Non-equilibrium dynamics in isolated many-particle quantum systems

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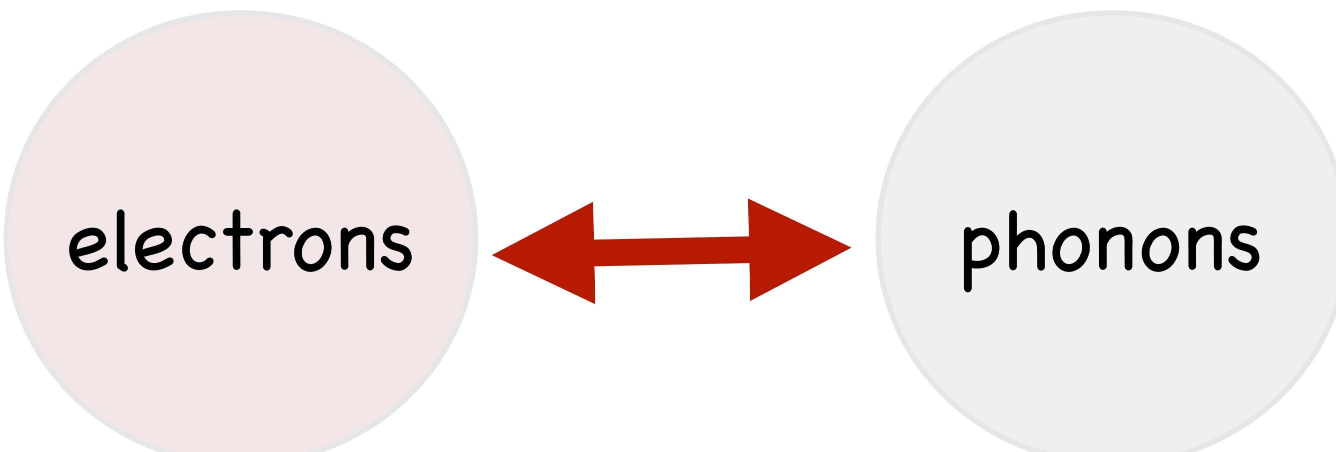
A. Isolated quantum many-particle systems

New area, as these are difficult to realize in the lab.

Electronic degrees of freedom in solids?

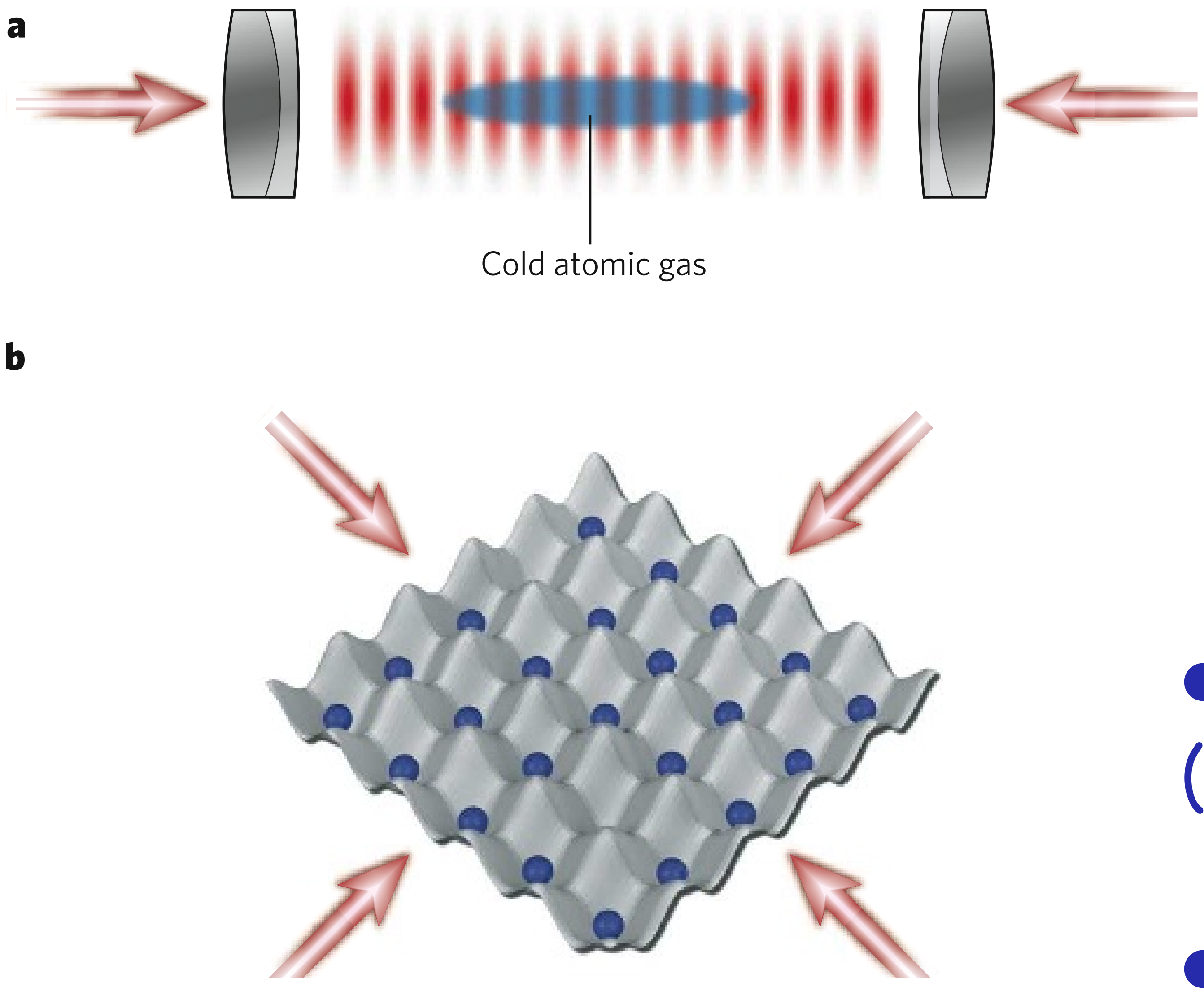
→ coupled e.g. to phonons.



Probability in electronic subsystem not conserved; non-unitary time evolution felt after short times

