

Ambitwistor strings and loop integrands

Lionel Mason

The Mathematical Institute, Oxford

`lmason@maths.ox.ac.uk`

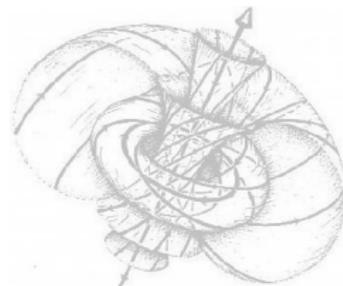
NBI Current Themes in HEP & Cosmology 20 August 2015

With David Skinner. [arxiv:1311.2564](https://arxiv.org/abs/1311.2564), and collaborations with Tim Adamo, Eduardo Casali, Yvonne Geyer, Arthur Lipstein, Ricardo Monteiro, Kai Roehrig, & Piotr Tourkine, [1312.3828](https://arxiv.org/abs/1312.3828), [1404.6219](https://arxiv.org/abs/1404.6219), [1405.5122](https://arxiv.org/abs/1405.5122), [1406.1462](https://arxiv.org/abs/1406.1462), [1506.08771](https://arxiv.org/abs/1506.08771), [1507.00321](https://arxiv.org/abs/1507.00321).

[Cf. also Cachazo, He, Yuan [arxiv:1306.2962](https://arxiv.org/abs/1306.2962), [1306.6575](https://arxiv.org/abs/1306.6575), [1307.2199](https://arxiv.org/abs/1307.2199), [1309.0885](https://arxiv.org/abs/1309.0885), [1412.3479](https://arxiv.org/abs/1412.3479)]

Ambitwistor spaces: spaces of complex null geodesics.

- Extends Penrose/Ward's gravity/Yang-Mills twistor constructions to non-self-dual fields.
- Yang-Mills Witten and Isenberg, et. al. 1978, 1985.
- Conformal and Einstein gravity LeBrun [1983,1991]
Baston & M. [1987] .



Ambitwistor Strings:

- Tree S-Matrices in all dimensions for gravity, YM etc. [CHY]
- From strings in ambitwistor space [M. & Skinner 1311.2564]
- New models for Einstein-YM, DBI, BI, NLS, etc. [Casali, Geyer, M.,
Monteiro, Roehrig 1506.08771].
- Loop integrands from the Riemann sphere [Geyer, M., Monteiro,
Tourkine, 1507.00321].

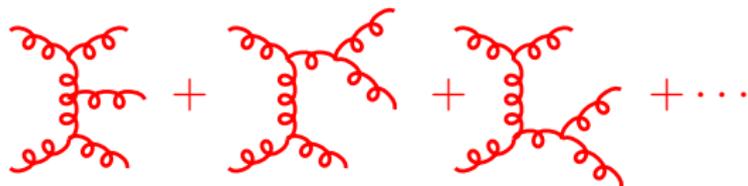
Provide string theories at $\alpha' = 0$ for field theory amplitudes.

Amplitudes from Feynman diagrams

Amplitudes are realized as sums of Feynman integrals.

Consider the five-gluon tree-level amplitude of QCD. Enters in calculation of multi-jet production at hadron colliders.

Described by following Feynman diagrams:



If you follow the textbooks you discover a disgusting mess.

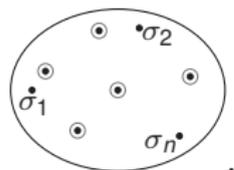
Trees \leftrightarrow classical, loops \leftrightarrow quantum.

The scattering equations

Take n null momenta $k_i \in \mathbb{R}^d$, $i = 1, \dots, n$, $k_i^2 = 0$, $\sum_i k_i = 0$,

- define $P : \mathbb{CP}^1 \rightarrow \mathbb{C}^d$

$$P(\sigma) := \sum_{i=1}^n \frac{k_i}{\sigma - \sigma_i}, \quad \sigma, \sigma_i \in \mathbb{CP}^1$$



- Solve for $\sigma_i \in \mathbb{CP}^1$ with the n scattering equations [Fairlie 197?]

$$\text{Res}_{\sigma_i} (P^2) = k_i \cdot P(\sigma_i) = \sum_{j=1}^n \frac{k_i \cdot k_j}{\sigma_i - \sigma_j} = 0.$$

$$\Rightarrow P^2 = 0 \quad \forall \sigma.$$

- For Möbius invariance $\Rightarrow P \in \mathbb{C}^d \otimes K$, $K = \Omega^{1,0} \mathbb{CP}^1$
- There are $(n-3)!$ solutions.

