Professor Hilke Schlichting

(1) **Eccentric Planetesimals**.

"Particle-in-a-box" accretion models approximate the relative motion of planetesimals as a random (or relative) velocity that is a function of the particles' eccentricities and/or inclinations. Here, we derive the most commonly used relation between the relative velocity $\vec{v}_{\rm rel}$ and eccentricity e. Consider a planetesimal on a Keplerian orbit so that

$$r = \frac{a\,(1-e^2)}{1+e\cos\theta}, \tag{1}$$

where $\theta \in [0, 2\pi]$ is the angle between the instantaneous position r and periapse (the true anomaly). The periapse velocity v_p is

$$v_p = \sqrt{\frac{GM}{a}}\sqrt{\frac{1+e}{1-e}}. \tag{2}$$

Compute an orbit-averaged $\langle v_{\rm rel}^2\rangle$, where $v_{\rm rel}$ is the relative velocity of the eccentric planetesimal compared to a co-planar, locally circular orbit. Here, "local" means that you are to compare the planetesimal's instantaneous velocity to that of a circular orbit with $a = r$, where r is the planetesimal's time-varying orbital radius (Fig. 1). Remember that angular momentum is conserved at every point on an orbit.

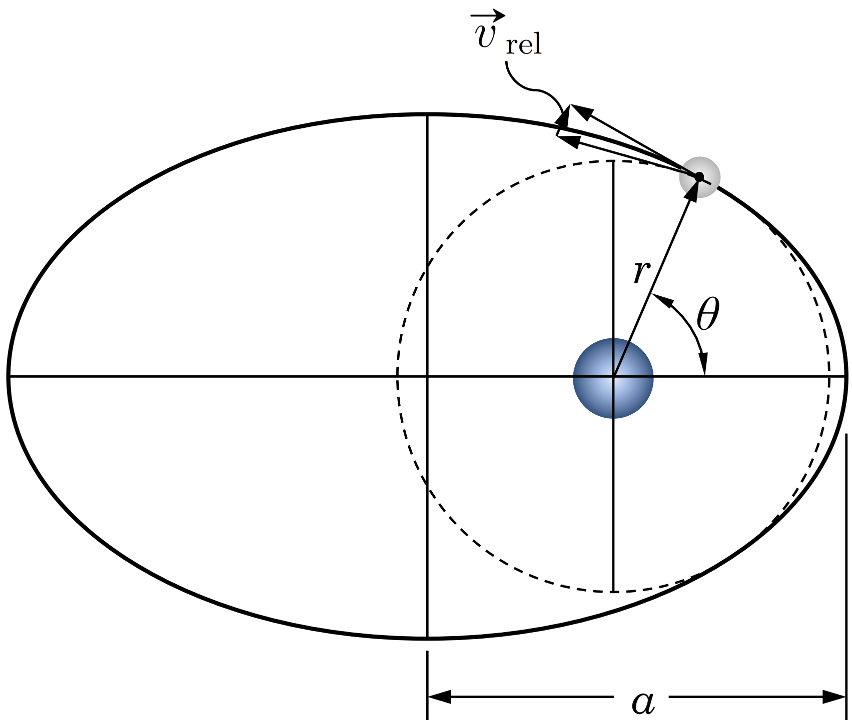

FIGURE 1. Orbit geometry for Problem 1.

(2) **Planet Formation.**

(a) What is dynamical friction and (qualitatively) how does it affect protoplanet dynamical evolution?

(b) What is viscous stirring and (qualitatively) how does it affect protoplanet dynamical evolution?

(c) In terms of the radius R of a protoplanet and time t, give an equation that describes the runaway growth regime and explain *briefly* in words what it means.

(d) What is a consequence of runaway growth?

(3) **Growing a Planet.**

(a) Derive the impact velocity $v_{\rm imp}$ of a test particle on a planet of mass M and radius R. Express $v_{\rm imp}$ as a function of the relative velocity at infinity v_∞ and the planet's escape velocity $v_{\rm esc}$. Show all steps.

(b) Qualitatively, what is gravitational focusing?

(c) Derive the gravitational focusing factor for the situation described in (a) using conservation of angular momentum.

(d) Why is there in practice a maximum value for the gravitational focusing factor that can be achieved as a test particle or planetesimal is accreted by a planet?

(4) **Isolation Masses.**

A growing planetary embryo will accumulate all of the material within an annulus of width $\Delta a = Br_H$, where B is a constant ($= 10$), r_H is the planet's Hill radius, and a is the planet's semimajor axis.

(a) Calculate the mass of the final planet, $M_{\rm iso}$, that will form from a disk with a uniform surface density in solids Σ_s. Show all steps.

(b) What would the radial dependence of the surface density need to be to produce an isolation mass that is independent of semimajor axis? How does this compare to the minimum mass Solar nebula?

(c) Compute M_{iso} for constant $\Sigma_s = 100\ \text{kg/m}^2$ at $1\ \text{AU}$. What does this imply about the mass and approximate total number of embryos $\mathcal{N}$ expected at the end of runaway growth in the region of the terrestrial planets? Assume the mass in solids equals the total final mass in the terrestrial planets, and use your calculation at $1\ \text{AU}$ to get an order of magnitude estimate for $\mathcal{N}$.