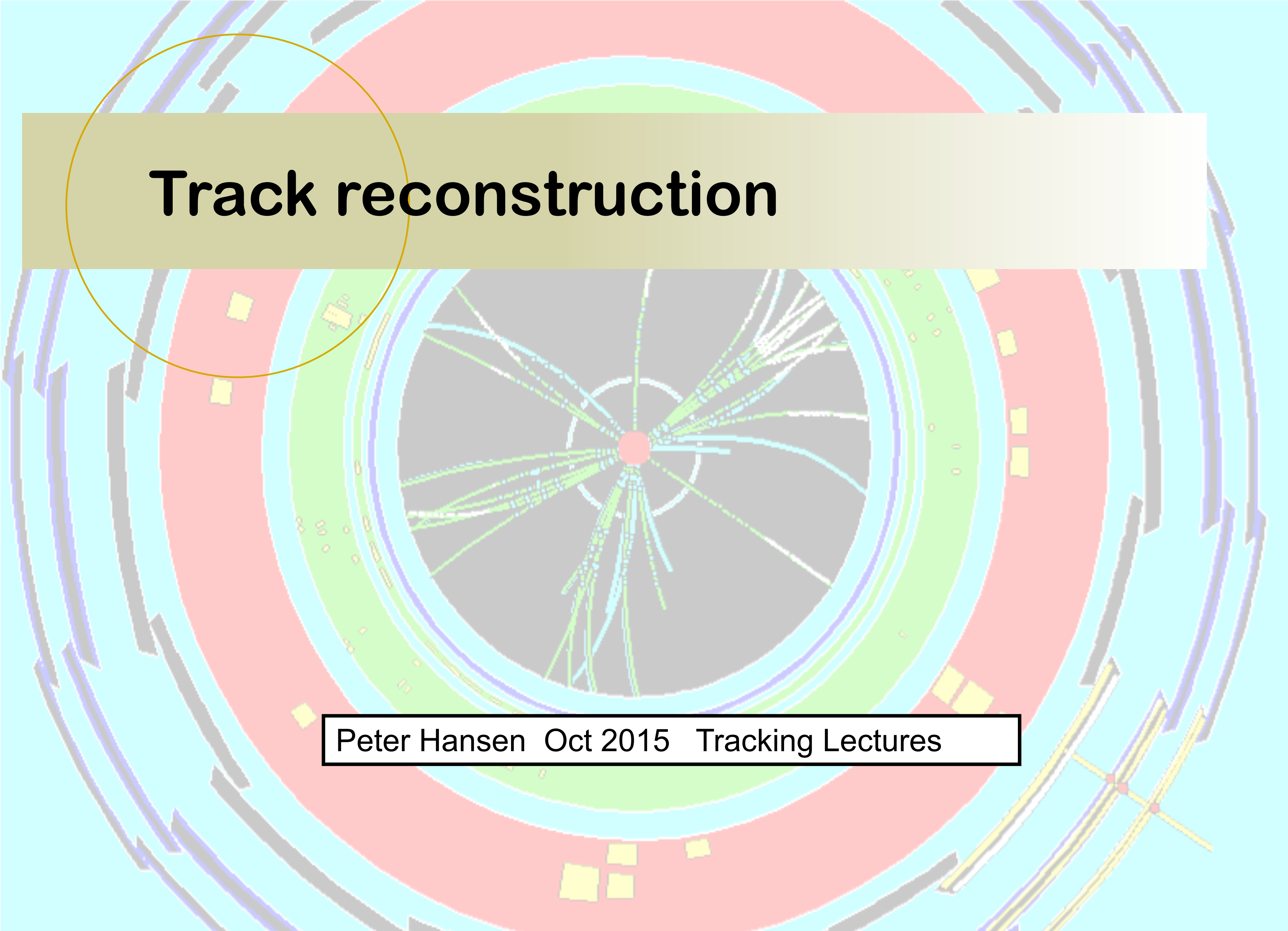
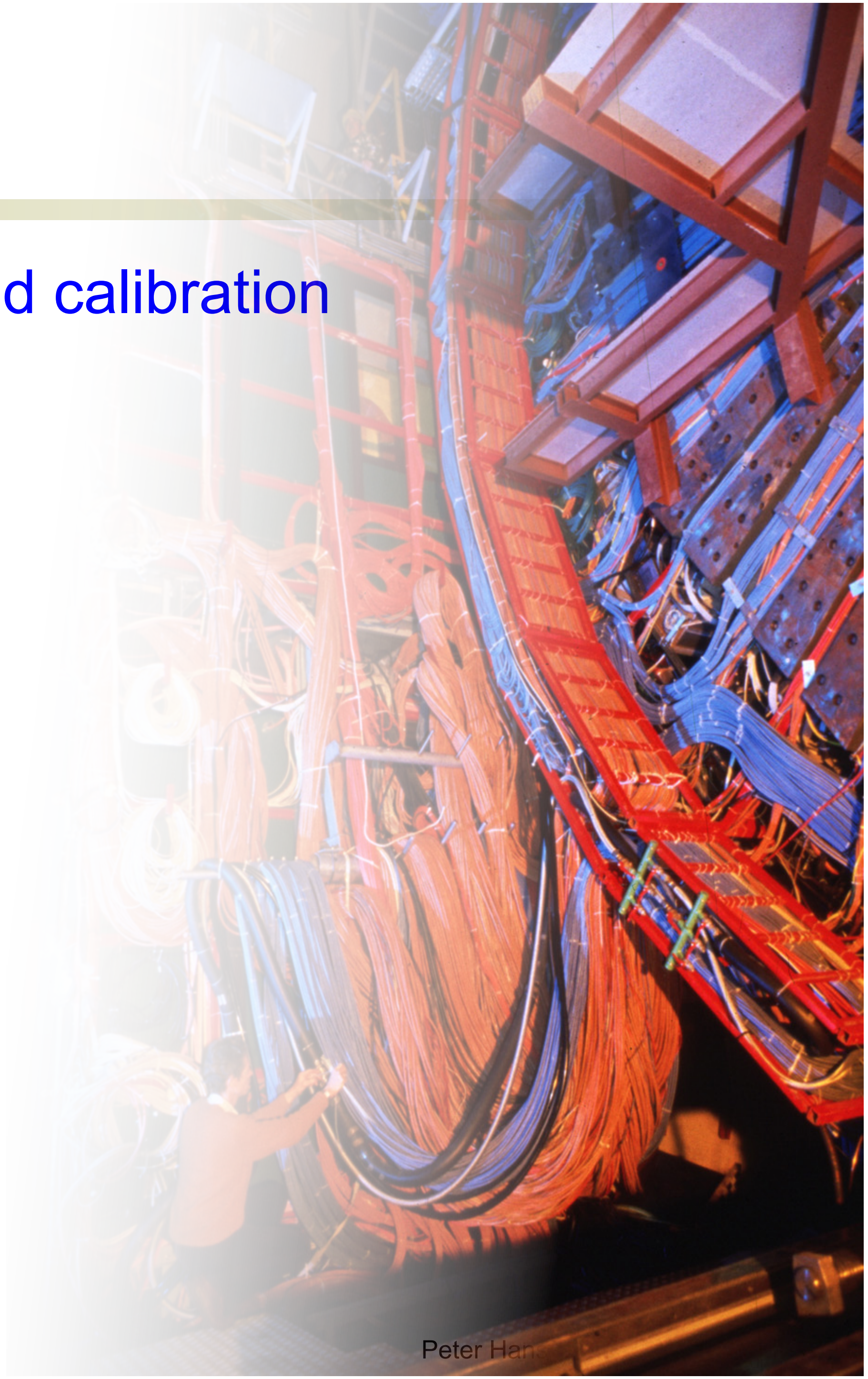
Track reconstruction



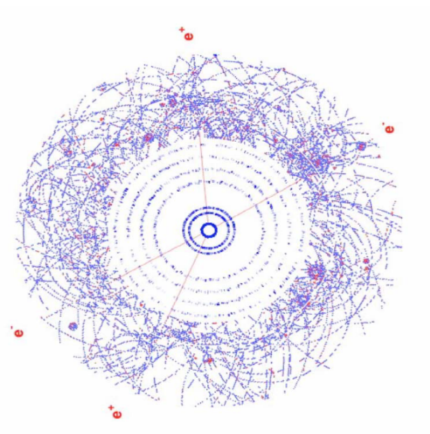
Peter Hansen Oct 2015 Tracking Lectures

Overview

- Spacepoint formation and calibration
- Pattern recognition
- Track fitting methods
- Vertexing
- Alignment



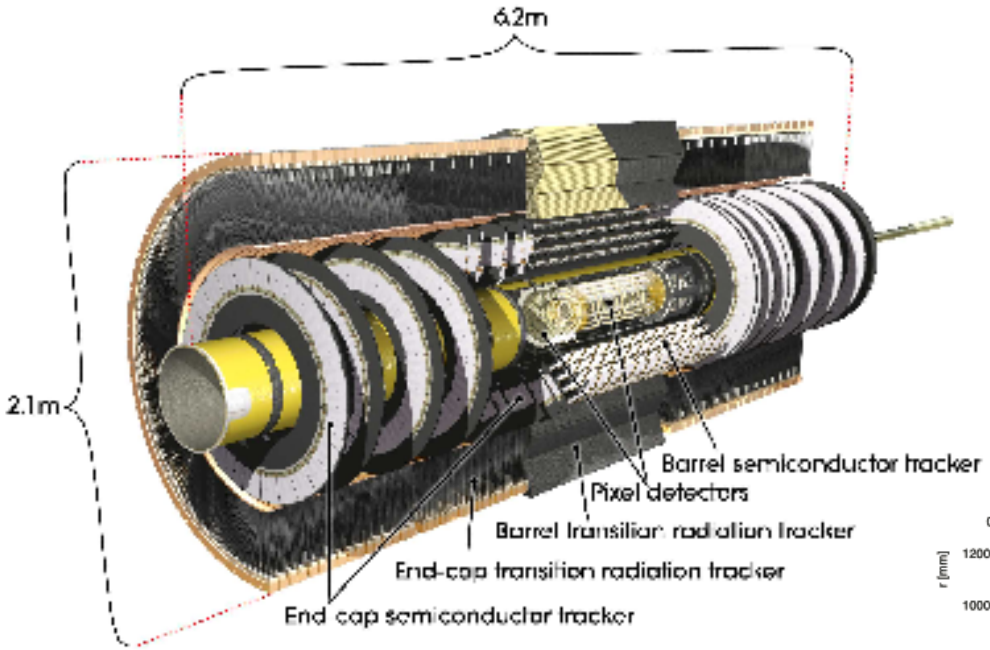
The tracking challenge

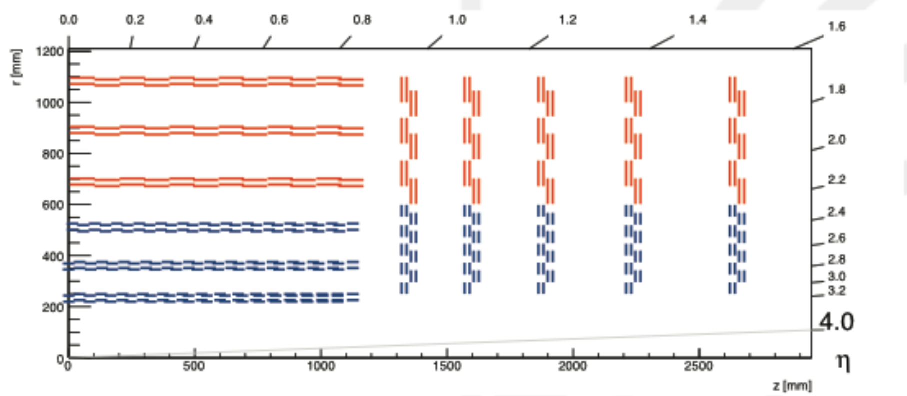


- Every second, 40 million beam-crossings are happening at the LHC, producing thousands of tracks from up to O(100) individual collisions. About 1 kHz of the crossings are to be selected for later processing.
- To cope with the high density and high momentum of the tracks very many channels are needed, causing rather large amounts of material in the tracking detectors.
- Thus, we need highly sophisticated and error-tolerant track-finders and –fitters, good calibration and alignment methods, robust vertexing and particle identification.

◆ This lecture covers standard tracking methods.

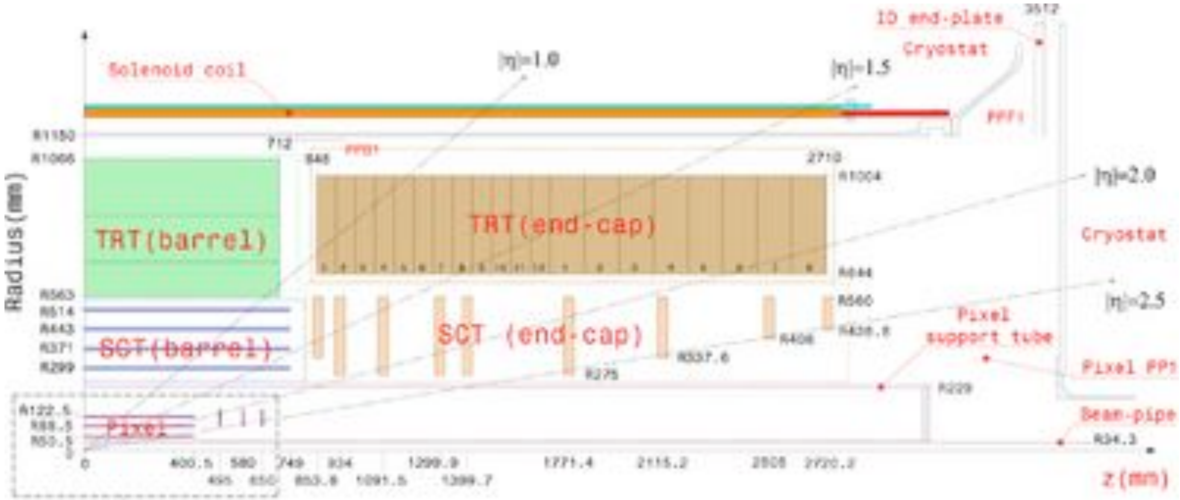
ATLAS and CMS Inner Trackers



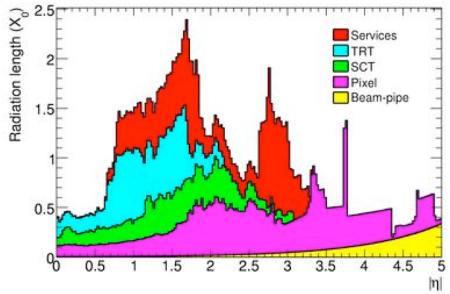


Many ATLAS and CMS examples are used this lecture. A general principle is to build detector planes roughly perpendicular to the tracks…

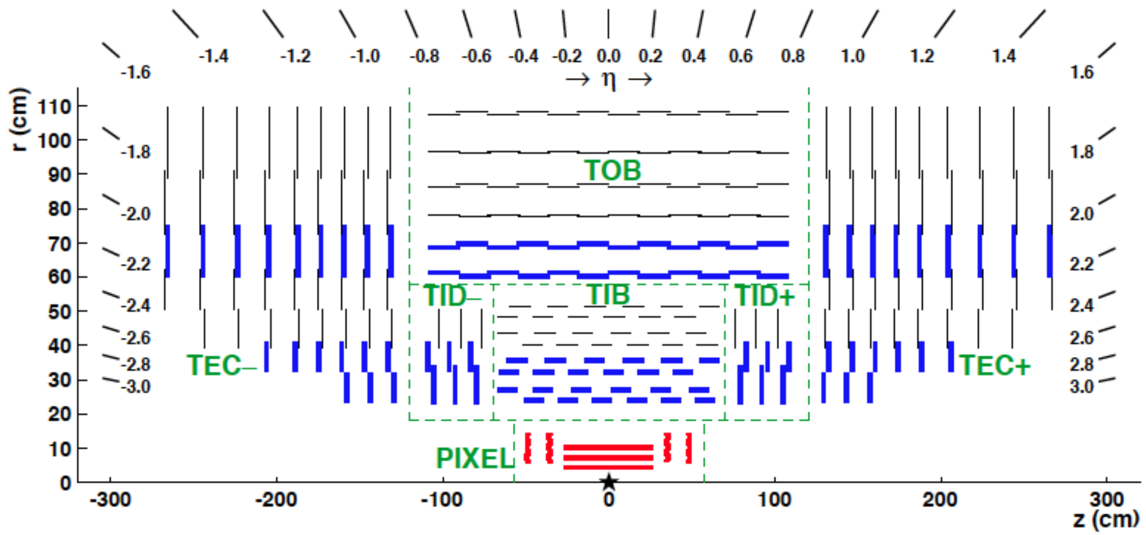
The Inner Detectors



The ATLAS ID

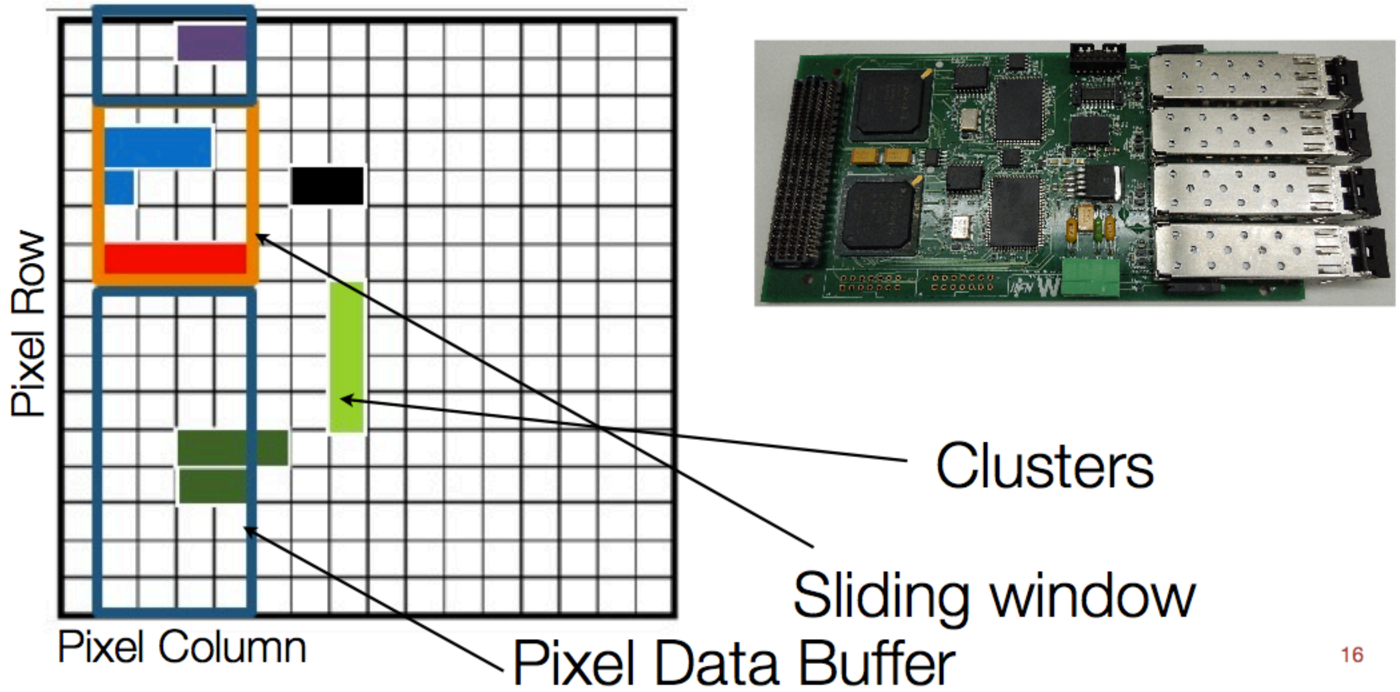


ATLAS ID material



The CMS silicon tracker

Space-point reconstruction



Clustering of pixel cells performed in hardware
by the ATLAS Fast Track Trigger

Spacepoint formation

- Tracking detectors register “hits” from signals induced on pickup electrodes by an *electron cloud* made by a track.
- In case of a hit on only one electrode, the precision is

$\Delta / \sqrt{12}$ (Δ = the electrode size).

- Much better is it if the signal is

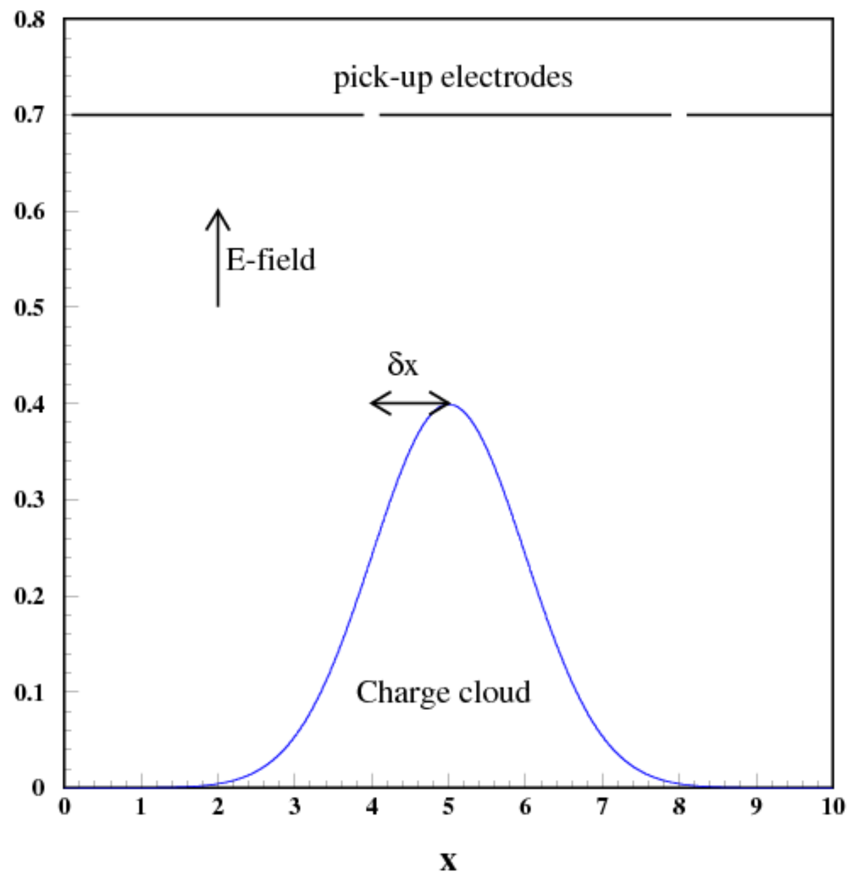
distributed over two electrodes.

In that case

$$\frac{\delta x}{\Lambda} = f(P_1, P_2, w)$$

This gives higher accuracy, but you need to know both P and the cloud width w.

