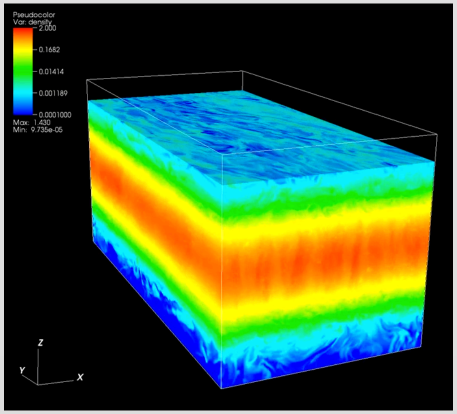
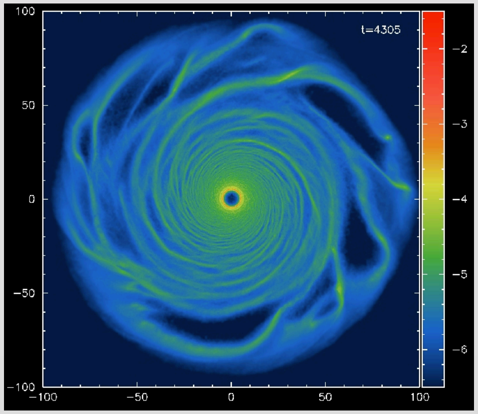
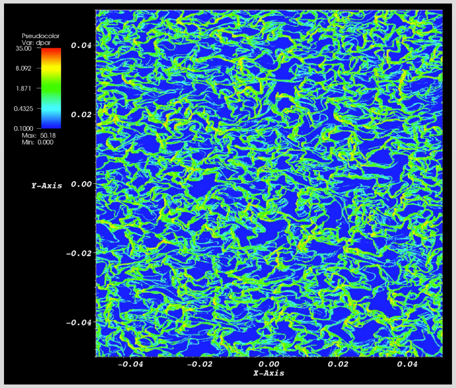
Protoplanetary disk structure



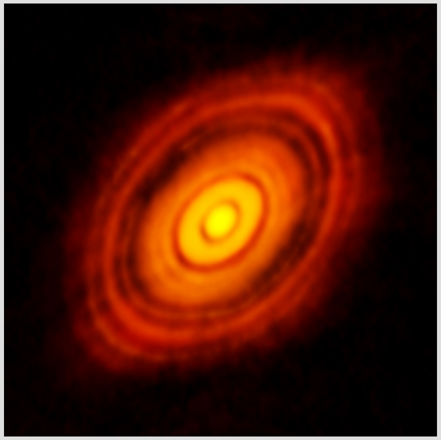




Phil Armitage
Colorado

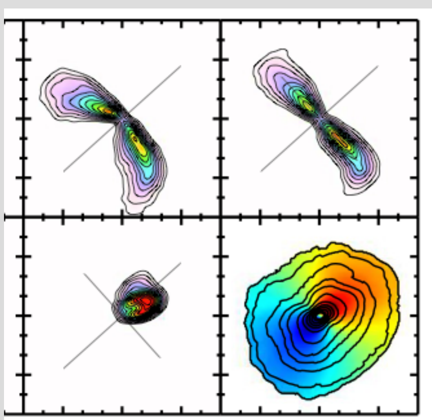
Motivation

What can we directly observe?



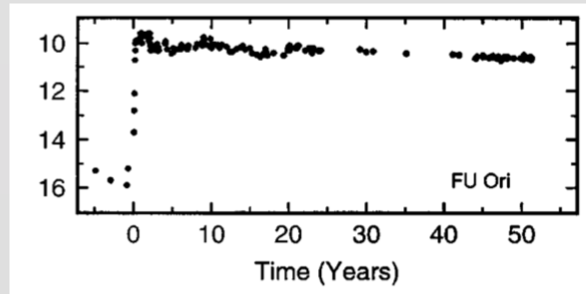
Dust
µm-mm

HL Tau, ALMA



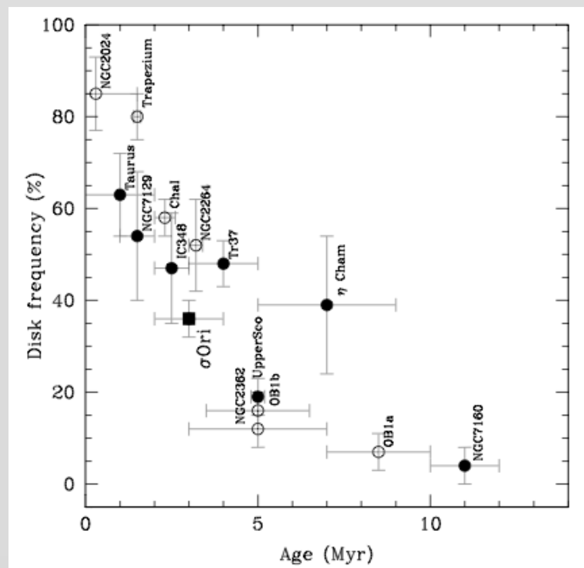
Gas – emission & kinematics
(CO + other molecules, H_2 hard)

Rosenfeld et al. '13



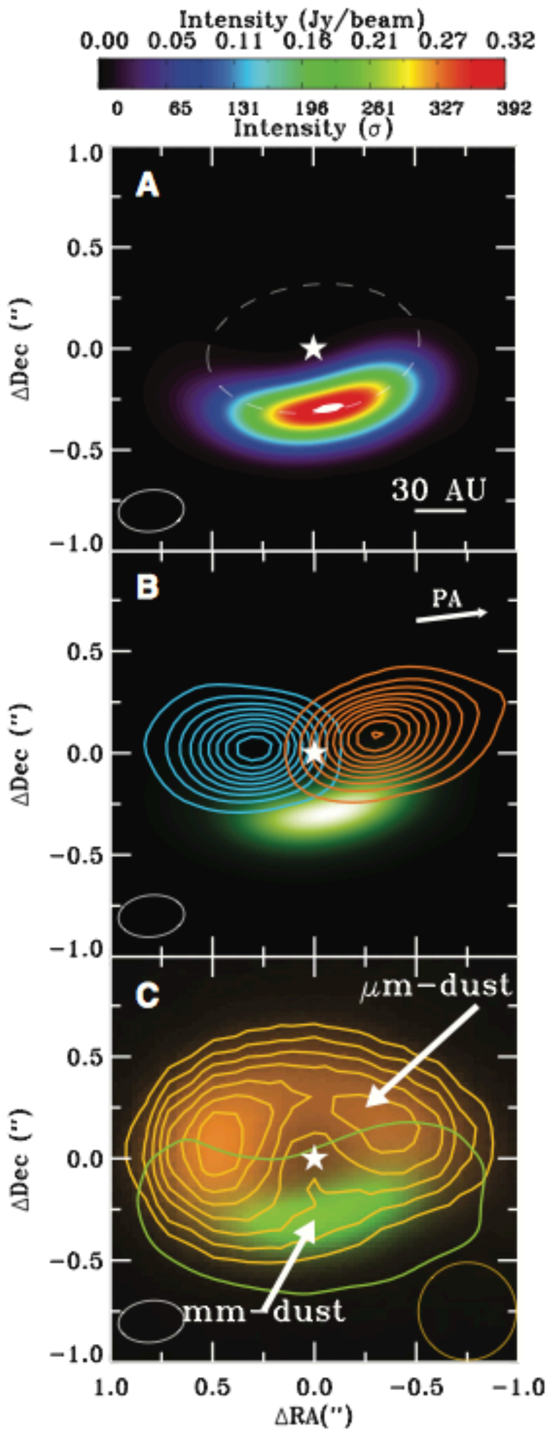
Limited direct time dependence

Hartmann & Kenyon '96



Evolution from population studies

Hernandez et al. '07



Motivation

What do we aim to learn?

- Why and how disk evolve
- Physical conditions for planet formation (density, temperature, level of turbulence…)
- Interaction and growth of solids within the gas
- Origin of structure within disks
- Planet-disk interactions

IRS 48, van der Marel et al. '13

Disk structure

Basic principles: disks are long-lived and (usually) well approximated by -

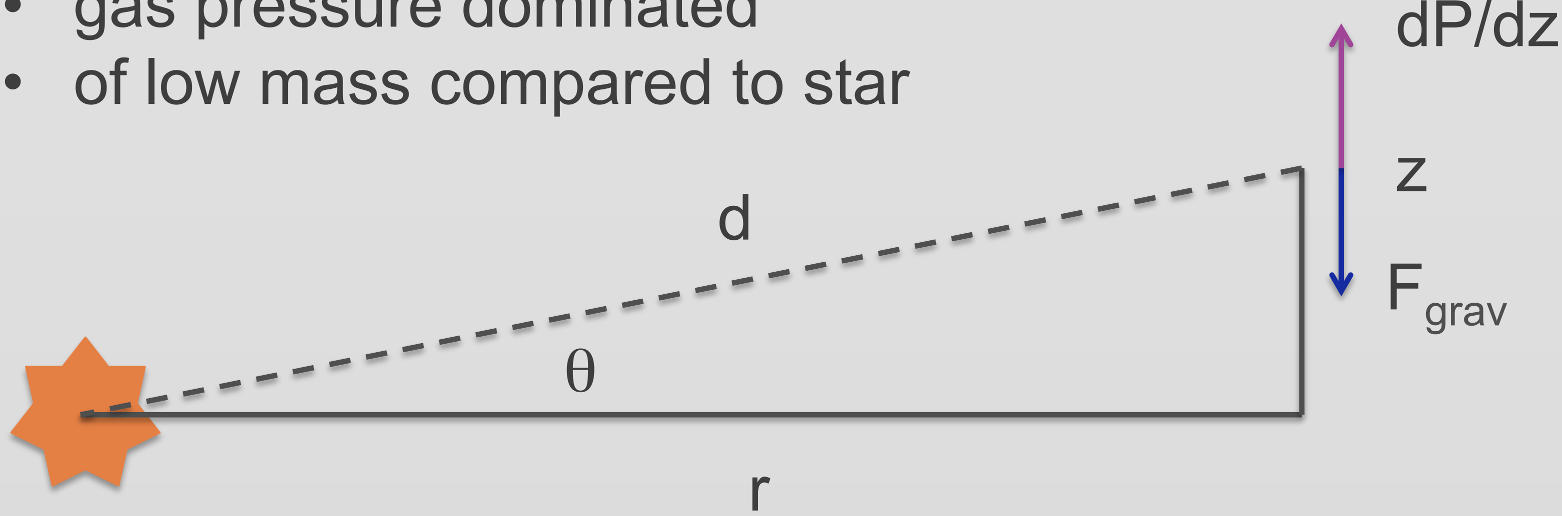
- hydrostatic equilibrium
- thermal equilibrium
- local ionization equilibrium

