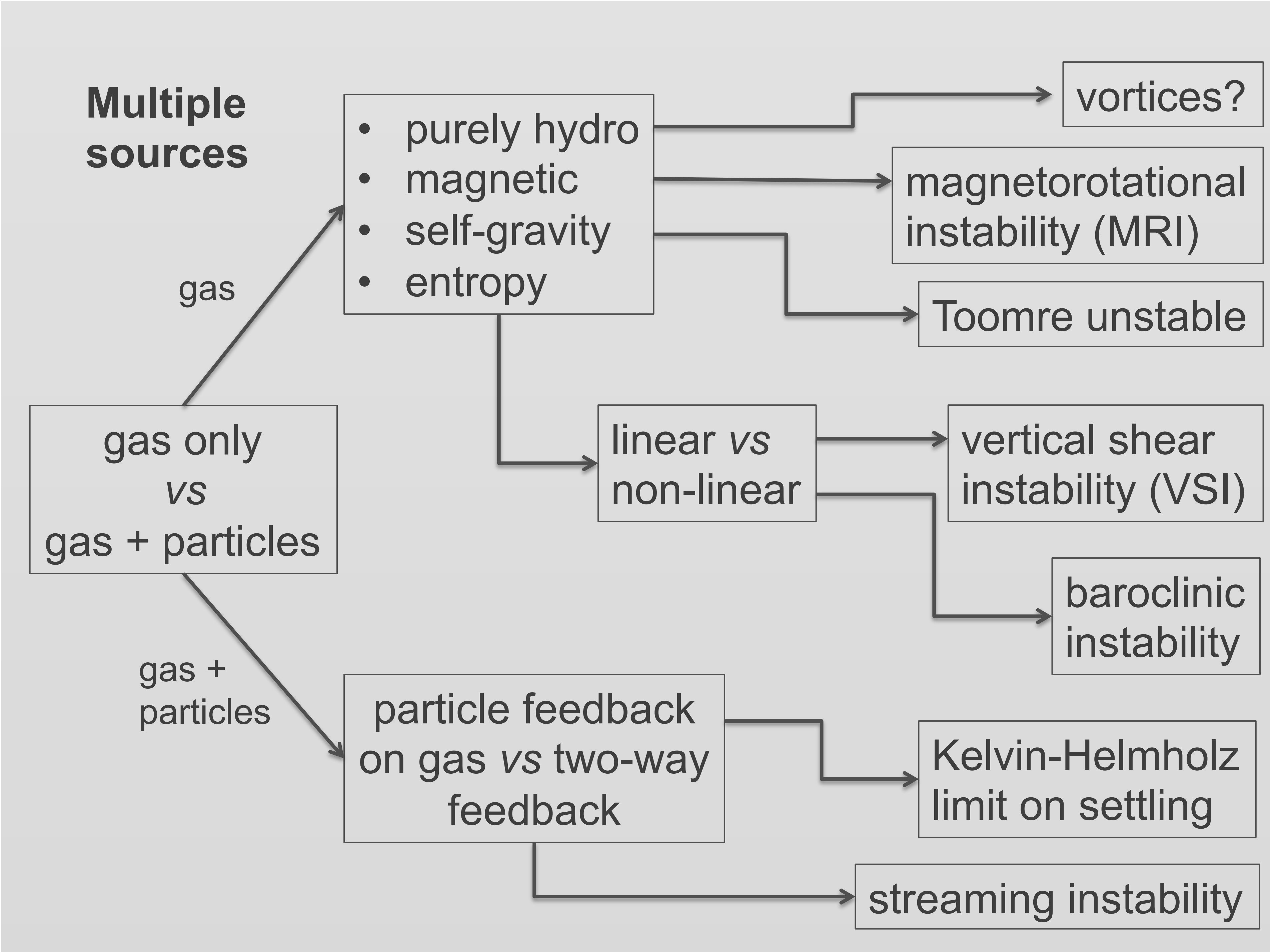
Protoplanetary disk evolution

Phil Armitage
Colorado

Multiple motivations

Turbulence

- **angular momentum transport**
  - disk evolution, episodic accretion, planet-disk interactions
- **radial and vertical mixing / diffusion**
  - dust settling, chemistry, *Stardust* sample interpretation…
- **concentration of particles**
  - observations of transition disks, prelude to planetesimal formation, meteoritics


Multiple sources
gas
gas only
vs
gas + particles
gas +
particles
purely hydro
magnetic
self-gravity
entropy
vortices?
magnetorotational instability (MRI)
Toomre unstable
linear vs non-linear
vertical shear instability (VSI)
baroclinic instability
particle feedback on gas vs two-way feedback
Kelvin-Helmholz limit on settling
streaming instability

Turbulence

Fluid turbulence:

Define **Reynolds number** $\mathrm{Re} = \frac{UL}{\nu_m}$

Velocity U ~ km s^{-1}, L ~ AU, $\nu_m \sim 10^6$ cm^2 s^{-1}

⇒ $\mathrm{Re} \sim 10^{12}$

Any turbulence present will be fully developed – large inertial range

BUT linear stability of shear flow is given by Rayleigh criterion:

$$\frac{\mathrm{d}l}{\mathrm{d}r} < 0$$

…for *instability*, $l \sim r^{1/2}$ for Keplerian flow so disks are linearly stable to infinitesimal hydrodynamic perturbations

Linear instabilities

Consider a background disk model that is:

- described by some set of physics (isothermal hydrodynamics, MHD, hydro + self-gravity…)
- in equilibrium

Linearize equations, perturb with e.g. $\mathbf{s} \propto e^{i(\omega t - k\mathbf{r})}$

System is unstable if there are growing modes: $\omega^2 < 0$

Main disk instabilities are also **local**, apply in a "patch" of disk where shear is linearized, do not depend on boundary conditions

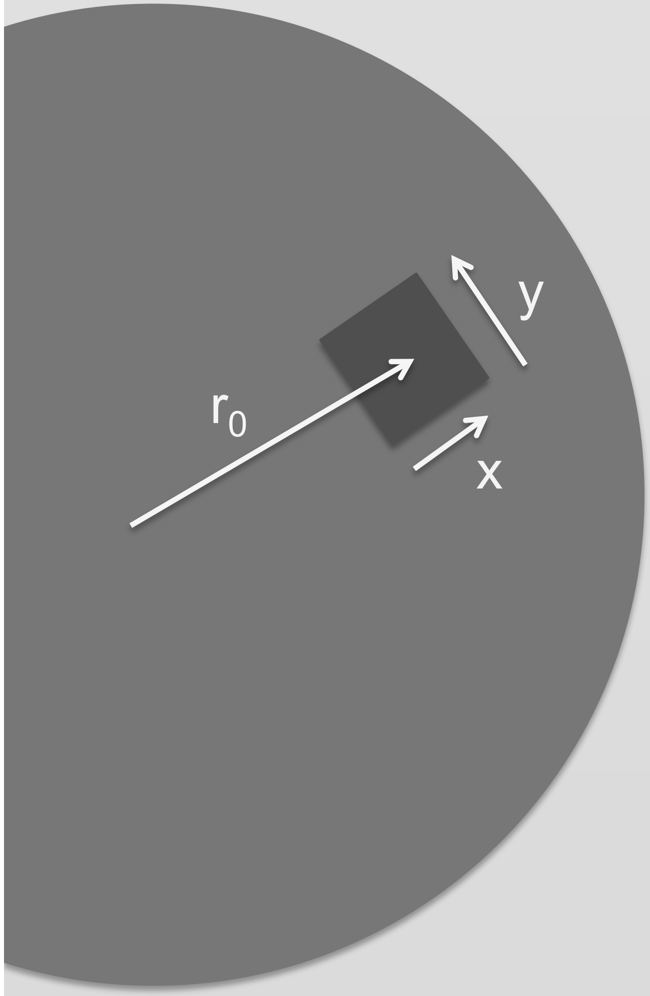
Interested in existence, growth rate, non-linear outcome

