“NBI summer School on Protoplanetary Disks and Planet Formation”

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Radial drift prevents particle growth beyond cm in the inner disc and mm in the outer disc. This is fully consistent with observations of protoplanetary discs. Fragmentation sets the limit of the growth within 10 AU. Turbulent density fluctuations lead to destruction of pre-planetesimals less than 1 to 10 km in radius in a dead zone and less than 1000 km in a region with active MRI turbulence.
Dust and ice particles in a protoplanetary disc coagulate to cm-sized pebbles and rocks.

Pebbles and rocks *sediment* to the mid-plane of the disc.

Further growth frustrated by high-speed collisions (>1–10 m/s) which lead to erosion and bouncing.

Layer *not* dense enough for gravitational instability.

⇒ Need some way for particle layer to get dense enough to initiate gravitational collapse.
How turbulence aids planetesimal formation

1. *Passive concentration* as particles pile up in long-lived pressure bumps and vortices excited in the turbulent gas flow.

2. *Active concentration* as particles make dense filaments and clumps to protect themselves from gas friction.
Particle concentration

Three ways to concentrate particles: (Johansen et al., 2014, arXiv:1402.1344)

- Between small-scale low-pressure eddies
  (Squires & Eaton, 1991; Fessler et al., 1994; Cuzzi et al., 2001, 2008; Pan et al., 2011)

- In pressure bumps and vortices
  (Whipple, 1972; Barge & Sommeria, 1995; Klahr & Bodenheimer, 2003; Johansen et al., 2009a)

- By streaming instabilities
  (Youdin & Goodman, 2005; Johansen & Youdin, 2007; Johansen et al., 2009b; Bai & Stone, 2010a,b,c)
Roche density

- Protoplanetary discs are gravitationally unstable if the parameter $Q$ is smaller than unity (Safronov, 1960; Toomre, 1964)

$$Q = \frac{c_s \Omega}{\pi G \Sigma} < 1$$

- The column density can be written in terms of the scale height and the mid-plane density

$$\Sigma \approx H \rho_0$$

- Turn the gravitational instability criterion into a criterion for the density

$$\rho_0 > \rho_R \approx \frac{\Omega^2}{G} \approx \frac{M_*}{r^3}$$

- The Roche density is $\rho_R \approx 6 \times 10^{-7}$ g/cm$^3$ at 1 AU, the mid-plane gas density is $\rho_0 \approx 1.4 \times 10^{-9}$ g/cm$^3$
Pressure bumps

Particles seek the point of highest pressure

⇒ Particles get trapped in pressure bumps

Achieve high enough local density for gravitational instability and planetesimal formation

(Figure from Whipple, 1972)
Pressure bumps in MRI turbulence

- Gas density shows the expected vertical stratification
- Gas column density shows presence of large-scale pressure fluctuations with variation only in the radial direction
- Pressure fluctuations of order 10%
Stress variation and pressure bumps

- Mass accretion rate and column density:
  \[ \dot{M} = 3\pi \Sigma \nu_t \implies \Sigma = \frac{\dot{M}}{3\pi \nu_t} \]
  \[ \nu_t = \alpha c_s H \]

  \[ \implies \text{Constant } \dot{M} \text{ and constant } \alpha \text{ yield } \Sigma \propto r^{-1} \]
  \[ \implies \text{Radial variation in } \alpha \text{ gives pressure bumps} \]
Particle trapping

- Strong correlation between high gas density and high particle density
  
  (Johansen, Klahr, & Henning, 2006)
Forming planetesimals in pressure bumps

(Johansen et al., 2011)
What sets the scale of pressure bumps?

- Pressure bumps reported in a number of MRI papers
  (Fromang & Stone, 2009; Davis et al., 2010; Simon et al. 2012)

- Pressure bumps cascade to the largest scales of local box simulations, but may stop at 5–10 scale heights  (Johansen et al., 2009; Dittrich, Klahr, & Johansen, 2013)

- More global simulations needed!
  (e.g. Fromang & Nelson, 2005; Lyra et al., 2008; Uribe et al., 2012)
Turbulence is a double-edged sword

- Turbulence can excite long-lived pressure bumps which trap particles.
- Turbulence excites high relative particle speeds between particles as well as between planetesimals.

