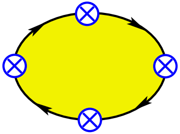
3. High-Energy Dynamics

- CCWZ Formalism
- Heavy Fields
- Low-Energy Constants
- Asymptotic Behaviour
- Signals of Heavy Scales

Energy Scale	Fields	Effective Theory
M_W	t, b, c $s, d, u\ ;\ G^a$	$\mathrm{QCD}^{N_f=6}$
$\lesssim m_c$	$s, d, u\ ;\ G^a$	$\mathrm{QCD}^{N_f=3}$
Λ_χ	V, A, S, P π, K, η	$\mathrm{R}\chi\mathrm{T}$
$\lesssim M_K$	π, K, η	$\chi\mathrm{PT}^{N_f=3}$
$\lesssim M_\pi$	π	$\chi\mathrm{PT}^{N_f=2}$

Large–N_C Counting Rules

$g_s \sim 1/\sqrt{N_C}$; $\alpha_s \sim 1/N_C$; $\langle T\,(J_1 \cdots J_n)\rangle \;\sim\; N_C$

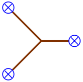
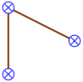


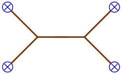
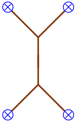
- Dominance of planar gluonic exchanges
- Non-planar diagrams suppressed by $1/N_C^2$
- Internal quark loops suppressed by $1/N_C$

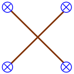
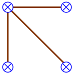
Colour Confinement $\Longrightarrow$ $J\,|0\rangle \;\sim\; |1 \text{ Meson}\rangle$

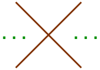
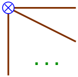
$$\boxed{\langle J(k)\, J(-k)\rangle \;=\; \sum_n \frac{f_n^2}{k^2 - M_n^2}}$$

- **Infinite** number of mesons $(\sim \ln k^2)$
- $f_n = \langle 0|J|n\rangle \sim \sqrt{N_C}$; $M_n \sim \mathcal{O}(1)$
- Mesons are **free**, **stable** and **non-interacting**

$\langle J\,J\,J \rangle \;=\; \Sigma$  $+\;\Sigma$ 

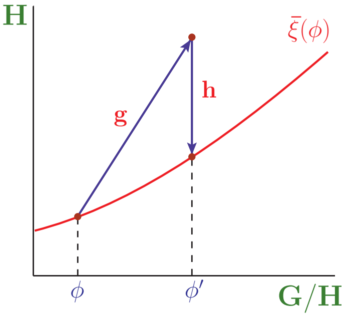
$\langle J\,J\,J\,J \rangle \;=\; \Sigma$  $+\;\Sigma$ 

$+\;\Sigma$  $+\;\Sigma$ 

 $\sim\; N_C^{1-\frac{n}{2}}$  $\sim\; N_C^{1-\frac{n}{2}}$

Crossing + Unitarity ➡ **Tree Approximation to some Local Effective Meson Lagrangian**

Coset Space Coordinates: $G \equiv SU(3)_L \otimes SU(3)_R \xrightarrow{SCSB} H \equiv SU(3)_V$



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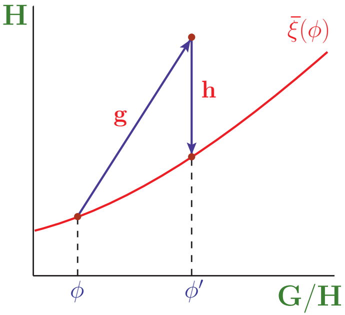

$$\bar{\xi}(\phi) \equiv (\xi_L(\phi), \xi_R(\phi)) \in G$$

$$\xi_L(\phi) \xrightarrow{G} g_L\, \xi_L(\phi)\, h^\dagger(\phi, g)$$

$$\xi_R(\phi) \xrightarrow{G} g_R\, \xi_R(\phi)\, h^\dagger(\phi, g)$$

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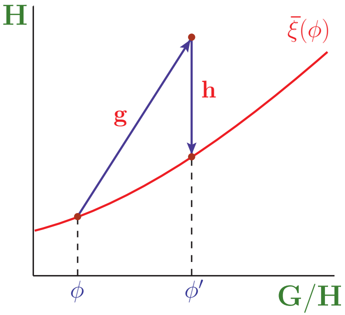