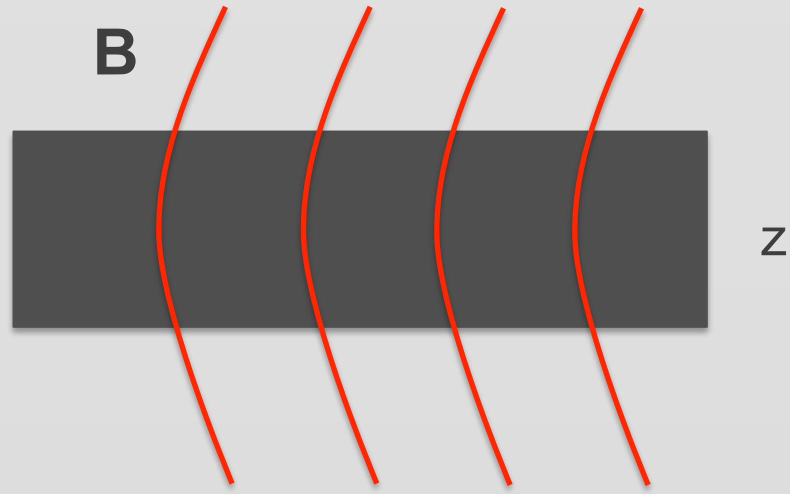
1. Magnetorotational instability

Balbus & Hawley '91, review Rev. Mod. Phys. '98

Physics: hydrodynamics ✔
magnetic fields ✔
self-gravity ✗
energy equation ✗

1. Magnetorotational instability

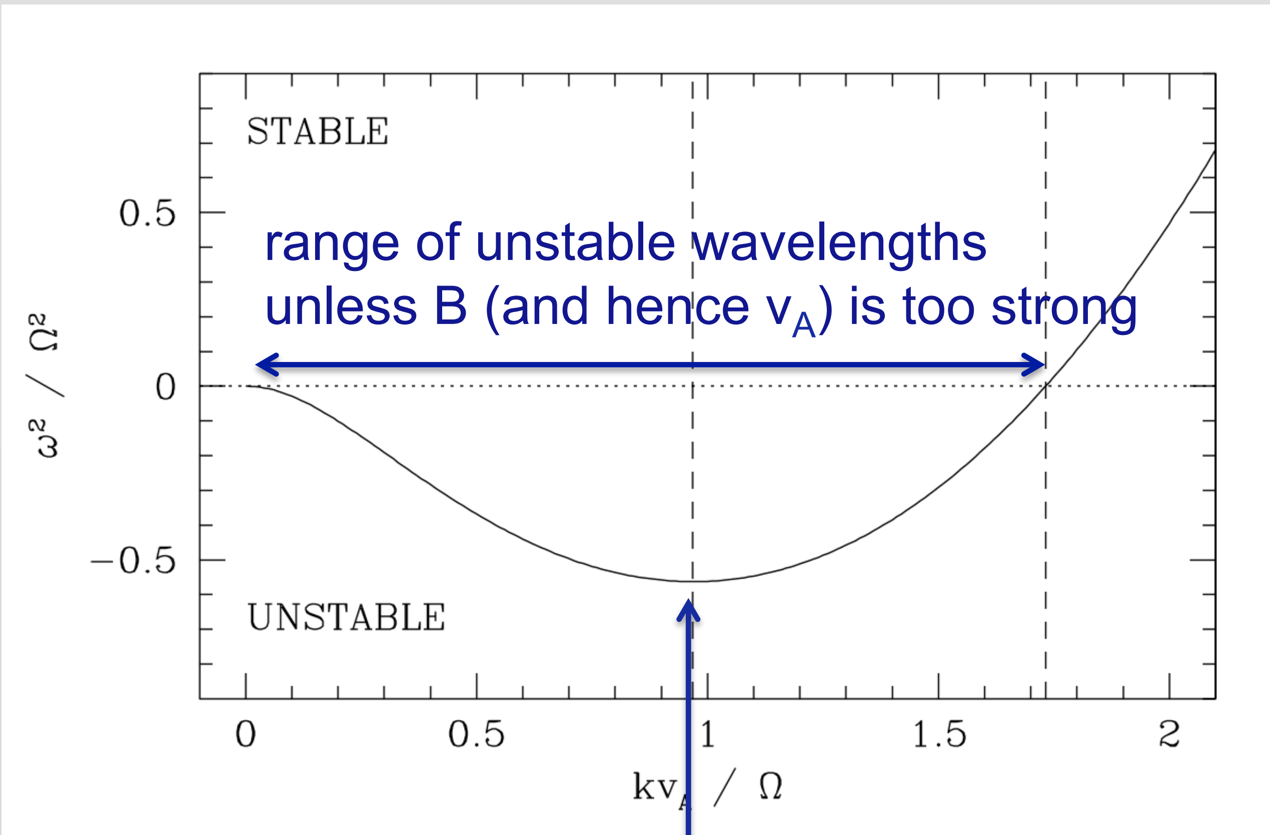




Initial equilibrium state has a weak, uniform vertical magnetic field B_z, assume ideal MHD

Dispersion relation:

$$\omega^4 - \omega^2 \left[\frac{\mathrm{d}\Omega^2}{\mathrm{d}\ln r} + 4\Omega^2 + 2\left(kv_A\right)^2 \right] + \left(kv_A\right)^2 \left[\left(kv_A\right)^2 + \frac{\mathrm{d}\Omega^2}{\mathrm{d}\ln r} \right] = 0$$



For a weak magnetic field, unstable if

$$\frac{\mathrm{d}}{\mathrm{d}r}\left(\Omega^2\right) < 0$$

In ideal MHD, always unstable

most unstable wavelength with growth rate $\omega \sim \Omega$ i.e. very fast!

6. Influence of the magnetic field on the stability of the rotating cloud

Some idea of the magnetic field's influence on the stability of the rotating cloud can be arrived at from certain results of Chandrasekhar (1961) for the motion of fluids between revolving cylinders (Couette flow) in the cases of a magnetic field H_z parallel to the axis of rotation and H_φ along the direction of rotation. For a field H_z of infinite conductivity, the stability condition is found to be

$$I_1 \frac{\mu H^2}{4\pi\rho} > -\int_{R_1}^{R_2} \frac{d\omega^2}{dR} R^2 \xi_R^2 \, dR. \qquad (27)$$

This result is somewhat unexpected, since the above does not reduce to Rayleigh's criterion when $H \to 0$. For a vanishingly small field when ω is a monotonic function of R, the necessary and sufficient condition for instability is that ω increase with R. In the protoplanetary cloud ω decreases with R

Safronov '72 – the MRI was *almost* discovered and understood by Chandrasekhar, Velikov & Safronov in the 1960s…

Conclusion that weakly magnetized disks are always violently unstable applies to *ideal MHD* (i.e. ionized disk), several complexities in protoplanetary disks

- currents decay due to collisions (**Ohmic** dissipation)
- magnetic field couples to charged particles, neutrals couple only via ion-neutral collisions (**ambipolar diffusion**)
- charge carriers moving in a magnetic field experience a Lorentz force, creating an additional electric field (**Hall effect**)

Very roughly, these “non-ideal” MHD effects damp the MRI in regions where the ionization fraction and / or density are very low

Ohmic **Hall** **Ambipolar**

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times \left[\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B} - \frac{\mathbf{J} \times \mathbf{B}}{e n_e} + \frac{(\mathbf{J} \times \mathbf{B}) \times \mathbf{B}}{c \gamma \rho_i \rho} \right]$$

Ideal MHD: MRI grows on scale h on time scale:

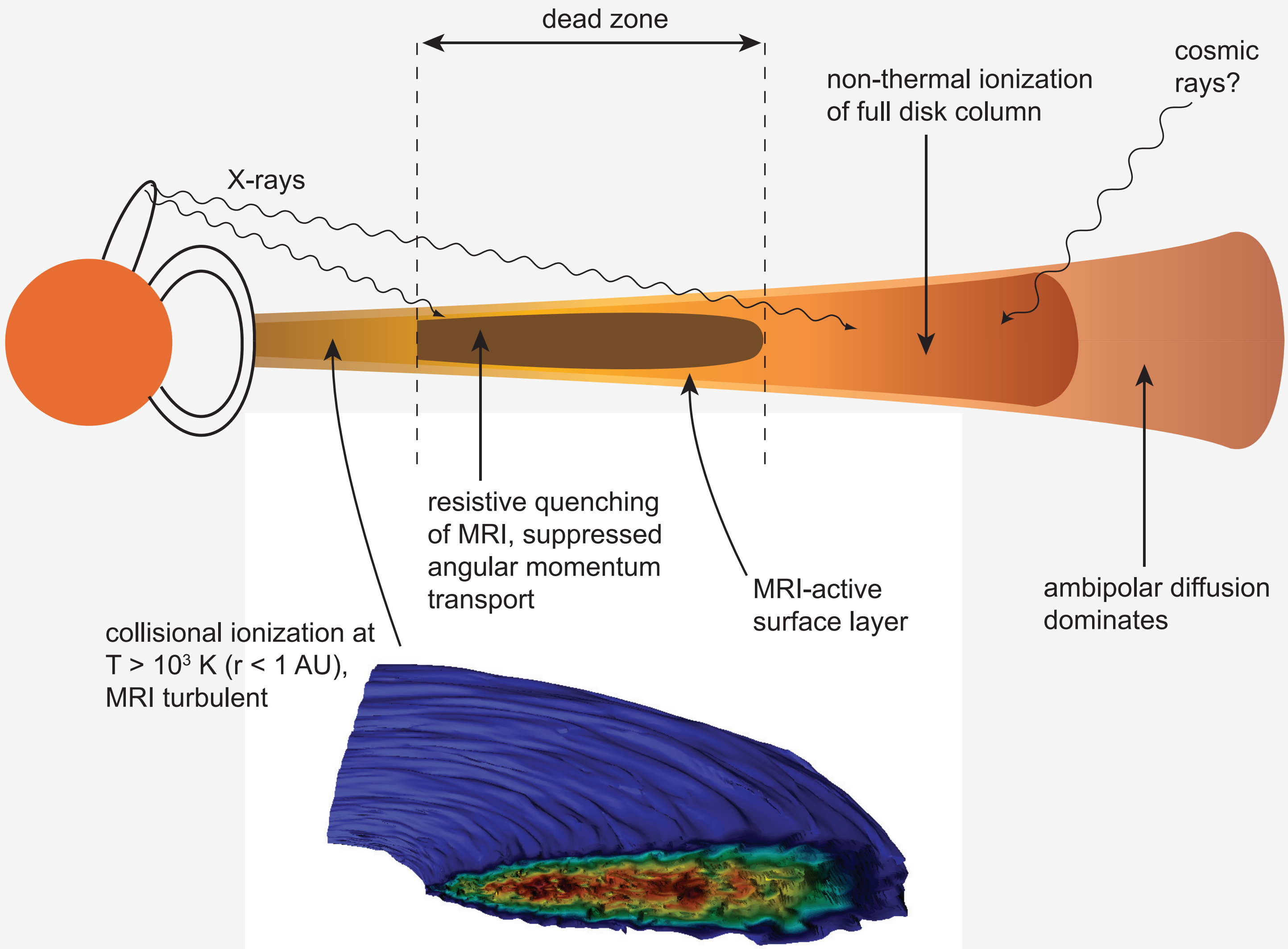
Diffusive term, leads to damping on time scale:

$$\tau \sim \frac{h}{v_A} = \tau \sim \frac{h^2}{\eta}$$

…with η the magnetic diffusivity ~ 1 / conductivity

MRI damped for $\eta > hv_A$
For inner disk $x_e > 10^{-12}$

Argument: *Gammie '96*



If MHD processes are dominant, expect suppressed turbulence near the mid-plane – a “dead zone”