Blum, Ronaldi: TPC tracking book



Spacepoint formation

- If 3 or more electrodes pick up signal for one track passing the detector layer, the barycenter is a popular estimator of the track position.
- The pulse-heights, P, must exceed a certain threshold and the electrodes must share a side or corner , forming a cluster. Summing over cluster cells, we get the barycenter:

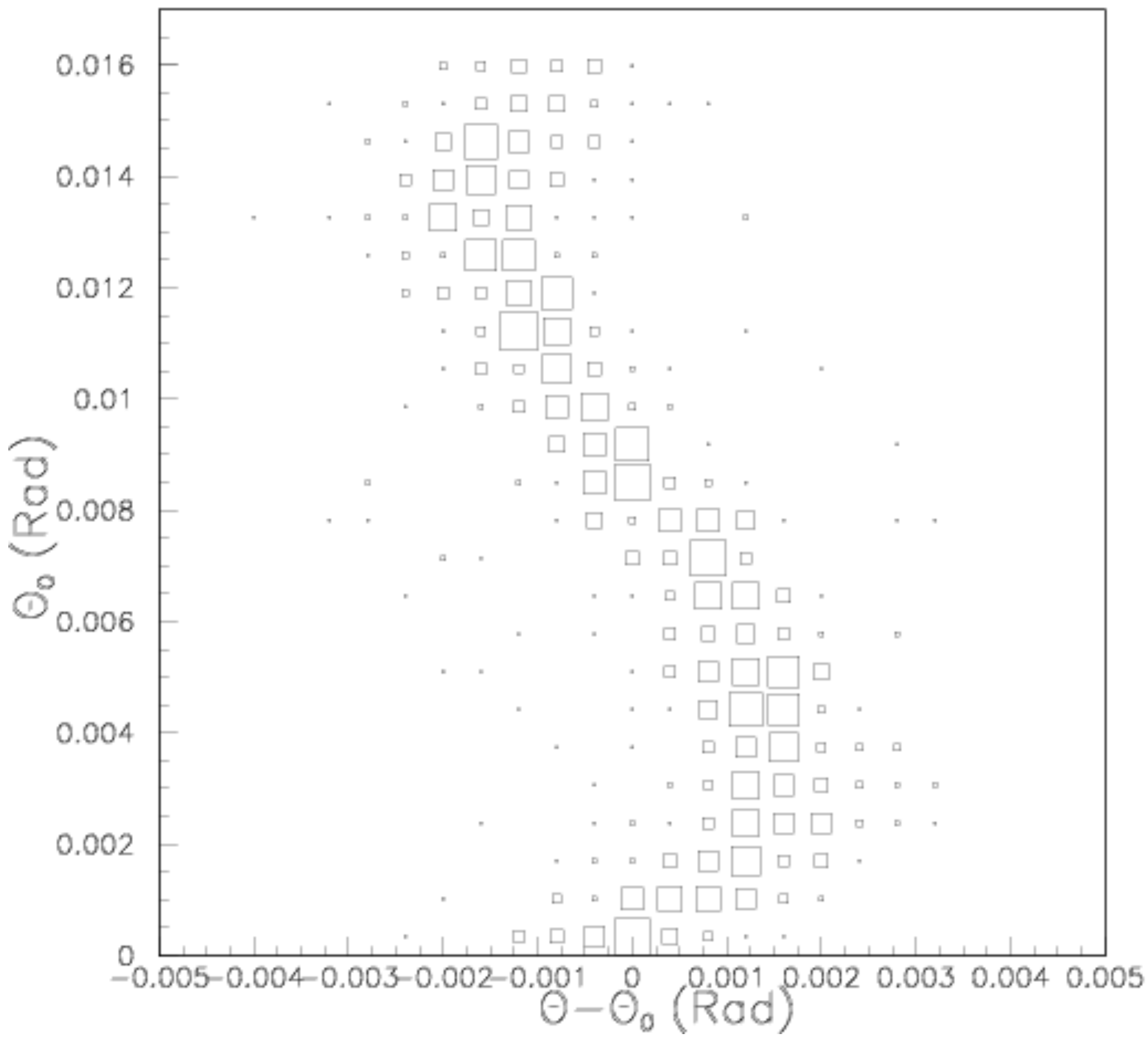
$$x = \sum P_i x_i \,/ \sum P_i$$

(sometimes only the cells at the *cluster edge* are used)

Spacepoint formation

- The barycenter needs correction because of the finite size of the electrodes.
- Example:

3x3cm electrodes in a lead-gas sampling calorimeter. The estimate is only unbiased at the border between two or at centre of one.

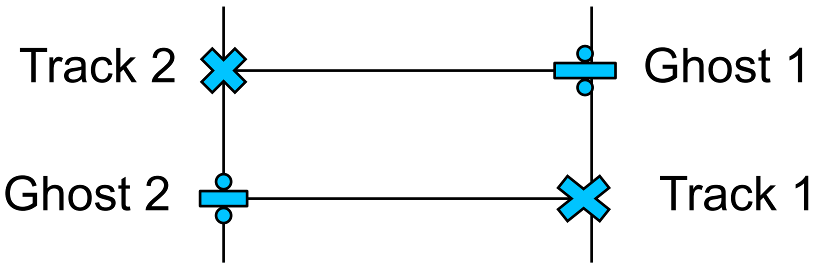


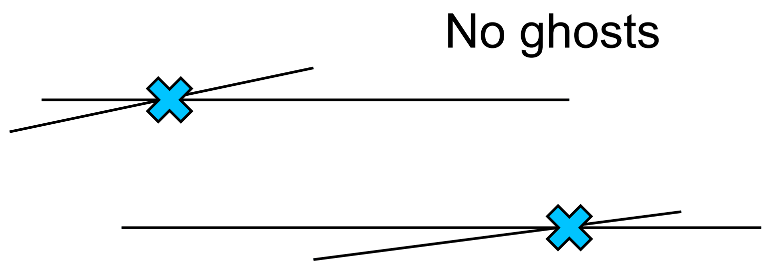
Aleph EM calorimeter

Stereo view

- If you do not have pixels, only wires or strips, what about the second coordinate?

- In strip detectors double sided wafers are often used with strips on both sides having an angle between them. But large angles gives ghost hits!
- At high track densities, 20-80 mrad is a good choice, avoiding too many ghost hits, having good resolution in the bending plane and still some resolution in the second coordinate.



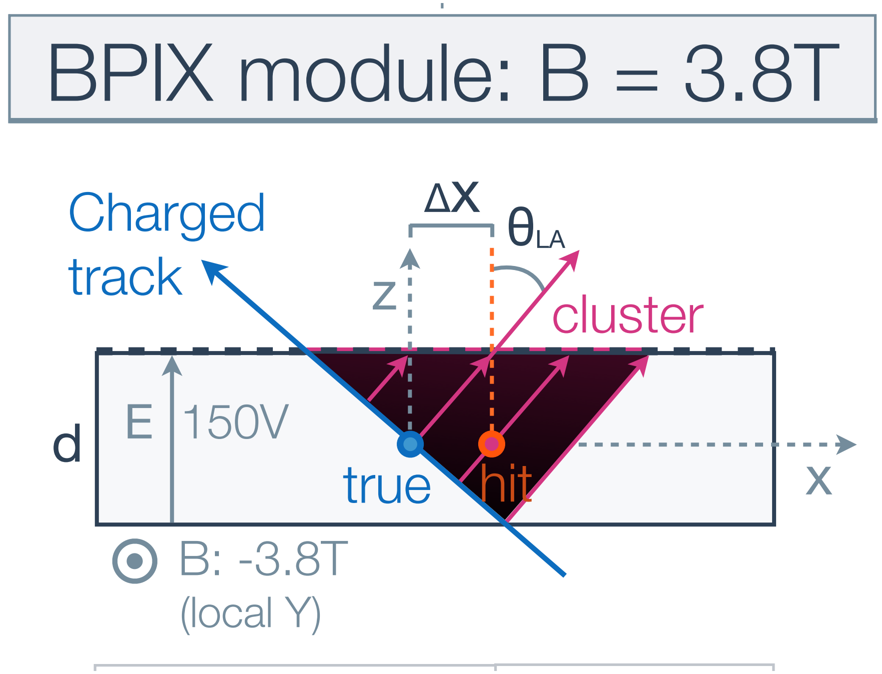


Spacepoint calibration

- In general we must know *the response function* , the probability distribution of induced pulse-heights for a given track impact
- (Actually, we would like the inverse: the pdf for the track impact, given the pulseheights. But we can not get that from testbeam..)
- The response function may vary from channel to channel and even vary in time. It must be *calibrated from data*.

Ex1: Lorentz angle and defects

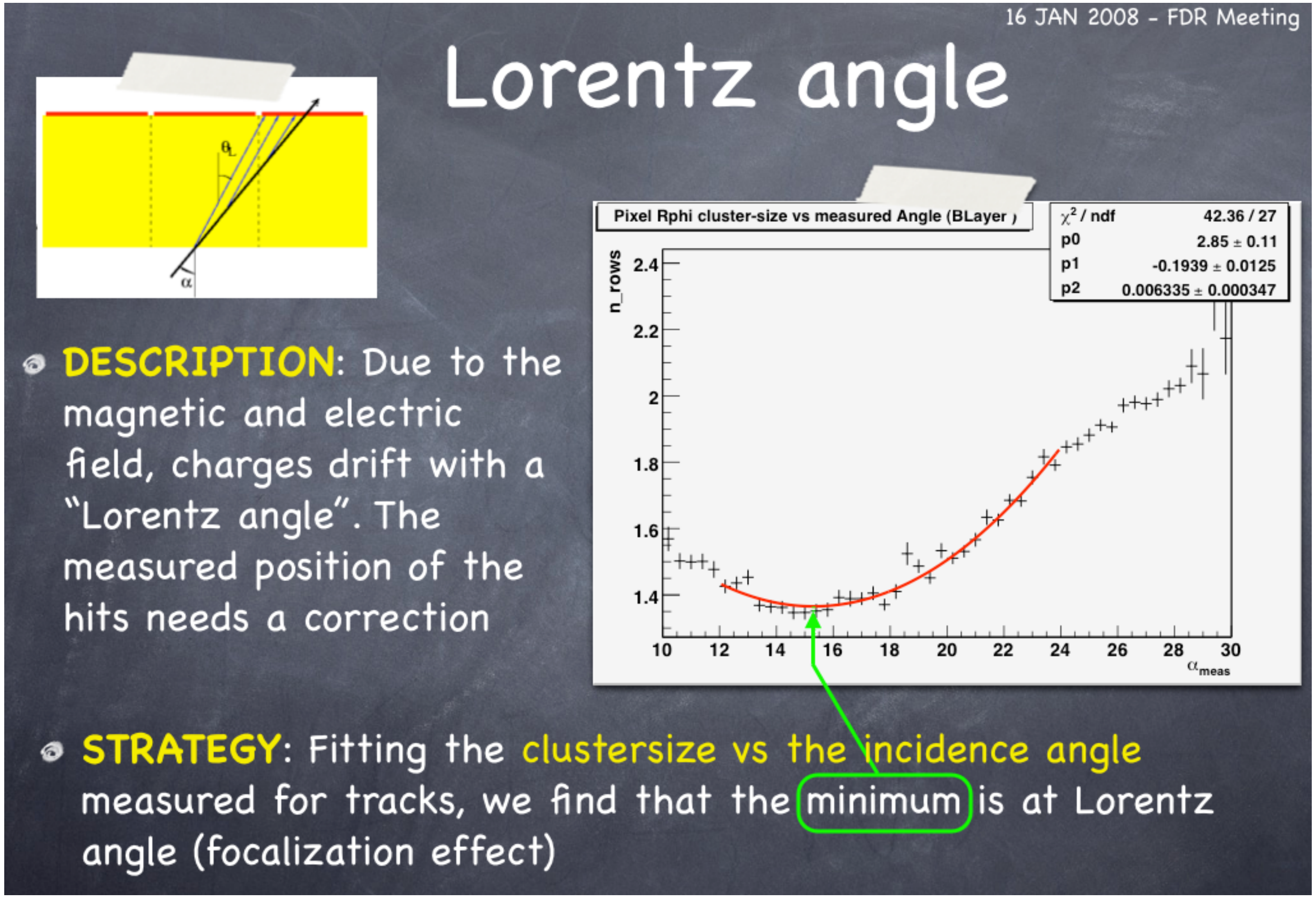
- Due to the Lorentz force from the B-field, the electron drift direction in silicon sensors is rotated by a Lorentz angle. This needs to be corrected for to get the true hit position.
- Another complication is the possibility of local radiation damage to pixels or strips biasing the barycenter.



- In CMS, all this is handled by comparing the observed charge distributions with a simulated template for each possible true track (sic!), where defects are accounted for.

Lorentz angle

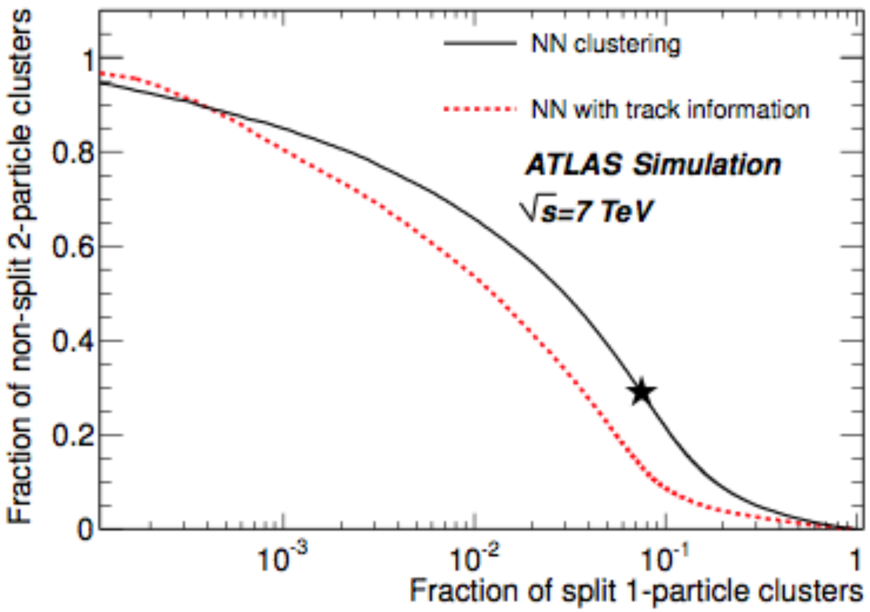
- In ATLAS, the Lorentz angle is extracted from the cluster size vs incident angle in the first tracking iteration. (from Simone Montesano)



Ex2: Splitting merged clusters

- At high track densities, clusters of fired detector cells from two different tracks may merge.
- For example, a jet with pT=1TeV has a typical distance of only 0.1mm between two tracks at the innermost ATLAS pixel layer.
- A NN algorithm has been developed to split pixel clusters again (Prokofieff and Selbach 2012)
- In the calorimeters similar MVA methods, or a simple search for local minima are used to split clusters.
- It is a must for NN or similar algorithms that pulse-height information is available (in the pixels this info is Time-over-Threshold).

Ex2: Splitting merged clusters



Improvements in Run2:

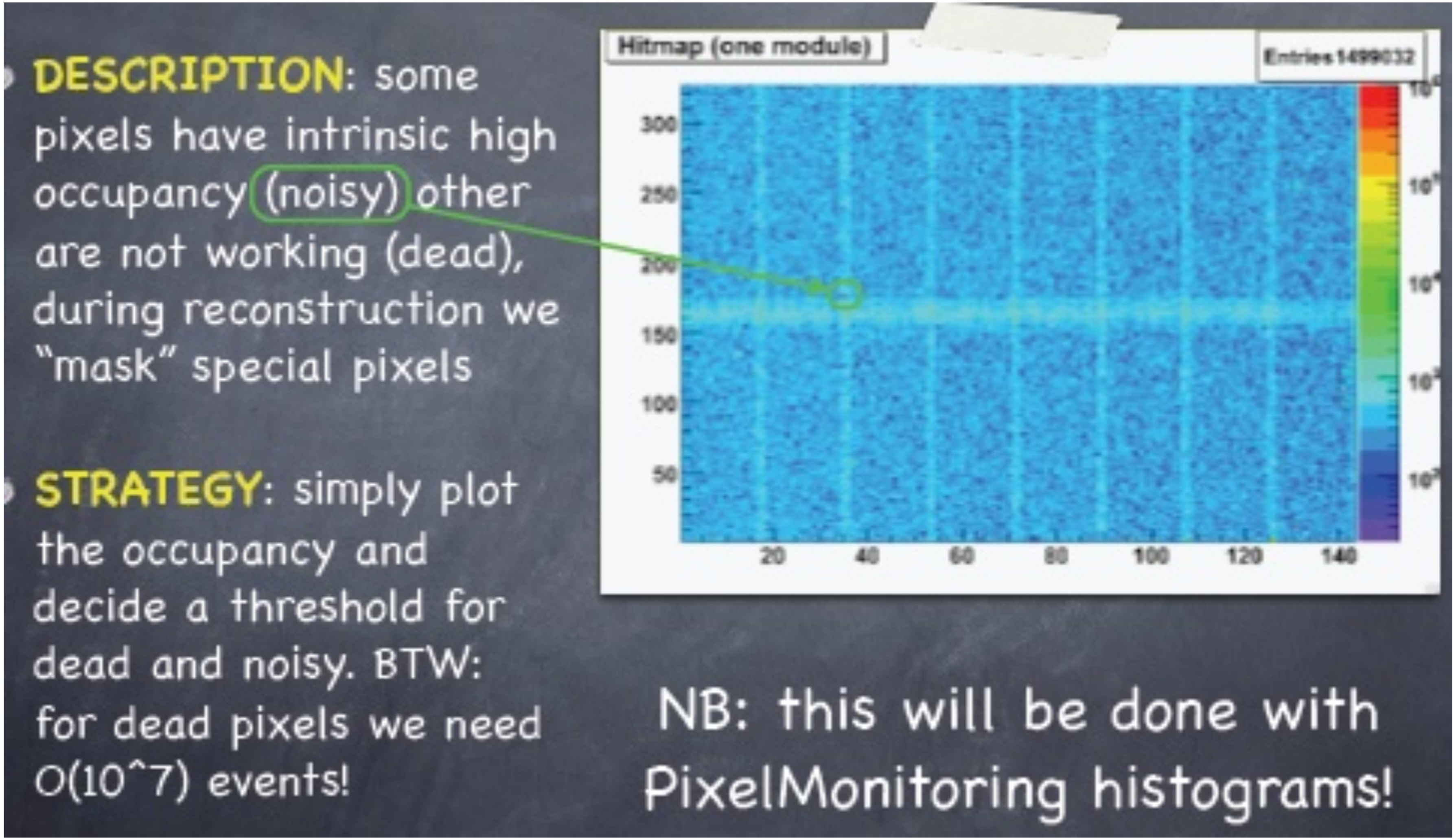
- NN (and other) evaluates if a pixel cluster is *shareable.*
- These can be shared without penalty in score (see later)
- Merged clusters are first split *after* Pass 1 track reco taking advantage of track info.

This yields:

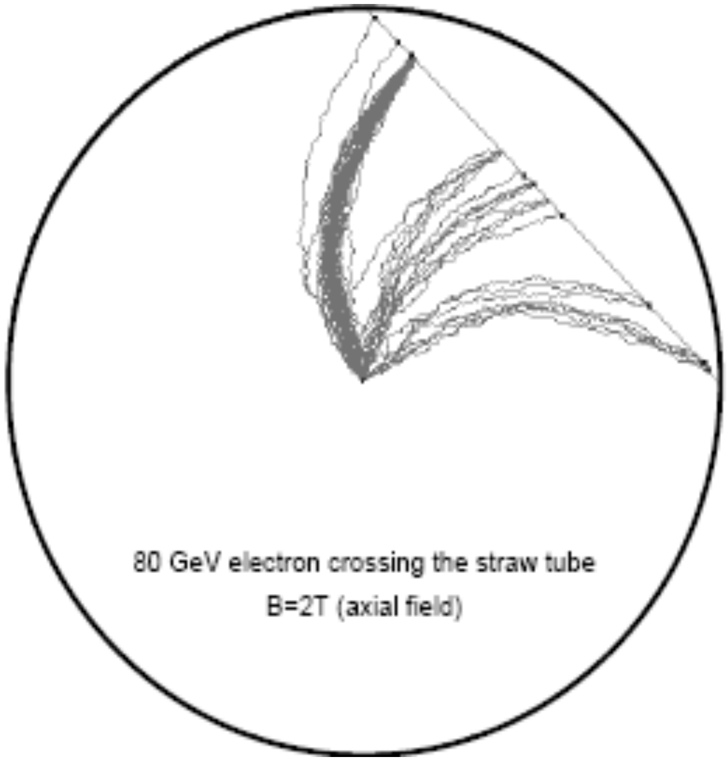
- a 10-17% improvement in track reconstruction efficiency in jet cores,
- a 7-13% increase in b-tagging efficiency
- a significant reduction of CPU (factor 4 when joined by other improvements in Run2 reco).

Ex3: Dead and noisy channels

- Any clustering algorithm must handle dead or noisy channels to avoid false clusters.

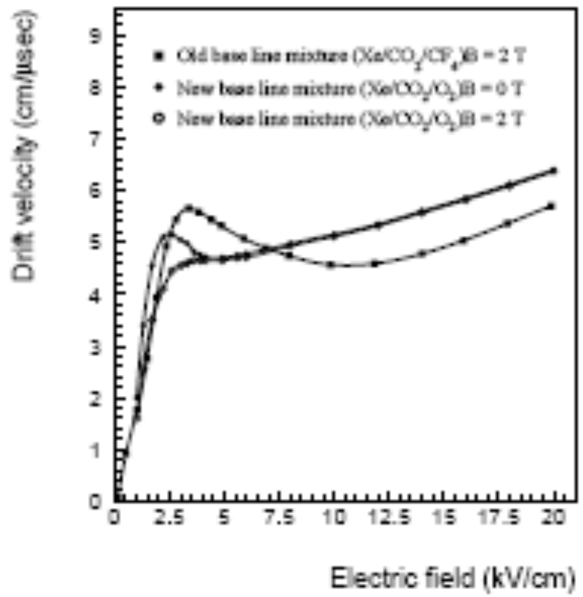


Ex4: Spacepoints in drift-tubes

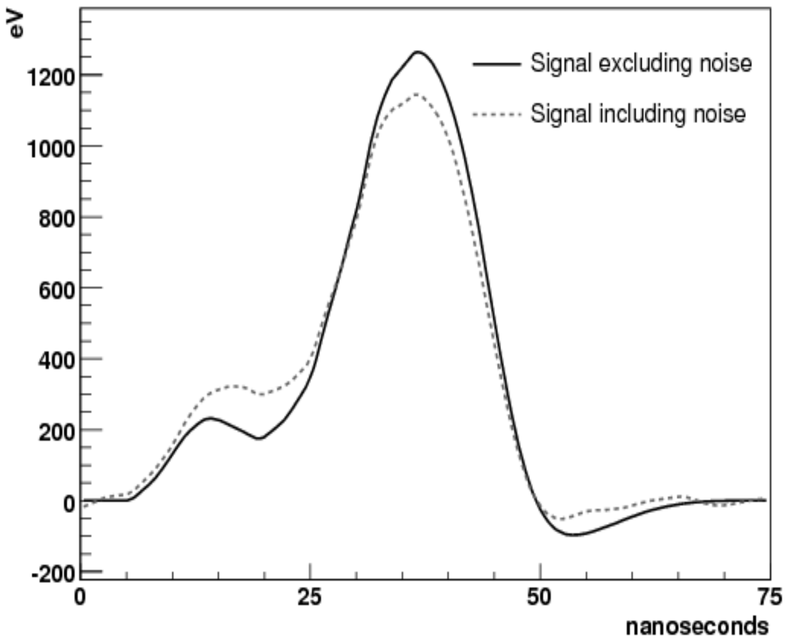


ATLAS TRT

The ATLAS TRT flags time-bin t where the signal exceeds some threshold. Must calibrate the distance R(t-t0) from the track to the each wire.