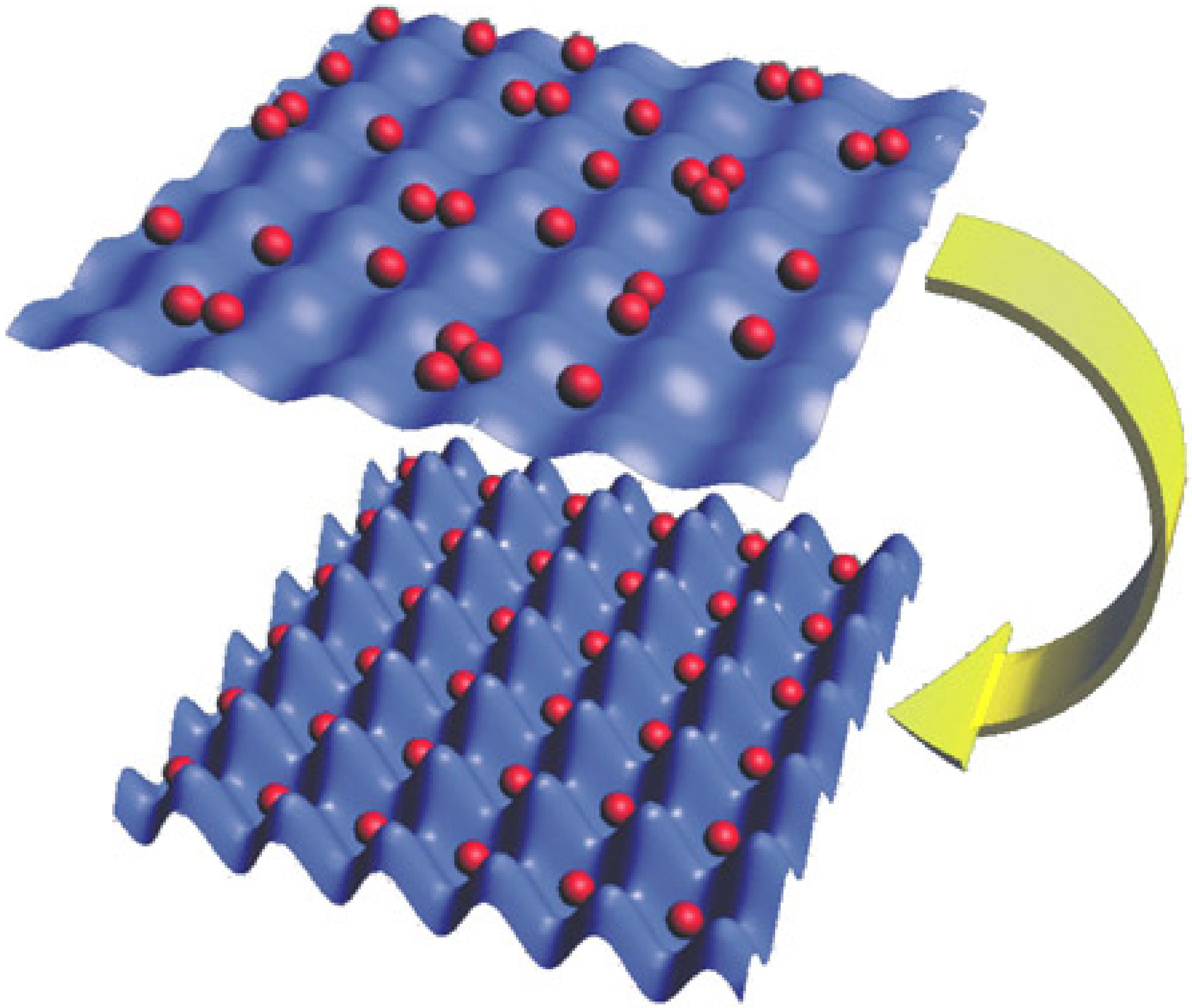
fundamentally
open systems

Ultra-cold atomic gases



Nice properties:

- weak coupling to environment (essentially unitary time evolution on long time scales)
- control over Hamiltonian
- easy to study non-equilibrium physics



prepare the system in ground state of some H(h)

"quench" h, i.e. time-evolve with different H(h′)

Quantum Quenches:

A. Consider a quantum many-particle system with Hamiltonian H(h)

B. Prepare the system in an initial density matrix $\rho(0)$ that does **not** correspond to an eigenstate, e.g. $\rho(0) = |\Psi_0\rangle\langle\Psi_0|$ GS of $H(h_0)$

C. Time evolution $\rho(t) = \exp(-iH(h)t)\ \rho(0)\ \exp(iH(h)t)$

D. Calculate **local** observables $\mathrm{Tr}[\rho(t)\ O(x)]$

Local Quantum Quenches

H(h) and $H(h_0)$ differ only in a small finite region

study propagation of a **local disturbance** e.g. via a 1-point function $\langle \psi(t)| \rho(x)| \psi(t)\rangle$

Injected a **microscopic** $O(L^0)$ amount of energy into the system.

real space probe of excitations over the ground state

Example: Local quench in the S=1/2 XXZ chain

Ganahl, Rabel, Essler& Evertz, '12

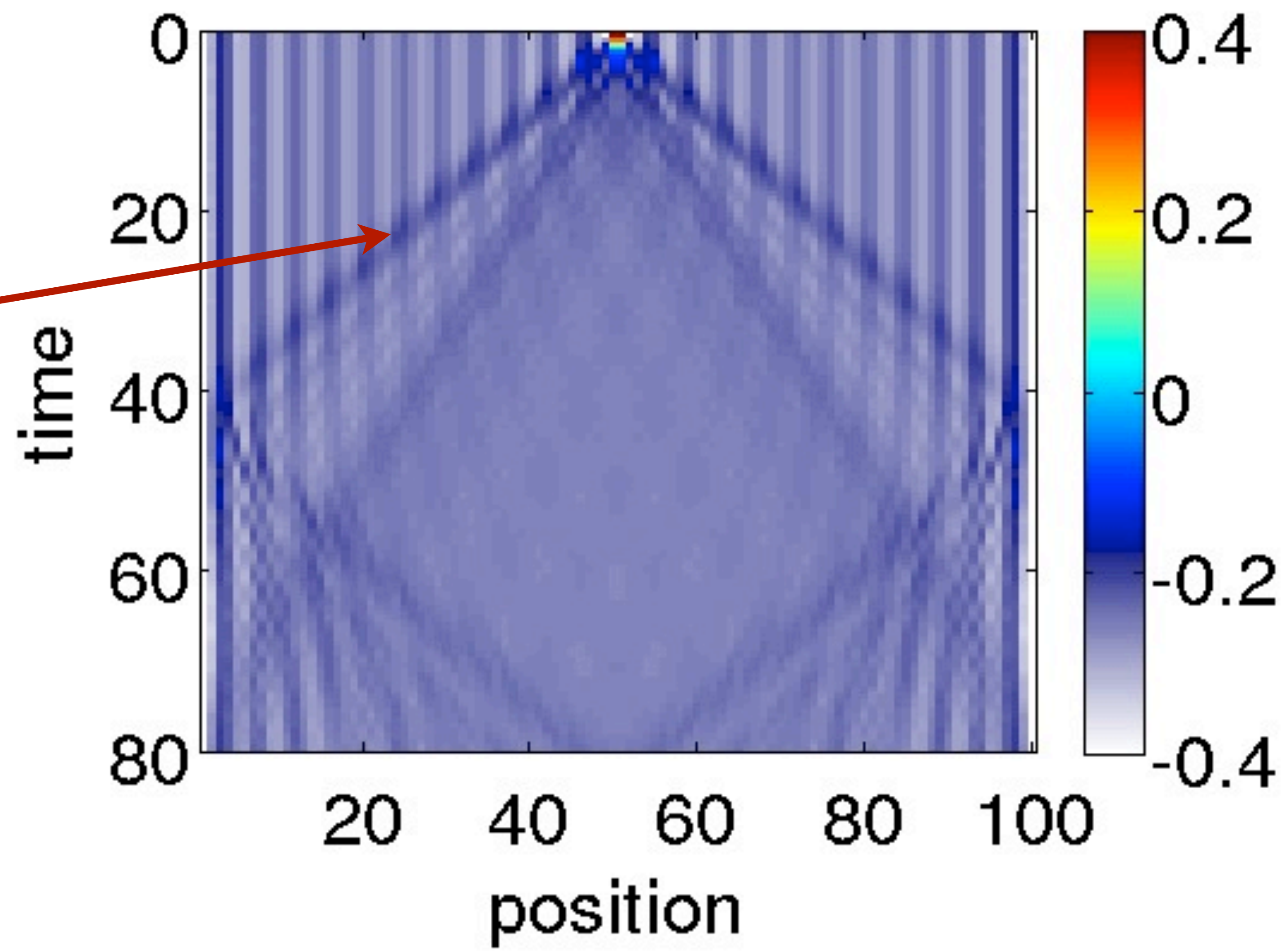
$$H(B) = J\sum_{j=-L/2}^{L/2} S_j^x S_{j+1}^x + S_j^y S_{j+1}^y + \Delta S_j^z S_{j+1}^z + hS_j^z$$