2. Self-gravity

Physics: hydrodynamics ✔
magnetic fields ✗
self-gravity ✔
energy equation ✗

Linear instability if "Toomre Q" is below critical value:

$$Q = \frac{c_s \Omega}{\pi G \Sigma} \lesssim Q_{\rm crit} \sim 1$$

Roughly this requires $M_{disk} / M_* > h / r$,
expect self-gravity to matter for massive disks

3. Vertical shear instability

Physics: hydrodynamics ✔
magnetic fields ✗
self-gravity ✗
energy equation ✔

$$\frac{\partial l^2}{\partial r} - \frac{k_r}{k_z}\frac{\partial l^2}{\partial z} < 0$$

and thermal diffusion due radiation, heating + cooling processes is “fast”

↑ radial gradient of specific angular momentum

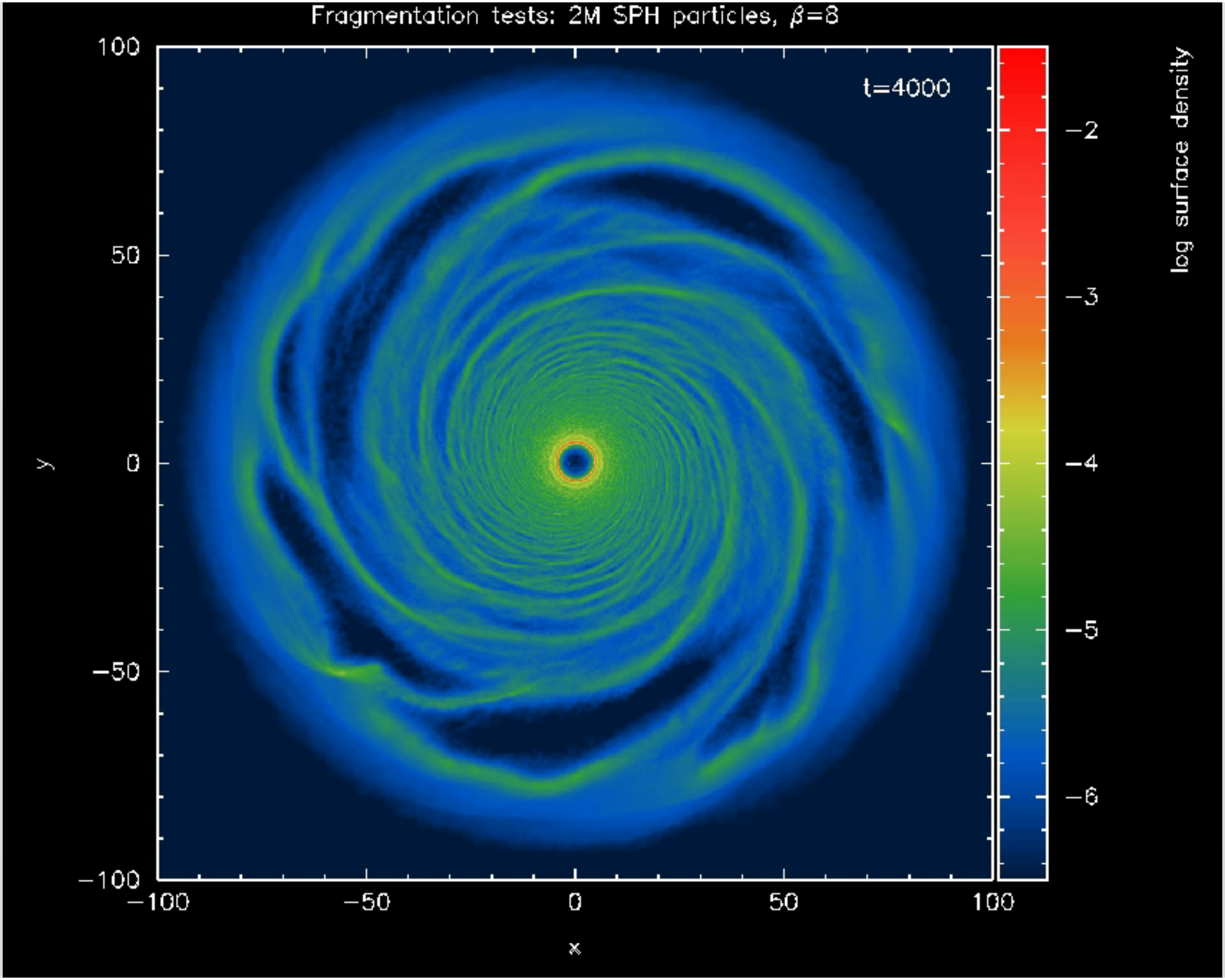
↑ vertical gradient of specific angular momentum

Nelson, Gressel & Umurhan ‘13

Non-linear evolution

With few exceptions, need numerical simulations to assess:

- non-linear evolution of disk instabilities
- saturation level
- nature of turbulence (waves, vortices…)


Fragmentation tests: 2M SPH particles, β=8
t=4000
log surface density
x
y
100
50
0
−50
−100
−2
−3
−4
−5
−6

Self-gravity: trailing spiral arms, “gravito-turbulence”

Assuming locality, condition that Q ~ Q_{crit} can be used to *analytically* estimate the efficiency of angular momentum transport *(Gammie ‘01)*

$$Q = \frac{c_s \Omega}{\pi G \Sigma} \lesssim Q_{\rm crit} \sim 1$$

Heating rate $(9/4)\nu\Sigma\Omega^2$ must balance cooling σT^4

$$\alpha = \frac{4}{9\gamma(\gamma-1)} \frac{1}{t_{\rm cool}\Omega}$$

where t_{cool} is the thermal energy per unit area / cooling rate, and γ is the adiabatic index of the gas

“Clumpiness” of the disk increases as t_{cool} decreases, generally thought that fragmentation occurs for low $t_{cool}\Omega$

Vertical shear instability

Fluid stresses with $\alpha \sim 10^{-3}$

Nelson et al. '13

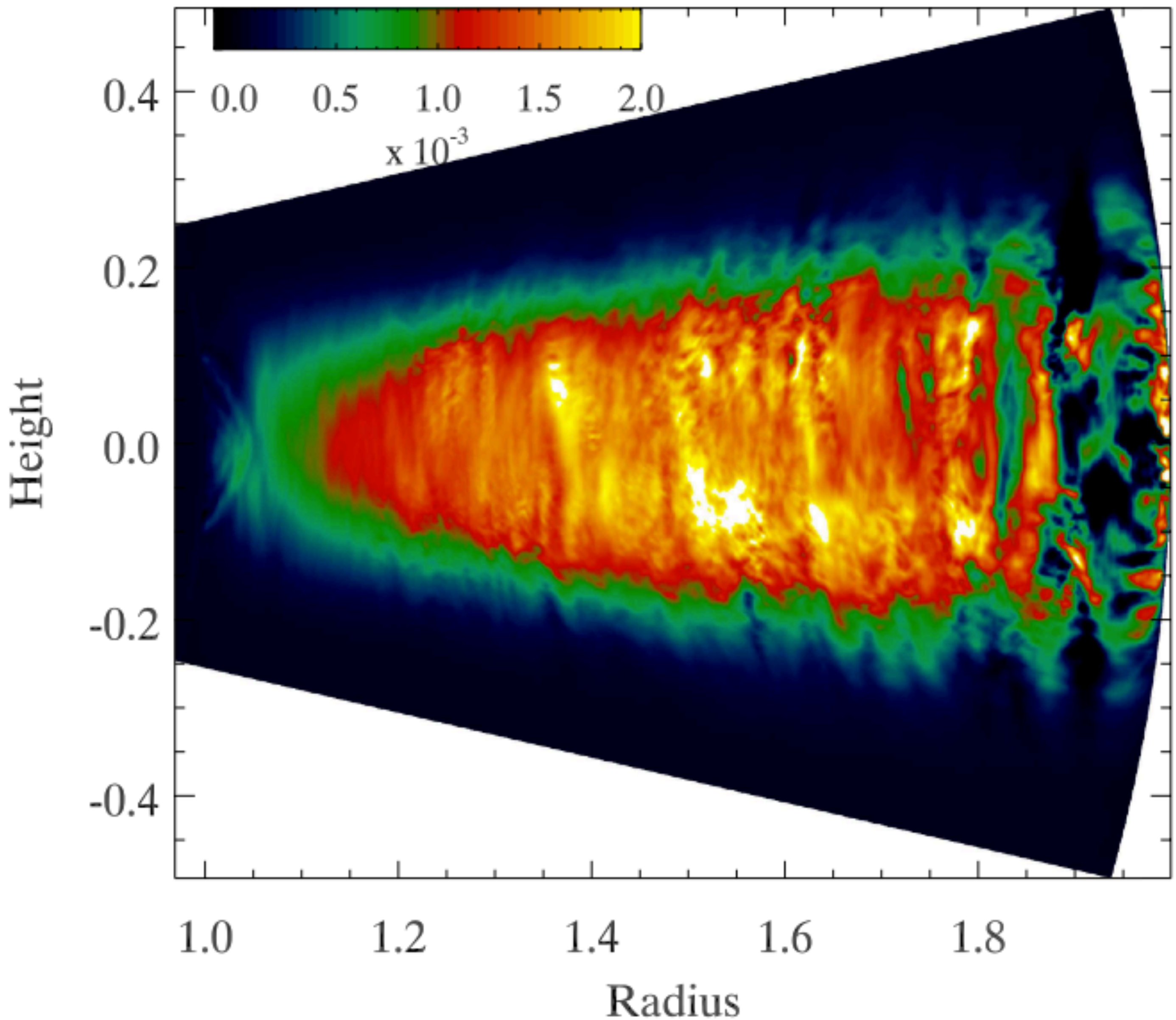
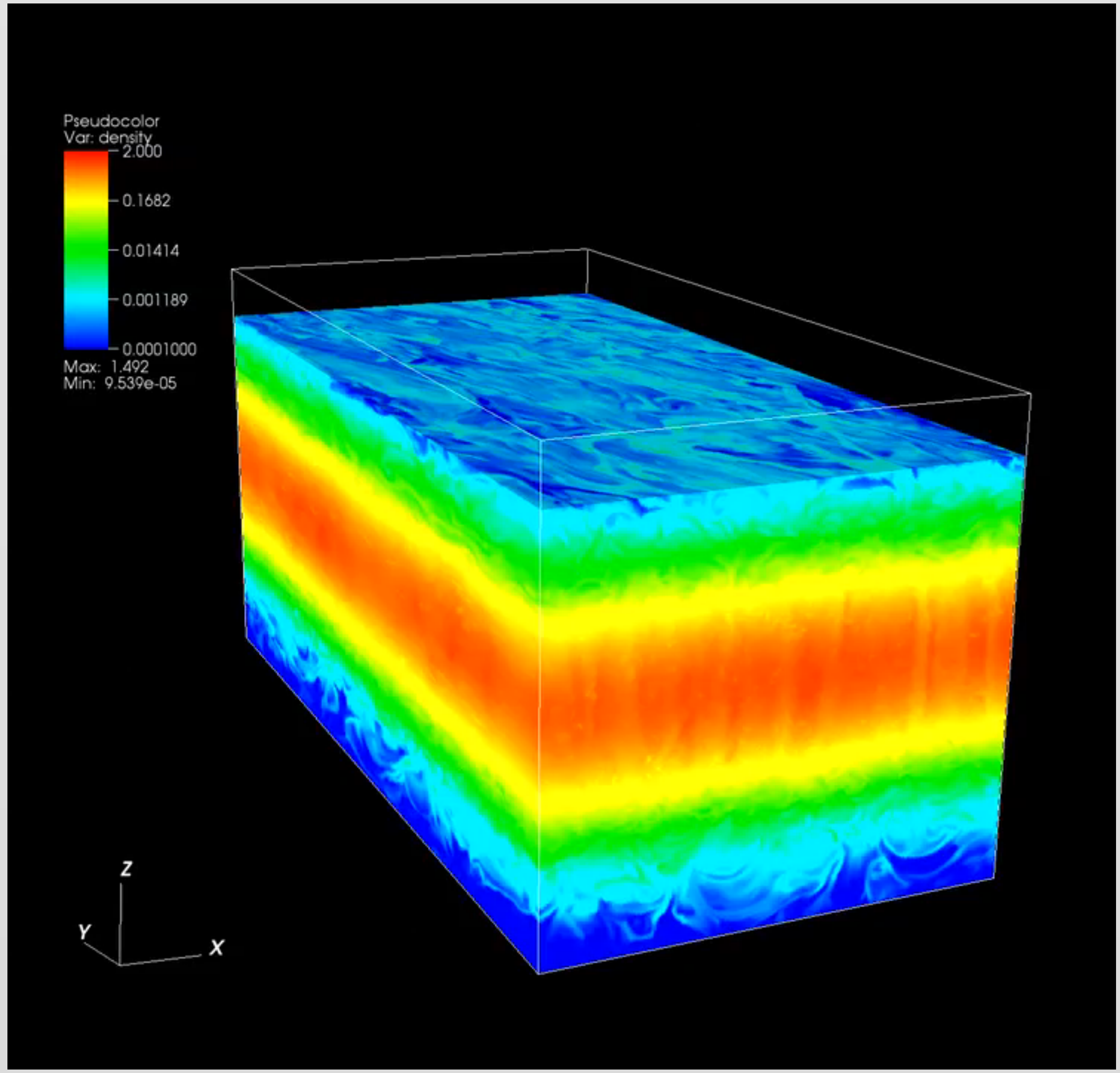


