# Lecture: Orbital Dynamics

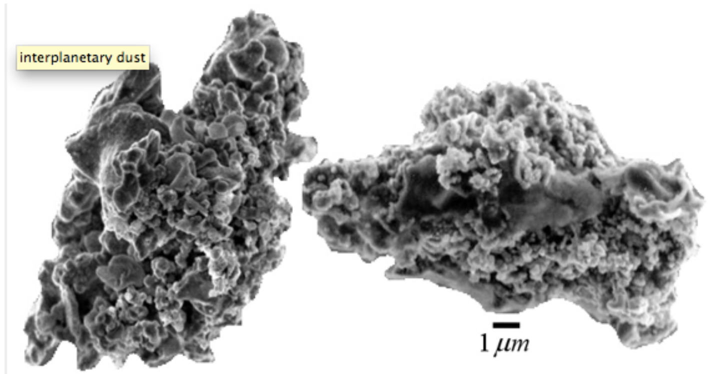
## Summer School 2015

Professor Schlichting (MIT)

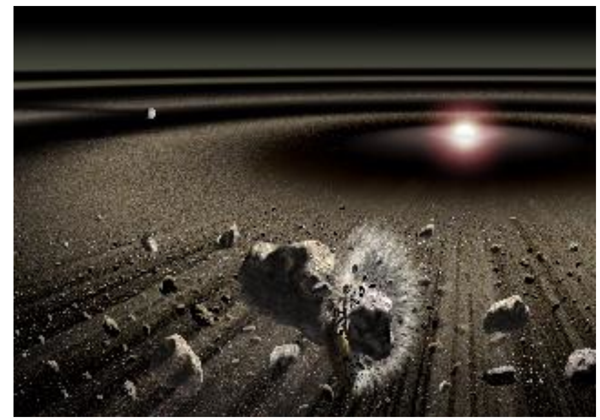
6th August 2015

# Solid Bodies: 3 Regimes

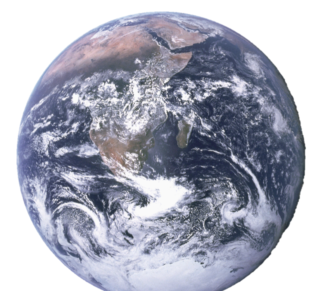
- Dust



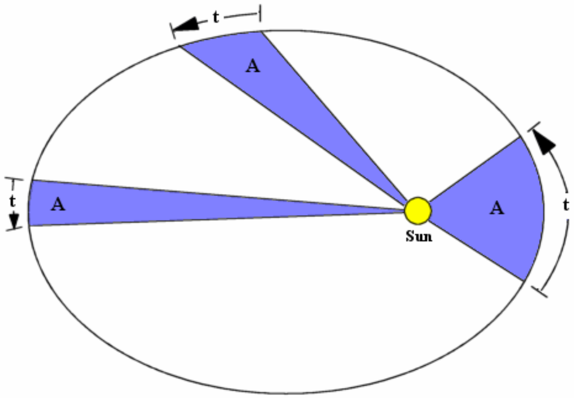

- Planetesimals

