Effective Field Theory

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IFIC, Valencia

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Outline

hep-ph/9806303

1) General Aspects of EFT

- Dimensional Analysis
- Relevant, Irrelevant, Marginal
- Quantum Loops
- Decoupling. Matching
- Scaling

2) χPT

- Goldstone Theorem
- Chiral Symmetry
- Effective Goldstone Theory
- Chiral Symmetry Breaking
- Phenomenology

3) High-Energy Dynamics

- CCWZ Formalism
- Heavy Fields
- Low-Energy Constants
- Asymptotic Behaviour
- Signals of Heavy Scales

4) EW Effective Theory

- Higgs Mechanism
- Custodial Symmetry
- Equivalence Theorem
- EW Effective Theory
- New Physics Scales

Euler-Heisenberg Lagrangian

Light-by-light scattering in QED at very low energies $(\mathbf{E_\gamma \ll m_e})$

- Gauge, Lorentz, Charge Conjugation & Parity constraints
- Energy expansion (E_γ / m_e)

$$\mathcal{L}_{\mathrm{eff}} = -\frac{1}{4} F^{\mu\nu} F_{\mu\nu} + \frac{a}{m_e^4} (F^{\mu\nu} F_{\mu\nu})^2 + \frac{b}{m_e^4} F^{\mu\nu} F_{\nu\sigma} F^{\sigma\rho} F_{\rho\mu} + \mathcal{O}(F^6/m_e^8)$$

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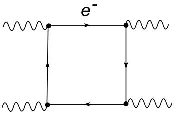

$$\longrightarrow \qquad a = -\frac{1}{36}\,\alpha^2 \qquad , \qquad b = \frac{7}{90}\,\alpha^2$$

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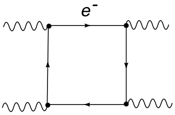

$$a = -\frac{1}{36}\,\alpha^2 \qquad , \qquad b = \frac{7}{90}\,\alpha^2$$

$$\sigma(\gamma\gamma \to \gamma\gamma) \propto \frac{\alpha^4 E^6}{m_e^8}$$

Why the sky looks blue?

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Rayleigh scattering

Low-energy scattering of photons with neutral atoms

$$E_\gamma \;\ll\; \Delta E \sim \alpha^2 m_e \;\ll\; a_0^{-1} \sim \alpha m_e \;\ll\; M_A$$

- Neutral atom + gauge invariance ➡ $F^{\mu\nu} = (\vec{E}, \vec{B})$
- Non-relativistic description

$$\mathcal{L}_{\mathrm{int}} \;=\; a_0^3\; \psi^\dagger\psi\, \left(c_1 \vec{E}^2 + c_2 \vec{B}^2\right) + \ldots \qquad , \qquad c_i \sim \mathcal{O}(1)$$

$$\mathcal{M} \;\sim\; c_i\, a_0^3\, E_\gamma^2 \qquad \Longrightarrow \qquad \boldsymbol{\sigma} \;\propto\; \mathbf{a_0^6\, E_\gamma^4}$$

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Blue light is scattered more strongly than red one

Dimensions

$$S \;=\; \int d^4x\, \mathcal{L}(x) \qquad \longrightarrow \qquad [\,\mathcal{L}\,] \;=\; E^4$$

$$\mathcal{L}_{\mathrm{KG}} \;=\; \partial_\mu \phi^\dagger \partial^\mu \phi - m^2\, \phi^\dagger \phi \qquad \longrightarrow \qquad [\,\phi\,] \;=\; [\,V^\mu\,] \;=\; [\,A^\mu\,] \;=\; E$$

$$\mathcal{L}_{\mathrm{Dirac}} \;=\; \bar{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi \qquad \longrightarrow \qquad [\,\psi\,] \;=\; E^{3/2}$$

$$[\,\sigma\,] \;=\; E^{-2} \qquad , \qquad [\,\Gamma\,] \;=\; E$$

Scalar Field Theory

- $\mathcal{L}_I = -\frac{\lambda}{3!}\,\phi^3 \quad \longrightarrow \quad [\lambda] = E$

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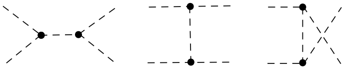

