Searching for the Sharpe-Singleton scenario in the Wilson Dirac spectrum

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Motivation

- Understand low energy behaviour of QCD, and chiral symmetry breaking
- Behaviour described by Chiral Perturbation Theory
- Comparison to lattice results needed
- Need to take into account the effects of nonzero lattice spacing

Wilson fermions (K.G. Wilson, 1974)

- Explicitly break chiral symmetry
- $D_W^\dagger \neq -D_W$
- $D_W^\dagger = \gamma_5 D_W \gamma_5$
- Eigenvalues of the Wilson Dirac operator come in complex conjugate pairs or are purely real
- Chiral symmetry breaking closely related to the smallest eigenvalues of the Wilson Dirac operator (T. Banks, A. Casher, 1980, L. Giusti, M. Lüscher, 2009)

Wilson Chiral Perturbation Theory

- Chiral perturbation theory with added terms that describe discretisation effects (S.R. Sharpe & R.L. Singleton, 1998)

$$
\begin{aligned}
S =& \frac{m}{2}\Sigma V \operatorname{Tr}\left(U + U^\dagger\right) + \frac{\zeta}{2}\Sigma V \operatorname{Tr}\left(U - U^\dagger\right) - a^2 V \Delta, \\
\Delta =& W_8 \left[\operatorname{Tr}\left(U + U^\dagger\right)\right]^2 + W_7 \left[\operatorname{Tr}\left(U - U^\dagger\right)\right]^2 + W_8 \operatorname{Tr}\left(U^2 + U^{2^\dagger}\right)
\end{aligned}
$$

- Three new low energy constants constrained by

$$W_8 > 0, W_6 < 0, W_7 < 0$$

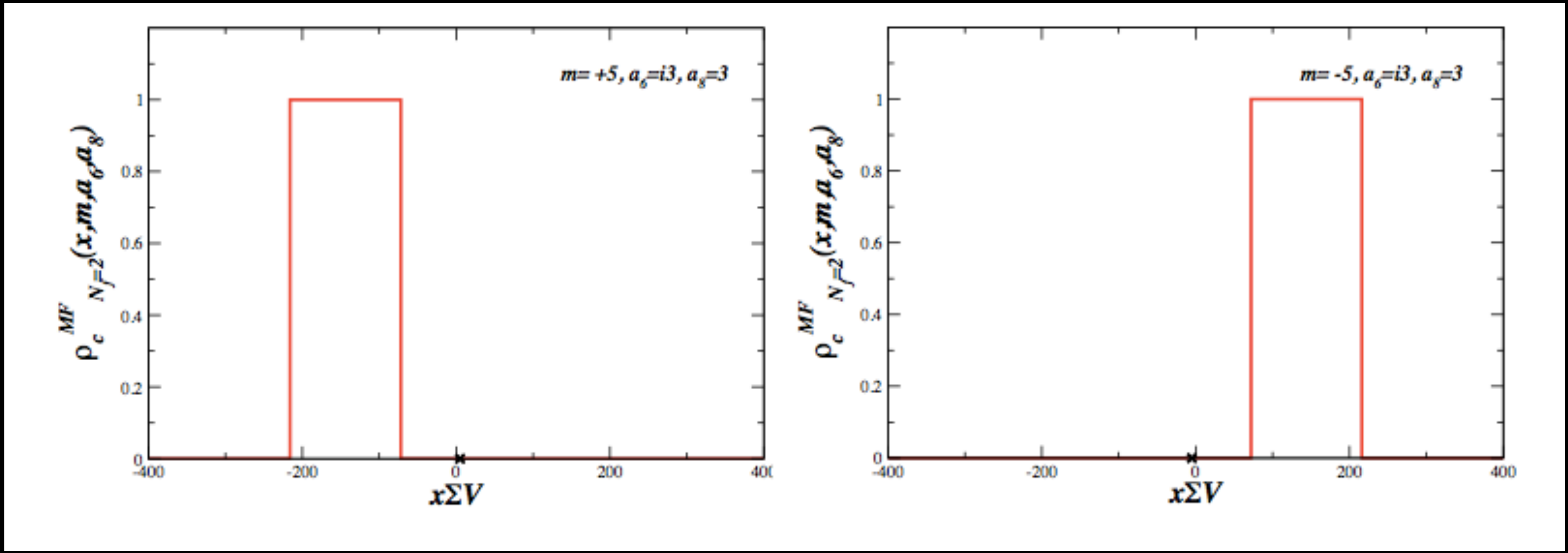
(M. Kieburg, K. Splittorff, J.J.M. Verbaarschot, 2012)