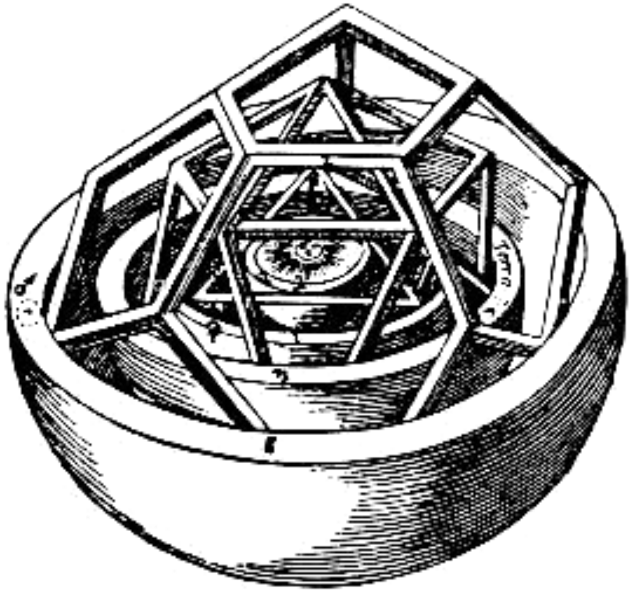


- Planetary cores/Terrestrial Planets



# Johannes Kepler
# (1571-1630)



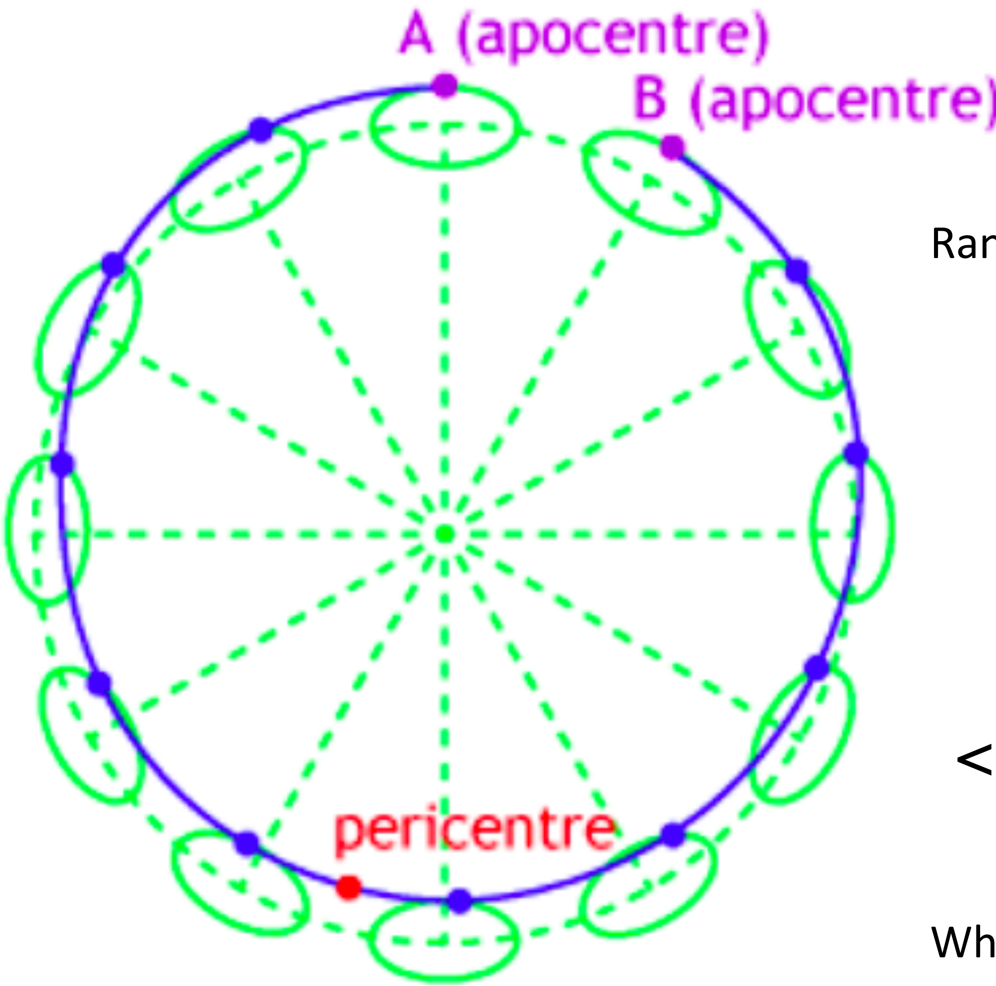
## Kepler's Laws

1. Each planet moves in an ellipse with the Sun at one focus.

2. The line connecting a planet and the Sun sweeps out equal areas in equal times.

