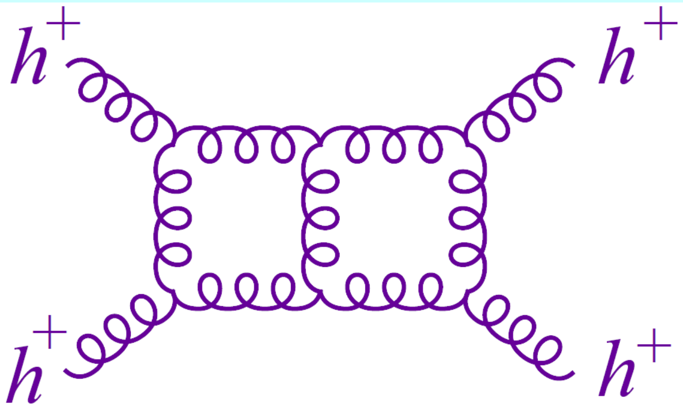
Are UV Poles Arbitrary in Quantum Gravity at Two Loops?



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Current Themes in HEP & Cosmology
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Introduction

- Quantum gravity nonrenormalizable by power counting: Newton's constant, $G_N = 1/M_{Pl}^2$ is dimensionful
- String theory cures divergences of quantum gravity – but particles are no longer pointlike.
- Is this necessary? Or could enough symmetry allow a point particle gravity theory to be perturbatively ultraviolet finite in D=4?
- N=8 supergravity (ungauged) DeWit, Freedman (1977); Cremmer, Julia, Scherk (1978); Cremmer, Julia (1978,1979)

verified explicitly to be finite through 4 loops

Bern, Carrasco, LD, Johansson, Roiban, 0905.2326, 1008.3327, 1201.5366

expected to be finite at least until 7 loops

Bossard, Howe, Stelle, 1009.0743; Beisert et al. 1009.1643;

- What about other theories, including pure Einstein gravity?

Other (point-like) proposals

- Asymptotic safety program:

 Gravity plus higher-dimension operators could flow to (conjectured?) nontrivial fixed points:

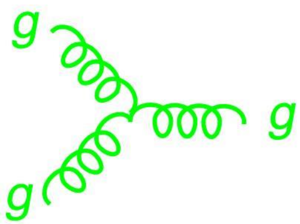
Weinberg (1979); ...; Niedermaier, Reuter, Liv. Rev. Rel. **9**, 5 (2006); …

- UV theory could be Lorentz asymmetric, but renormalizable   Hořava, 0812.4287, 0901.3775

- Here we perform a standard perturbative analysis

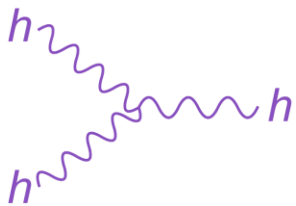
Why gravity should behave badly

gauge theory (spin 1) renormalizable



$$\supset \ell^{\mu}\,\eta^{\nu\rho} + \cdots$$

gravity (spin 2) nonrenormalizable



$$\supset \ell^{\mu_1}\,\ell^{\mu_2}\,\eta^{\nu_1\rho_1}\,\eta^{\nu_2\rho_2} + \cdots$$

Extra $\dfrac{\ell^2}{M_{\mathrm{Pl}}^2}$ per loop

Counterterm Basics

- Divergences associated with local counterterms
- On-shell counterterms are generally covariant
- Built out of products of Riemann tensor $R_{\mu\nu\sigma\rho}$ and covariant derivatives $\mathcal{D}_\mu$
- Terms containing Ricci tensor $R_{\mu\nu}$ and Ricci scalar R are removable by nonlinear field redefinitions (~eqns of motion) in Einstein action

$R^{\mu}_{\nu\sigma\rho} \sim \partial_\rho \Gamma^{\mu}_{\nu\sigma} \sim g^{\mu\kappa}\partial_\rho\partial_\nu g_{\kappa\sigma}$ has mass dimension 2

$G_N = 1/M_{Pl}^2$ has mass dimension -2

Each additional $R_{\mu\nu\sigma\rho}$ or $\mathcal{D}^2$ ←→ 1 more loop (in D=4)