Sharpe-Singleton scenario

- A phase in lattice QCD with Wilson fermions with no continuum analogue (S.R. Sharpe & R.L. Singleton, 1998)
- A result of the interplay between $a \to 0, m \to 0$
- The sign of $W_8 + 2W_6$ determines what phase the theory will be in
- The Sharpe-Singleton scenario will be realised for $W_8 + 2W_6 < 0$

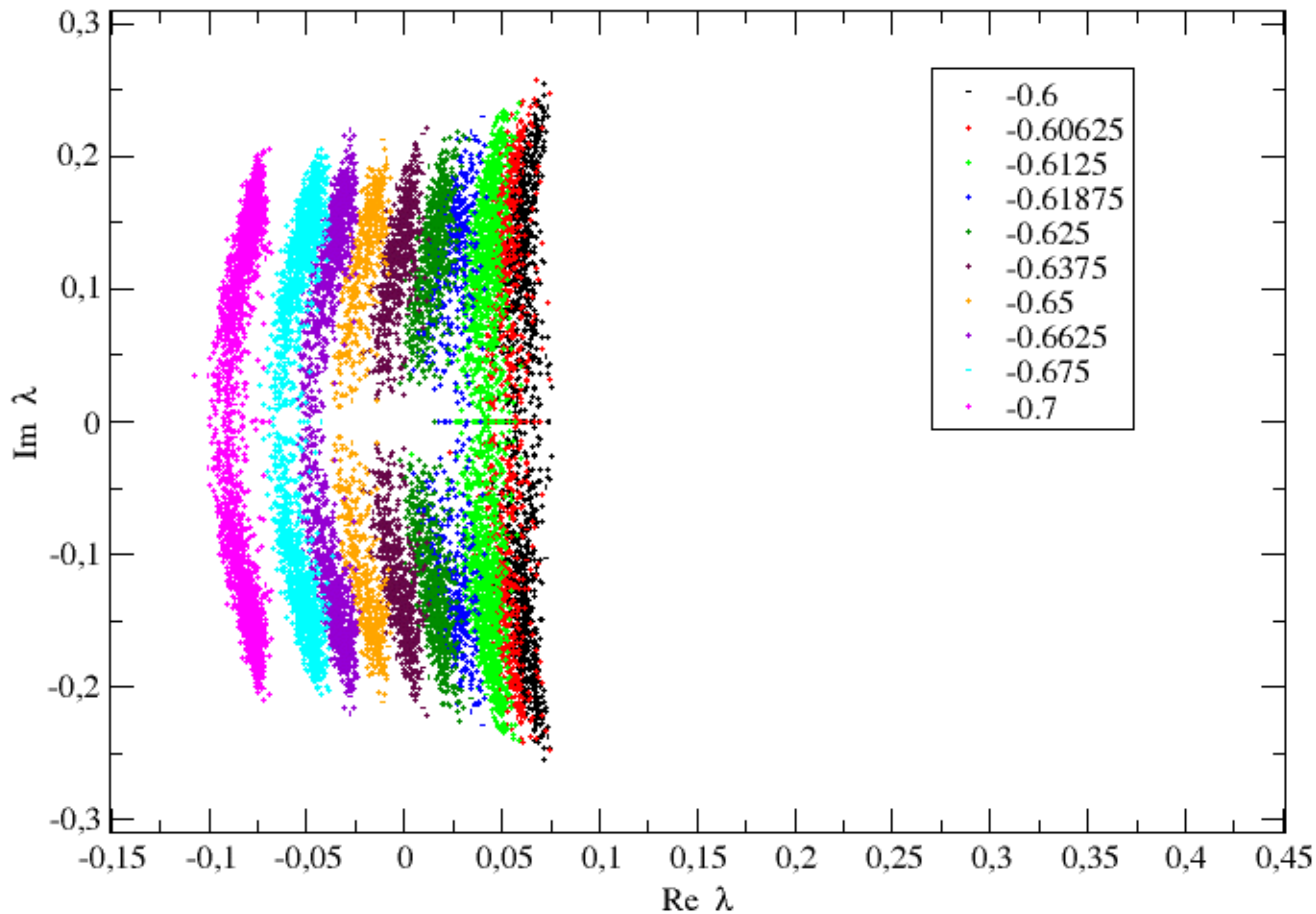
Sharpe-Singleton scenario

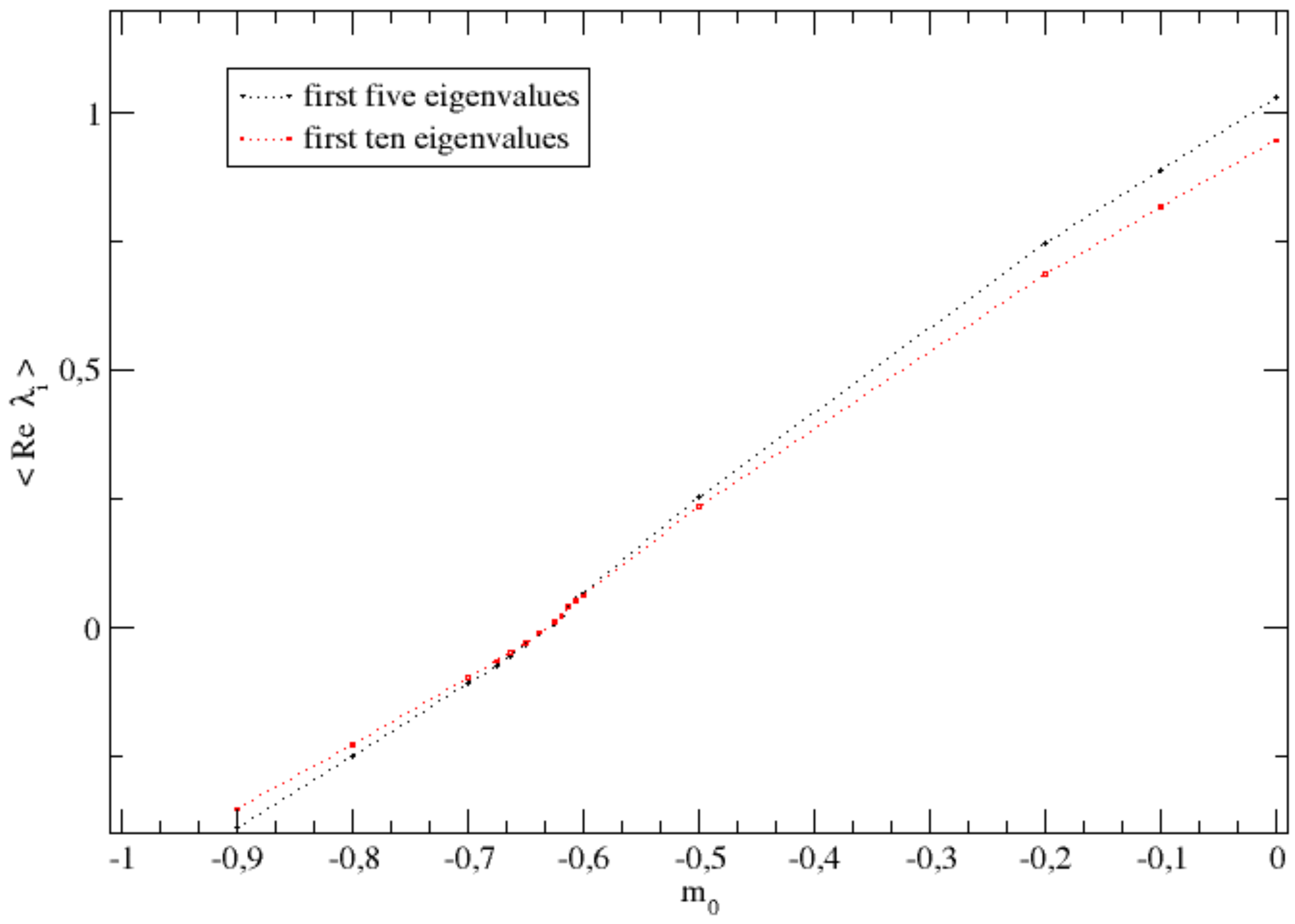
- Quark mass has a collective effect on the eigenvalue distribution (Figure from M. Kieburg, K. Splittorff, J.J.M. Verbaarschot, arXiv:1202.0620)