Figure 16. Spatial distribution of the time and horizontally averaged Reynolds stress (normalized by the mean pressure at each radius) for model T1R-0–3D.

Magnetorotational instability

In ideal MHD, leads to turbulence with $\alpha \sim 0.02$, with most of the stress coming from magnetic rather than fluid stresses

Relevant only to inner protoplanetary disks where $T > 10^3$ K



Simon, Beckwith & Armitage '12

Magnetorotational instability

Relationship of magnetic field instabilities
to turbulence in the cool part of the disk
where non-thermal ionization dominates…

Secure results:

- Ohmic diffusion damps the MRI near the mid-plane in the terrestrial planet-forming region
- ambipolar diffusion damps mid-plane turbulence in the outer disk, strongly if there is no net B_z

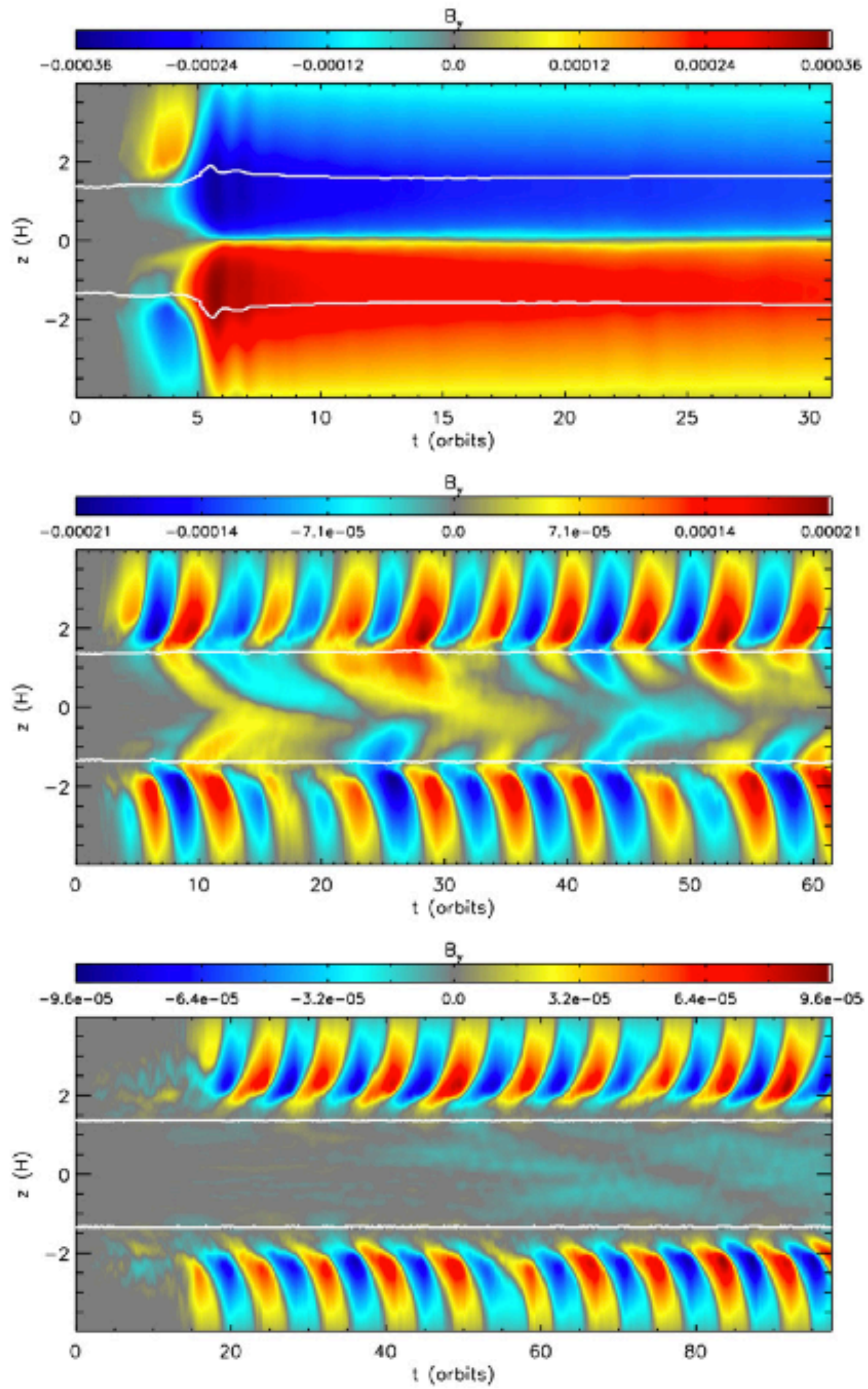
Provocative results:

- MRI + net field leads to disk winds
- where Hall effect dominates, get a **laminar** magnetic stress whose strength depends on **sign** of B_z

Simulations of the outer disk have turbulent surface layers, ambipolar damping near the mid-plane

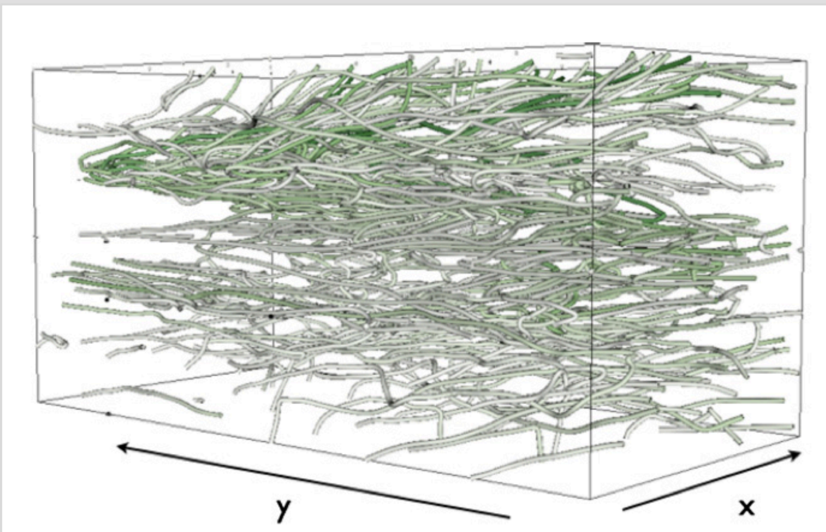
$$\beta_z = \frac{P_{\rm gas}}{B_z^2/8\pi} = 10^4$$

Stress and nature of the solution strong function of how much magnetic field threads the disk *(Simon et al. 2013)*