Arise in large α' strings [Gross-Mende 1988] & twistor-strings [Roiban, Spradlin,

Amplitude formulae for massless theories.

Proposition (Cachazo, He, Yuan 2013,2014)

Tree-level massless amplitudes in d -dims are integrals/sums

$$\mathcal{M}_n = \delta^d \left(\sum_i k_i \right) \int_{(\mathbb{CP}^1)^n} \frac{l^l l^r \prod_i \bar{\delta}(k_i \cdot P(\sigma_i))}{\text{Vol SL}(2, \mathbb{C}) \times \mathbb{C}^3}$$

where $l^{l/r} = l^{l/r}(\epsilon_i^{l/r}, k_i, \sigma_i)$ depend on the theory.

- polarizations ϵ_i^l for spin 1, $\epsilon_i^l \otimes \epsilon_i^r$ for spin-2 ($k_i \cdot \epsilon_i = 0 \dots$).
- Introduce skew $2n \times 2n$ matrices $M = \begin{pmatrix} A & C \\ -C^t & B \end{pmatrix}$,

$$A_{ij} = \frac{k_i \cdot k_j}{\sigma_i - \sigma_j}, \quad B_{ij} = \frac{\epsilon_i \cdot \epsilon_j}{\sigma_i - \sigma_j}, \quad C_{ij} = \frac{k_i \cdot \epsilon_j}{\sigma_i - \sigma_j}, \quad \text{for } i \neq j$$

and $A_{ii} = B_{ii} = 0$, $C_{ii} = \epsilon_i \cdot P(\sigma_i)$.

- For YM, $l^l = Pf'(M)$, $l^r = \prod_i \frac{1}{\sigma_i - \sigma_{i-1}}$.
- For GR $l^l = Pf'(M^l)$, $l^r = Pf'(M^r)$.

More CHY formulae:

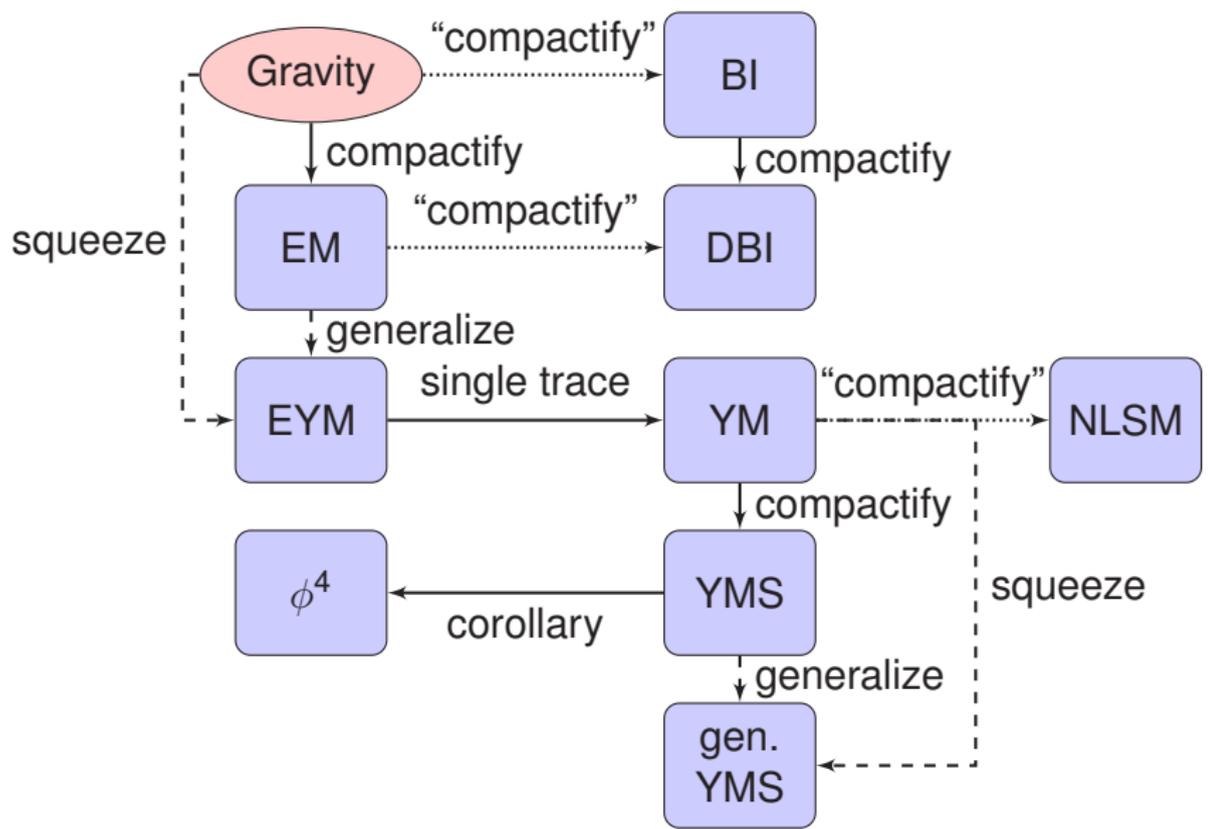


Figure: Theories studied by CHY and operations relating them.

Bosonic ambitwistor string action:

- Σ Riemann surface, coordinate $\sigma \in \mathbb{C}$
- Complexify space-time (M, g) , coords $X \in \mathbb{C}^d$, g hol.
- $(X, P) : \Sigma \rightarrow T^*M$, $P \in K$, holomorphic 1-forms on Σ .

$$S_B = \int_{\Sigma} P_{\mu} \bar{\partial} X^{\mu} - e P^2 / 2.$$

Underlying geometry:

- e enforces $P^2 = 0$,
- P^2 generates gauge freedom: $\delta(X, P, e) = (\alpha P, 0, 2\bar{\partial}\alpha)$.

So target is

$$\mathbb{A} = T^*M|_{P^2=0} / \{\text{gauge}\}.$$

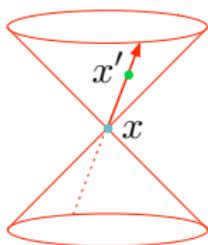
This is *Ambitwistor space*, space of complexified light rays.

The geometry of the space of light rays

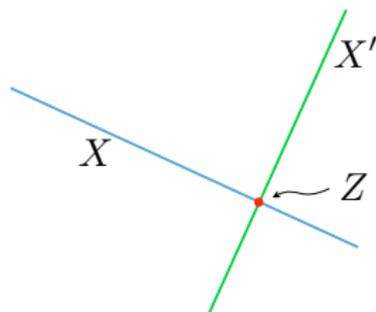
Ambitwistor space \mathbb{A} is space of complexified light rays.

- Light rays primary, events determined by lightcones $X \subset \mathbb{A}$ of light rays incident with x .
- Space-time $M =$ space of such $X \subset \mathbb{A}$.

Space-time



Twistor Space



Space-time geometry is encoded in complex structure of \mathbb{A} .

Theorem (LeBrun 1983 following Penrose 1976)

Complex structure of \mathbb{A} determines $(M, [g])$. Correspondence stable under deformations of $P\mathbb{A}$ that preserve $\theta = P_\mu dX^\mu$.

Quantize bosonic ambitwistor string:

- $(X, P) : \Sigma \rightarrow T^*M,$

$$S_B = \int_{\Sigma} P_{\mu}(\bar{\partial} + \tilde{e}\partial)X^{\mu} - e P^2/2.$$

- Gauge fix $\tilde{e} = e = 0, \rightsquigarrow$ ghosts & BRST Q
- Introduce vertex operators $V_i \leftrightarrow$ field perturbations.