In many cases chemistry is *not* necessarily in equilibrium

Disk structure

Consider a disk:

- heated from star (~vertically isothermal)
- gas pressure dominated
- of low mass compared to star



In hydrostatic equilibrium: $$\frac{\mathrm{d}P}{\mathrm{d}z} = -\rho g_z = -\rho \frac{GM_*}{d^2} \sin\theta$$

Write P = ρc_s^2, for z << r: $$\frac{GM_*}{d^2} \sin\theta \simeq \frac{GM_*}{r^3} z = \Omega_K^2 z$$

$$c_s^2 \frac{\mathrm{d}\rho}{\mathrm{d}z} = -\Omega_K^2 \rho z \quad \Rightarrow \quad \rho(z) = \rho_0 \exp[-z^2/2h^2]$$

Define **disk scale height** $h \equiv \frac{c_s}{\Omega_K}$

Vertical density profile is **gaussian**

Disk **surface density** $\Sigma = \int \rho \mathrm{d}z \qquad \rho_0 = \frac{1}{\sqrt{2\pi}} \frac{\Sigma}{h}$

Order of magnitude: at 1 AU, $\Sigma \sim 10^3$ g cm^{-2}, $h / r \sim 0.03$

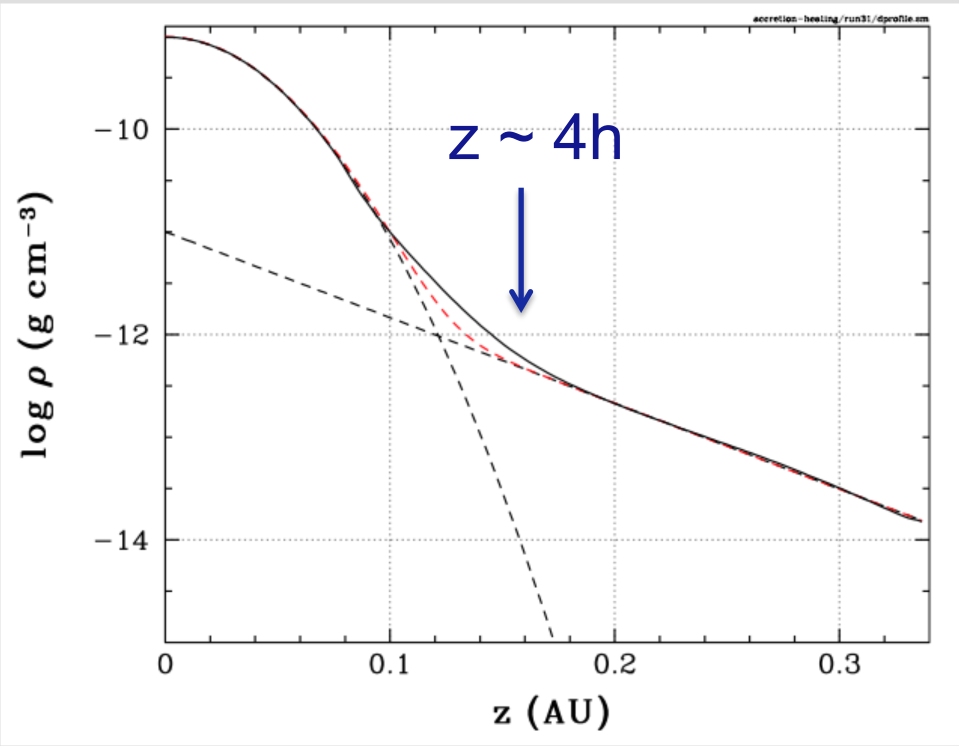
$\rho_0 = 10^{-9}$ g cm^{-3}
$c_s = 10^5$ cm s^{-1} (1 km s^{-1})

$\Rightarrow$

mean free path $\lambda \sim$ cm

What could alter vertical density profile?

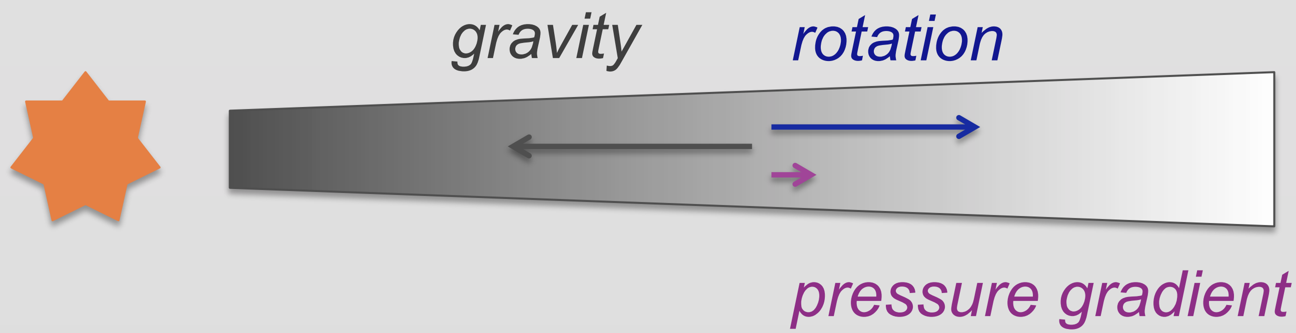
- self-gravity, condition is roughly $$\frac{M_{\mathrm{disk}}}{M_*} \gtrsim \frac{1}{2}\left(\frac{h}{r}\right)$$
- magnetic pressure $B^2 / 8\pi$



Hirose & Turner '11 find magnetic support above about z ~ 4h, leading to an exponential atmosphere (1 AU, flux-limited diffusion)

Depends on MHD processes in disk, in some cases **B**(z) may have larger effect

Velocity structure



For a slowly evolving disk, in axisymmetry:

$$\cancel{\frac{\partial v}{\partial t}} + (\mathbf{v} \cdot \nabla)\,\mathbf{v} = -\frac{1}{\rho}\nabla P - \nabla \Phi$$

$$\Rightarrow \quad \frac{v_\phi^2}{r} = \frac{GM_*}{r^2} + \frac{1}{\rho}\frac{\mathrm{d}P}{\mathrm{d}r}$$

$P = \rho c_s^2$, and $c_s \sim h$, so 2nd term $O(h/r)^2$

A specific example: $\Sigma \sim r^{-1}$, $T_c \sim r^{-1/2}$

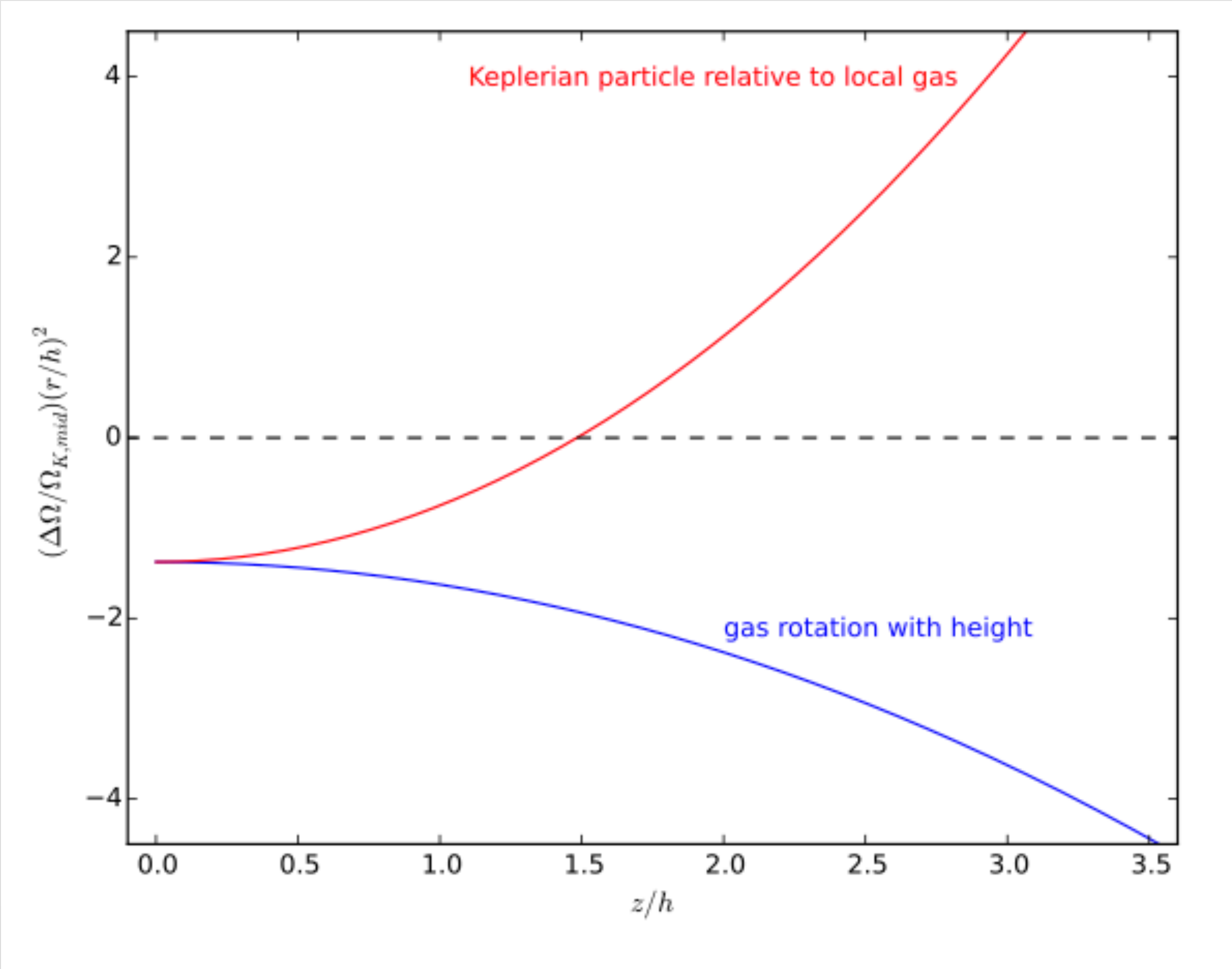
$$v_\phi = v_K \left[1 - \frac{11}{4} \left(\frac{h}{r} \right)^2 \right]^{1/2}$$