$$+ B[S_{-1}^z + S_0^z + S_1^z] \qquad B \gg J$$

$$|\Psi(0)\rangle = |GS \ \uparrow\uparrow\uparrow \ GS\rangle$$

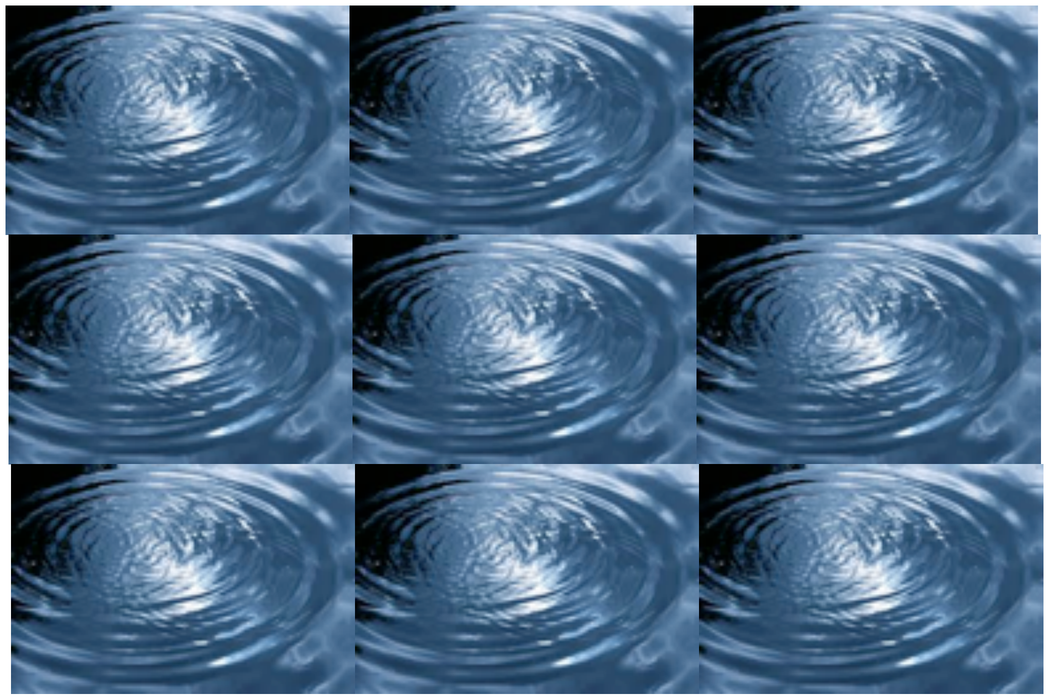
$$\langle\Psi(t)|S_j^z|\Psi(t)\rangle$$

disturbance appears to "spread" at a finite velocity.



Global Quantum Quenches

H(h) and H(h_0) differ everywhere



Shake up the entire system!

Here we have changed the system thermodynamically:

e.g.

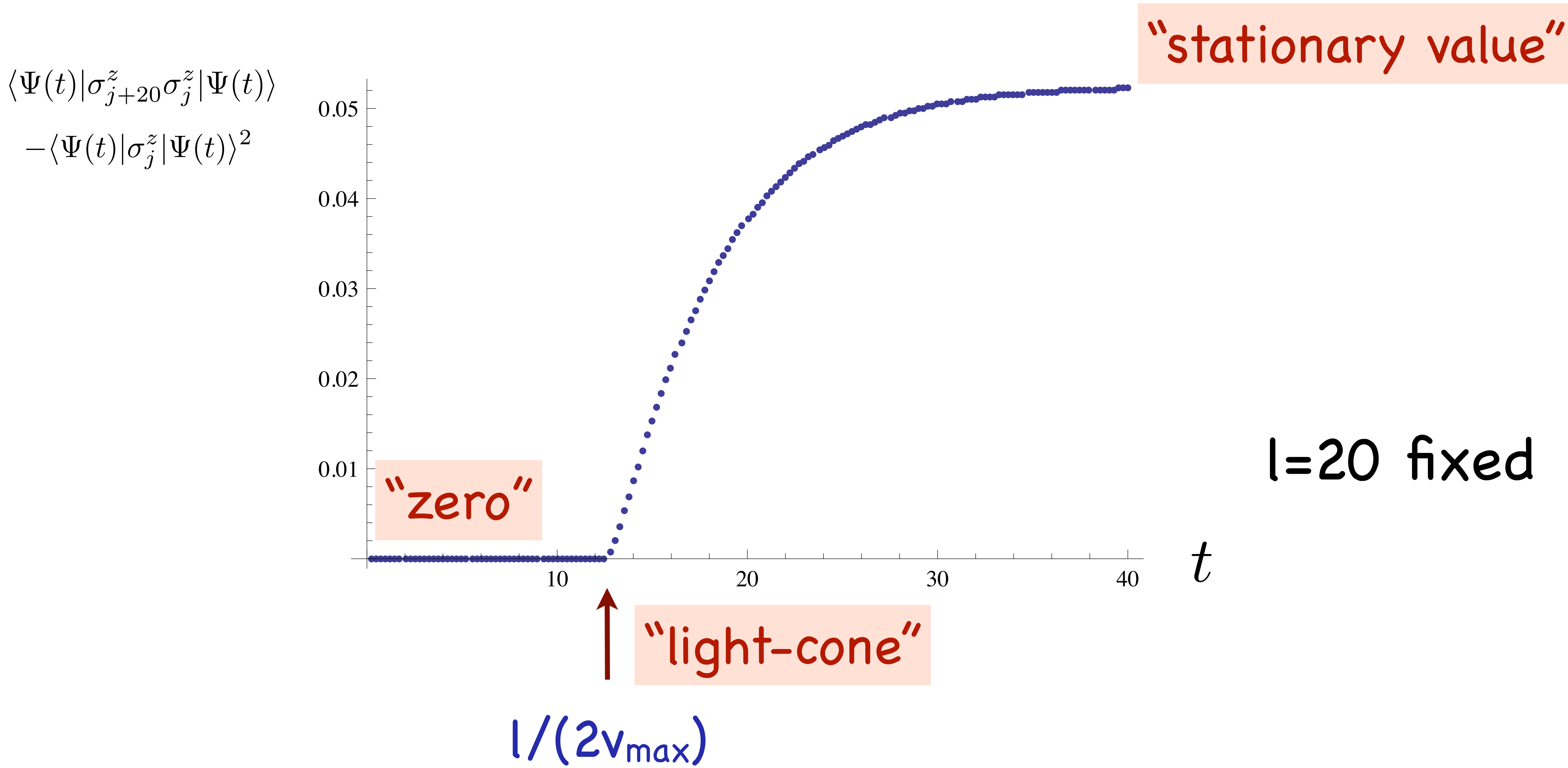
$$\lim_{L\to\infty}\frac{1}{L}\langle\Psi_0|H(h)|\Psi_0\rangle \neq \lim_{L\to\infty}\frac{1}{L}\langle\Psi_0|H(h')|\Psi_0\rangle$$

Example: transverse field Ising chain

Calabrese, Essler & Fagotti, '12

$$H(h) = -J\sum_j \sigma_j^z\sigma_{j+1}^z + h\sum_j \sigma_j^x$$

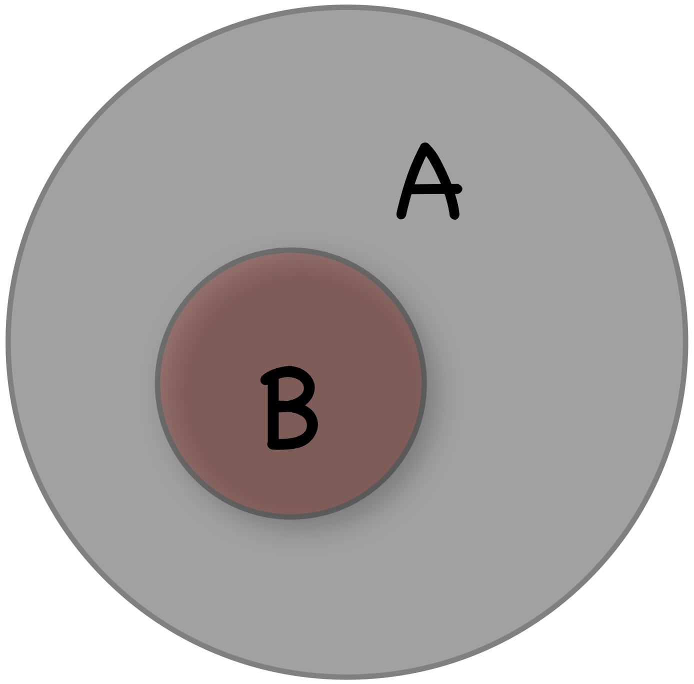
quench h=0.2 → h'=0.8



Late time behaviour

Given that we are considering an **isolated** system, it can never relax as a whole (∃ observables that remain time dependent)

- But it relaxes **locally** (in space).



