NBI exercises #1

(1) Consider a transiting planet, of mass M_p, in an orbit with semi-major axis a and period P. If the planet is itself orbited by a moon of mass m_s and orbital radius a_s (around the planet), find an expression for the magnitude of variations δt in the onset of transits caused by the presence of the moon. For a typical hot Jupiter, what sort of timing precision would be needed to detect the presence of moons similar to the Galilean satellites via their effect on transit times?

(2) The IAU definition of a Solar System "planet" includes the requirement that a planet be massive enough that gravity forces the body to adopt a "near-spherical" shape. Consider a rocky body that has a mean density ρ and a yield strength $P_{\rm yield}$ (the yield strength can be considered to be the maximum pressure that the rock can resist due to its internal strength before it deforms). By dimensional consideration of the hydrostatic equilibrium equation (or otherwise) determine an expression for the minimum radius of a body that would be expected to be roughly spherical.

For a density of 3.5 g cm^{-3} and a yield strength of 2×10^9 dynes cm^{-2} what is the minimum radius? Is this consistent with observations of Solar System objects?

(3) Suppose that the equation of state of gas in the protoplanetary disk can be written in polytropic form, $P = K\rho^\gamma$. For $\gamma \neq 1$, derive the vertical density profile assuming hydrostatic equilibrium (and $z \ll r$). How does the solution compare to the isothermal case we worked out previously?

(4) Consider a disk with mass $M_{\rm disk} \sim \pi r^2 \Sigma$ and thickness h, at radius r from a star of mass M_*. By approximating the self-gravity of the disk as that of an infinite sheet, estimate the minimum Σ such that disk self-gravity dominates the vertical acceleration at $z = h$. Hence, show that,

$$\frac{M_{\rm disk}}{M_*} > \frac{h}{r}$$

is a rough condition for when self-gravity matters for the vertical structure.

(5) Chondrules are mm-sized inclusions, found within meteorites, that appear to have solidified from a molten state following rapid heating (perhaps from a shock wave). One of the interesting properties of chondrules is that a small fraction (about 5%) are "compound", which perhaps suggests that the molten drops collided and stuck together before they solidified. If true, this places interesting limits on the density of the chondrule precursors within the disk.

(a) Assume that chondrules are spheres of radius s and density ρ, and that they have number density (number per cm^3) n and random velocities v. Write an expression for the characteristic collision time t.

(b) Various arguments based on the chemical properties and mineralogy of chondrules suggest that they cooled on a time scale of the order of 10^4 s. Assuming that $s = 1$ mm, $\rho = 3$ g cm^{-3} and that $v = 10$ cm s^{-1}, use the fact that 5% of chondrules are compound to estimate the volume density (in g cm^{-3}) of chondrules within the disk at the time when they were molten.

(c) At 1 AU the gas surface density is of the order of 10^3 g cm^{-2}. The vertical thickness of the protoplanetary disk at this radius is $h \sim 0.05r$, and a rough estimate is that the solid to gas ratio is 1%. Use these numbers to estimate the expected volume density of solids at 1 AU.

(d) By comparing your results from (b) and (c), determine by what factor chondrules need to be concentrated (compared to the mean solid density) to account for the observation of compound chondrules.