Amplitudes are computed as correlators of vertex ops

$$\mathcal{M}_n = \langle V_1 \dots V_n \rangle$$

For gravity add type II worldsheet susy $S_{\Psi_1} + S_{\Psi_2}$ where

$$S_{\Psi} = \int_{\Sigma} \Psi_{\mu} \bar{\partial} \Psi^{\mu} + \chi P \cdot \Psi.$$

From deformations of \mathbb{A} to the scattering equations

Gravitons \leftrightarrow vertex operators $V_i = \text{def'm of action } \delta S = \int_{\Sigma} \delta\theta.$

- θ determines complex structure on $P\mathbb{A}$ via $\theta \wedge d\theta^{d-2}$. So:
- Deformations of complex structure $\leftrightarrow [\delta\theta] \in H_{\bar{\partial}}^1(P\mathbb{A}, L)$.

Proposition

For perturbation $\delta g_{\mu\nu} = e^{ik \cdot X} \epsilon_{\mu} \epsilon_{\nu}$ of flat space-time

$$\delta\theta = \bar{\delta}(k \cdot P) e^{ik \cdot X} (\epsilon \cdot P)^2$$

Ambitwistor repr $\Rightarrow \bar{\delta}(k \cdot P) \Rightarrow$ scattering equs.

Proposition

CHY formulae for massless tree amplitudes e.g. YM & gravity arise from appropriate choices of worldsheet matter.

- Take $e^{ik_i \cdot X(\sigma_i)}$ factors into action to give

$$S = \frac{1}{2\pi} \int_{\Sigma} P \cdot \bar{\partial} X + 2\pi \sum_i ik \cdot X(\sigma_i).$$

- Gives field equations $\bar{\partial} X = 0$ and,

$$\bar{\partial} P = 2\pi \sum_i ik \delta^2(\sigma - \sigma_i).$$

- Solutions $X(\sigma) = X = \text{const.}$, $P(\sigma) = \sum_i \frac{k_i}{\sigma - \sigma_i} d\sigma$.

Thus path-integral reduces to

$$\mathcal{M}_n = \delta^d \left(\sum_i k_i \right) \int_{(\mathbb{CP}^1)^{n-3}} \frac{\prod_i (\epsilon_i \cdot P(\sigma_i))^2 \bar{\delta}(k_i \cdot P)}{\text{Vol } G}$$

We see $P(\sigma)$ appearing and scattering equations.

Unfortunately: amplitudes for $S \sim \int_M R + R^3$

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Unfortunately: amplitudes for $S \sim \int_M R + R^3$.

- Decorate null geodesics with spin vectors, vectors for internal degrees of freedom & other holomorphic CFTs.
- Take

$$S = S_B + S^l + S^r$$

where S^l, S^r are some worksheet matter CFTs.

- Total vertex operators given by

$$v^l v^r \bar{\delta}(k \cdot P) e^{ik \cdot X}$$

with v^l, v^r worksheet currents from S^l, S^r resp..

- Amplitudes become

$$\mathcal{M}_n = \delta^d \left(\sum_i k_i \right) \int_{(\mathbb{CP}^1)^n} \frac{\mathcal{I}^l \mathcal{I}^r \prod_i \bar{\delta}(k_i \cdot P)}{\text{Vol Gauge}}$$

where $\mathcal{I}^l, \mathcal{I}^r$ are worksheet correlators of v^l s, v^r s resp..

- In good situations, Q -invariance and discrete symmetries (GSO) rule out unwanted vertex operators.