At 1 AU, if (h/r) = 0.03:

- gas is sub-Keplerian by ~0.25%
- $|v_\phi - v_K| \sim 70 \text{ ms}^{-1}$

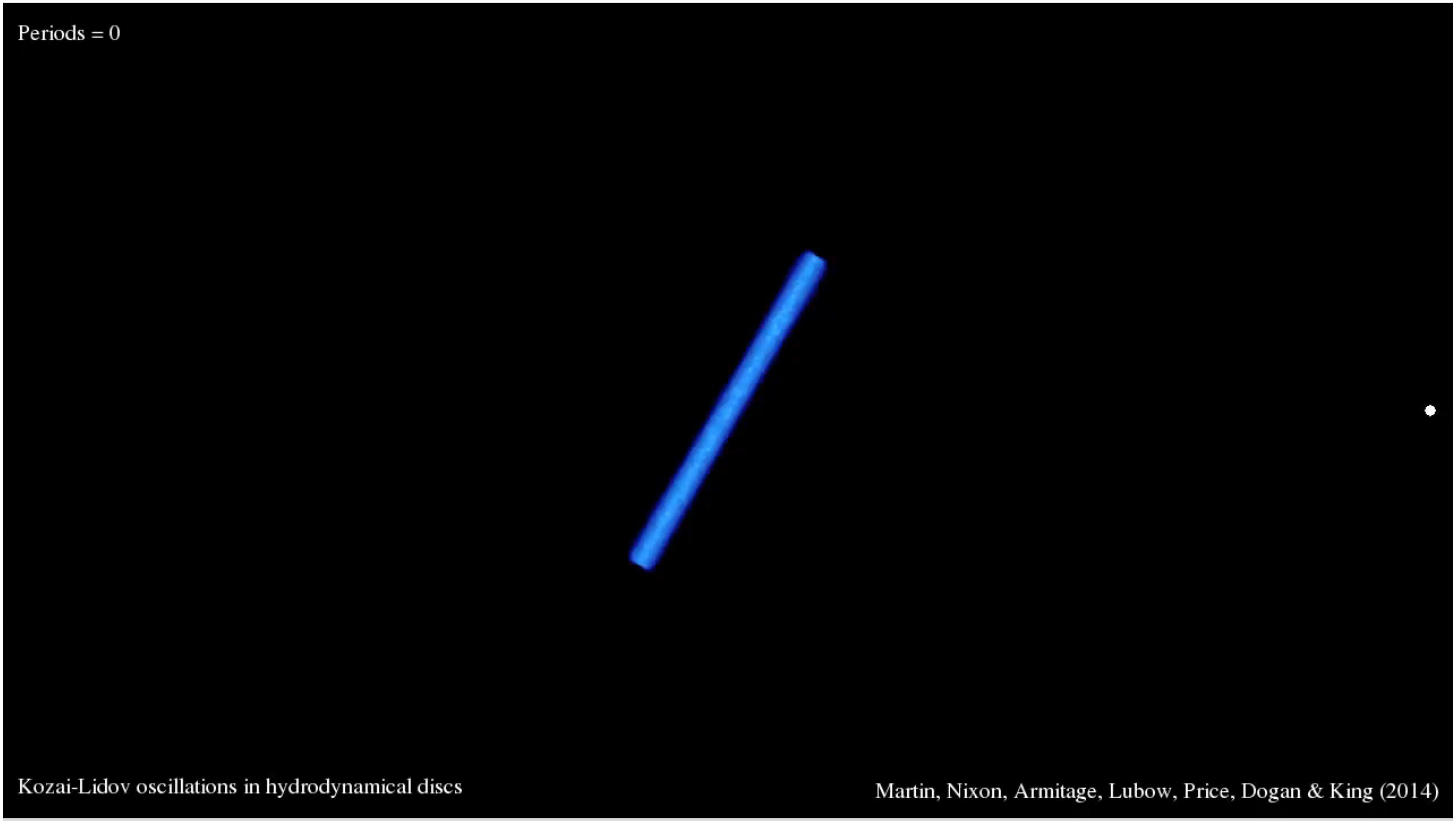
Non-self-gravitating disks can be treated as Keplerian for all practical (**gas** dynamical) purposes

Small deviation has outsized importance for **particle** dynamics!



At same order $(h/r)^2$ there is **vertical** shear in the equilibrium velocity field, even if disk is vertically isothermal

- see *Takeuchi & Lin '02* for calculation
- *Rosenfeld et al. '13* – effects at this order are potentially observable in molecular line kinematics



Disks in or around binaries may be eccentric and / or warped, are there *small* warps or eccentric gas flows in “normal” disks?

Temperature

Stellar irradiation *vs* heating by accretion

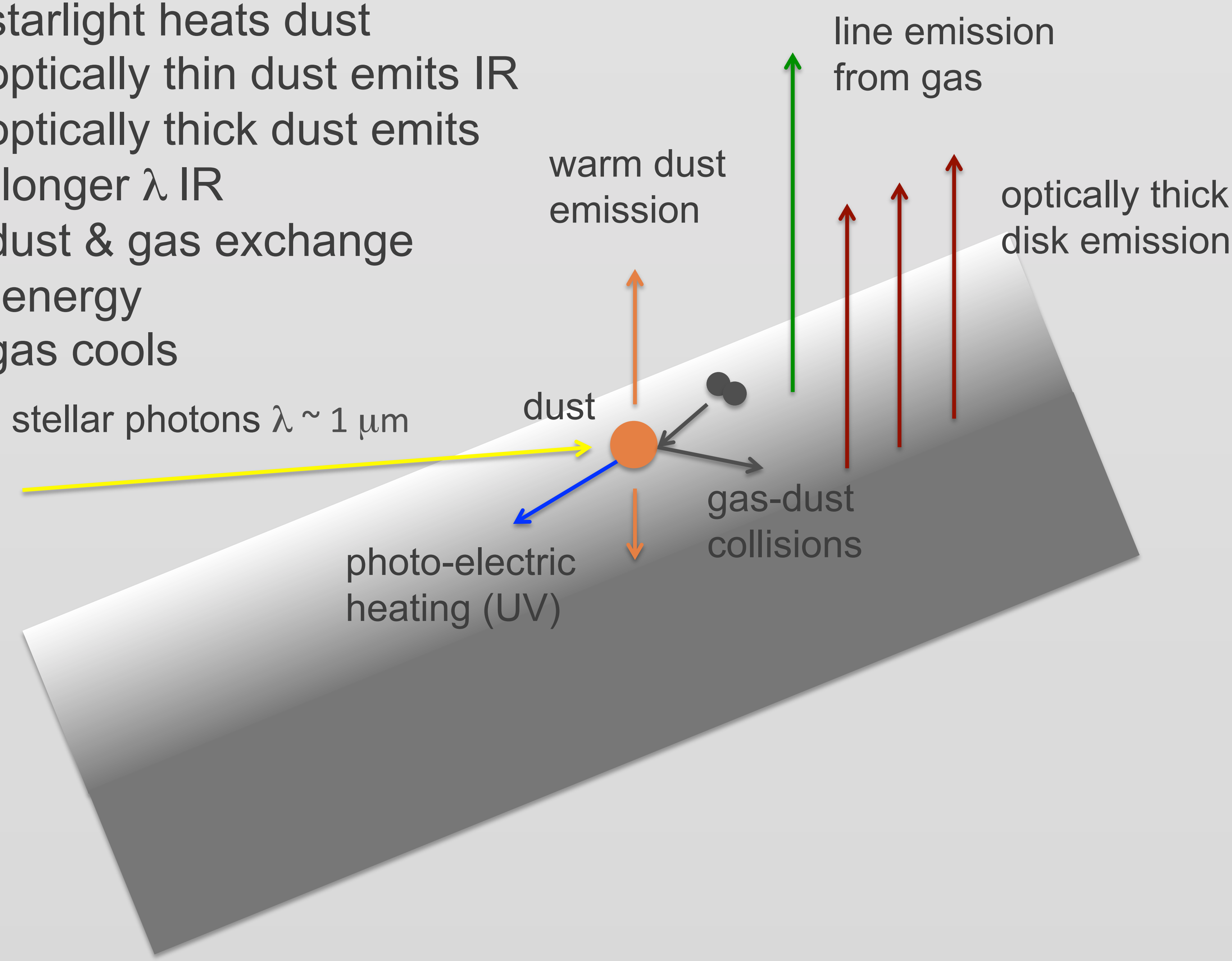
$$L_{\rm irr} = f L_* \qquad L_{\rm acc} = \frac{1}{2}\frac{G M_* \dot{M}}{R_{\rm in}}$$

Globally, accretion dominates if:

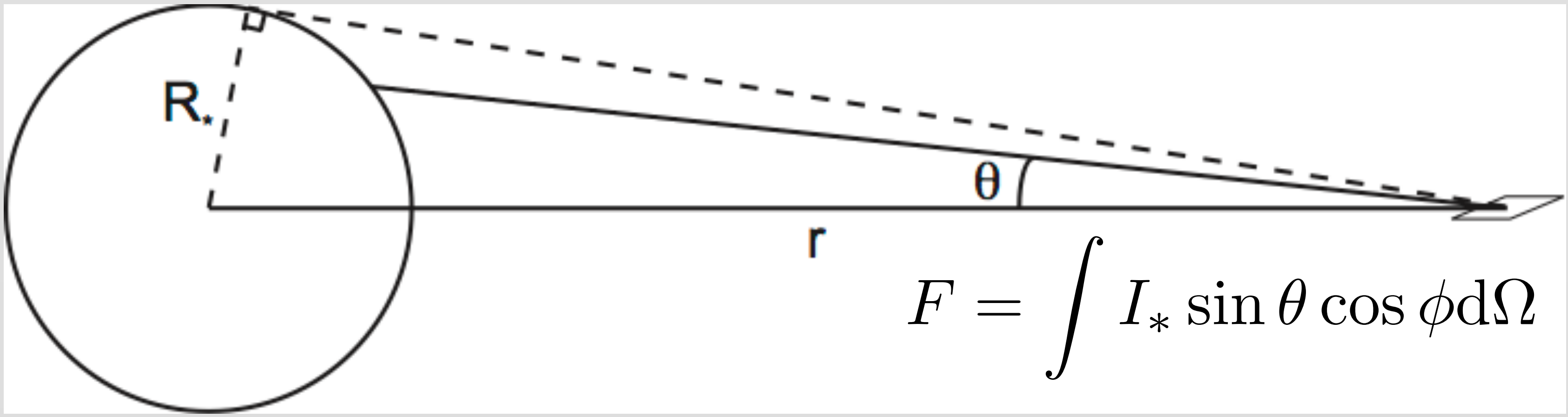
$$\dot{M} \gtrsim 2 \times 10^{-7} \left(\frac{L_*/M_*}{L_\odot/M_\odot} \right) \left(\frac{R_{\rm in}}{10\ R_\odot} \right) \left(\frac{f}{0.25} \right)\ M_\odot\ {\rm yr}^{-1}$$

Both heating sources can matter – depends on accretion rate, stellar mass, radius…

- starlight heats dust
- optically thin dust emits IR
- optically thick dust emits longer λ IR
- dust & gas exchange energy
- gas cools



Simple limit: razor-thin, optically thick disk, intercepts star light and re-emits locally as blackbody emission



$$F = \int I_* \sin\theta \cos\phi \, \mathrm{d}\Omega$$

Evaluates exactly to:

$$\left(\frac{T_{\rm disk}}{T_*}\right)^4 = \frac{1}{\pi}\left[\sin^{-1}\left(\frac{R_*}{r}\right) - \left(\frac{R_*}{r}\right)\sqrt{1-\left(\frac{R_*}{r}\right)^2}\right]$$

Asymptotically ($r \gg R_*$) $T_{\rm disk} \propto r^{-3/4}$

$$\frac{h}{r} \propto \frac{T^{1/2}}{v_K} \propto r^{1/8}$$

…disks usually flare to large distances

Additional physics for **dust** emission

1) Disk flares, intercepts more flux at large r than flat disk
(Kenyon & Hartmann 1987)

2) Surface dust radiates inefficiently (size smaller than the wavelength of re-emitted IR radiation)

warm optically thin emission from surface dust layer

$$T_{\mathrm{s}} = \frac{1}{\epsilon^{1/4}} \left(\frac{R_*}{2r} \right)^{1/2} T_*$$

Approximate analytic models:

$$T_\mathrm{i} \approx 150 \left(\frac{r}{1\ \mathrm{AU}}\right)^{-3/7}\ \mathrm{K}$$

50% of disk L in blackbody interior emission

$$T_\mathrm{s} \approx 550 \left(\frac{r}{1\ \mathrm{AU}}\right)^{-2/5}\ \mathrm{K}$$

50% in emission from a warmer surface dust layer

Chiang & Goldreich '97 – $\Sigma = 10^3\ (r / \mathrm{AU})^{-3/2}\ \mathrm{g\ cm^{-2}}$, $M = 0.5\ M_{Sun}$, $T = 4000$ K, $R_* = 2.5\ R_{Sun}$

Garaud & Lin '07 for version including accretion heating

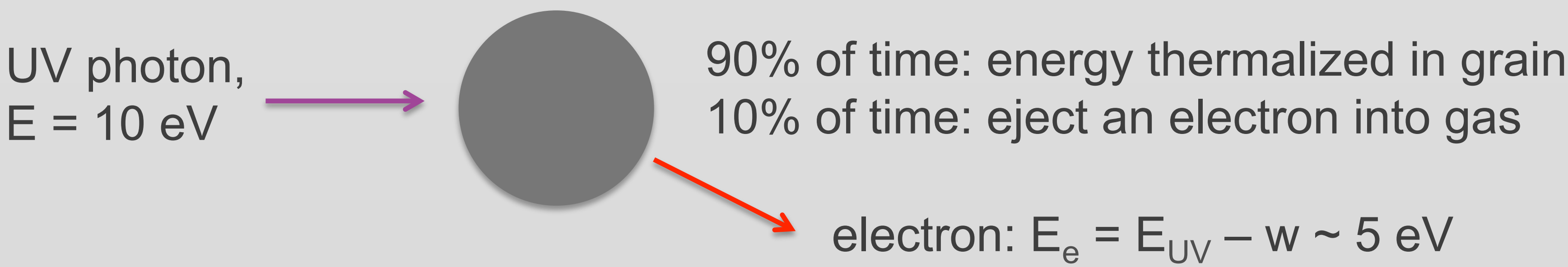
Passive disks have $T \sim r^{-0.5}$, flare to large r, any contribution from accretion heating greater at small r

What about the **gas** temperature?

Heating

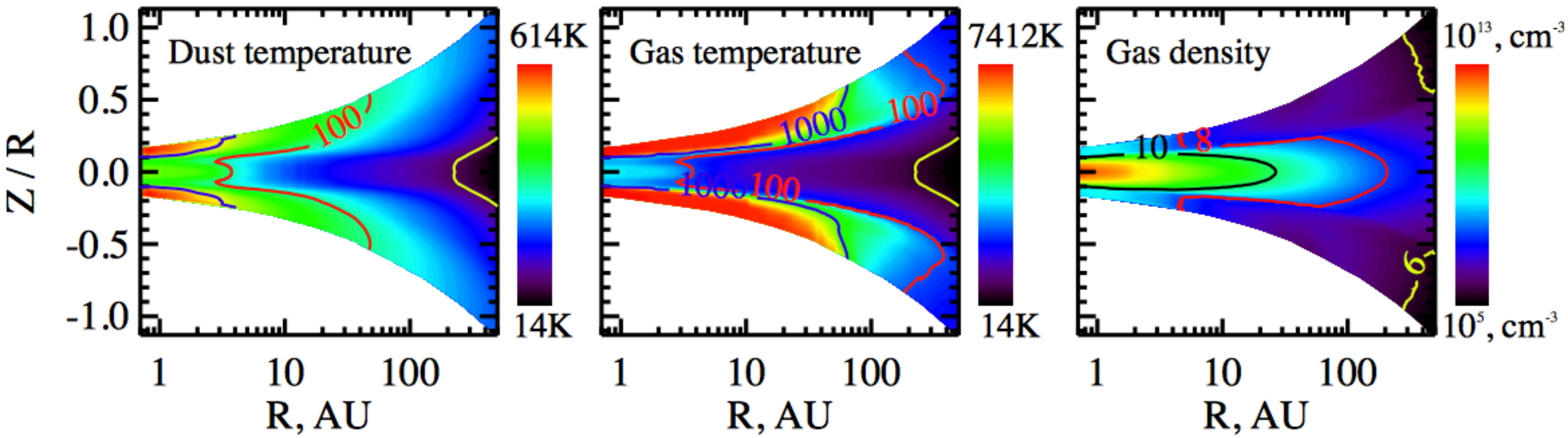
- collisions between molecules and dust particles
  - efficient equilibration at high density: $T_{gas} = T_{dust}$

- *photoelectric* heating near surface



~5% of incident UV flux goes into gas heating

Cooling – rotational transitions (e.g. CO), atomic fine structure lines



Henning & Semenov '13, from Akimkin et al. '13

General structure:

- optically thick, vertically isothermal structure near disk mid-plane
- warmer dust at surface
- still warmer molecular layer of gas

Need numerical codes to compute gas + dust models

Ionization

Ionization fraction $x_e = n_e / n_H$ affects:

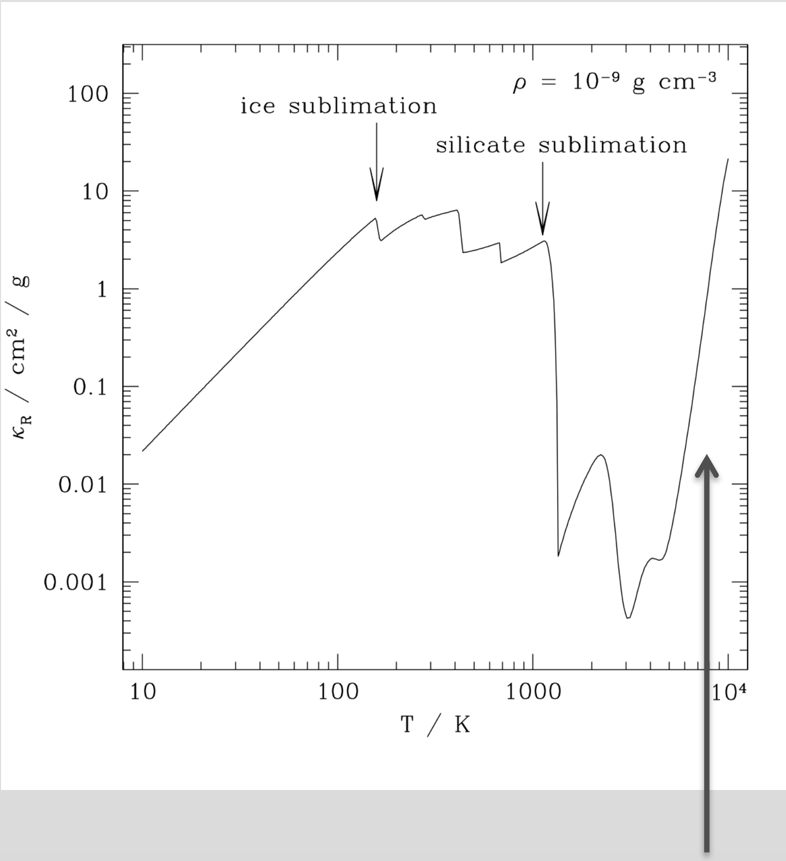
- chemistry
- coupling of gas to magnetic fields

Thermal ionization

Gas becomes fully ionized only at $T \sim 10^4$ K

These temperatures attained only in very high accretion rate states

Change in *opacity* when H is ionized is important for episodic accretion models

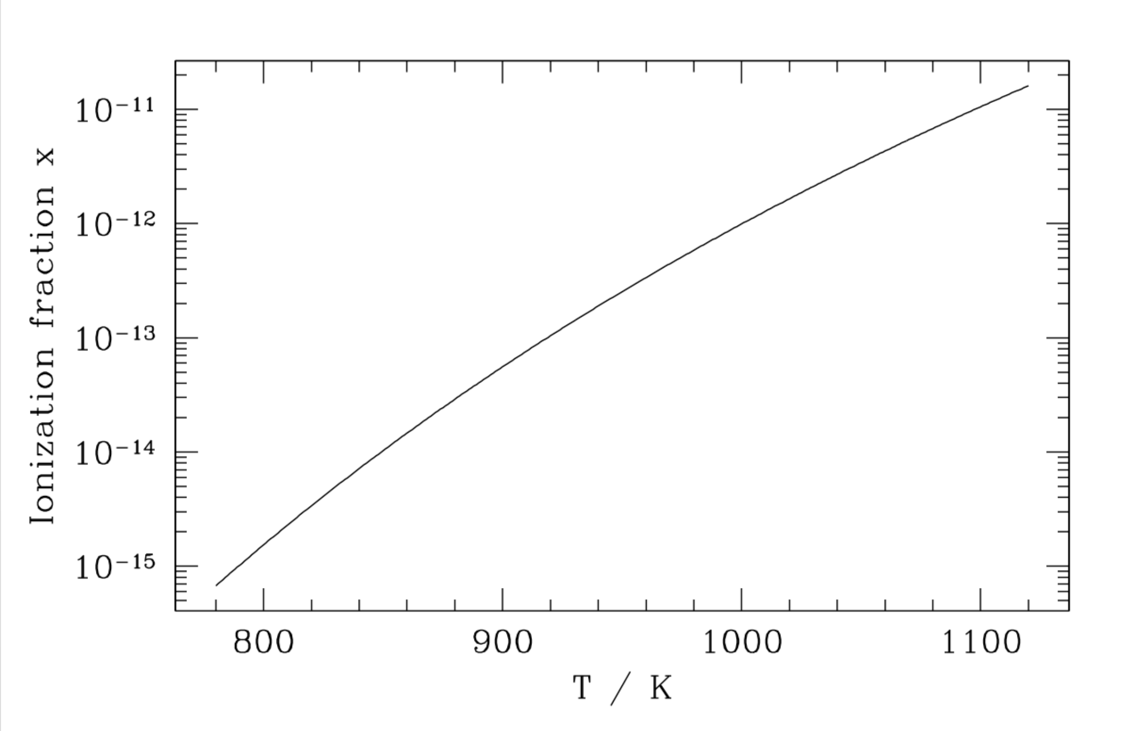


ionization of hydrogen
(opacity from *Semenov et al. '03)*

Thermal ionization

Ionization of the alkali metals occurs at $T \sim 10^3$ K

Yields an electron fraction $x_e \sim 10^{-12}$



Very low ionization degree, **but** comparable to the ionization needed to couple magnetic fields to gas

Calculation: use Saha equation, answer just $f(T,\rho)$

Non-thermal ionization – this is *much nastier!*

Need to know and balance:

- explicit ionization rates ζ from non-thermal processes
- specific reactions that lead to recombination

e.g. if dominant reaction is recombination with molecular ions ("dissociative recombination"), such as