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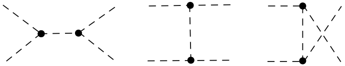

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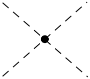

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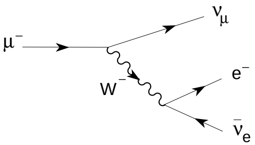
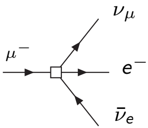
Fermi Theory

$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8\, M_W^2}$



$$\frac{-g_{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \quad \xrightarrow{q^2 \ll M_W^2} \quad \frac{g_{\mu\nu}}{M_W^2}$$



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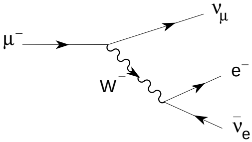

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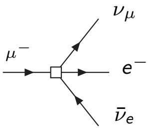

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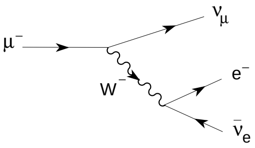

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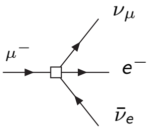

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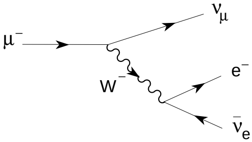

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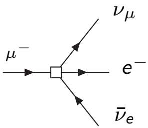

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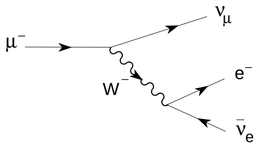

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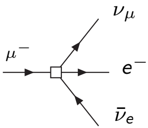

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Exp: $(17.83 \pm 0.04)\%$

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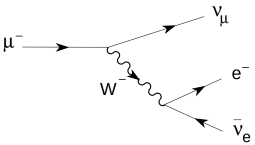

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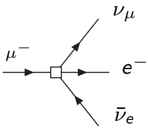

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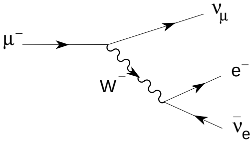

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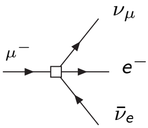

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- $\sigma(\nu_\mu e^- \to \mu^- \nu_e) \sim G_F^2\, s$ **Violates unitarity at high energies**

Relevant, Irrelevant & Marginal

$$\mathcal{L} = \sum_i c_i \, O_i \qquad , \qquad [O_i] = d_i \quad \longrightarrow \quad c_i \sim \frac{1}{\Lambda^{d_i - 4}}$$

Low-energy behaviour:

- **Relevant** $(\mathbf{d_i < 4})$**:** $\quad I \, , \, \phi^2 \, , \, \phi^3 \, , \, \bar\psi\psi$

  Enhanced by $(\Lambda/E)^{4-d_i}$

- **Marginal** $(\mathbf{d_i = 4})$**:** $\quad m^2\phi^2 \, , \, m\,\bar\psi\psi \, , \, \phi^4 \, , \, \phi\,\bar\psi\psi \, , \, V_\mu\,\bar\psi\gamma^\mu\psi$

- **Irrelevant** $(\mathbf{d_i > 4})$**:** $\quad \bar\psi\psi\,\bar\psi\psi \, , \, \partial_\mu\phi\,\bar\psi\gamma^\mu\psi \, , \, \phi^2\,\bar\psi\psi \, , \, \cdots$

  Suppressed by $(E/\Lambda)^{d_i-4}$

$$\boxed{\alpha(Q^2) \;=\; \frac{\alpha(Q_0^2)}{1-\beta_1\,\frac{\alpha(Q_0^2)}{2\pi}\;\log\left(Q^2/Q_0^2\right)}}$$

QED: $\beta_1 \;=\; \frac{2}{3}\sum_f Q_f^2\, N_f \;>\; 0$ $\;\Longrightarrow\;$ $\lim\limits_{Q^2\to 0}\alpha(Q^2) \;=\; 0$