- Entire System: A∪B
- Take A infinite, B finite
- Ask questions only about B:

Expectation values of **local** ops:

$$\lim_{t\to\infty}\lim_{L\to\infty}\frac{\langle\Psi_t|\mathcal{O}_B|\Psi_t\rangle}{\langle\Psi_t|\Psi_t\rangle} = \text{const}$$

Generic systems (only E conserved)

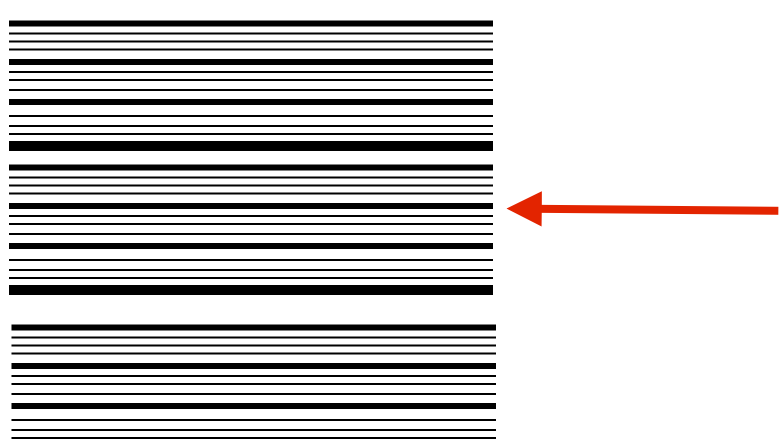
Deutsch '91
Srednicki '94

Locally the stationary state is **thermal:**

$$\boxed{\lim_{t\to\infty}\lim_{L\to\infty}\frac{\langle\Psi_t|\mathcal{O}_B|\Psi_t\rangle}{\langle\Psi_t|\Psi_t\rangle}=\lim_{L\to\infty}\mathrm{tr}\left[\rho_G\mathcal{O}_B\right]}$$

$$\rho_G=\frac{1}{Z}e^{-\beta H(h)} \qquad e=\lim_{L\to\infty}\frac{\mathrm{tr}\left[\rho_G H(h)\right]}{L}=\lim_{L\to\infty}\frac{\langle\Psi(0)|H(h)|\Psi(0)\rangle}{L}$$

How is this possible?



all states in the middle of the spectrum **look locally the same** i.e. thermal!

Systems with other conservation laws

Rigol, Dunjko, Yurosvki & Olshanii '07

$[I_m, I_n]=0, I_1=H(h)$

Locally the stationary state is described by a **Generalized Gibbs Ensemble:**

Jaynes '57

$$\lim_{t\to\infty}\lim_{L\to\infty}\frac{\langle\Psi_t|\mathcal{O}_B|\Psi_t\rangle}{\langle\Psi_t|\Psi_t\rangle}=\lim_{L\to\infty}\mathrm{tr}\left[\rho_{GGE}\mathcal{O}_B\right]$$

$$\rho_{GGE}=\frac{1}{Z}e^{-\sum_n\lambda_n I_n}$$

$$e_n=\lim_{L\to\infty}\frac{\mathrm{tr}\left[\rho_{GGE}I_n\right]}{L}=\lim_{L\to\infty}\frac{\langle\Psi(0)|I_n|\Psi(0)\rangle}{L}$$

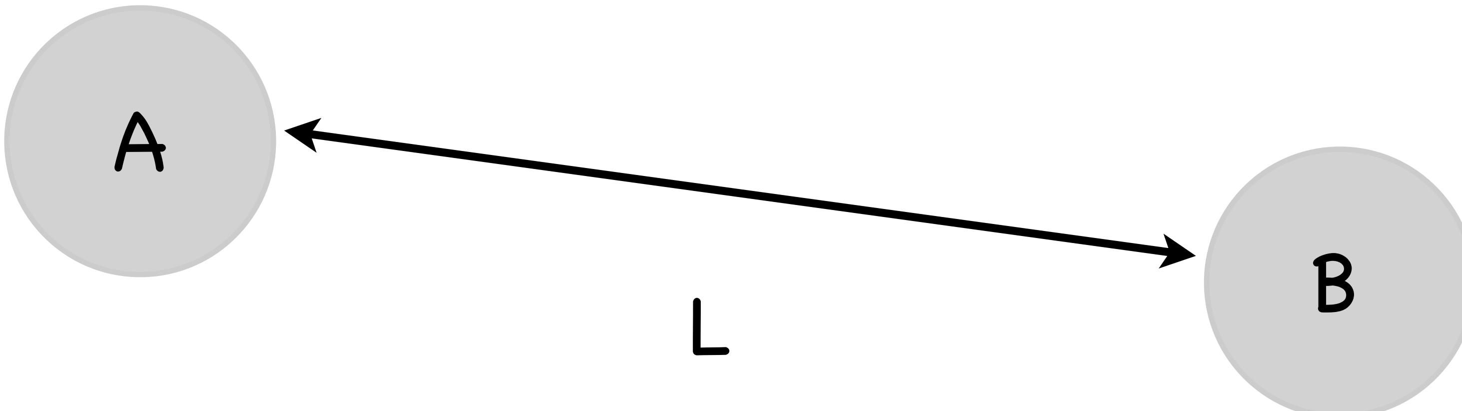
C. Causality and Lieb-Robinson bounds

We are dealing with non-relativistic QM $\rightarrow$ no "speed of light"

$\rightarrow$ no a priori reason why measurements should display **causal structures**.

How do perturbations/correlations spread?

Consider some non-relativistic quantum spin system with **short-ranged** Hamiltonian H



Let $O_{A/B}$ be operators acting only in subsystem A/B and $O_A(t)=\exp(iHt)\, O_A \exp(-iHt)$. Then the following bound holds

$$\| \, [O_A(t), O_B(0)] \, \| \leq c N_{min} \|O_A\| \, \|O_B\| \, \exp\left(-\frac{L - v|t|}{\xi}\right),$$