$$e^- + HCO^+ \longrightarrow H + CO$$

rate equation

$$\frac{\mathrm{d}n_e}{\mathrm{d}t} = \zeta n_n - \beta n_e n_{HCO^+} = 0$$

in ionization equilibrium

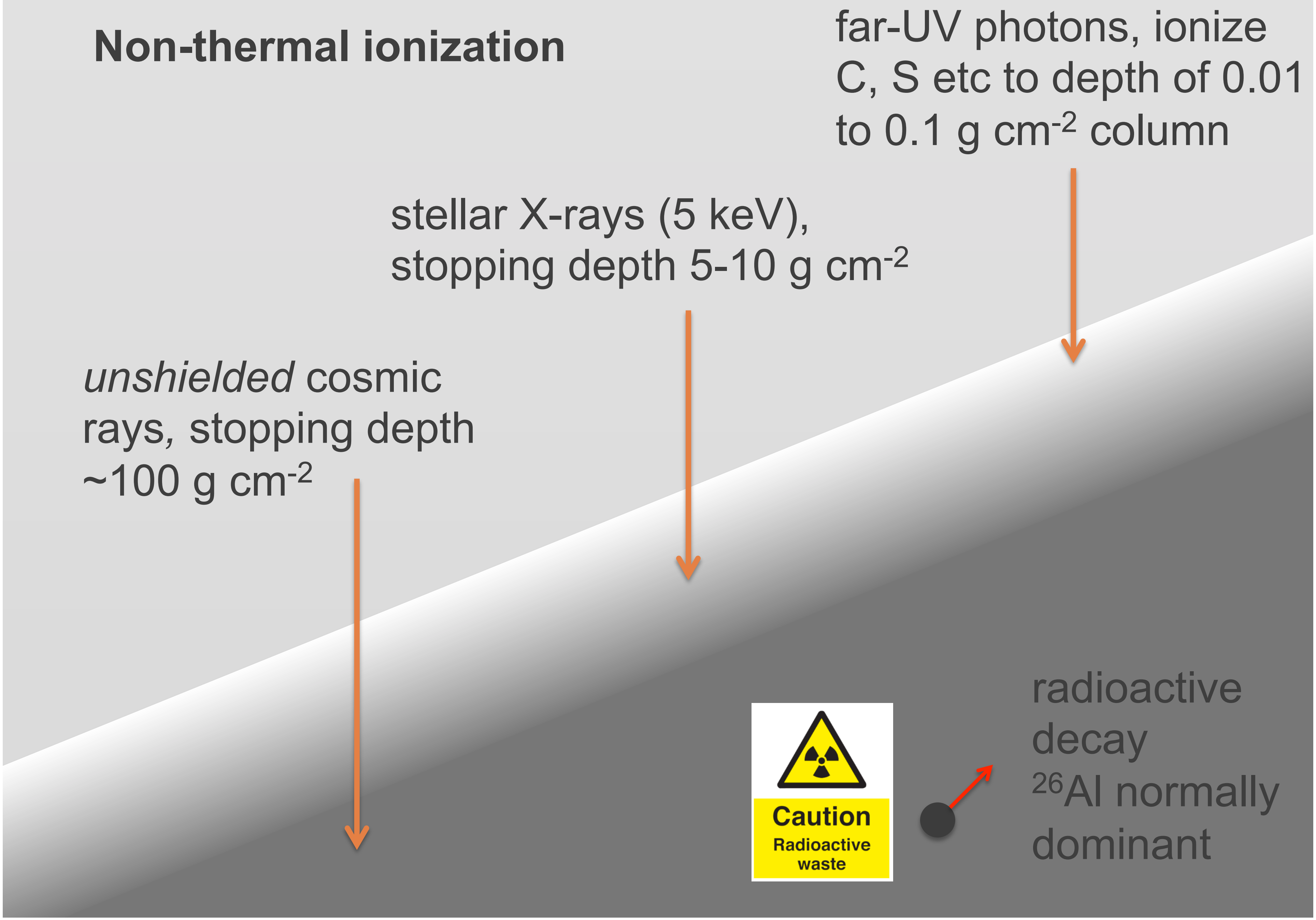
neutrals

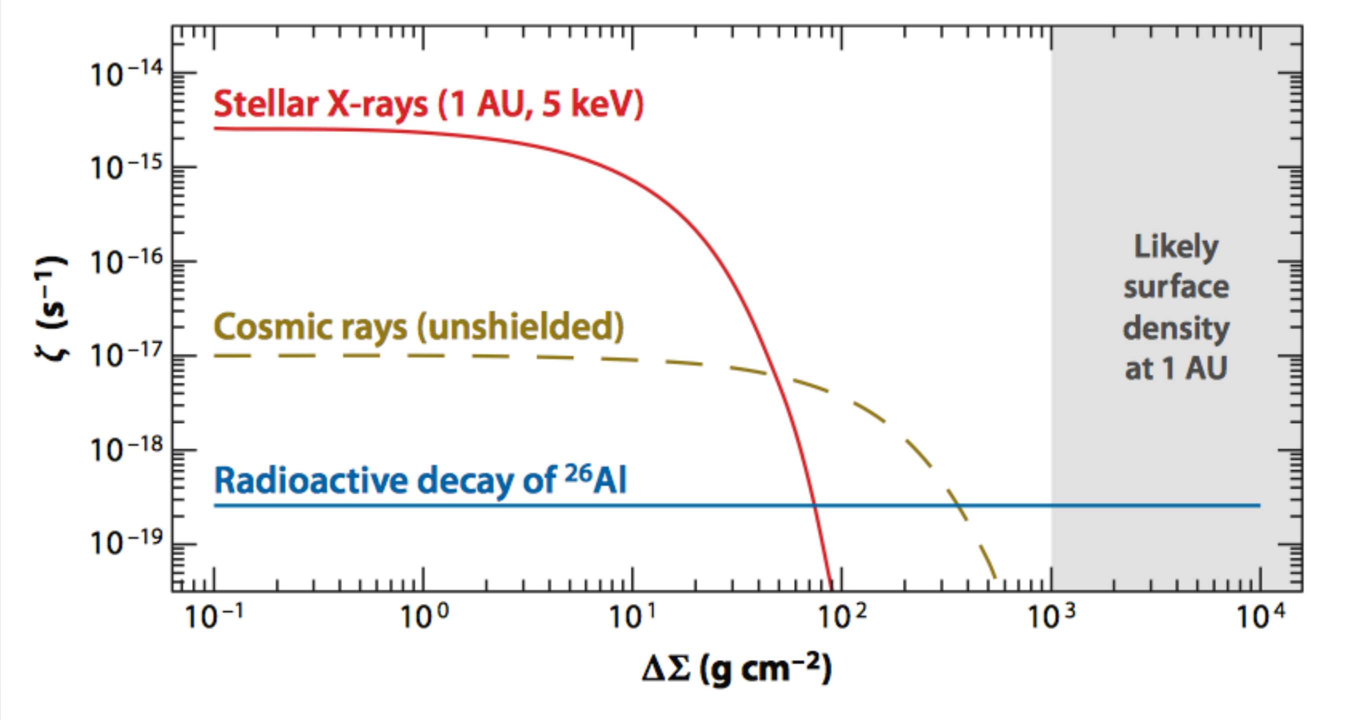
rate coefficient (function of T)

Solution assuming overall neutrality

$$x_e = \sqrt{\frac{\zeta}{\beta n_n}}$$

Non-thermal ionization





Simplified model for ionization rates after Turner & Sano '08

Mid-plane of the inner disk is too cold to be thermally ionized, dense enough to be shielded from external sources of ionizing radiation… very low x_e

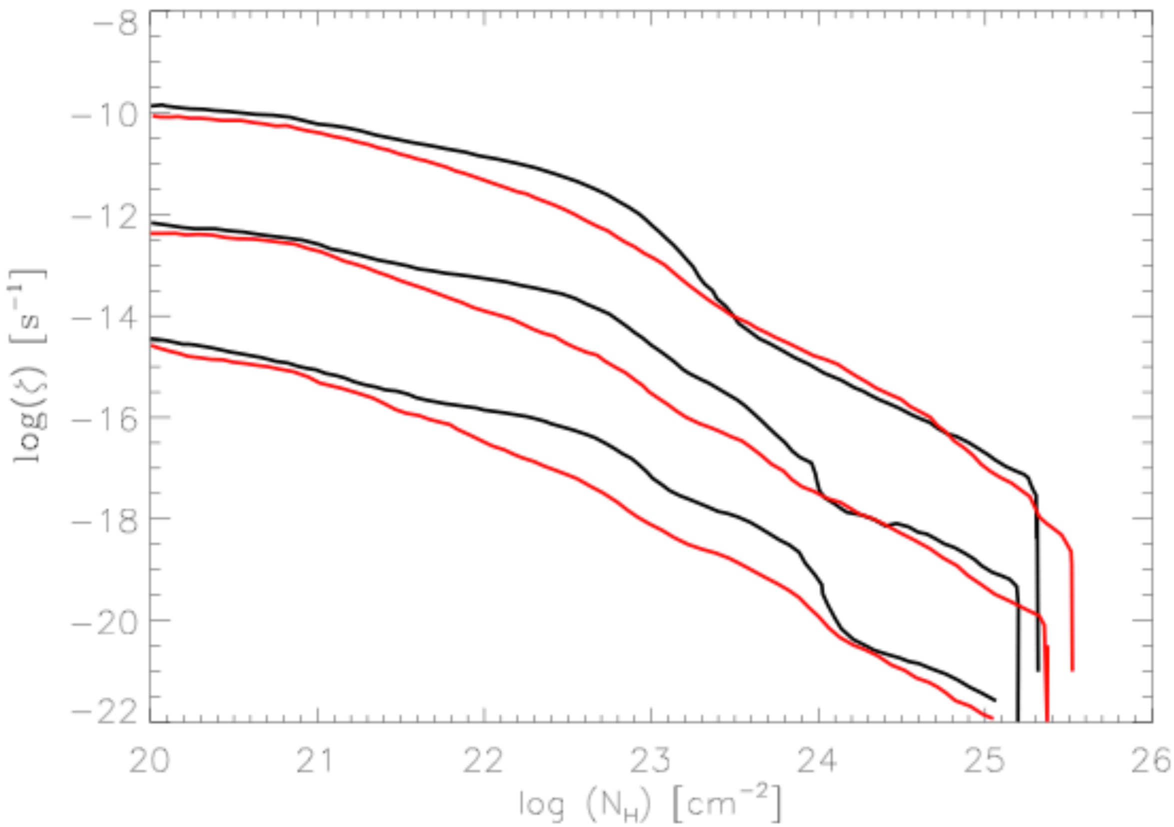


Figure 3. Ionization rates plotted vs. vertical column density for a two-temperature average representation of the COUP X-ray observations (Wolk et al. 2005) of the Orion Nebula Custer at 1 AU, 5 AU and 10 AU for solar abundances (black) and depleted interstellar abundances (red) given in Table 1. The X-ray luminosity is $L_X = 2 \times 10^{30}$ erg s^{-1}, and the other spectrum parameters are given in Table 2. For ease of viewing, the 5 AU and 10 AU curves have been shifted down by factors of 10 and 1,000 relative to those for 1 AU.

Stellar X-ray ionization from *Ercolano & Glassgold '13*

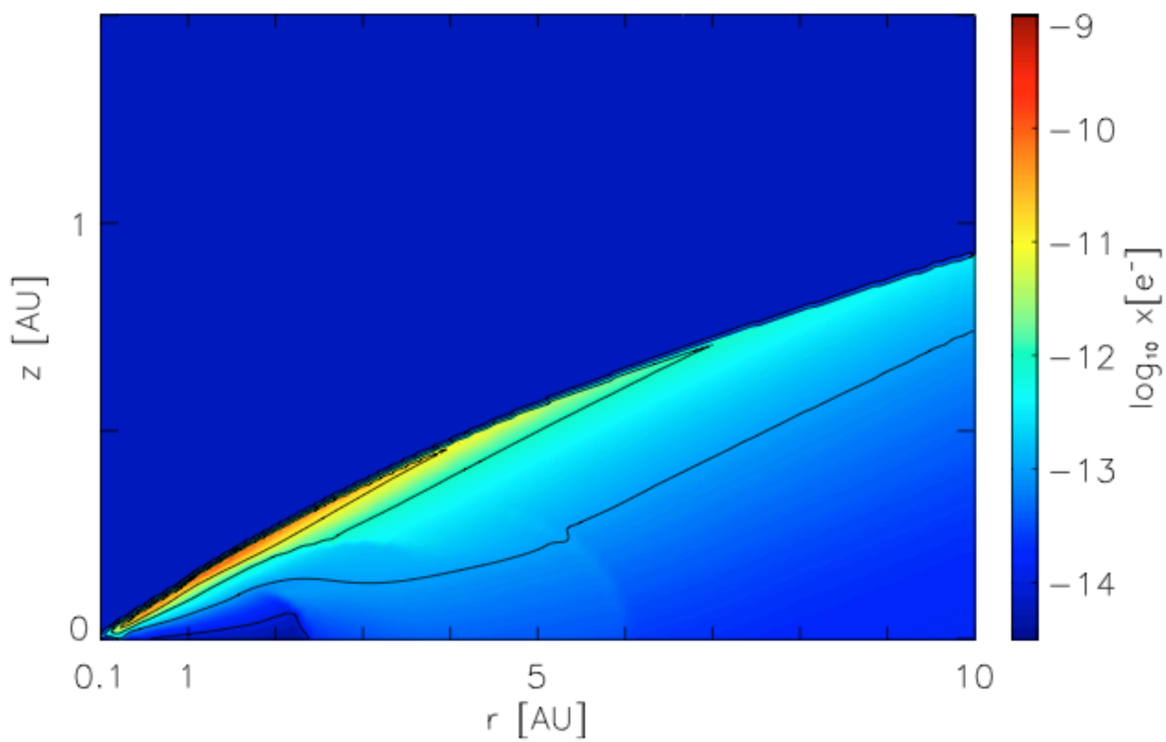


Fig. 20. This contour plot shows the electron distribution at $t = 10^5$ yrs for our $\alpha = 10^{-2}$, $\dot{M} = 10^{-7}\ M_\odot\,\mathrm{yr}^{-1}$ disk model by applying the UMIST model (`model3`) with $x_{\mathrm{Mg}} = 10^{-12}$. The contour lines refer to values $x[e^-]$ of 10^{-14}, 10^{-13}, 10^{-12}, and 10^{-11}.