Quantum corrections make QED irrelevant at low energies

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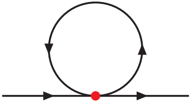
QCD: $\beta_1 = \frac{2\,N_F \,-\, 11\,N_C}{6} \;<\; 0 \quad\longrightarrow\quad \lim_{Q^2\to 0}\alpha_s(Q^2) \;=\; \infty$

Quantum corrections make QCD relevant at low energies

Quantum Loops

$$\mathcal{L} = \bar{\psi}\,(i\gamma^\mu\partial_\mu - m)\,\psi - \frac{a}{\Lambda^2}\,(\bar{\psi}\psi)^2 - \frac{b}{\Lambda^4}\,(\bar{\psi}\,\Box\psi)(\bar{\psi}\psi) + \cdots$$



$$\delta m \;\sim\; 2i\,\frac{a}{\Lambda^2}\,m\int\frac{d^4k}{(2\pi)^4}\,\frac{1}{k^2-m^2}$$

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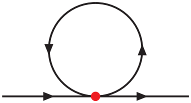

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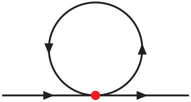

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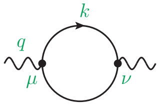
- **Dimensional regularization:** $\Delta_\infty(\mu) = \dfrac{2\,\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$

$$\delta m \;\sim\; 2a\,m\,\frac{m^2}{16\pi^2\Lambda^2}\left\{\Delta_\infty(\mu) + \log\left(\frac{m^2}{\mu^2}\right) - 1 + \mathcal{O}(D-4)\right\}$$

Well-defined expansion **(Mass independent)**

Vacuum Polarization $(m_f = 0)$



$$i\,\Pi^{\mu\nu}(q) \;=\; i\,\left(-q^2 g^{\mu\nu} + q^\mu q^\nu\right)\,\Pi(q^2)$$

$$\Pi(q^2) \;=\; -\frac{\alpha Q_f^2}{3\pi}\left\{\Delta_\infty(\mu) + \log\left(\frac{-q^2}{\mu^2}\right) - \frac{5}{3}\right\}$$

$$\equiv\; \Delta\Pi_\epsilon(\mu^2) \;+\; \Pi_R(q^2/\mu^2)$$

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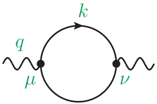

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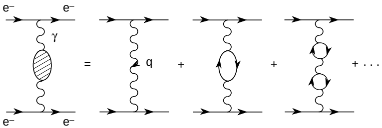
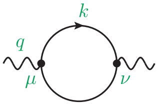


$$\alpha_0\,\left\{1 - \Delta\Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2)\right\}$$

$$\equiv\;\; \alpha_R(\mu^2)\;\left\{1 - \Pi_R(q^2/\mu^2)\right\}$$

Vacuum Polarization $(m_f = 0)$

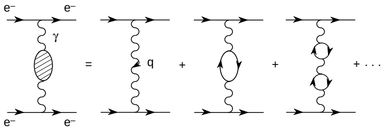


$$i\,\Pi^{\mu\nu}(q) \;=\; i\,\left(-q^2 g^{\mu\nu} + q^\mu q^\nu\right)\,\Pi(q^2)$$

$$\Pi(q^2) \;=\; -\frac{\alpha Q_f^2}{3\pi}\left\{\Delta_\infty(\mu) + \log\left(\frac{-q^2}{\mu^2}\right) - \frac{5}{3}\right\}$$

$$\equiv\; \Delta\Pi_\epsilon(\mu^2) \;+\; \Pi_R(q^2/\mu^2)$$

$$\Delta_\infty(\mu) = \frac{2\,\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$$

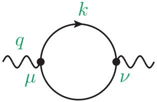


$$\alpha_0\,\left\{1 - \Delta\Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2)\right\}$$

$$\equiv\;\; \alpha_R(\mu^2)\;\left\{1 - \Pi_R(q^2/\mu^2)\right\}$$

$$\frac{\mu}{\alpha}\,\frac{d\alpha}{d\mu} \equiv \beta(\alpha) = \beta_1\,\frac{\alpha}{\pi} + \cdots \quad\Longrightarrow\quad \alpha(Q^2) \;\approx\; \frac{\alpha(Q_0^2)}{1 - \beta_1\,\frac{\alpha(Q_0^2)}{2\pi}\,\log\left(Q^2/Q_0^2\right)}$$

