4. Electroweak Effective Theory

- **Higgs Mechanism**
- **Custodial Symmetry**
- **Equivalence Theorem**
- **Goldstone Electroweak Effective Theory**
- **New Physics Scales**

Energy Scale	Fields	Effective Theory
$\Lambda_{NP} \sim$ TeV	S_n, P_n, V_n, A_n, F_n H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Underlying Dynamics
······ Energy Gap ······	↓	
M_W	H, W, Z, γ, g τ, μ, e, ν_i t, b, c, s, d, u	Standard Model

Effective Field Theory

$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(4)} + \sum_{D>4} \sum_i \frac{c_i^{(D)}}{\Lambda^{D-4}}\, \mathcal{O}_i^{(D)}$$

- **Most general Lagrangian with the SM gauge symmetries**
- **Light ($m \ll \Lambda_{NP}$) fields only**
- **The SM Lagrangian corresponds to $D = 4$**
- **$c_i^{(D)}$ contain information on the underlying dynamics:**

$$\mathcal{L}_{\rm NP} \doteq g_X\, (\bar q_L \gamma^\mu q_L)\, X_\mu \qquad \longrightarrow \qquad \frac{g_X^2}{M_X^2}\, (\bar q_L \gamma^\mu q_L)\, (\bar q_L \gamma_\mu q_L)$$

- **Options for H(126):**
  - **$SU(2)_L$ doublet (SM)**
  - **Scalar singlet**
  - **Additional light scalars**

Higgs Mechanism:

Gauge invariance

Massless $W^\pm$, Z (spin 1)

3×2 polarizations $= 6$

Higgs Mechanism: 3 additional degrees of freedom $\theta_i(x)$

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$+$

3 Goldstones $\theta_i(x)$

SSB ↓

Massive $W^\pm$, Z

3×3 polarizations $=$ 9

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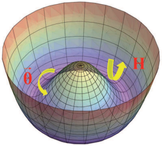
Spontaneous Symmetry Breaking

$$\mathcal{L}_\Phi = (D_\mu \Phi)^\dagger D^\mu \Phi - \mu^2 \Phi^\dagger \Phi - \lambda \left(\Phi^\dagger \Phi\right)^2$$



$\mu^2 < 0$

$$\Phi(x) = \exp\left\{ i \, \frac{\vec{\sigma}}{2} \cdot \vec{\theta}(x) \right\} \frac{1}{\sqrt{2}} \begin{bmatrix} 0 \\ v + H(x) \end{bmatrix}$$

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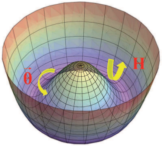

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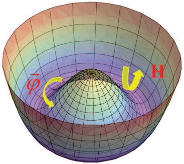
$D_\mu\Phi = (\partial_\mu + \frac{i}{2}\, g\, \vec{\sigma}\cdot\vec{W}_\mu + \frac{i}{2}\, g'\, B_\mu)\,\Phi$; $v^2 = -\mu^2/\lambda$

$$(D_\mu\Phi)^\dagger D^\mu\Phi \;\to\; M_W^2\, W_\mu^\dagger W^\mu + \frac{M_Z^2}{2}\, Z_\mu Z^\mu$$

$$\mathbf{M_W = M_Z \cos\theta_W = \frac{1}{2}\, g\, v}$$



$$\mathcal{L}_\Phi \;=\; (D_\mu\Phi)^\dagger D^\mu\Phi - \lambda\left(|\Phi|^2 - \frac{v^2}{2}\right)^2$$

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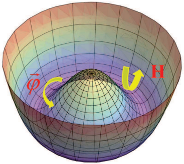

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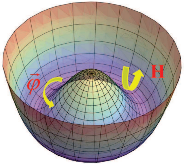

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$SU(2)_L \otimes SU(2)_R \to SU(2)_{L+R}$ **Symmetry:** $\Sigma \to g_L\,\Sigma\,g_R^\dagger$

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$$\Sigma \equiv (\Phi^c, \Phi) = \begin{pmatrix} \Phi^{0*} & \Phi^+ \\ -\Phi^- & \Phi^0 \end{pmatrix} \equiv \frac{1}{\sqrt{2}} (v + H)\, U(\vec{\theta}\,)$$

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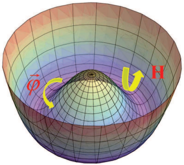

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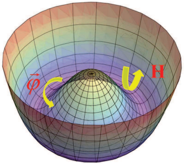