Calculation of ionization fraction from *Ilgner & Nelson '06*

Metal ions and dust grain reactions are both important factors determining ionization degree

Are cosmic rays shielded from disk? Solar wind modulates local flux of cosmic rays, do T Tauri stellar or disk winds have similar effect? *(Cleeves et al. '13)*

Is the microphysics of ionization balance fully understood? Ionization by electrons accelerated in MHD turbulence, non-linear relation between current and electric field strength… *(Inutsuka & Sano '05; McNally et al. '13; Okuzumi & Inutsuka '15)*

Protoplanetary disk evolution

The central problem

The gas orbital velocity is accurately Keplerian

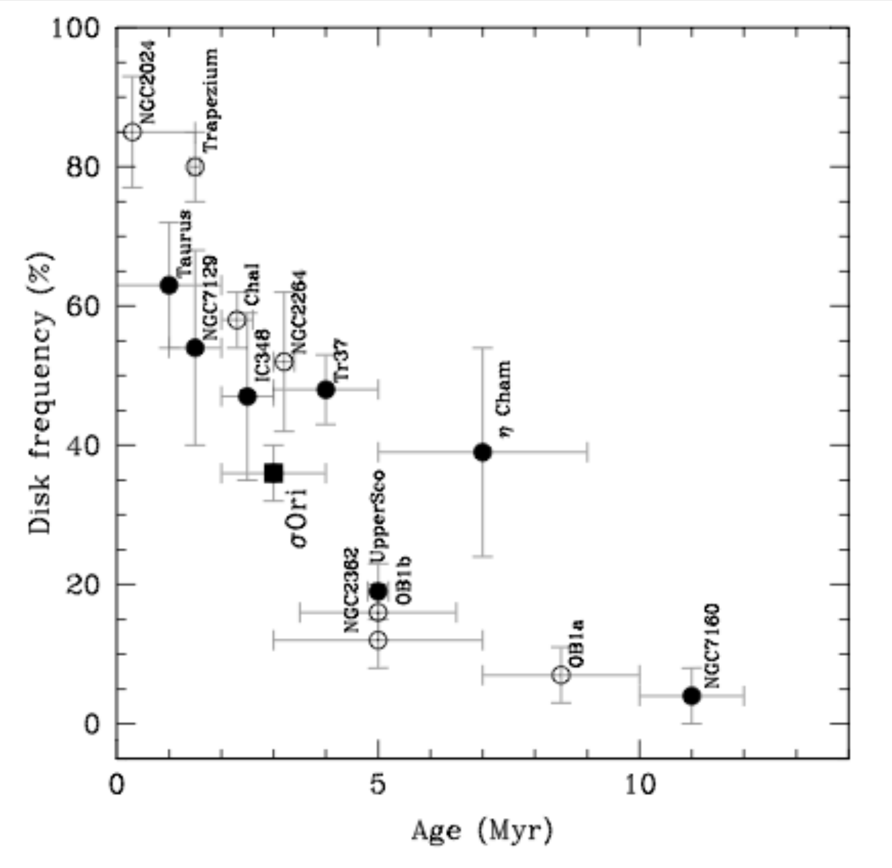
$$v_\phi = v_K + \mathcal{O}(h/r)^2$$

Specific angular momentum is robustly an increasing function of radius

$$l = \sqrt{GM_* r} \propto r^{1/2}$$

Even though lowest **energy** state favors gas accreting on to the star, **angular momentum** conservation forbids it

The central problem

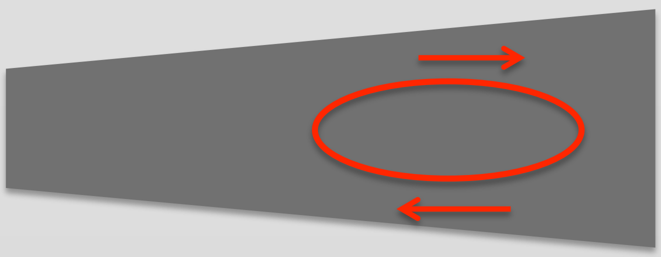


Hernandez et al. ‘07

Consistent with long observed disk lifetimes – disks are quasi-equilibrium structures that evolve slowly compared to dynamical time scale

The central problem

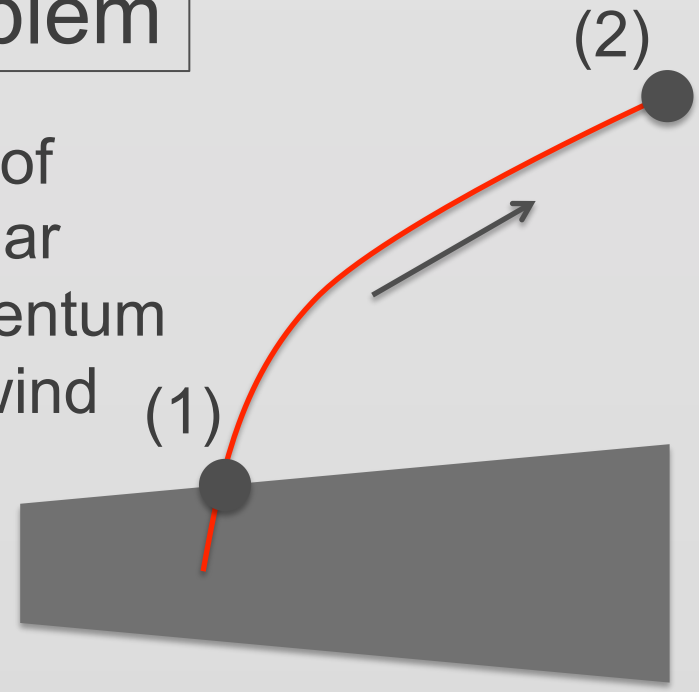
Redistribution of angular momentum within disk



"Viscous" disk: angular momentum mixed by internal turbulence

Not mutually exclusive!

Loss of angular momentum in a wind



$$l_1 = \Omega_1 r_1^2$$

if field line is like a rigid wire

$$l_2 = \Omega_1 r_2^2 \gg \Omega_{K,2} r_2^2$$

Classical disks

Lynden-Bell & Pringle '74; Shakura & Sunyaev '73 theory:

- disk is geometrically thin (h/r << 1), axisymmetric, planar
- angular momentum redistribution is modeled as a Navier-Stokes shear viscosity (kinematic viscosity ν)

continuity + angular momentum conservation

$$\left\{\begin{aligned} r\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial r}(r\Sigma v_r) &= 0 \\ r\frac{\partial}{\partial t}(r^2\Omega\Sigma) + \frac{\partial}{\partial r}(r^2\Omega \cdot r\Sigma v_r) &= \frac{1}{2\pi}\frac{\partial G}{\partial r} \end{aligned}\right.$$

specification of the torque G – **local**, scales linearly with shear

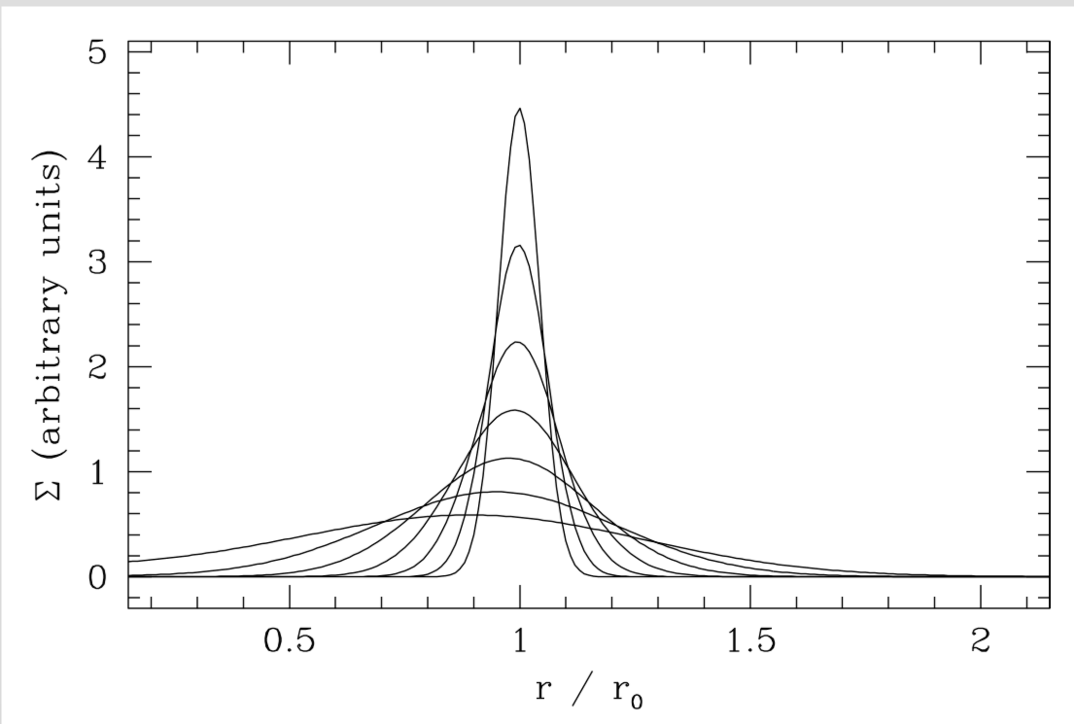
$$G = 2\pi r \cdot \nu\Sigma r \frac{\mathrm{d}\Omega}{\mathrm{d}r} \cdot r$$

Keplerian potential specializes to…

$$\boxed{\frac{\partial \Sigma}{\partial t} = \frac{3}{r}\frac{\partial}{\partial r}\left[r^{1/2}\frac{\partial}{\partial r}\left(\nu \Sigma r^{1/2}\right)\right]}$$

