Non-renormalization Theorems without Supersymmetry

Clifford Cheung

C. C. and Chia-Hsien Shen (1505.01844)

effective field theory lore

$$\mathcal{L} = \sum_i c_i \mathcal{O}_i$$

$c_i \sim 1$ in units of the cutoff Λ.

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- ultraviolet democracy of parameters
- couplings are generated by running

enhanced symmetry at

$$c_i = 0$$

e.g. chiral symmetry, supersymmetry,

$$\xrightarrow{\text{'t Hooft naturalness}}$$

radiative stability of

$$c_i \ll 1$$

e.g. electron mass, Higgs mass,

enhanced symmetry at	't Hooft naturalness	radiative stability of
$c_i = 0$	$\longrightarrow$	$c_i \ll 1$
e.g. chiral symmetry, supersymmetry,	(fact)	e.g. electron mass, Higgs mass,

True because c_i is a spurion for breaking, so

$$\frac{dc_i}{d\log\mu} \propto c_i$$

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$\longrightarrow$

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$$c_i = 0 \longleftarrow c_i \ll 1$$

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colloquial "naturalness"

$$\longleftarrow$$

(folklore)

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$$c_i \ll 1$$

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radiative stability of

$$c_i \ll 1$$

e.g. electron mass, Higgs mass,

Logically, this "naturalness" is equivalent to

no symmetry

"anything goes"

$$c_i \sim 1$$

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Radiative stability without symmetry???

standard model
effective field theory

Standard model effective theory is defined by

$$\mathcal{L} = \mathcal{L}_{\mathrm{SM}} + \Delta\mathcal{L}$$

$$\Delta\mathcal{L} = \sum_i c_i \mathcal{O}_i$$

where $[\mathcal{O}_i] = 6$ and B and L are assumed.

Caveat: operator basis $\{\mathcal{O}_i\}$ is not unique.

Inserting equations of motion is equivalent to a field redefinition at leading order, e.g.

$$\Box\phi \times (\cdot) \xrightarrow{\text{eq of motion}} -V'(\phi) \times (\cdot)$$

$$\phi \xrightarrow{\text{field redef}} \phi + (\cdot)$$

However, the S-matrix is left invariant.

Choose a basis of 59 operators, mod flavor.

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X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star\left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r)\tau^I\varphi W^I_{\mu\nu}$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar l_p\gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r)\varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar l_p\tau^I\gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r)\widetilde\varphi\, G^A_{\mu\nu}$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar e_p\gamma^\mu e_r)$
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$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r)\widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu\,\varphi)(\bar q_p\tau^I\gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r)\varphi\, G^A_{\mu\nu}$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar u_p\gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r)\tau^I\varphi\, W^I_{\mu\nu}$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu\,\varphi)(\bar d_p\gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r)\varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu\varphi)(\bar u_p\gamma^\mu d_r)$

Grzadkowski, Iskrzynski, Misiak, Rosiek (1008.4884)

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anomalous gauge vertex

anomalous magnetic moment

Higgs to di-gamma

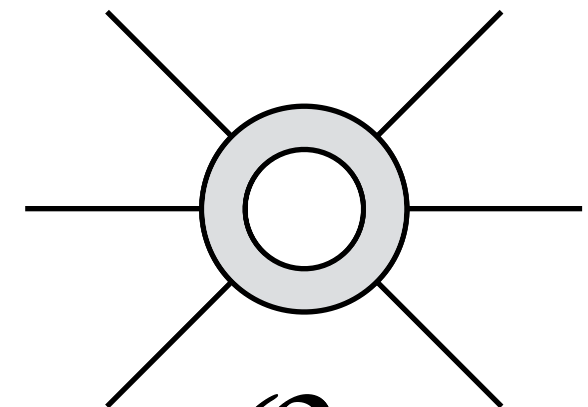
Grzadkowski, Iskrzynski, Misiak, Rosiek (1008.4884)

Manohar et. al computed the leading RG, encoded in the anomalous dimension matrix.

$$(4\pi)^2 \frac{dc_i}{d\log\mu} = \sum_j \gamma_{ij} c_j$$

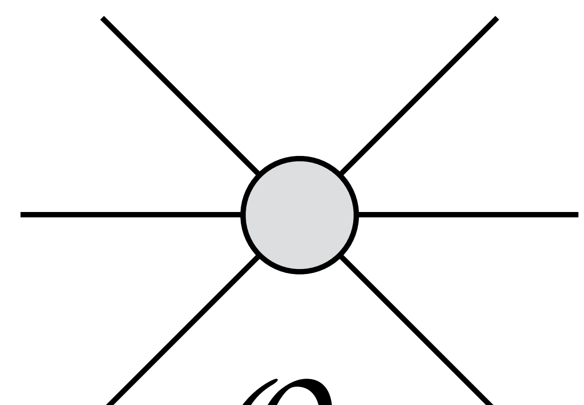
Here γ_{ij} is a dimensionless matrix made of the marginal couplings in the theory.

Alonso, Jenkins, Manohar, Trott (1308.2627, 1309.0819, 1310.4838, 1312.2014, 1409.0868) one-loop amplitude $= \sum_j c_j$



$\mathcal{O}_j$

$= \frac{1}{(4\pi)^2\epsilon}\sum_{ij}\gamma_{ij}c_j$



$\mathcal{O}_i$

$+$ finite

For convenience, express Lorentz covariance in terms of spinor indices, $\mu \leftrightarrow \alpha, \dot{\alpha}$.

scalars: ϕ

fermions: $\psi_\alpha, \bar{\psi}_{\dot{\alpha}}$

vectors: $F_{\alpha\dot{\alpha}\beta\dot{\beta}} = F_{\alpha\beta}\bar{\epsilon}_{\dot{\alpha}\dot{\beta}} + \bar{F}_{\dot{\alpha}\dot{\beta}}\epsilon_{\alpha\beta}$

derivatives: $D_{\alpha\dot{\alpha}}$

Mod Lorentz and flavor structures, reduce to 14 operator classes split into 3 groups.

holomorphic $\leftrightarrow \alpha, \beta, \ldots$

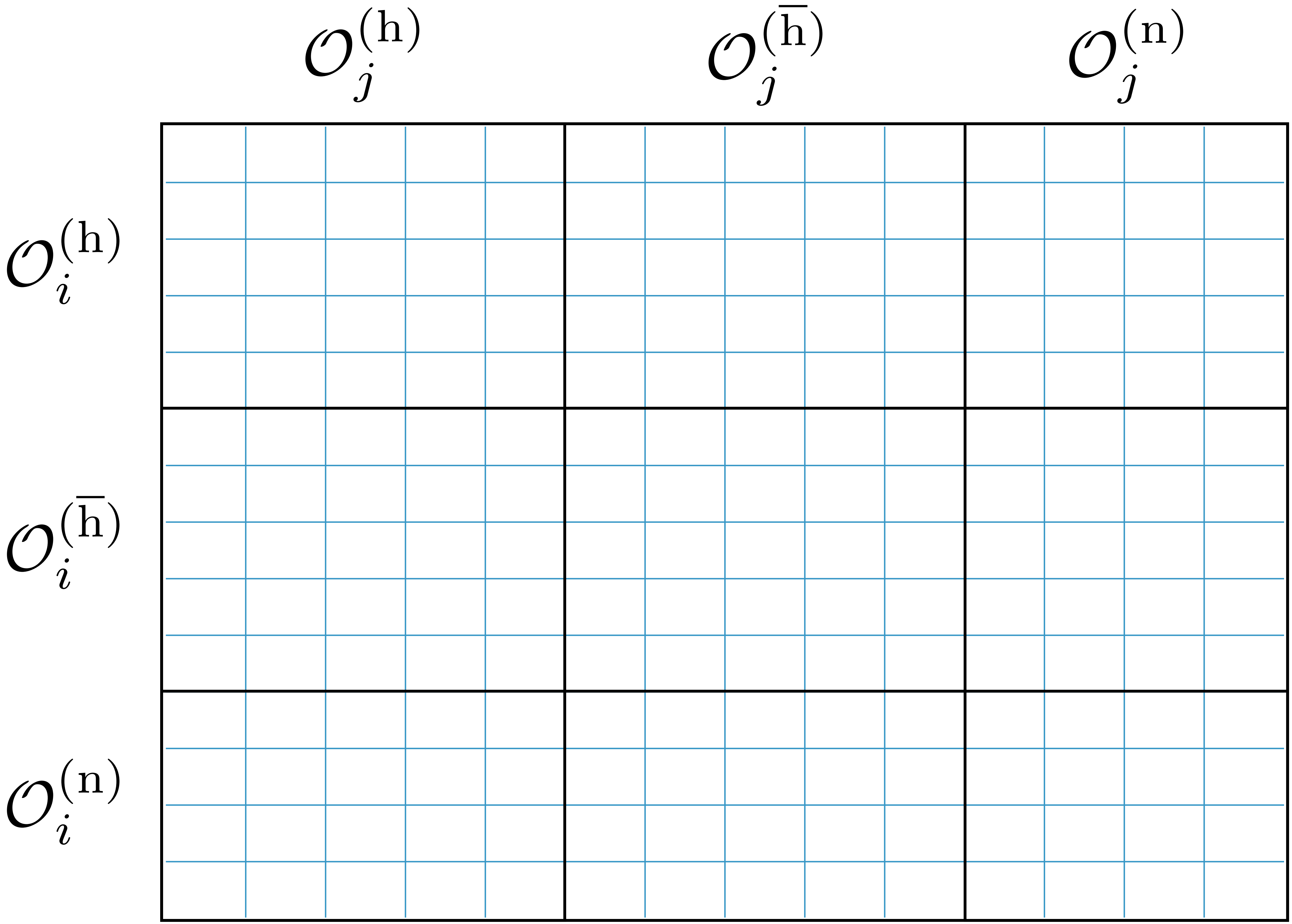
$$\mathcal{O}^{(\mathrm{h})} = F^3,\ F^2\phi^2,\ F\psi^2\phi,\ \psi^4,\ \psi^2\phi^3$$