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$$\mathbf{U}(\phi) \;\equiv\; \xi_R(\phi)\, \xi_L^\dagger(\phi) \;\xrightarrow{G}\; g_R\, \mathbf{U}(\phi)\, g_L^\dagger$$

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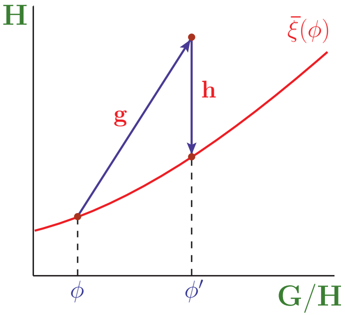

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$$\mathbf{U}(\phi) \equiv \xi_R(\phi)\, \xi_L^\dagger(\phi) \xrightarrow{G} g_R\, \mathbf{U}(\phi)\, g_L^\dagger$$

Canonical choice:

$$\boxed{\begin{array}{c} \xi_R(\phi) = \xi_L(\phi)^\dagger \equiv \mathbf{u}(\phi) \quad \xrightarrow{G} \quad g_R\, \mathbf{u}(\phi)\, h^\dagger(\phi, g) = h(\phi, g)\, \mathbf{u}(\phi)\, g_L^\dagger \\ \mathbf{U}(\phi) = \mathbf{u}(\phi)^2 = \exp\left\{ i\, \frac{\sqrt{2}}{f}\, \mathbf{\Phi} \right\} \end{array}}$$

CCWZ Formalism

Callan–Coleman–Wess–Zumino

$$\mathbf{u}(\varphi) \quad \xrightarrow{G} \quad g_R\, \mathbf{u}(\varphi)\, h^\dagger(\phi, g) \;=\; h(\phi, g)\, \mathbf{u}(\phi)\, g_L^\dagger$$

$SU(3)_V$ octets: $\qquad \mathbf{X} \quad \xrightarrow{\mathbf{G}} \quad \mathbf{h}(\phi, \mathbf{g})\; \mathbf{X}\; \mathbf{h}(\phi, \mathbf{g})^\dagger$

$$\boxed{\begin{array}{lcl} \mathbf{R} \equiv \frac{1}{2}\lambda^a R^a & , & \nabla_\mu R = \partial_\mu R + [\Gamma_\mu, R] \\ u_\mu \equiv i\, u^\dagger D_\mu U\, u^\dagger = u_\mu^\dagger & , & h^{\mu\nu} = \nabla^\mu u^\nu + \nabla^\nu u^\mu \\ f_\pm^{\mu\nu} = u\, F_L^{\mu\nu} u^\dagger \pm u^\dagger F_R^{\mu\nu}\, u & , & \chi_\pm \equiv u^\dagger \chi\, u^\dagger \pm u\, \chi^\dagger u \end{array}}$$

$$\Gamma_\mu = \frac{1}{2}\left\{ u^\dagger(\partial_\mu - i\, r_\mu)\, u + u\,(\partial_\mu - i\,\ell_\mu) u^\dagger \right\}$$

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$$\Gamma_\mu = \frac{1}{2}\left\{ u^\dagger(\partial_\mu - i\, r_\mu)\, u + u\,(\partial_\mu - i\,\ell_\mu) u^\dagger \right\}$$

$$\boxed{\mathcal{L}_2 = \frac{f^2}{4}\, \langle u^\mu u_\mu + \chi_+ \rangle}$$

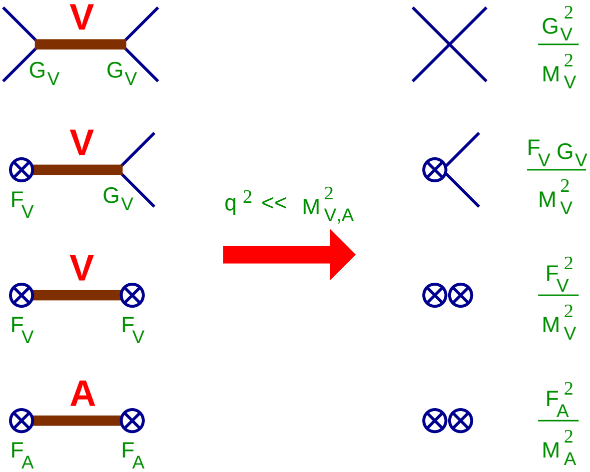
RχT

Resonance Nonet Multiplets: **V**(1^{--}) , **A**(1^{++}) , **S**(0^{++}) , **P**(0^{-+})