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Same Goldstone Lagrangian as QCD pions:

$$f_\pi \to v \qquad , \qquad \vec{\pi} \to \vec{\varphi} \to W_L^\pm, Z_L$$

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Derivative Coupling

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Derivative Coupling

Goldstones become free at zero momenta

Electroweak Symmetry Breaking

$$\mathcal{L}_2 = \frac{v^2}{4}\,\mathrm{Tr}\left(D_\mu U^\dagger D^\mu U\right) \quad \xrightarrow{U=1} \quad \mathcal{L}_2 = M_W^2\, W_\mu^\dagger W^\mu + \frac{1}{2} M_Z^2\, Z_\mu Z^\mu$$

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(explicit symmetry breaking)

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- **EW Goldstones are responsible for $\mathbf{M_{W,Z}}$** **(not the Higgs!)**
- **QCD pions also generate small $\mathbf{W, Z}$ masses:** $\delta_\pi \mathbf{M_W} = \dfrac{1}{2}\,\mathbf{g\, f_\pi}$

Goldstone interactions are determined by the underlying symmetry

$$
\begin{aligned}
\frac{v^2}{4}\,\langle\partial_\mu U^\dagger \partial^\mu U\rangle \;&=\; \partial_\mu\varphi^-\partial^\mu\varphi^+ + \frac{1}{2}\,\partial_\mu\varphi^0\partial^\mu\varphi^0 \\
&+\; \frac{1}{6v^2}\left\{\left(\varphi^+\overleftrightarrow{\partial}_\mu\,\varphi^-\right)\left(\varphi^+\overleftrightarrow{\partial}^\mu\varphi^-\right) + 2\left(\varphi^0\overleftrightarrow{\partial}_\mu\,\varphi^+\right)\left(\varphi^-\overleftrightarrow{\partial}^\mu\varphi^0\right)\right\} \\
&+\; O\left(\varphi^6/v^4\right)
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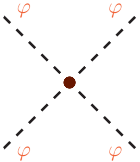

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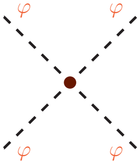

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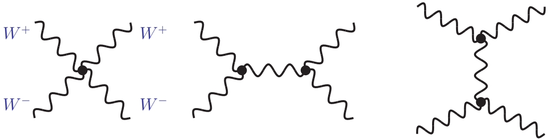
Non-Linear Lagrangian: $2\varphi\to 2\varphi,\ 4\varphi\ \cdots$ related

Equivalence Theorem



Cornwall–Levin–Tiktopoulos

Vayonakis

Lee–Quigg–Thacker

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right)$$
$$= T(\varphi^+\varphi^- \to \varphi^+\varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)$$

The scattering amplitude grows with energy

Goldstone dynamics ⟷ **derivative interactions**

Tree-level violation of unitarity

Longitudinal Polarizations

$$k^\mu = \left(k^0, 0, 0, |\vec{k}|\right) \quad \longrightarrow \quad \epsilon_L^\mu(\vec{k}) = \frac{1}{M_W}\,\left(|\vec{k}|, 0, 0, k^0\right) \;=\; \frac{k^\mu}{M_W} + O\left(\frac{M_W}{|\vec{k}|}\right)$$

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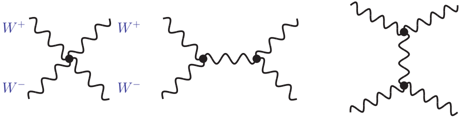
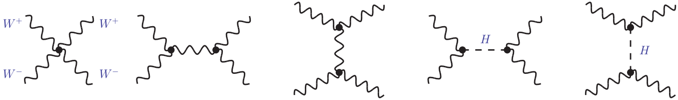


Gauge Cancelation

$$T(W_L^+ W_L^- \to W_L^+ W_L^-) = \frac{s+t}{v^2} + O\left(\frac{M_W}{\sqrt{s}}\right)$$

$$= T(\varphi^+ \varphi^- \to \varphi^+ \varphi^-) + O\left(\frac{M_W}{\sqrt{s}}\right)$$

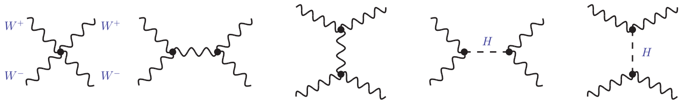
$W_L^+ W_L^- \to W_L^+ W_L^-$:



$$T_{\rm SM} = \frac{1}{v^2}\left\{ s + t - \frac{s^2}{s-M_H^2} - \frac{t^2}{t-M_H^2}\right\} = -\frac{M_H^2}{v^2}\left\{\frac{s}{s-M_H^2} + \frac{t}{t-M_H^2}\right\}$$