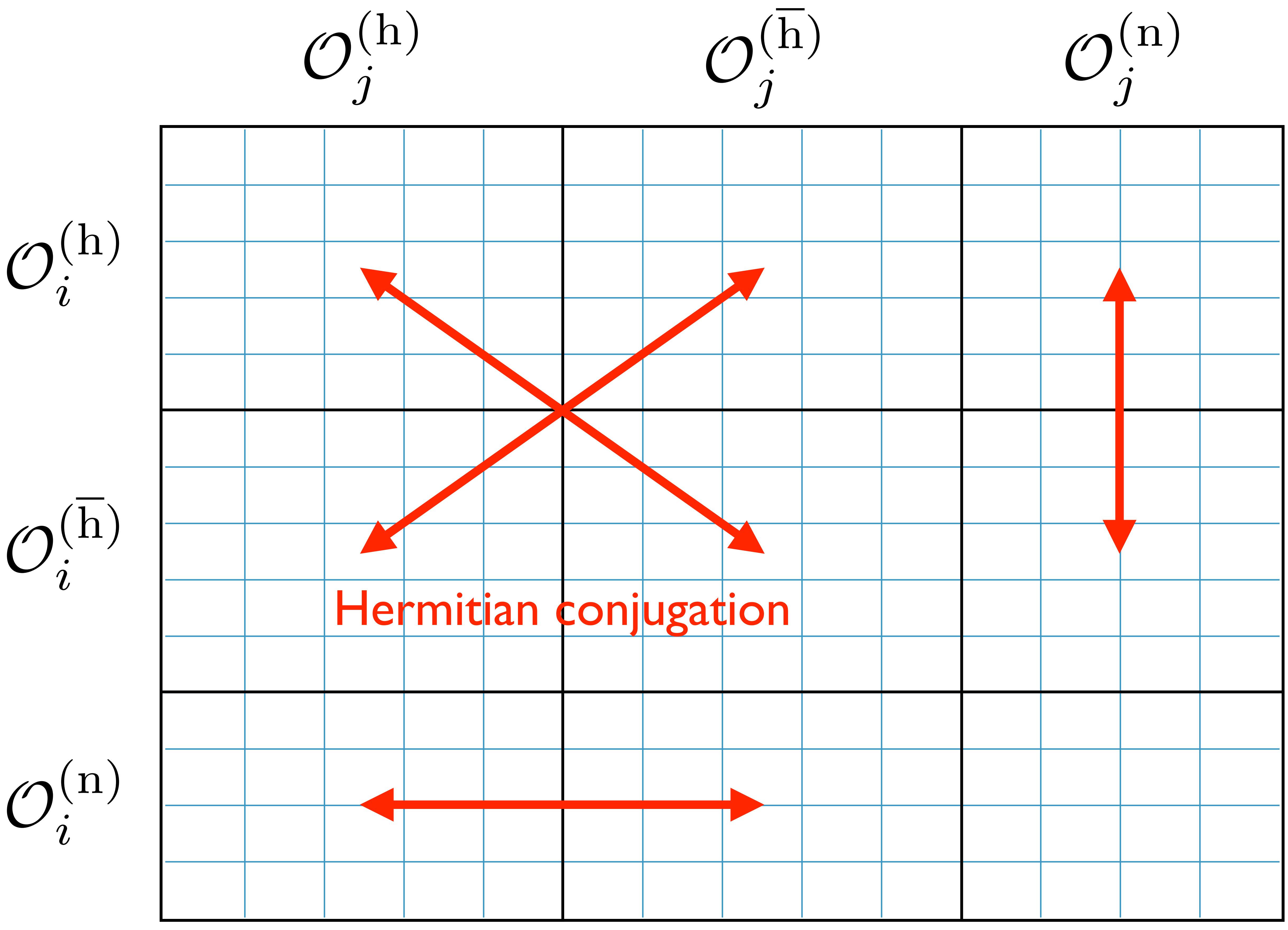
anti-holomorphic $\leftrightarrow \dot{\alpha}, \dot{\beta}, \ldots$

$$\mathcal{O}^{(\overline{\mathrm{h}})} = \bar{F}^3,\ \bar{F}^2\phi^2,\ \bar{F}\bar{\psi}^2\phi,\ \bar{\psi}^4,\ \bar{\psi}^2\phi^3$$

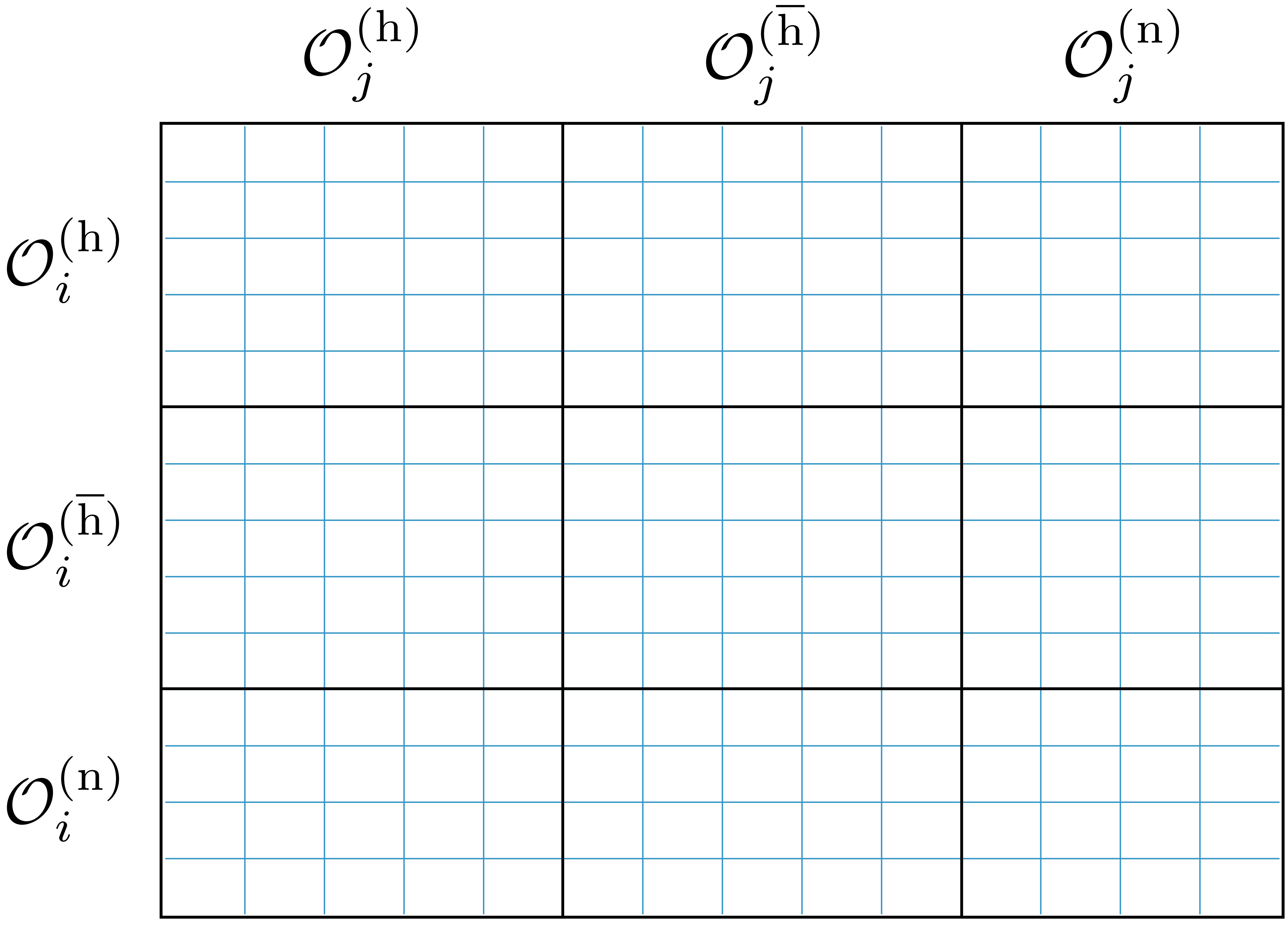
non-holomorphic $\leftrightarrow \alpha, \dot{\alpha}, \beta, \dot{\beta}, \ldots$

$$\mathcal{O}^{(\mathrm{n})} = \bar{\psi}^2\psi^2,\ \bar{\psi}\psi\phi^2 D,\ \phi^4 D^2,\ \phi^6$$


$\mathcal{O}_j^{(\mathrm{h})}$
$\mathcal{O}_j^{(\overline{\mathrm{h}})}$
$\mathcal{O}_j^{(\mathrm{n})}$
$\mathcal{O}_i^{(\mathrm{h})}$
$\mathcal{O}_i^{(\overline{\mathrm{h}})}$
$\mathcal{O}_i^{(\mathrm{n})}$



$\mathcal{O}_j^{(\mathrm{h})}$
$\mathcal{O}_j^{(\overline{\mathrm{h}})}$
$\mathcal{O}_j^{(\mathrm{n})}$
$\mathcal{O}_i^{(\mathrm{h})}$
$\mathcal{O}_i^{(\overline{\mathrm{h}})}$
$\mathcal{O}_i^{(\mathrm{n})}$
Hermitian conjugation



$\mathcal{O}_j^{(\mathrm{h})}$
$\mathcal{O}_j^{(\overline{\mathrm{h}})}$
$\mathcal{O}_j^{(\mathrm{n})}$
$\mathcal{O}_i^{(\mathrm{h})}$
$\mathcal{O}_i^{(\overline{\mathrm{h}})}$
$\mathcal{O}_i^{(\mathrm{n})}$

X = no diagram

	$\mathcal{O}_j^{(\mathrm{h})}$					$\mathcal{O}_j^{(\bar{\mathrm{h}})}$					$\mathcal{O}_j^{(\mathrm{n})}$			
$\mathcal{O}_i^{(\mathrm{h})}$			X	X	X			X	X	X	X	X	X	X
				X	X				X	X	X			X
									X				X	X
	X	X			X	X	X	X	X	X			X	X
	X													X
$\mathcal{O}_i^{(\bar{\mathrm{h}})}$			X	X	X			X	X	X	X	X	X	X
				X	X				X	X	X			X
				X									X	X
	X	X	X	X	X	X	X			X			X	X
						X								X
$\mathcal{O}_i^{(\mathrm{n})}$		X			X		X			X			X	X
														X
				X					X		X			X
	X		X	X		X		X	X		X			

X = no diagram

$\color{red}{X}$ = cancels!

	$\mathcal{O}_j^{(\mathrm{h})}$					$\mathcal{O}_j^{(\bar{\mathrm{h}})}$					$\mathcal{O}_j^{(\mathrm{n})}$			
$\mathcal{O}_i^{(\mathrm{h})}$		$\color{red}{X}$	X	X	X	$\color{red}{X}$	$\color{red}{X}$	X	X	X	X	X	X	X
				X	X	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	X	X	X	$\color{red}{X}$	$\color{red}{X}$	X
					$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	X	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	X	X
	X	X			X	X	X	X	X	X		$\color{red}{X}$	X	X
	X					$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$					X
$\mathcal{O}_i^{(\bar{\mathrm{h}})}$	$\color{red}{X}$	$\color{red}{X}$	X	X	X		$\color{red}{X}$	X	X	X	X	X	X	X
	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	X	X				X	X	X	$\color{red}{X}$	$\color{red}{X}$	X
	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	X	$\color{red}{X}$					$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	X	X
	X	X	X	X	X	X	X			X		$\color{red}{X}$	X	X
	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$		X								X
$\mathcal{O}_i^{(\mathrm{n})}$	$\color{red}{X}$	X	$\color{red}{X}$		X	$\color{red}{X}$	X	$\color{red}{X}$		X			X	X
	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$				X
	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	X	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	$\color{red}{X}$	X	$\color{red}{X}$	X			X
	X		X	X		X		X	X		X			

X = no diagram

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	$\mathcal{O}_j^{(\mathrm{h})}$					$\mathcal{O}_j^{(\overline{\mathrm{h}})}$					$\mathcal{O}_j^{(\mathrm{n})}$			
$\mathcal{O}_i^{(\mathrm{h})}$		**X**	X	X	X	**X**	**X**	X	X	X	X	X	X	X
				X	X	**X**	**X**	**X**	X	X	X	**X**	**X**	X
					X	**X**	**X**	**X**	X	**X**	**X**	**X**	X	X
	X	X			X	X	X	X	X	X	$y_u y_{d,e}$	**X**	X	X
	X					**X**	**X**	**X**	**X**	$y_u y_{d,e}$				X
$\mathcal{O}_i^{(\overline{\mathrm{h}})}$	**X**	**X**	X	X	X		**X**	X	X	X	X	X	X	X
	X	**X**	**X**	X	X				X	X	X	**X**	**X**	X
	X	**X**	**X**	X	**X**					**X**	**X**	**X**	X	X
	X	X	X	X	X	X	X			X	$y_u y_{d,e}$	**X**	X	X
	X	**X**	**X**	**X**	$y_u y_{d,e}$	X								X
$\mathcal{O}_i^{(\mathrm{n})}$	**X**	X	**X**	$y_u y_{d,e}$	X	**X**	X	**X**	$y_u y_{d,e}$	X			X	X
	X	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**	**X**				X
	X	**X**	**X**	X	**X**	**X**	**X**	**X**	X	**X**	X			X
	X		X	X		X		X	X		X			

X = no diagram

X = cancels!