Vacuum Polarization $(m_f \neq 0)$

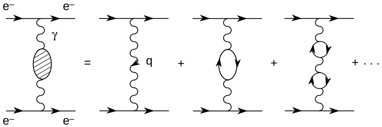


$$i\,\Pi^{\mu\nu}(q) \;=\; i\,\left(-q^2 g^{\mu\nu} + q^\mu q^\nu\right)\,\Pi(q^2)$$

$$\Pi(q^2) \;=\; -\frac{\alpha Q_f^2}{3\pi}\left\{\Delta_\infty(\mu) + 6\int_0^1 dx\; x(1-x)\,\log\left(\frac{m_f^2 - q^2 x(1-x)}{\mu^2}\right)\right\}$$

$$\equiv\; \Delta\Pi_\epsilon(\mu^2) \;+\; \Pi_R(q^2/\mu^2)$$

$$\Delta_\infty(\mu) = \frac{2\,\mu^{D-4}}{D-4} + \gamma_E - \log(4\pi)$$



$$\alpha_0\,\left\{1 - \Delta\Pi_\epsilon(\mu^2) - \Pi_R(q^2/\mu^2)\right\}$$

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Mass-Dependent Scheme: $\Delta\Pi_\epsilon(\mu^2) \;\equiv\; \Pi(-\mu^2)$

$$\Pi_R(q^2/\mu^2) \;=\; -Q_f^2\,\frac{\alpha}{3\pi}\,6\int_0^1 dx\,x(1-x)\,\log\left[\frac{m_f^2-q^2x(1-x)}{m_f^2+\mu^2x(1-x)}\right]$$

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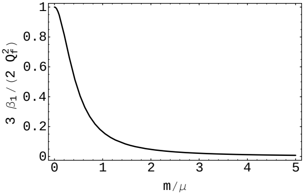
$$\beta_1 = 4\,Q_f^2\,\int_0^1 dx\;\frac{\mu^2x^2(1-x)^2}{m_f^2+\mu^2x(1-x)}$$



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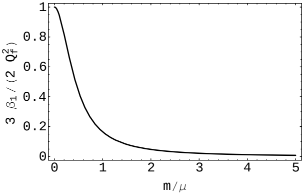

- $\mathbf{m_f^2 \ll \mu^2}$ **,** $\mathbf{q^2}$**:** $\beta_1 = \frac{2}{3}\,Q_f^2$, $\Pi_R(q^2/\mu^2) = -Q_f^2\,\frac{\alpha}{3\pi}\,\log\left(-q^2/\mu^2\right)$

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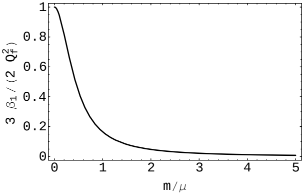

- $\mathbf{m_f^2 \ll \mu^2\, ,\, q^2}$: $\quad \beta_1 = \frac{2}{3} Q_f^2 \quad , \quad \Pi_R(q^2/\mu^2) = -Q_f^2 \frac{\alpha}{3\pi} \log\left(-q^2/\mu^2\right)$
- $\mathbf{m_f^2 \gg \mu^2, q^2}$: $\quad \beta_1 \sim \frac{2}{15} Q_f^2 \frac{\mu^2}{m_f^2} \quad , \quad \Pi_R(q^2/\mu^2) \sim Q_f^2 \frac{\alpha}{15\pi} \frac{q^2+\mu^2}{m_f^2}$

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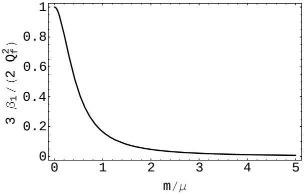

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$\longrightarrow$ **DECOUPLING** **(Appelquist-Carazzone Theorem)**

$\overline{\text{MS}}$ Scheme: $\quad \Delta\Pi_\epsilon(\mu^2) \,\equiv\, -Q_f^2\,\frac{\alpha_0}{3\pi}\,\Delta_\infty(\mu)$

$$\Pi_R(q^2/\mu^2) \;=\; -Q_f^2\,\frac{\alpha}{3\pi}\,6\int_0^1 dx\; x(1-x)\,\log\left[\frac{m_f^2-q^2x(1-x)}{\mu^2}\right]$$

