Alchemy for the 21st Century: Topological phenomena in periodically driven systems

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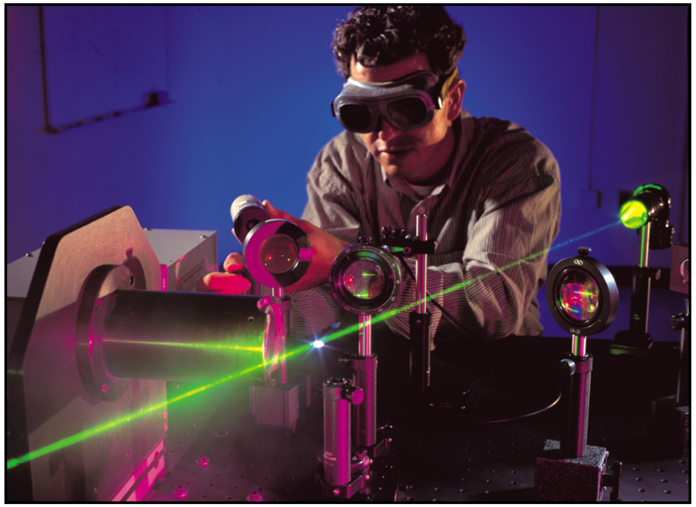


~100 years after Bohr, the basic laws and players are established

1913

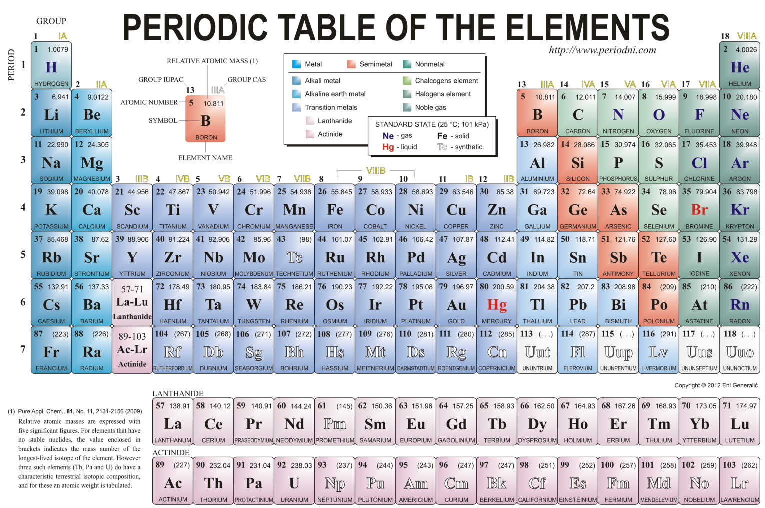
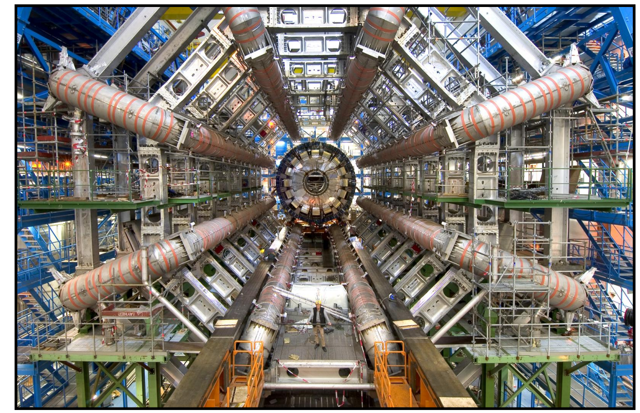
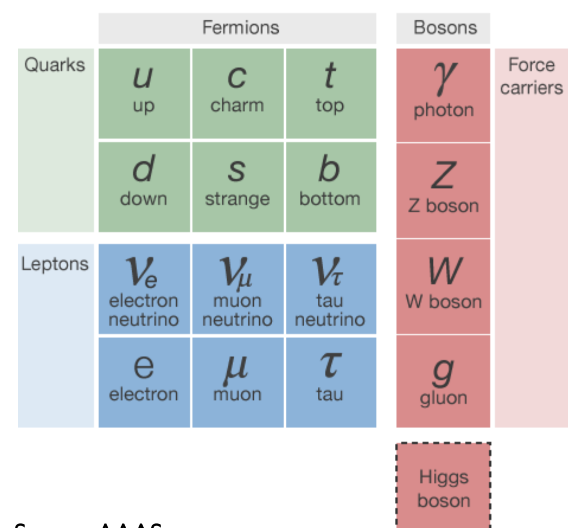


Image from www.periodni.com

2013





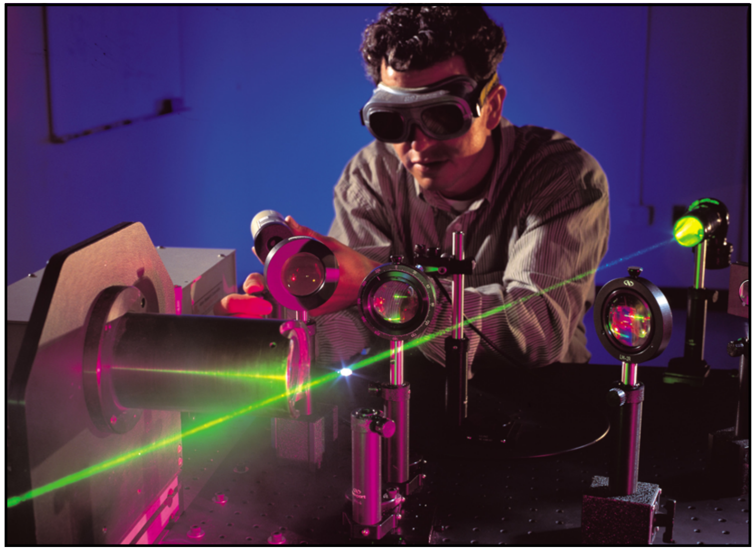
Source: AAAS

What new robust quantum phenomena can we realize through time-dependent driving?

Past

$Pb \rightarrow Au$

Present



$Si \rightarrow HgTe?$

$\rightarrow \quad \ldots \quad ?$

The Plan

I. Robust quantization and topology

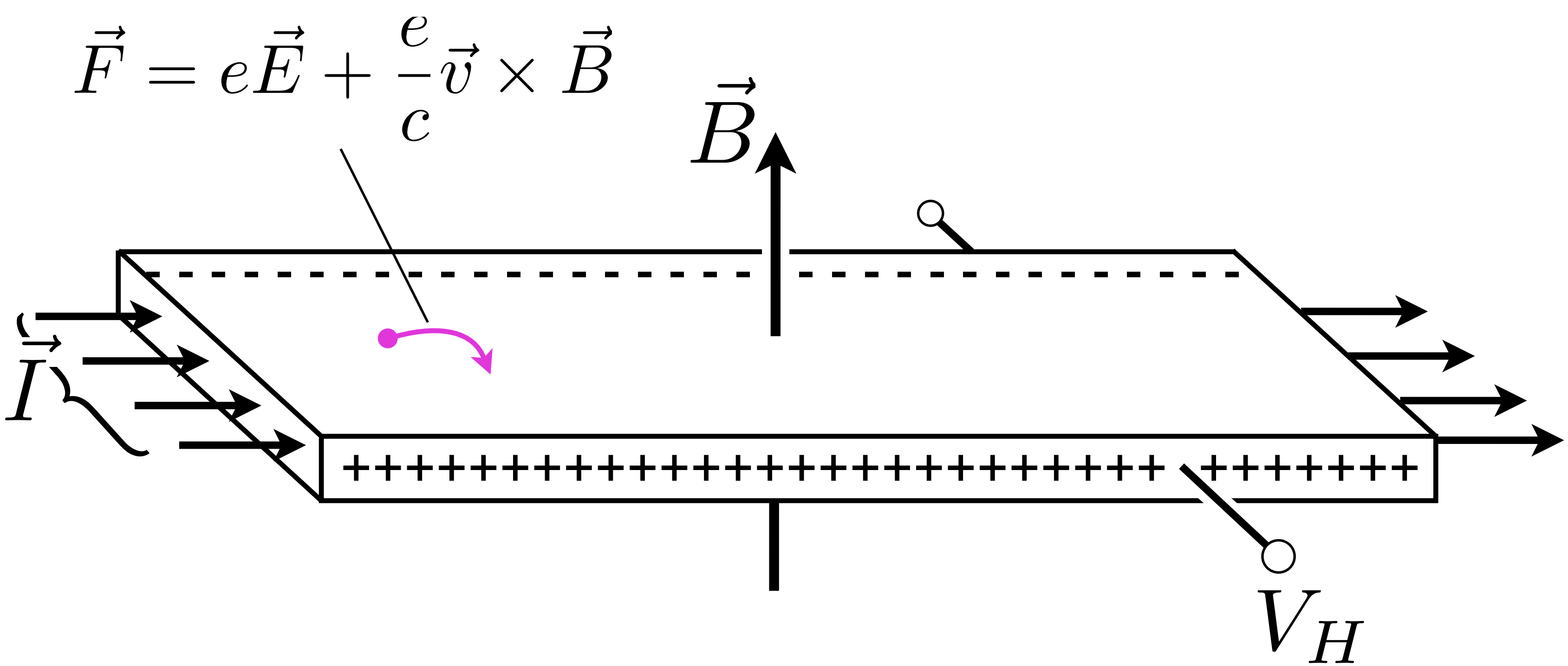
II. New concepts in periodically driven systems

