Four lectures on

COSMOLOGY

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Lecture 1: The large picture

observations, cosmological principle, Friedmann model, Hubble diagram, thermal history

Lecture 2: From quantum to classical

cosmological inflation, isotropy & homogeneity, causality, flatness, metric & matter fluctuations

Lecture 3: Hot big bang

radiation domination, hot phase transitions, relics, nucleosythesis, cosmic microwave radiation

Lecture 4: Cosmic structure

primary and secondary cmb fluctuations, large scale structure, gravitational instability

Inflationary ΛCDM model

this is the current "standard model", it is the "minimal model"

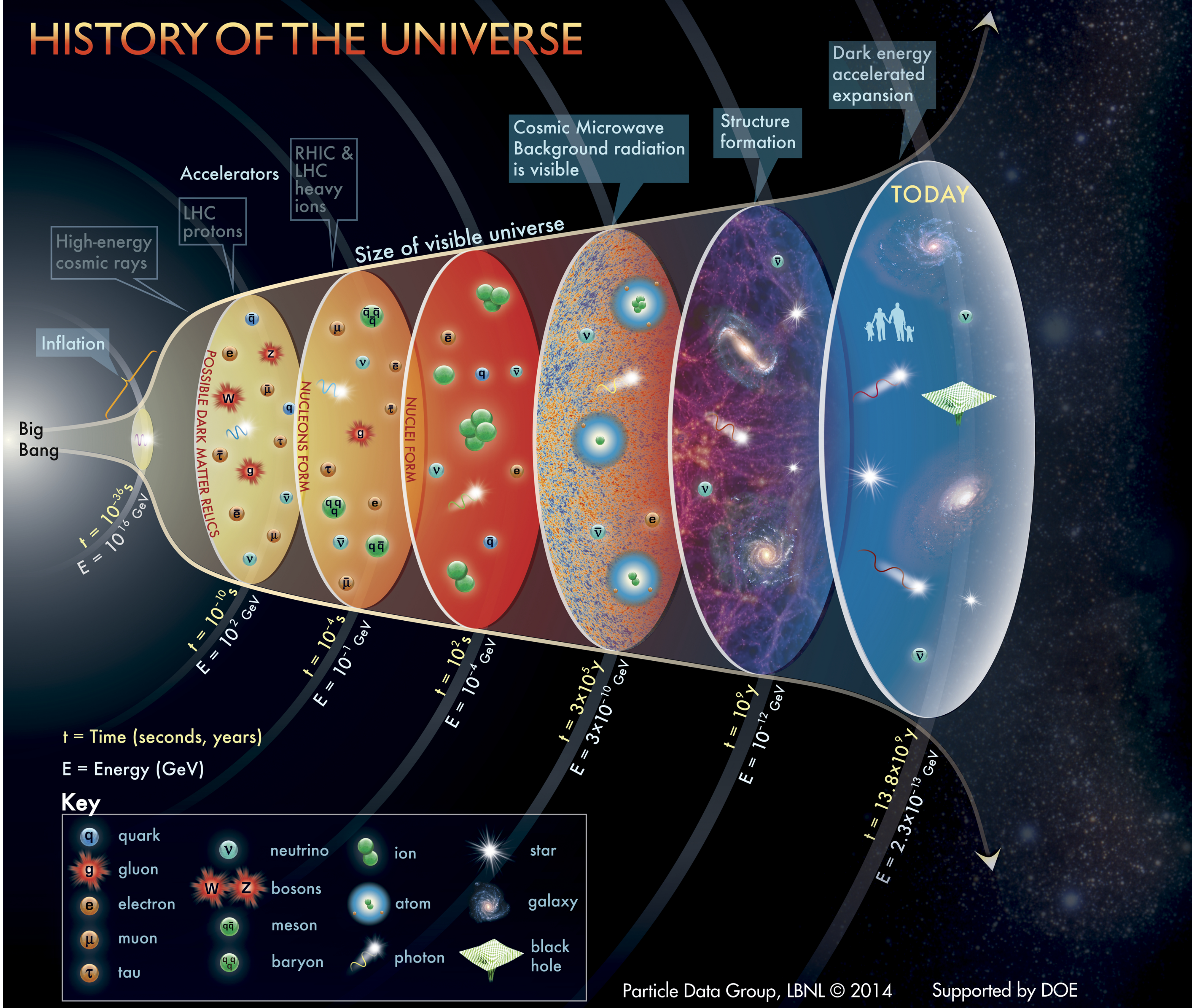
topology: trivial

geometry: flat Friedmann model

components: $\Lambda > 0$, cold dark matter, baryons, γ, ν (minimal masses)

small fluctuations of matter and metric: slow-roll inflation

minimal set of parameters necessary to study structure formation:
$h, T_0, \omega_b, \omega_m, A, n-1$ plus some astrophysical parameters $(\tau, b, Q_{nl}, \sigma_v, \ldots)$


HISTORY OF THE UNIVERSE
Dark energy accelerated expansion
Structure formation
Cosmic Microwave Background radiation is visible
RHIC & LHC heavy ions
Accelerators
LHC protons
High-energy cosmic rays
Inflation
Size of visible universe
TODAY
Big Bang
POSSIBLE DARK MATTER RELICS
NUCLEONS FORM
NUCLEI FORM
$t = 10^{-36}$ s
$E = 10^{16}$ GeV
$t = 10^{-10}$ s
$E = 10^{2}$ GeV
$t = 10^{-4}$ s
$E = 10^{-1}$ GeV
$t = 10^{2}$ s
$E = 10^{-4}$ GeV
$t = 3x10^{5}$ y
$E = 3x10^{-10}$ GeV
$t = 10^{9}$ y
$E = 10^{-12}$ GeV
$t = 13.8x10^{9}$ y
$E = 2.3x10^{-13}$ GeV
t = Time (seconds, years)
E = Energy (GeV)
Key
quark
gluon
electron
muon
tau
neutrino
bosons
meson
baryon
ion
atom
photon
star
galaxy
black hole
Particle Data Group, LBNL © 2014
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Cosmological perturbations

small fluctuations of Friedmann cosmology

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \quad T_{\mu\nu} = \bar{T}_{\mu\nu} + \delta T_{\mu\nu}$$

split up into scalar, vector and tensor perturbations (analogous for $\delta T_{\mu\nu}$)

$$\delta g_{00} = A, \quad \delta g_{0i} = \bar{\nabla}_i B + B_i^{\perp},$$

$$\delta g_{ij} = \bar{g}_{ij} C_1 + \bar{D}_{ij} C_2 + \bar{\nabla}_{(i} C_{j)}^{\perp} + C_{ij}^{\mathsf{TT}}, \quad \bar{D}_{ij} \equiv \bar{\nabla}_{(i} \bar{\nabla}_{j)} - \frac{1}{3} \bar{g}_{ij} \bar{\nabla}^2$$

$$\bar{\nabla}^i B_i^{\perp} = \bar{\nabla}^i C_i^{\perp} = 0, \quad \bar{\nabla}^j C_{ij}^{\mathsf{TT}} = 0, \quad \bar{g}^{ij} C_{ij}^{\mathsf{TT}} = 0$$

gauge fix two scalar and two vector degrees of freedom

linear regime: scalar, vector & tensor perturbations decouple
(2 dof per each type)

restrict for lecture to $K = 0$ and neglect anisotropic pressure (important for νs)

