pi} d\theta' \dot{Z}(\theta)\dot{Z}(\theta') \;\ln(1 - cos[\phi(\theta) - \phi(\theta')]).$$

- $\theta = \theta(\phi)$ is reparametrization of curve.
- The minimal area is obtained by minimizing w.r.t. $\theta(\phi)$ or, alternatively, wrt $\phi(\theta)$, demanding

$$\int_0^{2\pi} d\theta' \dot{Z}(\theta)\dot{Z}(\theta') \;\cot\frac{\phi_\star(\theta) - \phi_\star(\theta')}{2} = 0,$$

integral is principal value, $\phi_\star$ minimizes integral for given Z.

What we need in order to compute the QCD Amplitude:

Summary: Combine everything to get

$$G(p_1, ..., p_M) \propto \int_0^\infty d\tau \tau^{M-1} e^{-m\tau} \prod_{i=1}^{M-1} \int_0^{\phi_{i+1}} d\phi_i \int Dk(\phi)\ \mathcal{F}$$

with $\mathcal{F}$ given by

$$\int_{Z(0)=Z(2\pi)=0} DZ(\phi) \mathrm{tr}\ \mathrm{P} e^{i \int_0^{2\pi} d\phi [(k(\phi)+p(\phi))\dot{Z}(\phi) - \tau\gamma(\phi)k(\phi)/2\pi]} W[Z(\phi)]$$

$$W[Z(\phi)] = \mathcal{SP}_\phi \int D\phi(\theta) e^{K/2\pi \int_0^{2\pi} d\theta \int_0^{2\pi} d\theta' \dot{Z}(\theta)\dot{Z}(\theta')\ \ \ln(1-\cos[\phi(\theta)-\phi(\theta')])}.$$

$\mathcal{SP}$ means that only the saddle point is kept
so to the first order we only keep the minimal area behavior.

Postponement of $\mathcal{SP}$ from $Z-$ to $k-$integral $\Rightarrow$ Gaussian

- The $\mathcal{SP}$ operation is equivalent to the "classical" limit so it can be postponed from the Z to the k integration.

- This postponement also follows since the $\phi(\theta)$ dependence is only in the logarithm in the k integration.

The relevant integral over Z (all curves!) is thus $Gaussian$. Easily doable, left with k, ϕ_i, τ integrations. Result:

$$G(p_1, ..., p_M) = \prod_{1}^{M-1} \int_0^{\phi_{i+1}} d\phi_i e^{1/4\pi K \sum p_i p_j \ln(1-\cos(\phi_i - \phi_j))} \mathcal{K}.$$

$\mathcal{K}$ is function of ϕ_i, p_j. Poles come when logarithm is integrated, Mass^2 $\sim$integer. Valid for $K << t << s$.

Large # of external particles ⇒ the Veneziano model

If M is large, the τ integrand $\tau^{M-1}e^{-m\tau}$ is dominated by large $\tau = (M-1)/m$. The kernel $\mathcal{K}$ contains a factor, the exponent of which is

$$\frac{1}{4\pi K}\int_0^{2\pi} d\theta \int_0^{2\pi} d\theta' k(\dot{\theta})k(\dot{\theta}') \ln(1-\cos(\phi(\theta)-\phi(\theta')))$$
$$+\text{similar factor} - \frac{i\tau}{2\pi}\int_0^{2\pi} d\phi\ \gamma(\phi)k(\phi).$$

Scaling k in the last factor, $k \to \tau k$, the first terms are of the order $1/\tau^2$ and can be ignored. So the integral over k in $\mathcal{K}$ is not important to leading order.

The kernel then simplifies

$$\mathcal{K} \propto \int D\phi(\theta) = \prod_{1}^{M} \frac{\sin(\phi_{i+1}/2)\sin(\phi_i/2)}{\sin((\phi_{i+1}-\phi_i)/2)}.$$

The Veneziano model essentially follows ($s = \tan\phi/2$)

$$G(\text{many } p_i'\text{s}) \propto \int_{-\infty}^{s_{i+1}} \frac{ds_i}{1+s_i^2} \times \text{ a Koba} - \text{Nielsen integrand.}$$

The unsual factor $\frac{ds_i}{1+s_i^2}$ does not destroy Regge pole/Regge asymptotic behavior.

Remarks and conclusions

- Our approach valid only when the area behavior dominates the Wilson loop. Thus

  -no tachyon, it is a short distance phenomenon [Arvis 1983],
  lowest energy$= 1/(2\pi\alpha')\sqrt{R^2 - R_c^2}, R_c^2 = \pi^2\alpha'(d-2)/6$.

  -for the four point function our results are valid for
  $\frac{1}{K} < |t| << s$. Higher points obvious generalizations.

  -no requirement $D = 26$, the $D \neq 26$ anomaly is not relevant at large distances [Olesen 1985],
  $[L^{1i}, L^{1j}] = -iL^{ij} + (d-26)/R^2 \times$ stuff independent of R,
  L^{ij} =rotation generators.

  Thus: restriction to large distances makes the standard dual models more healthy.

- Strong exponential decrease for $|t| \sim s$ follows
  from the area behavior of W,
  like in the Veneziano case, but is not physically significant in our approach$-$
  for the $|t| \sim s$ case the perturbative behavior is much more important, decreases only like power.
  In the relation between the scattering amplitude and the Wilson loop we always need to insert the $dominant\, W$.

- Regge trajectory $\alpha' t + \alpha(0)$. In our case we need $\alpha'|t| >> 1$, so the intercept is not fixed here.

- In some kinematical domain our results are valid in all dimensions where W is area behaved.
  We can check with 2D: 't Hooft found for large masses Mass$^2 \sim$ integer, good agreement!

- From lattice gauge theory one finds that the potential has a subdominant Lüscher-term. We are now working on the inclusion of this.

- Hope that some of the results may survive approximately also for $N = 3$.
  If so at LHC one should see tracks of the Veneziano amplitude
  in the collider data, where huge # of particles are produced.
  Perhaps heavy ions would be a good place...