	$\mathcal{O}_j^{(h)}$					$\mathcal{O}_j^{(\bar{h})}$					$\mathcal{O}_j^{(n)}$			
$\mathcal{O}_i^{(h)}$		X	X	X	X	X	X	X	X	X	X	X	X	X
				X	X	X	X	X	X	X	X	X	X	X
					X	X	X	X	X	X	X	X	X	X
	X	X			X	X	X	X	X	X	$y_u y_{d,e}$	X	X	X
	X					X	X	X	X	$y_u y_{d,e}$				X
$\mathcal{O}_i^{(\bar{h})}$	X	X	X	X	X		X	X	X	X	X	X	X	X
	X	X	X	X	X				X	X	X	X	X	X
	X	X	X	X	X					X	X	X	X	X
	X	X	X	X	X	X	X			X	$y_u y_{d,e}$	X	X	X
	X	X	X	X	$y_u y_{d,e}$	X								X
$\mathcal{O}_i^{(n)}$	X	X	X	$y_u y_{d,e}$	X	X	X	X	$y_u y_{d,e}$	X			X	X
	X	X	X	X	X	X	X	X	X	X				X
	X	X	X	X	X	X	X	X	X	X	X			X
	X		X	X		X		X	X		X			

holomorphic non-renormalization?

A critical clue for the underlying mechanism:

"impurities" in holomorphic non-renormalization $\sim$ $y_u y_d, \; y_u y_e$

which are spurions for holomorphy violation,

$$\mathcal{L}_{\rm SM} \supset y_u q h u^c + y_d q h^\dagger d^c + y_e \ell h^\dagger e^c$$

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As we will see, this is simply the result of:

i) unitarity

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ii) helicity

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As we will see, this is simply the result of:

i) unitarity

&

ii) helicity (4D + on-shell)!

statement of new non-renormalization theorems

We derive one-loop non-renormalization theorems for general 4D QFTs.

Our proof centers on a gauge invariant, field reparameterization invariant observable:

$$A = \text{on-shell amplitude!}$$

definition of amplitude weight

$$w(A) = n(A) - h(A) \qquad \text{(weight)}$$

$$\overline{w}(A) = n(A) + h(A) \qquad \text{(anti-weight)}$$

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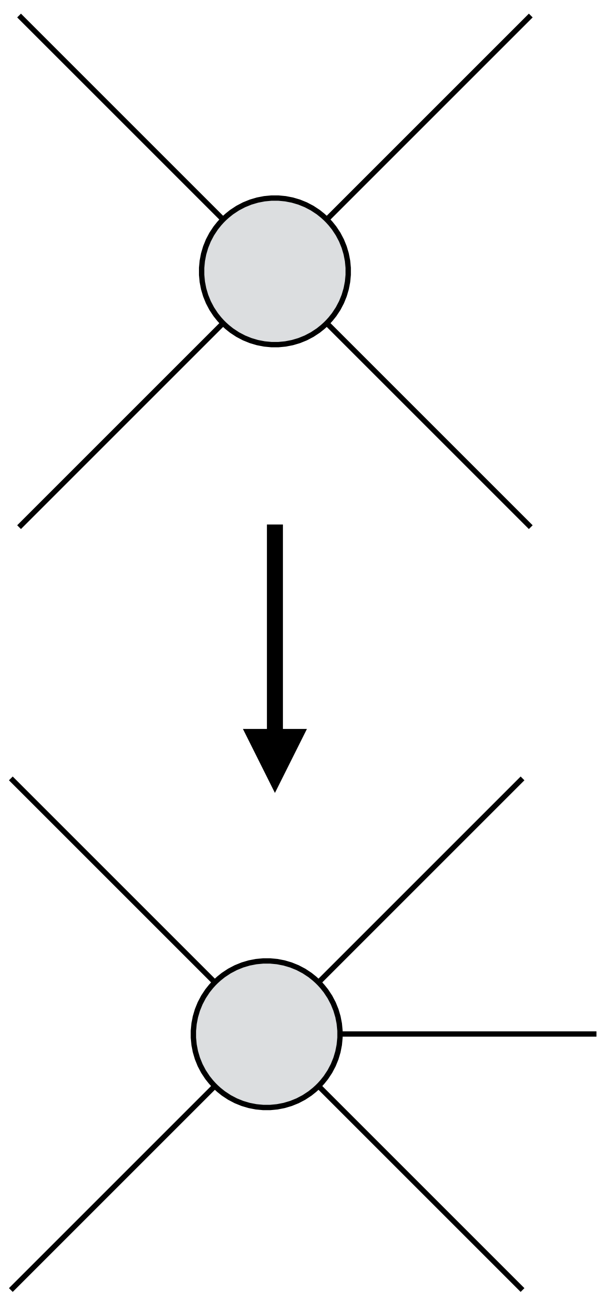
$$w(A) = n(A) - h(A) \qquad \text{(weight)}$$

$$\overline{w}(A) = n(A) + h(A) \qquad \text{(anti-weight)}$$

total number
of particles

total helicity
of particles

$w, \overline{w}$ monotonically increase when tacking on new vectors, fermions, and scalars.



$$\Delta n = 1$$

$$-1 \leq \Delta h \leq 1$$

$$0 \leq \Delta w, \Delta\overline{w} \leq 2$$

definition of operator weight

$$w(\mathcal{O}) = \min\{w(A)\} = n(\mathcal{O}) - h(\mathcal{O})$$

$$\overline{w}(\mathcal{O}) = \min\{\overline{w}(A)\} = n(\mathcal{O}) + h(\mathcal{O})$$

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marginalize
over all A≠0
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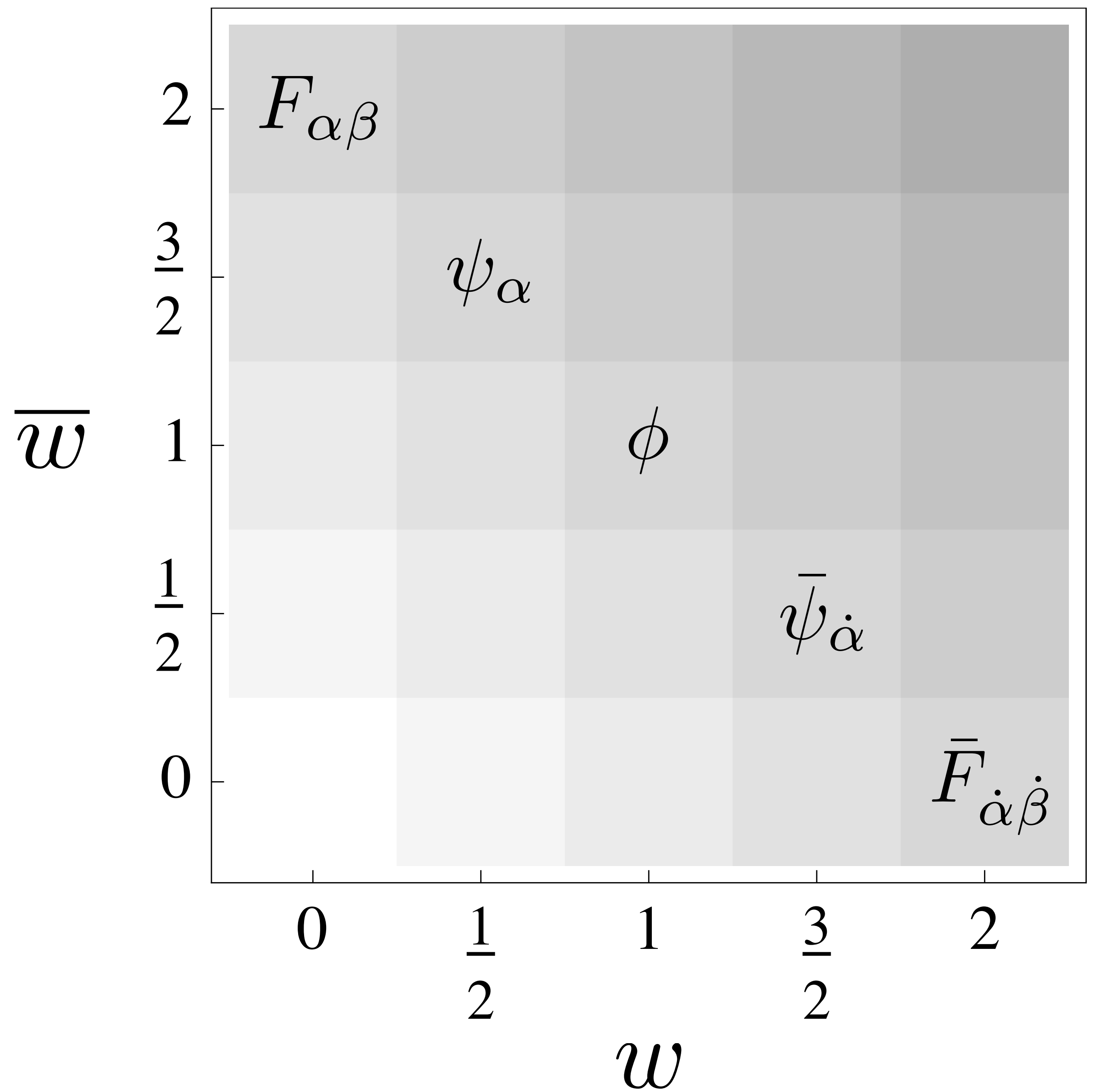
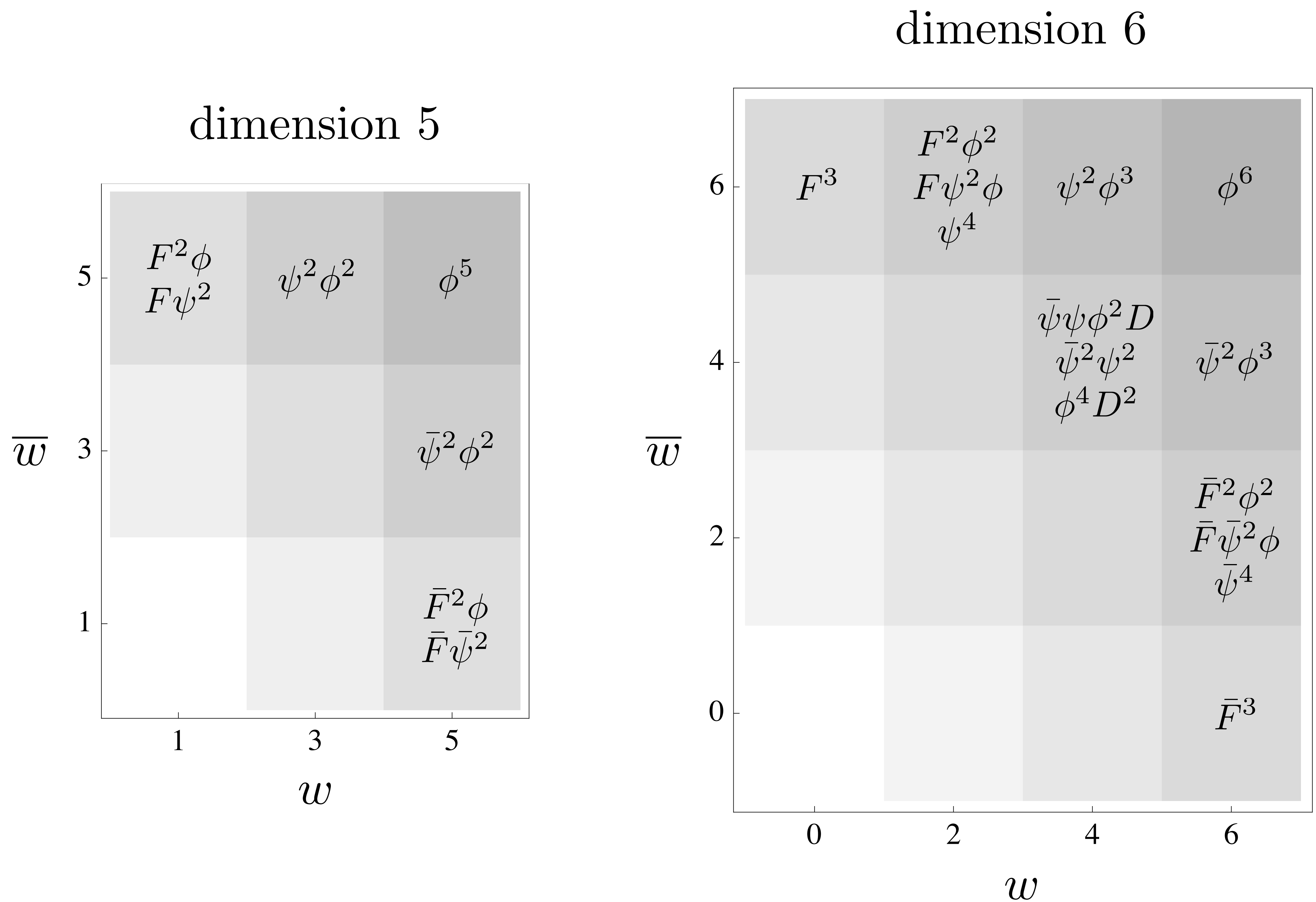
marginalize over all A≠0 involving O

