Lecture: Terrestrial Planet Formation

Summer School 2015

Professor Schlichting (MIT)

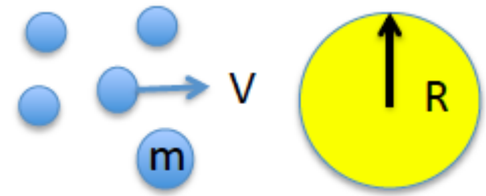
6th August 2015

Growth Rates

Collision Rate $= nV\pi R^2$

Growth Rate $\frac{dM}{dt} = nmV\pi R^2$

Mass Doubling Rate $\frac{1}{M}\frac{dM}{dt} \sim \frac{\sigma\Omega}{\rho R}$



@ 1 AU t_{coll} = M dt/dM ~ 10^7 years

@ 20 AU t_{coll} = M dt/dM ~ 10^{11} years

Need to enhance collisional Cross-sections

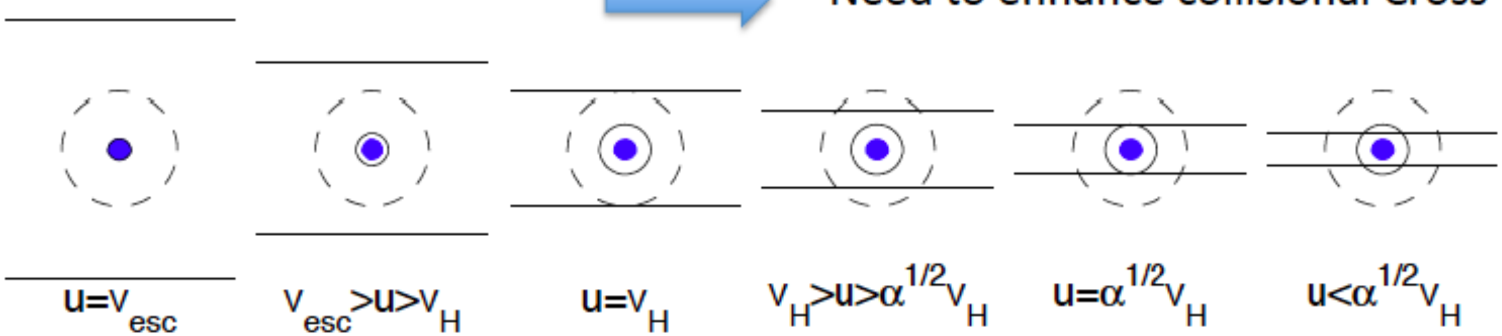


Figure 1: Geometry of disk scale height (*solid horizontal line*), body size (*filled circle*), its Hill sphere (*dashed circle*), and its effective size for accretion (*solid circle*)

Goldreich et al. 2004

Gravitational Focusing

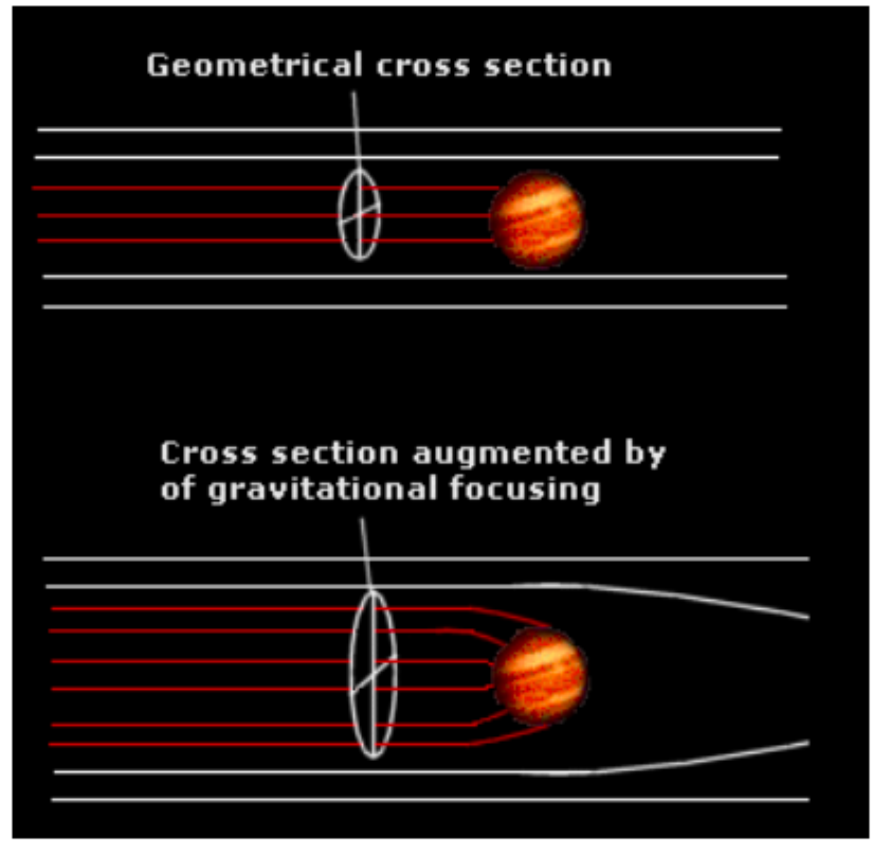
$$\frac{dm_p}{dt} = \rho_s V \pi b^2$$

Without gravitational focusing:

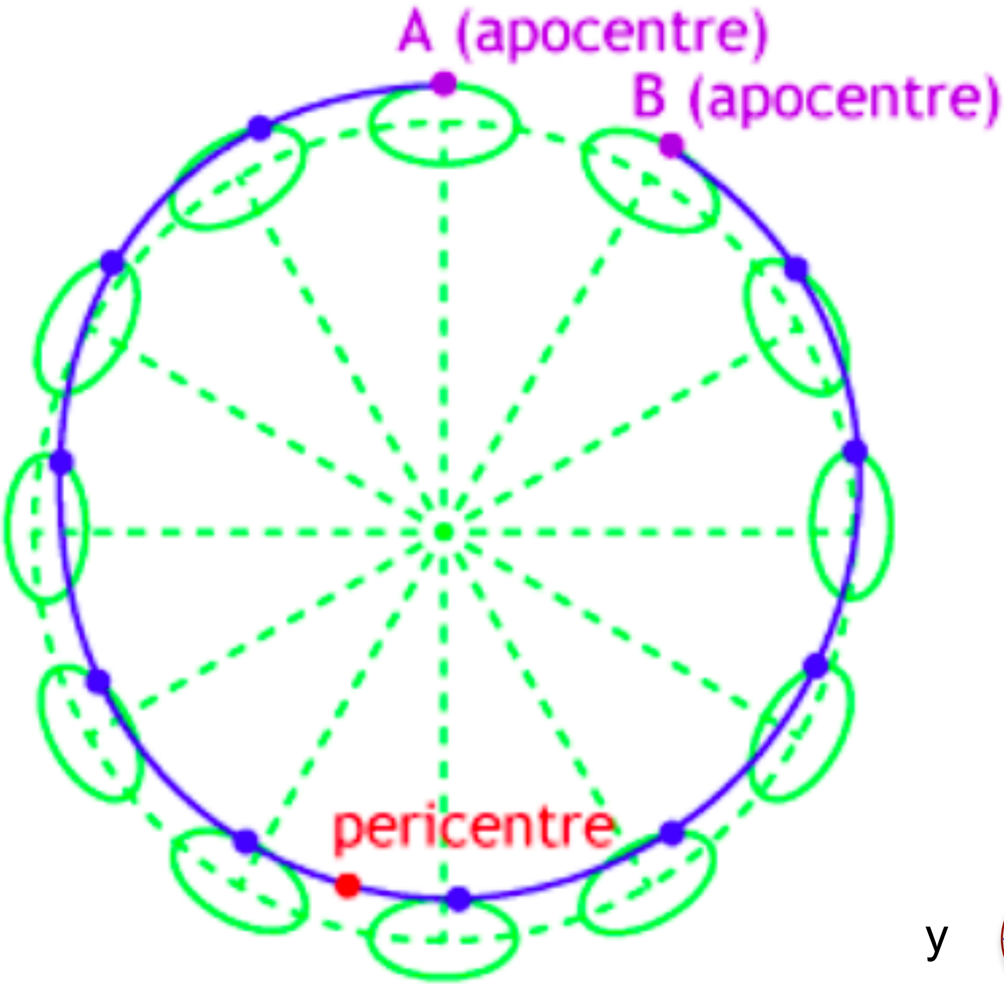
$$\frac{dm_\mathrm{p}}{dt} = \rho_\mathrm{s} \cdot V \cdot \pi a^2$$

With gravitational focusing:

$$\frac{dm_\mathrm{p}}{dt} = \rho_\mathrm{s} \cdot V \cdot \pi a^2 \left[1 + \frac{v_e^2}{V^2} \right]$$



Motion in the Rotating Frame



Positions

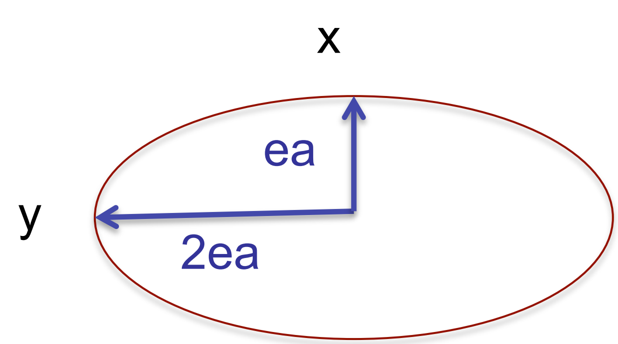
$$x = ae\sin\Omega t$$

$$y = 2ae\cos\Omega t$$

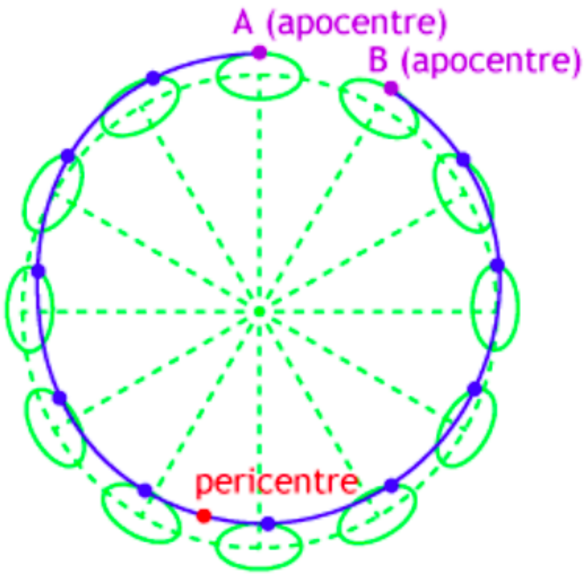
Velocities

$$\dot{x} = ae\Omega\cos\Omega t$$

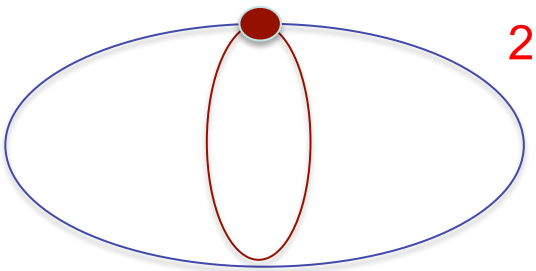
$$\dot{y} = -2ae\Omega\sin\Omega t$$



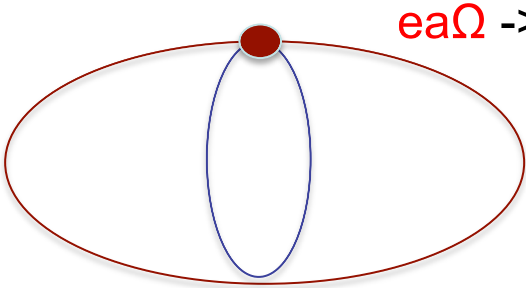
Viscous Stirring



- Velocity doubles at quadrature



- Velocity halves at periapse and apoapse



Safronov (1972) assumed an equal number of collisions in each direction:

Average Increase in k.e. =(4+1/4)/2=17/8

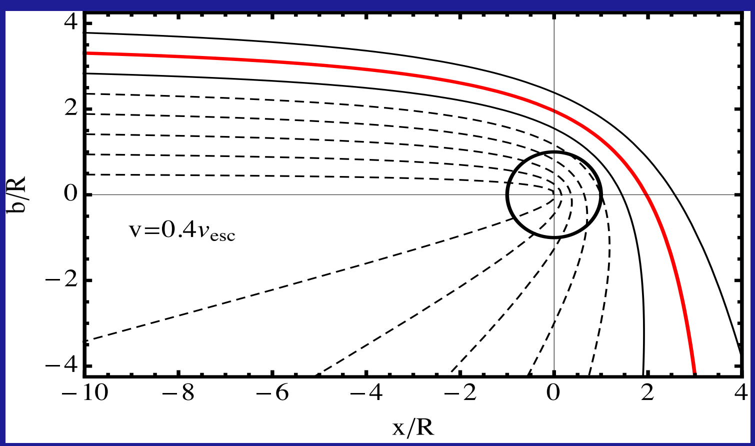
$$cross\,section = \pi R^2 \left(\frac{v_{esc}}{v_{\infty}} \right)^4$$

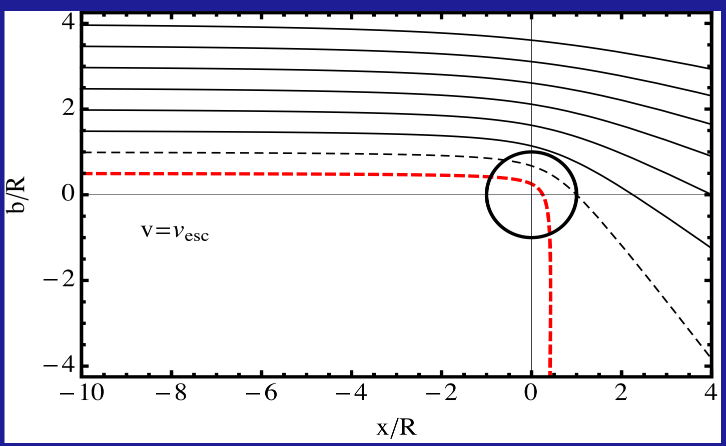
Viscous Stirring

Viscous stirring tends to increase the random kinetic energy all all bodies in the disk

For v <<v_{esc} $\sigma_{stirr} >> \sigma_{coll}$

For v >>v_{esc} $\sigma_{stirr} << \sigma_{coll}$



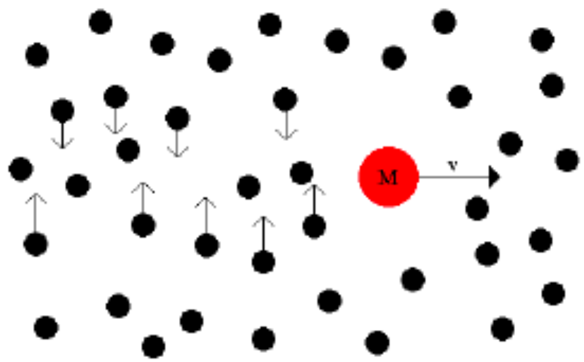


$$\sigma_{stirr} \sim \pi R^2 \left(\frac{v}{v_{esc}}\right)^{-4}$$

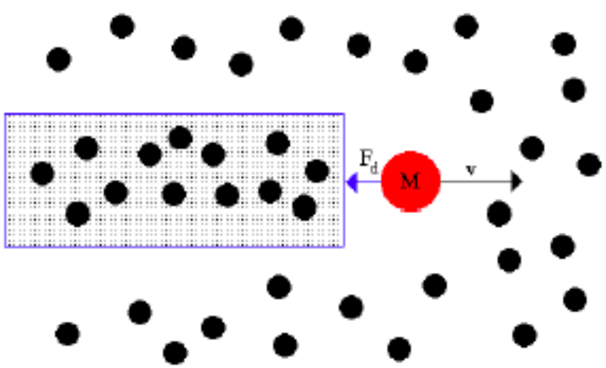
$$\sigma_{coll} \sim \pi R^2 \left(1 + \left(\frac{v}{v_{esc}}\right)^{-2}\right)$$

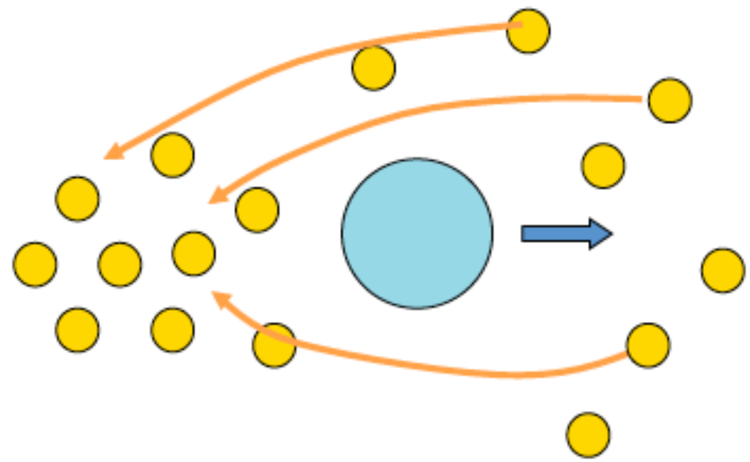
Dynamical Friction

consider a mass, M, moving through a uniform sea of stars. Stars in the wake are displaced inward.



this results in an enhanced region of density behind the mass, with a drag force, F_d known as dynamical friction

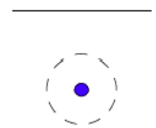




$$F_{df} \sim \left(\frac{GM}{v}\right)^2 \frac{\Sigma}{b} \sim GM\Sigma \qquad \text{2D}$$