$$\boxed{\begin{aligned}
\mathcal{L}_2^V &= \frac{F_V}{2\sqrt{2}}\,\langle V_{\mu\nu}\, f_+^{\mu\nu}\rangle \;+\; \frac{i\,G_V}{\sqrt{2}}\,\langle V_{\mu\nu}\, u^\mu\, u^\nu\rangle \\
\mathcal{L}_2^A &= \frac{F_A}{2\sqrt{2}}\,\langle A_{\mu\nu}\, f_-^{\mu\nu}\rangle \\
\mathcal{L}_2^S &= c_d\,\langle S\, u^\mu\, u^\nu\rangle \;+\; c_m\,\langle S\,\chi_+\rangle \\
\mathcal{L}_2^P &= i\, d_m\,\langle P\,\chi_-\rangle
\end{aligned}}$$

$$u_\mu \;=\; i\, u^\dagger\, D_\mu U\, u^\dagger \;=\; u_\mu^\dagger \qquad ; \qquad U \;=\; u^2$$

$$f_\pm^{\mu\nu} \;=\; u\, F_L^{\mu\nu}\, u^\dagger \pm u^\dagger\, F_R^{\mu\nu}\, u \qquad ; \qquad \chi_\pm \;=\; u^\dagger\,\chi\, u^\dagger \pm u\,\chi^\dagger\, u$$



$$\left\{ \mathbf{V}\,[M_V, G_V, F_V]\ ,\ \mathbf{A}\,[M_A, F_A] \right\} \longleftrightarrow \left\{ \mathbf{S}\,[M_S, c_d, c_m]\ ,\ \mathbf{P}\,[M_P, d_m] \right\}$$

$O(N_C)$:

$$2\,L_1 \;=\; L_2 \;=\; \sum_i \frac{G_{V_i}^2}{4\,M_{V_i}^2} \qquad ; \qquad L_3 \;=\; \sum_i \left\{ -\frac{3\,G_{V_i}^2}{4\,M_{V_i}^2} + \frac{c_{d_i}^2}{2\,M_{S_i}^2} \right\}$$

$$L_5 \;=\; \sum_i \frac{c_{d_i}\,c_{m_i}}{M_{S_i}^2} \qquad ; \qquad L_8 \;=\; \sum_i \left\{ \frac{c_{m_i}^2}{2\,M_{S_i}^2} - \frac{d_{m_i}^2}{2\,M_{P_i}^2} \right\}$$

$$L_9 \;=\; \sum_i \frac{F_{V_i}\,G_{V_i}}{2\,M_{V_i}^2} \qquad ; \qquad L_{10} \;=\; \frac{1}{4} \sum_i \left\{ \frac{F_{A_i}^2}{M_{A_i}^2} - \frac{F_{V_i}^2}{M_{V_i}^2} \right\}$$

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$\mathbf{O(1)}$:

$$2\,L_1 - L_2 = L_4 = L_6 = 0 \qquad ; \qquad L_7 = -\frac{\tilde{d}_m^{\,2}}{2\,M_{\eta_1}^2}$$

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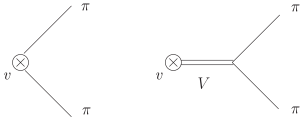
BUT $\qquad M_{\eta_1}^2 \;\sim\; O\left(\frac{1}{N_C}, \mathcal{M}\right)$

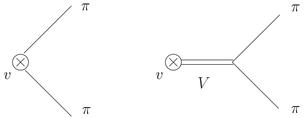
Vector Form Factor



$\langle\pi|\mathbf{v}_\mu|\pi\rangle$ **:**

$$F_V(t) \;=\; 1 \;+\; \sum_i \frac{F_{V_i}\, G_{V_i}}{f^2}\; \frac{t}{M_{V_i}^2 - t}$$

Vector Form Factor



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Short-distance QCD constraint: $\lim_{t\to\infty} F_V(t) = 0$

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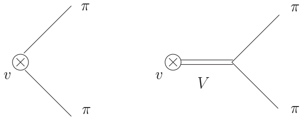

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$$\longrightarrow \quad \boxed{\sum_i F_{V_i}\, G_{V_i} \;=\; f^2}$$