III. Many-body dynamics and open questions

Part 1

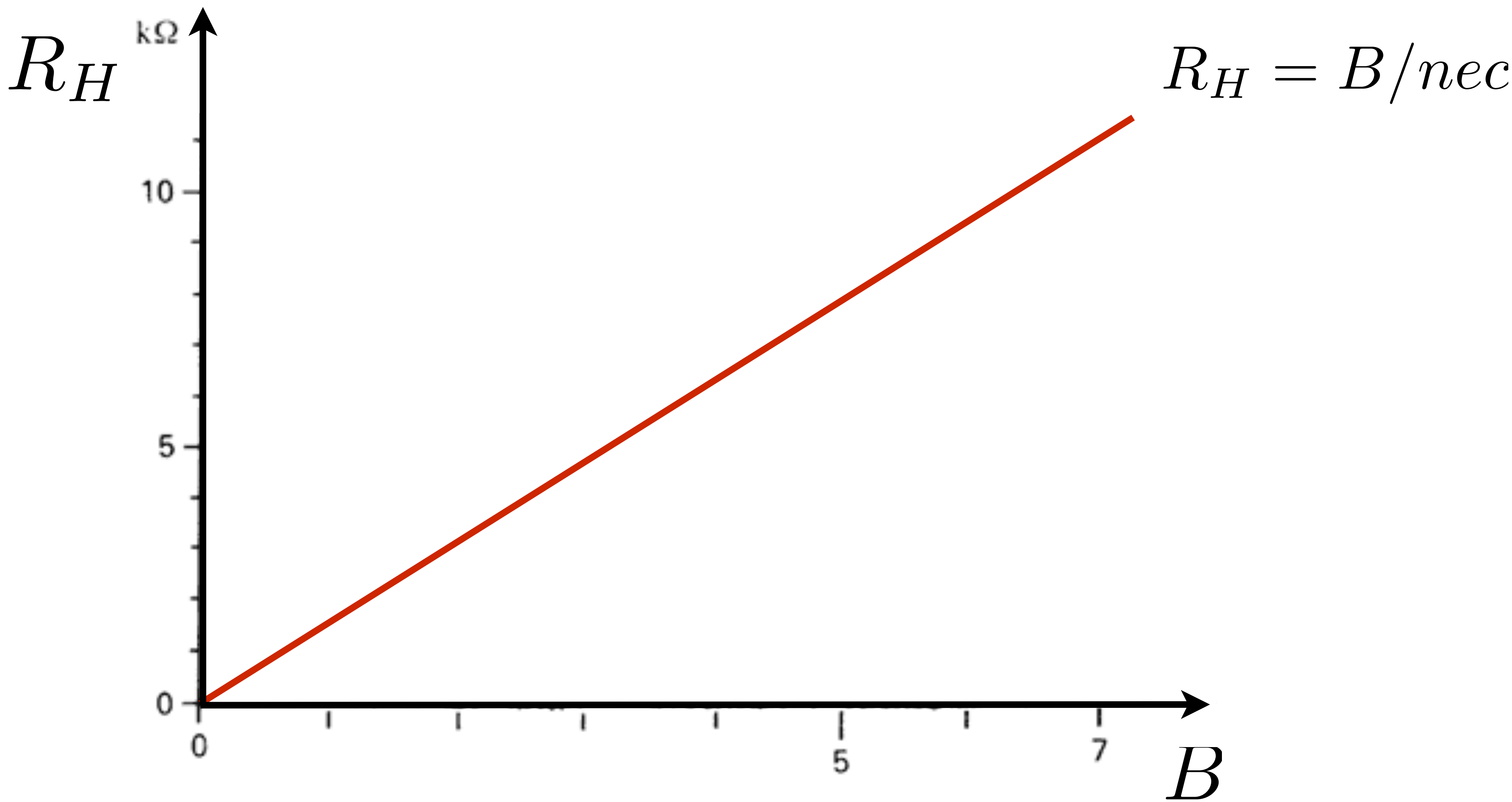
Robust quantization is linked to topology

Out-of-plane magnetic field generates voltage transverse to applied current



$$R_H \equiv V_H / I$$

Classically, Hall resistance is proportional to B



Hall resistance features extremely flat steps at low T, high B

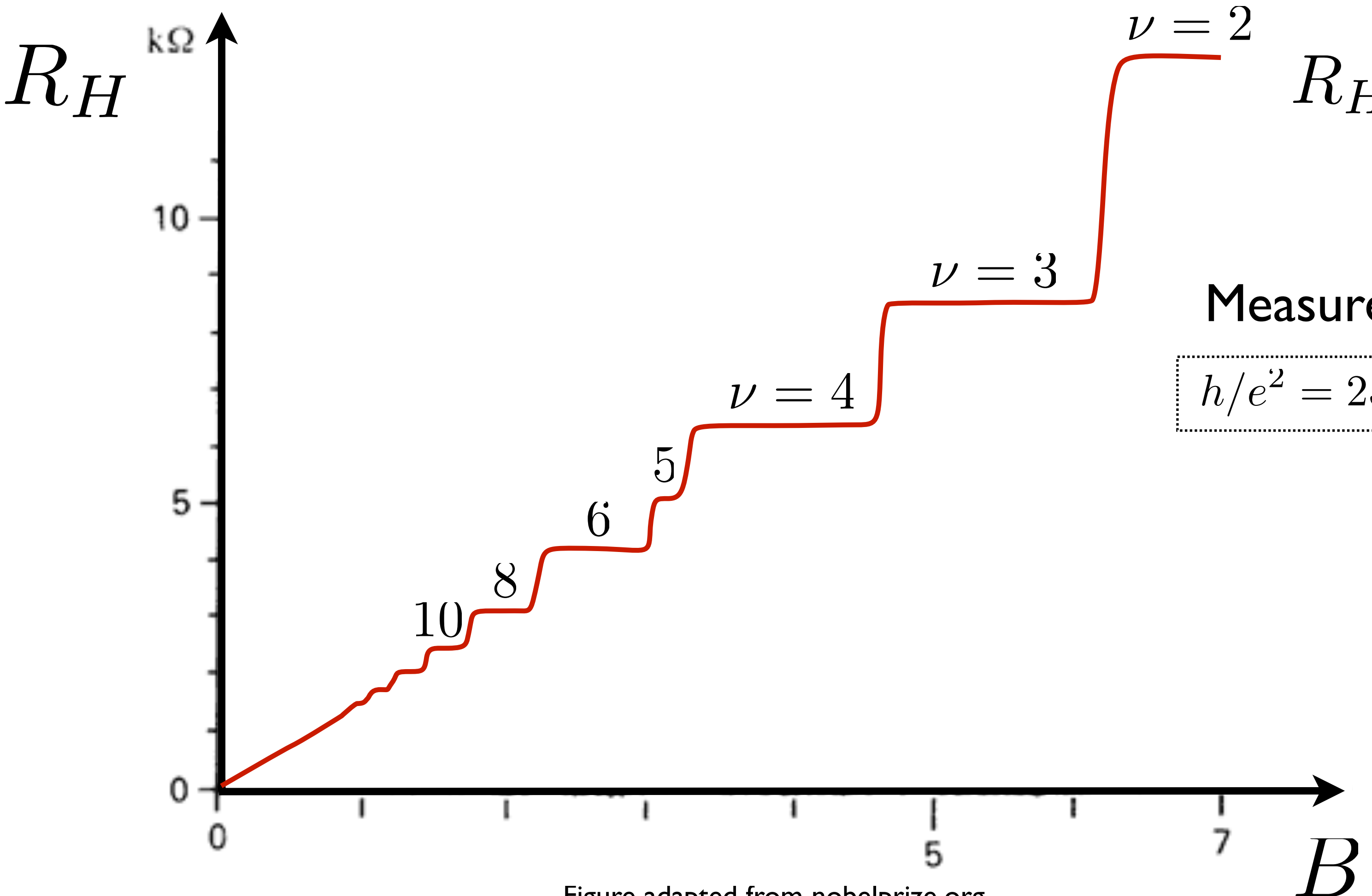


Figure adapted from nobelprize.org

$R_H = h/\nu e^2$

Measurement Standard

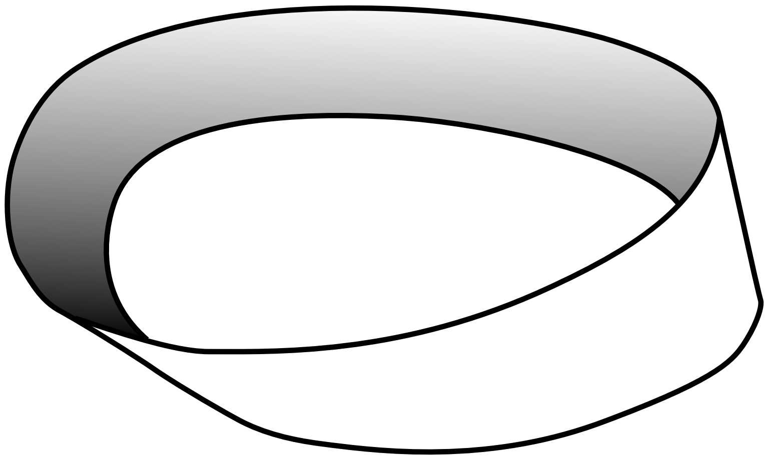
$h/e^2 = 25812.807557(18)\,\Omega$

Key theoretical insight:

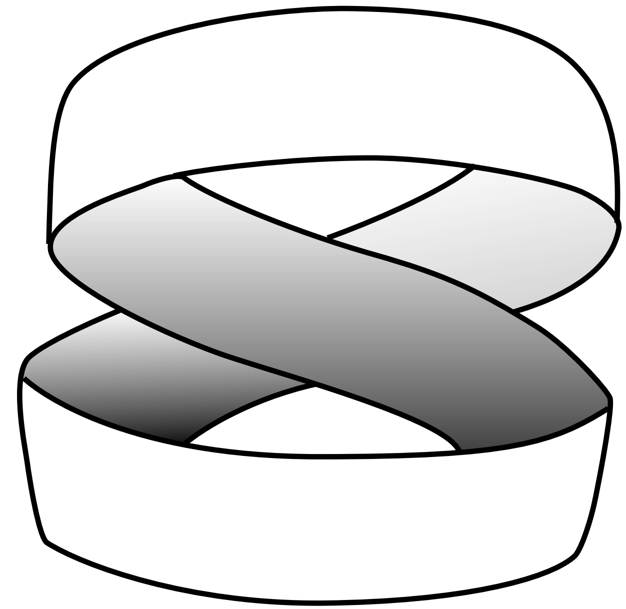
D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405 (1982).