Refinements of drift radii



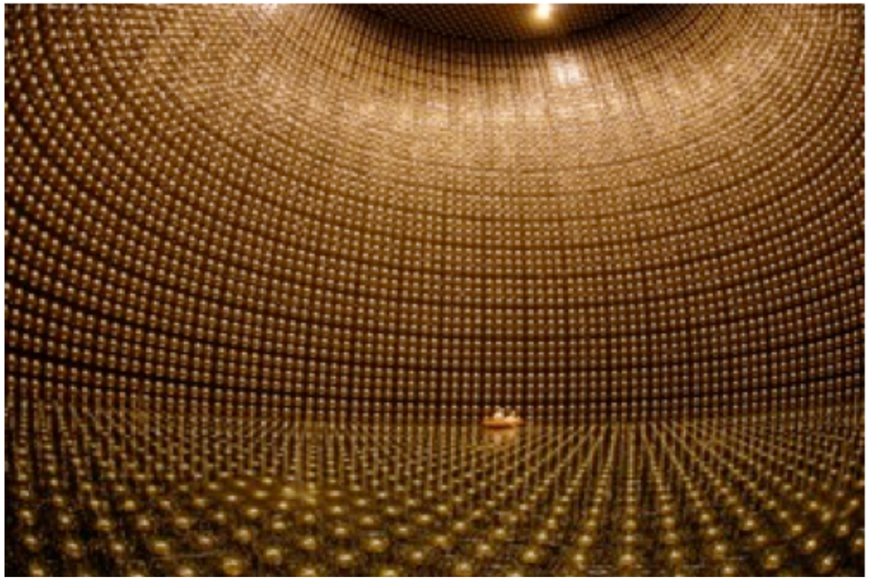
ATLAS TRT

Large pulses will trigger the threshold sooner for the same track impact -> small correction for large time-over-threshold or High-Threshold hit.

At a track refit, the track impact along the wire, angle and other info is available.

Small corrections for time-of-flight, signal propagation and other effects can be made at this point.

Ex5: Big water Cherenkov detectors



The “*space-points*” are here the signals on each PMT: the charges on the anode and their arrival times.

For calibration Super-K needs *for each single PMT:*

1) The gain = charge / photo-electron.

2) The quantum and collection efficiency.

3) The timing calibration and resolution.

4) The background level.

In addition it needs the water transparency, temperature, the geo-magnetic field etc at each point in space.

A variety of light sources, radioactive sources and even small linear accelerators are used in the calibration.

From space-points to tracks

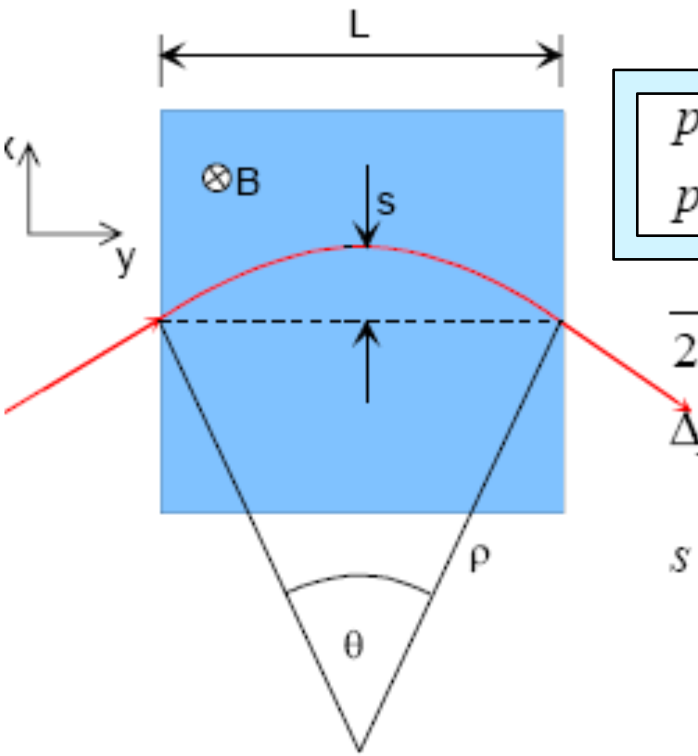
- Given a collection of space-points we need to group together those space-points that belong to a track and determine the tracks features.

- The important feature of a track is its momentum, so we open a parenthesis on momentum measurement

- Then we will look at pattern recognition

- Then study two track fit algorithms: the Kalman filter and the global chi-squared fit.

Momentum measurement (..



$$\boxed{\begin{aligned} &p_T = qB\rho \\ &p_T\ (\mathrm{GeV/c}) = 0.3B\rho \quad (\mathrm{T\cdot m}) \end{aligned}}$$

$$\frac{L}{2\rho} = \sin\theta/2 \approx \theta/2 \quad \rightarrow \quad \theta \approx \frac{0.3L\cdot B}{p_T}$$

$$\Delta p_T = p_T \sin\theta \approx 0.3L\cdot B$$

$$s = \rho(1-\cos\theta/2) \approx \rho\frac{\theta^2}{8} \approx \frac{0.3}{8}\frac{L^2 B}{p_T}$$

This and next three slides are from Christian Jorams summer student lectures

Momentum accuracy

the sagitta s is determined by 3 measurements with error $\sigma(x)$:

$$s = x_2 - \frac{x_1 + x_3}{2}$$

$$\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} = \frac{\sigma(s)}{s} = \frac{\sqrt{\frac{3}{2}}\sigma(x)}{s} = \frac{\sqrt{\frac{3}{2}}\sigma(x)\cdot 8p_T}{0.3\cdot BL^2}$$

for N equidistant measurements, one obtains

(R.L. Gluckstern, NIM 24 (1963) 381)

$$\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} = \frac{\sigma(x)\cdot p_T}{0.3\cdot BL^2}\sqrt{720/(N+4)}$$ (for $N \geq \approx 10$)

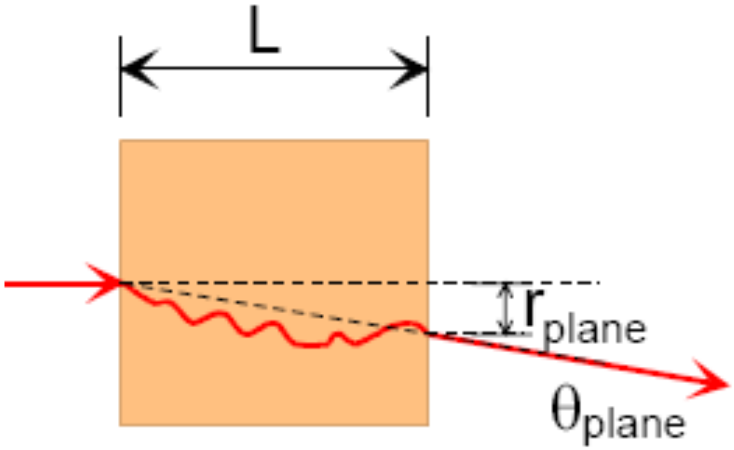
ex: p_T=1 GeV/c, L=1m, B=1T, $\sigma(x)$=200μm, N=10

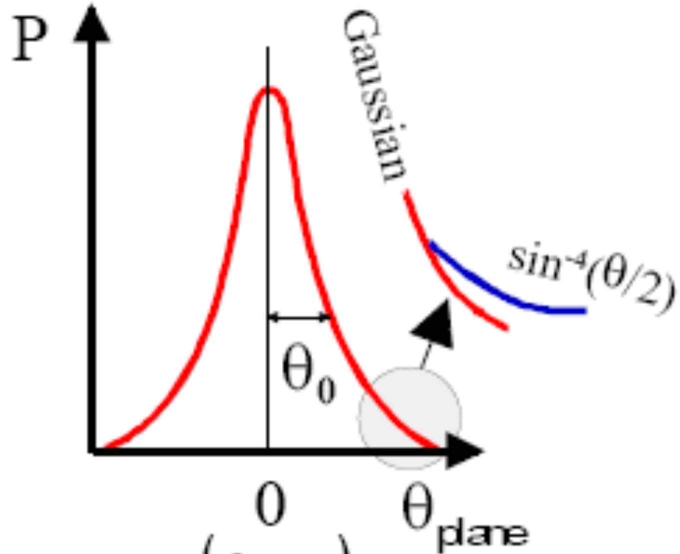
$$\left.\frac{\sigma(p_T)}{p_T}\right|^{meas.} \approx 0.5\%$$ ($s \approx 3.75$ cm)

Multiple scattering

Sufficiently thick material layer

→ the particle will undergo multiple scattering.





$$\theta_0 = \theta_{plane}^{RMS} = \sqrt{\langle \theta_{plane}{}^2 \rangle} = \frac{1}{\sqrt{2}} \theta_{space}^{RMS}$$

$$P(\theta_{plane}) = \frac{1}{\sqrt{2\pi}\theta_0} \exp\left\{ -\frac{\theta_{plane}{}^2}{2\theta_0^2} \right\}$$

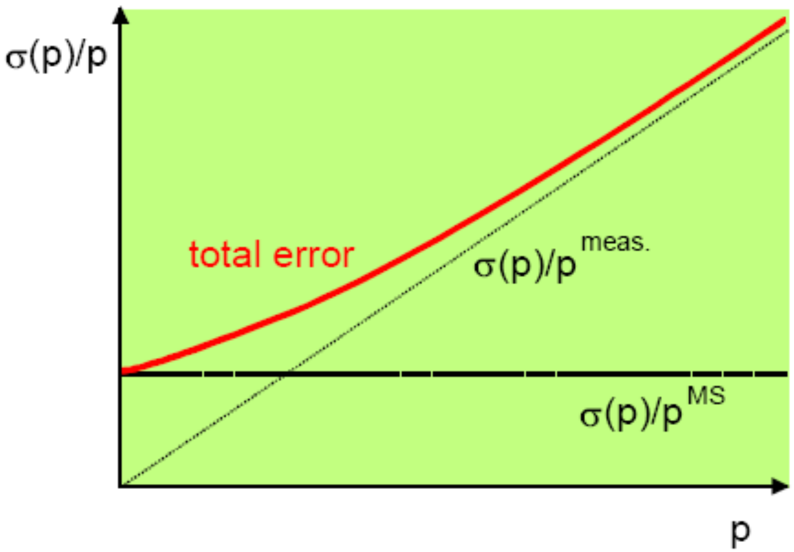
Total momentum error ..)

contribution from multiple scattering

$$\Delta p^{MS} = p \sin\theta_0 \approx p \cdot 0.0136 \frac{1}{p}\sqrt{\frac{L}{X_0}}$$

$$\left.\frac{\sigma(p)}{p_T}\right|^{MS} = \frac{\Delta p^{MS}}{\Delta p_T} = \frac{0.0136\sqrt{\frac{L}{X_0}}}{0.3BL} = 0.045\frac{1}{B\sqrt{LX_0}}$$

independent of p !

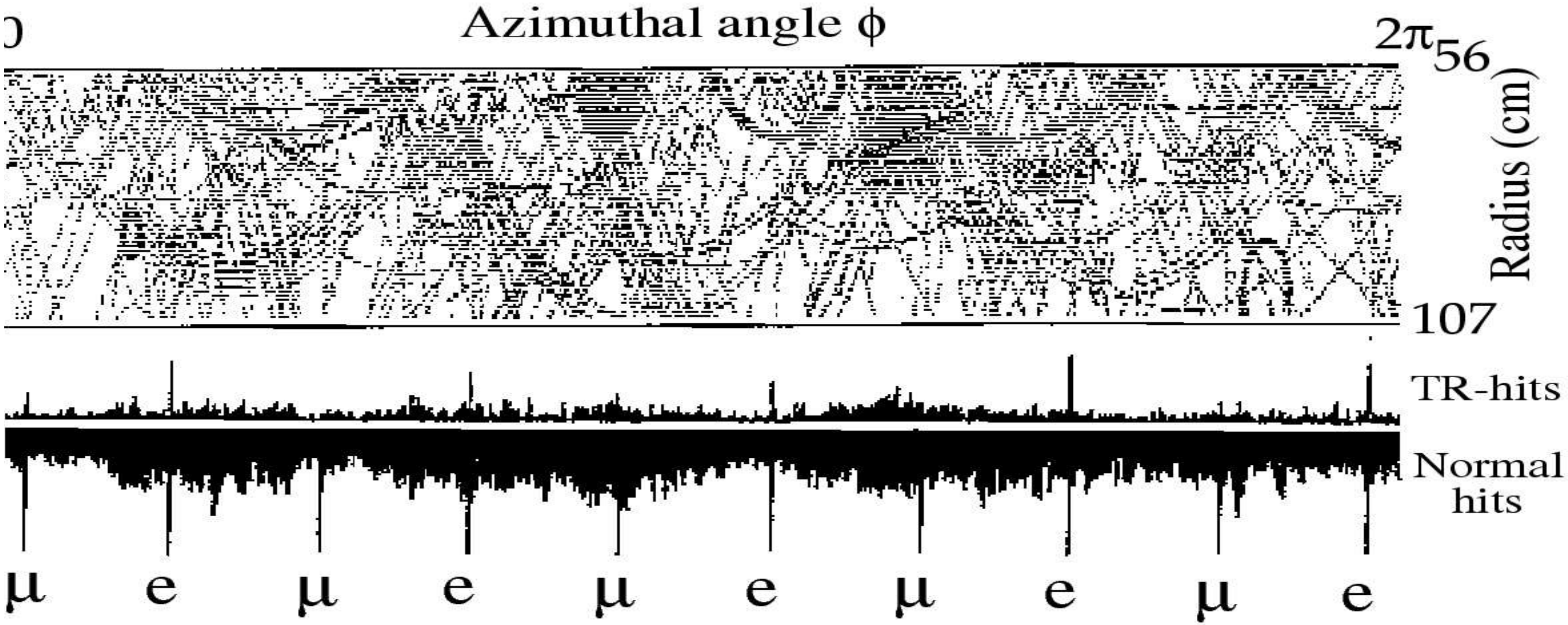


Fast pattern recognition

- How to fast associate a subset of the hits to a track?
- *Predefined templates*, i.e. patterns of fired cells defining an allowed track. Used in fast trigger algorithms. The *cell tower* is an example from the calorimeter world.
- *Hough transform* is another method. For straight tracks in two dimensions, each hit corresponds to a straight line in the slope-intercept plane. Peaks in this plane where many lines intercept reveal the hits-on-tracks. This is also relatively fast.

Simple Hough transform

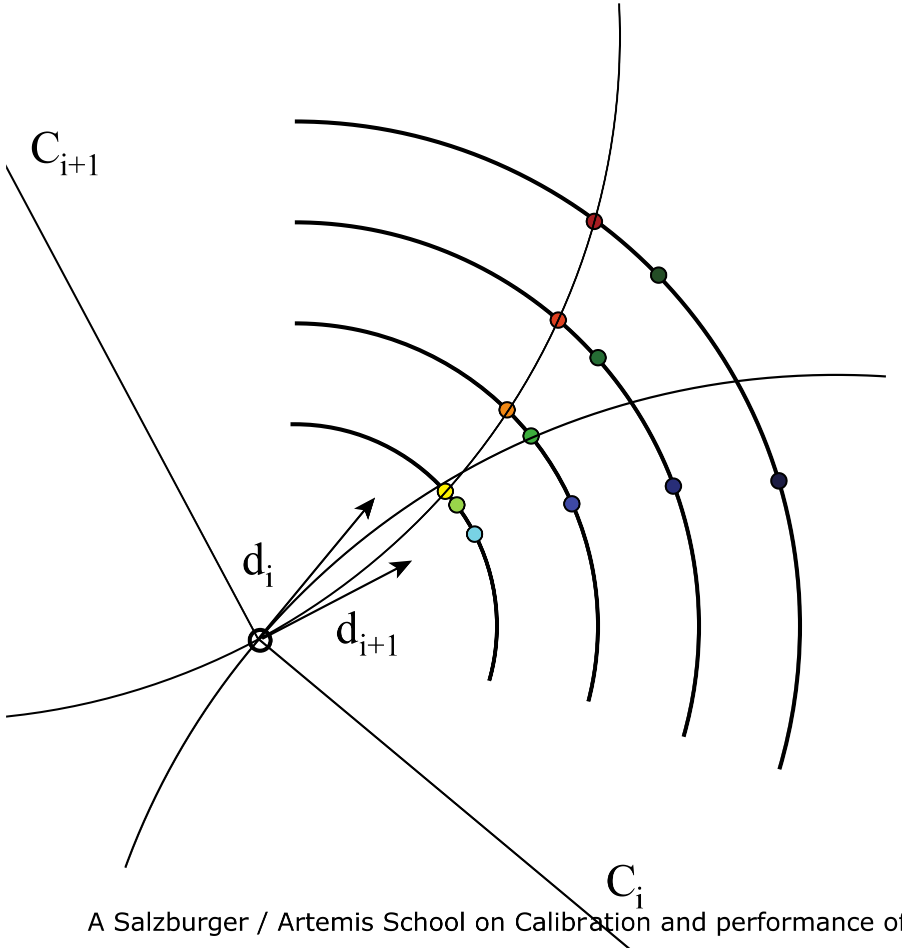
- Histogram methods may provide fast seeds for high momentum tracks – here an example:

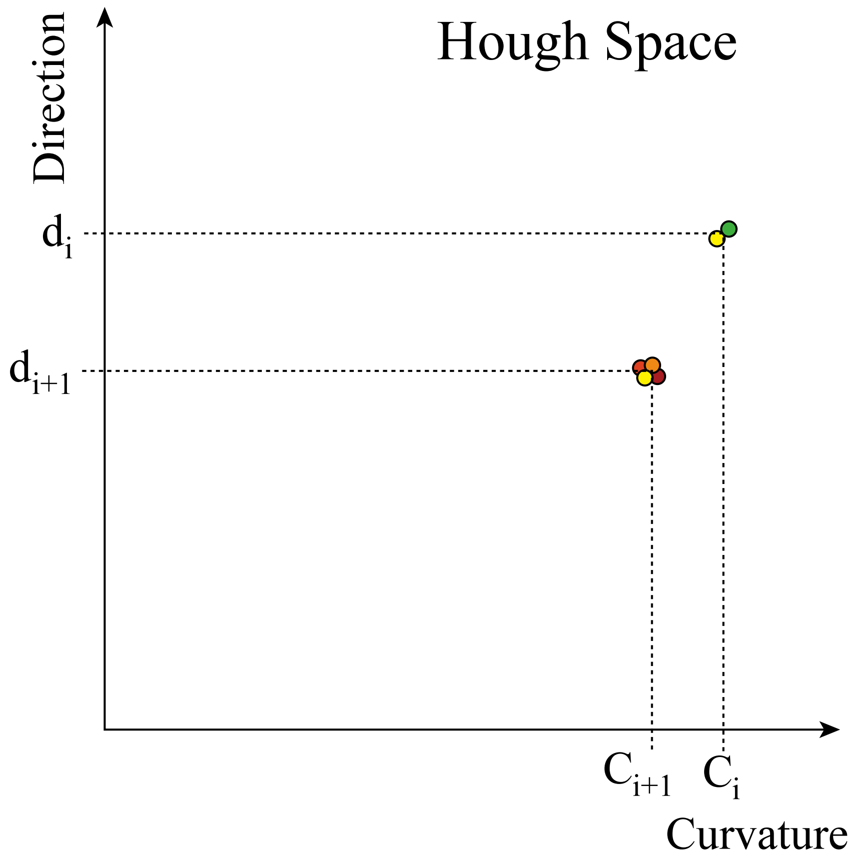


ATLAS TRT

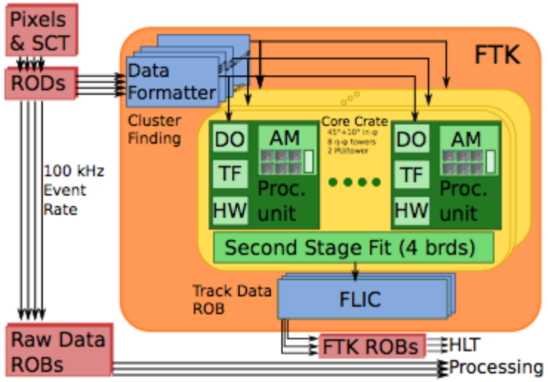
General Hough transform

- Scan in two dimensions (d_i=-$C_i r+d_{hit}$)
- Count number of compatible hits.

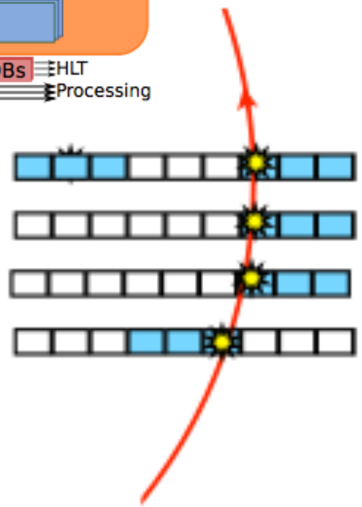




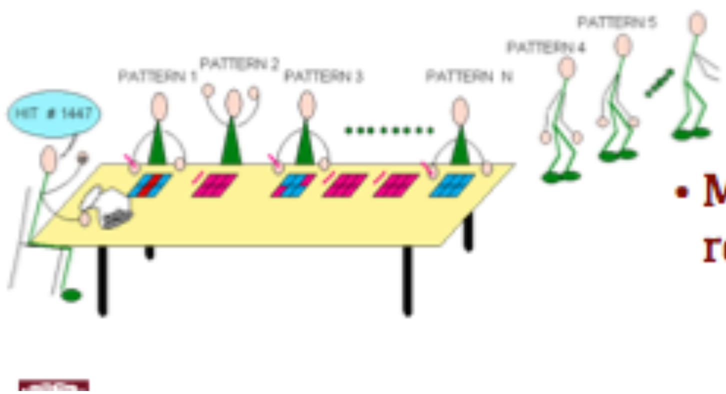
Associative memory – ATLAS FTK



The *associative memory* where each hit is seen by all possible templates is the most advanced example.