Weinberg Sum Rules

Chiral Symmetry:

$$\Pi_{LR}^{\mu\nu}(q) \equiv \int d^4x \, \mathrm{e}^{iqx} \, \langle 0| T(J_L^\mu(x) \, J_R^\nu(0)^\dagger)|0\rangle = (-g^{\mu\nu} q^2 + q^\mu q^\nu) \, \Pi_{LR}(q^2) = 0$$

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OPE: $\Pi_{LR}^{\mu\nu}(q) \neq 0$ only through **order parameters of EWSB**
(operators invariant under **H** but not under **G**)

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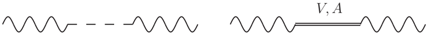
$$\to \quad \boxed{\begin{array}{ll} \frac{1}{\pi}\int_0^\infty \mathrm{d}s\, \left[\mathrm{Im}\Pi_{VV}(s) - \mathrm{Im}\Pi_{AA}(s)\right] = f^2 & (1^{\mathrm{st}}\ \mathrm{WSR}) \\ \frac{1}{\pi}\int_0^\infty \mathrm{d}s\, s\, \left[\mathrm{Im}\Pi_{VV}(s) - \mathrm{Im}\Pi_{AA}(s)\right] = 0 & (2^{\mathrm{nd}}\ \mathrm{WSR}) \end{array}}$$

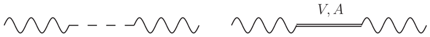
WSRs @ LO



$$\Pi_{LR}(s) \;=\; \frac{f^2}{s} + \sum_i \frac{F_{V_i}^2}{M_{V_i}^2 - s} - \sum_i \frac{F_{A_i}^2}{M_{A_i}^2 - s}$$

- **1st WSR:** $\sum_i F_{V_i}^2 - \sum_i F_{A_i}^2 \;=\; f^2$
- **2nd WSR:** $\sum_i F_{V_i}^2\, M_{V_i}^2 - \sum_i F_{A_i}^2\, M_{A_i}^2 \;=\; 0$

WSRs @ LO



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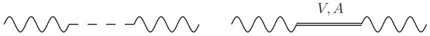
SRA $\longrightarrow$ $F_V^2 = v^2 \dfrac{M_A^2}{M_A^2 - M_V^2} \quad , \quad F_A^2 = v^2 \dfrac{M_V^2}{M_A^2 - M_V^2} \quad , \quad M_A > M_V$

WSRs @ LO



$$\Pi_{LR}(s) \; = \; \frac{f^2}{s} + \sum_i \frac{F_{V_i}^2}{M_{V_i}^2 - s} - \sum_i \frac{F_{A_i}^2}{M_{A_i}^2 - s}$$

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SRA ➡ $F_V^2 \; = \; v^2 \frac{M_A^2}{M_A^2 - M_V^2} \quad , \quad F_A^2 \; = \; v^2 \frac{M_V^2}{M_A^2 - M_V^2} \quad , \quad M_A > M_V$

1st WSR likely valid also in gauge theories with non-trivial UV fixed points

2nd WSR questionable (not valid) in walking (conformal) TC scenarios

Appelquist–Sannino, Orgogozo–Rychkov

Short-Distance Constraints

Vector Form Factor $\langle \pi | v_\mu | \pi \rangle$**:**

$$F_V(t) = 1 + \sum_i \frac{F_{V_i}\, G_{V_i}}{f^2}\, \frac{t}{M_{V_i}^2 - t}$$

$$\lim_{t\to\infty} F_V(t) = 0 \quad \longrightarrow \quad \boxed{\sum_i F_{V_i}\, G_{V_i} = f^2}$$

Axial Form Factor $\langle \gamma | a_\mu | \pi \rangle$**:**

$$G_A(t) = \sum_i \left\{ \frac{2\, F_{V_i}\, G_{V_i} - F_{V_i}^2}{M_{V_i}^2} + \frac{F_{A_i}^2}{M_{A_i}^2 - t} \right\}$$

$$\lim_{t\to\infty} G_A(t) = 0 \quad \longrightarrow \quad \boxed{\sum_i \left(2\, F_{V_i}\, G_{V_i} - F_{V_i}^2\right) / M_{V_i}^2 = 0}$$