Linearised Einstein equations: scalars I

several perfect fluids $a =$ b, cdm, rad, . . .

$$\Delta_a \equiv \frac{\delta\epsilon_a}{(\epsilon+p)_a}, \quad \Delta = \sum_a \frac{(\epsilon+p)_a}{\epsilon+p}\Delta_a, \quad v_a \equiv -i\hat{\mathbf{k}}\mathbf{v}_a, \quad v = \sum_a \frac{(\epsilon+p)_a}{\epsilon+p} v_a$$

sum of perfect fluids makes up one imperfect fluid:

entropy (isocurvature) perturbations

$$\mathcal{S} \equiv \frac{\delta p - c_s^2 \delta\epsilon}{\epsilon+p} = \sum_a c_a^2 \frac{(\epsilon+p)_a}{\epsilon+p}(\Delta_a - \Delta)$$

isentropic initial conditions: $\mathcal{S} = \mathcal{S}' = 0 \Rightarrow \Delta_a = \Delta$ and $v_a = v$

Newtonian longitudinal gauge: $A = -2a^2\phi, B = 0, C_1 = -2a^2\psi, C_2 = 0$

Linearised Einstein equations: scalars II

continuity and Euler equations ($\mathcal{H} = a'/a, \eta$ conformal time)

$$\Delta_a' = kv_a + 3\psi', \quad v_a' + (1 - 3c_a^2)\mathcal{H}v_a = -c_a^2 k\Delta_a - k\phi$$

$\zeta_a \equiv \Delta_a/3 - \psi$ is constant on large scales ($k \ll \mathcal{H}$) Bardeen 1989

Poisson equation

$$-k^2\psi - 3\mathcal{H}\psi' - 3\mathcal{H}^2\phi = (\mathcal{H}' - \mathcal{H}^2)\Delta$$

vanishing of anisotropic pressure: $\phi = \psi$

dominant mode on superhorizon scales:
$\zeta \simeq -(5 + 3w)/[3(1 + w)]\phi$, with $w \equiv p/\epsilon$

$\zeta \simeq -\frac{5}{3}\phi(t > t_{\rm eq}) = -\frac{3}{2}\phi(t < t_{\rm eq})$ ϕ decreases by factor of 9/10 at equality

Linearised Einstein equations: scalars III

superhorizon scales: $\zeta_a \simeq$ const

subhorizon scales:

$$\Delta_{\mathrm{r}}'' + c_{\mathrm{r}}^2 k^2 \Delta_{\mathrm{r}} \simeq 0, \quad \Delta_{\mathrm{m}}'' + \mathcal{H}\Delta_{\mathrm{m}}' \simeq \frac{3}{2}(1+w)\mathcal{H}^2\Delta$$

until decoupling: $\Delta_{\mathrm{r}} \propto \cos(c_{\mathrm{r}} k\eta)$ — acoustic oscillations

radiation era: $\Delta \approx \Delta_{\mathrm{r}}$; $\Delta_{\mathrm{m}} \propto b_1 + b_2 \log \eta$ — supression of growth

matter era: $\Delta \approx \Delta_{\mathrm{m}}$; $\Delta_{\mathrm{m}} \propto \eta^2 \propto a$ — growth of structure

Λ era: $\Delta \approx \Delta_{\mathrm{m}}$; $\Delta_{\mathrm{m}} \simeq$ const — stop of structure formation

Anisotropy of cosmic microwave background (CMB)

photon decoupling at $t \sim 350\,000$ years

temperature fluctuations $\delta T/T$ Sachs & Wolfe 1967

$$\frac{\delta T^{\mathsf{S}}}{T}(\vec{e}) = [\frac{\delta T_\gamma}{T_\gamma} + \phi - e^i v^{\mathsf{S}}_{\gamma i}]_{\mathsf{dec}} + \int_{\eta_{\mathsf{dec}}}^{\eta_0} \mathsf{d}\bar{\eta} \frac{\partial}{\partial \bar{\eta}}(\phi + \psi)$$

$$\frac{\delta T^{\mathsf{T}}}{T}(\vec{e}) = -\frac{1}{2} e^i e^j \int_{\eta_{\mathsf{dec}}}^{\eta_0} \mathsf{d}\bar{\eta} \frac{\partial}{\partial \bar{\eta}} h_{ij}$$

scalar: temperature fluctuation, graviational red-shift, Doppler effect at decoupling;

scalar & tensor: integrated SW effect

extra astrophysical parameter:

optical depth τ or redshift of reionization

Predictions for cosmic microwave background

$\delta T(\mathbf{e}) = \sum a_{\ell m} Y_{\ell m}(\mathbf{e})$ with $a^*_{\ell m} = (-1)^m a_{\ell -m}$ (reality condition)
$\Rightarrow 2\ell + 1$ degrees of freedom for ℓth moment

statistical isotropy:

$\langle \delta T(\mathrm{R}\mathbf{e}_1) \ldots \delta T(\mathrm{R}\mathbf{e}_n) \rangle = \langle \delta T(\mathbf{e}_1) \ldots \delta T(\mathbf{e}_n) \rangle, \quad \forall \mathrm{R} \in \mathrm{SO}(3),\ \forall n > 0$

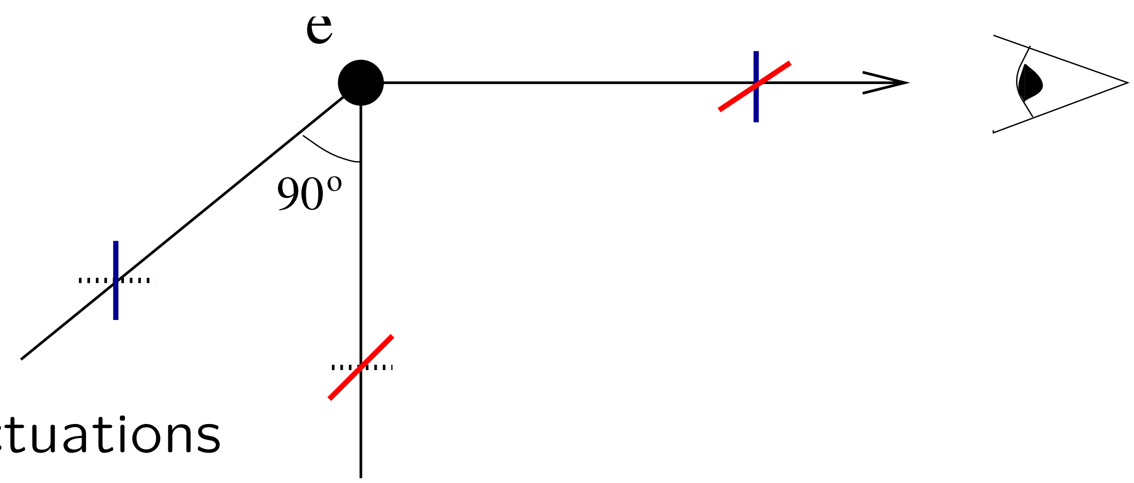
- $\langle \delta T(\mathbf{e}) \rangle = 0$ and $\langle a_{\ell m} \rangle = 0$
- $\langle \delta T(\mathbf{e}_1) \delta T(\mathbf{e}_2) \rangle = f(\mathbf{e}_1 \cdot \mathbf{e}_2) = \frac{1}{4\pi} \sum_\ell (2\ell + 1) C_\ell P_\ell(\cos\theta), \quad \cos\theta \equiv \mathbf{e}_1 \cdot \mathbf{e}_2$ with $\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \delta_{\ell \ell'} \delta_{m m'}$, C_ℓ multipole moments