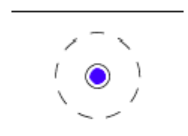
Growth Rates & Stirring Rates

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}$$



$u=v_{esc}$

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}\left(\frac{u}{v_{esc}}\right)^{-2}$$

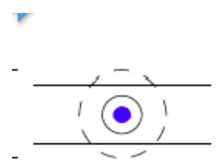


$v_{esc}>u>v_H$

$$\frac{1}{u}\frac{du}{dt} \sim \frac{\Sigma\Omega}{\rho R}\left(\frac{u}{v_{esc}}\right)^{-4}$$

$u>v_H$

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}\alpha^{-1/2}\frac{v_{esc}}{u}$$

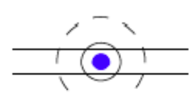


$v_H>u>\alpha^{1/2}v_H$

$$\frac{1}{u}\frac{du}{dt} \sim \frac{\Sigma\Omega}{\rho R}\alpha^{-3/2}\left(\frac{u}{v_{esc}}\right)^{-1}$$

$u<v_H$

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}\alpha^{-3/2}$$

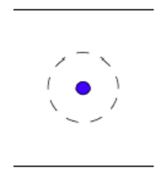


$u<\alpha^{1/2}v_H$

Growth Regimes

$$\frac{1}{R}\frac{dR}{dt} \propto R^{-1}$$

Orderly Growth

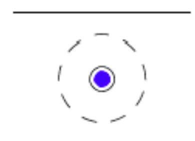


$u=v_{esc}$

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}$$

$$\frac{1}{R}\frac{dR}{dt} \propto R$$

Runaway Growth

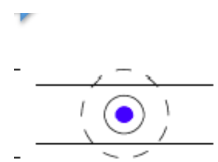


$v_{esc}>u>v_H$

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}\left(\frac{u}{v_{esc}}\right)^{-2}$$

$$\frac{1}{R}\frac{dR}{dt} \propto 1$$

Neutral Growth

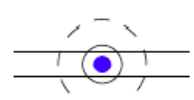


$v_H>u>\alpha^{1/2}v_H$

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}\alpha^{-1/2}\frac{v_{esc}}{u}$$

$$\frac{1}{R}\frac{dR}{dt} \propto R^{-1}$$

Orderly Growth



$u<\alpha^{1/2}v_H$

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}\alpha^{-3/2}$$

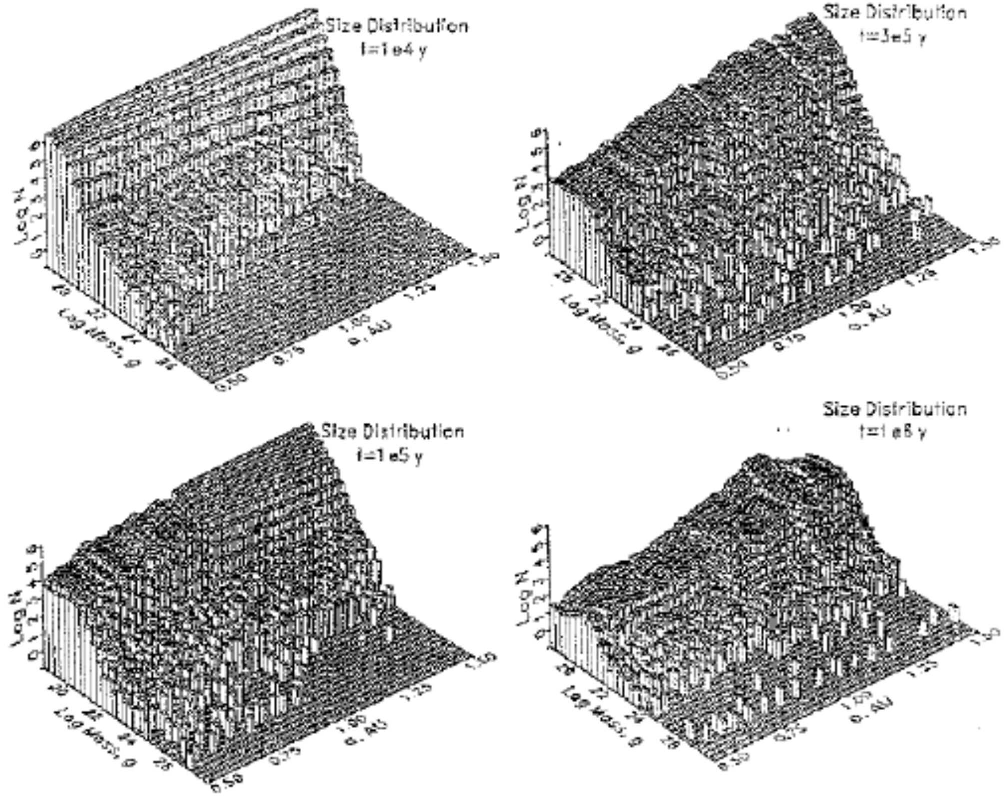
Runaway Growth

$$\frac{1}{R_1}\frac{dR_1}{dt} \propto R_1$$

For $v_{esc} > u$ gravitational focusing enhances the accretion rate

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}\left(\frac{v_{esc}}{u}\right)^2 \propto R$$

Small number of large bodies run-away from the rest of the planetesimal population