J. E. Avron, R. Seiler, and B. Simon, Phys. Rev. Lett. **51**, 51 (1983).

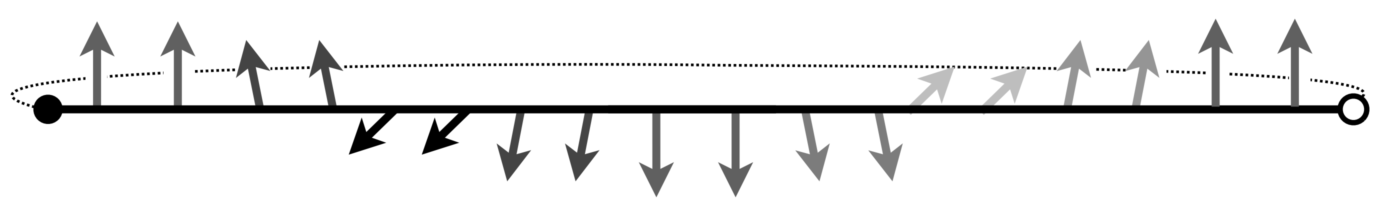
Topologically distinct objects cannot be smoothly interconverted



Simple Loop



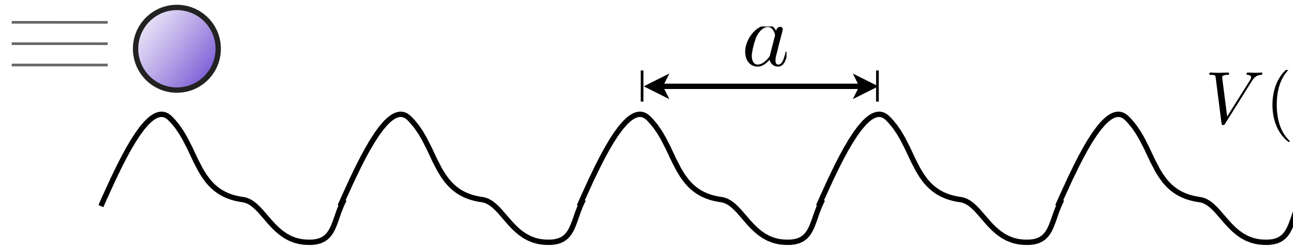
Twisted Strip



Momentum is conserved for particle in constant potential

$$V(x) = V_0$$

Crystal momentum is conserved for particle in a <u>periodic</u> potential



$$V(x+a) = V(x)$$

Translation operator T_a commutes with Hamiltonian, can be simultaneously diagonalized:

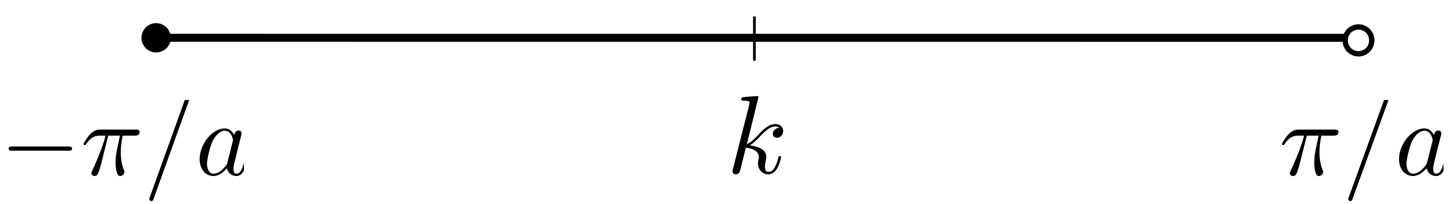
$$H|\,\psi_{nk}\,\rangle = E_{nk}|\,\psi_{nk}\,\rangle$$

$$T_a|\,\psi_{nk}\,\rangle = e^{ika}|\,\psi_{nk}\,\rangle$$

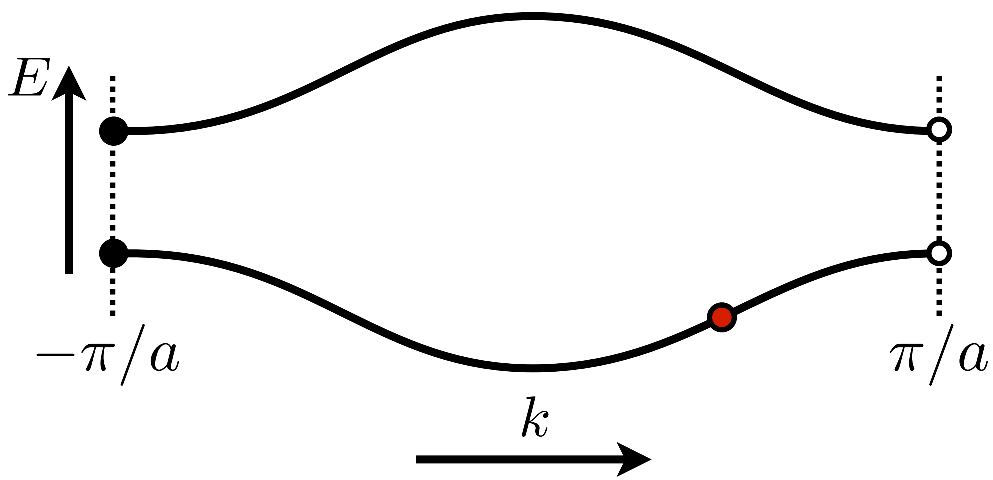
Crystal momentum lives on a circle, $-\pi/a \leq k < \pi/a$

$$T_a|\,\psi_{nk}\,\rangle = e^{ika}|\,\psi_{nk}\,\rangle$$

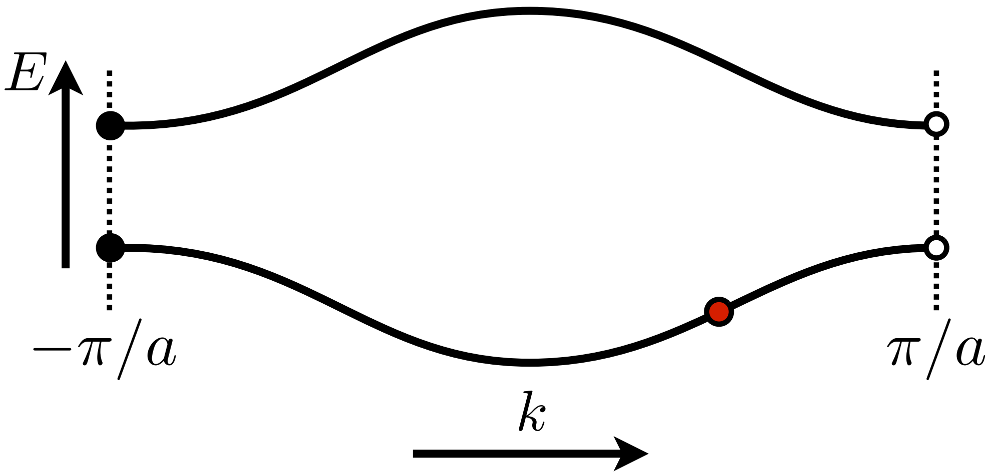
Eigenvalue, state invariant under $k \to k + 2\pi N/a$