Weinberg Sum Rules:

$$\Pi_{LR}(t) = -\frac{f^2}{t} + \sum_i \frac{F_{V_i}^2}{M_{V_i}^2 + t} - \sum_i \frac{F_{A_i}^2}{M_{A_i}^2 + t}$$

$$\lim_{t\to\infty} t\; \Pi_{LR}(t) = 0$$

$$\lim_{t\to\infty} t^2\, \Pi_{LR}(t) = 0$$

$$\longrightarrow \quad \boxed{\begin{array}{c} \sum_i \left(F_{V_i}^2 - F_{A_i}^2\right) = f^2 \\ \sum_i \left(M_{V_i}^2\, F_{V_i}^2 - M_{A_i}^2\, F_{A_i}^2\right) = 0 \end{array}}$$

Scalar FF: $$F^S_{K\pi}(s) = 1 + \sum_i \frac{4 c_{m_i}}{f^2} \left[c_{d_i} + (c_{m_i} - c_{d_i}) \frac{M_K^2 + M_\pi^2}{M_{S_i}^2} \right] \frac{s}{M_{S_i}^2 - s}$$

$$\lim_{s\to\infty} F^S_{K\pi}(s) = 0 \quad \longrightarrow \quad \boxed{4 \sum_i c_{d_i} c_{m_i} = f^2 \quad ; \quad \sum_i \frac{c_{m_i}}{M_{S_i}^2} (c_{m_i} - c_{d_i}) = 0}$$

S − P Sum Rules: $$\Pi_{SS-PP}(t) = 16 B_0^2 \left\{ \sum_i \frac{c_{m_i}^2}{M_{S_i}^2 + t} - \sum_i \frac{d_{m_i}^2}{M_{P_i}^2 + t} - \frac{f^2}{8\, t} \right\}$$

$$\lim_{t\to\infty} t \;\Pi_{SS-PP}(t) = 0 \quad \longrightarrow \quad \boxed{8 \sum_i (c_{m_i}^2 - d_{m_i}^2) = f^2}$$

Pseudoscalar Nonet:

$$\mathcal{L}_2 \doteq \frac{f^2}{4} \langle \chi\, U^\dagger + U\, \chi^\dagger \rangle \sim -i \frac{f}{\sqrt{24}} \eta_1 \langle \chi_- \rangle \quad \longrightarrow \quad \boxed{\tilde{d}_m = -\frac{f}{\sqrt{24}}}$$

1–Resonance Approximation:

Ecker, Gasser, Leutwyler, Pich, de Rafael

$$F_V = 2\, G_V = \sqrt{2}\, F_A = \sqrt{2}\, f \quad ; \quad M_A = \sqrt{2}\, M_V \quad ; \quad d_m = \frac{1}{2\sqrt{2}}\, f$$

$$c_m = c_d = \frac{1}{2}\, f$$

Jamin, Oller, Pich

$$M_P \approx \sqrt{2}\, M_S$$

$\Rightarrow$

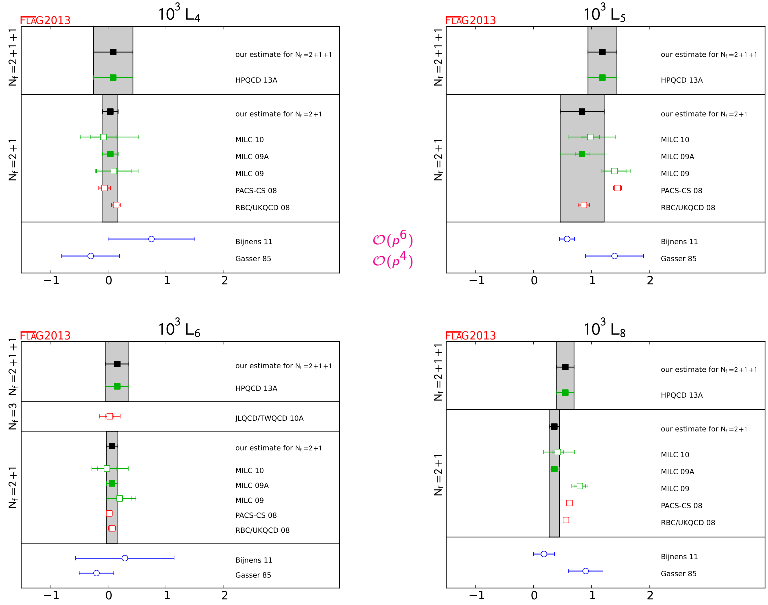
$$\boxed{\begin{array}{c} 2\, \mathbf{L}_1 = \mathbf{L}_2 = \frac{1}{4}\, \mathbf{L}_9 = -\frac{1}{3}\, \mathbf{L}_{10} = \frac{\mathbf{f}^2}{8\, \mathbf{M}_V^2} \\[2ex] \mathbf{L}_3 = -\frac{3\, \mathbf{f}^2}{8\, \mathbf{M}_V^2} + \frac{\mathbf{f}^2}{8\, \mathbf{M}_S^2} \quad ; \quad \mathbf{L}_5 = \frac{\mathbf{f}^2}{4\, \mathbf{M}_S^2} \\[2ex] \mathbf{L}_8 = \frac{\mathbf{f}^2}{8\, \mathbf{M}_S^2} - \frac{\mathbf{f}^2}{16\, \mathbf{M}_P^2} \quad ; \quad \mathbf{L}_7 = -\frac{\mathbf{f}^2}{48\, \mathbf{M}_{\eta_1}^2} \end{array}}$$

