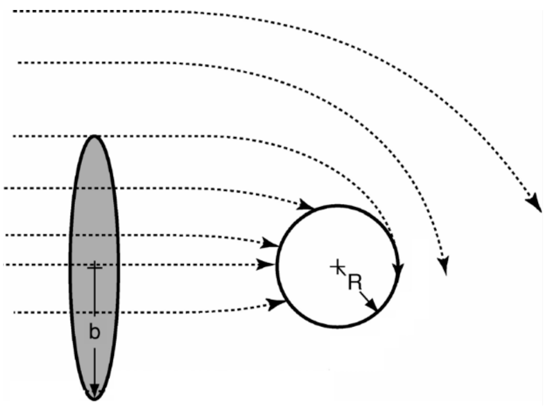
NBIA Summer School on Protoplanetary Disks and Planet Formation
Exercise session #2, Tuesday, August 4th 2015

1. Radii and masses measured for five hypothetical transiting planets are listed below. State the ranges in composition consistent with these measurements. In some cases, the ranges allowed by the error bars include some unlikely or unphysical compositions. Which cases are these, and why?

   (a) $R_p = 3 \pm 1\ R_\oplus$, $M_p = 3 \pm 1\ M_\oplus$
   (b) $R_p = 1 \pm 0.5\ R_\oplus$, $M_p = 3 \pm 1\ M_\oplus$
   (c) $R_p = 12 \pm 2\ R_\oplus$, $M_p = 300 \pm 100\ M_\oplus$
   (d) $R_p = 3 \pm 1\ R_\oplus$, $M_p = 30 \pm 10\ M_\oplus$
   (e) $R_p = 2 \pm 0.2\ R_\oplus$, $M_p = 10 \pm 1\ M_\oplus$

2. Derive the two-body gravitational accretion cross-section of a planetary embryo of radius R and mass M for small planetesimals whose velocity at ∞ relative to the embryo is v. (Hint: First determine the maximum unperturbed impact parameter of a planetesimal which collides with a planetary embryo. Note that the periapsis of this planetesimal's orbit is at a distance R from the planetary embryo's center. Use conservation of angular momentum and energy.)



3. Consider a few relatively large planetesimals in a swarm of much smaller bodies. Assume that the densities of all bodies are the same and that the velocity dispersion is comparable to the escape speed of the small bodies.

   (a) Show that the cross-sections (and the accretion rates) of the large planetesimals are proportional to the fourth power of their radii.

   (b) Use this result to demonstrate that the largest planetesimal doubles in mass the fastest and thus 'runs away' from the rest of the distribution of bodies.

4. Find the initial temperature profile of the Earth, assuming that it was homogeneous and it accreted so fast (or that large impacts buried the heat so deep) that radiation losses were negligible. Assume zero conductivity, so the heat is not redistributed.