total number of particles created by O

total helicity of particles created by O

field operator



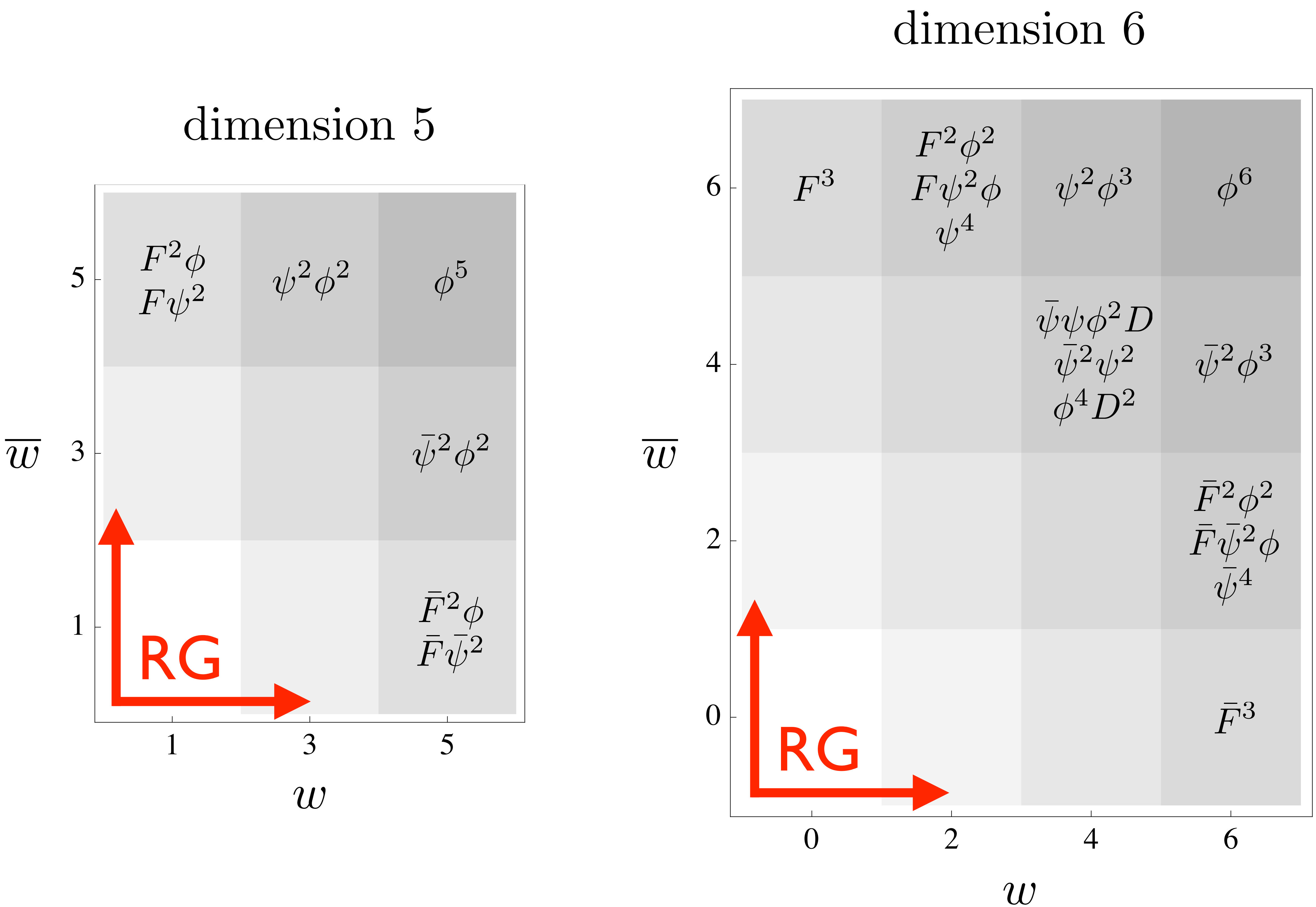

dimension 5
$F^2\phi$
$F\psi^2$
$\psi^2\phi^2$
ϕ^5
$\bar\psi^2\phi^2$
$\bar F^2\phi$
$\bar F\bar\psi^2$
$\overline{w}$
w
1
3
5
dimension 6
F^3
$F^2\phi^2$
$F\psi^2\phi$
ψ^4
$\psi^2\phi^3$
ϕ^6
$\bar\psi\psi\phi^2 D$
$\bar\psi^2\psi^2$
$\phi^4 D^2$
$\bar\psi^2\phi^3$
$\bar F^2\phi^2$
$\bar F\bar\psi^2\phi$
$\bar\psi^4$
$\bar F^3$
$\overline{w}$
w
0
2
4
6

new non-renormalization theorem

Leading irrelevant deformation of a 4D QFT with marginal interactions satisfies:

$\mathcal{O}_i$ cannot be renormalized by $\mathcal{O}_j$ of greater weight or anti-weight.

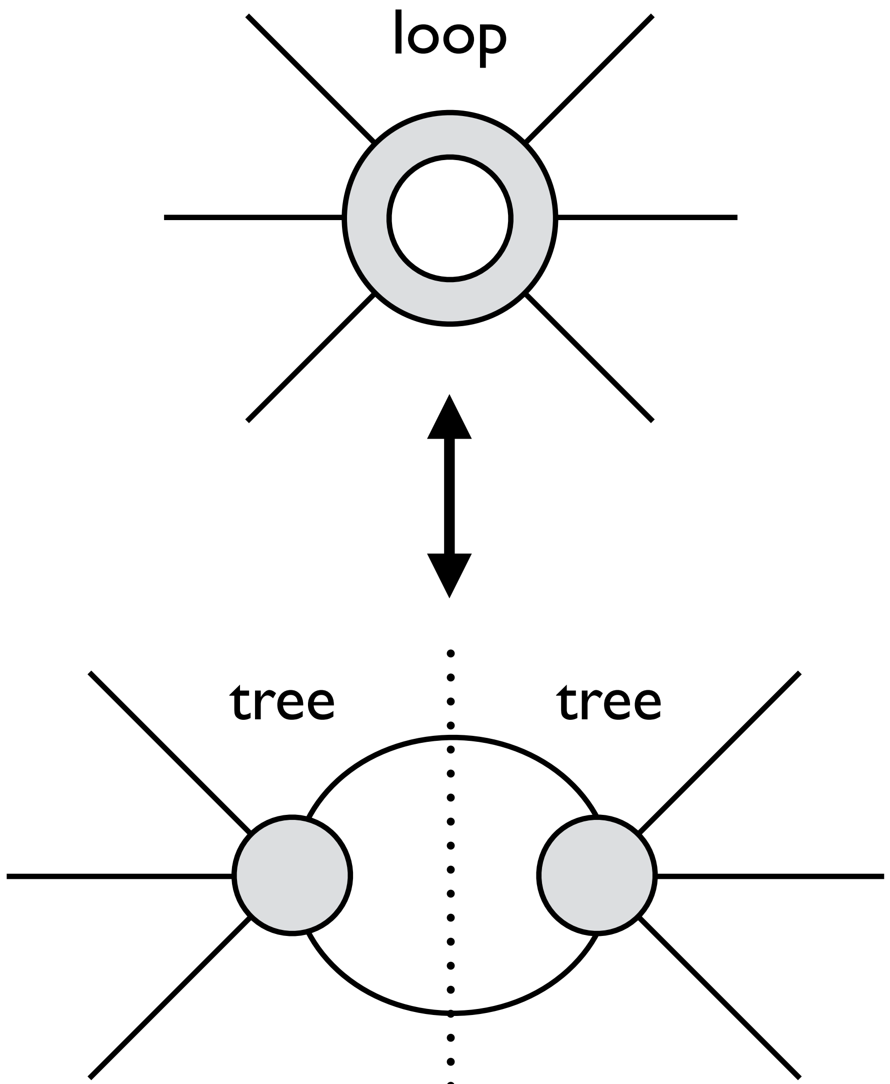
$$\gamma_{ij} = 0 \quad \text{if} \quad w_i < w_j \quad \text{or} \quad \overline{w}_i < \overline{w}_j$$



dimension 5
$F^2\phi$
$F\psi^2$
$\psi^2\phi^2$
ϕ^5
$\bar{\psi}^2\phi^2$
$\bar{F}^2\phi$
$\bar{F}\bar{\psi}^2$
RG
$\overline{w}$
w
dimension 6
F^3
$F^2\phi^2$
$F\psi^2\phi$
ψ^4
$\psi^2\phi^3$
ϕ^6
$\bar{\psi}\psi\phi^2 D$
$\bar{\psi}^2\psi^2$
$\phi^4 D^2$
$\bar{\psi}^2\phi^3$
$\bar{F}^2\phi^2$
$\bar{F}\bar{\psi}^2\phi$
$\bar{\psi}^4$
$\bar{F}^3$
RG
$\overline{w}$
w

proof of new non-renormalization theorems

i) unitarity

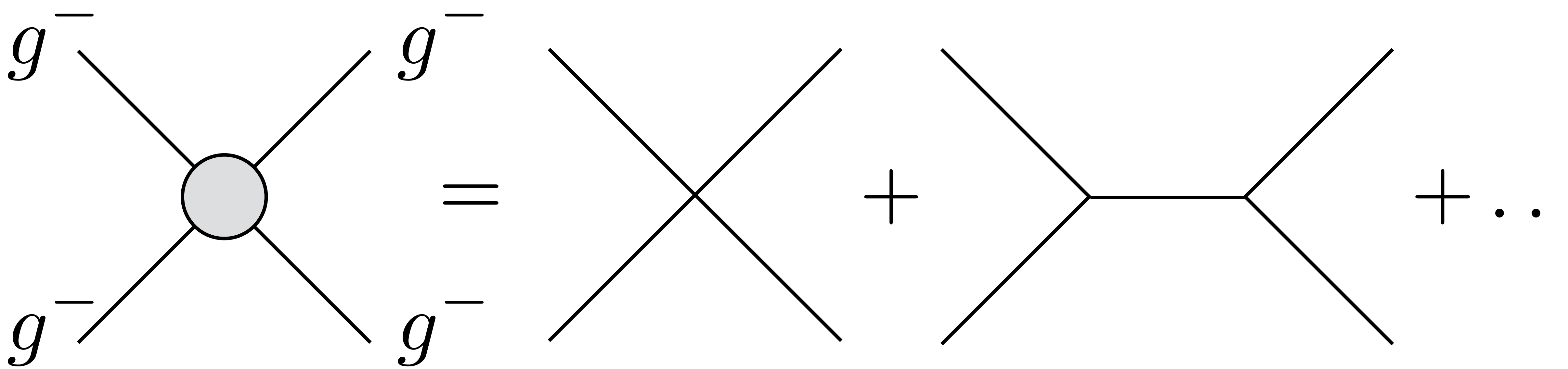


running coupling

physical discontinuity

No cut means no running.

ii) helicity



$= 0$ (on-shell, renormalizable)

Feynman diagrams exist, but vanish on-shell!

tree amplitudes

At renormalizable level, nearly every n-point tree amplitude A_n satisfies

$$w_n, \bar{w}_n \geq 4 \qquad \text{for} \qquad n \geq 4$$

Weights monotonically increase, so it suffices to consider all $w_4 < 4$ amplitudes.