- Hits are ganged into Super Strips (SS)
  - Roughly 15x36 pixels/16strips per SS @ 70 int/x-ing
- **Custom associative memory chips** are used to compare hits to $O(10^9)$ patterns **simultaneously**
  - Pattern matching finished as soon as all hits are read
- Matched patterns (Roads) are then fit to reject bad roads
  - Most matches are fake, need fits to reduce bad rate



The state vector

- Let at each detector surface the track be given by a vector (position, direction, 1/p), along with its uncertainties:

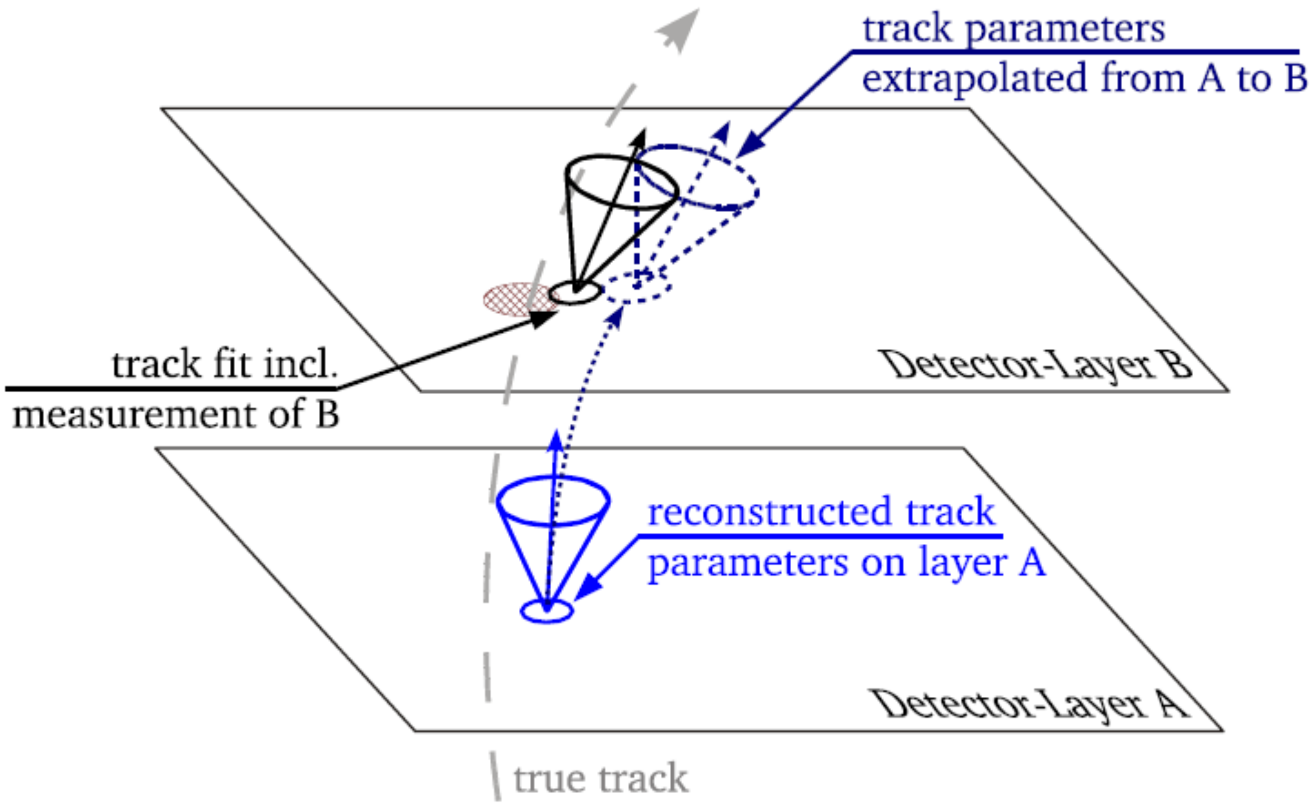


Figure from ATLAS reconstruction group

The helix

- An example of a *state vector* is *helix parameters*, where 90^o-λ is the track angle to the B field, R is the radius, s the path length and h is a sign. This gives the trajectory:

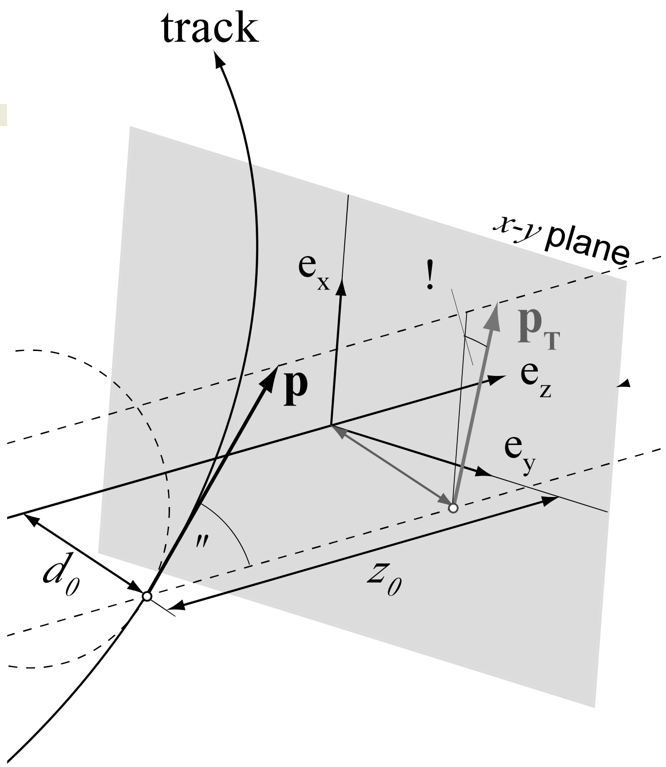
$$
\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} x^0 + R\cdot(\cos(\alpha_0 + \frac{hs}{R}\cdot\cos\lambda) - \cos\alpha_0) \\ y^0 + R\cdot(\sin(\alpha_0 + \frac{hs}{R}\cdot\cos\lambda) - \sin\alpha_0) \\ z^0 + s\cdot\sin\lambda \end{pmatrix}
$$

- A track in a detector with cylinder symmetry is a collection of helices at each “cylinder surface”.

Perigee parameters

- The perigee parameters

$$x = (\varphi_0, d_0, z_0, \theta, \frac{q}{p})$$



are often used to describe the track state at the closest point to the beam (z) axis.

- Use q/p because q/p is approximately measured with a gaussian uncertainty.
- d_0 has some sign convention according to the angular momentum of the track wrt the z axis

The projection matrix H

- To compare with measurements $\underline{m}$, the track state $\underline{x}$ needs to be mapped onto "measurement space". We linearize:
- H=δm/δx, where H is the projection matrix

(assuming for simplicity that $\underline{x}=0$ corresponds to $\underline{m}=0$)

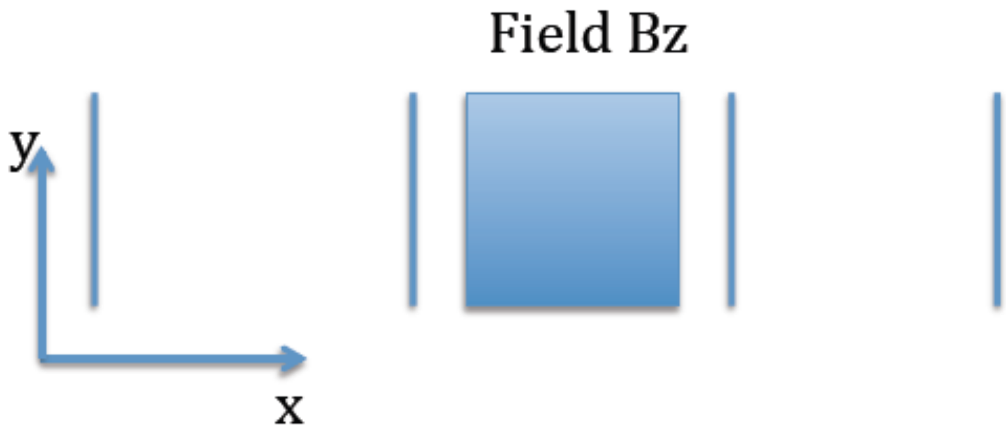
- Consider, for example, a set of strips forming a small angle α with the x axis. Let the track parameters be x and y at each plane of strips. Then we must do

$$H\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -\sin\alpha & \cos\alpha \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

in order to arrive at the y' -coordinate perpendicular to the tilted strip. This y' measures the hit strip numbers which are the raw measurements $\underline{m}$.

Spectrometer example

4 y-z pixel planes   distance between planes d



$$\tan(\Delta\varphi) = 0.3 \times \int B_z dx \times q/p = b(q/p)$$

Using units of Tesla,m, and GeV/c

Spectrometer example

Projection matrix (linearized)

measurements

State vector at plane 1

$$m = \begin{bmatrix} z_1 \\ \dots \\ z_4 \\ y_1 \\ \dots \\ y_4 \end{bmatrix}, \qquad x = \begin{bmatrix} z \\ z' \\ y \\ y' \\ q/p \end{bmatrix}, \qquad H = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 1 & d & 0 & 0 & 0 \\ 1 & 2d & 0 & 0 & 0 \\ 1 & 3d & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d & 0 \\ 0 & 0 & 1 & 2d & bd/2 \\ 0 & 0 & 1 & 3d & 3bd/2 \end{bmatrix}$$

The Kalman filter

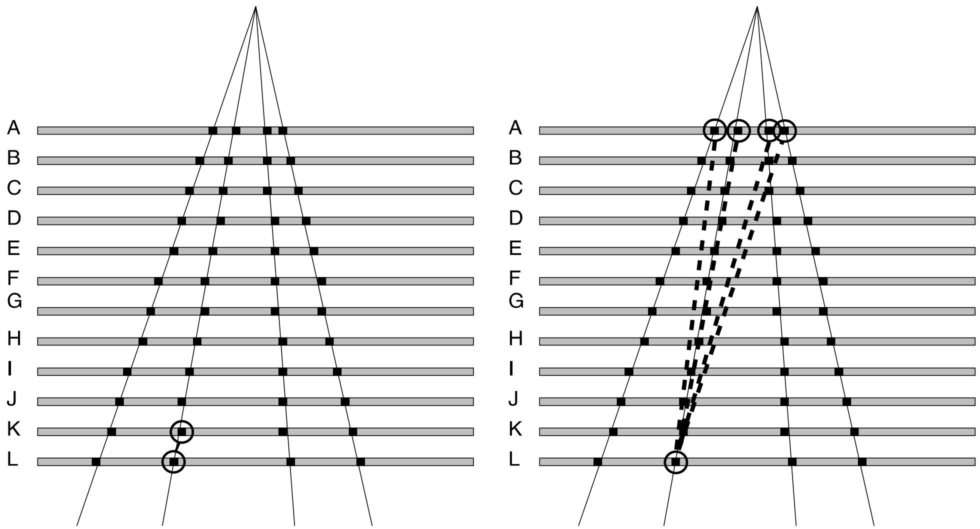
- Determines the track state vector dynamically from measurements at each detector surface.
- These are either discarded or used to update the existing state vector.
- Needs only inversion of small matrices. Fast.
- Can account for noise, multiple scattering and energy loss at each surface. Efficient.
- Is equivalent to the least squares fit, but provides pattern recognition integrated in the fit.

Seeding

- We need TRACK SEEDs with a high efficiency and modest fake rate. Many different strategies:

Figure from Mankel arXiv:0402039v1

Start from two outer layers:



Connect all inner layer hits to each outer

- ATLAS and CMS use the inner pixel layers for seeds and then proceed outwards for track finding

The propagator F

- Let the track transport from layer k-1 to k be given by

$$x_k = f(x_{k-1})$$

- Let the predicted state be denoted by a tilde. If f is not already linear in x, we Taylor expand it:

$$\tilde{x}_k = F_k x_{k-1},$$

$$C_k^{k-1} = F_k C_{k-1} F_k^T + Q_k$$

where C_k is the covariance matrix for the predicted state and Q contains the additional random perturbations in the step, such as multiple scattering and energy loss.

Covariance matrices V and C

- A pair of random variables x_i and x_j has the covariance matrix:

$$V_{ij} = E((x_i - E(x_i)) \times (x_j - E(x_j)))$$

It is symmetric and have diagonal elements equal to the variances of the x'es.

Off-diagonal elements describe the degree of correlation between x_i and x_j.

- Any set of functions f_i of the x's has (to lowest order in a Taylor expansion) the covariance matrix:

$$C_{ij}^{f} = \sum_{kl} \frac{\partial f_i}{\partial x_k} \frac{\partial f_j}{\partial x_l} V_{kl}$$

- This is the chain rule.

The propagator F – simple example

- The F propagator is exactly the same as the transfer matrix of accelerator physics.
- For our example spectrometer we have the z projection propagation from the second to the third plane (a drift space in accelerator language).

$$\tilde{z}_3 = F_3 z_2 = \begin{bmatrix} 1 & d \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_2 \\ z'_2 \end{bmatrix},$$

$$C_3 = F_3 C_2 F_3^T + Q.$$

$$C_2 = \begin{bmatrix} \sigma_m^2 & \sigma_m^2 / d \\ \sigma_m^2 / d & 2\sigma_m^2 / d^2 \end{bmatrix}, \qquad Q = \begin{bmatrix} \theta_{MS}^2 d^2 & \theta_{MS}^2 d \\ \theta_{MS}^2 d & \theta_{MS}^2 \end{bmatrix}$$

The propagator F – complex case

- In regions with an inhomogenous B field, the preferred method is Runge-Kutta integration. Here, the trajectory derivatives are sampled at a number of intermediate positions, weighted so that the error is 5th power in h, the small time-step to the next plane:

$$y' = f(t, y), \qquad y_0 = y_0(t_0)$$

$$y_{n+1} = y_n + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$k_1 = f(t_n, y_n)$$

$$k_2 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_1)$$

$$k_3 = f(t_n + \frac{h}{2}, y_n + \frac{h}{2}k_2)$$

$$k_4 = f(t_n + h, y_n + hk_3)$$

The propagator F – complex case

- Let us try a grossly nonlinear case y=exp(t):

$$y' = y, \qquad y_0 = 1$$

$$k_1 = 1$$

$$k_2 = 1 + \frac{h}{2}$$

$$k_3 = 1 + \frac{h}{2}(1 + \frac{h}{2})$$

$$k_4 = 1 + h + \frac{h^2}{2}(1 + \frac{h}{2})$$

$$y_1 = 1 + \frac{h}{6}(k_1 + 2k_2 + 2k_3 + k_4) = 1 + h + \frac{h^2}{2} + \frac{h^3}{6} + \frac{h^4}{24}$$

- Victory!

The residual r

- The difference between a measurement m and its prediction by the track state, Hx, is called the residual:

$$r_k^{k-1} = m_k - H_k \tilde{x}_k, \;\; R_k^{k-1} = V_k + H_k C_k^{k-1} H_k^T$$

V is the *covariance matrix* of the measurements

R is the *covariance matrix* of the residuals.

(Note that the contribution from the track is here *added* to the measurement variance. The measurement is not used yet. If the hit contributes to the track, the track variance is instead *subtracted* from the residual variance).

At this point you can reject a measurement m_k on the basis of r_k^2/R_{kk} . This is the *pattern recognition* part.

Updating the state vector:

- A way to update the track state with the newly added hit is to take a weighted average of the predicted track state and the state suggested by the new measurement:

$$x_k = C_k((C_k^{k-1})^{-1} x_k^{k-1} + H^T V_k^{-1} m_k)$$

$$C_k^{-1} = (C_k^{k-1})^{-1} + H^T V_k^{-1} H$$

Updating the state vector:

- An equivalent way is to use the gain matrix K:

$$K_k = C_k^{k-1} H^T (V_k + H_k C_k^{k-1} H_k^T)^{-1}$$

$$\tilde{x}_k = \tilde{x}_k^{k-1} + K_k (m_k - H_k \tilde{x}_k^{k-1})$$

$$C_k = (1 - K_k H_k) C_k^{k-1}$$

Filtered residuals

- The filtered residual, its covariance and χ2 are

$$r_k = (1 - H_k K_k) r_k^{k-1}, \; R_k = (1 - H_k K_k) V_k, \; \chi^2 = r^T R^{-1} r$$

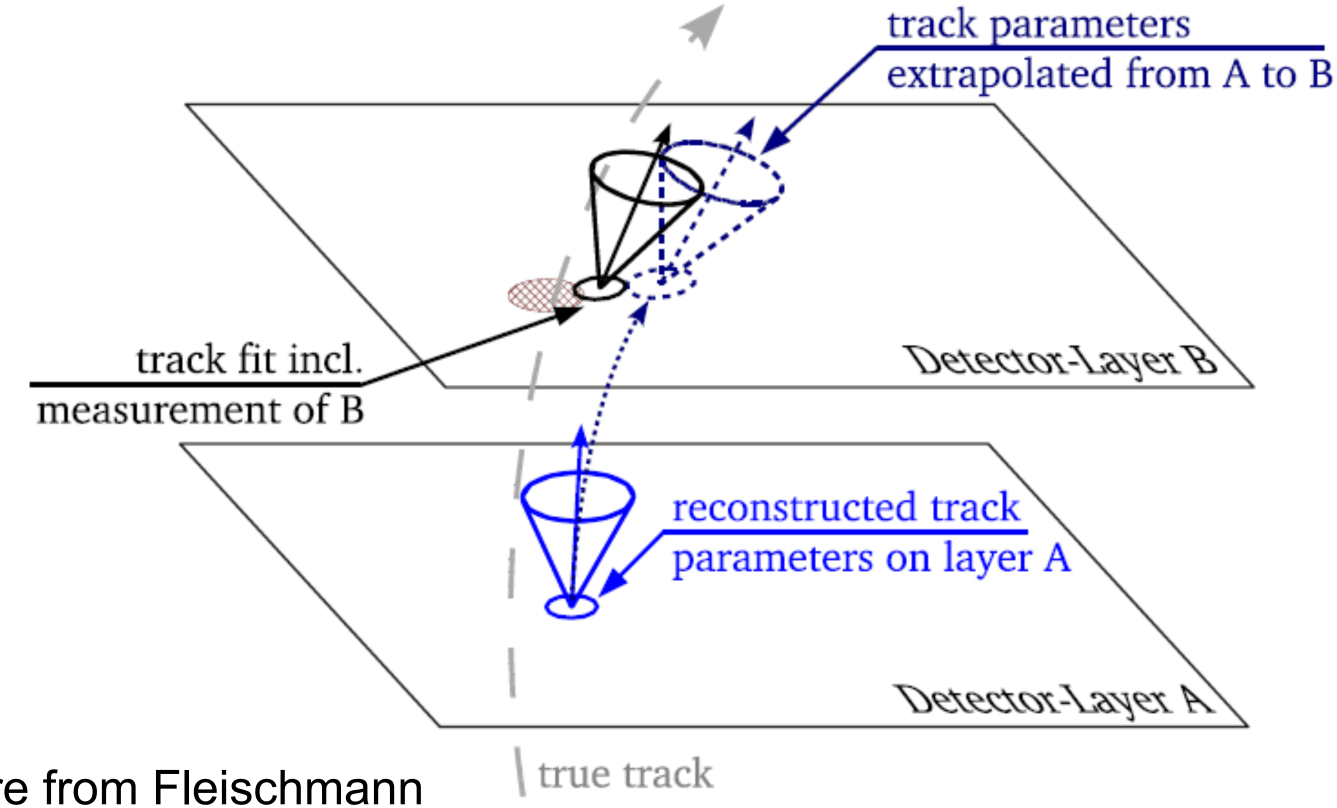


Figure from Fleischmann

Smoothing

- We have reached the end with n hits. Now the procedure is repeated backwards. This is used to update the state at each surface k with the information from all the other:

$$C_{k|n}{}^{-1} = C_{k|k}{}^{-1} + (C^{b}_{k|k+1})^{-1}$$

$$x_{k|n} = C_{k|n}(C_{k|k}{}^{-1} x_{k|k} + (C^{b}_{k|k+1})^{-1} x_{k|k+1})$$

- Finally the state at the innermost surface is extrapolated to the perigee, and this result is used in further analysis.

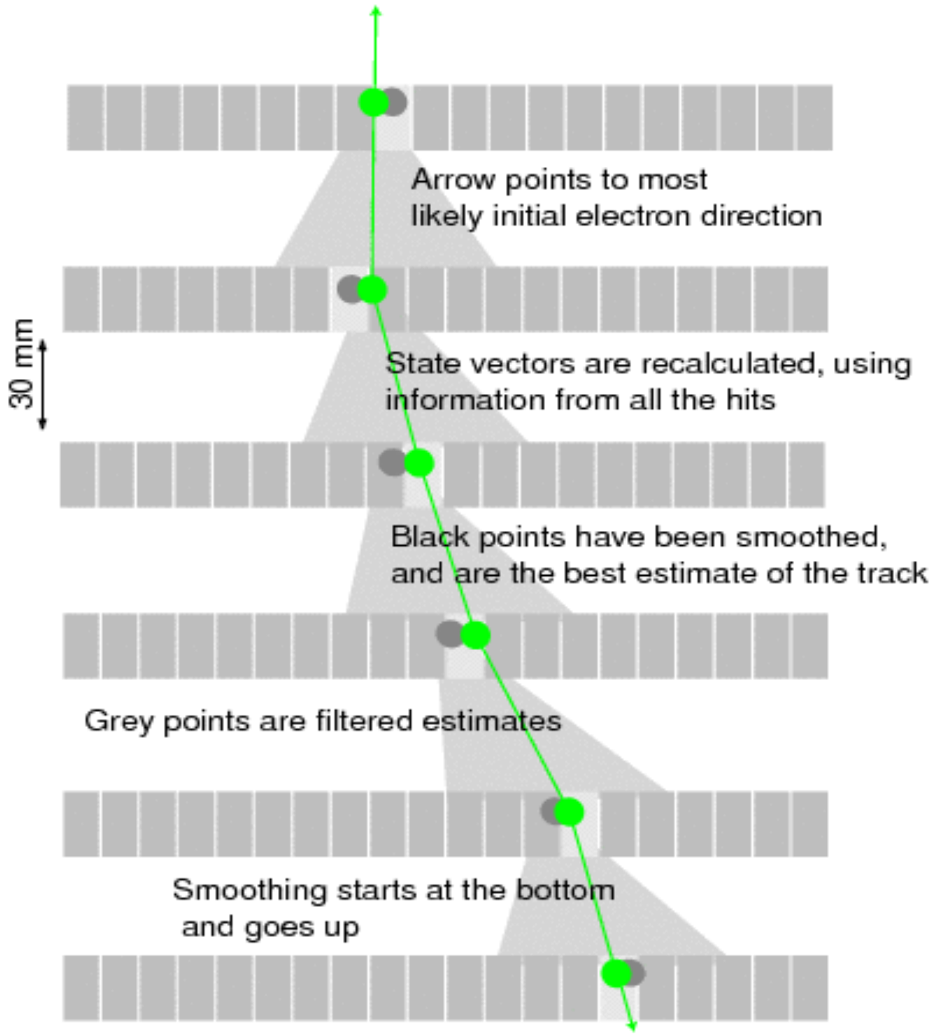
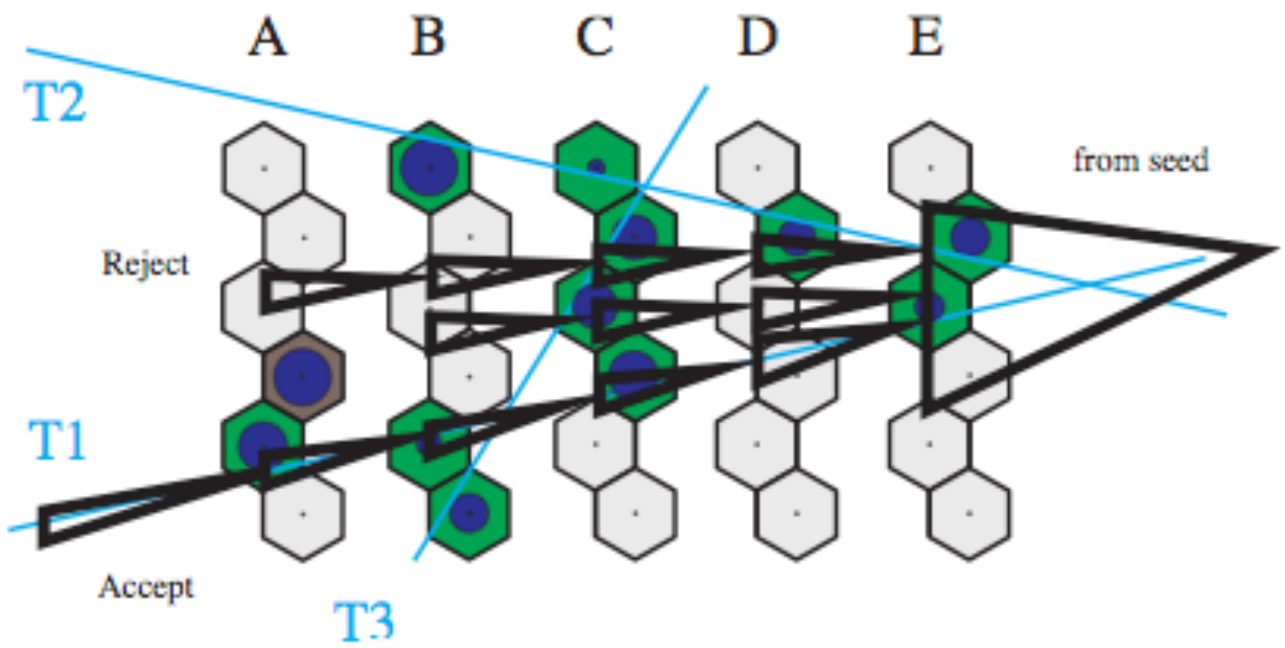


Fig. 2.— The Kalman smoothing process.

From the GLAST Science Prototype

Combinatorial Kalman Filter

- For tracking in dense track environments, the nearest hit might not be the best.
- The Combinatorial Kalman Filter (Mankel 1997) keeps many options open for propagating a seed until one of them conquers



Outlier removal and iterations

- Several iterations are normally used:

- The smoother fit can use updated space-points (applying small corrections depending on track parameters).

- At the smoother step, outliers contributing a large $\chi 2$ can be removed (due to δ-electrons, nearby tracks, misalignment or noise)

- The easiest tracks (with high pT and many hits-on-track) are reconstructed first and their hits removed from the list of hits.