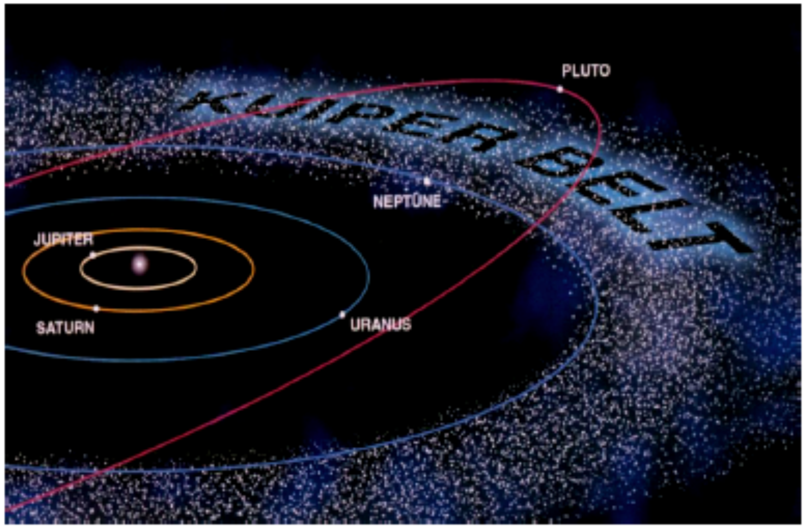
Runaway Growth in Solar System

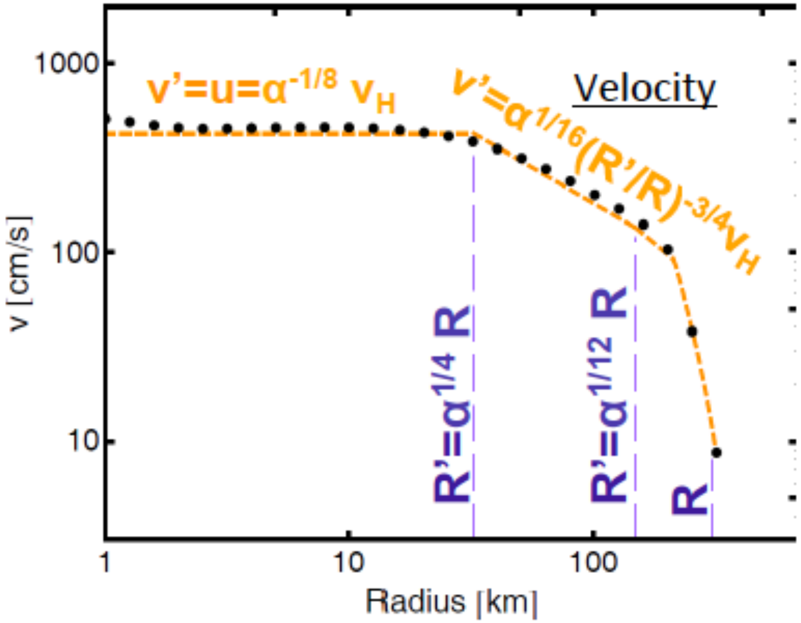
$$\alpha \sim R_{Sun} / a$$

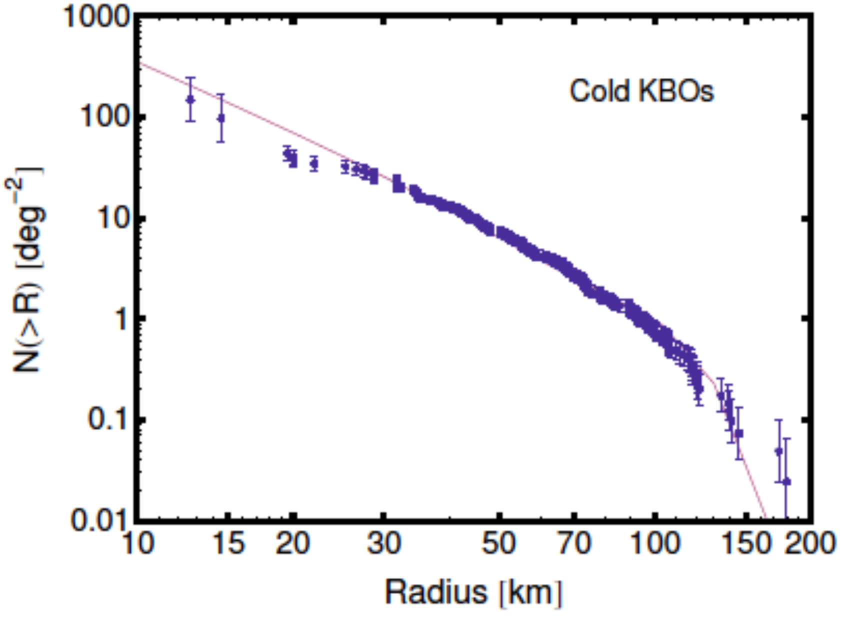
$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}\left(\frac{u}{v_{esc}}\right)^{-2} + \frac{\Sigma\Omega}{\rho R}\alpha^{-3/2}$$