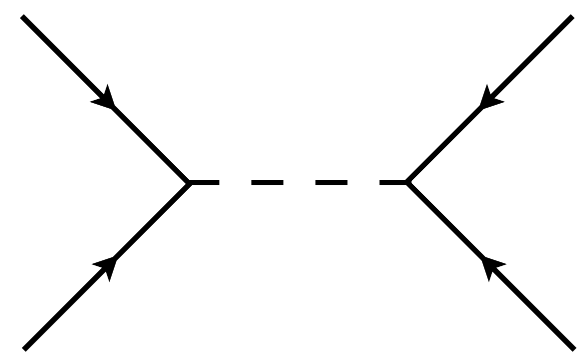
Most $w_4 = 1, 3$ amplitudes do not have any corresponding Feynman diagrams.

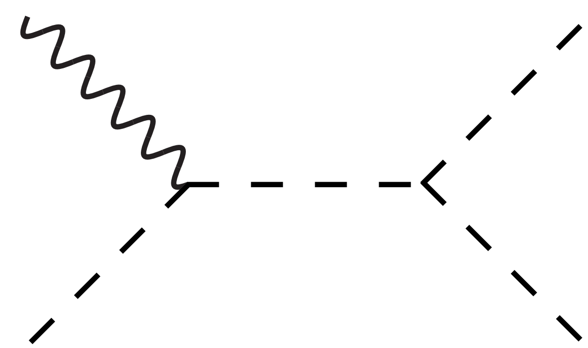
$$\begin{aligned} 0 &= A(F^+F^+F^\pm\phi) = A(F^+F^+\psi^\pm\psi^\pm) \\ &= A(F^+F^-\psi^+\psi^+) = A(F^+\psi^+\psi^-\phi) \\ &= A(\psi^+\psi^+\psi^+\psi^-). \end{aligned}$$

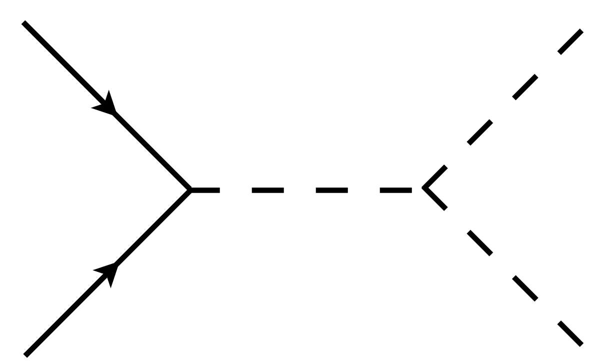
Most $w_4 = 0, 2$ amplitudes have Feynman diagrams but vanish on-shell.

$$\begin{aligned} 0 &= A(F^+F^+F^+F^\pm) = A(F^+F^+\psi^+\psi^-) \\ &= A(F^+F^+\phi\,\phi) = A(F^+\psi^+\psi^+\phi). \end{aligned}$$

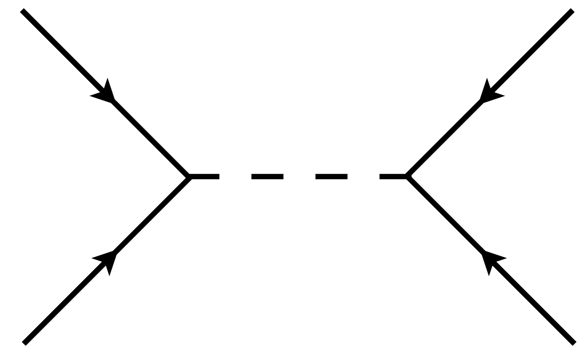
Lastly, we have three non-zero "exceptional" diagrams with $w_4 < 4$.

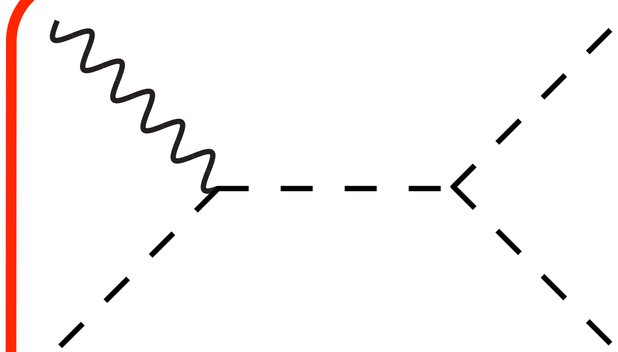
$A(\psi^+\psi^+\psi^+\psi^+) \quad =$  $(w_4 = 2)$

$A(F^+\phi\,\phi\,\phi) \quad =$  $(w_4 = 3)$

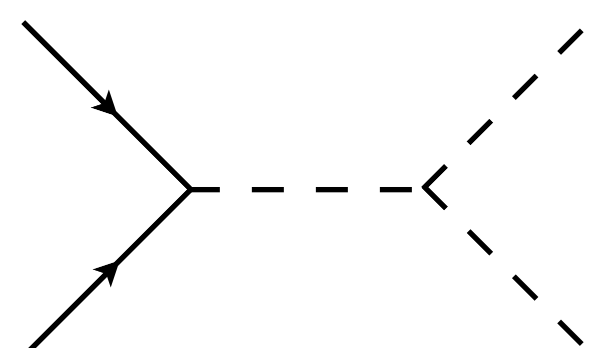
$A(\psi^+\psi^+\phi\,\phi) \quad =$  $(w_4 = 3)$

Lastly, we have three non-zero "exceptional" diagrams with $w_4 < 4$.

$A(\psi^+\psi^+\psi^+\psi^+) =$  $(w_4 = 2)$

$A(F^+\phi\,\phi\,\phi) =$  $(w_4 = 3)$

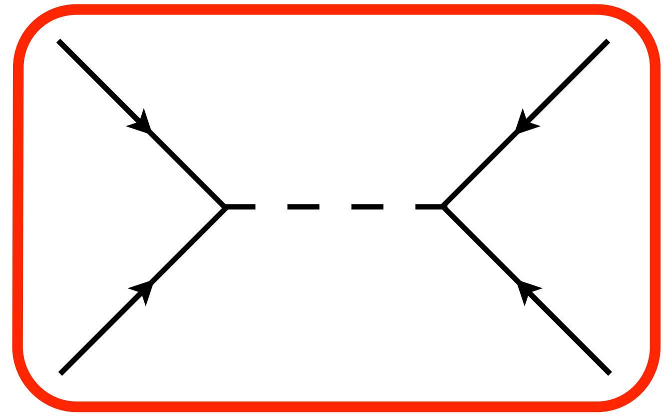
not in SM

$A(\psi^+\psi^+\phi\,\phi) =$  $(w_4 = 3)$

Lastly, we have three non-zero "exceptional" diagrams with $w_4 < 4\,.$

in SM

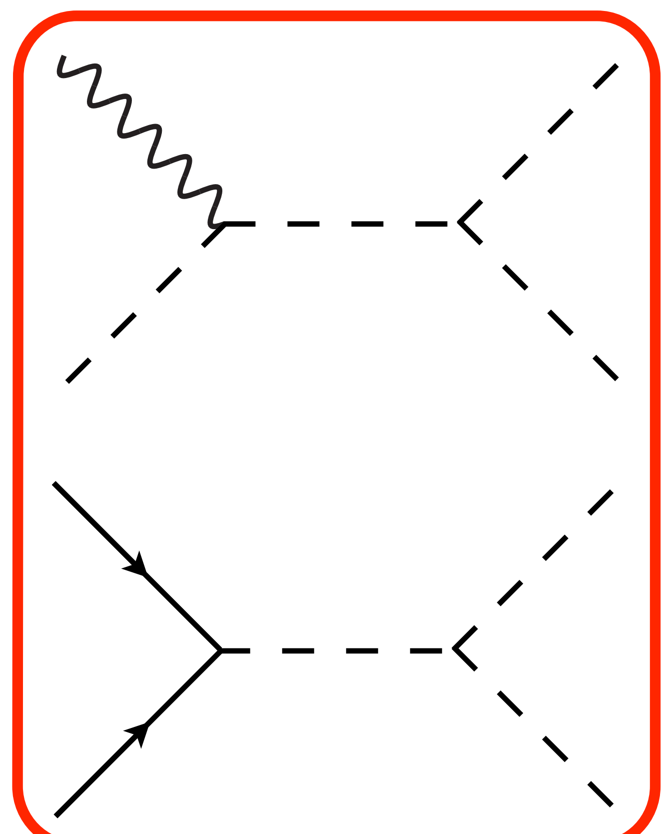
$$A(\psi^+\psi^+\psi^+\psi^+) \quad = \qquad (w_4 = 2)$$