(Johansen et al., 2014)
Cosmic rays do not penetrate to the mid-plane of the disc, so the ionisation fraction in the mid-plane is too low to sustain MRI

⇒ Accretion in active surface layers or by disc winds

⇒ Weak turbulence and low collision speeds in the dead zone
Disc wind model

- Mid-plane is decoupled from the magnetic field by ohmic diffusion and surface layers by ambipolar diffusion \((\text{Bai} \& \text{Stone}, 2013)\)
- Threading magnetic field enters a wind configuration \((\text{Blandford} \& \text{Payne}, 1982)\)
- Angular momentum transported vertically away from the mid-plane
- Thin but rapid accretion flow where azimuthal magnetic field changes sign about 3 \(H\) from the mid-plane \((\text{Gressel et al.}, 2015)\)
- Mid-plane is completely laminar with no turbulent motion

\((\text{Gressel et al.}, 2015)\)
Streaming instability

- Gas orbits slightly slower than Keplerian
- Particles lose angular momentum due to headwind
- Particle clumps locally reduce headwind and are fed by isolated particles

\[ v_{\text{Kep}}(1-\eta) \]

⇒ Youdin & Goodman (2005): “Streaming instability”
The streaming feeds off the velocity difference between gas and particles

- Particles move faster than the gas and drift inwards, pushing the gas outwards
- In total there are 8 linear modes (density waves modified by drag)
- One of the modes is unstable (Youdin & Goodman, 2005; Jacquet, Balbus, & Latter, 2011)
- Requires both radial and vertical displacements
- Fastest growth for large particles and local dust-to-gas ratio above unity
Clumping

Linear and non-linear evolution of radial drift flow of meter-sized boulders:

⇒ Strong clumping in non-linear state of the streaming instability

(Youdin & Johansen, 2007; Johansen & Youdin, 2007; done with Pencil Code [pencil-code.googlecode.com])
Why clump?
Sand dunes

- **Barchan** sand dunes form when sparse sand moves over bedrock and wind has a dominant direction.
- Experiments show that larger sand dunes move slower than smaller sand dunes.
  \[ \Rightarrow \] Small sand dunes melt together to larger and larger sand dunes.
- Similar dynamics to what drives formation of dense filaments of particles in protoplanetary discs...

\[ \text{(Groh et al. 2008)} \]
Bai & Stone (2010a) presented high-resolution convergence tests of non-stratified 2-D simulations.

- Maximum particle density increases with resolution, converging at $1024^2$ or $2048^2$.
- Confirmation of Pencil Code results with independent code (Athena).
Stratified simulations

- Johansen, Youdin, & Mac Low (2009) presented first stratified simulations of streaming instabilities
- Particles sizes $\Omega \tau_f = 0.1, 0.2, 0.3, 0.4$ (3–12 cm at 5 AU, 1–4 cm at 10 AU)
- Dust-to-gas ratio no longer a free parameter, but column density $Z = \Sigma_p/\Sigma_g$ is

Copenhagen 2015 (Lecture 2)
Convergence tests – stratified

- Particle density up to 10,000 times local gas density
- Criterion for gravitational collapse: \( \rho_p \gtrsim \Omega^2 / G \sim 100 \rho_g \)
- Maximum density increases with increasing resolution

(Johansen, Mac Low, Lacerda, & Bizzarro, 2015)
Scale-by-scale convergence

$\Omega \tau_f = 0.3$, $Z = 0.02$, $\varepsilon = 0.3$

- Plot shows maximum density over a given scale (averaged over time)
- Points for $64^3$ and $128^3$ almost on top of each other
  $\Rightarrow$ Streaming instability overdensities converge scale-by-scale
- Increasing the resolution increases the maximum density because density at grid-cell level gains structure at increased resolution

(Johansen, Youdin, & Lithwick, 2012)
Sedimentation of 10 cm rocks

- Streaming instability relies on the ability of solid particles to accelerate the gas towards the Keplerian speed

⇒ Efficiency increases with the metallicity of the gas

- Solar metallicity: turbulence caused by the streaming instability puffs up the mid-plane layer, but no clumping

- Dense filaments form spontaneously above $Z \approx 0.015$
Metallicity matters

Copenhagen 2015 (Lecture 2)  Planetesimal formation  26 / 38
Why is metallicity important?