- The remaining hits are now fitted with more relaxed requirements (in particular a lower pT cut).

Finally the filter is repeated outside-in, starting with seeds in the outer layers, in particular to pick up long-lived decays and photon conversions.

Global (Newton-Raphson) fits

- The global least-squares fit assumes we know in advance which hits belong to the track.
- It minimizes the weighted sum of distances between the fitted track and the assigned hits, adjusting the track states at each surface.
- It is mathematically equivalent to the Kalman filter-smoother for a fixed selection of hits on a track.
- If all measurement errors are Gaussian, it is also equal to the maximum likelihood fit.

Global fits

- In the approximation where the expected measurements are linear in the track parameters x, we minimize:

$$\chi^2 = (\overline{m} - H\overline{x})^T V^{-1} (\overline{m} - H\overline{x})$$

where m is a vector of measurements at *all the surfaces.*

- The solution is:

$$\overline{x} = (H^T V^{-1} H)^{-1} H^T V^{-1} \overline{m}$$

For normally distributed m, this is also the maximum likelihood estimate of the parameters.

The factor

$$(H^T V^{-1} H)^{-1} = \left(\frac{1}{2} \frac{\delta^2 \chi^2}{\delta \overline{x}^2}\right)^{-1}$$

is also the covariance matrix C of the track parameters.

Newton-Raphson fit

- If the projection h(x) is not linear, we can Taylor expand around an initial value x0 obtaining approximately:

$$\frac{d\ \chi^2}{dx}(x_0) = 2H^T V^{-1}(m - h(x_0))$$

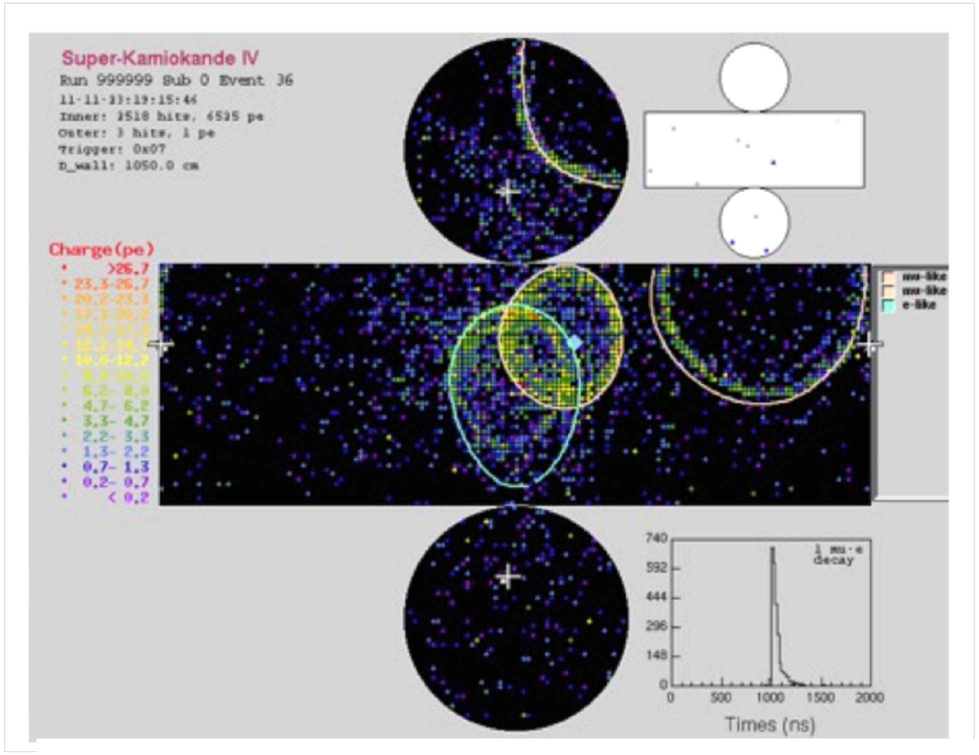
$$\frac{d^2\chi^2}{dx^2}(x_0) = 2H^T V^{-1} H = Cov^{-1}(x_0)$$

we insert now instead $$x_1 = x_0 - \left(\frac{d^2\chi^2}{dx^2}\right)^{-1} \frac{d\chi^2}{dx}$$

x_1 may not exactly minimize χ^2 – but x_1 is, after all, better than x_0. Thus we iterate until convergence:

(|x1 -x0|<small number).

Track fits in Super-Kamiokande



- Reconstruct Cherenkov rings
- Extract track parameters
- X=
- Vertex position
- Momentum and direction
- Particle ID

By maximizing the total likelihood:

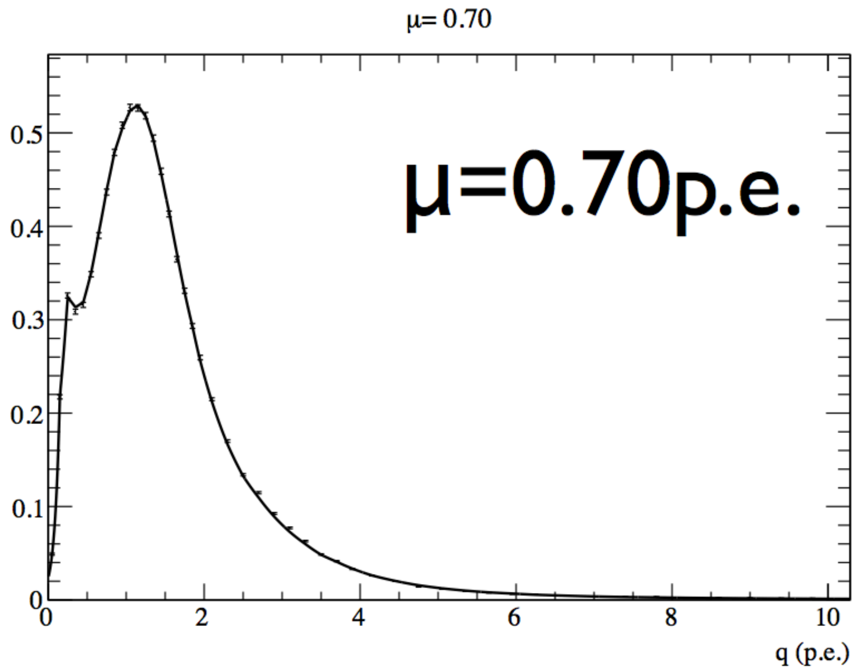
i-th PMT's charge, time

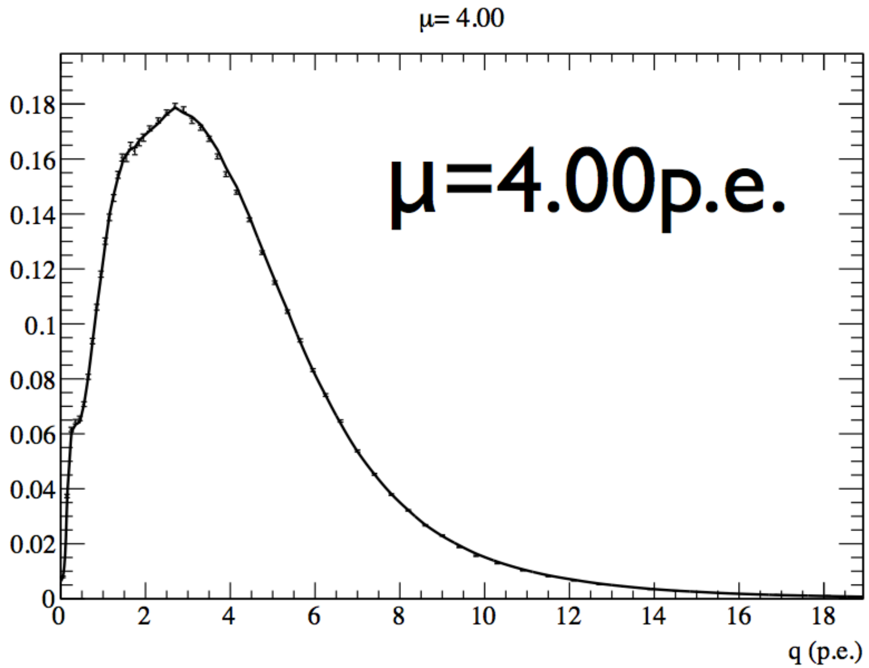
$$L(\mathbf{x}) = \prod_{i}^{\text{unhit}} P(i\text{unhit}|\mathbf{x}) \prod_{i}^{\text{hit}} P(i\text{hit}|\mathbf{x}) f_q(q_i|\mathbf{x}) f_t(t_i|\mathbf{x})$$

Unhit probability   Hit probability   Charge likelihood   Time likelihood

Track fits in Super-Kamiokande

- The individual pdf’s are far from Gaussian ! See f_q below.
- So good seeding is very important to avoid local minima and CPU consumption is very high





Dealing with multiple scattering

- The global chi-squared track fit can allow at each scattering plane a MS angle treated as an extra track parameter with a contribution to chi-squared of $(\theta / \theta_0)^2$
- Alternatively we can introduce correlations between surfaces in the covariance matrix V. This is what we do in the exercise:

$$V \rightarrow V + S\Theta S^T \qquad S = \partial r / d\theta \qquad \Theta_{jj} = \theta_{0\,j}^2$$

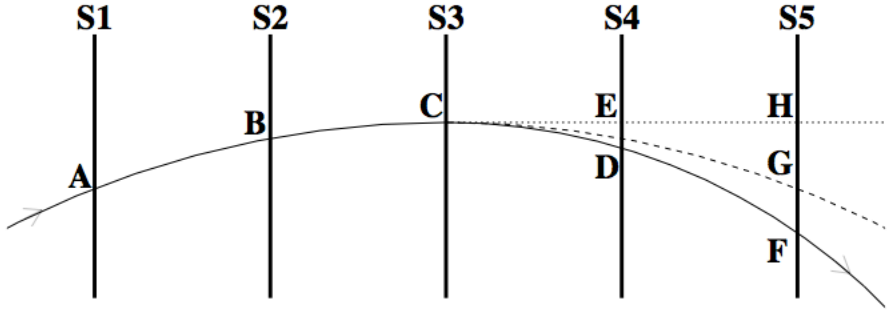
- MS is approximately Gaussian.
- The Global Chisquared and the Kalman Filter only work efficiently with Gaussian deviations from expectations.

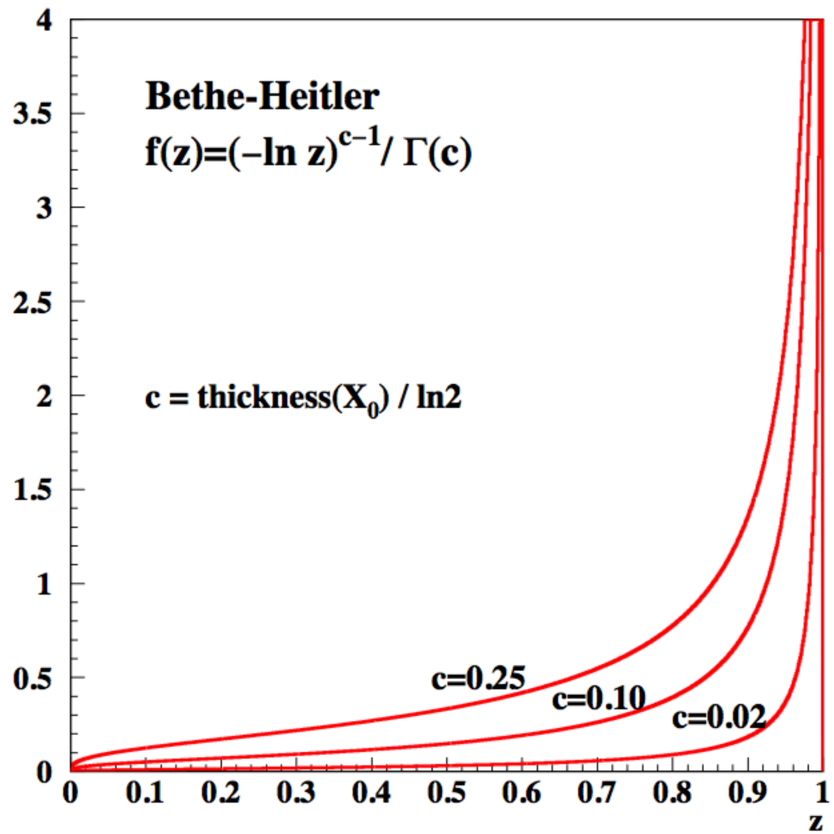
Dealing with Non Gaussian errors

- Special methods are needed to take care of *non-gaussian* influences. Typical example is hard photon radiation where the probability density for the electron to retain a fraction z of its energy follows the Bethe-Heitler law

$$f(z) = (-\ln z)^{c-1} / \Gamma(c)$$

$$c = X_{rl} / \ln 2$$





Gaussian Sum Filter

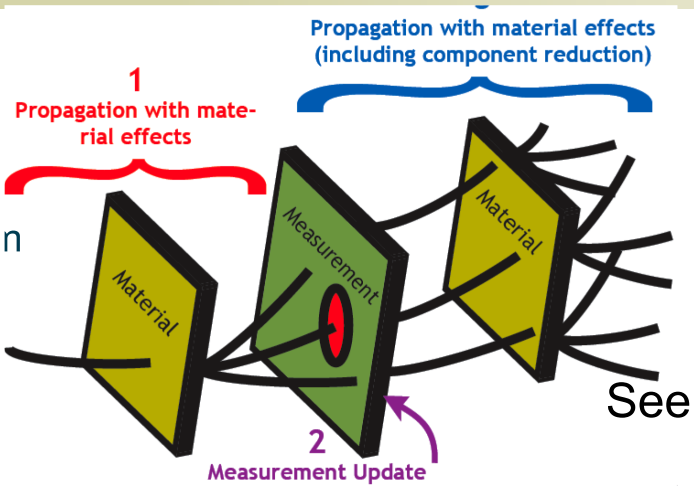


Figure from Salzburger lectures

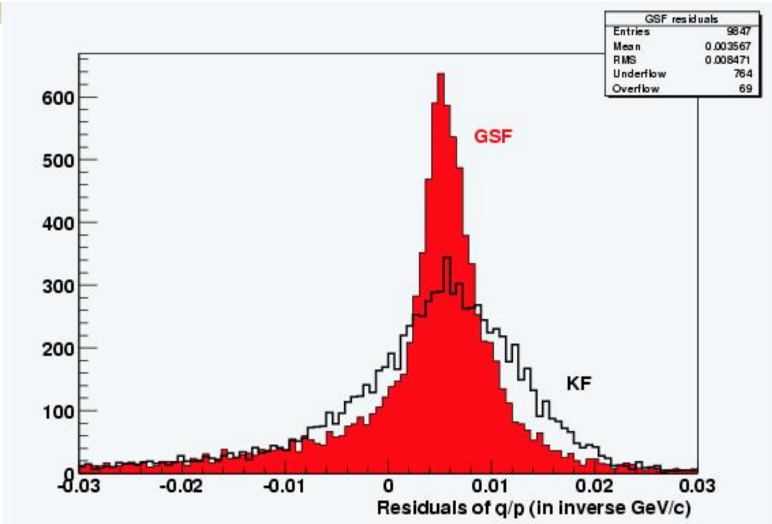
See eg R. Frühwirth and S. Frühwirth-Schnatter, 1998

- Branch the Kalman filter at each surface into parallel paths using a finite number of different Gaussian errors.
- This is the same as modelling e.g. the Bethe-Heitler as a sum of Gaussians

$$f(z) = \sum_{i}^{Nmax} g_i \phi(z; \mu_i, \sigma_i)$$

where the weights g_i, the average, μ_i, and variance, σ_i^2, of the energy are determined beforehand from simulation.

Gaussian Sum Filter



From A. Strandlie, CMS simulation 2003

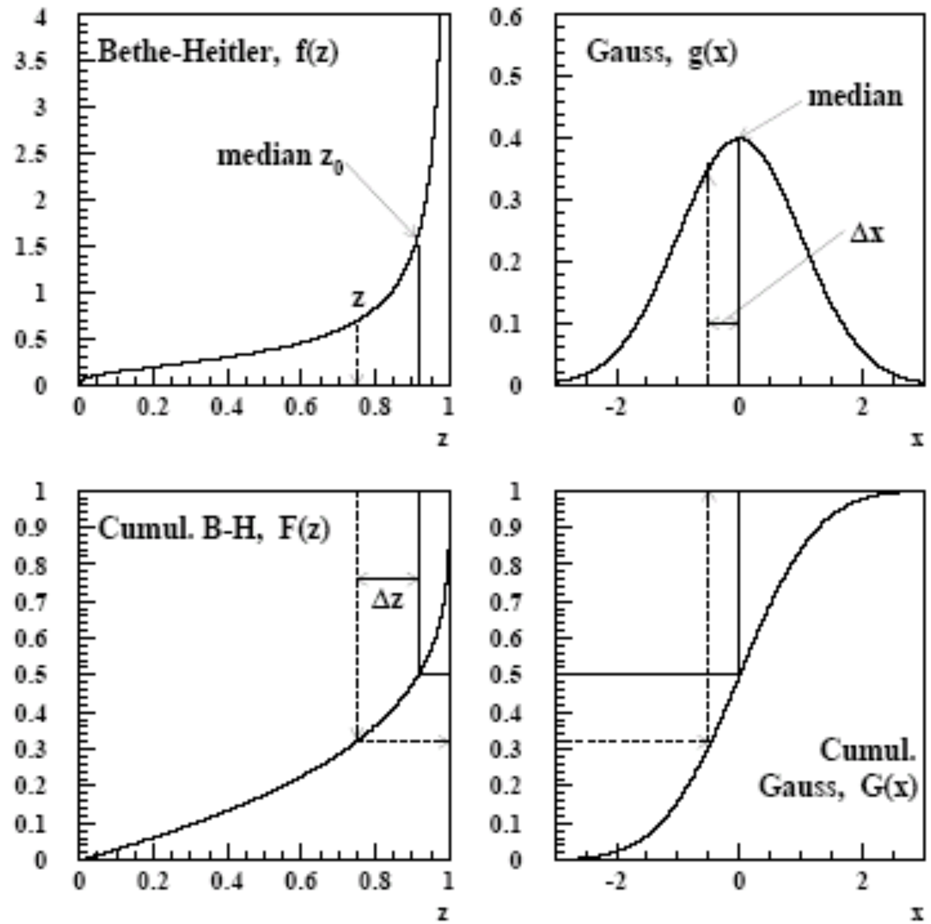
- Effectively the track state branches out into a number of possibilities at each plane.
- *Component reduction*, must be carried out at some point to keep the number of branches from exploding.
- The resulting algorithm is very efficient in recovering from hard bremsstrahlung, but is also very CPU consuming. Often restricted to electron candidates.

Dynamic Noise Adjustment

- DNA offers an alternative recovery from bremsstrahlung.
- First, the z retained after a particular surface is estimated using the hits in a few following planes.
- Then an adjustable noise level $\sigma(z)$ is calculated so that the Bethe-Heitler probability this z equals that for $z=z(\text{median})+x\sigma(z)$, where x is drawn from a unit Gaussian.

Dynamic Noise Adjustment

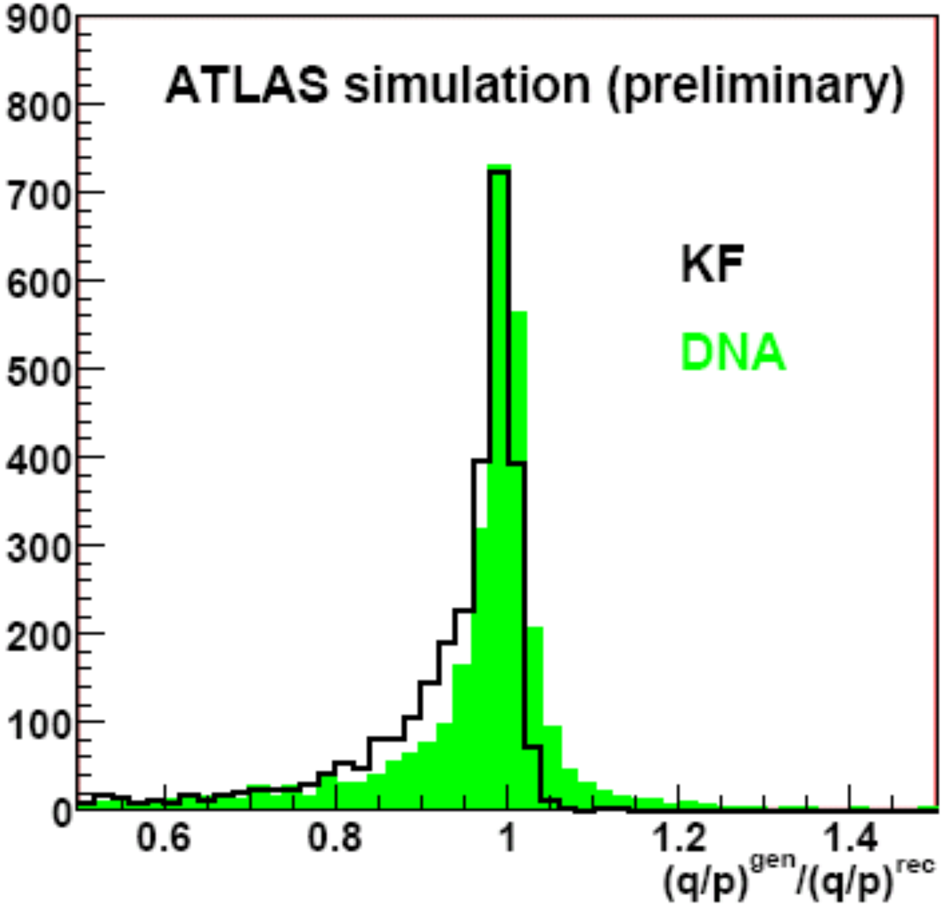
- The dynamically adjusted $\sigma(z)=\Delta z/\Delta x$ noise term is fed back to the Kalman Filter covariance matrix, just like for multiple scattering, and allows for an increase of 1/p.



from Kartvilishvili

DNA filter

- Using the DNA filter instead of just a fixed noise of width $<(z - z_{median})^2>$ helps for electrons, because z is not Gaussian:



From Kartvilishvili

Global optimization

- The problem arises of competing assignments of hits to the different track candidates or to noise. What is the optimal assignment?
- In essence this is the travelling salesman's problem. It is not sure that the nearest track is the best. The problem should be tackled by minimizing a total energy function.
- In the Elastic Arms Algorithm a number of "deformable track templates" must first be found. These should also include a "noise template". The number of tracks stays fixed, but their parameters can change during the procedure.

Global optimization

- A "metric" M_{ia} is now defined, typically the squared distance from hit i to track template a.
- One could try to minimise to minimize a "total energy":

$$E = \sum_{a}^{Tracks} \sum_{i}^{Hits} \left[S_{ia} M_{ia} \right]$$

where the "assignment strength" S_{ia} is either 0 or 1.

- However, optimizing the S_{ia}'s is tricky since the energy-landscape is very "spiky" with lots of local minima.