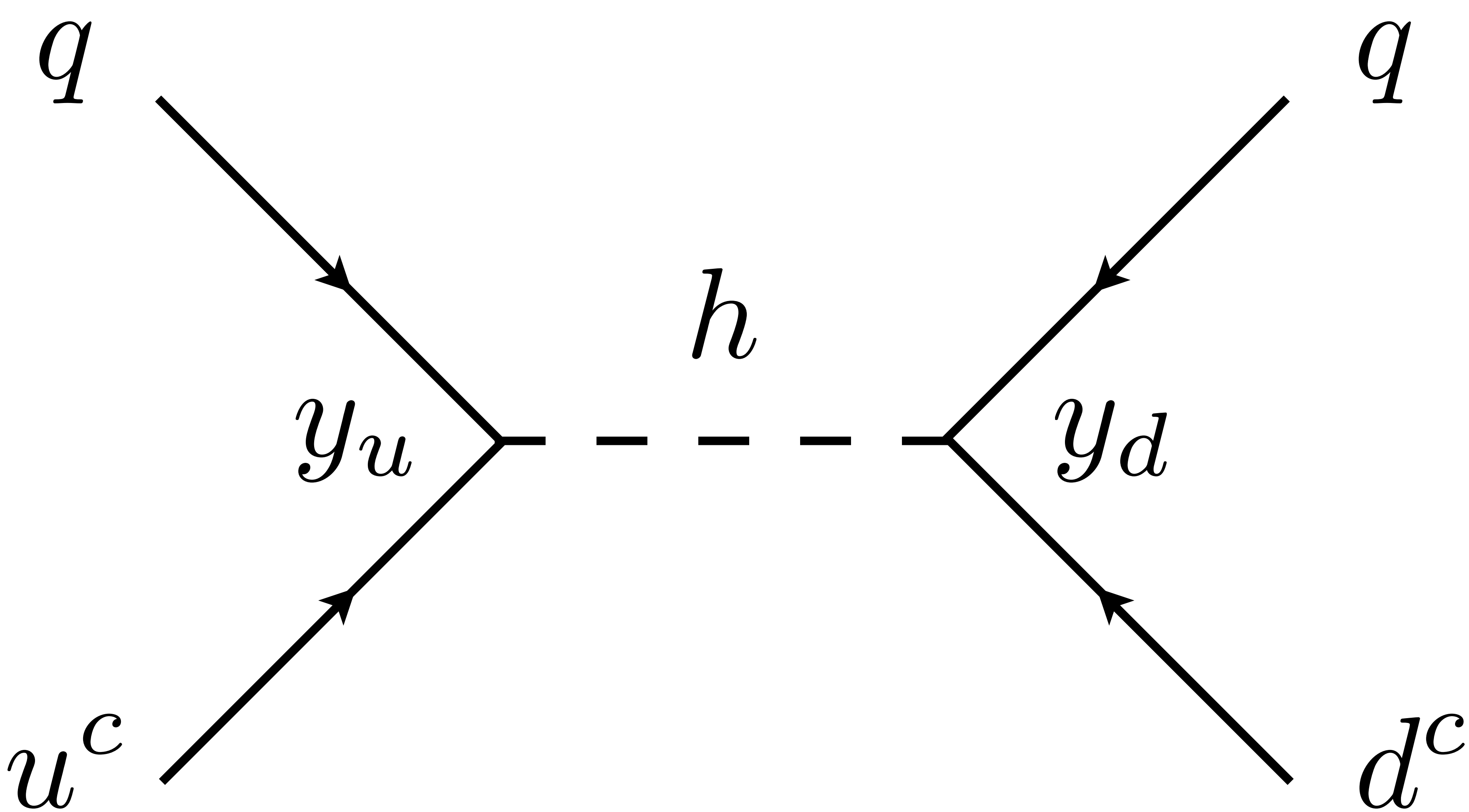
$$A(F^+\phi\,\phi\,\phi) \quad = \qquad (w_4 = 3)$$

not in SM

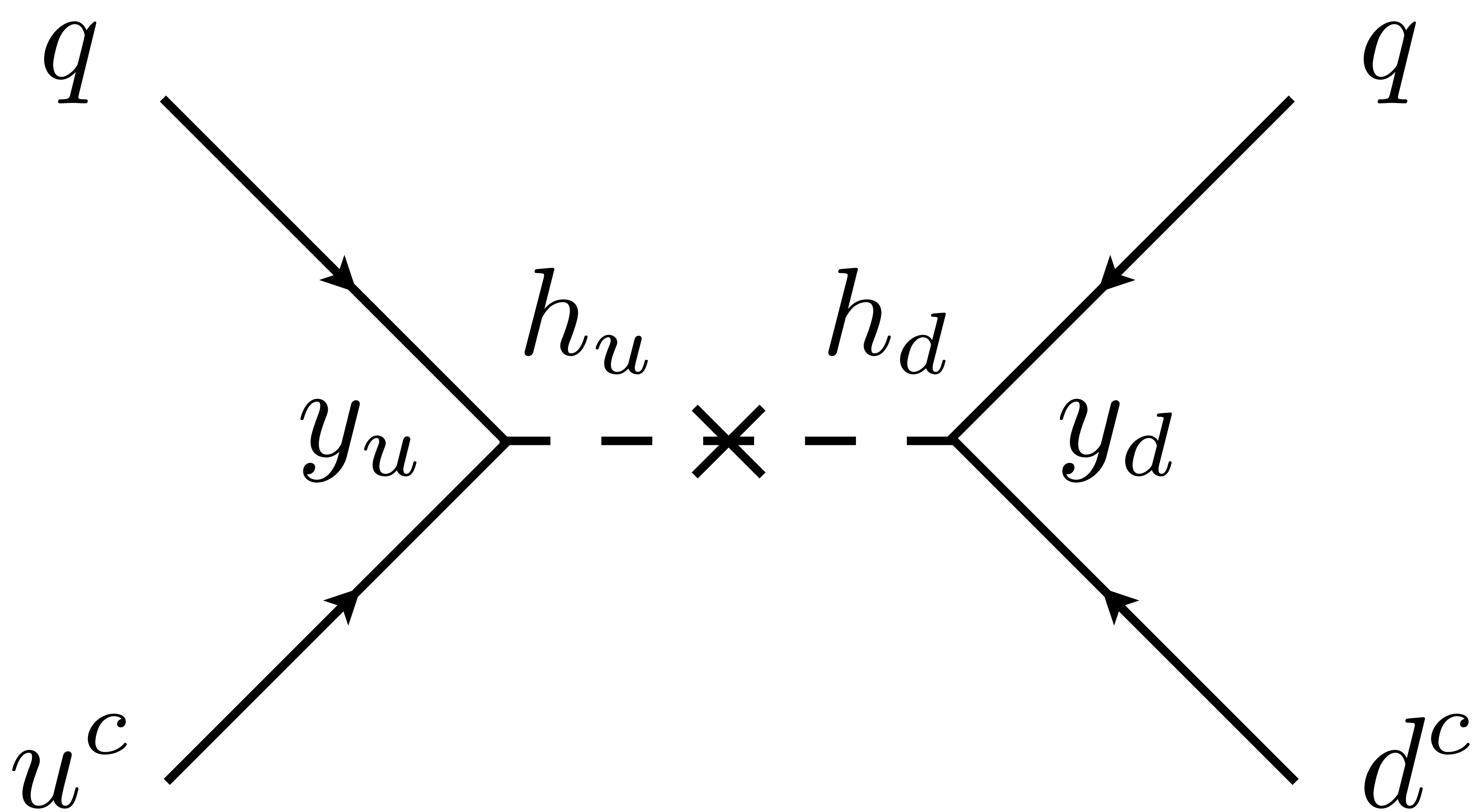
$$A(\psi^+\psi^+\phi\,\phi) \quad = \qquad (w_4 = 3)$$



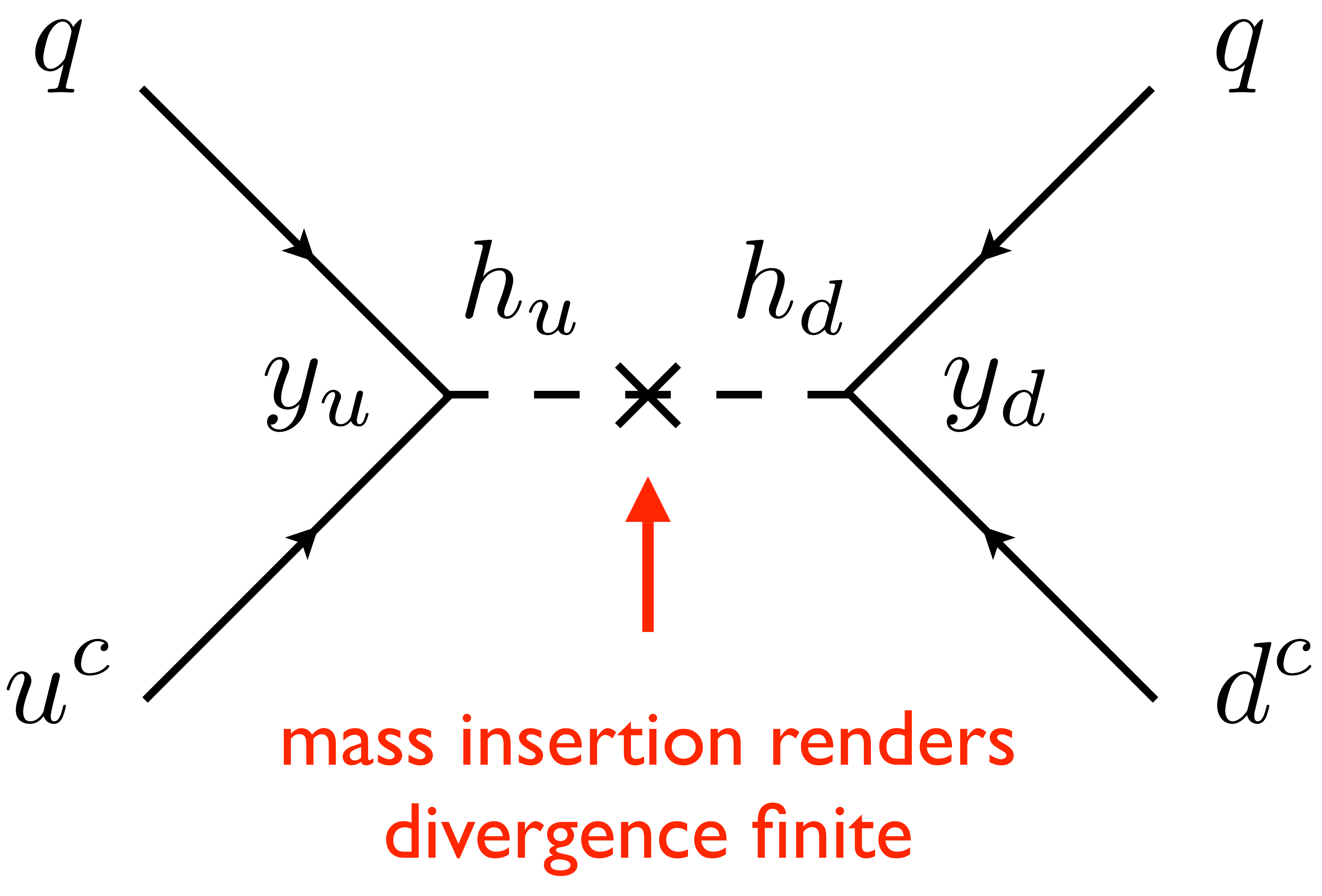
In SM, exceptional diagram is generated…



… but not in the holomorphic 2HDM.



… but not in the holomorphic 2HDM.



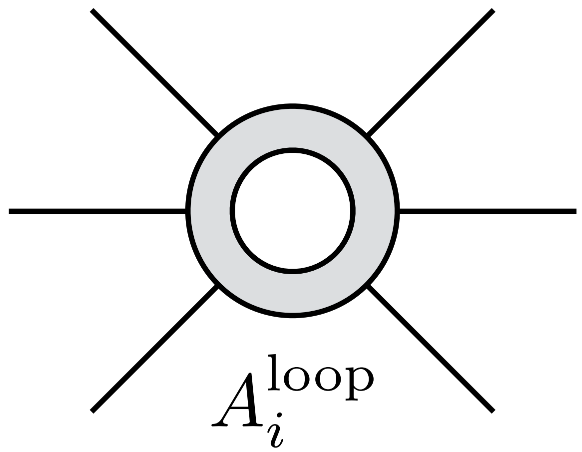
Since weights cannot decrease from adding particles, we find that

$$w_n, \bar{w}_n \geq 4 \qquad \text{for} \qquad n \geq 4$$

mod the exceptional amplitudes discussed.