Higgs-exchange exactly cancels the $O(s,t)$ terms in the SM

$W_L^+ W_L^- \to W_L^+ W_L^-$:



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When $s \gg M_H^2$, $\qquad T_{\rm SM} \approx -\frac{2M_H^2}{v^2} \quad , \qquad a_0 \equiv \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta \; T_{\rm SM} \approx -\frac{M_H^2}{8\pi v^2}$

Unitarity:

Lee–Quigg–Thacker

$$|a_0| \le 1 \quad \longrightarrow \quad \mathbf{M_H} < \sqrt{8\pi}\,\mathbf{v}\; \underbrace{\sqrt{2/3}}_{W^+W^-,\, ZZ,\, HH} \approx \mathbf{1\ TeV}$$

What happens in QCD?

- **QCD satisfies unitarity** (it is a renormalizable theory)
- **Pion scattering unitarized by exchanges of resonances** (composite objects):
  - P-wave $(J = 1)$ unitarized by ρ exchange
  - S-wave $(J = 0)$ unitarized by σ exchange
- **The σ meson is the QCD equivalent of the SM Higgs**
- **BUT**, the σ is an ‘effective’ object generated through π rescattering (summation of pion loops)

Does not seem to work this way in the EW case, but . . .

Higher-Order Goldstone Interactions

$$\mathcal{L}_{\mathrm{EW}}^{(4)}\Big|_{\mathrm{CP-even}} = \sum_{i=0}^{14} a_i\,\mathcal{O}_i$$

(Appelquist, Longhitano)

$$\mathcal{O}_0 = v^2\,\langle T_L V_\mu\rangle^2$$

$\mathcal{O}_1 = \langle U\,\hat{B}_{\mu\nu}\,U^\dagger\,\hat{W}^{\mu\nu}\rangle$	$\mathcal{O}_2 = i\,\langle U\,\hat{B}_{\mu\nu}\,U^\dagger\,[V^\mu, V^\nu]\rangle$
$\mathcal{O}_3 = i\,\langle \hat{W}_{\mu\nu}\,[V^\mu, V^\nu]\rangle$	$\mathcal{O}_4 = \langle V_\mu V_\nu\rangle\,\langle V^\mu V^\nu\rangle$
$\mathcal{O}_5 = \langle V_\mu V^\mu\rangle^2$	$\mathcal{O}_6 = 4\,\langle V_\mu V_\nu\rangle\,\langle T_L V^\mu\rangle\,\langle T_L V^\nu\rangle$
$\mathcal{O}_7 = 4\,\langle V_\mu V^\mu\rangle\,\langle T_L V_\nu\rangle^2$	$\mathcal{O}_8 = \langle T_L\,\hat{W}_{\mu\nu}\rangle^2$
$\mathcal{O}_9 = -2\,\langle T_L\,\hat{W}_{\mu\nu}\rangle\,\langle T_L\,[V^\mu, V^\nu]\rangle$	$\mathcal{O}_{10} = 16\,\{\langle T_L V_\mu\rangle\,\langle T_L V_\nu\rangle\}^2$
$\mathcal{O}_{11} = \langle (D_\mu V^\mu)^2\rangle$	$\mathcal{O}_{12} = 4\,\langle T_L D_\mu D_\nu V^\nu\rangle\,\langle T_L V^\mu\rangle$
$\mathcal{O}_{13} = 2\,\langle T_L D_\mu V_\nu\rangle^2$	$\mathcal{O}_{14} = -2i\,\varepsilon^{\mu\nu\rho\sigma}\,\langle \hat{W}_{\mu\nu} V_\rho\rangle\,\langle T_L V_\sigma\rangle$

$$V_\mu \equiv D_\mu U\,U^\dagger \quad , \quad D_\mu V_\nu \equiv \partial_\mu V_\nu - i\,[\hat{W}_\mu, V_\nu] \quad , \quad (V_\mu, D_\mu V_\nu, T_L) \to g_L\,(V_\mu, D_\mu V_\nu, T_L)\,g_L^\dagger$$

Symmetry breaking: $T_L \equiv U\,\frac{\sigma_3}{2}\,U^\dagger \quad , \quad \hat{B}_{\mu\nu} \equiv -g'\,\frac{\sigma_3}{2}\,B_{\mu\nu}$

EW Resonance Effective Theory

- **Towers of heavy states are usually present in strongly-coupled models of EWSB:** **Technicolour, Walking TC. . .**