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- $\beta_1 = \frac{2}{3}\,Q_f^2$ **Independent of $\mathbf{m_f}$**

Heavy fermions do not decouple

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Perturbation theory breaks down

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Perturbation theory breaks down

SOLUTION: Integrate Out Heavy Particles

Matching

μ

EFT with heavy fermion $\qquad \beta_1 = \frac{2}{3}\, Q_f^2 + \beta_1^{\text{light}}$

$\mathbf{m_f}$ -

EFT without heavy fermion $\qquad \beta_1 = \beta_1^{\text{light}}$

- **Two different EFTs** **(with and without the heavy fermion f)**
- **Same S-matrix elements for light-particle scattering at** $\mu = \mathbf{m_f}$

Effective Field Theory

$$\mathcal{L}(\phi,\Phi) \;=\; \frac{1}{2}(\partial\phi)^2+\frac{1}{2}(\partial\Phi)^2-\frac{1}{2}m^2\phi^2-\frac{1}{2}M^2\Phi^2-\frac{\lambda}{2}\phi^2\Phi$$

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$$\sigma(\phi\phi\to\phi\phi) \;\sim\; \frac{1}{E^2} \;\times\; \begin{cases} (\lambda/E)^4 \;, & (m \ll M \ll E) \\ (\lambda/M)^4 \;, & (m, E \ll M) \end{cases}$$

Effective Field Theory

$$\mathcal{L}(\phi, \Phi) \;=\; \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}m^2\phi^2 - \frac{1}{2}M^2\Phi^2 - \frac{\lambda}{2}\phi^2\Phi$$

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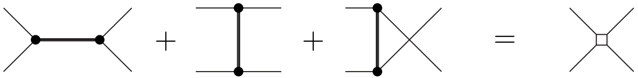

$$\mathbf{E, m \ll M}$$

$$\frac{\lambda^2}{s - M^2} = -\frac{\lambda^2}{M^2} \sum_{n=0} \frac{s}{M^2} \qquad \Rightarrow \qquad \mathcal{L}_{\mathrm{eff}}(\phi) \;=\; \sum_i c_i\, O_i(\phi)$$

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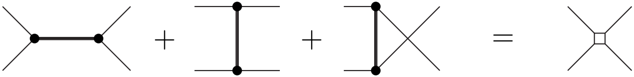

$\mathbf{E, m \ll M}$

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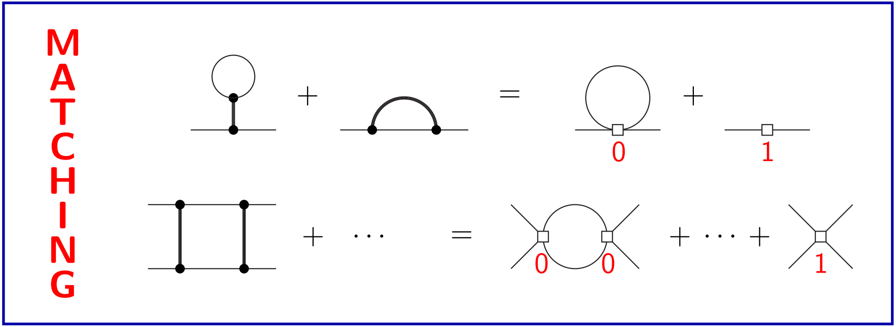
$$[O_i] = d_i \qquad ; \qquad c_i \sim \frac{\lambda^2}{M^2}\,\frac{1}{M^{d_i-4}}$$