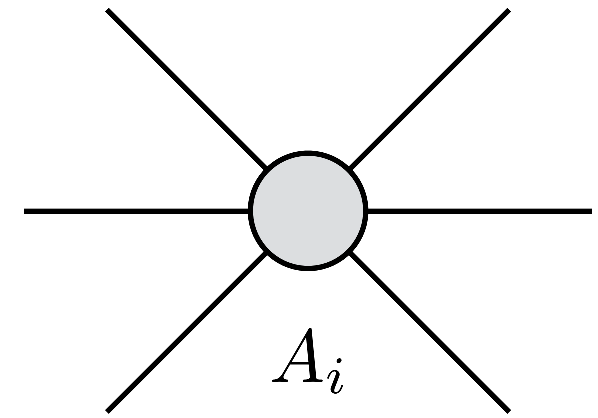
Next, we consider one-loop amplitudes…

one-loop amplitudes

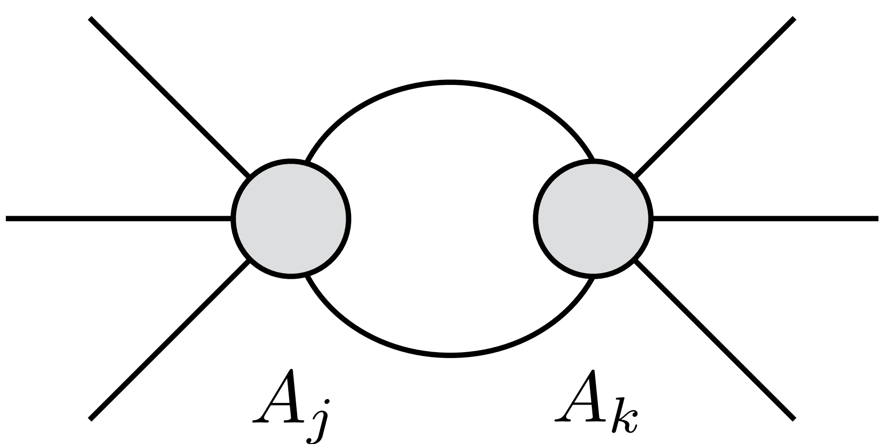


divergent piece

$$\frac{1}{\epsilon}$$



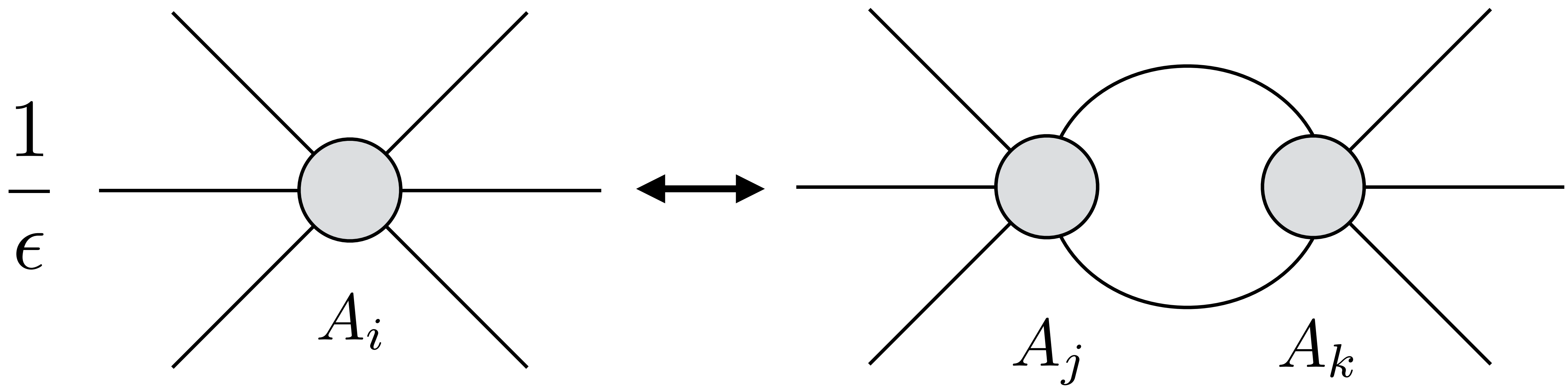
unitarity cut



$$n_i = n_j + n_k - 4$$

$$h_i = h_j + h_k$$

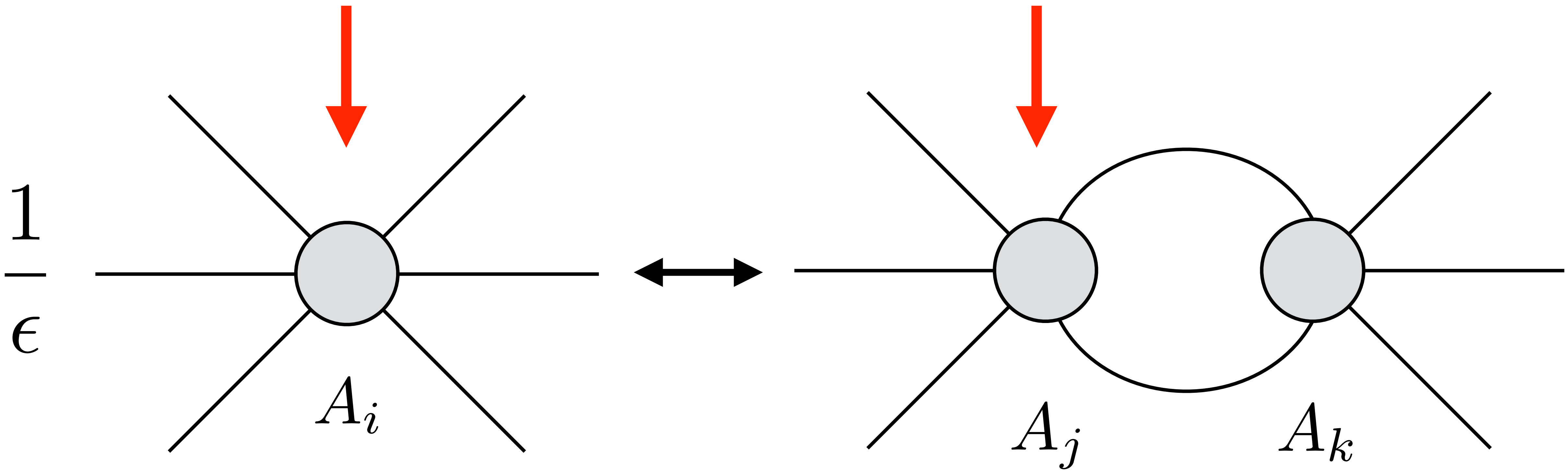
$$w_i = w_j + w_k - 4$$



$$w_i = w_j + w_k - 4$$

tree amplitude with insertion of $\mathcal{O}_i$

tree amplitude with insertion of $\mathcal{O}_j$

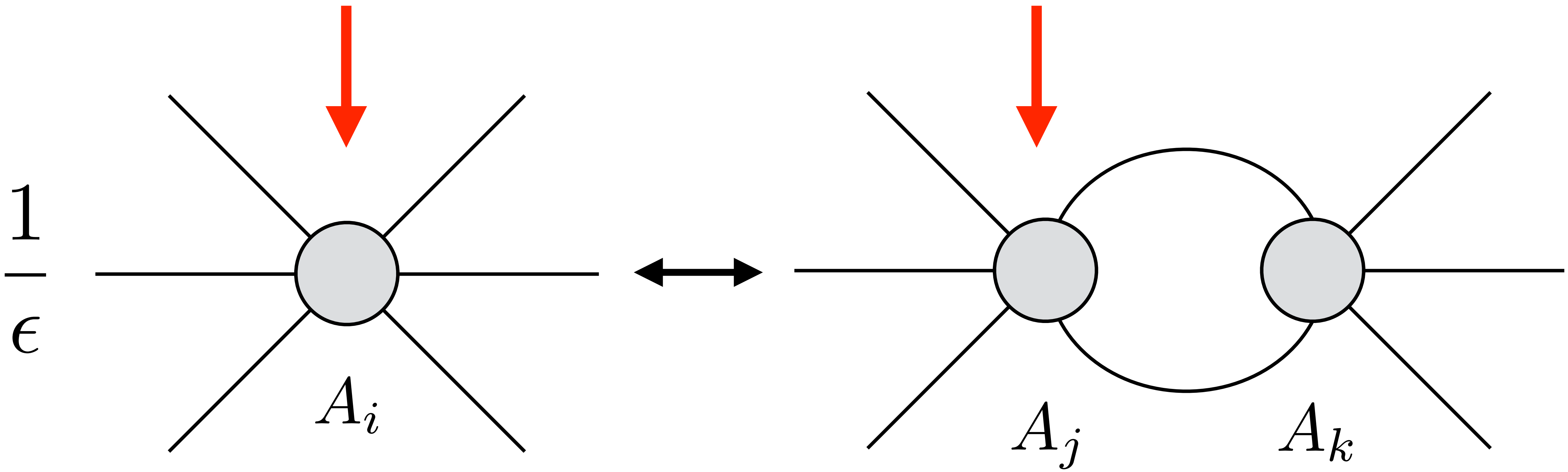


$$w_i = w_j + w_k - 4$$

renormalizable tree amplitude $(w_k \geq 4)$

tree amplitude with insertion of $\mathcal{O}_i$

tree amplitude with insertion of $\mathcal{O}_j$



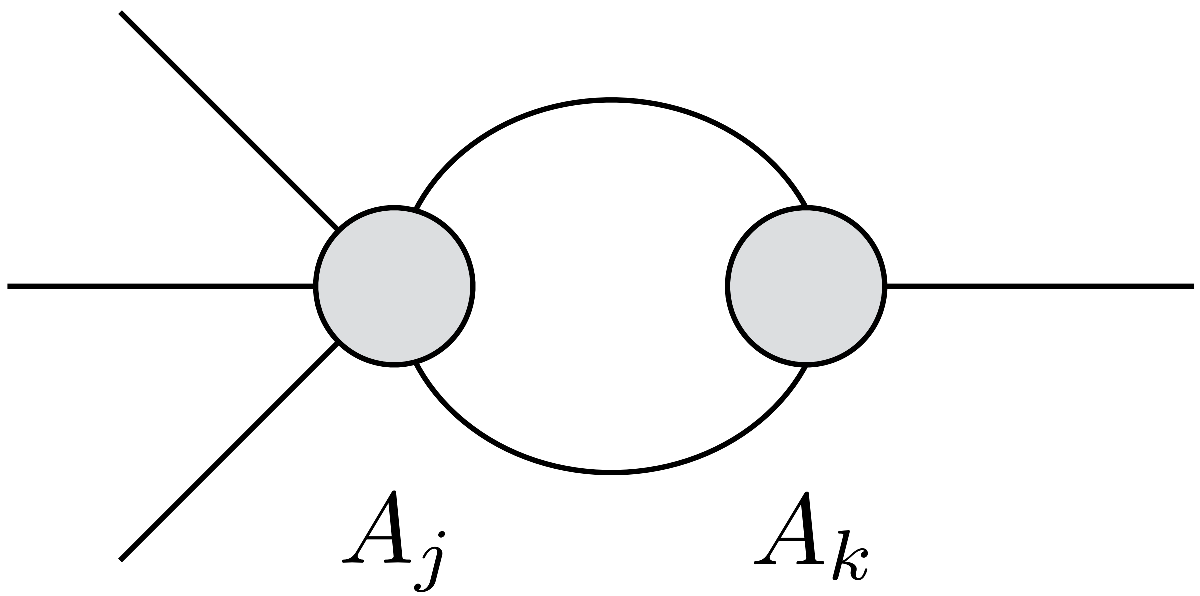
$$w_i = w_j + w_k - 4$$

$$\boxed{w_i \geq w_j}$$

QED.

renormalizable tree amplitude $(w_k \geq 4)$

Caveat: we've dropped 3-point contributions, which vanish in dim reg but are IR divergent.