gaussianity: no extra information in higher correlation functions

(best) estimator: $\widehat{C}_\ell = 1/(2\ell + 1) \sum_m |a_{\ell m}|^2$ (assumes statistical isotropy)
cosmic variance: $\mathrm{Var}(\widehat{C}_\ell) = 2C_\ell^2/(2\ell + 1)$ (assumes gaussianity)

Polarisation of CMB

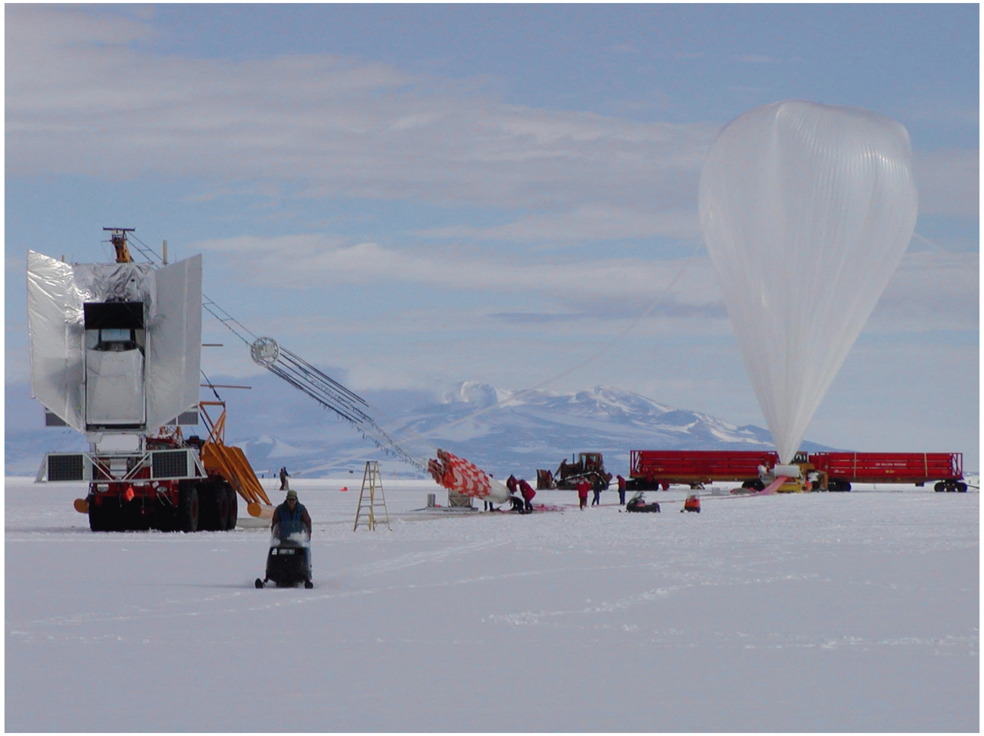
quadrupole at
photon decoupling induces
linear polarisation



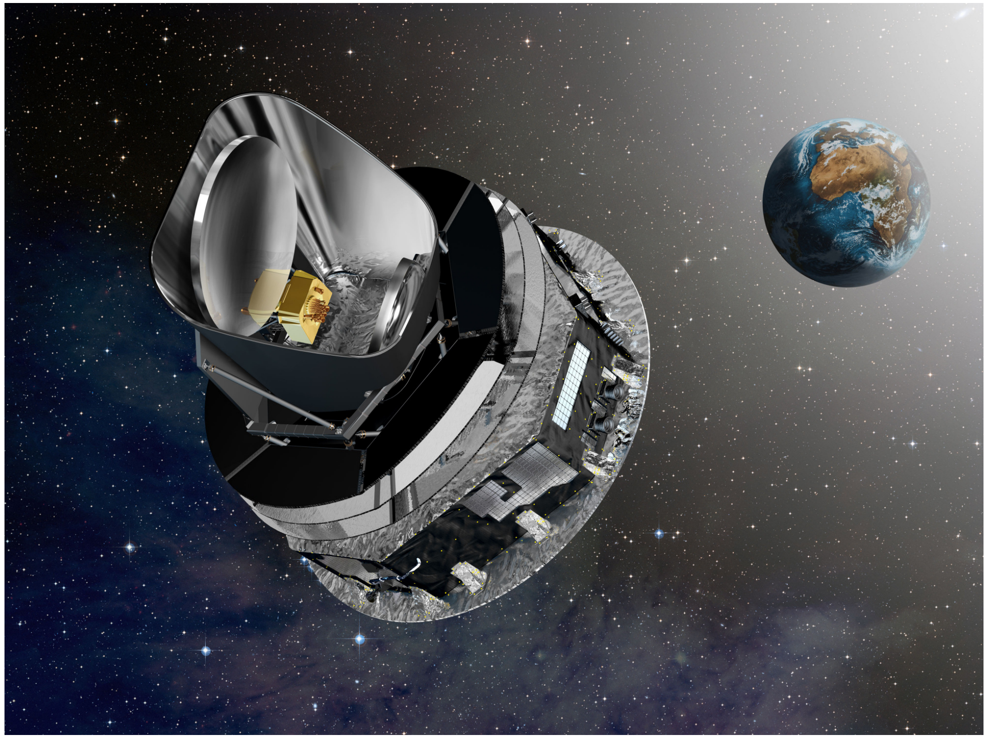
direct proof
of primordial nature of fluctuations

E- and B-modes (gradient and rotor field)
density fluctuations (E) and gravitational waves (E & B)