- Gas orbits slightly slower than Keplerian
- Particles lose angular momentum due to headwind
- Particle clumps locally reduce headwind and are fed by isolated particles

\[ \nu_{\text{Kep}}(1-\eta) \]

- Clumping relies on particles being able to accelerate the gas towards Keplerian speed
Metallicity of exoplanet host stars

- First planet around solar-type star discovered in 1995 ([Mayor & Queloz, 1995])
- Today several thousand exoplanets known
- Exoplanet probability increases sharply with metallicity of host star

⇒ Expected due to efficiency of core accretion and pebble accretion ([Ida & Lin, 2004; Mordasini et al., 2009; Lambrechts & Johansen, 2014])
⇒ ... but planetesimal formation may play equally big part ([Johansen et al., 2009; Bai & Stone, 2010b])
**Dependence on headwind parameter**

- *Bai & Stone* (2010c) searched for the critical metallicity for clumping as a function of the headwind parameter $\Pi = \Delta \nu / c_s$

  $\Rightarrow$ Slow headwind (close to star or in pressure bumps) gives lower threshold

  $\Rightarrow$ Careful when using pressure bumps to stop radial drift – streaming instability leads to strong clumping when headwind is slow
Gravitational collapse

- Particle concentration by streaming instabilities reach at least 10,000 times the gas density

- Filaments fragment to bound pebble clumps, with contracted radii 25-200 km
  (Johansen, Mac Low, Lacerda, & Bizzarro, 2015)

⇒ Initial Mass Function of planetesimals
Planetaryesimal birth sizes

\[ N_\geq = K M^{-0.8} \exp\left[-\left(M/M_{\text{exp}}\right)^{4/3}\right] \]
\[ N_\geq = K M^{-0.6} \exp\left[-\left(M/M_{\text{exp}}\right)^{4/3}\right] \]
\[ N_\geq = K M^{-0.4} \exp\left[-\left(M/M_{\text{exp}}\right)^{4/3}\right] \]

- Cumulative size distribution is less affected by noise than the differential size distribution
- Well-fitted by an exponentially tapered power law
- Most of the mass resides around the knee
- Small planetesimals dominate in number
- Can be compared to the asteroid belt (next lecture)

(Johansen et al., 2015)
Concentrating chondrules

- Typical particle sizes considered for the streaming instability are of size 10 cm (when scaled to the asteroid belt)
- Meteorites contain up to 80% mass in *chondrules* of sizes 0.1–1 mm (e.g. Krot *et al.*, 2009)

⇒ Smaller particles can be concentrated at higher metallicity (Carrera, Johansen, & Davies, 2015)

- Metallicity increase by photoevaporation or drifting particles? (Alexander *et al.*, 2006; Alexander & Armitage, 2007)
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Achieving the conditions for the streaming instability

Possible to form pebble sizes needed for streaming instability outside of the ice line (Drazkowska & Dullemond, 2014)

But bouncing stalls silicate particles at mm sizes inside of the ice line

About half of the solid mass remains in tiny grains unable to participate in the streaming instability

Photoevaporation can increase the dust-to-gas ratio to close to $Z \sim 0.1$ already before inner hole is formed (Gorti et al., 2015)

Need global disc wind models including dust (Armitage et al., 2013)
Stirring of the mid-plane

- Baroclinic instability is similar to radial convection (Klahr & Bodenheimer, 2003)
- Produces vortices after extended growth phase
- Particles are trapped in the vortices
- Back-reaction friction force nevertheless destroys the vortices (Rättig et al., 2015)
- Vertical shear instability could also stir the mid-plane (Nelson et al., 2013)
Planetesimal formation by particle concentration and gravitational collapse

1. Dust growth by coagulation to a few cm

2. Spontaneous clumping through streaming instabilities and in pressure bumps and vortices

3. Gravitational collapse to form 100–1000 km radius planetesimals
Summary of planetesimal formation

- Particles can be concentrated in the gas to reach the Roche density
- Concentration mechanisms include pressure bumps, vortices and streaming instability
- The streaming instability leads to very strong particle concentration, to more than a factor 10,000 times the gas density
- Planetesimals form with a wide range of sizes – from up to Ceres size, down to 25 km at the highest resolution reached
- The particle sizes and metallicities needed for the streaming instability can be achieved outside of the ice line
- Other sources of turbulence (baroclinic instability and vertical shear instability) likely relevant but still under exploration