One loop

- Pure gravity has only one available counterterm:

$$R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$$

- However, $R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\sigma\rho}R^{\mu\nu\sigma\rho}$

the Gauss-Bonnet term, is a total derivative in four dimensions

- So pure gravity is UV finite at one loop

't Hooft, Veltman (1974)

- Matter → $T_{\mu\nu}T^{\mu\nu}$ → one loop divergences – for amplitudes with 4 external scalars

Two loops

$$R^3 \equiv R^{\lambda\rho}{}_{\mu\nu} R^{\mu\nu}{}_{\sigma\tau} R^{\sigma\tau}{}_{\lambda\rho}$$

- Unique counterterm available for pure Einstein gravity, using on-shell conditions (field redefinitions) in D=4

- UV pole in 3-point function computed by Goroff, Sagnotti, Phys. Lett. B160, 81 (1985), Nucl. Phys. B266, 709 (1986)

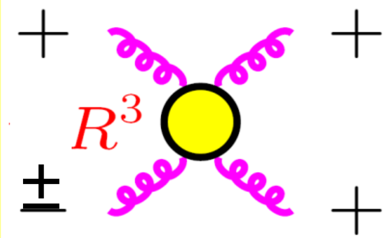
$$\mathcal{L}_{R^3} = -\frac{209}{1440}\left(\frac{\kappa}{2}\right)^2 \frac{1}{(4\pi)^4}\frac{1}{\epsilon}\sqrt{-g}\, R^3$$

where $\kappa^2 = 32\pi G_N$

- Confirmed by van de Ven, Nucl. Phys. B378, 309 (1992)

Aside: Pure supergravity ($\mathcal{N} \geq 1$):
Divergences deferred to at least three loops

R^3 can't be supersymmetrized:
helicity amplitudes (±+++) incompatible with SUSY Ward identities
Grisaru; Deser, Kay, Stelle; Tomboulis (1977)



Three loops → supersymmetric counterterm, abbreviated R^4
plus (many) other terms containing other fields in SUSY multiplet
Deser, Kay, Stelle (1977); Howe, Lindström (1981); Kallosh (1981); Howe, Stelle, Townsend (1981)

R^4 produces first subleading term in low-energy limit of 4-graviton scattering in (N=8 supersymmetric) type II string theory:

$$\alpha'^3 R^4 \Rightarrow \alpha'^3 stu \underbrace{M_4^{\text{tree}}(1,2,3,4)}_{\text{4-graviton amplitude in (super)gravity}}$$

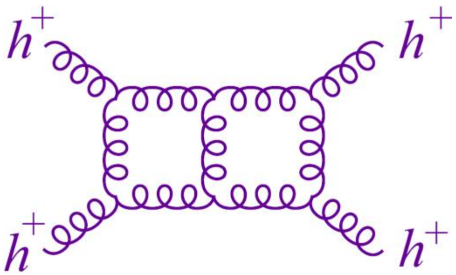
Gross, Witten (1986)

Despite this, it doesn't cause a divergence in $\mathcal{N} \geq 4$ SUGRA
Bern, Davies, Dennen, Huang, 1202.3423, 1209.2472

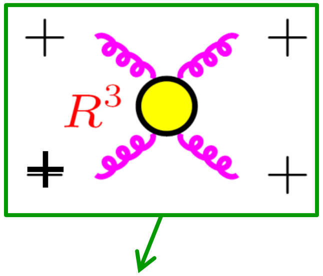
Back to pure gravity @ 2 loops

- Using unitarity in *D=4-2ε* dimensions, we computed the bare two-loop 4-graviton amplitude

$M_4^{(2)}(++++)$ =



+ ...