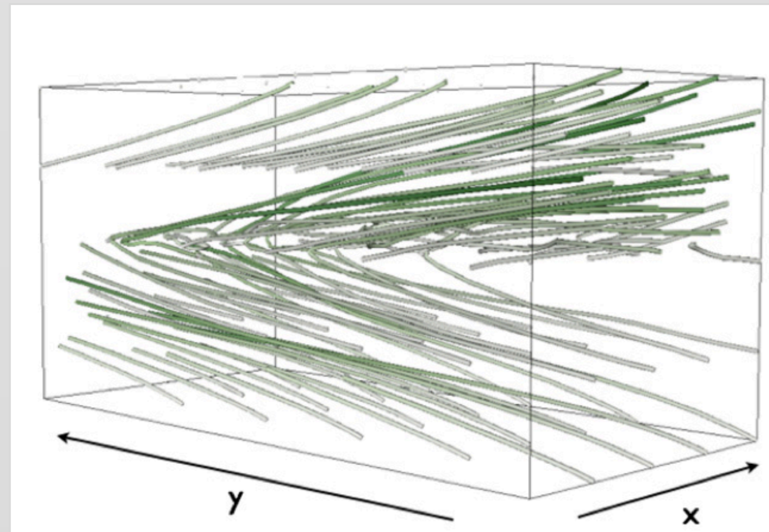


Vertical fields lead to a transition from turbulence to disk winds, which *may* carry away significant mass and angular momentum

$\beta_z = 10^4$



$\beta_z = 10^3$

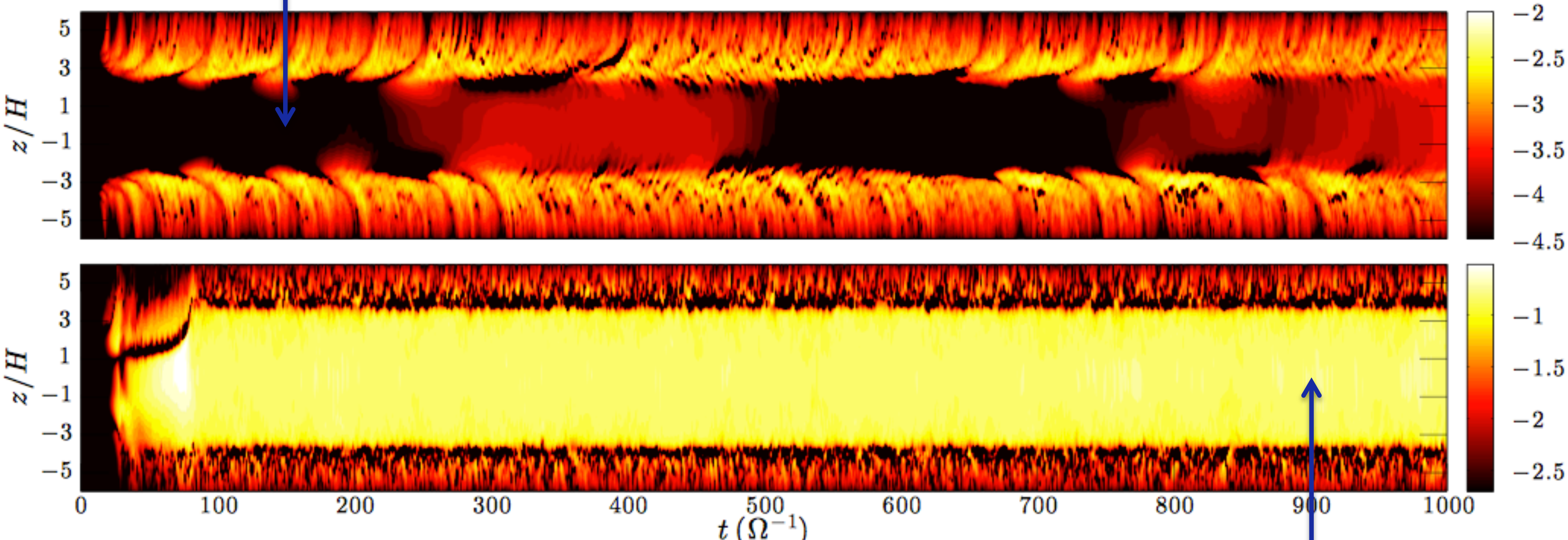


Simon et al. 2013

Generic result (Suzuki & Inutsuka ’09; Fromang et al. ‘13; Bai & Stone ‘13; Lesur et al. ‘13)

At r ~ AU, expect Hall effect to be dominant term

Ohmic diffusion only, get a dead zone with very weak mid-plane stress / turbulence

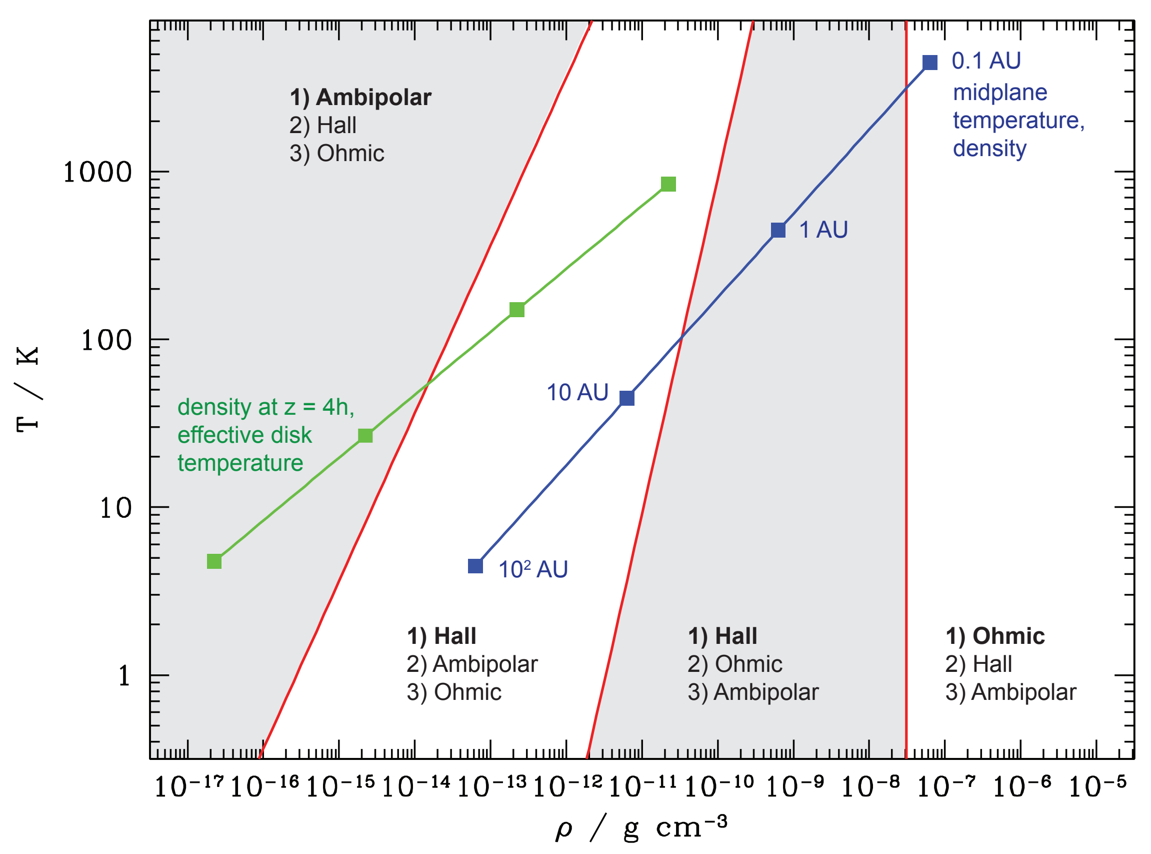


with the Hall effect, strong laminar transport of angular momentum due to Maxwell stress $B_r B_\phi$

Lesur, Kunz & Fromang '14

Hall effect depends on the **sign** of (Ω.**B**)

Predict different levels of stress and turbulence in disks where the net field is aligned / anti-aligned with rotation



Is disk structure bimodal on ~AU scales?

Armitage, ARA&A '11

Disk winds

Photoevaporation – basic physics

- heat surface of disk to sound speed c_s
- where $c_s > v_K$, gas is unbound
- flows away from disk in a thermal wind

Naively: $$c_s = \sqrt{\frac{GM_*}{r_g}}$$

$$\Rightarrow \quad r_g = \frac{GM_*}{c_s^2}$$

gas escapes beyond a critical radius

c_s = 10 km s^{-1} implies r_g ~ 10 AU (corrected to ~2 AU when account is taken of rotation