- **Worksheet SUSY:** Let $\Psi^\mu \in K^{1/2}$, spin 1/2 fermions on Σ ,

$$S_\Psi = \int g_{\mu\nu} \Psi^\mu \bar{\partial} \Psi^\nu - \chi P_\mu \Psi^\mu$$

Replace $v = \epsilon \cdot P$ by $v = \epsilon \cdot P + \epsilon \cdot \Psi k \cdot \Psi$ (or $u = \delta(\gamma) \epsilon \cdot \Psi$).
Worksheet correlator

$$\mathcal{I}^{l/r} = \langle u_1 u_2 v_3 \dots v_n \rangle = P f'(M).$$

- **Free fermions and current algebras:** Free ‘real’ Fermions $\rho^a \in \mathbb{C}^m \otimes K^{1/2}$

$$S_\rho = \int_\Sigma \delta_{ab} \rho^a \bar{\partial} \rho^b, \quad a = 1, \dots, m,$$

With Lie alg structure const f^{abc} , set $v = t_a f^{abc} \rho^b \rho^c$.

Correlators \rightsquigarrow ‘Parke-Taylor’ + unwanted multi-trace terms

$$\langle v_1 \dots v_n \rangle = \frac{\text{tr}(t_1 \dots t_n)}{\sigma_{12} \sigma_{23} \dots \sigma_{n1}} + \dots$$

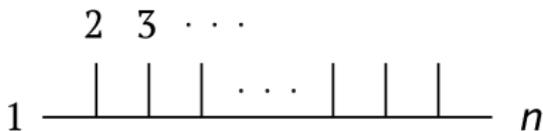
where $\sigma_{ij} = \sigma_i - \sigma_j$.

Use fermions $\tilde{\rho}^a, \rho^a \in \mathfrak{g} \otimes K^{1/2}$, bosons $q^a, y^a \in \mathfrak{g} \otimes K^{1/2}$

$$S_{CS} = \int_{\Sigma} \tilde{\rho}_a \bar{\partial} \rho^a + q_a \bar{\partial} y^a + \chi \operatorname{tr} \rho \left(\frac{[\tilde{\rho}, \rho]}{2} + [q, y] \right).$$

- Gauge fix $\chi = 0 \rightsquigarrow$ ghosts $(\beta, \gamma) \rightsquigarrow$ two fixed vertex operators to end chain of structure constants 'comb'.
- Vertex ops: $u = \delta(\gamma) t \cdot \rho, \quad \tilde{u} = \delta(\gamma) t \cdot \tilde{\rho}, \quad (\text{fixed})$
 $v = t \cdot [\rho, \rho], \quad \tilde{v} = t \cdot ([\tilde{\rho}, \rho] + [q, y]).$
- To be nontrivial, correlator must have just one untilded VO

$$\langle u_1 \tilde{u}_2 \tilde{v}_3 \dots \tilde{v}_n \rangle = C(1, \dots, n) := \frac{\operatorname{tr}(t_1 [t_2, [t_3, \dots [t_{n-1}, t_n] \dots]])}{\sigma_{12} \dots \sigma_{n1}}.$$



The 2013 CHY formulae & ambitwistor models

Above lead essentially to original models & formulae:

- $(S', S^r) = (S_{\tilde{\psi}}, S_{\psi}) \rightsquigarrow$ type II gravity,
- $(S', S^r) = (S_{CS}, S_{\psi}) \rightsquigarrow$ heterotic with YM,
- $(S', S^r) = (S_{CS}, S_{CS}) \rightsquigarrow$ bi-adjoint scalar.

The latter two come with unphysical gravity.

S_{CS} improves on current algebras in avoiding multi-trace terms and all models critical in 10d.

Combined matter systems

$S_{\Psi_1, \Psi_2} = S_{\Psi_1} + S_{\Psi_2}$ two worldsheet susy's for S^l or S^r . This is maximum. It gives VO currents

$$u = \delta(\gamma_1)k \cdot \Psi_2, \quad v = k \cdot \Psi_1 k \cdot \Psi_2.$$

$S_{\Psi, \rho} = S_{\Psi} + S_{\rho}$ combines 'real' Fermions with susy, \rightsquigarrow VO currents as usual for S_{Ψ} and

$$u_t = \delta(\gamma)t \cdot \rho, \quad v_t = k \cdot \Psi t \cdot \rho.$$

$$S_{\Psi, CS} = \int_{\Sigma} \Psi \cdot \bar{\partial} \Psi + \tilde{\rho}_a \bar{\partial} \rho^a + q_a \bar{\partial} y^a + \chi \left(P \cdot \Psi + \text{tr} \rho \left(\frac{[\tilde{\rho}, \rho]}{2} + [q, y] \right) \right).$$