- Expected to find:

$$M_4^{(2)}(++++)|_{\text{UV pole}} = A_{R^3} = \frac{209}{24}\frac{\mathcal{K}}{\epsilon}$$

- Found:

$$M_4^{(2)}(++++)|_{\text{UV pole}} = -\frac{3431}{5400}\frac{\mathcal{K}}{\epsilon}$$

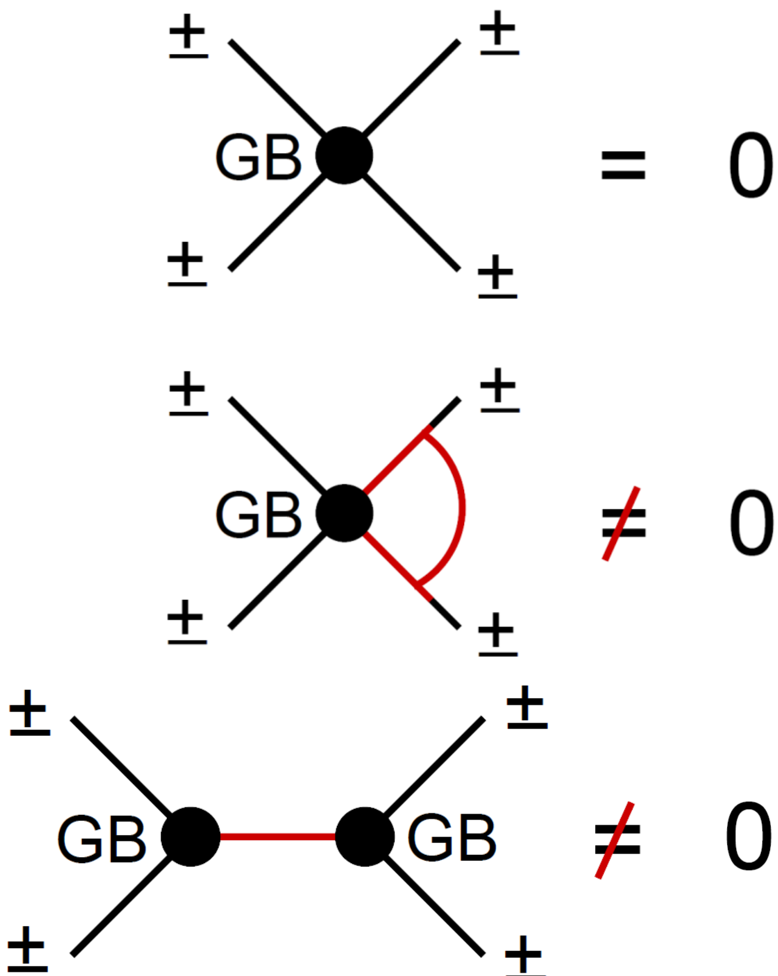
$$\mathcal{K} = \frac{i(\kappa/2)^6\, stu}{(4\pi)^4} \times \text{phase}$$

???

Then we remembered

- Although ’t Hooft and Veltman told us we could ignore the Gauss-Bonnet term because it was a total derivative in *D*=4, they also gave us dimensional regularization – and GB is not a total derivative for arbitrary *D*.
- Example of an evanescent operator, well-studied in gauge theory, especially for higher-order corrections to anomalous dimensions, … Buras, Weisz, Nucl. Phys. B 333, 66 (1990); Dugan, Grinstein, Phys. Lett. B 256, 239 (1991); Jack, Jones, Roberts hep-ph/9401349; Herrlich, Nierste, hep-ph/9412375; Harlander, Kant, Mihaila, Steinhauser, hep-ph/0607240
- Need to identify its divergent coefficient and insert it into one-loop amplitudes and trees.

D=4 vs. D=4-2ε



1-Loop Coefficient of GB well-known

Trace anomaly [also related to "conformal anomaly"]
Capper, Duff, (1974,1975); Tsao (1977); Gibbons, Hawking, Perry (1978); Critchley (1978); Duff, hep-th/9308075

Computed for arbitrary fields in the loop by
Duff, van Nieuwenhuizen, Phys. Lett. B 94, 179 (1980)
"Quantum Inequivalence of Different Field Representations"

scalars

2-forms
a.k.a. axions(?)

3-forms
a.k.a. nothing(?)

$$\mathcal{L}_{\mathrm{GB}} = \frac{1}{(4\pi)^2}\frac{1}{\epsilon}\left[\frac{53}{90} + \frac{1}{360}n_0 + \frac{91}{360}n_2 - \frac{1}{2}n_3\right] \times\sqrt{-g}[R^2 - 4R_{\mu\nu}^2 + R_{\mu\nu\rho\sigma}^2]$$

Followed promptly by Siegel, Phys .Lett. B103, 107 (1981)
"Quantum Equivalence of Different Field Representations"

What's going on?