Elastic arms and annealing

- This is tackled by annealing and fuzzy assignment strength:

$$S_{ia} = \frac{e^{-\beta M_{ia}}}{e^{-\beta\lambda} + \sum_{a=1}^{Tracks} e^{-\beta M_{ia}}}$$

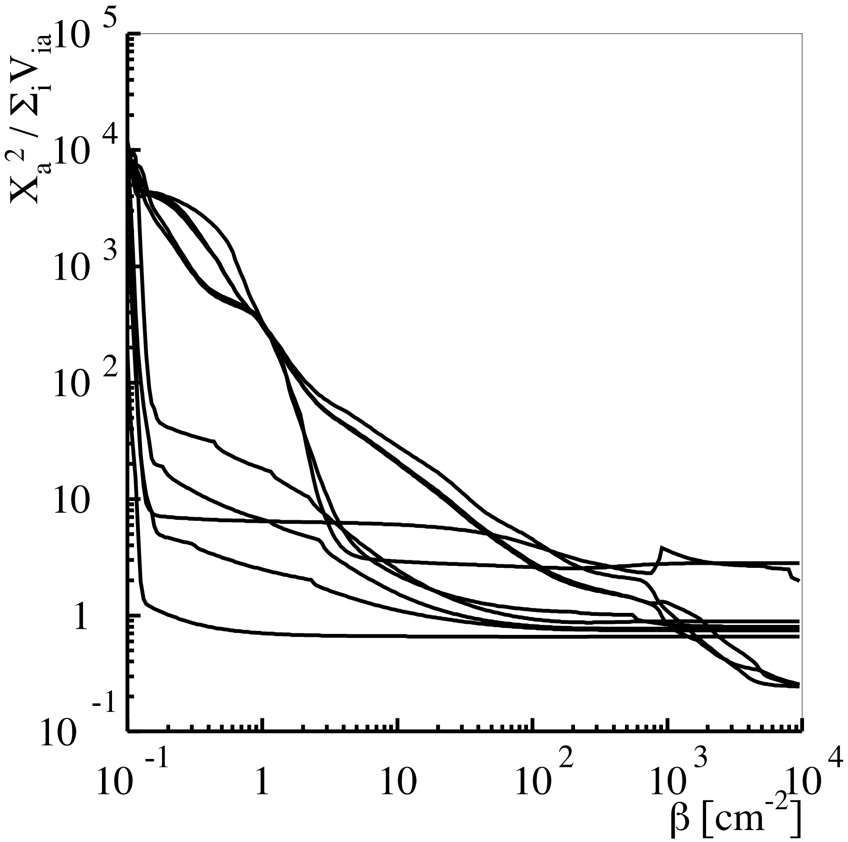
where $\beta=1/T$ and λ is a "chisquared cut" of the order 10.

S_{i0} is here the assignment strength to noise ($M_{i0}=\lambda$)

- We now start at a high "temperature" where the S_{ia}'s are relatively large, even for distant hits. Few local minima.
- The track parameters are then iterated to the global minimum of E using its derivatives a la Newton-Raphson

Elastic arms and annealing

- We then lower the temperature (by eg 5%), repeat and continue until T<<1.
- At this point, all the S_{ia}'s take ~discreet values of 0 and 1.



Chisquared for 10 muon tracks in HERA-B with decreasing temperature. (From Borgmeier Diploma Thesis 1996)

More is found in R.Mankels review (arXiv:040239v1) from 2004.

Deterministic Annealing Filter

- A problem with a *global method* like Elastic Arms is that the approximate number of tracks must be known beforehand. It gives you only better hit sharing, not better track finding efficiency.
- Therefore Frühwirth and Strandlie proposed to modify the (*local*) Kalman filter using an assignment probability S_{ik} for assigning hit i in plane k to the current track.
- Thus all hits have a say in the propagation of a given seed.
- Investigated for ATLAS in S. Fleischmanns thesis

DAF assignment probabilities

- The assignment probability for each of n_k measurements in layer k to the current track is assumed to be proportional to a multivariate Gaussian:

$$\phi_k^i = \phi(m_k^i ; H_k x_k , TV_k^i + H_k C_k^* H_k^T)$$

σ

where x here is the smoothed track state, but without involving layer k in the fit, and T a temperature parameter (the last term is the “track contribution” to the error, which can often be ignored).

This is nothing but the likelihood for a track to produce a given hit using scaled measurement errors. However, what we want is the posterior probability of the track parameters.

DAF assignment probability

- Allowing for the hypothesis that no hit is produced by the track in layer k, we normalise the assignment probability as:

$$S_i^k = \frac{\phi_k^i}{\sum_j^{n_k} \left(\Lambda_k^j + \phi_k^j \right)}$$

- The cut term may be parametrized as

$$\Lambda_k^j = \frac{1}{(2\pi)^{\dim(m)} \sqrt{T \det V_k^i}} \exp(-\frac{\lambda}{2T})$$

where λ acts as a χ^2 cut-off at low temperature.

(Frühwirth and Strandlie, Comp.Phys.Comm 120,197 (1999))

DAF algorithm

- The filtered state can take several measurements per detector layer into account by using their weighted mean.

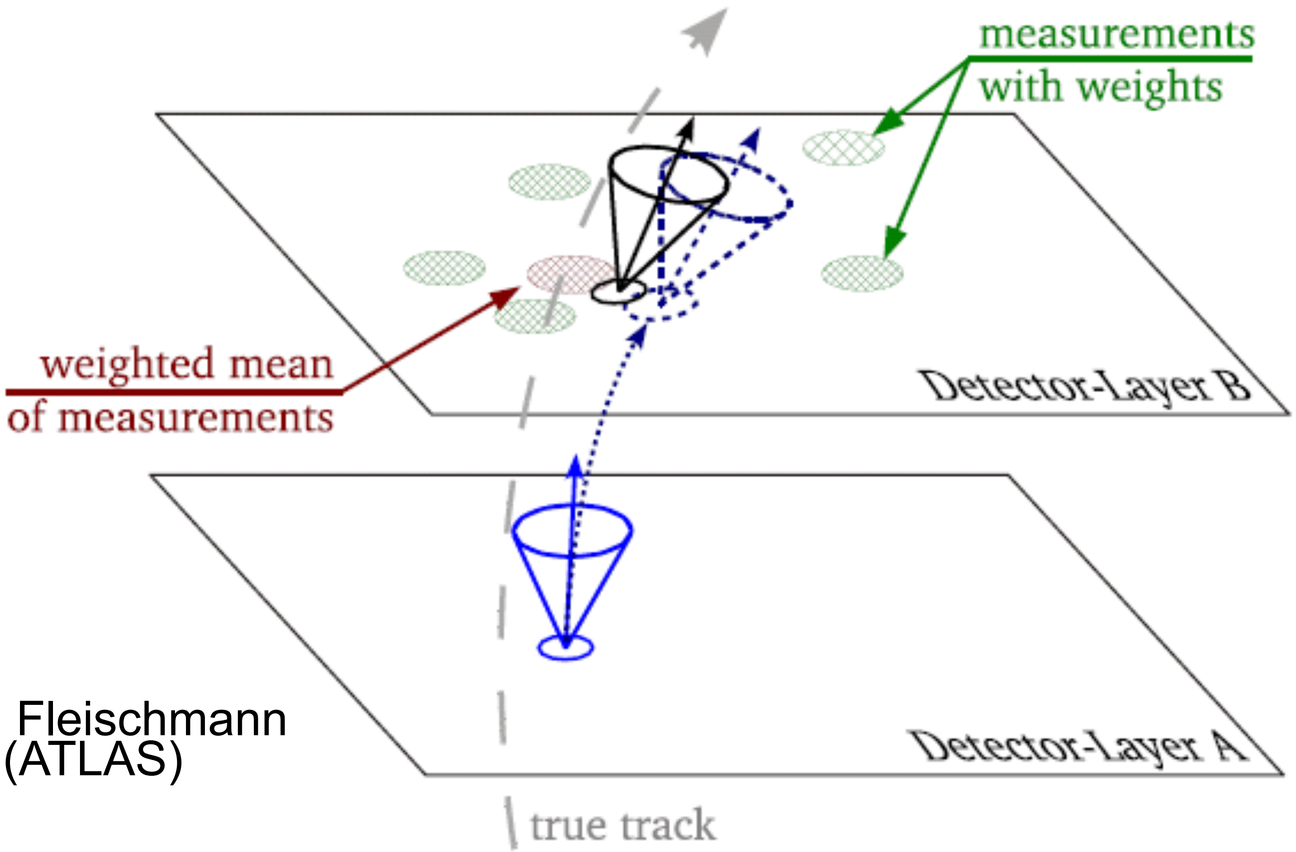
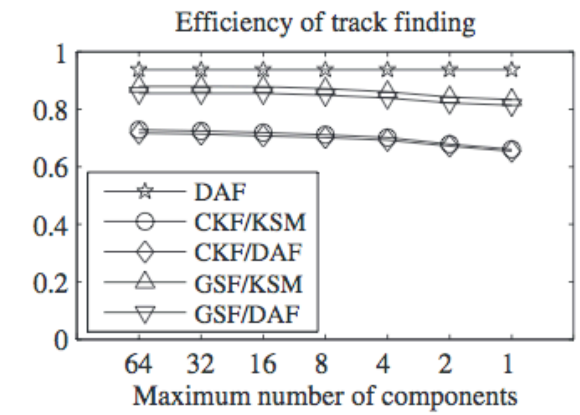


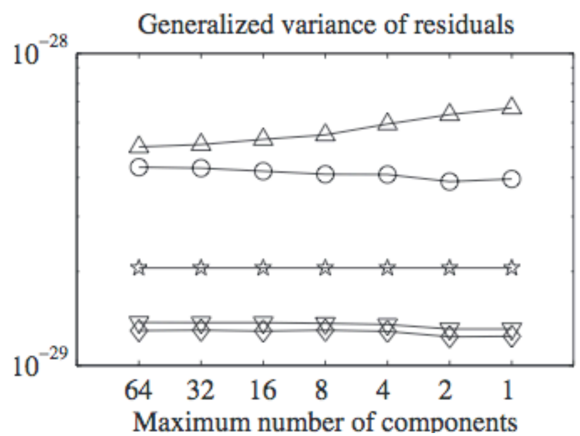
Figure from Fleischmann
PhD thesis (ATLAS)

DAF in practice

- The Deterministic Annealing Filter has turned out especially effective in finding the best left-right choices in drift tubes.
- It can be used as an ”afterburner” and may significantly improve momentum resolution.



Simulated performance in an “ATLAS-like” setup of the DAF, either in standalone mode or as a track fitter following a CKF or GSF track finder. (Frühwirth and Strandlie, 2006)



DAF as a multi-track fitter

- It can be extended it to a multi-track fitter with built-in pattern recognition

  (Frühwirth and Strandlie, Comp.Phys.Comm,133(2000)34).
- In this case, the normalisation of assignment probabilities needs to be changed so that the sum runs over all accepted tracks competing for the measurements.
- As in the Elastic Arms, the procedure starts at a high temperature and iterates with decreasing tolerance, but without working with a fixed number of tracks.

Track scoring

- Competition among tracks for the same hits call for a quality estimator (or “score”) used to reject or accept the track.
- A combination of sub-estimators are used:
  - Number of precision hits
  - Number of outlier hits
  - Holes (track passing through live sensor no signal)
  - Shared hits (penalized if hit is not “shareable”).
  - Total χ^2 per degree of freedom
- In a *second pass* the hits not yet assigned to a track may be reconsidered with larger tolerances to form, for example, low pT tracks or tracks from secondary interactions (long lived decays).

Finding the primary vertex

- Typically a limited ”beam-spot” is given by the machine-parameters , beam-position monitors or pre-processing.
- Hereafter, just two tracks suffices to provide an accurate seed for the vertex

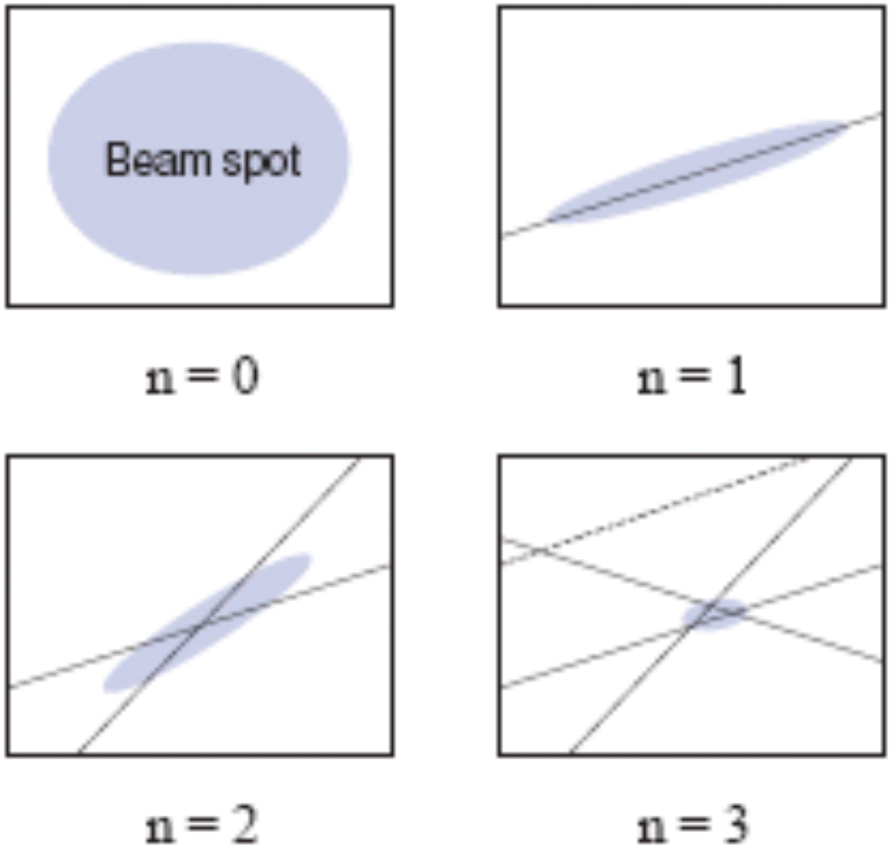


Figure from Mankel

Finding the first vertex seed

- The danger is of course that the initial seed is wrong, so great care must be taken in this very first step.
- In ZEUS, all candidate track pairs were checked for compatibility with a common vertex on the beam-line. They were then ranked according to how many other pairs they agreed with. The best pair then started the chi-squared fit.
- CMS also finds the coordinates with the highest density of track pair crossings. Each track pair is weighted by a decreasing function of the distance between their two perigees. The position with the largest weight is the seed.

ATLAS Multi Vertex Finder

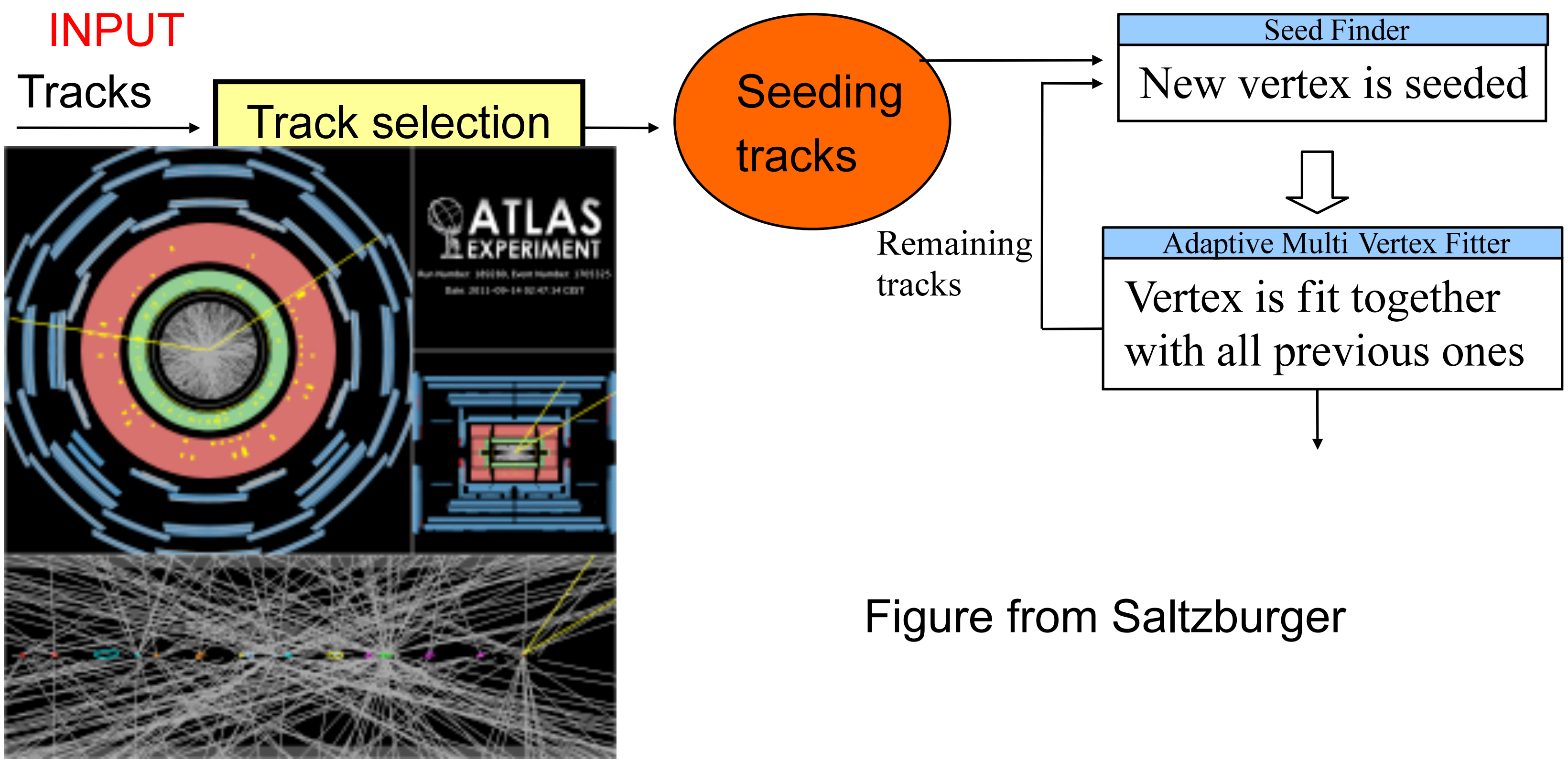


Figure from Saltzburger

- Several vertices fitted simultaneously (20 vertices above)
- Several iterations with decreasing tolerance for assigning a track to a vertex a la the Multitrack DAF.

Billoir vertex fit

- The alternative to a Kalman Filter is the Newton-Raphson least squares fit (for vertices called a Billoir fit):
- It requires that the collection of tracks associated with the vertex is known in advance.
- It can be done e.g. after the Kalman Filter vertex finder.
- Not only the vertex is fitted, but also the track momenta, this time with the constraint that they should all come from the same vertex point. This yields improved momenta.

Billoir vertex fit

- Let $\overline{v} = (x_v, y_v, z_v)$ be the vertex position for n tracks
- Let $\overline{p}_i = (p_{xi}, p_{yi}, p_{zi})$ be the i'th track momentum
- Let $\overline{x}_i = F(\overline{v}, \overline{p}_i)$ be the 5 track parameters of the i'th track at some reference surface.
- To first order in a Taylor series: $F = F(\overline{v}_0, \overline{p}_{0i}) + D_i\delta\overline{v} + E_i\delta\overline{p}_i$
- v_0 and p_{0i} are estimates of the vertex and track momenta.
- Let $\delta\overline{x}_i = \overline{x}_{i,meas} - F(\overline{v}_0, \overline{p}_{0i})$ and V_i be the δx_i covariance matrix.
- Then $\chi^2 \approx \sum(\delta\overline{x}_i - D_i\delta\overline{v} - E_i\delta\overline{p}_i)^T V_i^{-1}(\delta\overline{x}_i - D_i\delta\overline{v} - E_i\delta\overline{p}_i)$

Billoir vertex fit

- Just like in the track fit (slide 49), we minimize χ^2 by:

$$\bar{v} = \bar{v}_0 + \left(A - \sum B_i C_i^{-1} B^T{}_i\right)^{-1} \left(\bar{t} - \sum (B_i C_i^{-1})^T \bar{u}_i\right)$$

$$\bar{p}_i = \bar{p}_{0i} + C_i^{-1}(\bar{u}_i - B_i^T \delta\bar{v})$$

$$A = \sum D_i^T V_i^{-1} D_i \qquad B_i = D_i^T V_i^{-1} E_i \qquad C_i = E_i^T V_i^{-1} E_i$$

$$\bar{t} = \sum D_i^T V_i^{-1} \delta\bar{x}_i \qquad \bar{u}_i = E_i^T V_i^{-1} \delta\bar{x}_i$$

- Where the real work for the programmer is in the initial calculation of D and E - and in the initial guess of v_0

Billoir vertex fit

- Now interchange $\overline{v}_0$ with $\overline{v}$ and continue until convergence
- The covariance of the fitted parameters is at each step:

$$\mathrm{cov}(\overline{v}) = \left(A - \sum B_i C_i^{-1} B_i^T\right)$$

$$\mathrm{cov}(\overline{p}_i) = C_i^{-1} + (B_i C_i^{-1})^T \,\mathrm{cov}(\overline{v}) B_i C_i^{-1}$$

$$\mathrm{cov}(\overline{v},\overline{p}_i) = -\mathrm{cov}(\overline{v}) D_i E_i^{-1}$$

- We also get correlations between the track momenta:

$$\mathrm{cov}(\overline{p}_i,\overline{p}_j) = \delta_{ij} E_j^{-1} - E_i^{-1} D_i^T \,\mathrm{cov}(\overline{v},\overline{p}_j)$$

Exploiting external constraints

If you have some prior knowledge about the beam collision position b, just add an extra contribution to the chisquared which effectively changes the derivatives D_i:

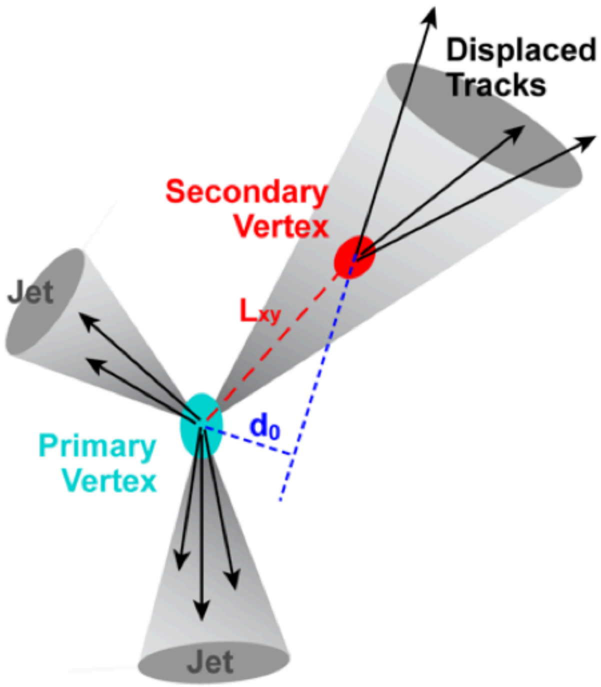
$$\delta\chi^2 = (\bar{v} - \bar{b})^T V_b (\bar{v} - \bar{b})$$

- If we deal with a secondary vertex and expect the sum of track momenta to point back to the primary vertex (exact constraint), then we can use the method of Lagrange Multipliers:

$$\delta\chi^2 = -\bar{\lambda} \bullet \bar{d}(\bar{v}, \sum \bar{p}_i)$$

- Where λ are 3 new arbitrary fit parameters and d is the minimal vector distance between the primary vertex and the line pointed by the momentum sum (more about that later).

b-jet tagging



- Jets with a B-hadron can be identified by the lifetime (1.5 ps) and high mass of the b quark (about 4.2 GeV)

From the ATLAS B-physics group

using secondary vertices

- One way is to (partly) reconstruct the decay chain:

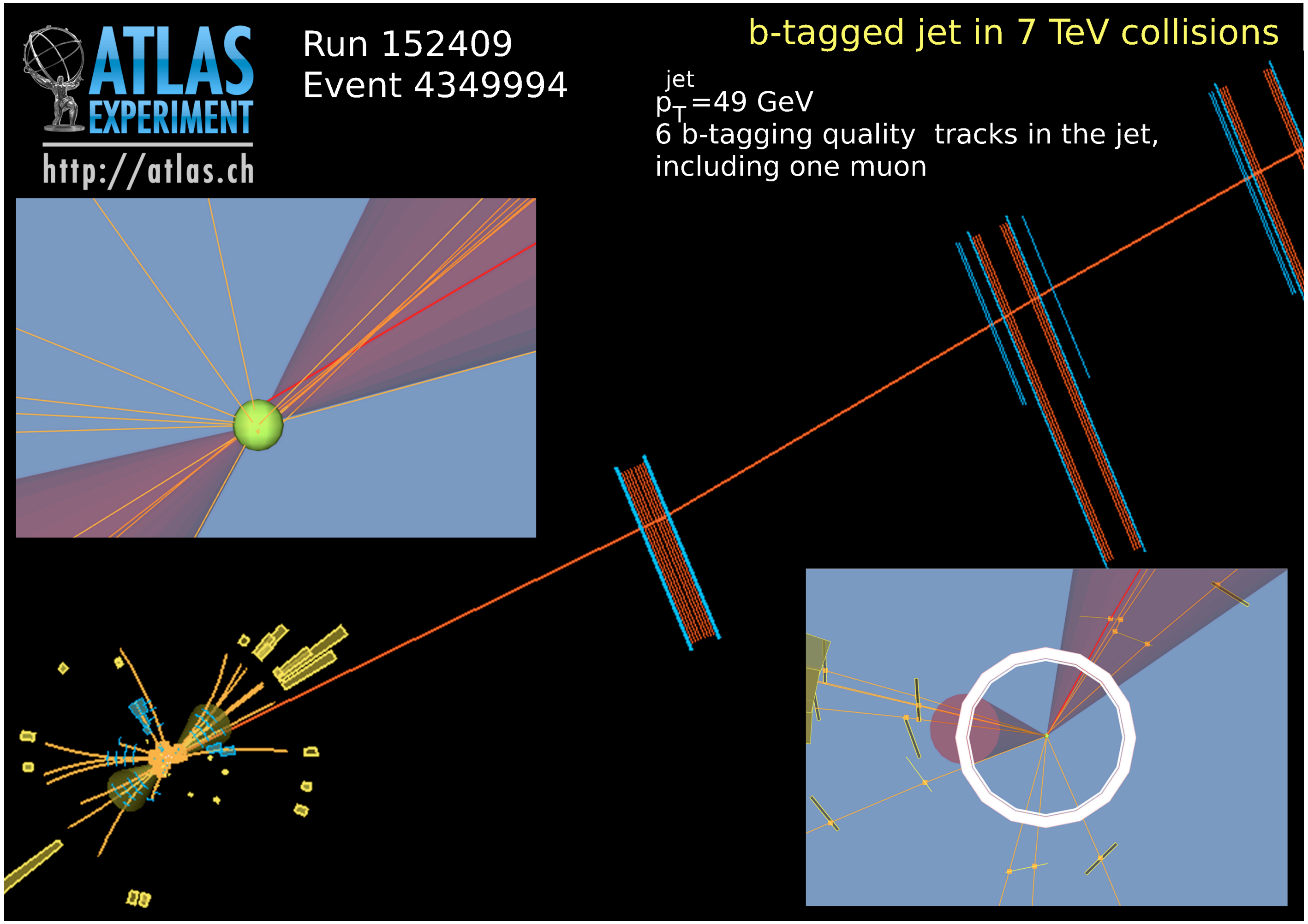
  b-jet -> B^-+X -> D^0+X+Y -> K^-+X+Y+Z

Where X are b-quark fragmentation particles, Y are other particles from B^- decay and Z other particles from D decay.