Accretion of small bodies

Accretion of big bodies





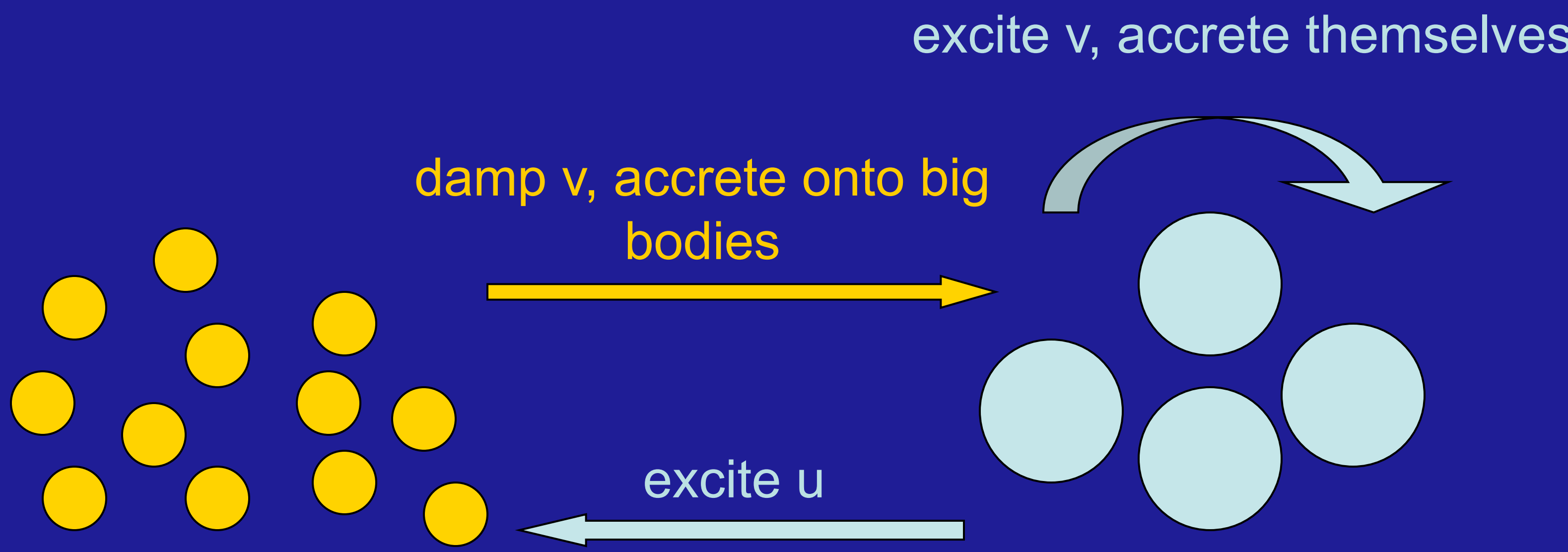


(Schlichting & Sari, 2011)