Photoevaporation – many variations on a theme…

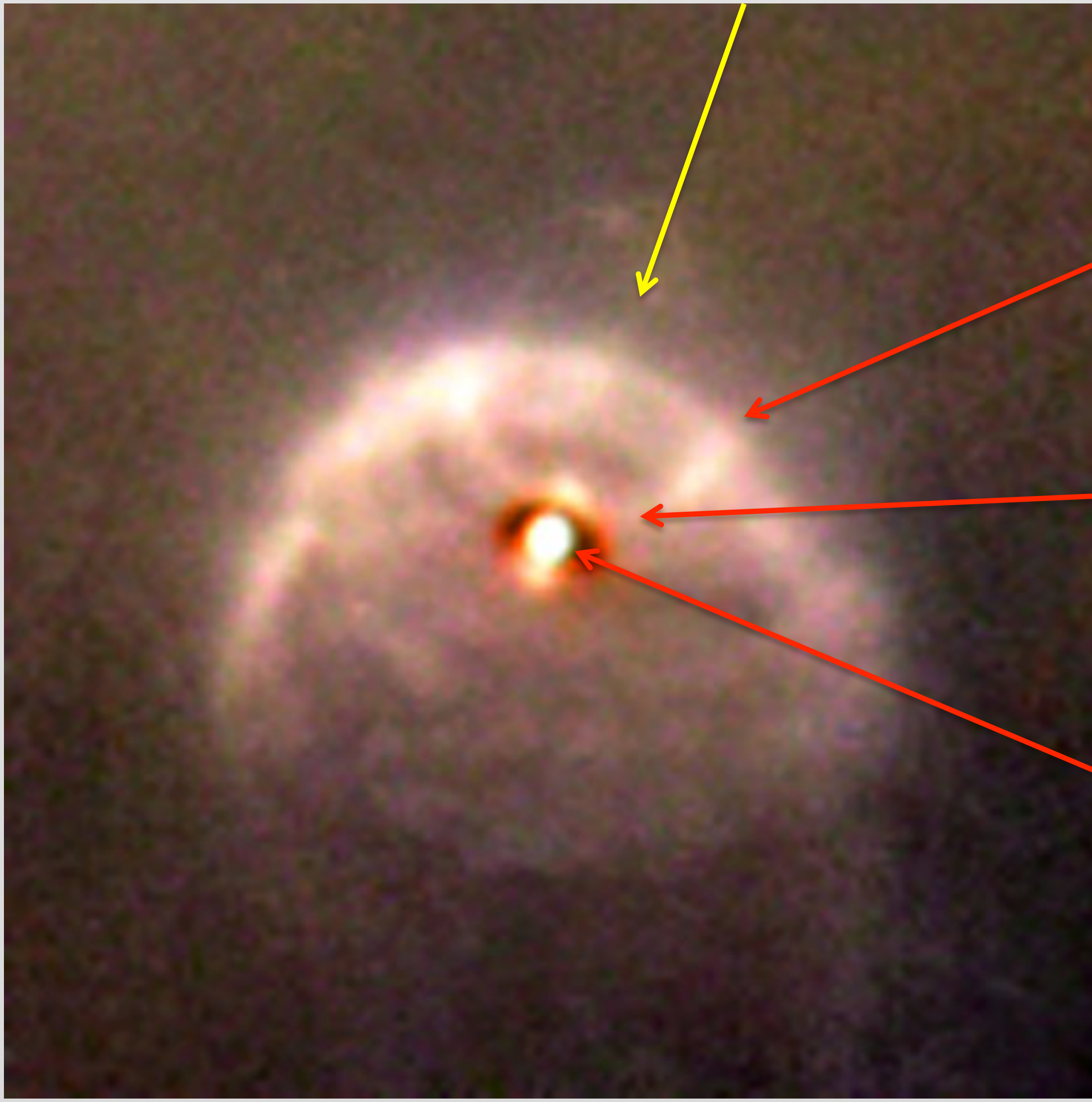
Heating by central star:

- EUV radiation – $h\nu > 13.6$ eV
- FUV radiation – $6.0 \text{ eV} < h\nu < 13.6$ eV
- X-rays

External photoevaporation:

- FUV radiation from *massive* stars in a cluster

Review: Alexander, Pascucci et al. '14, PP6



HST / ACS – c.f. Ricci et al. '08

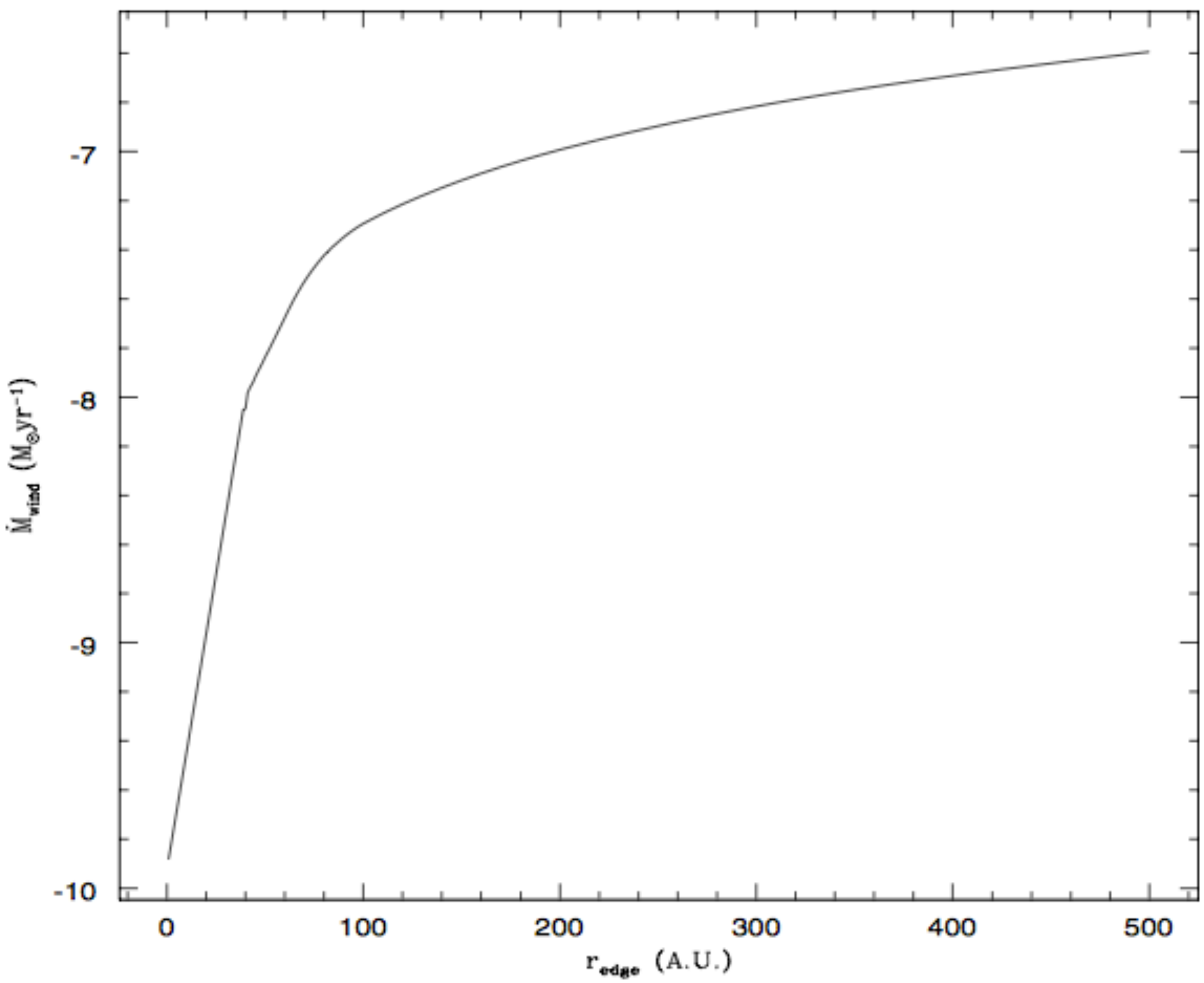
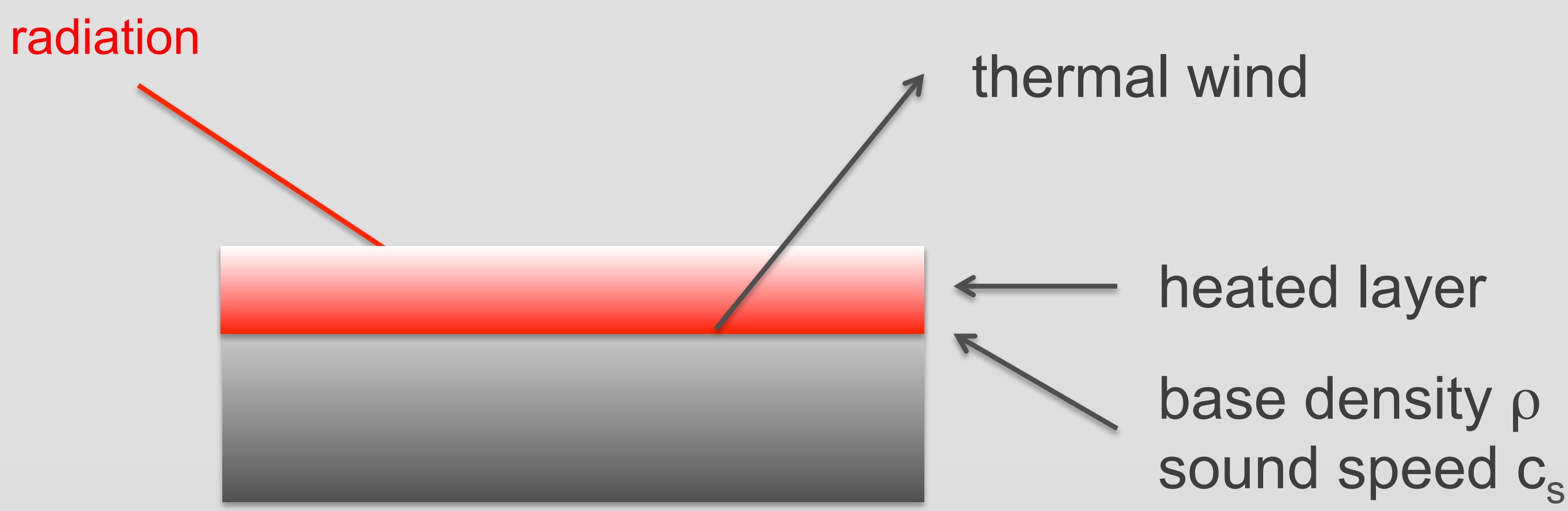


Figure 1. Assumed photoevaporative mass loss rate as a function of outer disc radius (equations 3–6). Adapted from Adams et al. (2004) for the case $M_* = 1\,M_\odot$ and ultraviolet radiation field intensity $G_0 = 3000$.