Observations of the microwave sky

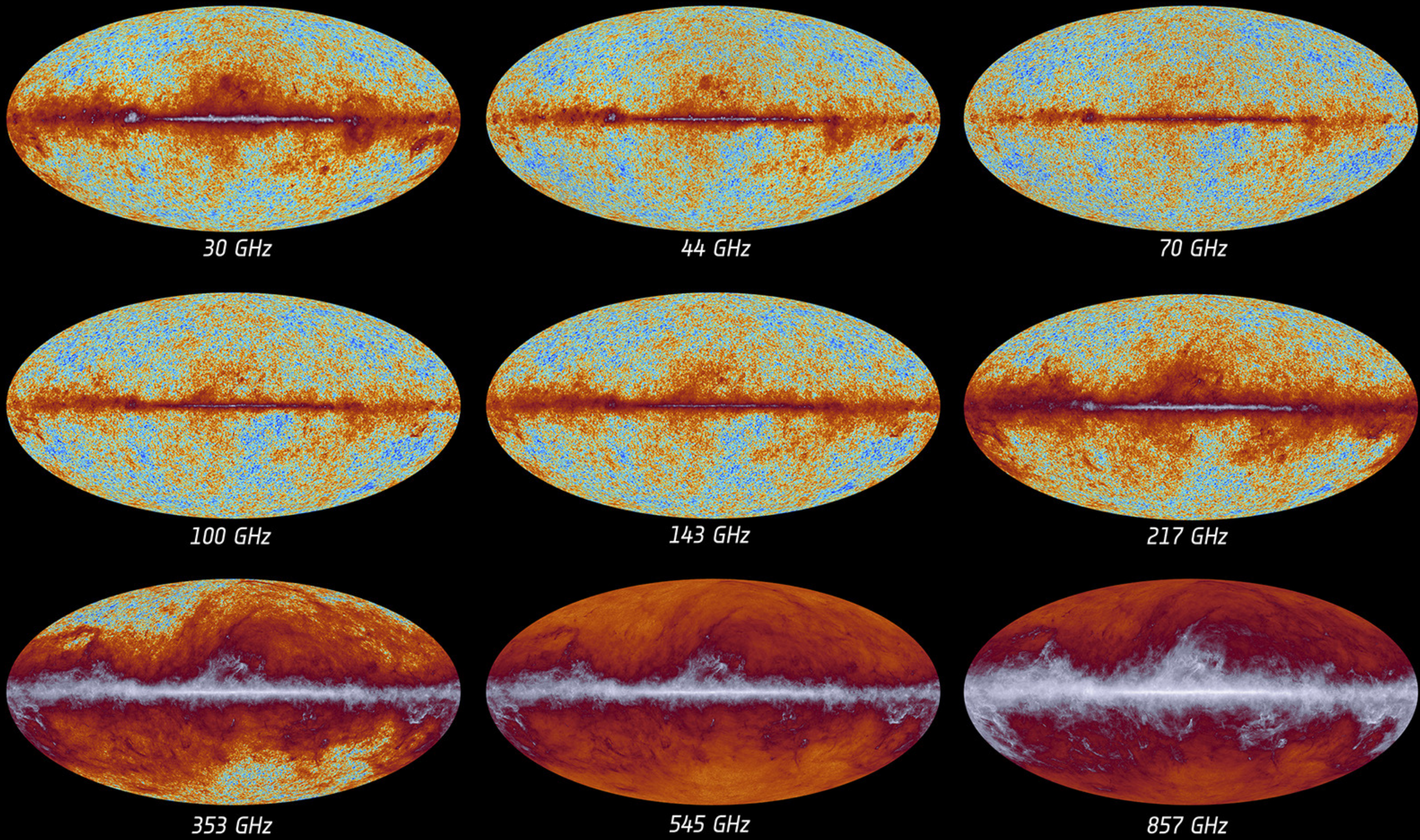


BOOMERanG

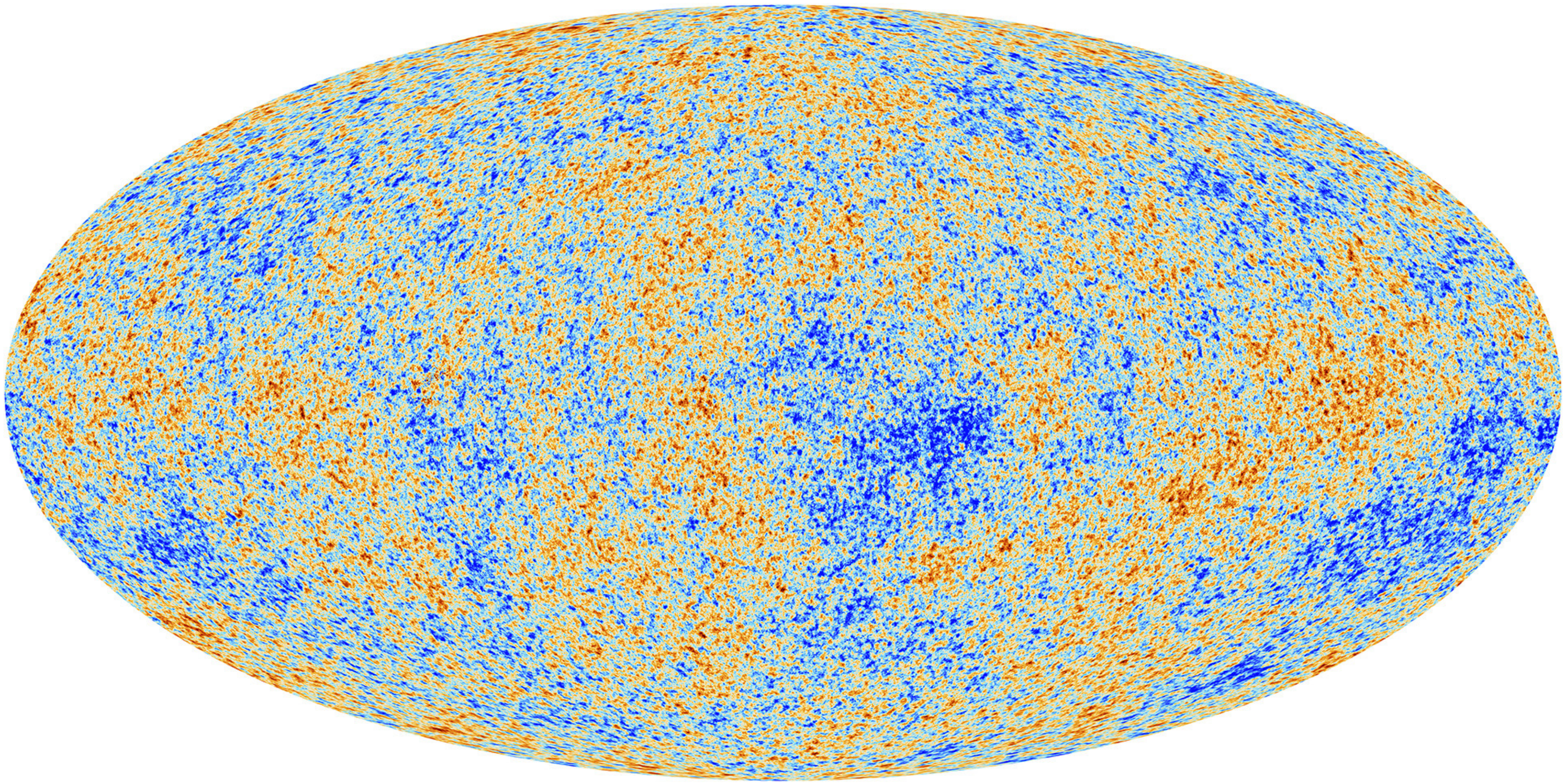


Planck

The sky as seen by Planck



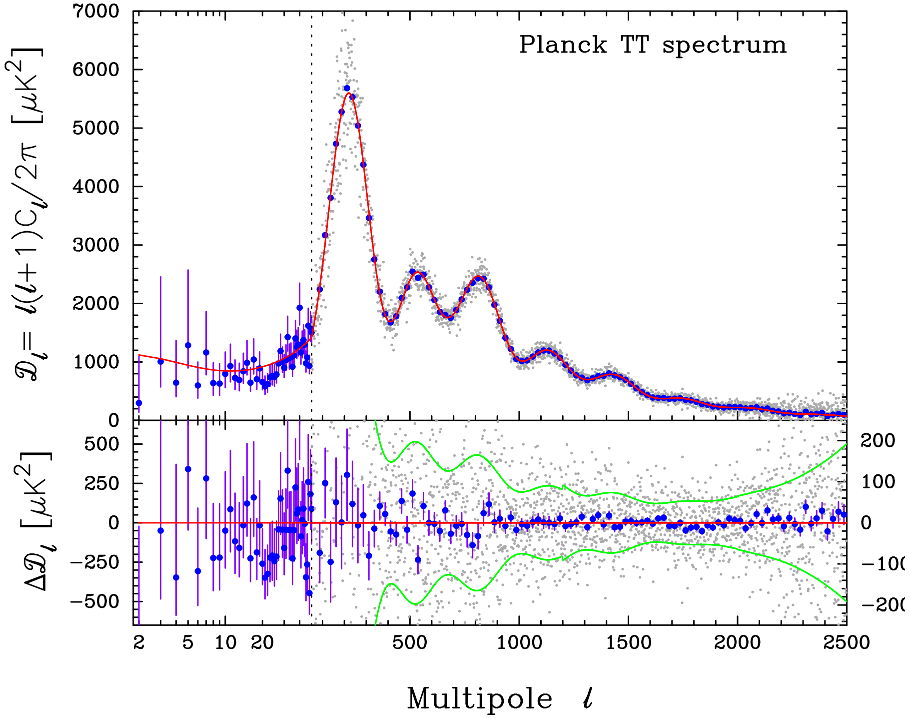
Foreground cleaned map

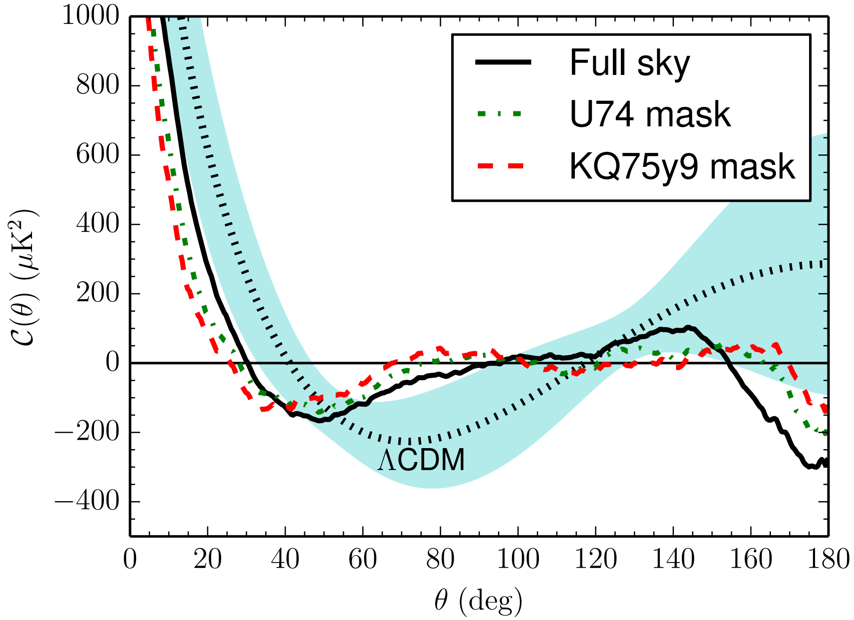


SMICA

Planck collaboration, Ade et al. 2014

Angular power spectrum and two-point correlation functions



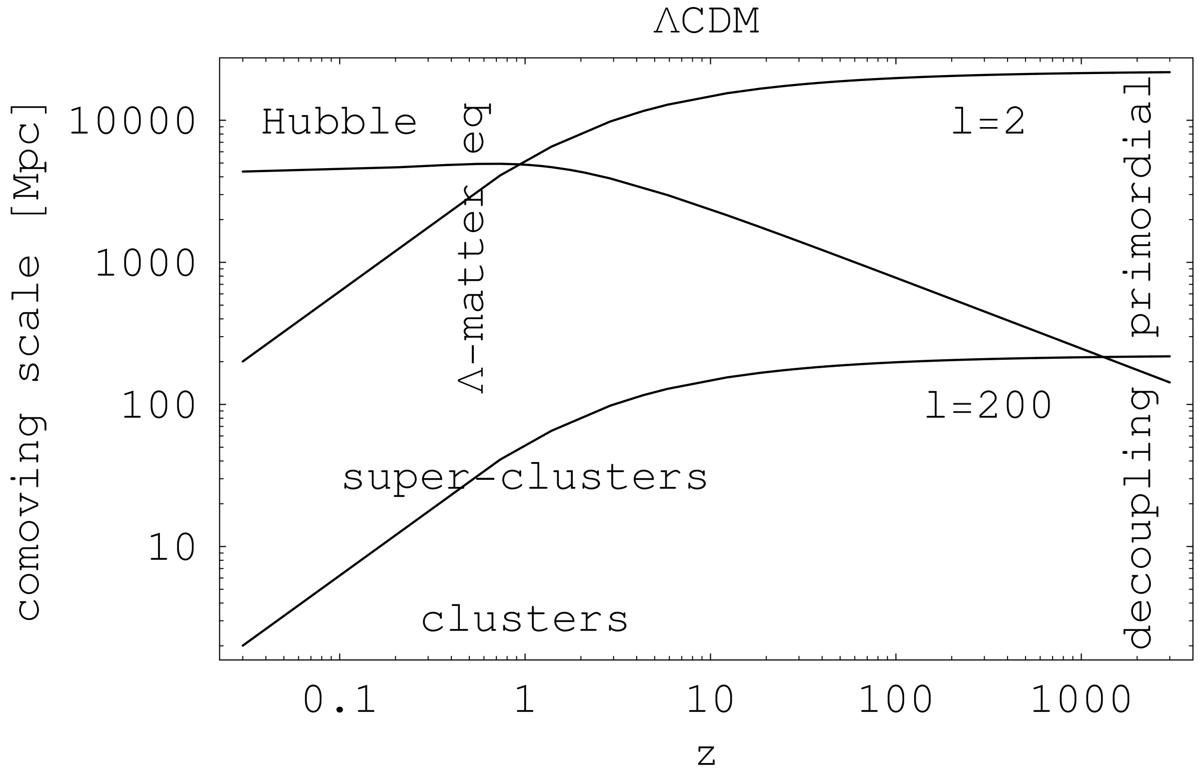


Copi et al. 2013

Planck collaboration, Ade et al. 2014