One Loop: $\mathcal{L}_{\rm eff} = \frac{1}{2}\, a\, (\partial\phi)^2 - \frac{1}{2}\, b\, \phi^2 + c\, \frac{\lambda^2}{8M^2}\, \phi^4 + \cdots$



$$a = 1 + a_1\, \frac{\lambda^2}{16\pi^2 M^2} + \cdots \qquad ; \qquad b = m^2 + b_1\, \frac{\lambda^2}{16\pi^2} + \cdots$$

$$c = 1 + c_1\, \frac{\lambda^2}{16\pi^2 M^2} + \cdots \qquad ; \qquad \cdots$$

Principles of Effective Field Theory

- **Low-energy dynamics** independent of details at high energies
- Appropriate physics description at the analyzed scale **(degrees of freedom)**
- **Energy gaps:** $\mathbf{0 \leftarrow m \ll E \ll M \rightarrow \infty}$
- Non-local heavy-particle exchanges replaced by a **tower of local interactions** among the light particles
- **Accuracy:** $(E/M)^{(d_i-4)} \gtrsim \epsilon \quad \Longleftrightarrow \quad d_i \lesssim 4 + \frac{\log(1/\epsilon)}{\log(M/E)}$
- **Same infrared** (but different ultraviolet) **behaviour** than the underlying fundamental theory
- The only remnants of the high-energy dynamics are in the **low-energy couplings** and in the **symmetries** of the EFT

Evolution from High to Low Scales

Large μ

$\mathcal{L}(\phi_i) + \mathcal{L}(\phi_i, \Phi)$ ϕ_i, Φ

$\downarrow$ **Renormalization Group**

$\mu = M$ **Matching**

$\mathcal{L}(\phi_i) + \delta\mathcal{L}(\phi_i)$ ϕ_i

$\downarrow$ **Renormalization Group**

Small μ

Wilson Coefficients: $\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}}\, O_i$

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$$\langle O_i \rangle_B = Z_i(\epsilon,\mu)\, \langle O_i(\mu)\rangle_R \qquad ; \qquad \mu \frac{d}{d\mu} \langle O_i \rangle_B = 0$$

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$$\left(\mu\, \frac{d}{d\mu} + \gamma_{O_i}\right) \langle O_i \rangle_R = 0 \qquad ; \qquad \gamma_{O_i} \equiv \frac{\mu}{Z_i}\, \frac{dZ_i}{d\mu} = \gamma_{O_i}^{(1)}\, \frac{\alpha}{\pi} + \gamma_{O_i}^{(2)} \left(\frac{\alpha}{\pi}\right)^2 + \cdots$$

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$$\mu\,\frac{d}{d\mu}\, c_i(\mu)\,\langle O_i\rangle_R = 0 \qquad \longrightarrow \qquad \left(\mu\,\frac{d}{d\mu} - \gamma_{O_i}\right) c_i(\mu) = 0$$

$$\longrightarrow \qquad \boxed{\begin{aligned} c_i(\mu) &= c_i(\mu_0)\ \exp\left\{\int_{\alpha_0}^{\alpha} \frac{d\alpha}{\alpha}\,\frac{\gamma_{O_i}(\alpha)}{\beta(\alpha)}\right\} \\ &= c_i(\mu_0)\left[\frac{\alpha(\mu^2)}{\alpha(\mu_0^2)}\right]^{\gamma_{O_i}^{(1)}/\beta_1}\left\{1 + \cdots\right\}\end{aligned}}$$

Operator Mixing:

$\mathcal{L} = \sum_i \frac{c_i}{\Lambda^{d_i-4}}\, O_i$

$$\langle O_i \rangle_B = \sum_j \mathbf{Z}_{ij}(\epsilon,\mu)\,\langle O_j(\mu)\rangle_R \qquad ; \qquad \gamma_O \equiv \mathbf{Z}^{-1}\,\mu\,\frac{d}{d\mu}\,\mathbf{Z}$$

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Diagonalization: $\left(\mathbf{U}^{-1}\gamma_O^T\,\mathbf{U}\right)_{ij} = \tilde{\gamma}_{O_i}\,\delta_{ij} \qquad ; \qquad \tilde{c}_i = \mathbf{U}^{-1}_{ij}\, c_j$

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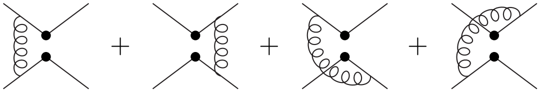
$$\rightarrow \qquad \boxed{c_i(\mu) = \sum_{j,k} \mathbf{U}_{ij} \, \exp\left\{ \int_{\alpha_0}^{\alpha} \frac{d\alpha}{\alpha} \, \frac{\tilde{\gamma}_{O_j}(\alpha)}{\beta(\alpha)} \right\} \mathbf{U}_{jk}^{-1} \, c_k(\mu_0)}$$