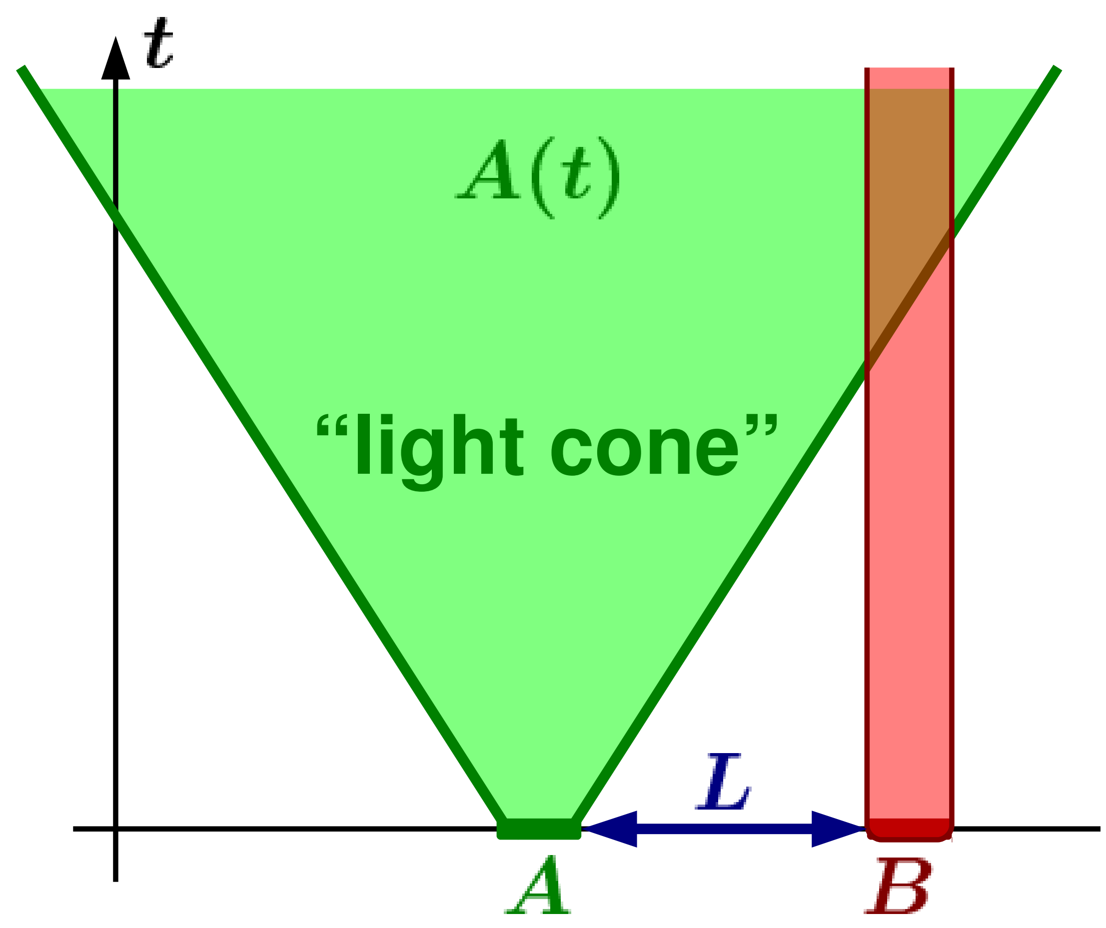
Lieb & Robinson '72

$$\| [O_A(t), O_B(0)] \| \leq c N_{min} \|O_A\| \, \|O_B\| \, \exp\left(-\frac{L - v|t|}{\xi}\right),$$

Lieb & Robinson '72

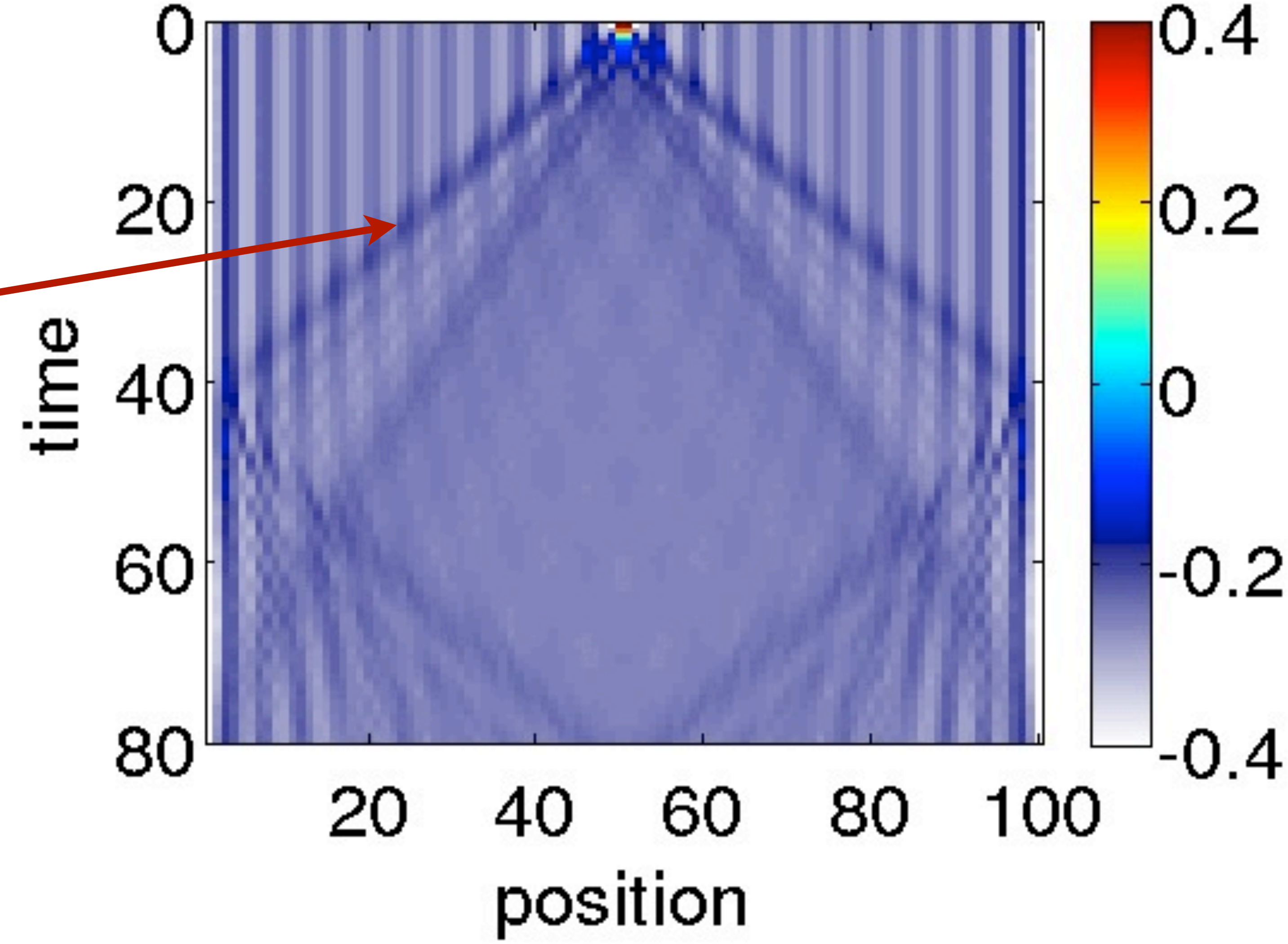
- RHS exponentially small until L≈vt → operators ≈ **commute**
- perturbation in A does not affect measurement in B **significantly** until **at least** vt **for some v.**

there can be, and **is,** an exponentially small effect **immediately,**
→"approximate causality"