Angular scales of cosmic microwave background

CMB probes physics back to photon decoupling $z_{\rm dec} \approx 1100$



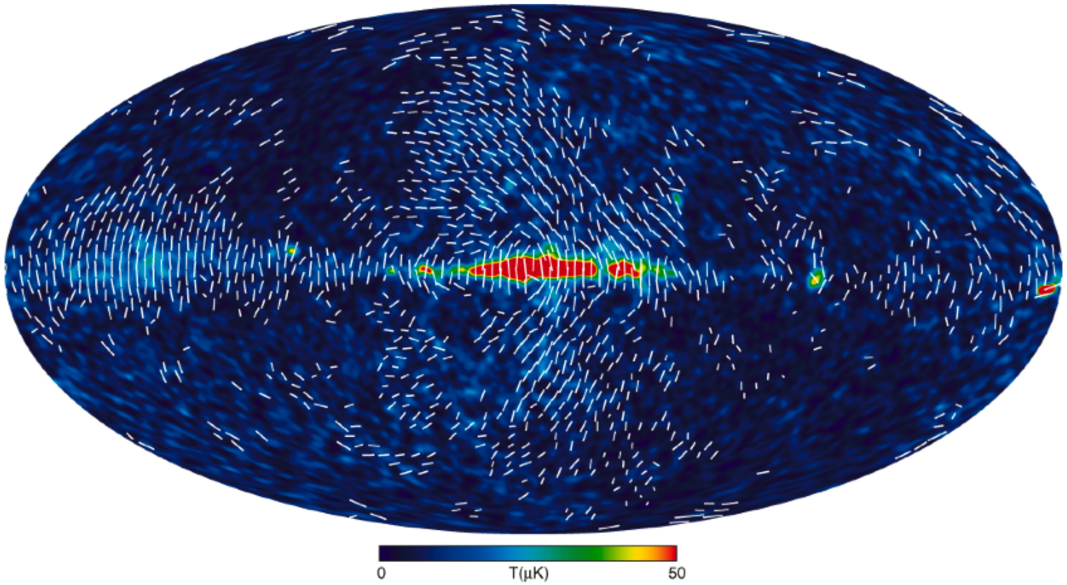
Geometry of the Universe

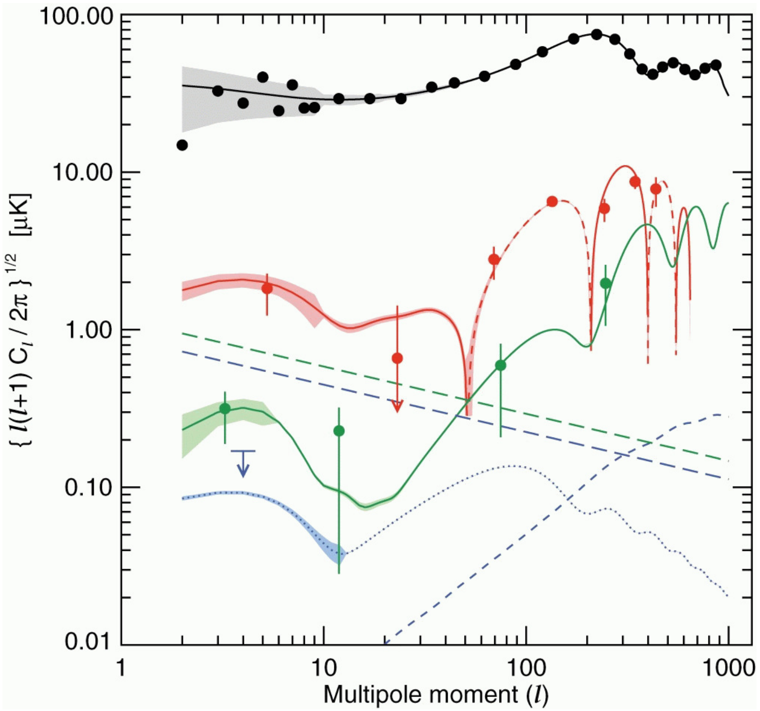
acoustic oscillations of photon-baryon plasma

$\lambda_{\text{ph}}/2 = (c_s/H)_{\text{dec}}$ and t_{dec} fixed (H-atom) $\Rightarrow$
triangle with all sides and one angle known determines the geometry

CMB & BAO $\Rightarrow$
$\Omega - 1 = -0.0010^{+0.0062}_{-0.0065}$ Planck collaboration, Ade et al 2014

First full sky maps of polarisation





WMAP Q band polarisation
dominated by foreground, low S/N

Page et al. 2006

Cosmological parameters: power-law Λ CDM

inflationary parameters:

$\mathcal{P}_\zeta = (2.196^{+0.051}_{-0.060}) \times 10^{-9}$, $n = 0.9603 \pm 0.0073$

dynamic parameters:

$h = 0.673 \pm 0.012$, $\Omega_{\rm cdm} h^2 = 0.1199 \pm 0.0027$, $\Omega_{\rm b} h^2 = 0.02205 \pm 0.00028$

astrophysical parameter:

$\tau = 0.089^{+0.012}_{-0.014}$

fit to Planck TT & WMAP TE   Planck collaboration, Ade et al. 2014

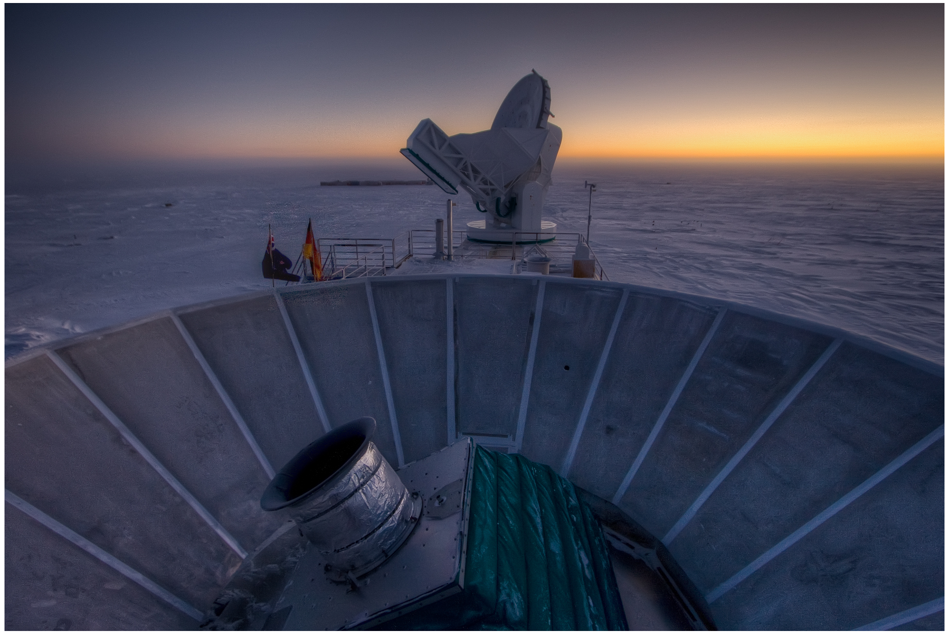
Planck 2013 (& 2014): Minimal model fits data very nicely!

5% atoms, 26% cold dark matter and 69% dark energy

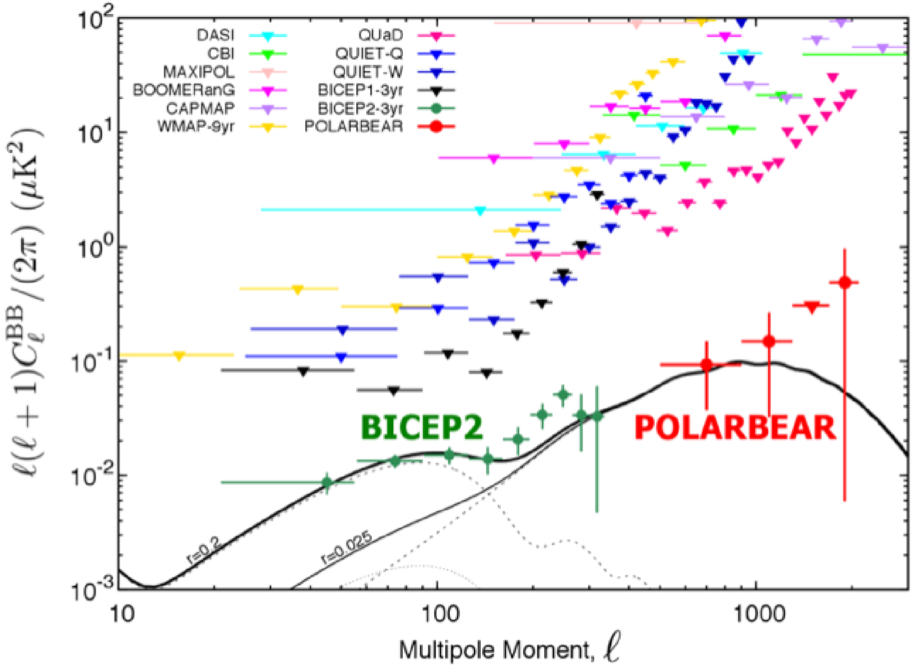
preliminary results as presented at 2014 Ferrara conference:

calibration now consistent with WMAP, story remains the same

Observations of B-Polarisation



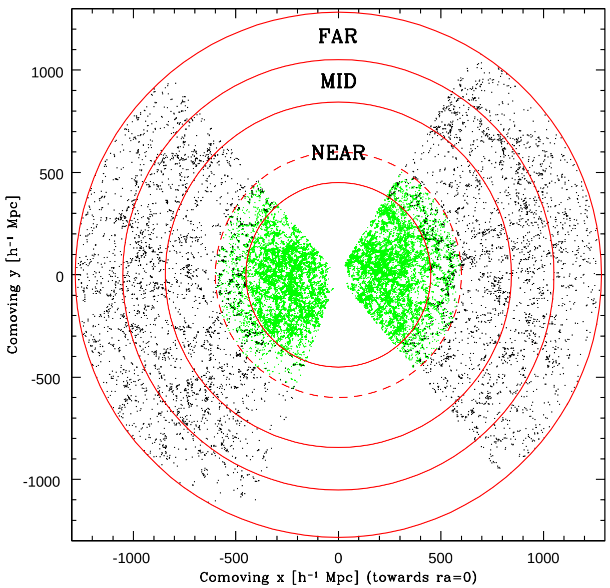
BICEP2 and SPT



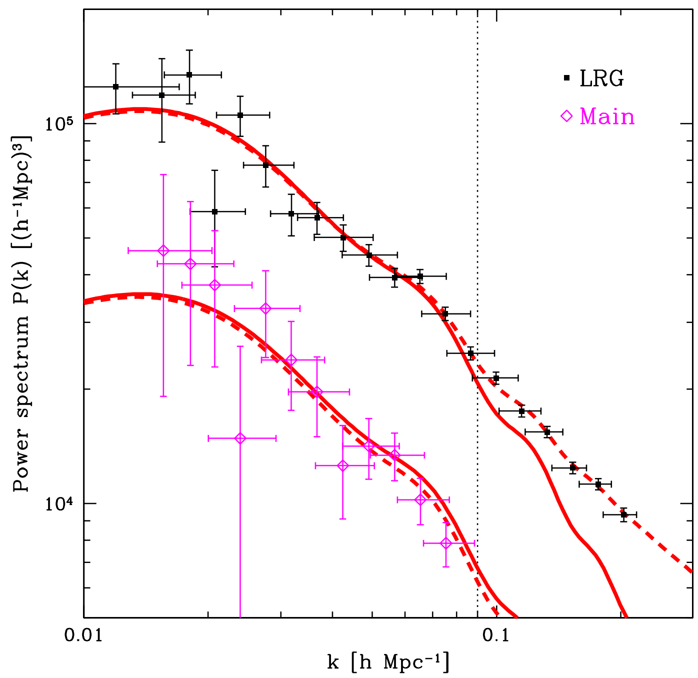
tensor modes, dust or both?