L_i'S from Resonance Exchange

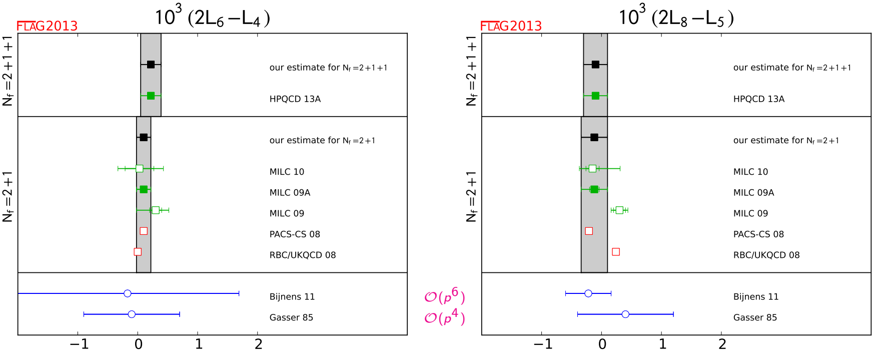
i	$10^3 \cdot L_i^r(M_\rho)$	V	A	S	η_1	Total	Total[b)]
1	0.7 ± 0.3	0.6	0	0	0	0.6	0.9
2	1.3 ± 0.3	1.2	0	0	0	1.2	1.8
3	-3.5 ± 1.1	-3.6	0	0.6	0	-3.0	-4.3
4	-0.3 ± 0.5	0	0	0	0	0.0	0.0
5	1.4 ± 0.5	0	0	1.4[a)]	0	1.4	2.1
6	-0.2 ± 0.3	0	0	0	0	0.0	0.0
7	-0.4 ± 0.2	0	0	0	-0.3	-0.3	-0.3
8	0.9 ± 0.3	0	0	0.9[a)]	0	0.9	0.8
9	6.9 ± 0.7	6.9[a)]	0	0	0	6.9	7.2
10	-5.5 ± 0.7	-10.0	4.0	0	0	-6.0	-5.4

a) Input b) Short-Distance Constraints

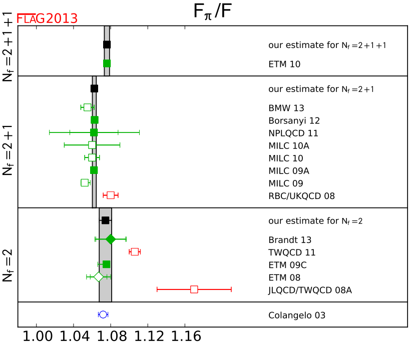
Lattice Determination of SU(3) LECs

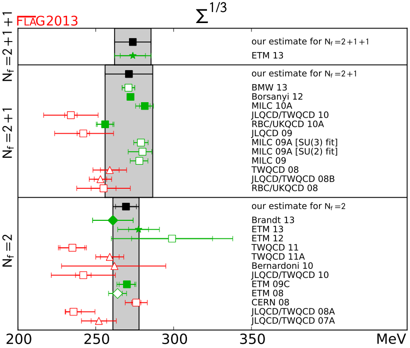


Lattice Determination of SU(3) LECs



Lattice Determination of SU(2) LECs





$$F = F_\pi\big|_{m_u=m_d=0} \qquad , \qquad \Sigma = -\langle 0|\bar{u}u|0\rangle\big|_{m_u=m_d=0}$$

Lattice Determination of SU(2) LECs