3. For all planets, the orbital period squared divided by the semimajor axis cubed is constant.

# Relative velocities due to e & i




Random velocity due to e & i

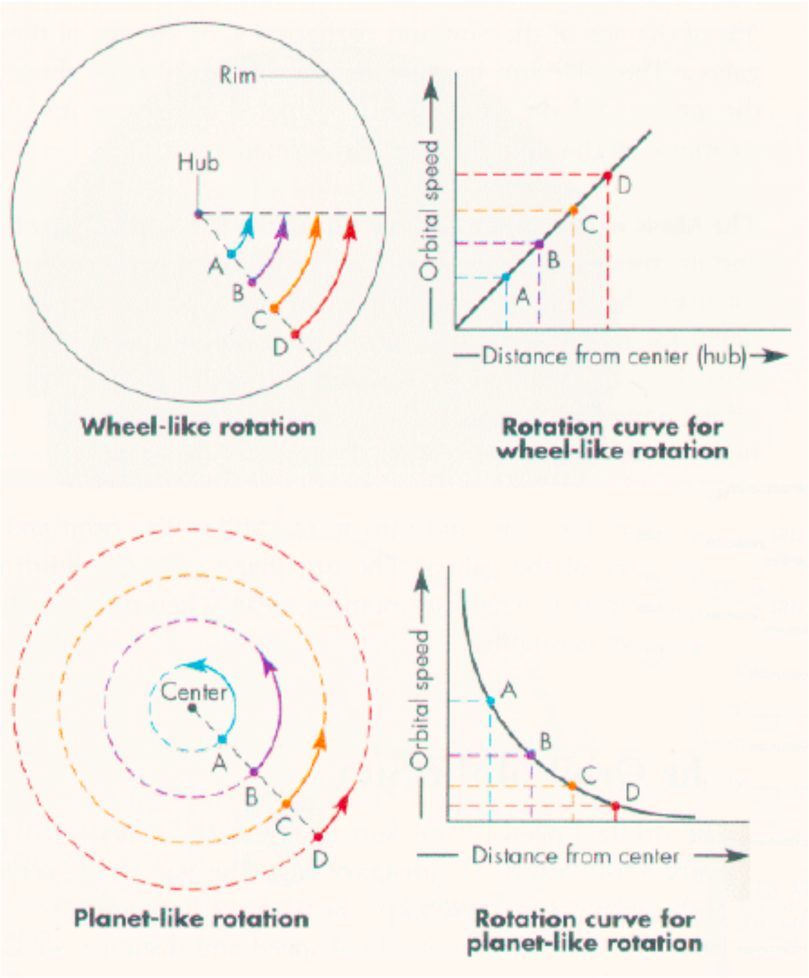
$$v_{ran}^2 = v_r^2 + v_z^2$$

$$< v_z^2 >= \frac{v_k^2 i^2}{2}$$

$$< v_{ran}^2 >= v_k^2 (i^2 / 2 + ze^2)$$

What is the value of z?