Two-Group-Approximation

Small bodies

Big bodies



Mass surface density = σ

mass = m, radius = r

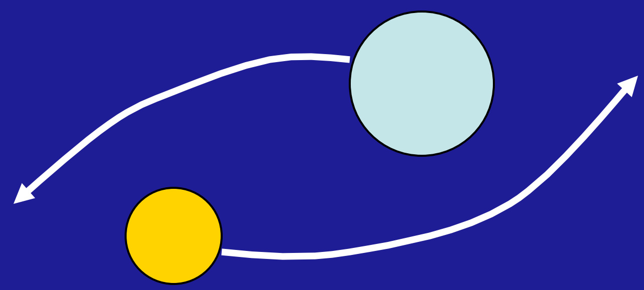
velocity dispersion = u

Mass surface density = Σ

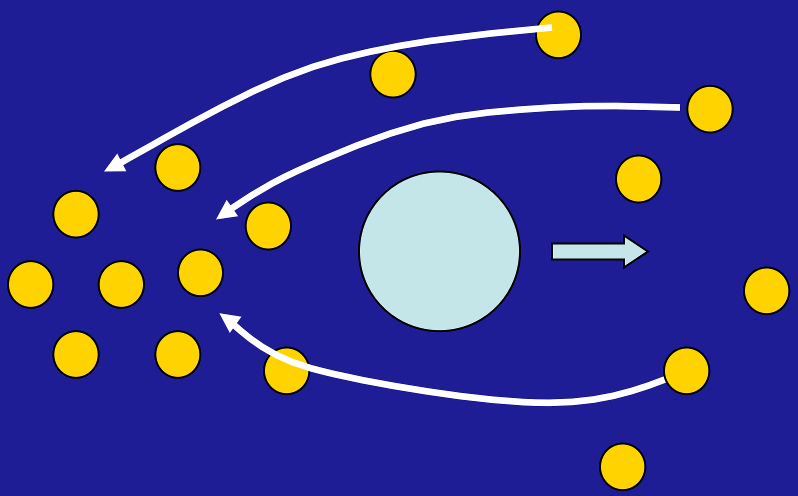
Mass = M, Radius = R

velocity dispersion = v

Velocity Evolution

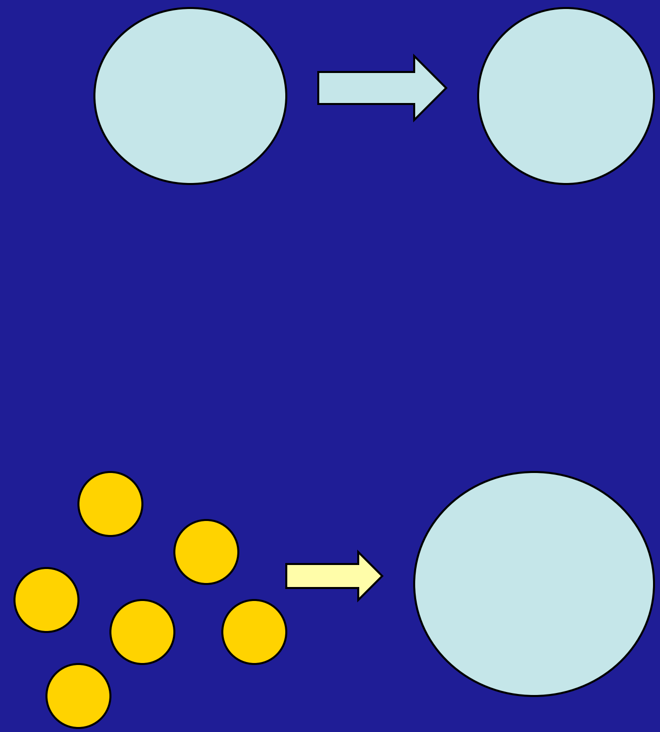


Viscous Stirring

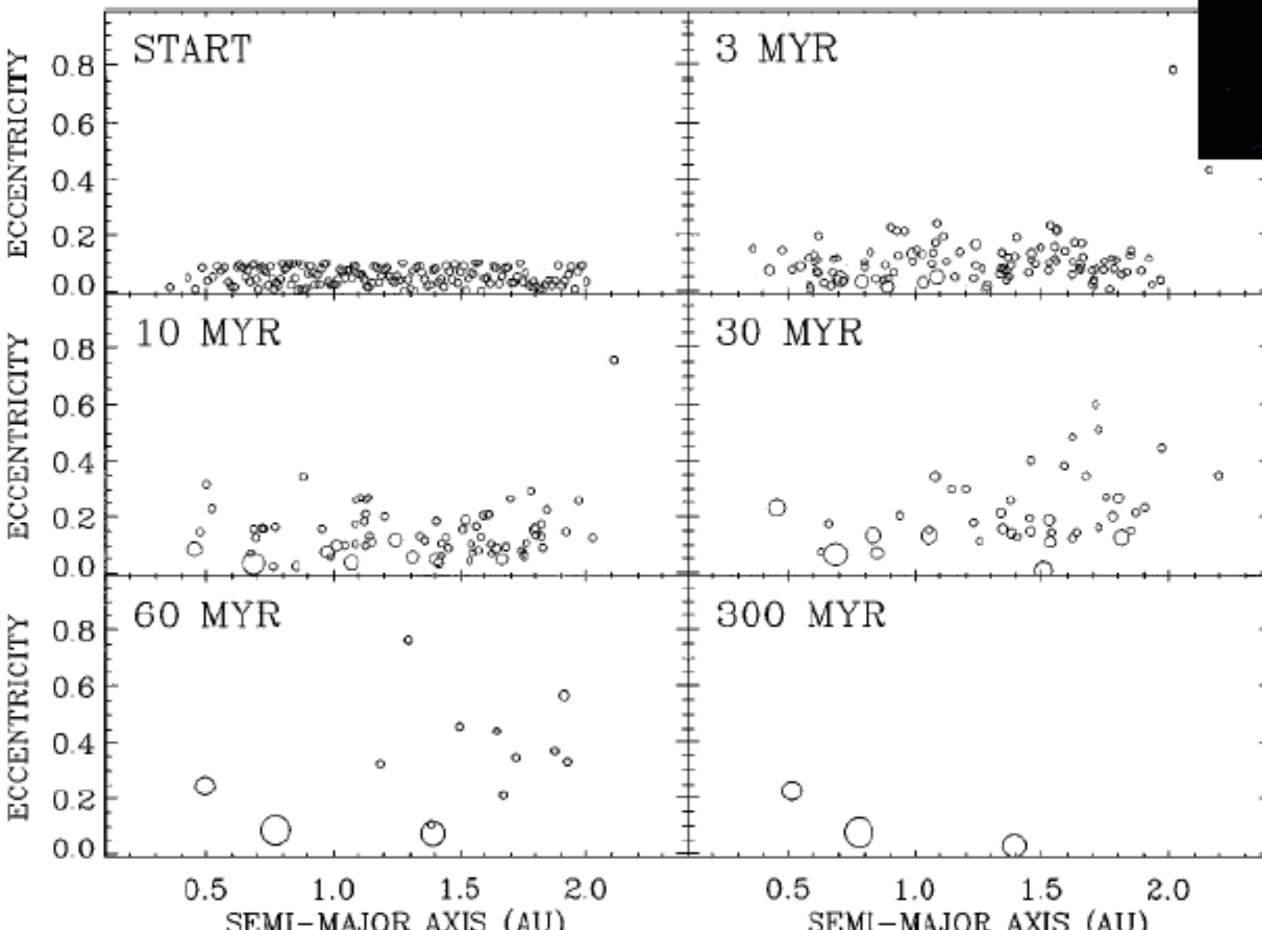


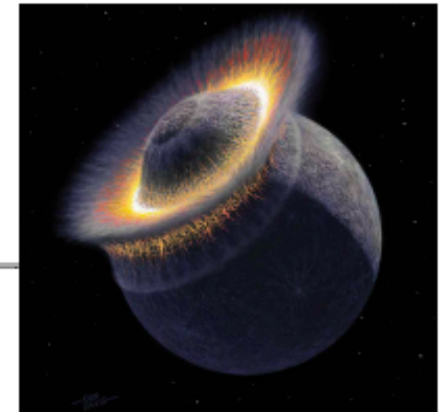
Dynamical Friction

Growth



Giant Impacts





Chambers 2002

Oligarchic Growth

- Runaway growth ends when
  - a) velocities of small bodies become sub-Hill
  - b) velocities of small bodies become super-escape
  - c) when Oligarchic growth begins

When Oligarchic growth ends $\Sigma \sim \sigma$

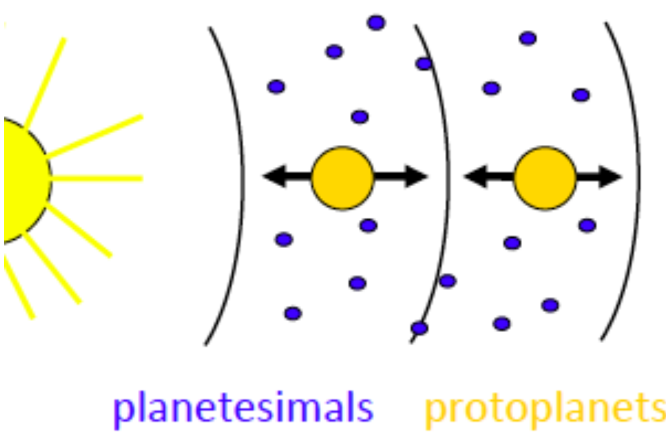
Oligarchic growth maybe the final stage of planet formation in the outer Solar system

Isolation Masses

$$r_H = r\left(\frac{m_p}{3M_*}\right)^{1/3}$$

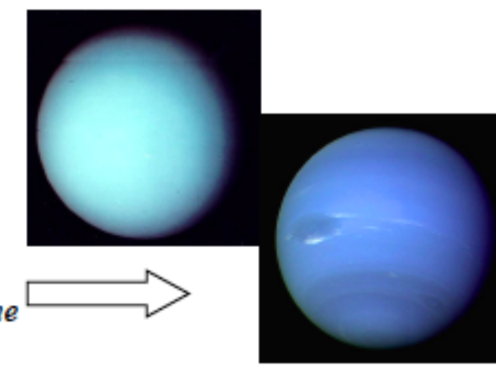
$$m_p = 2\pi r \cdot 2Br_H \cdot \Sigma_s = 4\pi Br^2\Sigma_s\left(\frac{m_p}{3M_*}\right)^{1/3}$$