FUV radiation fields in cores of rich clusters (e.g. Orion) leads to rapid mass loss if disks are large (r_d ~ 100 AU)

Clarke '07

Internal photoevaporation



Rough estimate for mass loss rate per unit area:

$$\dot{\Sigma} \sim \rho c_s$$

Integrating over the disk surface gives total mass loss rate

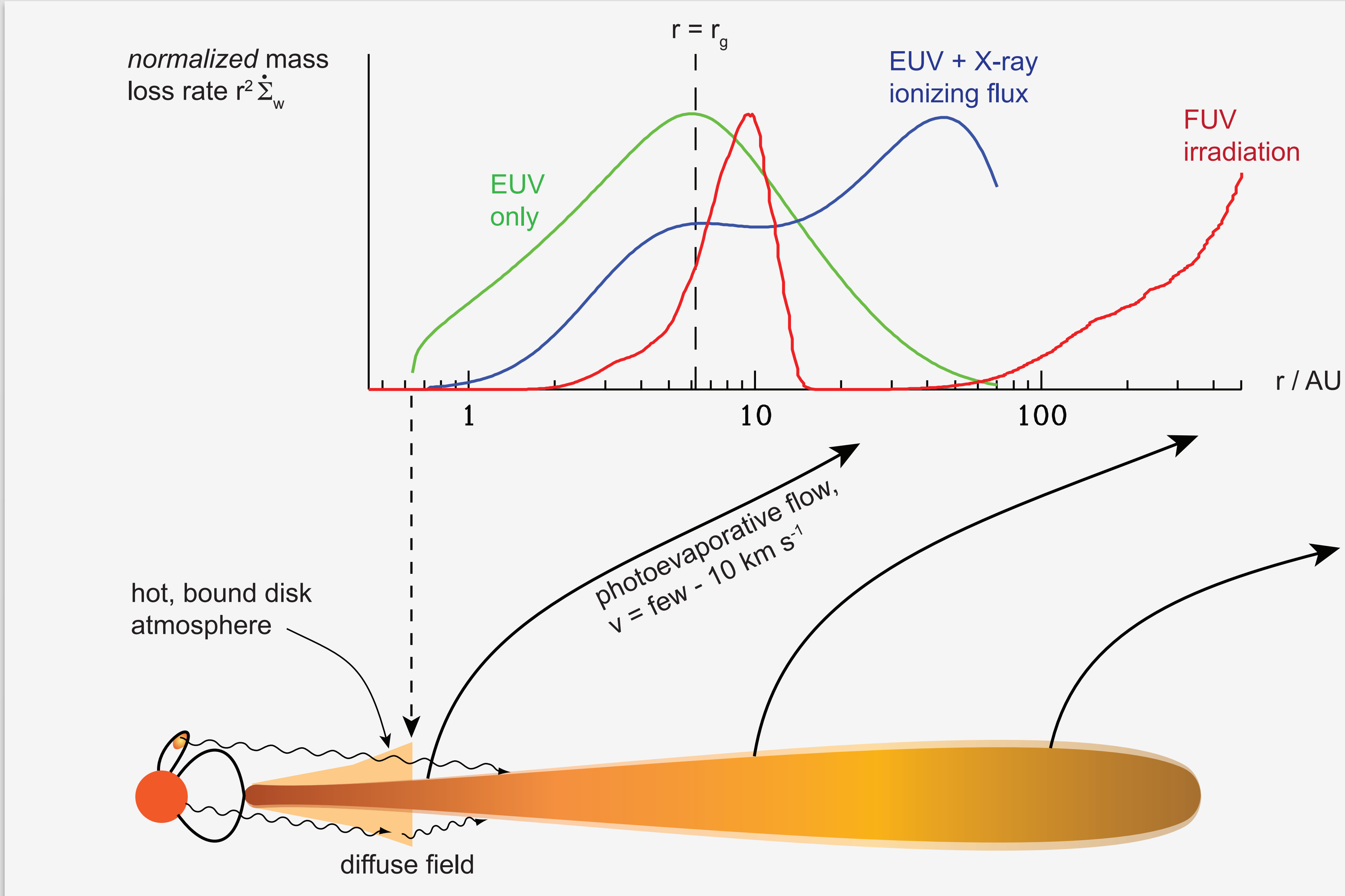
Internal photoevaporation

Mass loss rate can be computed fairly easily for EUV case:

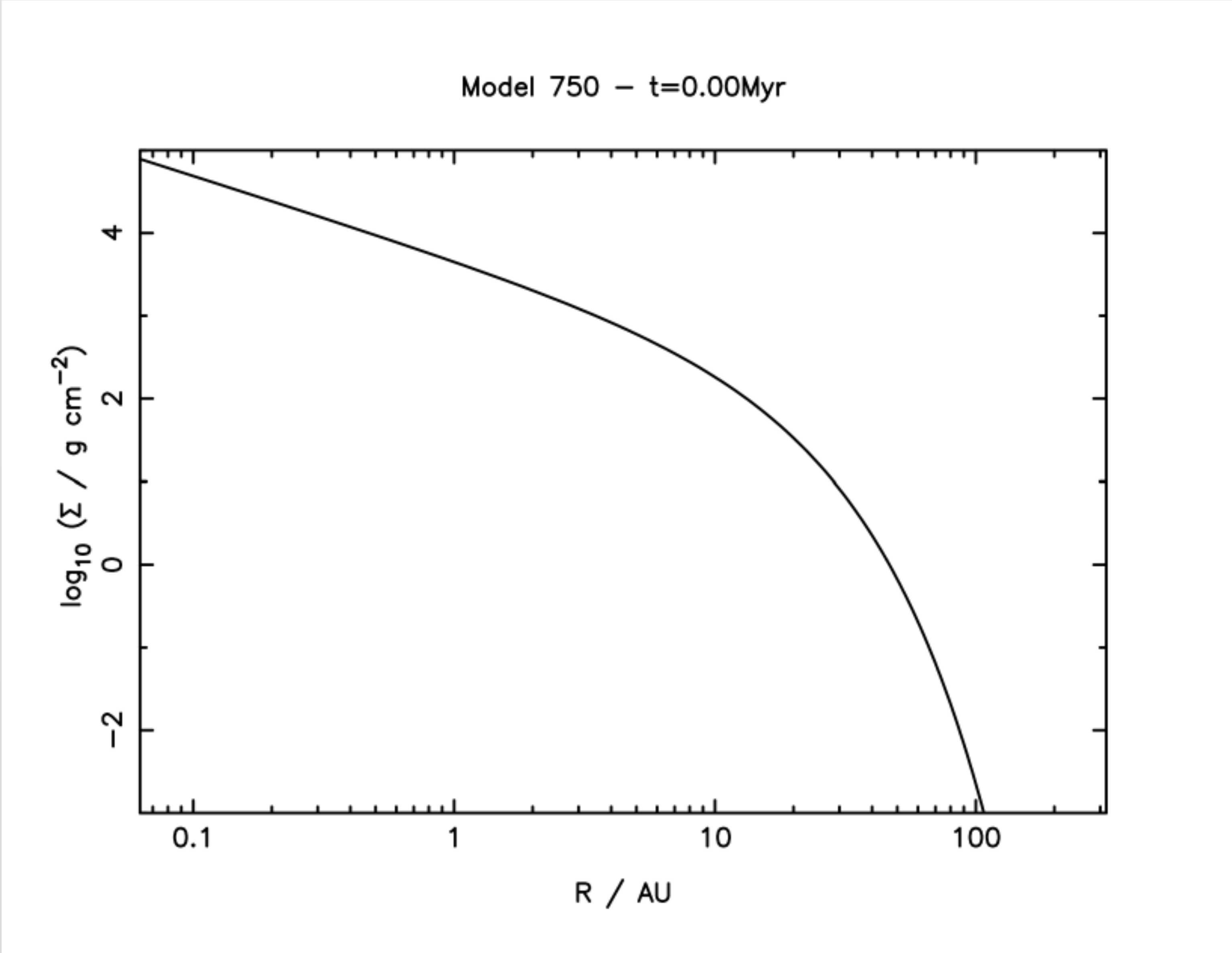
$$\dot{M}_{\rm wind} \sim 10^{-10} \left(\frac{\Phi}{10^{41}\ {\rm s}^{-1}} \right)^{1/2} \left(\frac{M_*}{M_\odot} \right)^{1/2} \ M_\odot\ {\rm yr}^{-1}$$

Font et al. '04

Mass loss rates are much harder to compute for the FUV and X-ray cases, but are generally estimated to be 1-2 orders of magnitude greater *(Gorti & Hollenbach '09; Owen et al. '10)*



r = r_g
normalized mass loss rate r² Σ̇_w
EUV + X-ray ionizing flux
FUV irradiation
EUV only
r / AU
1
10
100
photoevaporative flow, v = few - 10 km s⁻¹
hot, bound disk atmosphere
diffuse field

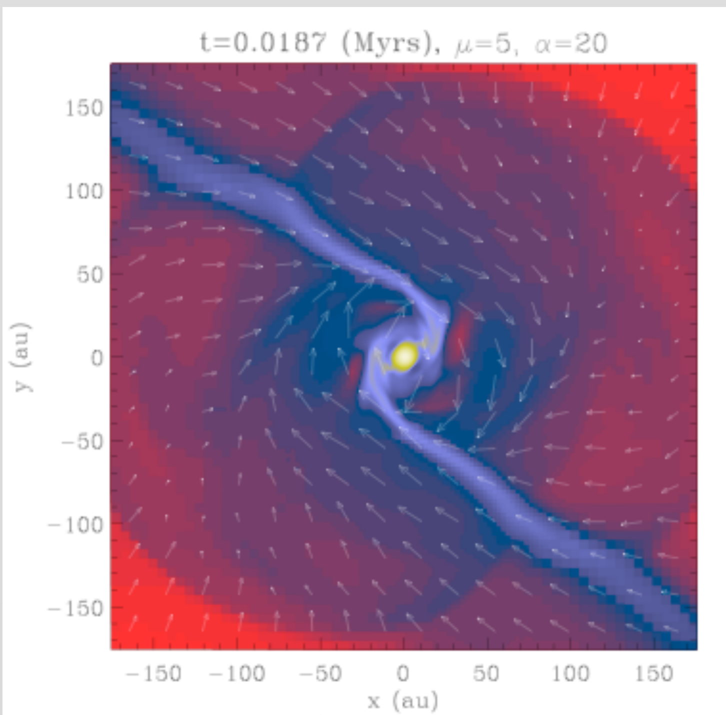


Model of disk evolution with photoevaporation (and a massive planet!) – *Alexander & Armitage '09*