- The number of found vertices along the jet-axis, their distances from PV, the mass and number of tracks at each vertex are all examples of variables with power to discriminate between b-quarks and lighter partons.

Using a lepton tag

- Due to the high B-meson mass, its leptonic decay (~10%) has a higher p_T^{rel} than leptons from light parton jets

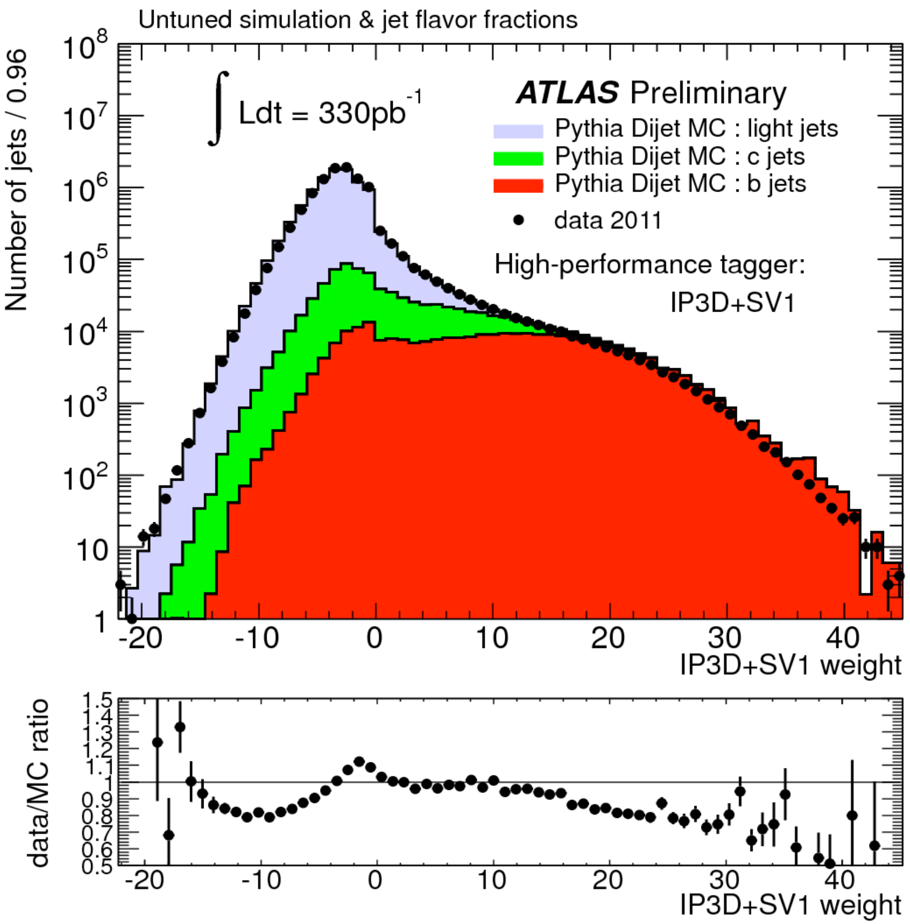


Using impact parameter prob.

- Combine impact parameter significances for jet tracks

$$P(jet) = \Pi \cdot \sum_{i=0}^{N_{trk}} \frac{-\ln(\Pi)^i}{i!} \quad where \quad \Pi = \prod_{i \in jet} \int_{S_i}^{+\infty} f(S)dS$$

S is the impact parameter significance and f(S) its light jet probability.
The P(jet) estimator has many nice properties.



- Finally combine everything:
combined Likelihood
or Multi-Variate Analysis

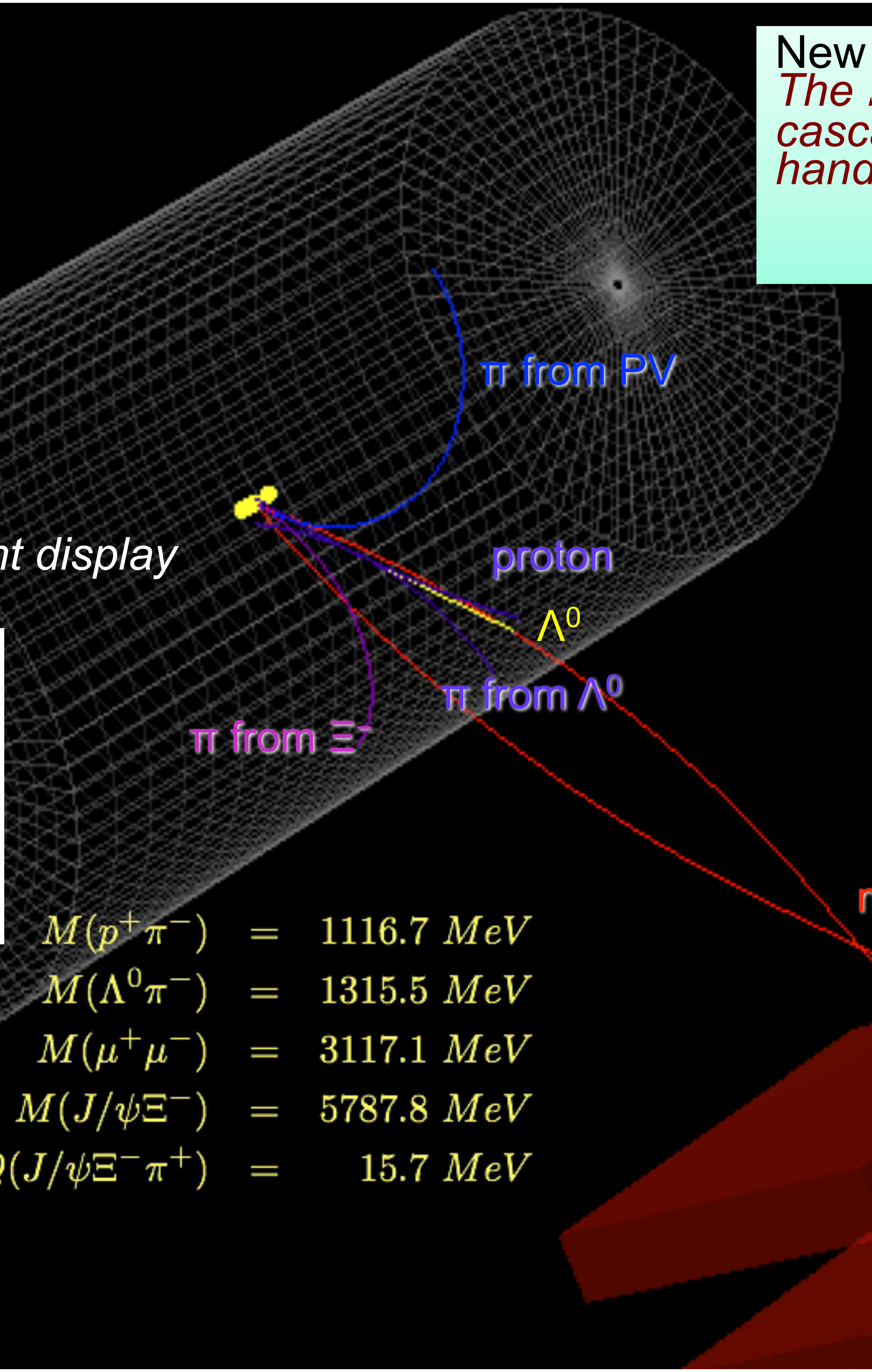
Constraints from priors

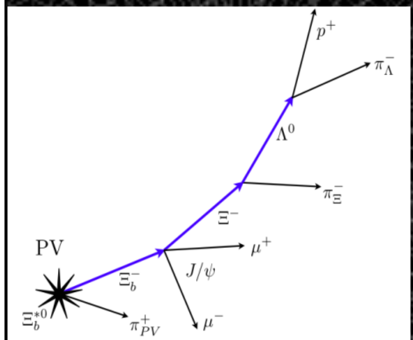
- *a priori* knowledge can be used with great advantage.
- One example is the beam energy constraint used in the reconstruction of tracks from e^+e^- collisions.
- Another is the reconstruction of B meson cascade decay where the known masses of the D mesons are used as a constraint.
- Another is e^+e^- tracks from photon conversions. A vanishing photon mass can be imposed using Lagrange Multipliers.
- Another on the next slide.
- Details on implementation are found in

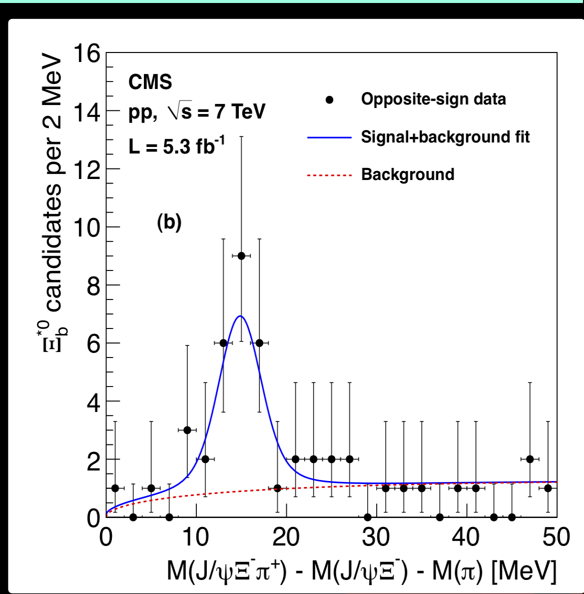
  www.phy.ufl.edu/~avery/fitting/kinfit_talk1.pdf

New ParticleDiscovery

The Ξ_b^{*0} involves elegant cascade that CMS tracker handles beautifully.







$$M(p^+\pi^-) = 1116.7\ MeV$$
$$M(\Lambda^0\pi^-) = 1315.5\ MeV$$
$$M(\mu^+\mu^-) = 3117.1\ MeV$$
$$M(J/\psi\Xi^-) = 5787.8\ MeV$$
$$Q(J/\psi\Xi^-\pi^+) = 15.7\ MeV$$

Lagrange Multipliers

- Let again x be the track parameters of the two tracks that form a conversion candidate. The constraints must be expressed as some functions, H(x), being exactly zero.
- We again expand around an approximate solution x_A:

$$\frac{\partial \overline{H}}{\partial \overline{x}}(\overline{x}-\overline{x}_A)+\overline{H}(\overline{x}_A)=\overline{D}\Delta\overline{x}+\overline{d}=\overline{0}$$

- If the two tracks should emerge parallel from a common point, the expression would be something like

$$\overline{d}=\overline{H}(\overline{p}_1,\overline{p}_2,\overline{r}_1,\overline{r}_2)_A=\left(\frac{\overline{p}_1}{E_1}-\frac{\overline{p}_2}{E_2},\overline{r}_1-\overline{r}_2\right)_A$$

- Where the p's and r's refer to the start points of the tracks.

Lagrange Multipliers

- The function to be minimized is now (dropping vector bars):

$$\chi^2 = (x - x_0)^T V_0^{-1}(x - x_0) + 2\lambda^T(D(x - x_A) + d)$$

- The minimum is found in the space of the track parameters x=(p,r) and the real constants λ.
- The ”0” refer to the unconstrained solution from the track fits and the “A” to the previous iteration of this fit.
- The solutions have to be iterated since the constraint equations were linearized.

Solution to the constrained fit

- (put derivatives of chi2 to zero and solve by substitution):

$$x = x_0 - V_0 D^T \lambda$$

$$\lambda = V_D (D(x_0 - x_A) + d)$$

$$V_D = (D V_0 D^T)^{-1}$$

$$V_x = V_0 - V_0 D^T V_D V_0$$

$$\chi^2 = \lambda^T V_D^{-1} \lambda$$

Notes to the constrained fit

- Once we have determined D and d, the rest is automatic
- x_0 and V_0 are given by the independent track fits
- We do not have to invert V_0, only DV_0D^T
- The $\chi 2$ is a sum of terms, one for each constraint.
  You can choose to cut on them individually

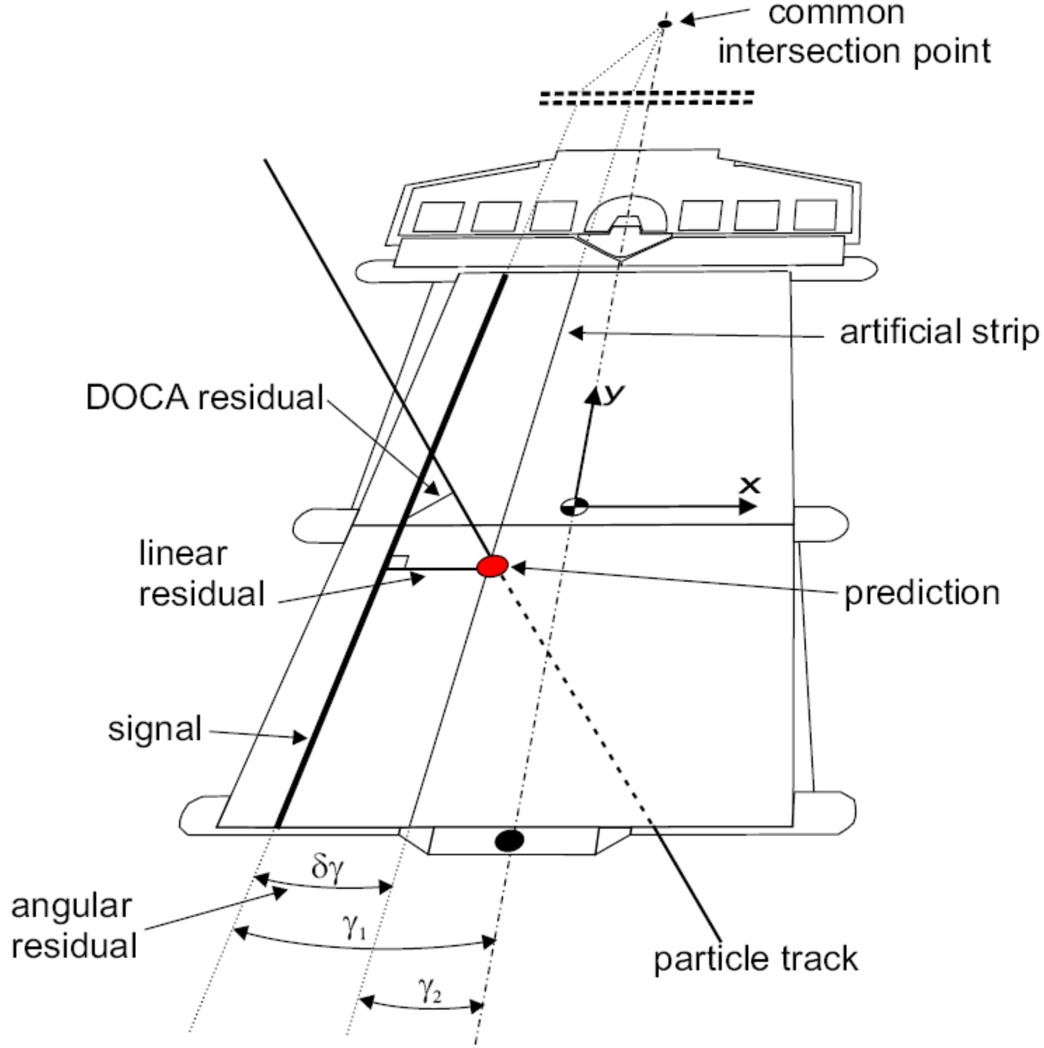
Alignment

- In order to have high resolution unbiased tracking, the detector elements must be correctly aligned.
- This is partly achived by optical survey, and for example laser alignment systems, to track short-term movements.
- The ultimate alignment precision is best achieved by using the fitted tracks themselves.

Alignment with tracks

- Consider our example spectrometer. From the reconstructed tracks, we want to determine the alignment corrections to the position and orientation of each plane.
- Consider a single measured coordinate y_i and a track model $y=h(\mathbf{x},\boldsymbol{\alpha})$, where $\mathbf{x}$ are the track parameters and $\boldsymbol{\alpha}$ are the alignment corrections.
- A straight forward estimate of $\delta\alpha_i$ is simply the average residual $\langle r_i = y_i - h(x,\alpha)\rangle$, averaged over all fitted tracks.
- If the considered plane does not take part in the fitted track, r_i is called an unbiased residual.
- This *"local"* approach requires in general many iterations because the correlations between planes induced by the fitted tracks are ignored with this method.

Alignment with tracks



From ATLAS alignment paper

Global alignment with tracks

- In the *"global"* approach we define a total *chi2* of a large track sample:

$$\chi^2 = \sum_{tracks} r^T R^{-1} r$$

$$r(x,\alpha,m) = m - h(x,\alpha)$$

where r are the residuals, α the alignment parameters of the detector elements and x the individual track parameters.

What we want is to simultaneously minimise *chi2* both with respect to the millions of x's and to the many α's.

Sounds impossible, but it isn't!

Global alignment with tracks

- After fitting for the track parameters, x, we have to first order the total derivative wrt the alignment :

$$\frac{d\chi^2}{d\overline{\alpha}} = 2\sum_{tracks} A^T R^{-1}\overline{r}$$

where *A* is the *partial* derivative of *r wrt α.*

- If R is diagonal, the derivative wrt some α receives only contributions from the local detector element for which the partial derivative A=δr/δα is non-zero.
- Finding δα so that the sum of the derivatives over all tracks be zero thus results in M coupled equations, just like for the Billoir vertex fit.

Global alignment with tracks

- If the summed $\chi 2$ is not already at minimum, we linearize the problem to these M equations:

$$\frac{d\chi^2}{d\overline{\alpha}} = -\frac{d^2\chi^2}{d\overline{\alpha}^2}\Delta\overline{\alpha}$$

- Several algorithms exists for solving them iteratively. MILLIPEDE is a well-known example (google Blobel).
- Another example is MINRES, minimising the distance between the two sides of the equation (used by CMS).
- Others calculate eigenvectors and eigenvalues of the second derivative exploiting the sparseness of this matrix (used by ATLAS).

Global alignment with tracks

- The explicit solution to the alignment problem is thus

$$\Delta\alpha = -\left(\frac{d^2\chi^2}{d\alpha^2}\right)^{-1}\frac{d\chi^2}{d\alpha}$$

or, assuming r is linear in α,:

$$\Delta\alpha_i = -\left[\sum_{tracks}\frac{\partial\overline{r}}{\partial\alpha_i}R^{-1}\frac{\partial\overline{r}^T}{\partial\alpha_j}\right]^{-1}\sum_{tracks}\left[\frac{\partial\overline{r}}{\partial\alpha_j}R^{-1}r^T\right]$$

where $R = V - HCH^T$ is the covariance matrix of the residual vector of a track. See ATL-INDET-PUB-2007-009.

Eigen-modes of distortion

- Let

$$b = -\frac{1}{2}\frac{d\chi^2}{d\overline{\alpha}}, \qquad A = \frac{1}{2}\frac{d^2\chi^2}{d\overline{\alpha}^2}$$

Then the covariance of the fitted alignment corrections is $C(\Delta\alpha)=A^{-1}$. Since this matrix has an inverse, it can be diagonalized and written in terms of its eigenvectors u:

$$C_{kl}(\Delta\overline{\alpha}) = \sum_{j}^{M} \frac{1}{d_j} u_k^{(j)} u_l^{(j)}$$

where

$$\Delta\overline{\alpha} = \sum_{j}^{M} \frac{1}{d_j} (u^{(j)T} b) u^{(j)}$$

Eigen-modes of distortion

- The eigenvectors are collective orthogonal distortions of the detector. The change in the χ2 due to the correction Δα receives independent contributions from each mode:

$$\Delta\chi^2 = -2\sum_{j}^{M}\frac{1}{d_j}(u^{(j)T}b)^2$$

- Thus we can identify and correct these contributions independently of each other.

Weak modes

Clearly there is a problem if $d_j=0$.
Small eigenvalues of A correspond to small $d\chi^2/d\alpha$. These are called ”weak modes”, poorly constrained by the data.

So there are some distortions which cannot be seen in the residuals but still may spoil the momentum measurement.

Weak modes

The only way out is to use external constraints.