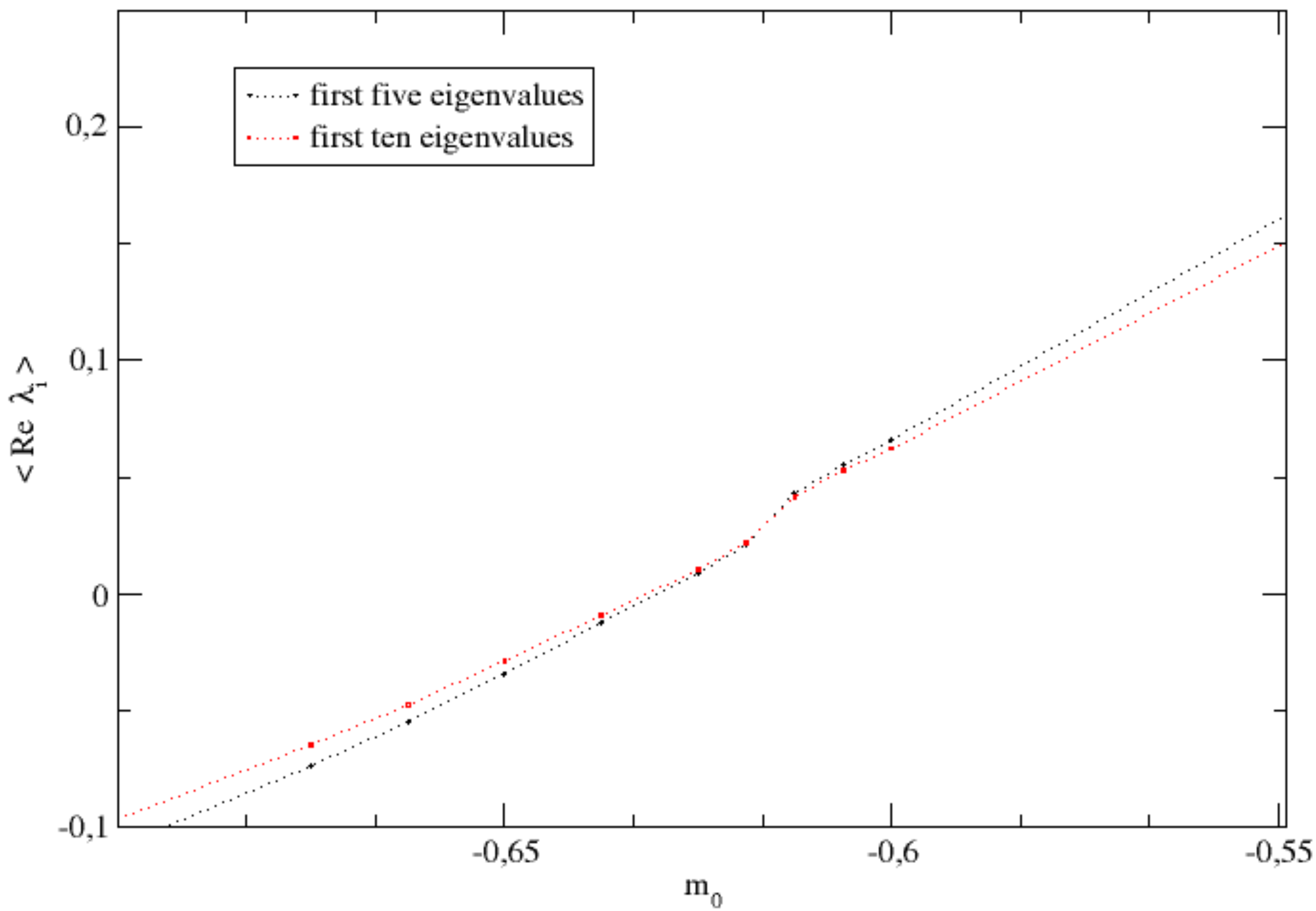


Lattice setup

- Simulate $SU(3), \ N_f = 2$
- Wilson fermions with clover improvement
- $\beta = 5.47 \qquad V = 16^4 \qquad m_0 = 0.0, \ldots, -0.9$
- Calculate 50 of the lowest lying eigenvalues of the Wilson Dirac operator with the Arnoldi algorithm as ordered by the magnitude of the real part


Im λ
Re λ
0,3
0,2
0,1
0
-0,1
-0,2
-0,3
-0,15
-0,1
-0,05
0
0,05
0,1
0,15
0,2
0,25
0,3
0,35
0,4
0,45
-0.6
-0.60625
-0.6125
-0.61875
-0.625
-0.6375
-0.65
-0.6625
-0.675
-0.7


first five eigenvalues
first ten eigenvalues
< Re λ_i >
m_0
1
0,5
0
-1
-0,9
-0,8
-0,7
-0,6
-0,5
-0,4
-0,3
-0,2
-0,1
0


first five eigenvalues
first ten eigenvalues
0,2
0,1
0
-0,1
< Re λ_1 >
-0,65
-0,6
-0,55
m_0

Conclusions and outlook

- The quark mass changing sign has a quantifiable effect on the eigenvalue distribution
- Need to vary parameters to see if the effect is persistent