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This program works in QCD: RχT (Ecker–Gasser–Leutwyler–Pich–de Rafael)

Good dynamical understanding at large N_C

LO Resonance EW Lagrangian:

Pich–Rosell–Sanz-Cillero

$$\mathcal{L}_{\rm eff} \;=\; \mathcal{L}_\varphi \;+\; \sum_R \mathcal{L}_R \;+\; \sum_{R,R'} \mathcal{L}_{RR'} \;+\; \cdots$$

Triplets: $\mathbf{V}(1^{--})$, $\mathbf{A}(1^{++})$; **Singlet:** $\mathbf{S_1}(0^{++})$

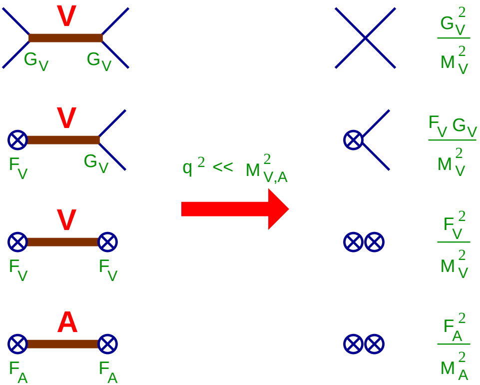
$$\mathcal{L}_{S_1} + \mathcal{L}_A + \mathcal{L}_V \;=\; \frac{v}{2}\,\kappa_W\; S_1\,\langle u^\mu u_\mu\rangle \;+\; \frac{F_A}{2\sqrt{2}}\,\langle A_{\mu\nu} f_-^{\mu\nu}\rangle \;+\; \frac{F_V}{2\sqrt{2}}\,\langle V_{\mu\nu}\, f_+^{\mu\nu}\rangle \;+\; \frac{i\,G_V}{2\sqrt{2}}\,\langle V_{\mu\nu}\;[u^\mu u^\nu]\rangle$$

$$\mathcal{L}_{S_1 A} \;=\; \sqrt{2}\;\lambda_1^{SA}\;\partial_\mu S_1\,\langle A^{\mu\nu} u_\nu\rangle$$

Antisymmetric $V_{\mu\nu}$ and $A_{\mu\nu}$ fields **(better UV properties):**

$$\mathcal{L}_{\rm Kin} = -\frac{1}{2}\sum_{R=V,A}\langle \nabla^\lambda R_{\lambda\mu}\nabla_\nu R^{\nu\mu} - \frac{1}{2}\,M_R^2\,R_{\mu\nu}R^{\mu\nu}\rangle \;+\; \frac{1}{2}\,\partial^\mu S_1\,\partial_\mu S_1 \;-\; \frac{1}{2}\,M_{S_1}^2\;S_1^2$$

Resonance Exchange



$$a_1 = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2} \quad , \quad a_2 = a_3 = \frac{F_V G_V}{4M_V^2} \quad , \quad a_4 = -a_5 = \frac{G_V^2}{4M_V^2}$$

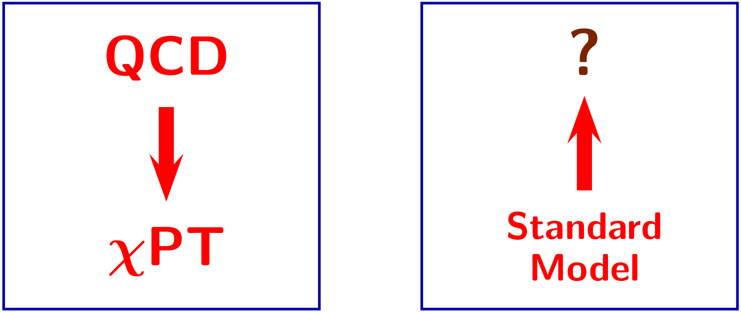
OUTLOOK

- **Effective Field Theory:** powerful low-energy tool
- **Mass Gap:** $E, m_{\text{light}} \ll \Lambda_{NP}$
- **Assumption:** relevant symmetries (breakings) & light fields
- Most general $\mathcal{L}_{\text{eff}}(\phi_{\text{light}})$ allowed by symmetry
- Short-distance dynamics encoded in LECs
- LECs constrained phenomenologically
- **Goal:** get hints on the underlying fundamental dynamics

→ **New Physics**

Learning from QCD experience. EW problem more difficult

Fundamental Underlying Theory unknown



Additional dynamical input (fresh ideas!) needed