Tallies nicely with what we have seen for **local quenches**

$\langle \Psi(t) | S_j^z | \Psi(t) \rangle$

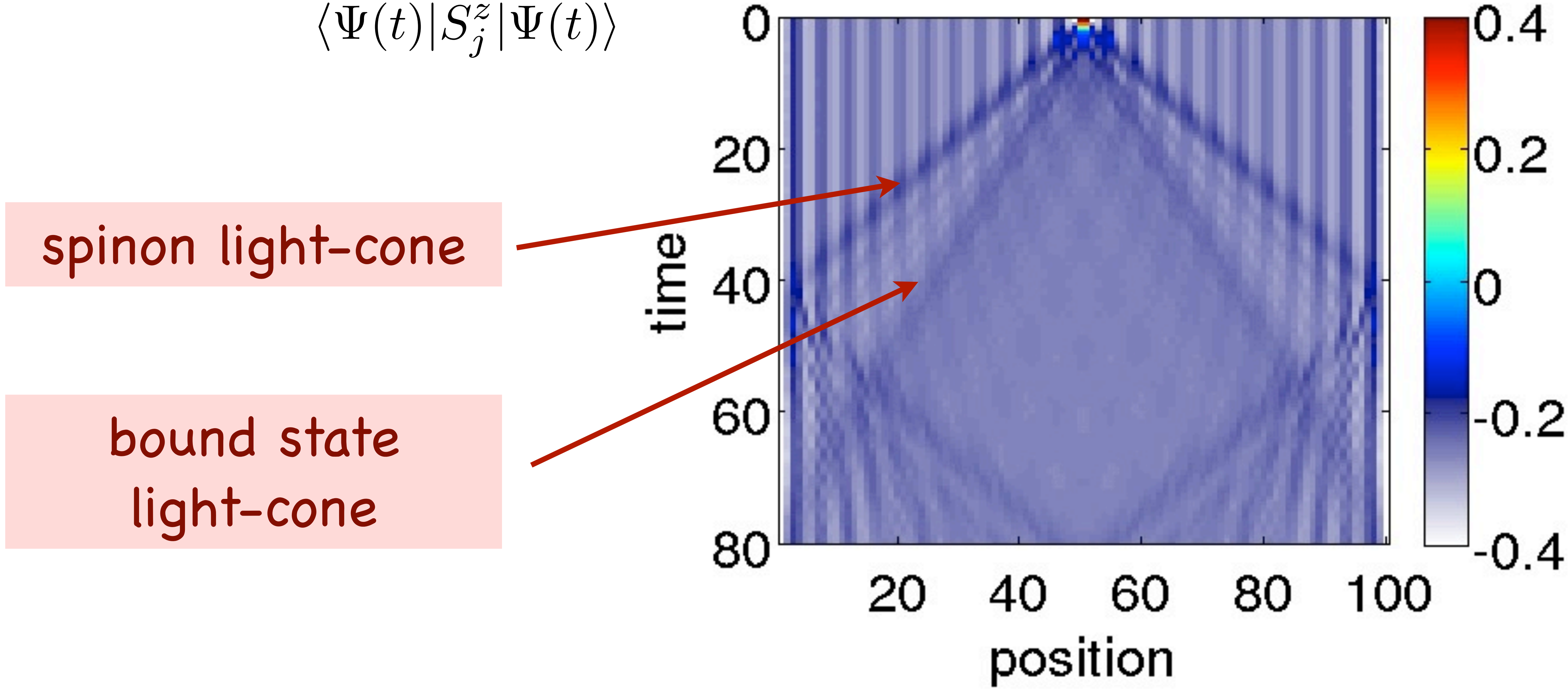


disturbance appears to "spread" at a finite velocity.

Velocity: max group velocity of elementary excitations **over the ground state of H** (here max v_{spinon}).

Ganahl, Rabel,
Essler& Evertz ‘12

$\langle\Psi(t)|S_j^z|\Psi(t)\rangle$

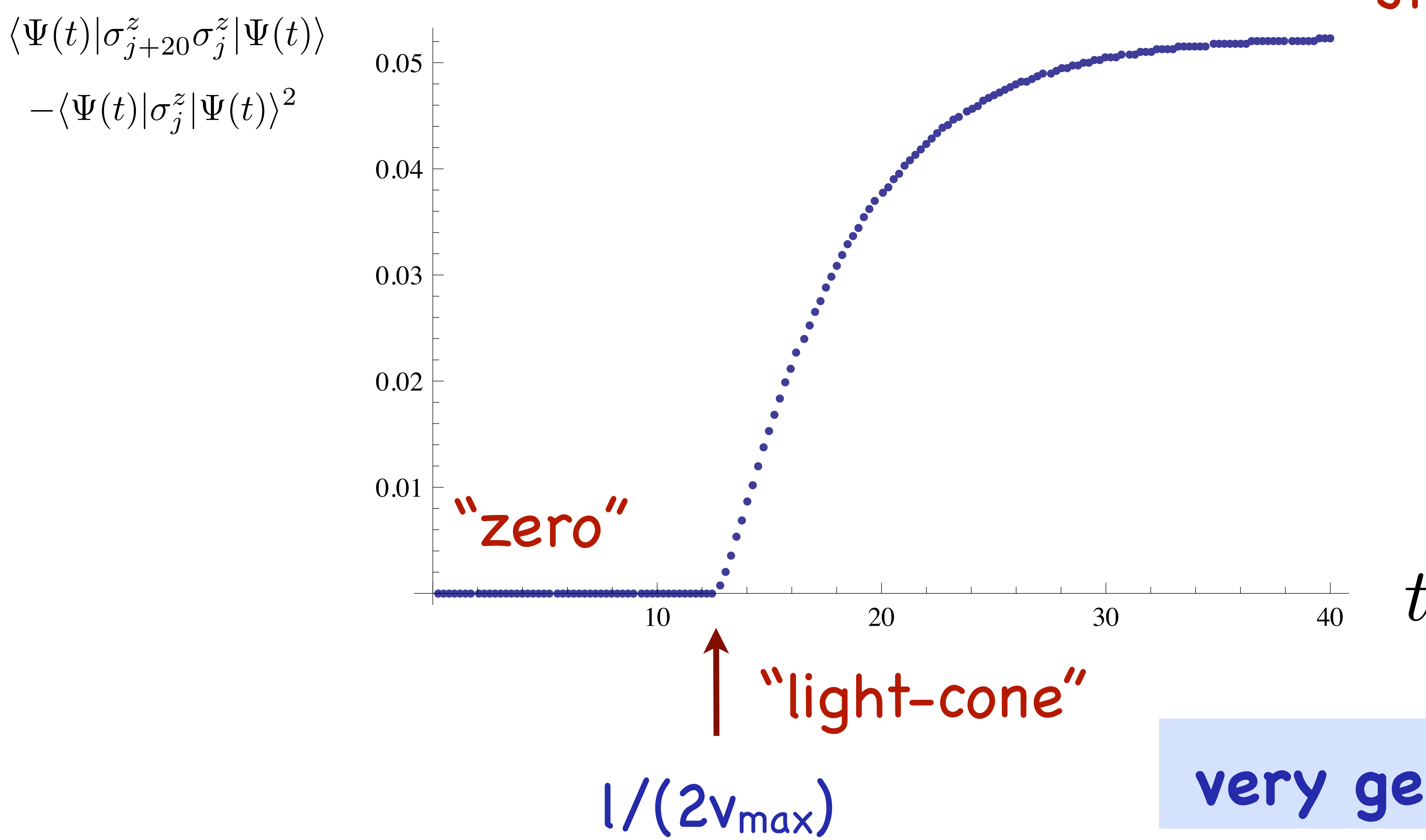


Light cones in global quenches ?

Transverse field Ising chain order parameter 2-point function

$$H(h) = -J \sum_j \sigma_j^z \sigma_{j+1}^z + h \sum_j \sigma_j^x$$

“stationary value”



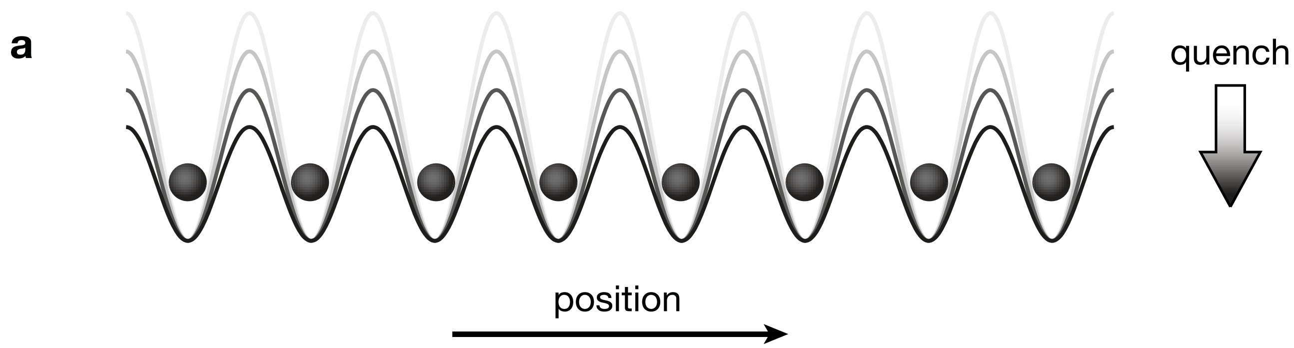
quench
h=0.2 → h’=0.8

l=20 fixed

very general !