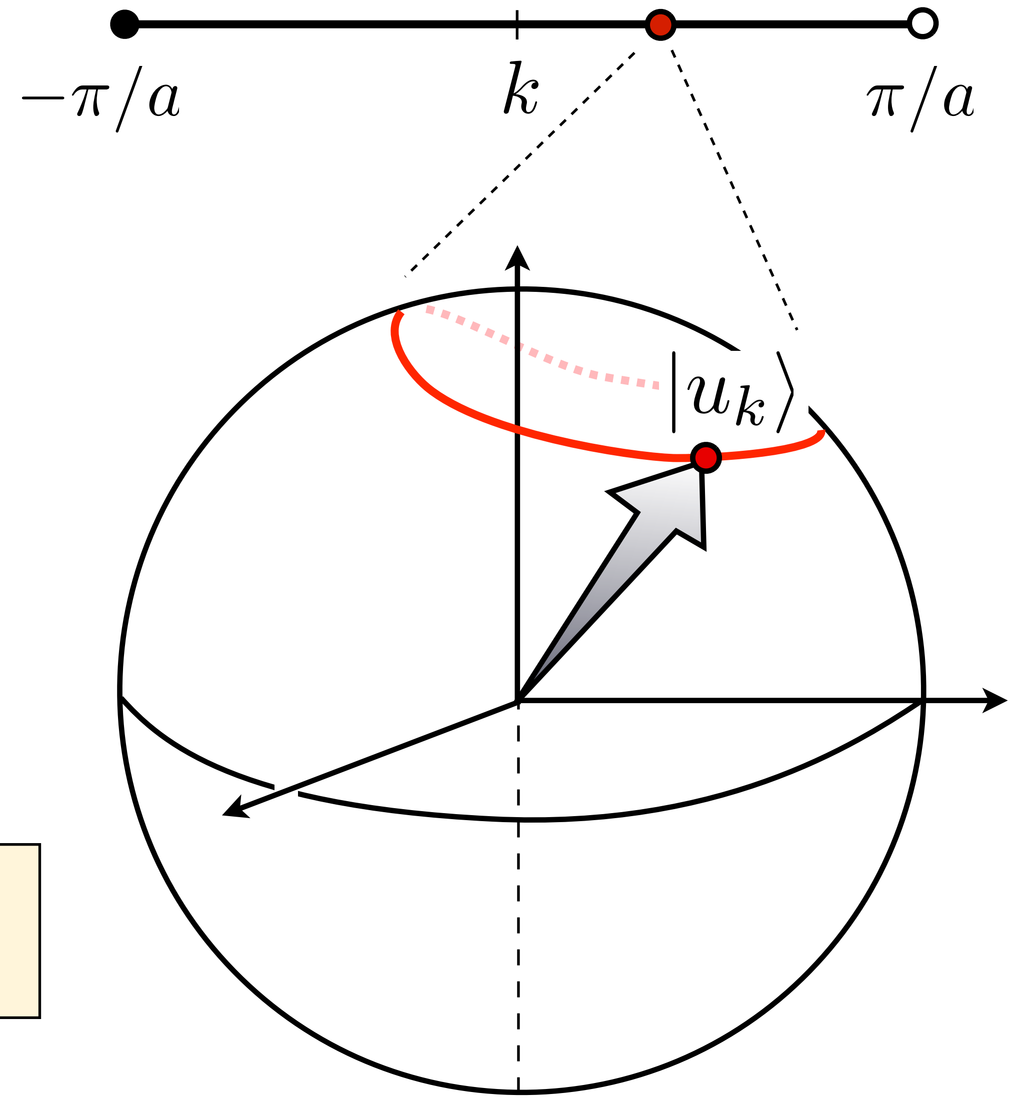


Eigenvalues, eigenvectors are <u>periodic</u> in crystal momentum



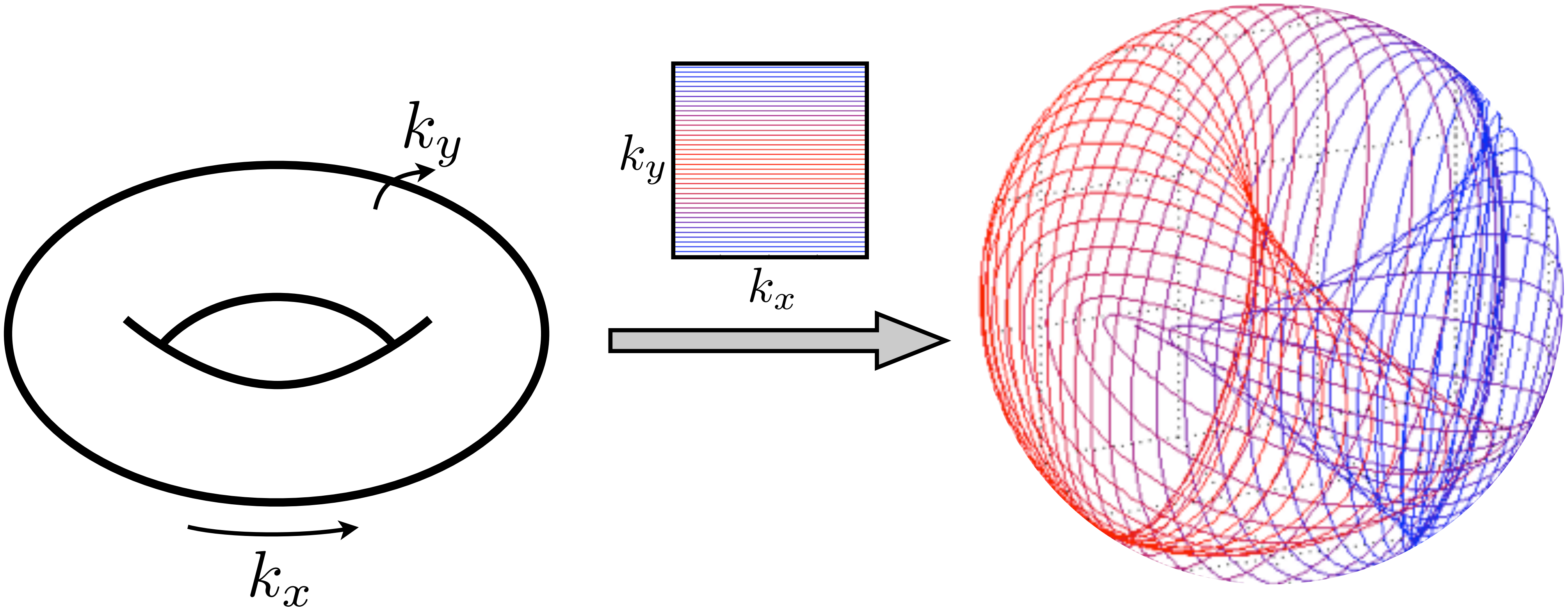
Eigenvalues, eigenvectors are <u>periodic</u> in crystal momentum





Quantum Hall systems, topological insulators: eigenvectors twisted in non-trivial way

Example: wrapping of Bloch sphere for 2D system



TKNN: Hall conductance directly related to Chern invariant!

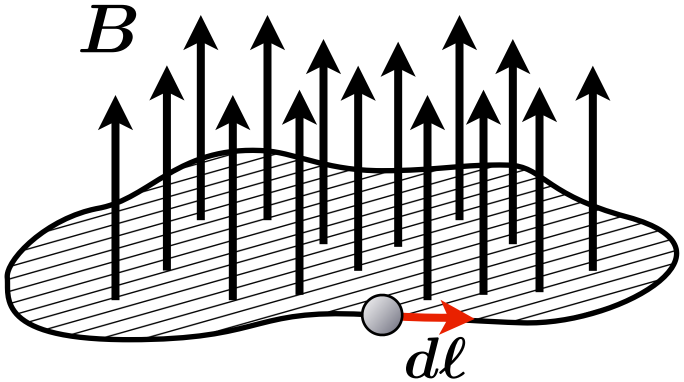
Classic Papers:

D. J. Thouless, M. Kohmoto, M. P. Nightingale, and M. den Nijs, Phys. Rev. Lett. **49**, 405 (1982).

F. D. M. Haldane, Phys. Rev. Lett. **61**, 2015 (1988).

Berry curvature acts as a magnetic field for *phase space*

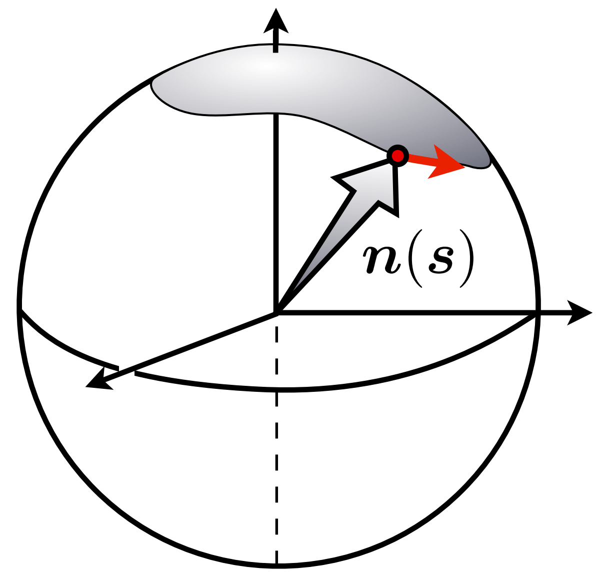
Aharonov-Bohm effect



$$\varphi_{\mathrm{AB}} = \oint \boldsymbol{d\ell} \cdot \boldsymbol{A}$$

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

Berry phase



$$\varphi_{\mathrm{B}} = \oint \boldsymbol{ds} \cdot \mathcal{A}$$

$$\mathcal{A} = \langle \psi_{\boldsymbol{n}} | i \nabla_s | \psi_{\boldsymbol{n}} \rangle, \quad \mathcal{F} = \nabla \times \mathcal{A}$$

Berry curvature acts as a magnetic field for *phase space*

$$H = \varepsilon(p) + V(x)$$

Total energy → H; Kinetic energy → $\varepsilon(p)$; Potential energy → $V(x)$

Semiclassical equations of motion

$$\dot{\boldsymbol{x}} = \frac{d\varepsilon}{d\boldsymbol{p}} + \dot{\boldsymbol{p}} \times \mathcal{F}$$

"Anomalous velocity" → $\dot{\boldsymbol{p}} \times \mathcal{F}$

$$\dot{\boldsymbol{p}} = -\frac{dV}{d\boldsymbol{x}} + \dot{\boldsymbol{x}} \times \boldsymbol{B}$$

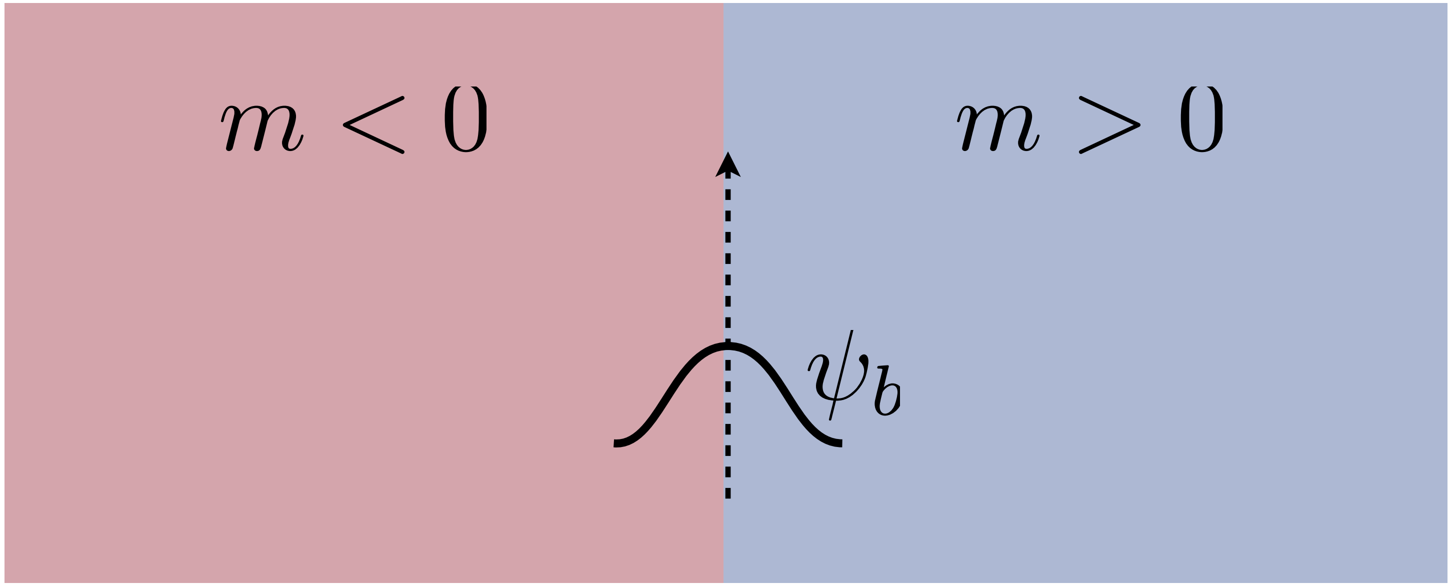
Lorentz force → $\dot{\boldsymbol{x}} \times \boldsymbol{B}$

For reference, see:

D. Xiao, M.-C. Chang, Q. Niu, Rev. Mod. Phys. **82**, 1959 (2010).

Nontrivial topology revealed by “protected” boundary modes

Dirac equation with mass domain wall



$$H = -iv(\sigma_x \partial_x + \sigma_y \partial_y) + m(x)\sigma_z$$

Altand-Zirnbauer symmetry classes define "Periodic table of topological insulators and superconductors"

		TRS	PHS	SLS	$d=1$	$d=2$	$d=3$
standard (Wigner-Dyson)	A (unitary)	0	0	0	-	$\mathbb{Z}$	-
	AI (orthogonal)	$+1$	0	0	-	-	-
	AII (symplectic)	-1	0	0	-	$\mathbb{Z}_2$	$\mathbb{Z}_2$
chiral (sublattice)	AIII (chiral unitary)	0	0	1	$\mathbb{Z}$	-	$\mathbb{Z}$
	BDI (chiral orthogonal)	$+1$	$+1$	1	$\mathbb{Z}$	-	-
	CII (chiral symplectic)	-1	-1	1	$\mathbb{Z}$	-	$\mathbb{Z}_2$
BdG	D	0	$+1$	0	$\mathbb{Z}_2$	$\mathbb{Z}$	-
	C	0	-1	0	-	$\mathbb{Z}$	-
	DIII	-1	$+1$	1	$\mathbb{Z}_2$	$\mathbb{Z}_2$	$\mathbb{Z}$
	CI	$+1$	-1	1	-	-	$\mathbb{Z}$

Table from A. P. Schnyder et al., Phys. Rev. B **78**, 195125 (2008).
See also A.V. Kitaev, arXiv:0901.2686 (2009).