Disk winds

Thermal winds (photoevaporation) – no torque on the remaining disk so affect evolution only via mass loss

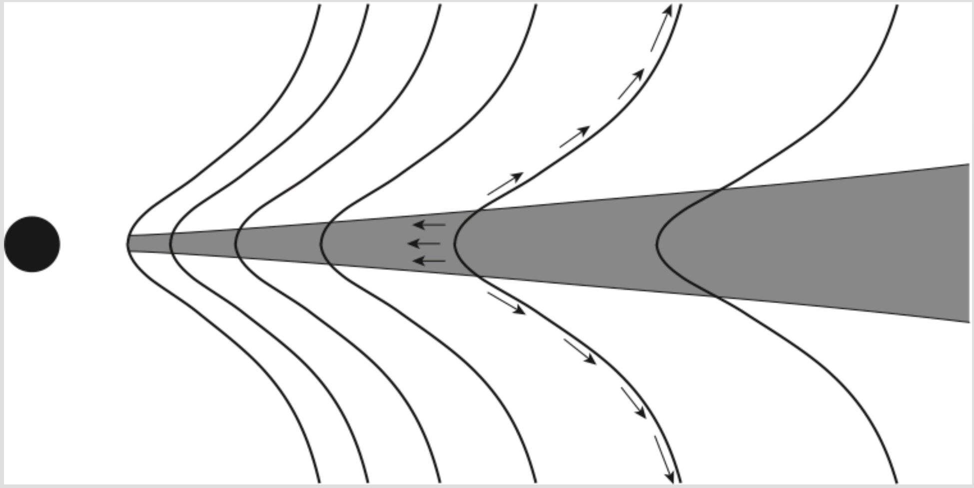
Magnetic outflows can lead to a torque



Star formation simulations suggest magnetic removal of angular momentum can be efficient (too efficient?!) during initial collapse

Expect disks to retain some net magnetic field after formation

Hennebelle & Ciardi '09



Disk, threaded by field B, with magnetic wind, density ρ(r,z), velocity v(r,z)

Flow rotates ~rigidly provided: $\rho v^2 \lesssim \frac{B^2}{8\pi}$

Out to “Alfven surface”, outflowing gas **gains** angular momentum, magnetic field exerts a torque on disk surface

At r_A, specific angular momentum: $l_A = r_A^2 \Omega_{\rm disk}$

If r_A >> radius where field line meets disk surface, a small mass outflow can remove all the angular momentum needed for accretion

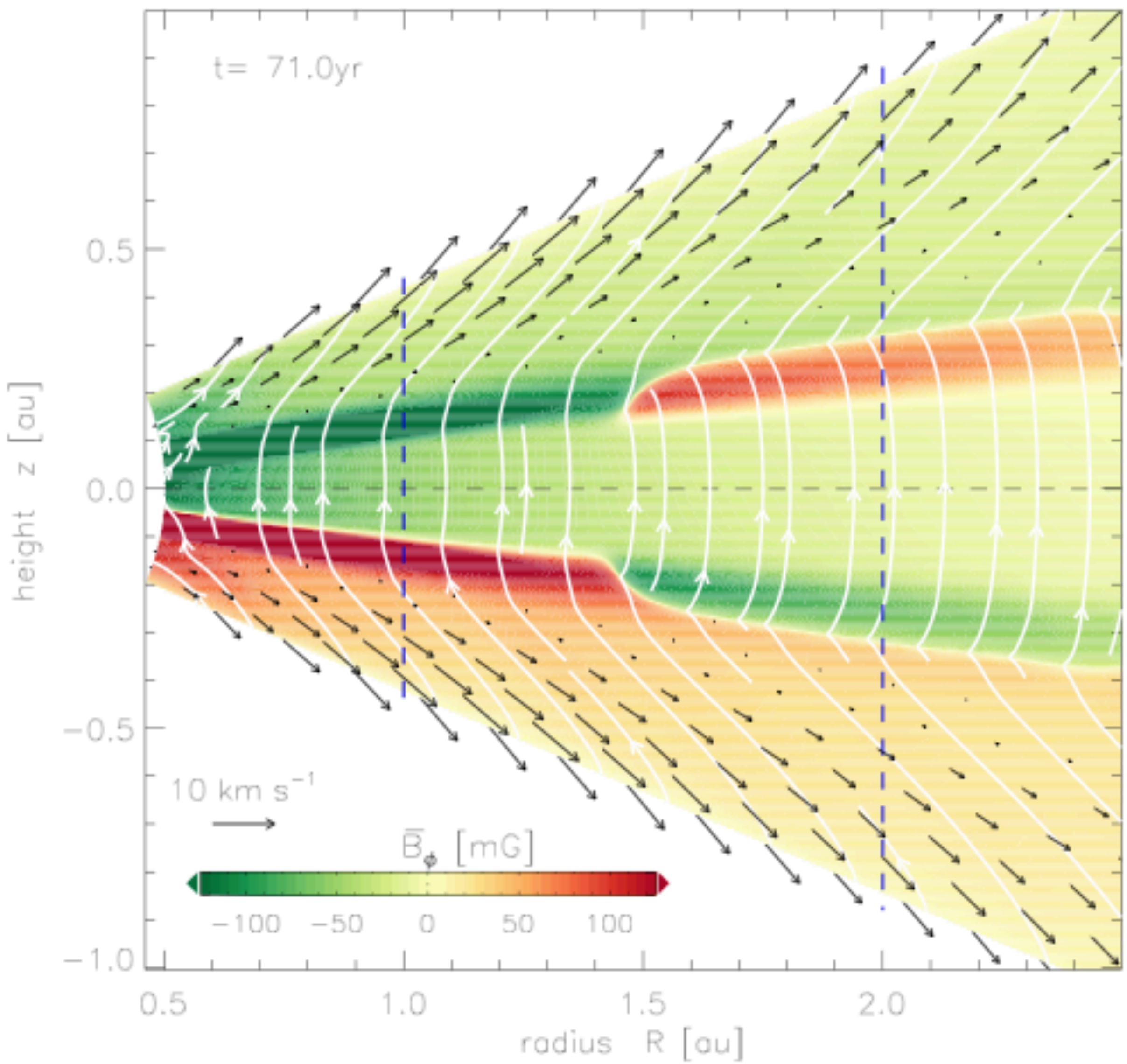


Figure 5. Field topology of our fiducial simulation at different evolution times. The azimuthal magnetic field (color) has been restricted to values $|B_\phi| < 125\,\mathrm{mG}$ for clarity; peak values are a few hundred mG. We also show projected magnetic field lines (white) and velocity vectors (black). Additional lines indicate the position, z_b, of the wind base (dot-dash), and the radial location of the profiles plotted in Figs. 6 and 7 (dashed lines).

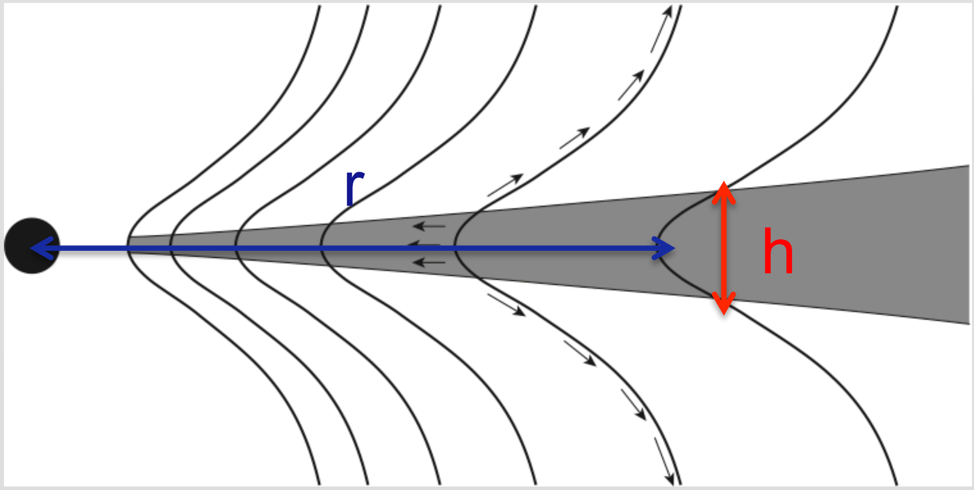
Gressel et al. '15: disk models with net field defined via poloidal pressure with respect to gas pressure:

$$\beta_z \equiv \frac{B_z^2/8\pi}{\rho c_s^2}$$

Here $\beta_z \sim 10^5$

Observational constraints on this scenario?

If net field is important, what determines how B_z evolves?



Standard answer: assume turbulence provides both an effective viscosity and an effective magnetic resistivity

Radial scale for accretion r >> h vertical scale for magnetic reconnection… expect poloidal flux ψ to diffuse radially faster than it is "dragged" inward *(Lubow et al. 1994)*

$$\frac{\partial \psi}{\partial t} + r v_{\mathrm{adv}} B_z + r v_{\mathrm{diff}} B_{rs} = 0$$

$v_{\mathrm{adv}} \rightarrow -\nu/r$

$v_{\mathrm{diff}} \rightarrow +\eta/h$

c.f. Guilet & Ogilvie '14

Standard thinking:

- magnetic winds may be strong during formation of the protostar / disk phase
- may occasionally prevent formation of a long-lived disk at all (“magnetic braking catastrophe”)
- most net flux subsequently escapes
- a *small* net flux plays a role in stimulating MRI later in disk lifetime
- disk dispersal is thermally driven

Testing theory

Conversely we are likely to learn more about the properties of accretion discs in astrophysics by observation and subsequent modeling than by pure theorizing.

Pringle, ARA&A, 1981