Cold atom Experiments

Cheneau et al '12



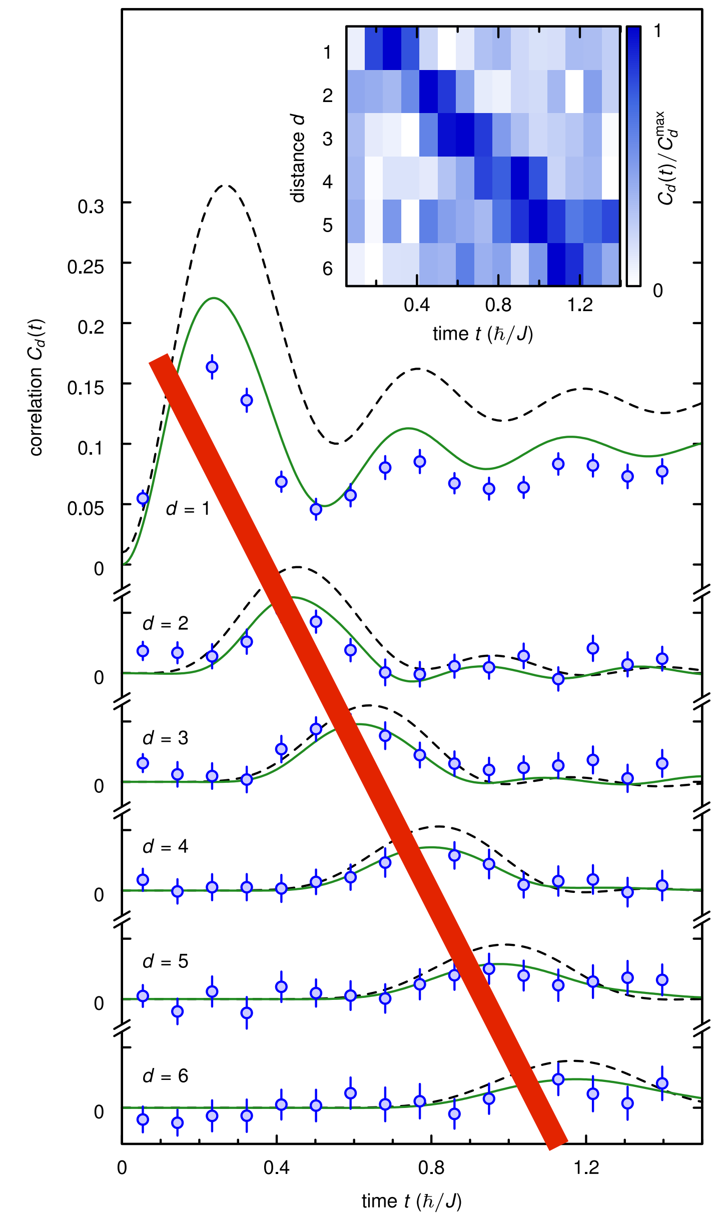
quench
$U_0/J=40 \rightarrow U/J=9$

$$\hat{H} = \sum_j \left\{ -J\left(\hat{a}_j^\dagger \hat{a}_{j+1} + \mathrm{h.c.}\right) + \frac{U}{2}\hat{n}_j(\hat{n}_j - 1) \right\},$$

occupation parity 2-point function

$$C_d(t) = \langle \hat{s}_j(t)\hat{s}_{j+d}(t)\rangle - \langle \hat{s}_j(t)\rangle\langle \hat{s}_{j+d}(t)\rangle\ ,$$

$$\hat{s}_j(t) = \exp(i\pi[\hat{n}_j(t) - \bar{n}])$$



This also follows from Lieb-Robinson bounds:

$$\| [O_A(t), O_B(0)] \| \leq c N_{min} \|O_A\| \, \|O_B\| \exp\left(-\frac{L - v|t|}{\xi}\right),$$

Lieb & Robinson '72

Bravyi, Hastings & Verstraete '06

exponentially decaying correlations in initial state

$$\langle \mathcal{O}_A(t) \mathcal{O}_B(t) \rangle_{\text{conn}} < \bar{c}(|A| + |B|) e^{-(L-2vt)/\xi}$$

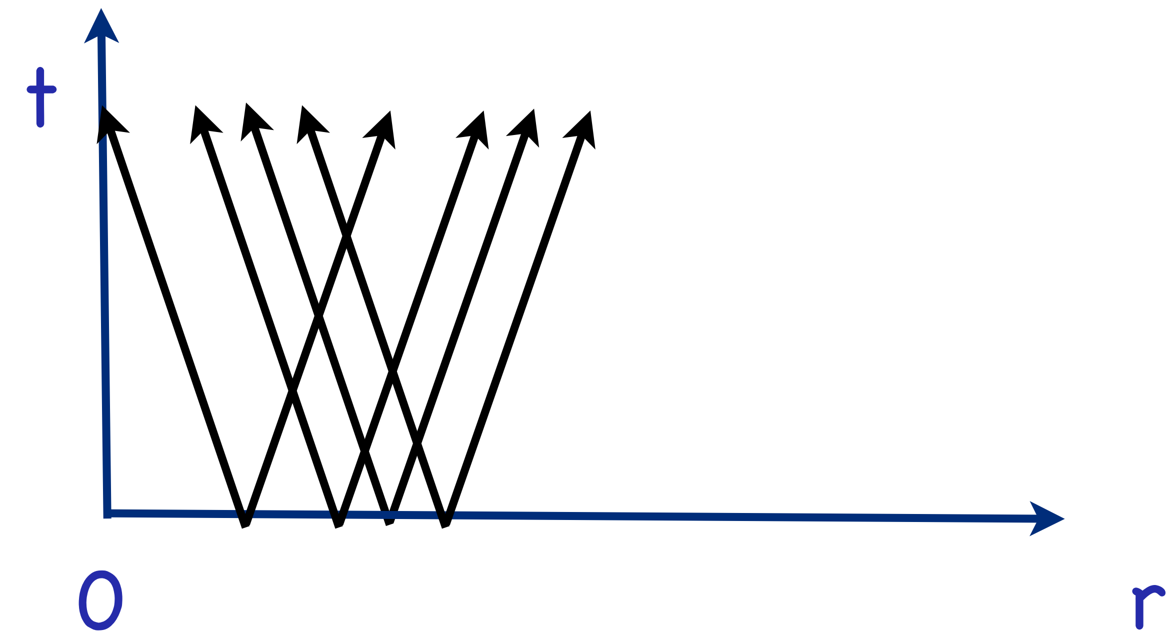
$\bar{c}, \xi, v$ are constants

There is a kind of "speed limit" for **"sizeable"** connected correlations to emerge.