If, for example, an optical survey yields the alignment shift α_{survey} with precision σ, you would add a piece

$$\Delta\chi^2 = (\alpha - \alpha_{survey})^2 / \sigma^2$$

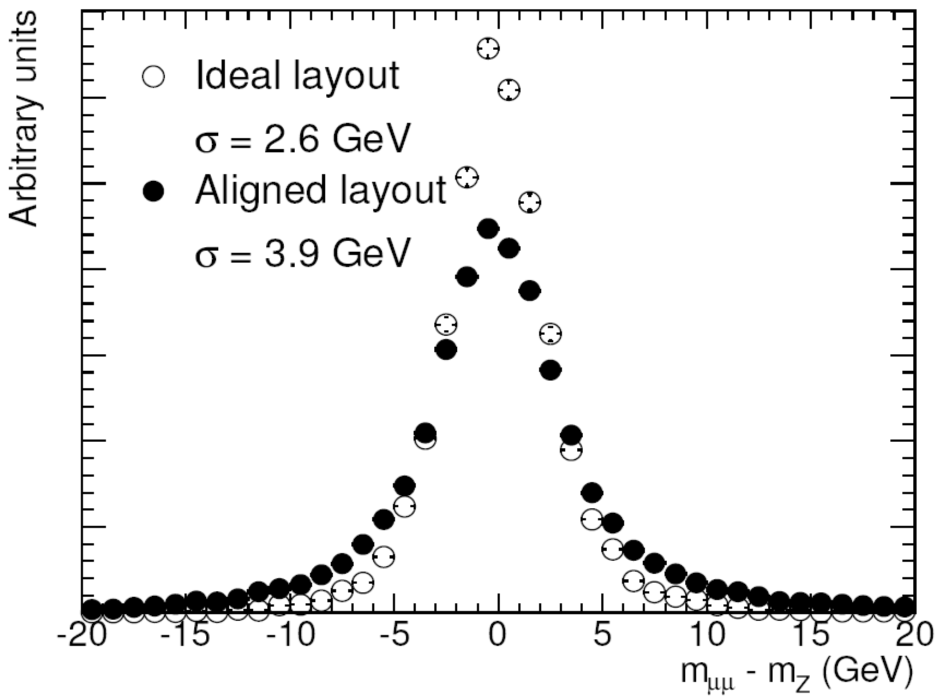
If in general relations g(α)=0 exist with covariance G:

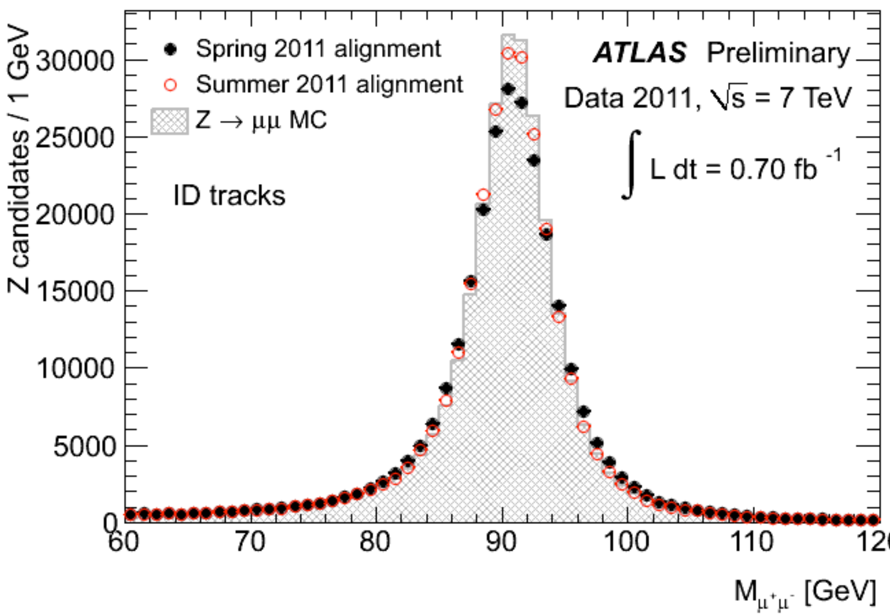
$$\Delta\chi^2 = g^T G g$$

Exact constraints, like known resonance masses, are taken into account with Lagrange multipliers.

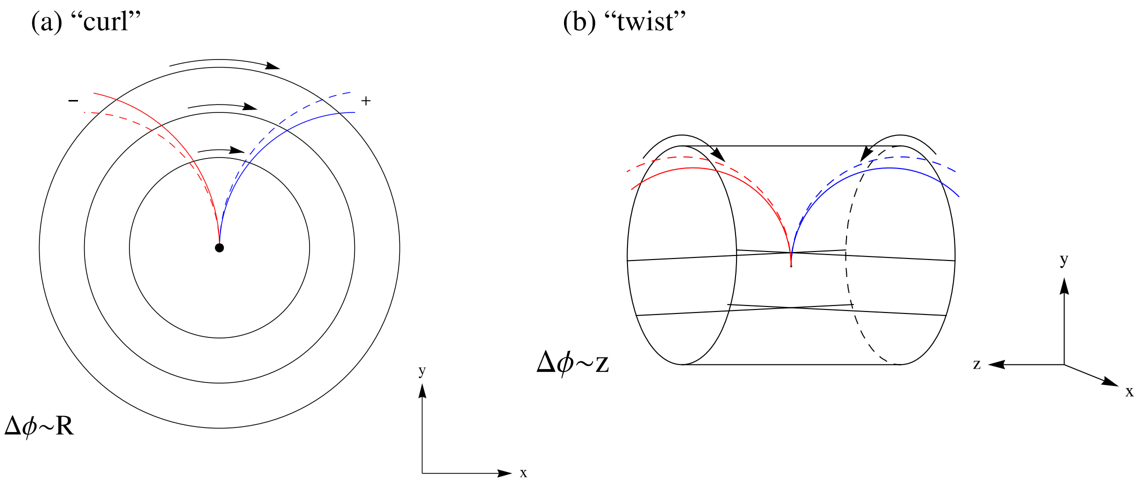
Effect of weak mode misalignment

- ATLAS saw the troubles from weak modes already in simulation. Extra constraints from cosmics, surveys, resonances and combined detectors can correct for weak modes.
- 2008 2011





Curls and twists (q anti-sym)

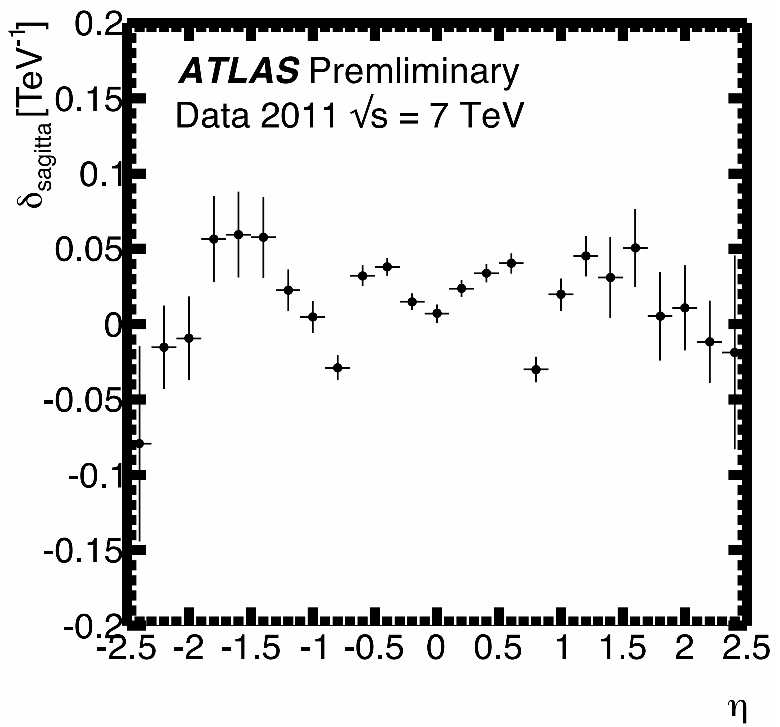
- Imaging a rotation of the various layers in phi proportional with R. That would approximately conserve the helix-shape but bias the momentum (different for positive and negative charge). That is called a curl.
- 

- Imagine a rotation of the end of a layer cylinder. That would approximately conserve the helix-shape but bias the momentum in an eta dependent way. That is called a twist.

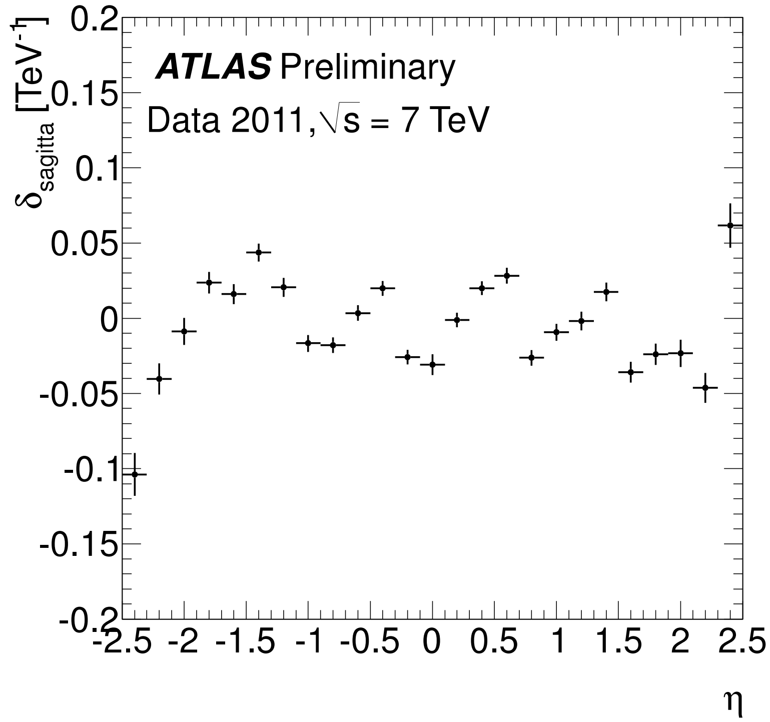
From the ATLAS Silicon alignment group, Bruckman et al

Checking for twists and curls

- No effect of twists and curls seen after alignment



E/p (e+ e-)

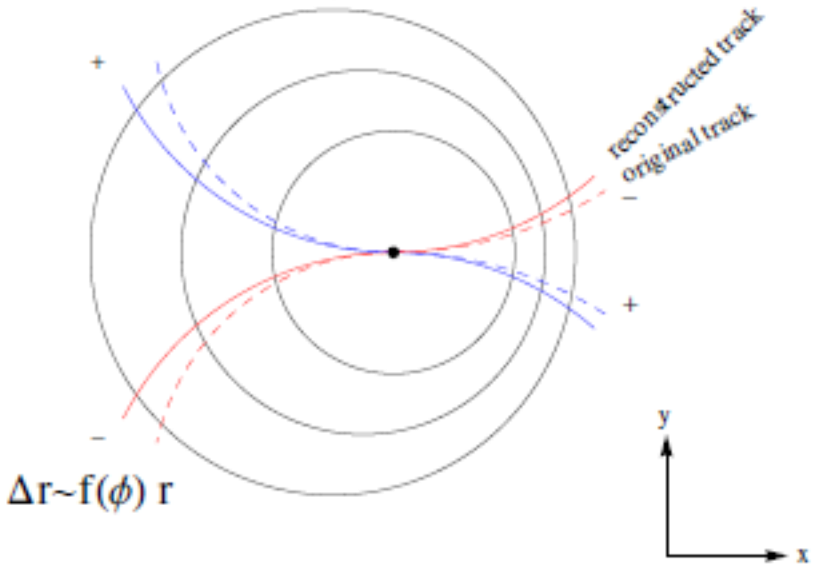


Z->mu+mu- mass

$$q/p_{Corrected} = q/p_{\mathrm{Re}\,constructed}(1 - qp_T\delta_{Sagitta})$$

Radial deformations (q sym)

- There are also weak mode distortions affecting charges symmetrically – but different at different phi.



$$p_T \longrightarrow p_T(1 + 2\epsilon_{\mathrm{radial}})$$

From the ATLAS Silicon alignment group, Bruckman et al

Checking radial deformations

- Phi dependence of low mass resonances

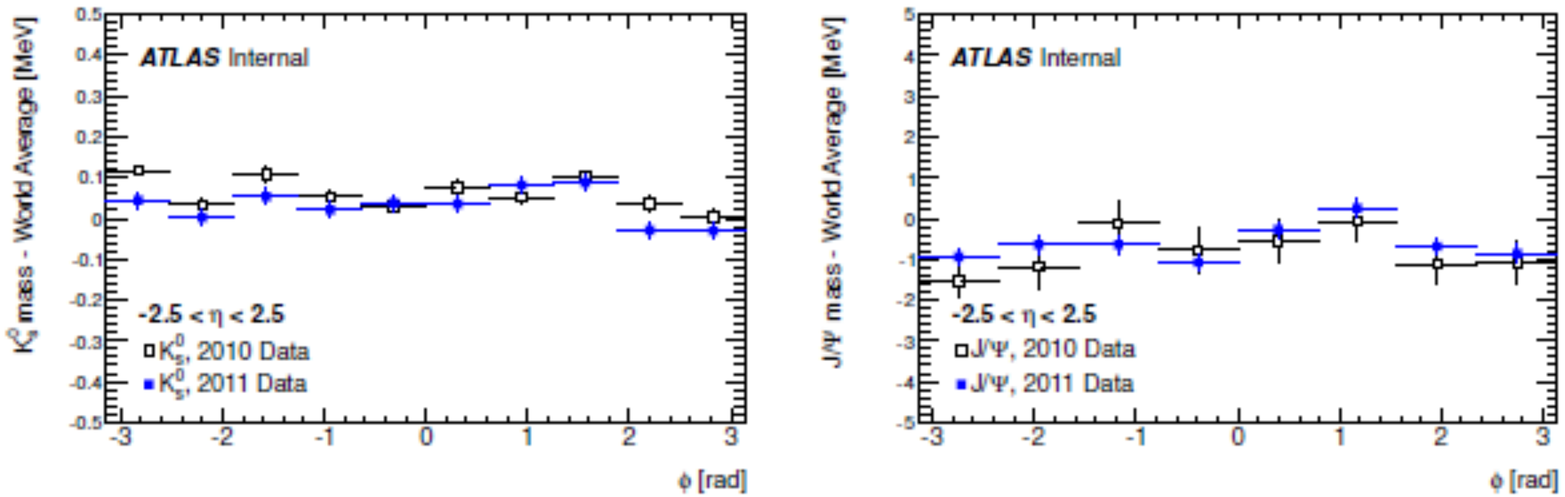
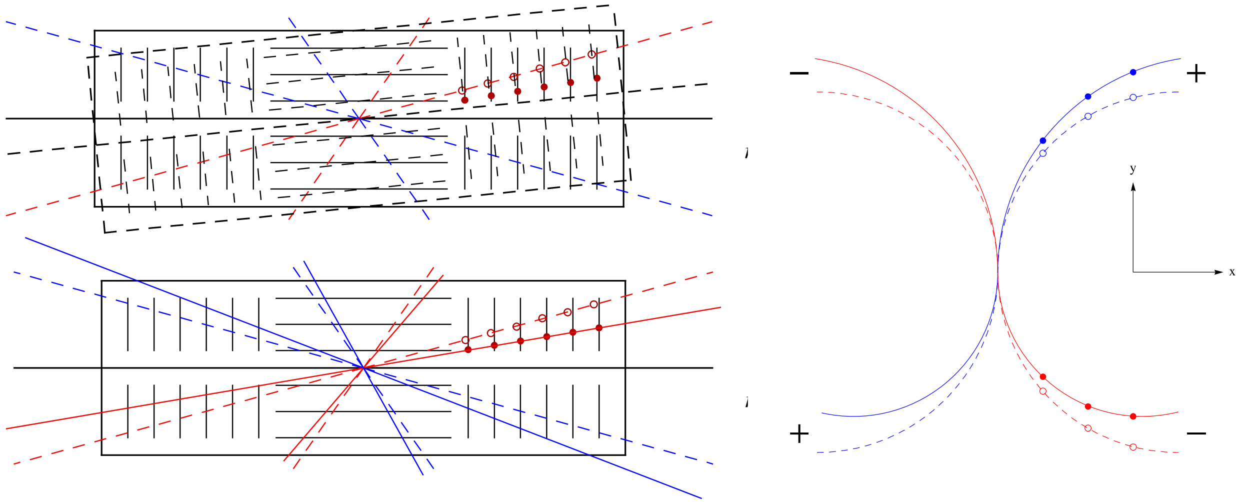


Figure 8: $m_{K_S^0}(\phi_{K_S^0})$ (left) and $m_{J/\psi}(\phi_{J/\psi})$ (right) as a function of ϕ-direction of the resonance.

From the ATLAS Silicon alignment group, Bruckman et al

B field rotations

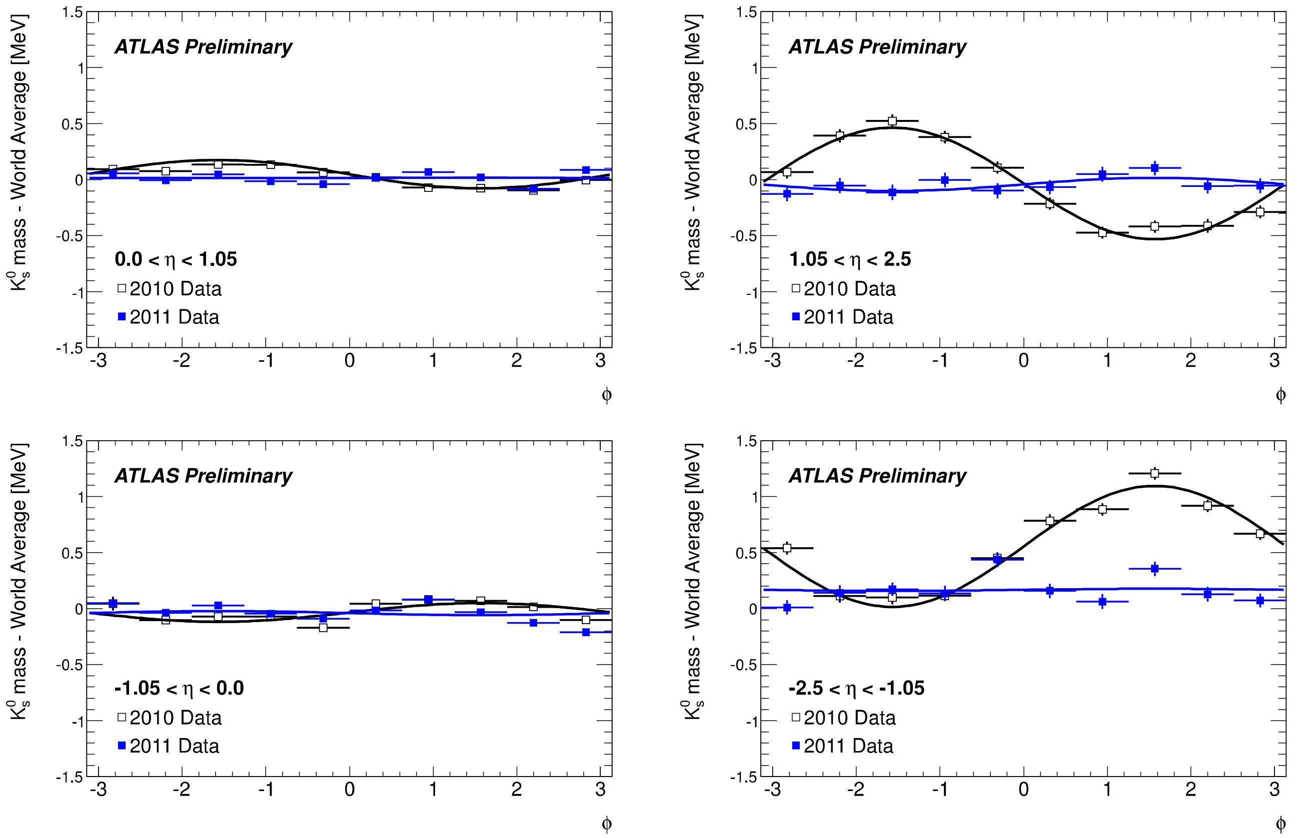


$$p \rightarrow p(1 - \cot\theta \sin\varphi \alpha_{rot})$$

From the ATLAS Silicon alignment group, Bruckman et al

Correcting for B field rotation

- The needed extra constraint is here provided by the K0 mass.
- The rotations in data are found from interpolation among simulated rotated samples.



Checking for B field rotation

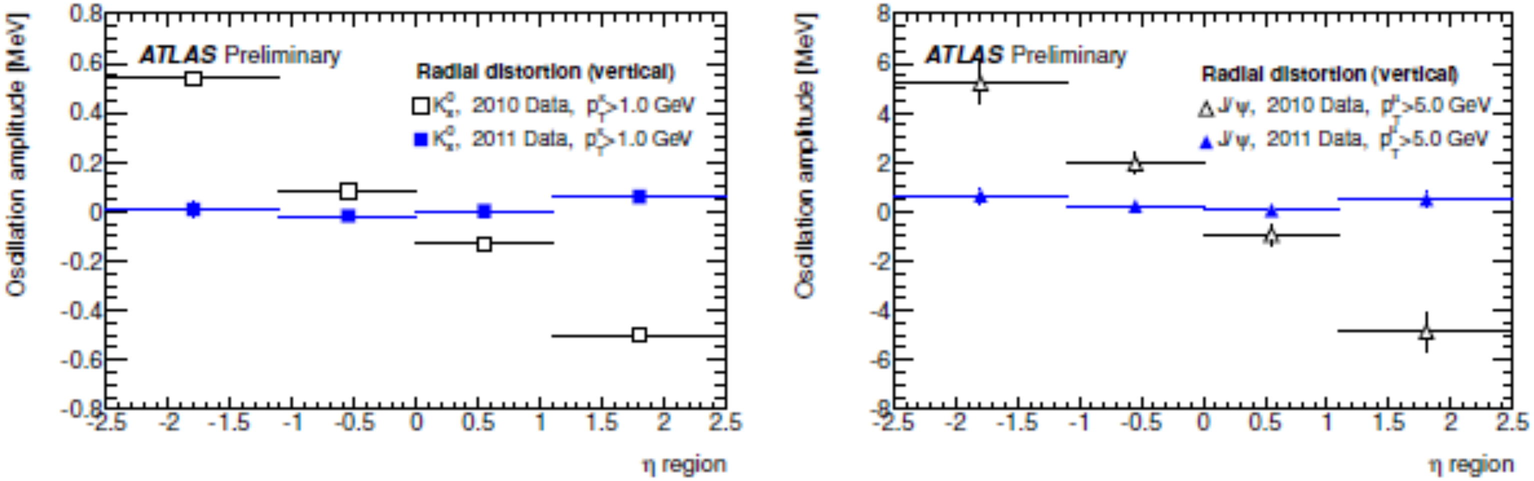


Figure 11: Comparison of the fitted radial-deformation amplitudes in 2010 and 2011 data for the K_S^0 (left) and the J/ψ (right) resonances.

Table 2: Rotation values obtained in different detector regions. Quoted errors are statistical only (from the amplitude fit).

	End-cap C	Barrel Minus	Barrel Plus	End-cap A
R_x^B from K_S^0 [mrad]	0.53 ± 0.02	0.26 ± 0.06	0.40 ± 0.04	0.67 ± 0.02
R_x^B from J/ψ [mrad]	0.48 ± 0.03	0.61 ± 0.11	0.41 ± 0.13	0.52 ± 0.03

From the ATLAS Silicon alignment group, Bruckman et al

Summary of particle tracking

- Good spacepoints is the most important thing!
- Constants need to be frequently CALIBRATED and ALIGNED!
- Pre-fabricated *templates* saves CPU time.
- Fits maximize the likelihood of track and vertex states, given the spacepoints. The Kalman Filter and Global Chisquared minimization are the standard methods.
- Non-linearities are handled by iteration and non-Gaussianity by procedures such as the GSF.
- Further refinements are possible by with global methods arbitrating competing track-hit asignments and using external constraints.
- Ultimate optimum is to use ALL the info in the data and ALL knowledge that you have otherwise – a bit like in big water Cherenkovs.