- If the one-loop $\mathcal{L}_{\rm GB}$ affects the 2-loop UV pole, will it depend on whether we add scalars (n_0) or 2-forms (n_2) which are supposed to be dual to (pseudo)scalars?

- If we add 3-forms (n_3), "evanescent fields" which do not even propagate in 4 dimensions, could that affect the pure gravity divergence at 2 loops – without messing up the 1 loop finiteness of pure gravity?

Aside on IR divergences

- Since we compute on-shell 4-point amplitudes, we have to remove IR poles in order to extract the UV ones.
- Because $M_4(++++)$ vanishes at tree level, its 2-loop IR poles are essentially equivalent to those of a one-loop amplitude, from single graviton exchange between pairs of external legs Weinberg (1965); Naculich, Schnitzer, 1101.1524; Naculich, Nastase, Schnitzer, 1301.2234

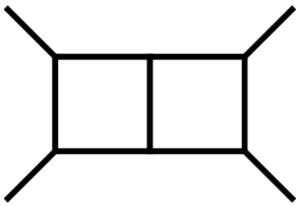
$$M_4^{(2)}(++++) = \frac{\kappa^2}{32\pi^2\,\epsilon}[s\ln s + t\ln t + u\ln u]M_4^{(1)}(++++)$$

- For finite parts, need $O(\varepsilon)$ terms in 1-loop amplitude, including overall factor of number of states in D dimensions [(D-2)(D-1)/2 – 1 = D(D-3)/2 for pure gravity]

First consider gravity + 3 forms

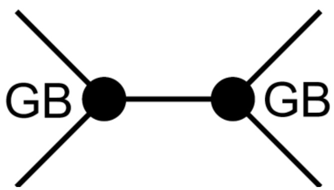
$$(\mu^2)^{L\epsilon} \int (d^{4-2\epsilon}\ell)^L$$

$$1/\epsilon \quad \rightarrow \quad \ln \mu^2$$



$$-\frac{3431}{5400} - \frac{199}{30} n_3 + 6 n_3^2 \quad \xrightarrow{\times 2} \quad -\frac{3431}{2700} - \frac{199}{15} n_3 + 12 n_3^2$$

$$\left(\frac{53}{90} - \frac{1}{2} n_3\right)\left(\frac{26}{15} + 24 n_3\right) \quad \xrightarrow{\times 1} \quad \frac{689}{275} + \frac{199}{15} n_3 - 12 n_3^2$$

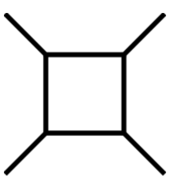
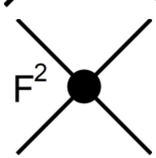


$$\left(\frac{53}{90} - \frac{1}{2} n_3\right)^2 \times 24 \quad \rightarrow \quad 0$$

Total $\quad \frac{209}{24} - \frac{15}{2} n_3 \qquad -\frac{1}{4}$

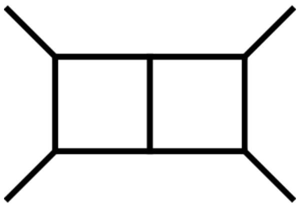
Conclusions so far

- A non-propagating field (in D=4) can change the leading $1/\varepsilon$ UV pole in a theory!
- At the same time, it doesn't affect the physics in the renormalized theory: the coefficient of $\ln \mu^2$ is independent of it (and is totally unrelated to the $1/\varepsilon$ pole)!
- Compare with textbook 1-loop situation: $(\mu^2)^\epsilon \int d^{4-2\epsilon}\ell$

	$1/\epsilon$		$\ln \mu^2$
	$\boxed{\beta_0}$	$\times 1$ $\rightarrow$	β_0
[	$-\beta_0$	$\rightarrow$	0]
Total	0		$\boxed{\beta_0}$