With ghosts etc., the VO currents are those for S_{Ψ} and

$$\begin{aligned} \tilde{u}_t &= \delta(\gamma)t \cdot \tilde{\rho}, & u_t &= \delta(\gamma)t \cdot \rho, \\ \tilde{v}_t &= k \cdot \Psi t \cdot \tilde{\rho} + t \cdot ([\tilde{\rho}, \rho] + [q, y]), & v_t &= k \cdot \Psi t \cdot \rho + t \cdot [\rho, \rho]. \end{aligned}$$

GSO now reverses signs of all fields in matter system.

Ambitwistor strings with combinations of matter

CGMMRS 150?

$S' \backslash S'$	S_Ψ	S_{Ψ_1, Ψ_2}	$S_{\rho, \Psi}^{(\tilde{m})}$	$S_{CS, \Psi}^{(\tilde{N})}$	$S_{CS}^{(\tilde{N})}$
S_Ψ	E				
S_{Ψ_1, Ψ_2}	BI	Galileon			
$S_{\rho, \Psi}^{(m)}$	EM $U(1)^m$	DBI	EMS $U(1)^m \times U(1)^{\tilde{m}}$		
$S_{CS, \Psi}^{(N)}$	EYM	ext. DBI	$EYMS$ $SU(N) \times U(1)^{\tilde{m}}$	$EYMS$ $SU(N) \times SU(\tilde{N})$	
$S_{CS}^{(N)}$	YM	Nonlinear σ	$EYMS$ $SU(N) \times U(1)^{\tilde{m}}$	<i>gen. YMS</i> $SU(N) \times SU(\tilde{N})$	<i>Biadjoint Scalar</i> $SU(N) \times SU(\tilde{N})$

Table: Theories arising from the different choices of matter models.

The string paradigm gives

$$\mathcal{M}_n = \text{[circle with 6 dots]} + \text{[torus with 6 dots]} + \dots + \text{[genus-2 surface with 6 dots]} + \dots$$

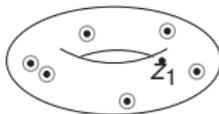
Can we make sense of this at 1 loop, i.e., on a torus?

Need critical model with all anomalies cancelling, i.e., type II super-gravity.

1-loop: the scattering equations on a torus

[Adamo, Casali, Skinner 2013, Casali Tourkine 2014 Geyer, M., Monteiro, Tourkine 2015]

On torus $\Sigma_q = \mathbb{C}/\{\mathbb{Z} \oplus \mathbb{Z}\tau\}$, $q = e^{2\pi i\tau}$, solve



$$\bar{\partial}P = 2\pi i \sum_i k_i \bar{\delta}(z - z_i) dz \quad \text{with}$$

$$P = 2\pi i \ell dz + \sum_i k_i \left(\frac{\theta'_1(z - z_i)}{\theta_1(z - z_i)} + \sum_{j \neq i} \frac{\theta'_1(z_{ij})}{n \theta_1(z_{ij})} \right) dz.$$

zero-modes $\ell \in \mathbb{R}^d \leftrightarrow$ loop momenta.

Scattering eqs:

$$\text{Res}_{z_i} P^2 := k_i \cdot P(z_i) = 0, \quad i = 2, \dots, n, \quad P(z_0)^2 = 0.$$