BICEP2 and Polarbear 2014

Observation of large scale structure and power spectrum



SDSS LRG (black) & main (green)



Tegmark et al. 2006

extra parameters:

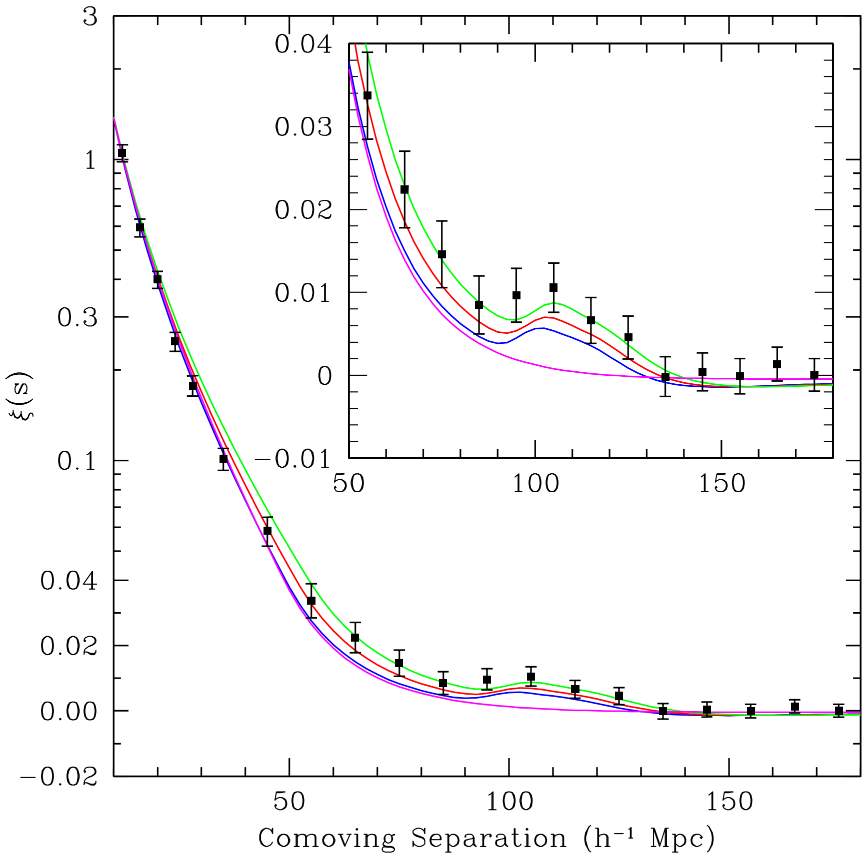
bias for each sample: $P_\mathrm{s} = b_s^2 P_\mathrm{m}$; nonlinear corrections (dashed): Q_nl

Baryon acoustic oscillations I

baryon acoustic oscillations
in matter power spectrum $P(k)$
and
peak in correlation function $\xi(r)$

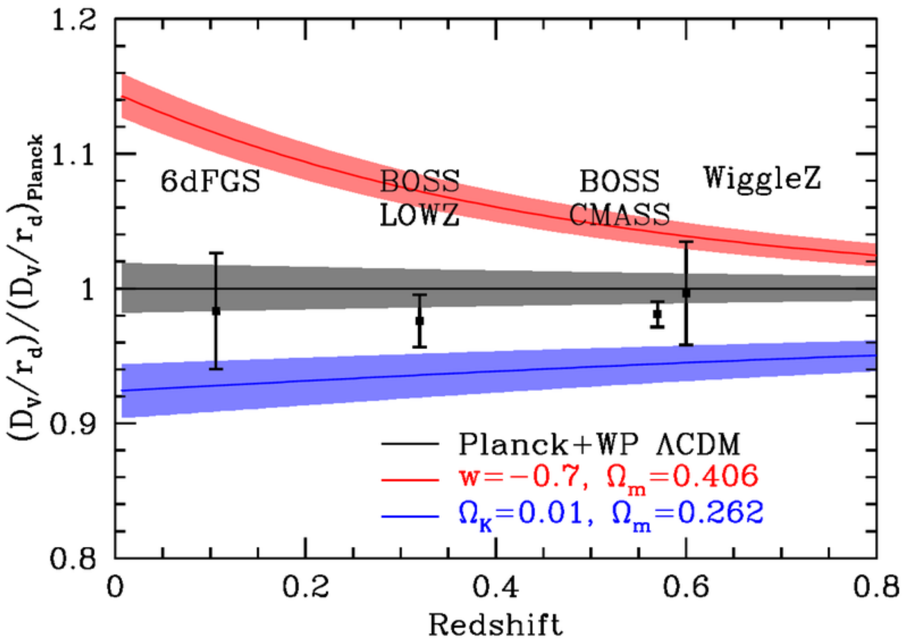
acoustic scale at $z \simeq 0.35$
constrain on
$d_{\mathsf{V}}(z) = [d_{\mathsf{a}}^2(z) d_{\mathsf{H}}(z) z]^{1/3}$

compare to acoustic scale
in CMB at $z \simeq 1100$

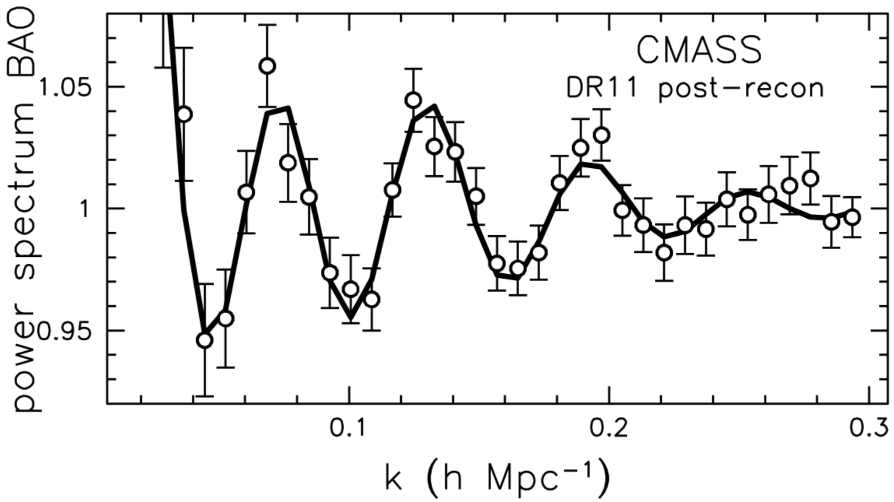


SDSS LRG Eisenstein et al. 2005

Baryon acoustic oscillations II



Anderson et al. 2013



Anderson et al. 2013

Summary of 4th lecture

structure forms via gravitational instability
seeds from quantum fluctuations during inflation

cosmic microwave background: most detailed and well defined probe

galaxy redshift surveys: less precise, extra parameter b

galaxy clusters, weak lensing surveys, Lyα forest, etc.

CMB polarisation (B-modes) interesting for fundamental physics
scale of inflation, higher accuracy thus additional cross-checks;
joint BICEP-Planck analysis, SPTPol, ACTPol, etc.

The last slide of the lecture

we arrived at a very successful model based on
standard model of particle physics & general relativity
idea of cosmological inflation
introduction of cosmological constant and dark matter

minimal set of well motivated physical parameters (9):
$T_0, m_\nu, \omega_\mathsf{b}, \omega_m, h, H_\mathsf{inf}, \varepsilon_1, \varepsilon_2, T_\mathsf{rh}$

minimal used set (6):
$T_0, \omega_\mathsf{b}, \omega_m, h, A, n-1$

astrophysical parameters

(follow from physical parameters, but cannot be calculated):

$\tau, b_s, Q_\mathsf{nl}, \sigma_v, \ldots$

What causes cosmological inflation?
What is the dark energy? What is the dark matter?