$$= \frac{c_{\rm UV}}{\epsilon} - \frac{c_{\rm IR}}{\epsilon} + \text{finite}$$

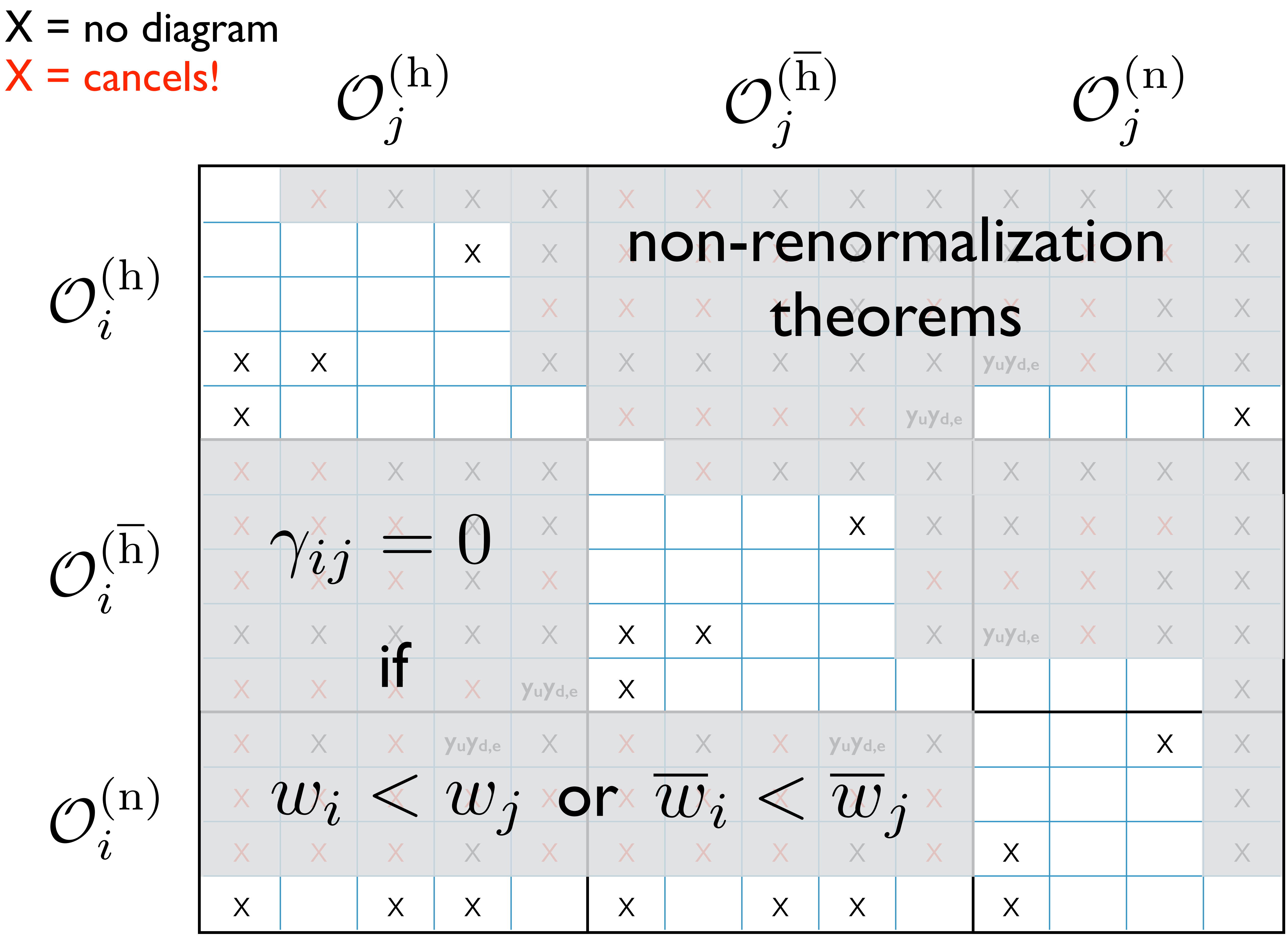
So $c_{\rm UV} = c_{\rm IR}$, but we can show $c_{\rm IR} = 0$ since there aren't IR divergent real emission diagrams for $w_i < w_j$ or $\overline{w}_i < \overline{w}_j$.

(back to)
standard model
effective field theory

X = no diagram

X = cancels!

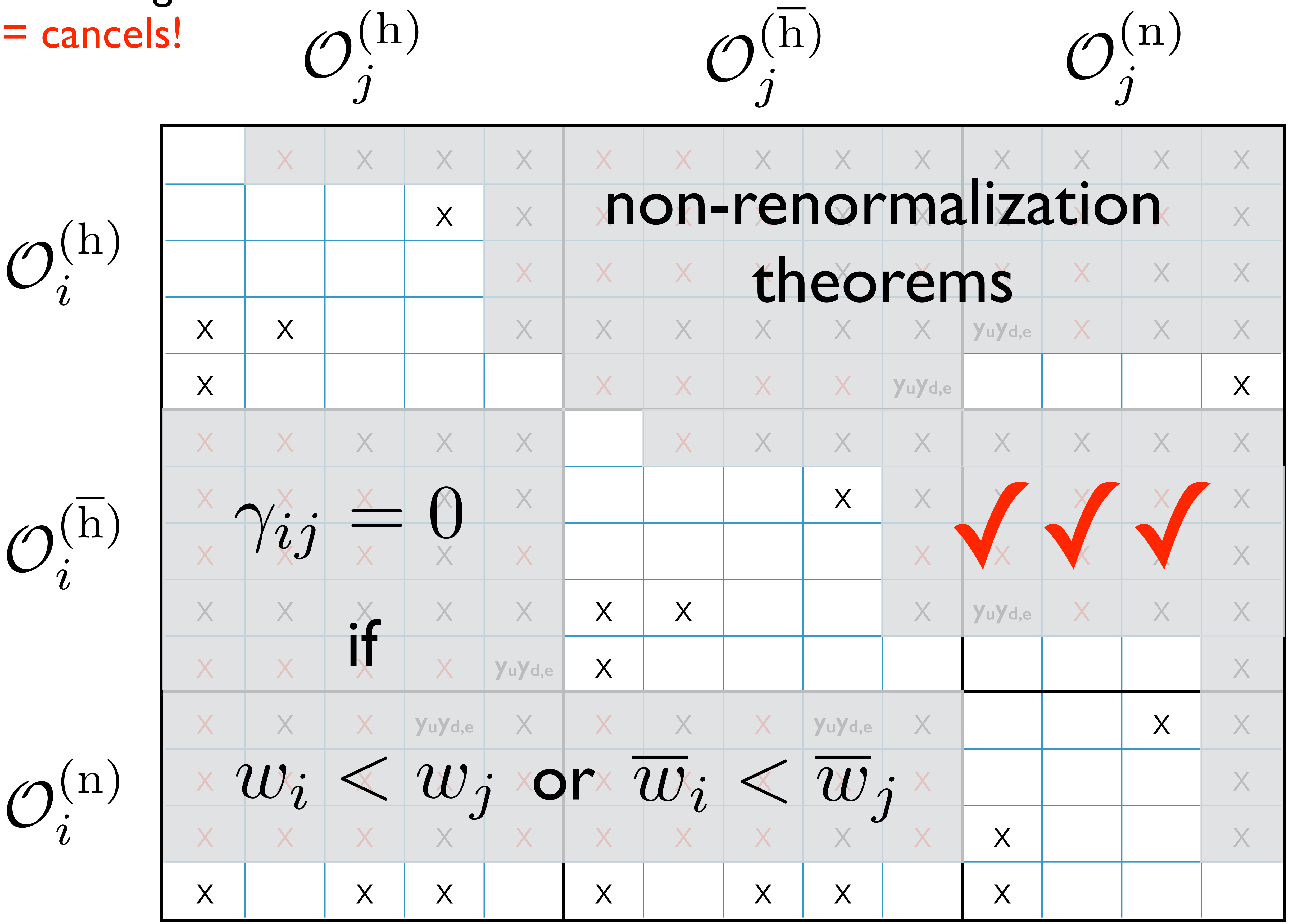
	$\mathcal{O}_j^{(\mathrm{h})}$					$\mathcal{O}_j^{(\overline{\mathrm{h}})}$					$\mathcal{O}_j^{(\mathrm{n})}$			
$\mathcal{O}_i^{(\mathrm{h})}$		X	X	X	X	X	X	X	X	X	X	X	X	X
				X	X	X	X	X	X	X	X	X	X	X
					X	X	X	X	X	X	X	X	X	X
	X	X			X	X	X	X	X	X	$y_u y_{d,e}$	X	X	X
	X					X	X	X	X	$y_u y_{d,e}$				X
$\mathcal{O}_i^{(\overline{\mathrm{h}})}$	X	X	X	X	X		X	X	X	X	X	X	X	X
	X	X	X	X	X				X	X	X	X	X	X
	X	X	X	X	X					X	X	X	X	X
	X	X	X	X	X	X	X			X	$y_u y_{d,e}$	X	X	X
	X	X	X	X	$y_u y_{d,e}$	X								X
$\mathcal{O}_i^{(\mathrm{n})}$	X	X	X	$y_u y_{d,e}$	X	X	X	X	$y_u y_{d,e}$	X			X	X
	X	X	X	X	X	X	X	X	X	X				X
	X	X	X	X	X	X	X	X	X	X	X			X
	X		X	X		X		X	X		X			X



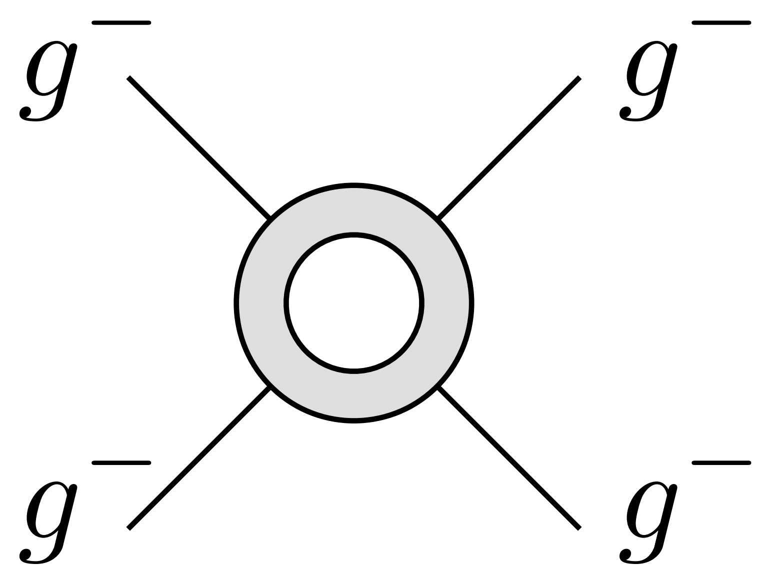
X = no diagram
X = cancels!
$\mathcal{O}_j^{(\mathrm{h})}$
$\mathcal{O}_j^{(\overline{\mathrm{h}})}$
$\mathcal{O}_j^{(\mathrm{n})}$
$\mathcal{O}_i^{(\mathrm{h})}$
$\mathcal{O}_i^{(\overline{\mathrm{h}})}$
$\mathcal{O}_i^{(\mathrm{n})}$
non-renormalization theorems
$\gamma_{ij} = 0$
if
$w_i < w_j$ or $\overline{w}_i < \overline{w}_j$
$y_u y_{d,e}$

X = no diagram

X = cancels!



What about higher loop order?



$$\neq \quad 0$$

Helicity selection rules fail at finite one-loop.

Non-renormalization should fail at two-loop!

conclusions

- New non-renormalization theorems arise from unitarity and helicity in 4D QFTs.

- Fully explains curious zeros in the RG of the SM EFT, all without off-shell symmetry.

- Our proof strongly suggests that observed non-renormalization will fail at higher loop!

thank you!