$$= \frac{(4\pi Br^2\Sigma_s)^{3/2}}{(3M_*)^{1/2}}.$$



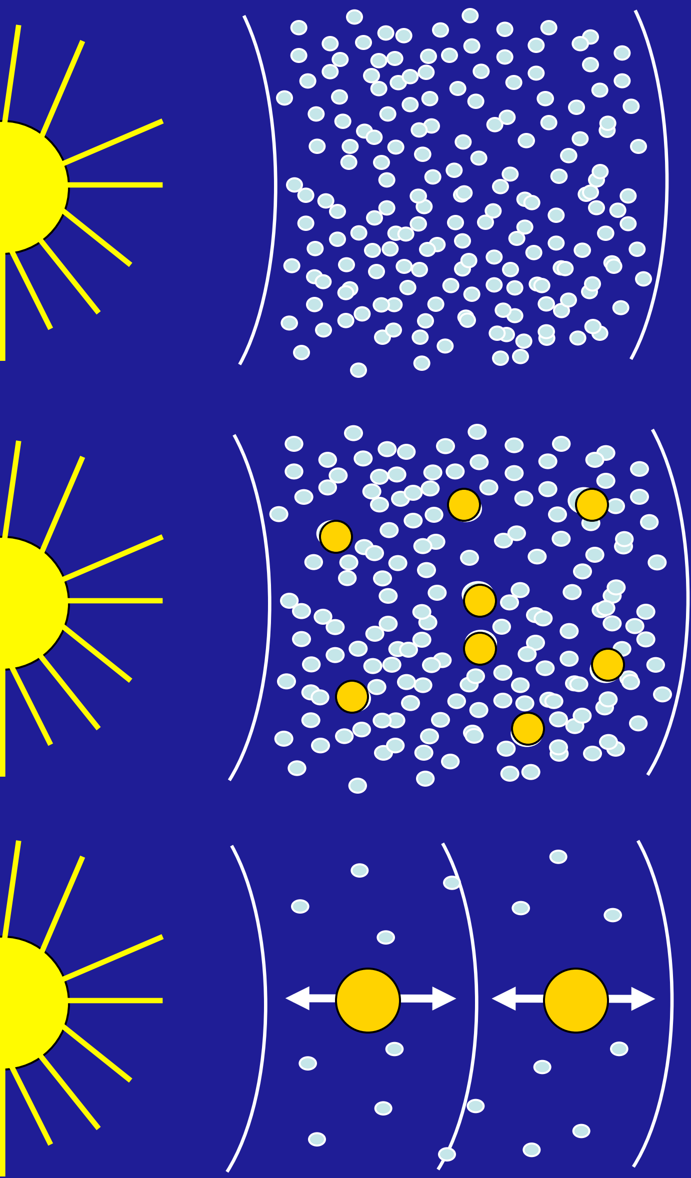
@ 20 AU

$$M_{iso} \approx 2\pi a(\Delta a_{zone})\Sigma \sim M_{Neptune}$$



@ 1 AU

$$M_{iso} \approx 2\pi a(\Delta a_{zone})\Sigma \sim 0.01 M_{Earth}$$



planetesimals protoplanets

u, σ v, Σ

1. Planetesimal formation:

2. Runaway growth:

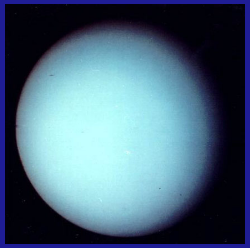
$$\frac{1}{R_1}\frac{dR_1}{dt} \propto R_1$$

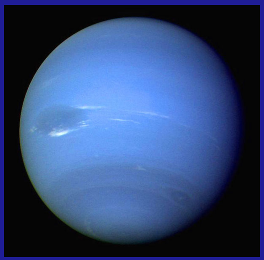
For $v_{esc} > u$ gravitational focusing enhances the accretion rate

$$\frac{1}{R}\frac{dR}{dt} \sim \frac{\sigma\Omega}{\rho R}\left(\frac{v_{esc}}{u}\right)^2 \longrightarrow t_{grow} \sim 10^7 \text{ years}$$

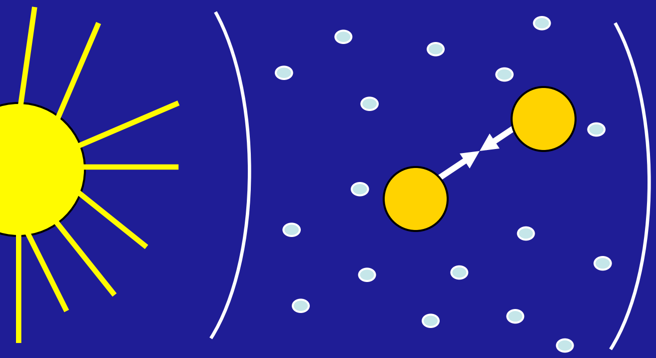
3. Oligarchic growth & Isolation:

$$M_{iso} \approx 2\pi a(\Delta a_{zone})\Sigma \sim M_{Neptune}$$





Last Stages of Terrestrial Planet Formation

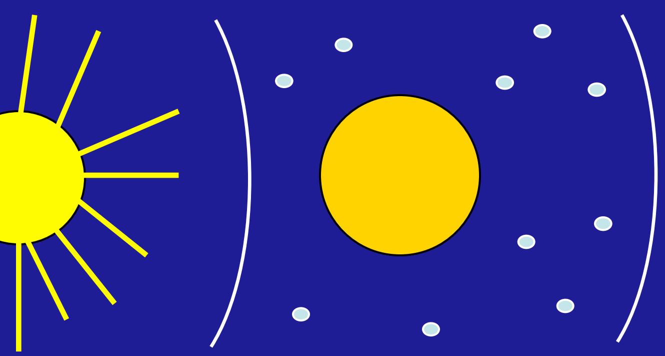


Giant Impacts:

Protoplanets' velocity dispersion increases

→ Giant Impacts

$t_{Giant\text{-}Impacts}$ ~10^8 years (1AU)



Clean up:

- Orbits planar & circular

- Accretion & ejection of remaining planetesimals