Gives amplitude formula

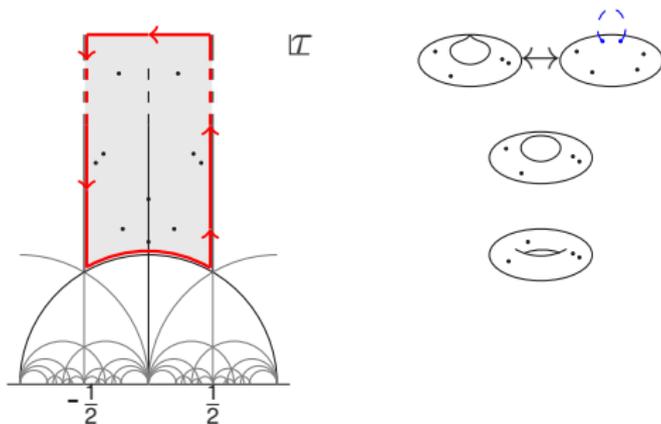
$$\mathcal{M}_{\text{SG}}^{(1)} = \int \mathcal{I}_q d^d \ell d\tau \bar{\delta}(P^2(z_0)) \prod_{i=2}^n \bar{\delta}(k_i \cdot P(z_i)) dz_i.$$

Localizes on discrete set of solutions to scattering eqs.

With $\mathcal{I}_q = 1$, conjectured to be permutations sum of n -gons.

From the elliptic curve to the Riemann sphere

[Geyer, M., Monteiro, Tourkine 1507.00321]



$\sum \{\text{residues at } P^2(z_0) = 0\} = \{\text{residue at } q = 0\}$ so

$$\begin{aligned} \mathcal{M}_n^{(1)} &= \int \mathcal{I}_q d^d \ell \frac{dq}{q} \bar{\partial} \frac{1}{P^2(z_0)} \prod_{i=2}^n \bar{\delta}(k_i \cdot P(z_i)) dz_i, \\ &= - \int \mathcal{I}_0 d^d \ell \frac{1}{\ell^2} \prod_{i=2}^n \bar{\delta}(k_i \cdot P(\sigma_i)) \frac{d\sigma_i}{\sigma_i^2}, \end{aligned}$$

Off-shell scattering eqs and n -gon conjecture

At $q = 0$

$$P(z) = P(\sigma) = \ell \frac{d\sigma}{\sigma} + \sum_{i=1}^n \frac{k_i d\sigma}{\sigma - \sigma_i}.$$

Set $S = P^2 - \ell^2 d\sigma^2/\sigma^2$, gives *off-shell scattering equations*:

$$0 = \text{Res}_{\sigma_i} S = k_i \cdot P(\sigma_i) = \frac{k_i \cdot \ell}{\sigma_i} + \sum_{j \neq i} \frac{k_j \cdot k_i}{\sigma_i - \sigma_j}.$$

The n -gon conjecture becomes

$$\mathcal{M}_{n\text{-gon}}^{(1)} = - \int d^{2n+2} \ell \frac{1}{\ell^2} \prod_{i=2}^n \bar{\delta}(k_i \cdot P(\sigma_i)) \frac{d\sigma_i}{\sigma_i^2},$$

which yields

$$\mathcal{M}_n^{(1)} = \frac{(-1)^n}{\ell^2} \sum_{\sigma \in \mathcal{S}_n} \prod_{i=1}^{n-1} \frac{1}{\ell \cdot K_{\sigma_i} + \frac{1}{2} K_{\sigma_i}^2}, \quad K_{\sigma_i} = \sum_{j=1}^i k_{\sigma_i(j)}$$

Partial fractions + shifts in ℓ gives permutation sum of n -gons.

Supergravity 1-loop integrand

For supergravity $\mathcal{I}_q = \mathcal{I}_q^L \mathcal{I}_q^R$ with $\mathcal{I}^{L/R} \equiv \mathcal{I}^{L/R}(k_i, \epsilon_i^{L/R}, z_i | q)$.
At $q = 0$

$$\mathcal{I}_0^{L/R} = 16 \left(\text{Pf}(M_2^{L/R}) - \text{Pf}(M_3^{L/R}) \right) - 2 \partial_{q^{1/2}} \text{Pf}(M_3^{L/R}),$$

So the 1-loop supergravity integrand is

$$\mathcal{M}_n^{(1)} = - \int \mathcal{I}_0^L \mathcal{I}_0^R \frac{1}{\ell^2} \prod_{i=2}^n \bar{\delta}(k_i \cdot P(\sigma_i)) \frac{d\sigma_i}{\sigma_i^2}.$$

Checked at 4 points algebraically and 5 points numerically.