The Plan

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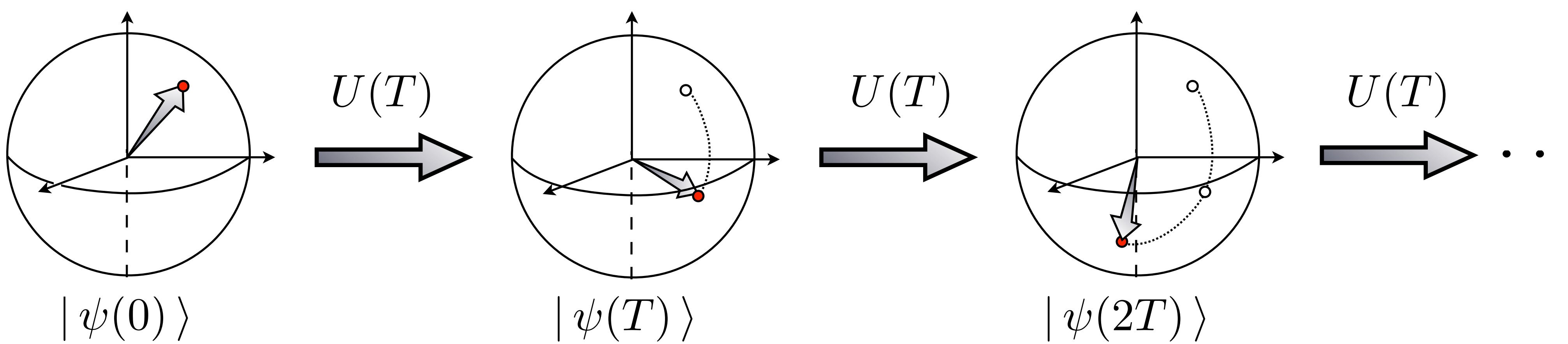
No ground state, energy conservation for driven system

$$i\frac{d}{dt}|\,\psi\,\rangle = H(t)|\,\psi\,\rangle; \quad H(t+T) = H(t)$$

↑ periodic driving

Quasi-energy is conserved for system with discrete time translation symmetry

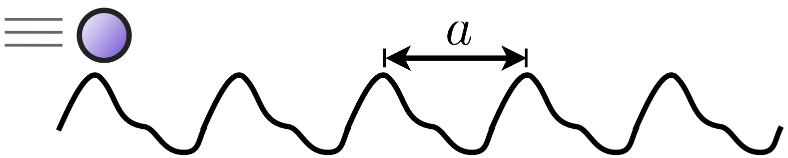
$$U(T)|\,\psi_n\,\rangle = e^{-i\varepsilon_n T}|\,\psi_n\,\rangle$$



$$U(T) = \mathcal{T} e^{-i\int_0^T H(t)dt}$$

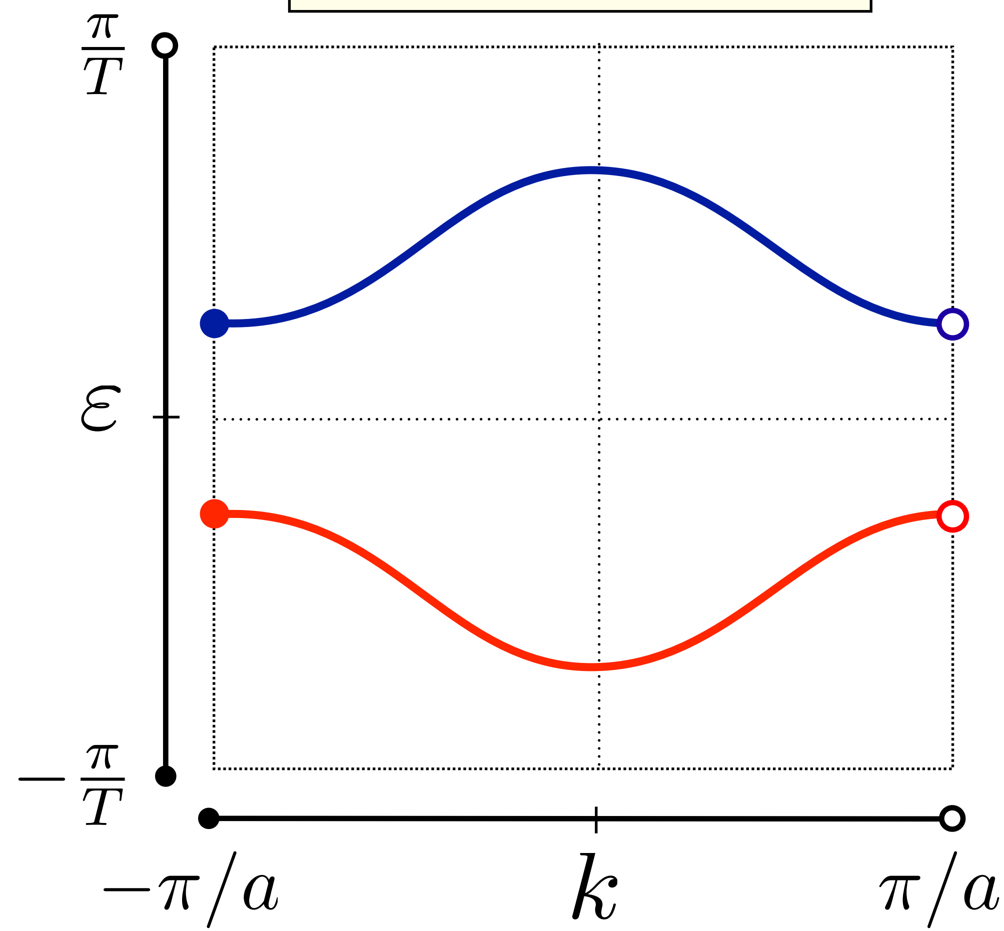
Eigenvalue invariant under $\varepsilon_n \to \varepsilon_n + 2\pi N/T$: quasi-energy lives on a circle

On a lattice find Floquet bands, similar to static system



$$V(x+a) = V(x)$$

$$U(T) \equiv e^{-iH_{\text{eff}}T}$$

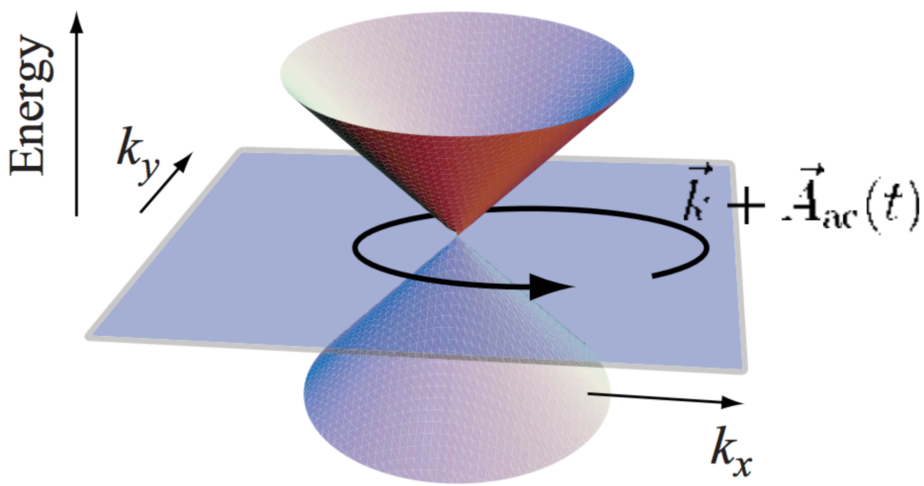


Suggests analogues of topological phenomena from static systems in driven systems

T. Kitagawa, E. Berg, MR, and E. A. Demler, Phys. Rev. B 82, 235114 (2010).

Optical control of band topology discussed for various setups

Circularly-polarized light opens Haldane gap in graphene



T. Oka and H. Aoki, Phys. Rev. B **79**, 081406 (2009).

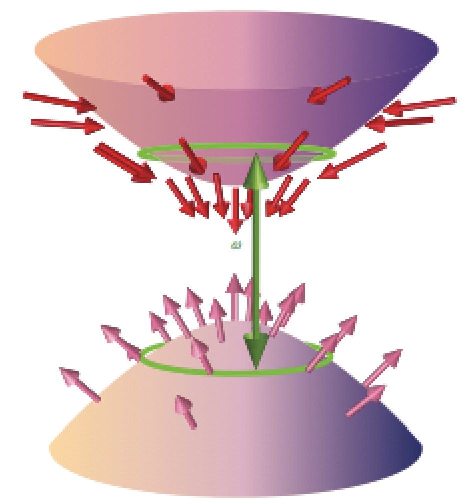
T. Kitagawa, et al., Phys. Rev. B **84**, 235108 (2011).