# Relative Velocities due to Keplerian shear




Total Random velocity in disk

$$v_{ran}^2 = v_{shear}^2 + v_{e,i}^2$$

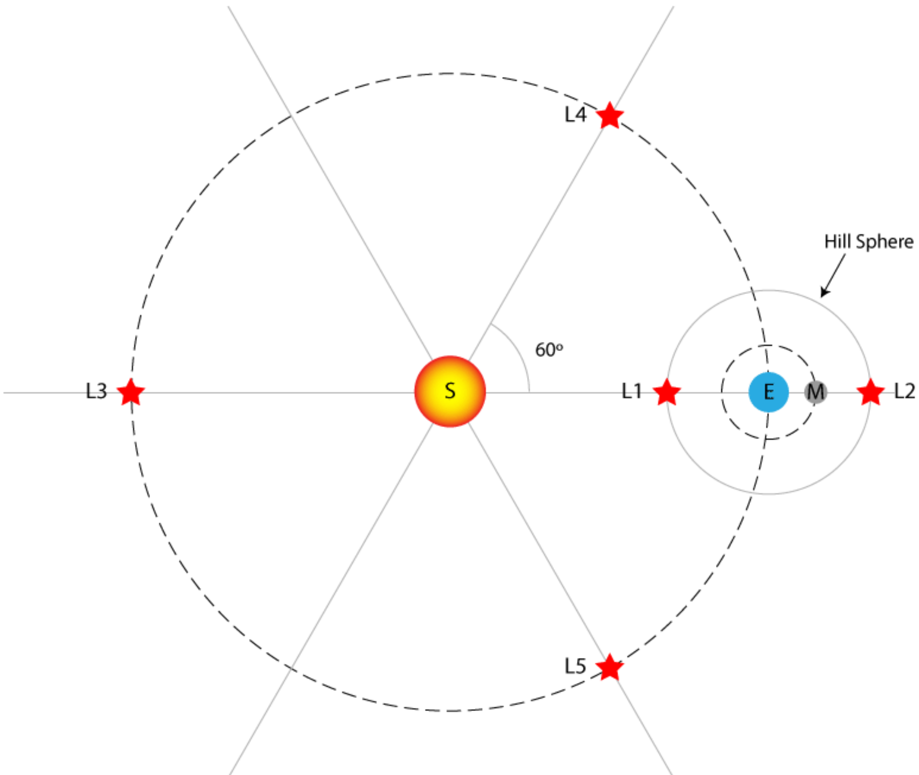
Random velocity due to Keplerian shear

$$\Delta v = a\Delta\Omega$$

$$\Delta v = \frac{3}{2}\Delta a\Omega$$

$$v_{ran} = \frac{3}{2}\Delta a\Omega$$

# Hill Sphere




$$R_H = a\left(\frac{M_P}{3M_*}\right)^{1/3}$$

Tidal forces from the sun are balanced by the gravitational force from planet

Satellites have to be within the Hill sphere

$$\alpha = R / R_H$$

$$v_H = \Omega R_H$$

Orbital frequency of satellite orbiting planet at distance $R_H$ is about the same as orbital frequency of planet around the sun

# 2D vs. 3D

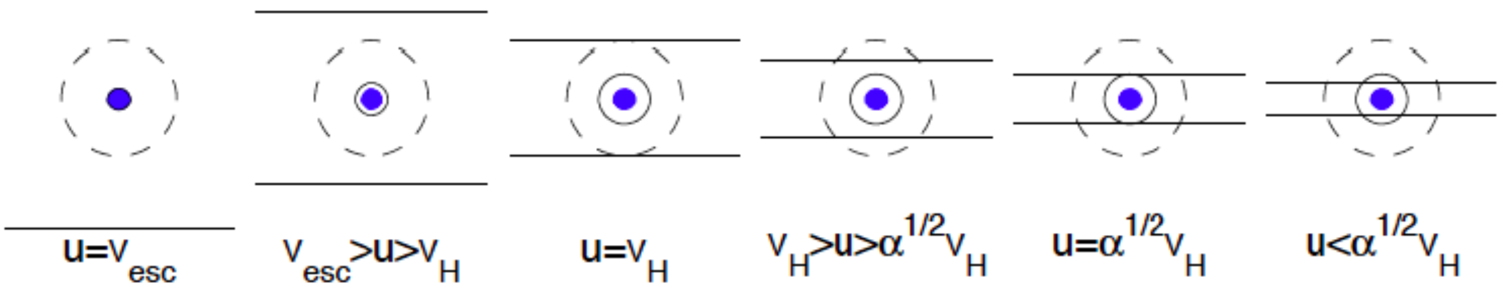



Figure 1: Geometry of disk scale height (*solid horizontal line*), body size (*filled circle*), its Hill sphere (*dashed circle*), and its effective size for accretion (*solid circle*)

Goldreich et al. 2004