Super Yang-Mills 1-loop integrand

This leads to conjecture for super Yang-Mills at 1 loop;

$$\mathcal{M}_n^{(1)} = \int \mathcal{I}_0^L PT_n \prod_{i=2}^n \bar{\delta}(k_i \cdot P(\sigma_i)) \frac{d\sigma_i}{\sigma_i} .$$

Here, \mathcal{I}_0^R factor is replaced by cyclic sum of Parke-Taylor's running through loop,

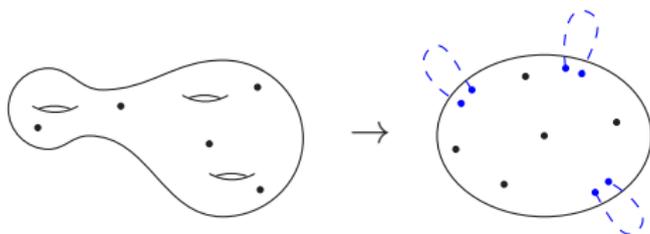
$$PT_n = \sum_{i=1, i \bmod n}^n \frac{\sigma_{0\infty}}{\sigma_{0i} \sigma_{ii+1} \sigma_{i+1i+2} \cdots \sigma_{i+n\infty}} .$$

Checked at 4 and 5 points.

PT_n^2 integrand remains uncertain.

Outlook: All-loop Scattering equations on \mathbb{CP}^1

Use residue thms to localize genus g moduli integrals to bdy cpt with g a -cycles contracted $\rightsquigarrow \mathbb{CP}^1$ with g nodes.



Fixes g moduli, remaining $2g - 3 \leftrightarrow 2g$ new marked points.
1-form P becomes

$$P = \sum_{r=1}^g \ell_r \omega_r + \sum_i k_i \frac{d\sigma}{\sigma - \sigma_i},$$

here ω_r is basis of g global holomorphic 1-forms on nodal \mathbb{CP}^1 .
Set $S(\sigma) := P^2 - \sum_{r=1}^g \ell_r^2 \omega_r^2$, off-shell scattering equations are

$$\text{Res}_{\sigma_i} S = 0, \quad i = 1, \dots, n + 2g, .$$

Outlook: All-loop integrands on \mathbb{CP}^1

Leads to proposal for all-loop integrand;

$$\mathcal{M}_n^{(g)} = \int_{(\mathbb{CP}^1)^{n+2g}} d^{dg} \ell \frac{\mathcal{I}_0^L \mathcal{I}_0^R}{\text{Vol } \mathbf{G}} \prod_{r=1}^g \frac{1}{\ell_r^2} \prod_{i=1}^{n+2g} \bar{\delta}(\text{Res}_{\sigma_i} \mathcal{S}(\sigma_i)),$$

$$\text{where } \mathcal{I}_0 = \begin{cases} \mathcal{I}_0^L \mathcal{I}_0^R, & \text{gravity} \\ \mathcal{I}_0^L PT_n, & \text{Yang-Mills} \\ PT_n PT'_n & \text{biadjoint scalar} \end{cases} .$$

Suggests n -point g -loop integrands have similar complexity to $n + 2g$ -point tree amplitudes.

Chiral $\alpha' = 0$ ambitwistor strings use LeBrun's correspondence to give theories underlying CHY formulae old & new.

- Incorporates colour/kinematics Yang-Mills/gravity ideas.
- Extends to many theories from DBI to Nonlinear Sigma models.
- Critical models extend to loops on a Riemann surface.
- Higher genus Riemann surface formulae reduce to simpler formulae on \mathbb{CP}^1 .
- *Off-shell scattering equations* on \mathbb{CP}^1 can be used to find loop integrands for non-critical models.
- We have canonical choice of loop momenta.

Can we do optimal powercounting for $\mathcal{N} = 8$ SUGRA?

Thank You