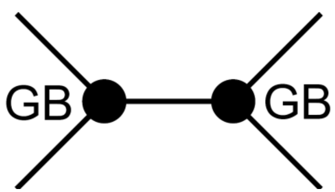
What about gravity + scalars [vs. 2 forms]?

$$1/\epsilon \quad \rightarrow \quad \ln \mu^2$$



$$-\frac{3431}{5400}-\frac{277}{10800}n_0+\frac{1}{5400}n_0^2 \quad \xrightarrow{\times 2} \quad -\frac{3431}{2700}-\frac{277}{5400}n_0+\frac{1}{2700}n_0^2$$

$$\left(\frac{53}{90}+\frac{n_0}{360}\right)\left(\frac{26}{15}-\frac{2n_0}{15}\right) \quad \xrightarrow{\times 1} \quad \frac{689}{275}-\frac{199}{2700}n_0-\frac{1}{2700}n_0^2$$



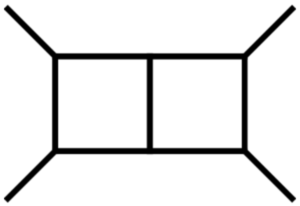
$$\left(\frac{53}{90}+\frac{n_0}{360}\right)^2 \times 24 \quad \rightarrow \quad 0$$

Total

$$\frac{209}{24}-\frac{1}{48}n_0 \qquad\qquad -\frac{1}{8}(2+n_0)$$

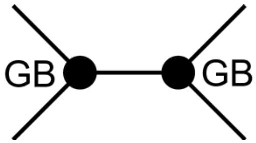
gravity + 2 forms

$$1/\epsilon \quad \rightarrow \quad \ln \mu^2$$

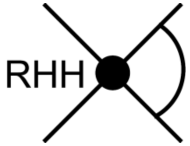


$$-\frac{3431}{5400}+\frac{8543}{10800}n_2+\frac{8281}{5400}n_2^2 \xrightarrow{\times 2} -\frac{3431}{2700}+\frac{8543}{5400}n_2+\frac{8281}{2700}n_2^2$$

$$\left(\frac{53}{90}+\frac{91}{360}n_2\right)\left(\frac{26}{15}-\frac{2\cdot 91 n_2}{15}\right) \xrightarrow{\times 1} \frac{689}{275}-\frac{18109}{2700}n_2-\frac{8281}{2700}n_2^2$$



$$\left(\frac{53}{90}+\frac{91}{360}n_2\right)^2 \times 24 \rightarrow 0$$



$$5n_2 \xrightarrow{\times 1} 5n_2$$

Total

$$\frac{209}{24}+\frac{299}{48}n_2 \qquad -\frac{1}{8}(2+n_2)$$

Conclusions part deux

- 2 forms and scalars → different $1/\varepsilon$ UV poles at 2 loops!
- At the same time, it doesn't affect the physics in the renormalized theory: the coefficient of $\ln \mu^2$ is independent of it (and is totally unrelated to the $1/\varepsilon$ pole)!
- "Quantum equivalence" under duality transformations holds only when that equivalence allows for the adjustment of coefficients of higher-dimension operators.
- This caveat is not found in previous arguments for quantum equivalence.

Siegel, Phys. Lett. B103, 107 (1981); Fradkin, Tseytlin, Ann. Phys. 162, 31 (1985); Grisaru et al., Nucl. Phys. B247, 157 (1984).

Conclusions part trois

- The trace anomaly and Gauss-Bonnet term play a key role in the UV pole structure of pure gravity.
- Remarkably, an evanescent operator can affect a leading UV pole.
- On the other hand, this pole is not really physical, compared with coefficient of log of renormalization scale.
- Can one establish this quantum equivalence beyond two loops? (Also check other helicities at two loops.)
- How would things look with a non-dimensional UV regulator?
- Nonvanishing 1-loop GB coefficient for pure N=1 supergravity, yet it should not diverge until 3 loops. [M. Duff]
- Does GB matrix element vanish at 2 loops in the supersum, or cancel against a bare divergence or other evanescent operator?

Ultraviolet Behavior Can Still Be Interesting Even at a Mere Two Loops