Wilson Coeff. in the Fermi EFT

$$\mathcal{L}_{\mathrm{eff}} = \frac{G_F}{\sqrt{2}} V_{12} V_{43}^* \, O_{\{1,2;3,4\}} \qquad ; \qquad O_{\{1,2;3,4\}} \equiv [\bar{q}_1 \gamma^\mu (1-\gamma_5) q_2] \, [\bar{q}_3 \gamma_\mu (1-\gamma_5) q_4]$$



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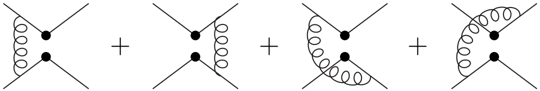

Colour: $$\sum_a \lambda^a_{ij} \lambda^a_{kl} = -\frac{2}{N_C} \, \delta_{ij} \, \delta_{kl} + 2 \, \delta_{il} \, \delta_{kj}$$

Fierz: $$[\gamma^\mu (1-\gamma_5)]_{\alpha\beta} \, [\gamma_\mu (1-\gamma_5)]_{\gamma\delta} = -[\gamma^\mu (1-\gamma_5)]_{\alpha\delta} \, [\gamma_\mu (1-\gamma_5)]_{\gamma\beta}$$

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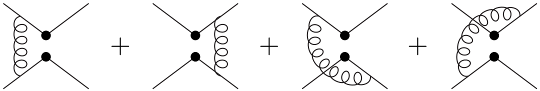

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$$\mathcal{L}_{\rm eff} = \frac{G_F}{2\sqrt{2}}\, V_{12} V_{43}^*\; \{c_+(\mu)\, Q_+ + c_-(\mu)\, Q_-\} \qquad ; \qquad Q_\pm \equiv O_{\{1,2;3,4\}} \pm O_{\{1,4;3,2\}}$$

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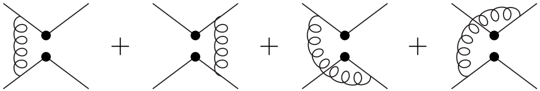

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$$\gamma_\pm = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \frac{\alpha_s}{\pi} \quad \Longrightarrow \quad \boxed{ c_\pm(\mu) \approx \left(\frac{\alpha_s(M_W^2)}{\alpha_s(\mu^2)} \right)^{a_\pm} \quad , \quad a_\pm = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \frac{6}{33 - 2 N_f} }$$

Backup Slides

QCD Matching

$$(\mu > M) \qquad \mathcal{L}_{\mathrm{QCD}}^{(N_F)} \quad \Longleftrightarrow \quad \mathcal{L}_{\mathrm{QCD}}^{(N_F-1)} + \sum_{d_i>4} \frac{c_i}{M^{d_i-4}}\, O_i \qquad (\mu < M)$$

QCD Matching

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$$\boxed{\begin{array}{c} \alpha_s^{(N_F)}(\mu^2) \;=\; \alpha_s^{(N_F-1)}(\mu^2)\, \left\{ 1 + \sum_{k=1}^{\infty} C_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi}\right]^k \right\} \\ L \equiv \ln\left(\mu^2/m_q^2\right) \\ m_q^{(N_F)}(\mu^2) \;=\; m_q^{(N_F-1)}(\mu^2)\, \left\{ 1 + \sum_{k=1}^{\infty} H_k(L) \left[\frac{\alpha_s^{(N_F-1)}(\mu^2)}{\pi}\right]^k \right\} \end{array}}$$

QCD Matching

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- Matching conditions known to 4 (3) loops: $\quad C_{1,2,3,4}\; ,\;\; H_{1,2,3}$

  (Schroder-Steinhauser, Chetyrkin et al, Larin et al)

- L dependence known to 4 loops: $\quad H_4(L)$
- **$\alpha_s(\mu^2)$ is not continuous at threshold**