Diffusive evolution of surface density Σ

Viscous time scale: $t_\nu = r^2/\nu$

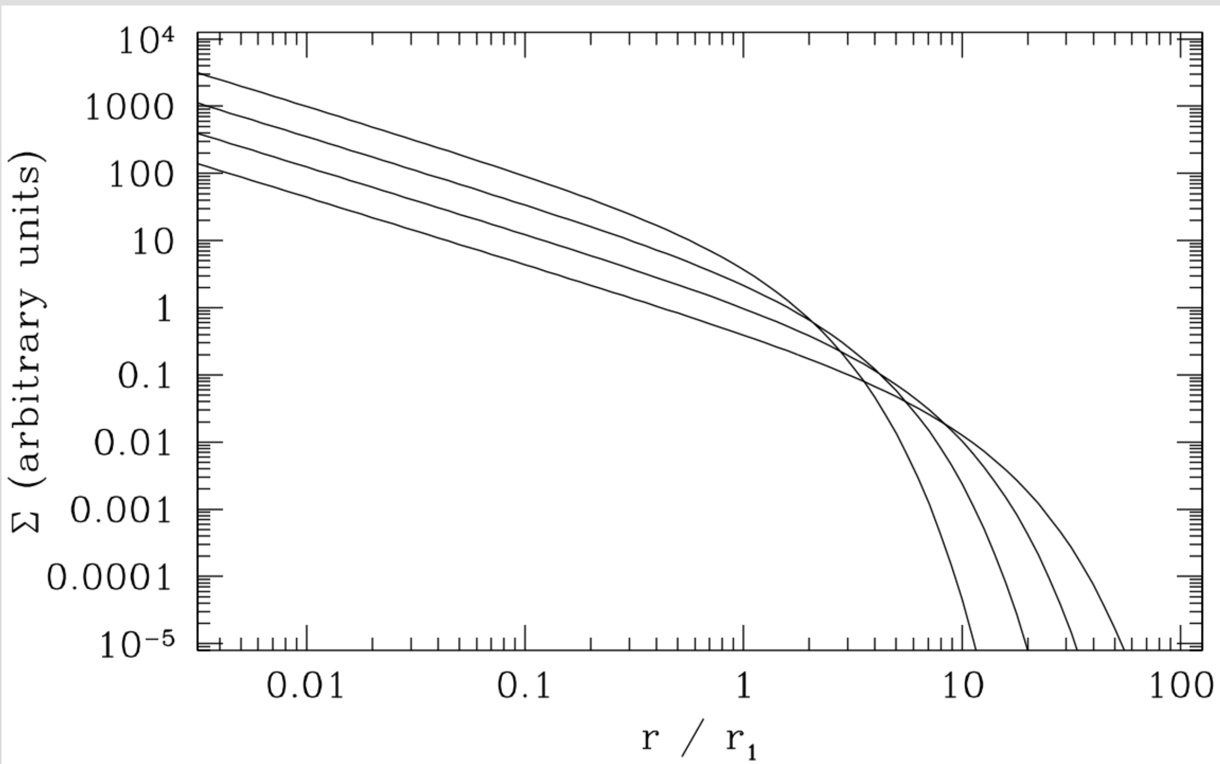


Green's function solution: mass flows to r = 0, while angular momentum carried by tail of mass to infinity

In steady-state, if: $\nu \propto r^{\gamma}$ ⇒ $\Sigma = \dot{M}/(3\pi\nu) \propto r^{-\gamma}$

Also have explicit self-similar solution:

$$\Sigma(\tilde{r}, T) = \frac{C}{3\pi\nu_1\tilde{r}^{\gamma}} T^{-(5/2-\gamma)/(2-\gamma)} \exp\left[-\frac{\tilde{r}^{(2-\gamma)}}{T}\right]$$



$$\tilde{r} \equiv r/r_1$$

$$\nu_1 \equiv \nu(r_1)$$

$$T \equiv \frac{t}{t_s} + 1$$

$$t_s \equiv \frac{1}{3(2-\gamma)^2}\frac{r_1^2}{\nu_1}$$

Simple model to fit to observations

How applicable is classical disk theory?

Angular momentum transport is *not* due to real “viscosity”

$$\nu_m \sim c_s \lambda \ \sim 10^6 \ \mathrm{cm}^2 \ \mathrm{s}^{-1} \quad \Rightarrow \quad t_\nu (5 \ \mathrm{AU}) \sim 10^{14} \ \mathrm{yr}$$

~km s^{-1} ~10 cm

However, obtain same one-dimensional evolution equation if transport is due to an average turbulent stress, provided it is locally defined. e.g. for a fluid with magnetic fields,

$$W_{r\phi} = \left\langle \delta v_r \delta v_\phi - \frac{B_r B_\phi}{4\pi\rho} \right\rangle_\rho$$

transport from fluid (“Reynolds”) and magnetic (“Maxwell”) stress

$$\nu\Omega$$

Balbus & Papaloizou ‘99

How applicable is classical disk theory?

Things will go wrong if we try to apply the theory when:

- transport mechanism is *non-local* (e.g. self-gravity when M_{disk} is not much smaller than M_*)
- mass loss (e.g. from photoevaporation) occurs on a time scale < viscous time scale
- 1D situations where Ω far from Keplerian
- time scales shorter than correlation time for turbulence
- **any** 2D or 3D situation (warps, eccentric disks, meridional circulation)…

α-model disks

Can make a predictive theory if we can write ν as a function of other disk parameters (T, r, ρ, x_e…)

$$\boxed{\nu = \alpha c_s h}$$

Shakura-Sunyaev '73 α-prescription

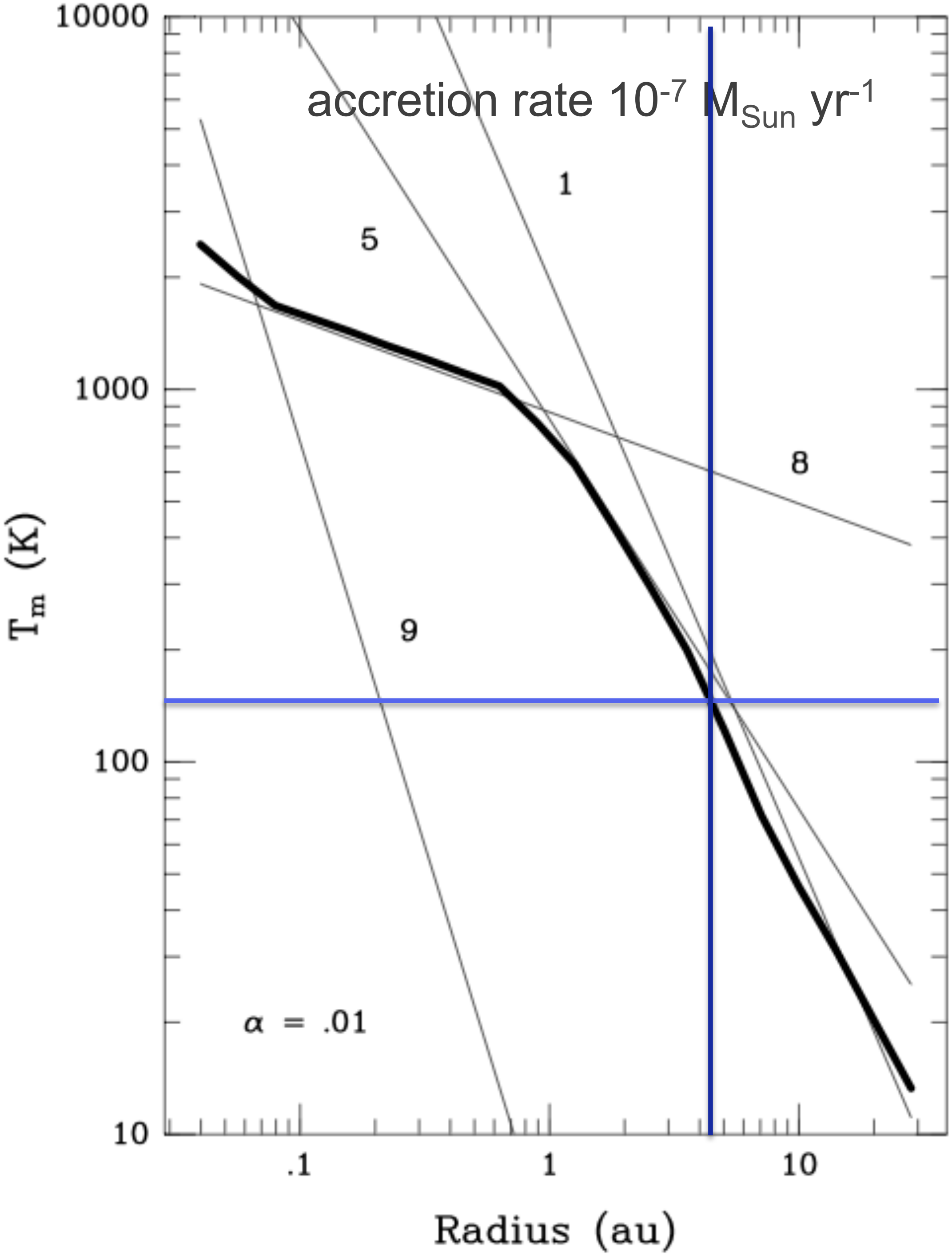
For α assumed constant, one parameter description of protoplanetary disk evolution

Identify disk lifetime with the viscous time at outer edge

$$t_\nu = \left(\frac{h}{r}\right)^{-2} \frac{1}{\alpha \Omega}$$

t_ν = 1 Myr at 30 AU, $(h / r) = 0.05$

$\Rightarrow$ $\boxed{\alpha = 0.01}$



If irradiation dominates, with fixed $T \sim r^{-1/2}$, then an α-disk is equivalent to $\nu \sim r$ (since $\nu = \alpha c_s^2 / \Omega$)

An α model predicts the time-varying radial (and vertical) structure for any accretion rate

e.g. snow line near 4 AU for this model

Bell et al. '97

$$\boxed{\nu = \alpha c_s h}$$

We can always choose to express the efficiency of angular momentum transport in terms of α

α-disk theory is useful **if** it encodes the "leading order" dependence of the stress on the local disk properties, i.e. so that α is a slowly varying function of Σ, r etc

Various caveats:

- α likely a strong function of T, Σ, *if* transport is due to MHD processes
- vertical structure also depends on how accretion energy is distributed vertically… even more uncertain
- for comparison against observations, reducing a possibly complex function to one number