Resonant driving used to create band inversion

ARTICLES
PUBLISHED ONLINE: 13 MARCH 2011 | DOI: 10.1038/NPHYS1926
nature physics

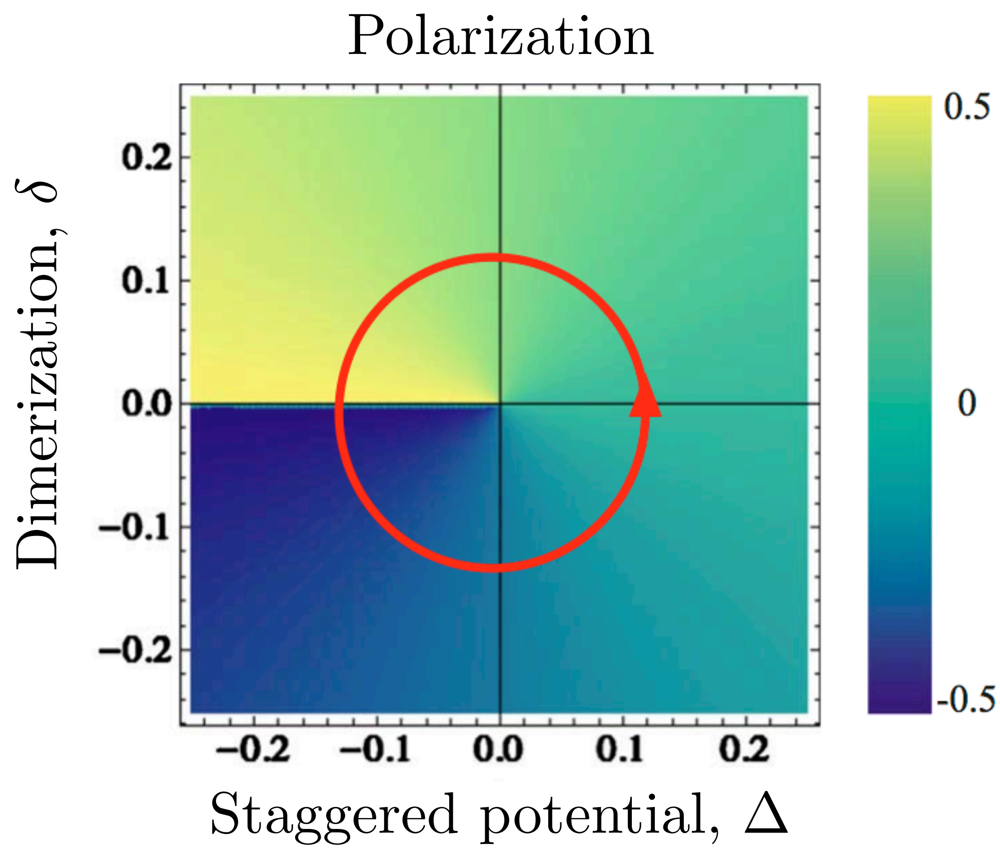
Floquet topological insulator in semiconductor quantum wells

Netanel H. Lindner[1,2]★, Gil Refael[1,2] and Victor Galitski[3,4]

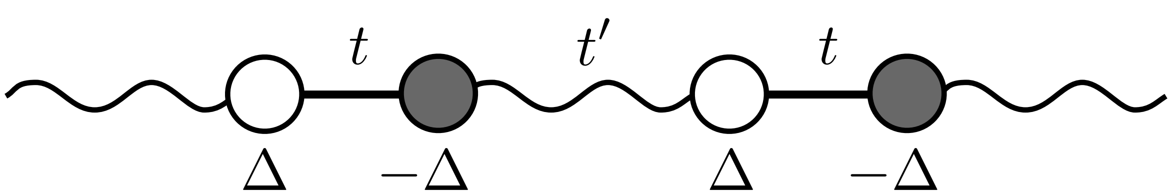


N. Lindner, G. Refael, and V. Galitski, Nature Physics **7**, 490 (2011).

Gapped system: charge pumped via adiabatic cycle is quantized



Rev. Mod. Phys. **82**, 1959 (2010)

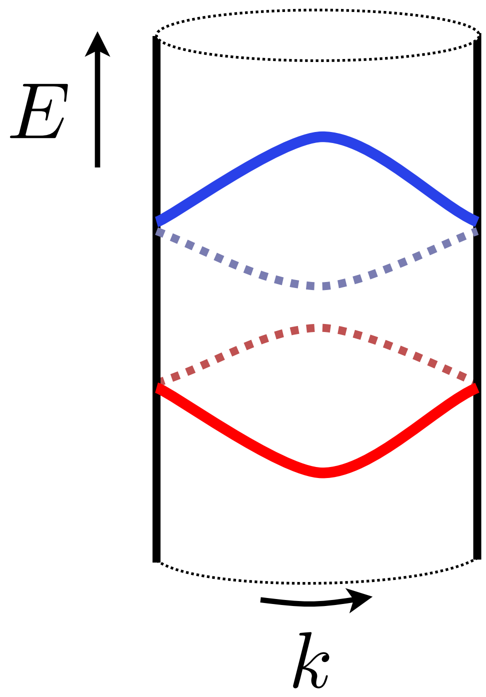


$$t = t_0 + \delta$$
$$t' = t_0 - \delta$$

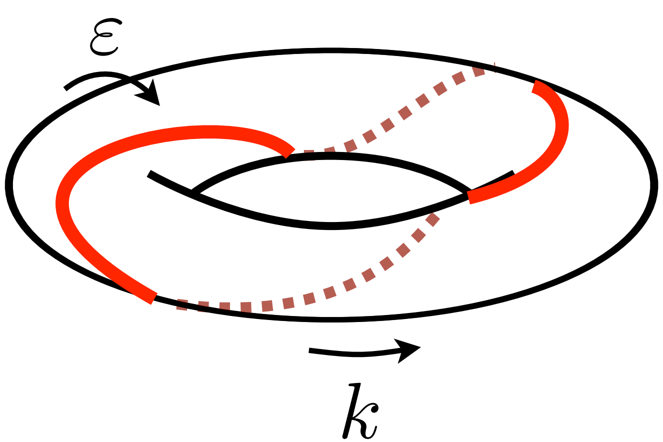
D. J. Thouless, Phys. Rev. B **27**, 6083 (1983).

New topological configurations possible in driven systems

Normal band structure: <u>cylinder</u>



Quasi-band structure: <u>torus</u>



Quasi-energy winding related to quantized adiabatic transport

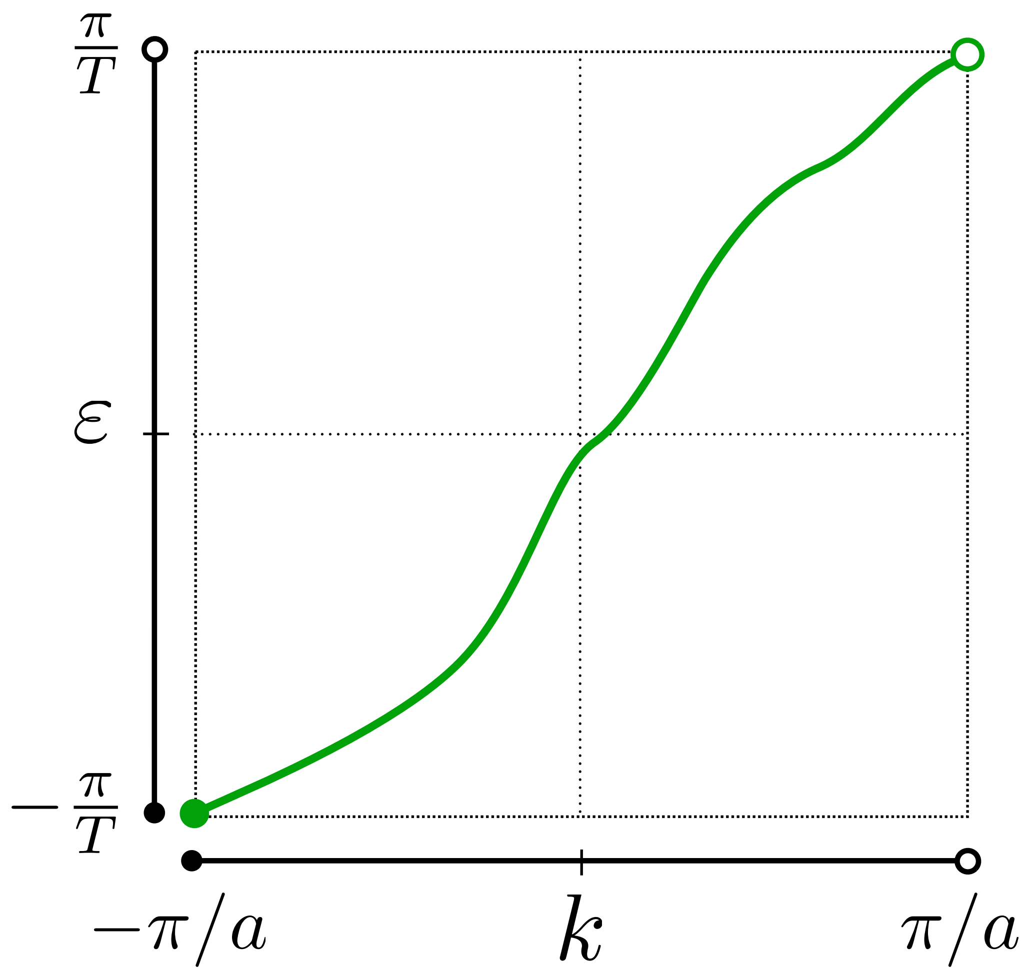
Average group velocity quantized

$$\bar{v}_g = \overline{\frac{d\varepsilon_k}{dk}} = a/T$$

Average displacement:

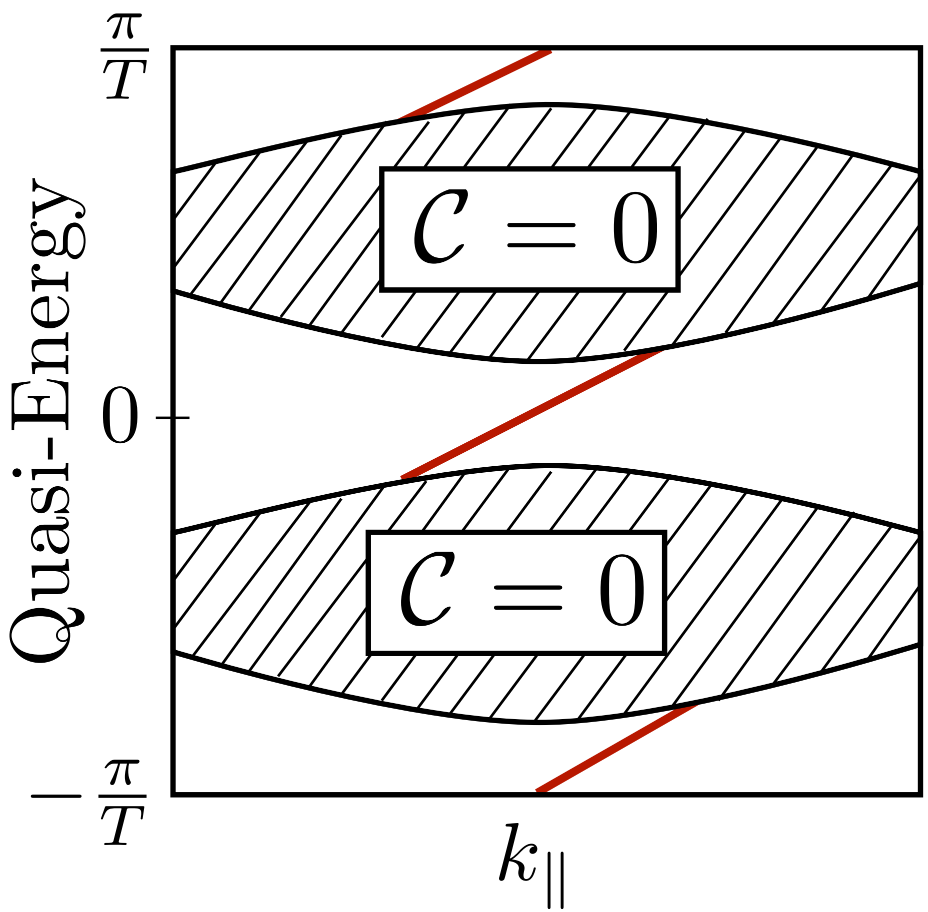
$$\overline{\Delta x} = \bar{v}_g\, T = a$$

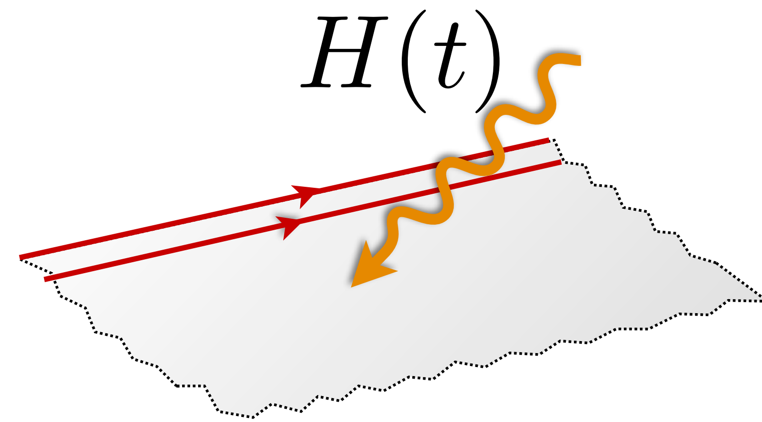
shift by one unit cell



T. Kitagawa, E. Berg, MR, and E. A. Demler, Phys. Rev. B 82, 235114 (2010).

Driven 2D systems may support chiral edge modes even when all Chern numbers are zero!





Chiral edge modes for $\mathcal{C} = 0$ bands

T. Kitagawa, E. Berg, MR, and E. A. Demler, Phys. Rev. B 82, 235114 (2010).
MR, N. Lindner, E. Berg, and M. Levin, Phys. Rev. X **3**, 031005 (2013).

Other examples (Floquet-Majorana, TRS, Chiral symmetry, ...):

L. Jiang *et al.*, Phys. Rev. Lett. **106**, 220402 (2011).
D. Carptentier *et al.*, arXiv:1407.7747 (2014). J. K. Asboth *et al.*, Phys. Rev. B **90**, 125143 (2014).

The Plan

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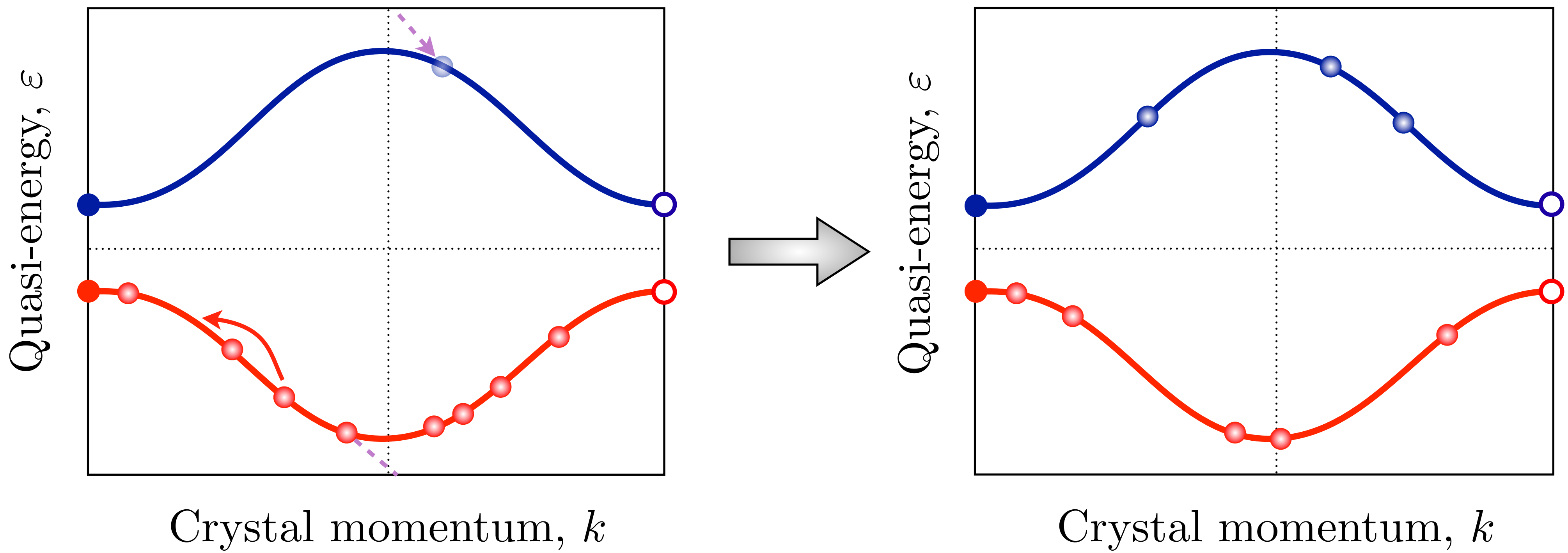
Quasienergy periodicity opens many new scattering channels



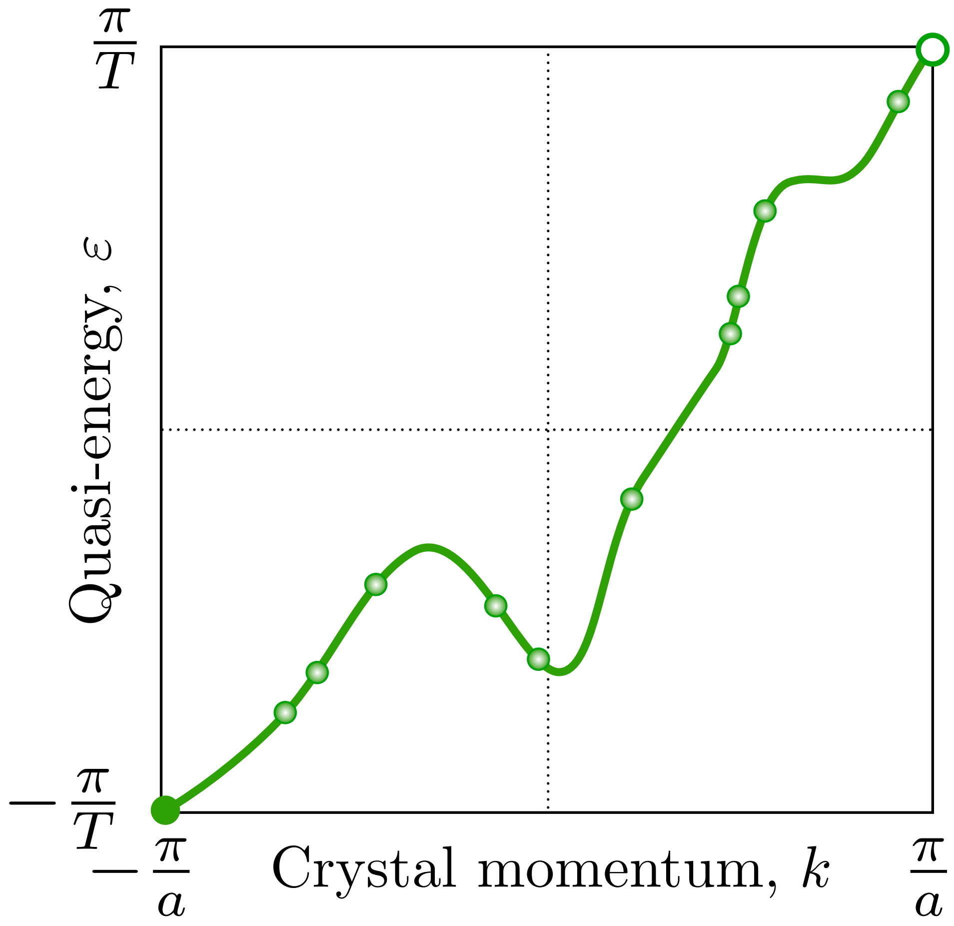
For steady states of closed, interacting systems, see for example:

A. Lazarides, A. Das, and R. Moessner, Phys. Rev. Lett. **112**, 150401 (2014).

L. D'Alessio, M. Rigol, Phys. Rev. X **4**, 041048 (2014).

P. Ponte, A. Chandran, Z. Papic, and D. A. Abanin, arXiv:1403:6480 (2014).

Current carried by partially-filled band can be anything



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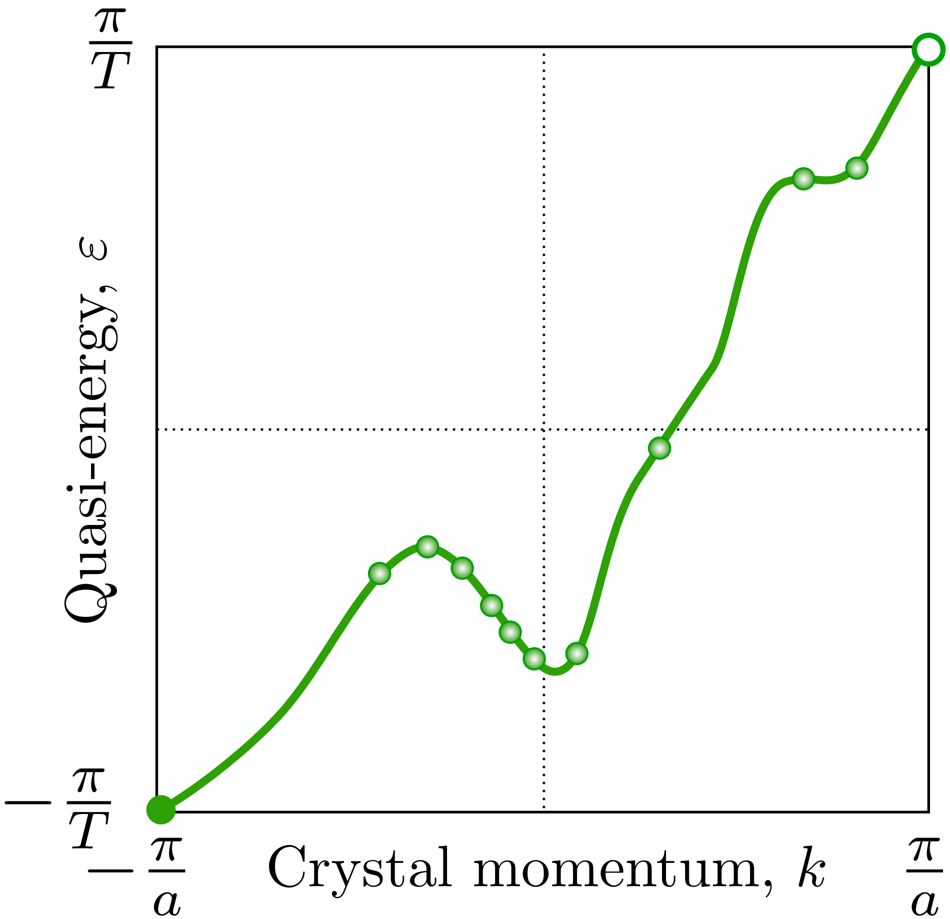
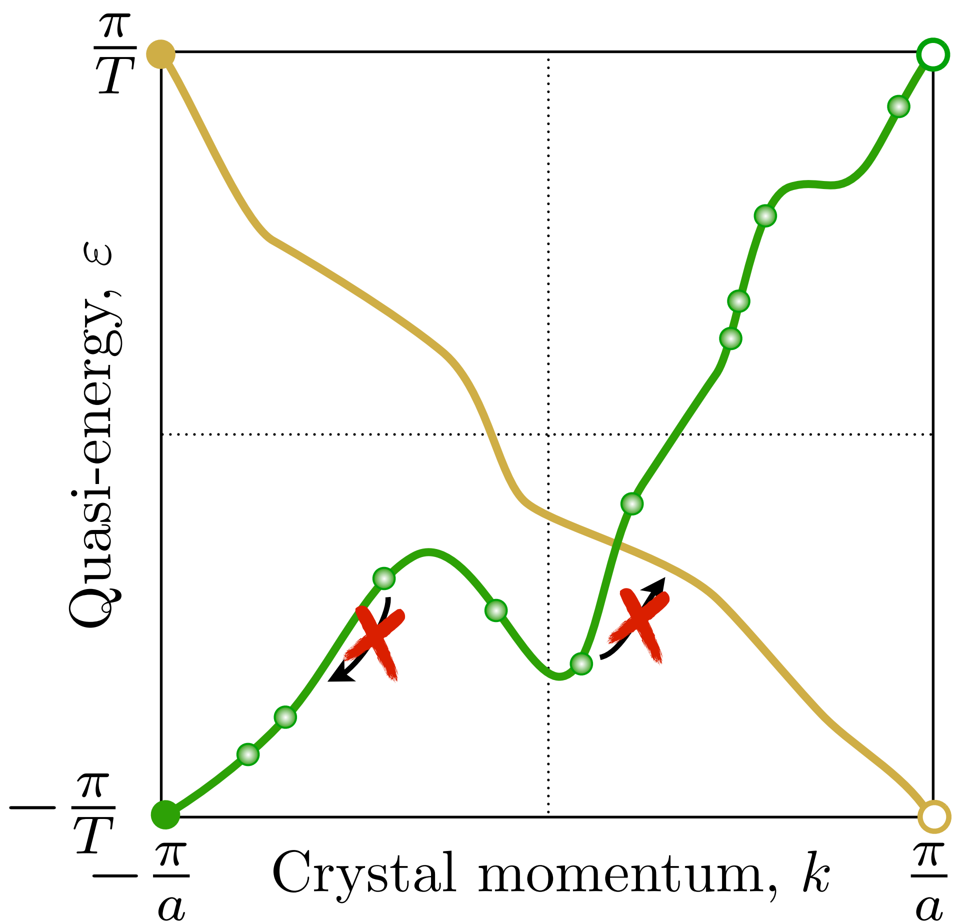


Prediction: chiral state forms at intermediate times with quantized pumping coefficient

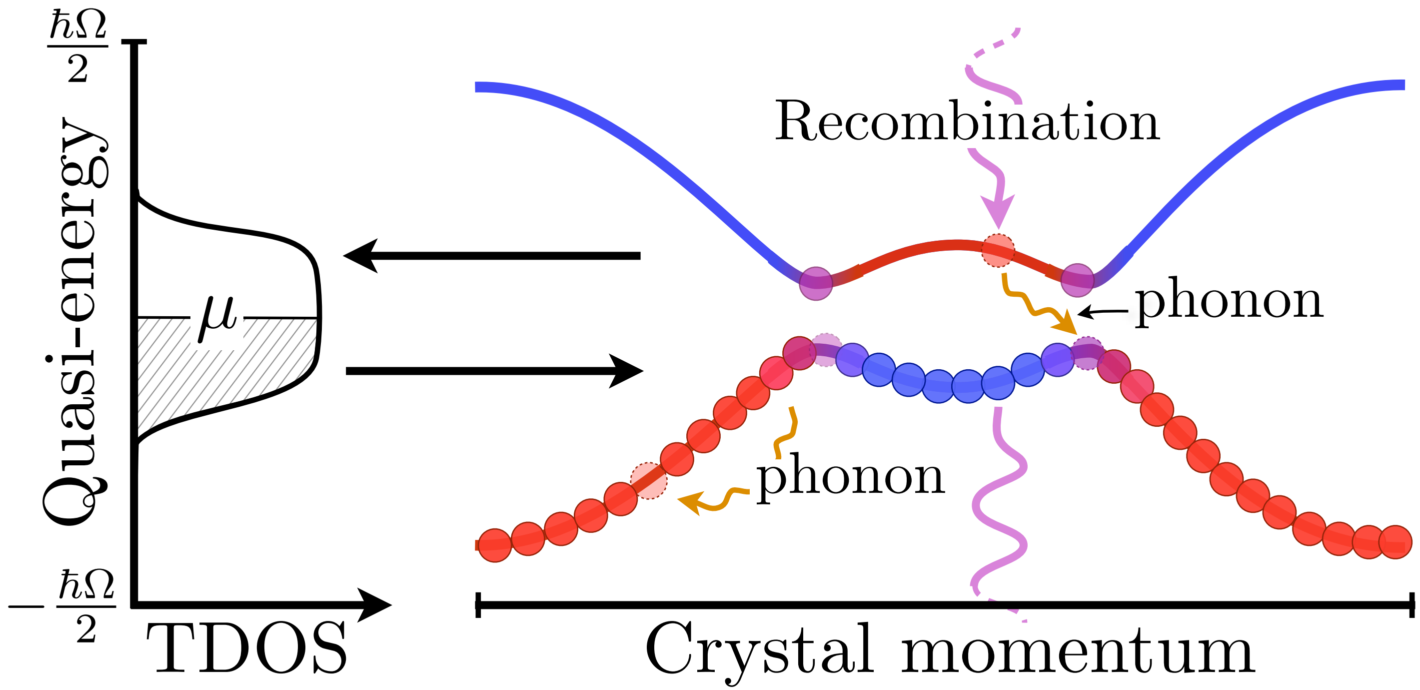


$$(\text{current}) = (1/T) \times (\text{density})$$

MR, E. Berg, and N. Lindner, *in preparation.*

Insulator-like steady states can be reached via bath coupling

Driven semiconductor + Bose, Fermi baths



K. Seetharam, C.-E. Bardyn, N. Lindner, MR, and G. Refael, arXiv:1502.02664 (2015).

see also, for example:

V. M. Galitskii, S. P. Goreslavskii, and V. F. Elesin, JETP **30**, 117 (1970).

T. Shirai, T. Mori, and S. Miyashita, Phys. Rev. E **91**, 030101(R) (2015). D. E. Liu, Phys. Rev. B **91**, 144301 (2015).

H. Dehghani, T. Oka, and A. Mitra, arXiv:1412.8469 (2014). T. Iadecola, T. Neupert, C. Chamon, arXiv:1502.05047 (2015).

Summary and open questions

New class of topological phases identified for periodically driven systems

What are the observable manifestations?

What other new phases are possible? Other dimensions?

Generalizations to interacting systems?

Experiments to identify/study anomalous edge states, etc.

Collaborators: Takuya Kitagawa, Eugene Demler, Netanel Lindner, Erez Berg, Michael Levin, and Gil Refael