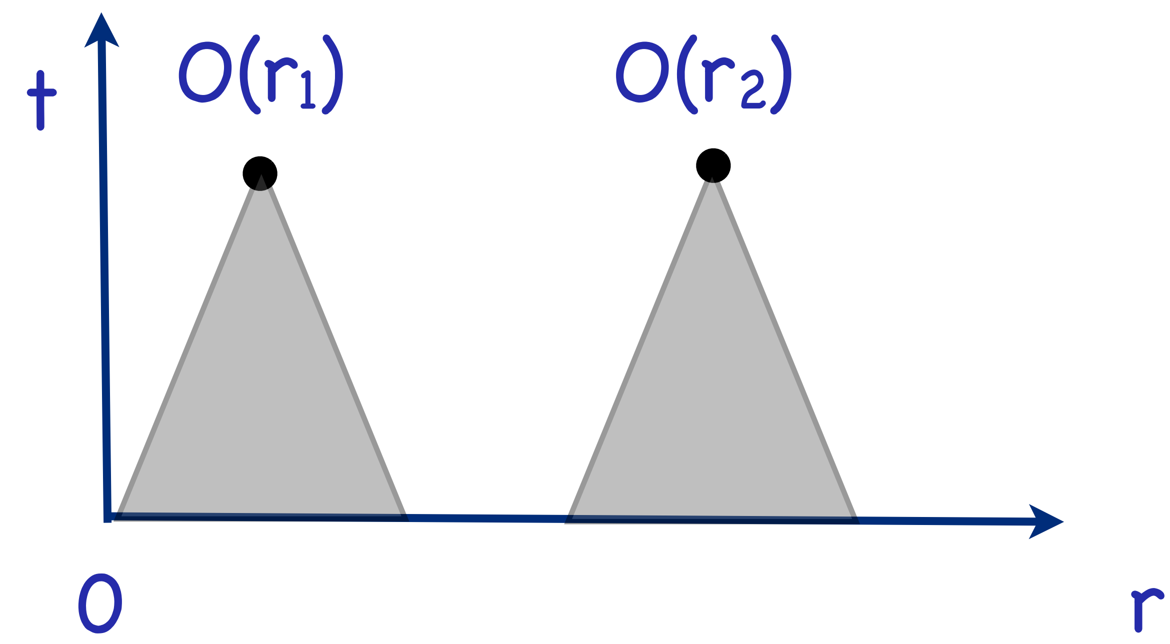
Physical ("Quasiparticle") picture:

Calabrese & Cardy '05, '06

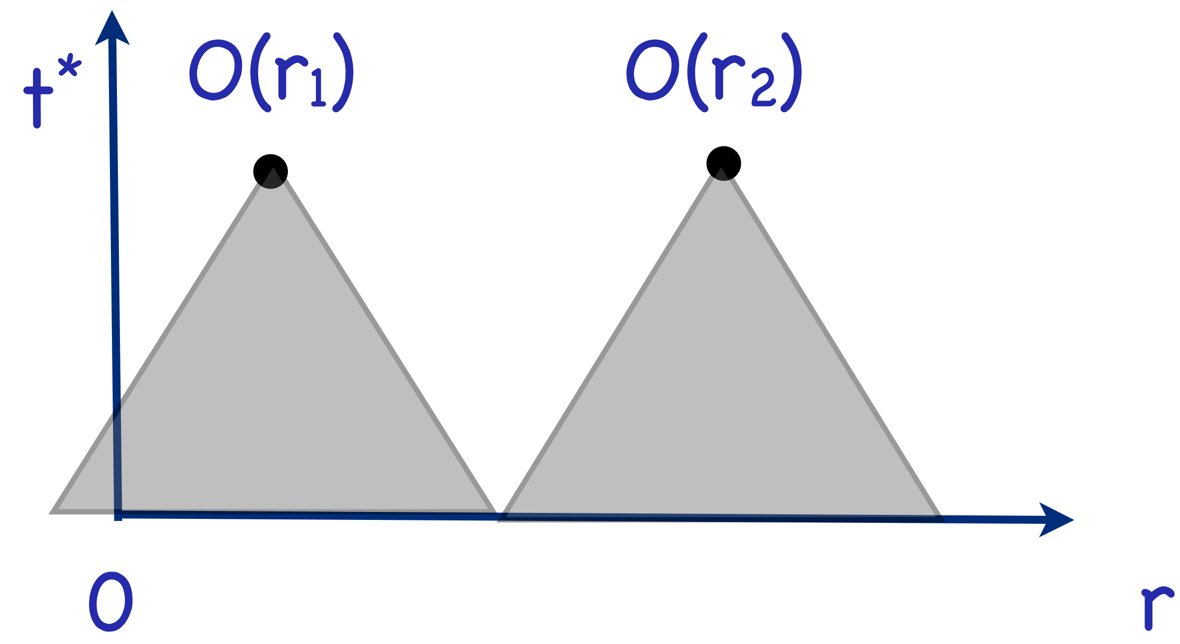
Quench creates quasiparticles at t=0, which start propagating with maximal velocity v



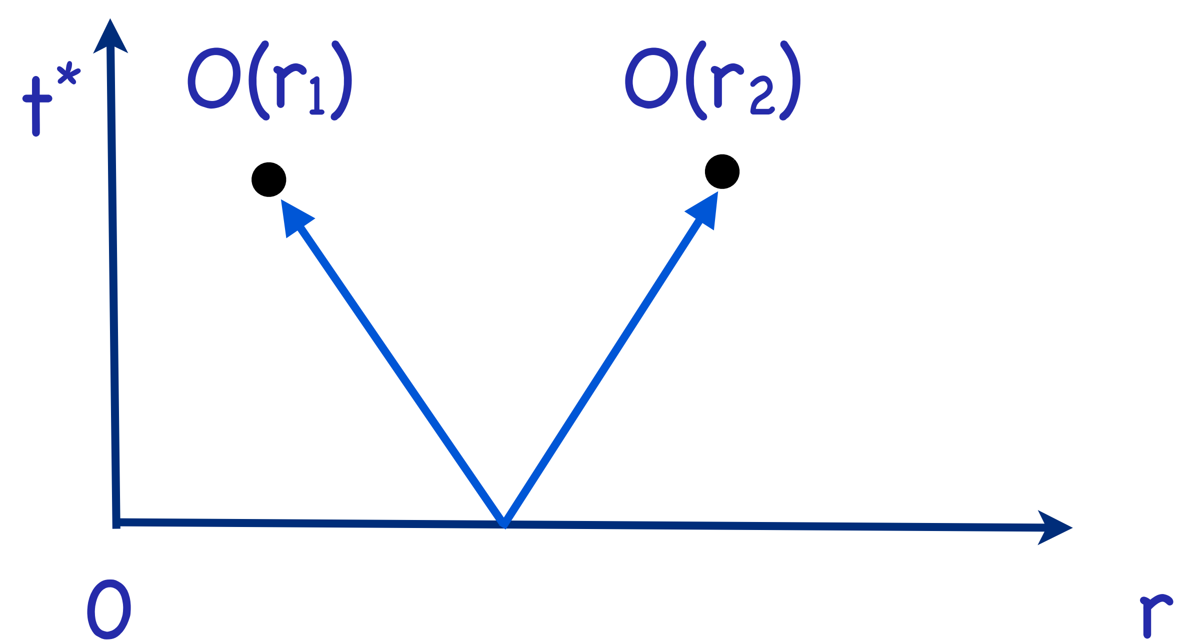
Operators at r_j get "hit" by quasiparticles from within the backwards light cone
→ dephasing of 1-point fns



At **$t^* = |r_2 - r_1|/(2v)$** the backwards light cones touch, and **connected correlations develop**



Correlations induced by **entangled quasiparticle pairs.**



What is the relevant velocity ???

D. Light cone structure after global quenches

Bonnes, Essler & Läuchli '14

The model:

$$H(\Delta) = J \sum_{i=1}^{L-1} \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right) .$$

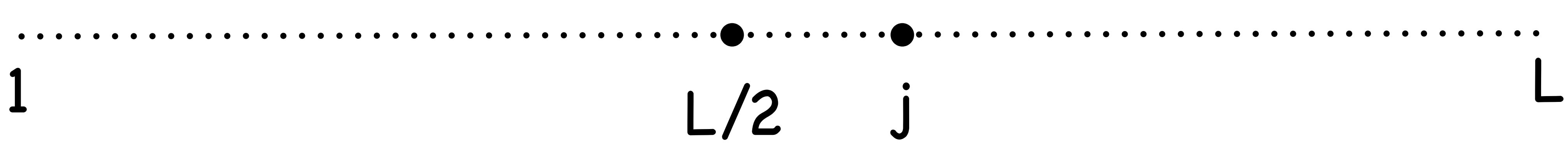
The quench:

$$\rho(t=0) = Z_\beta^{-1} \exp[-\beta H(\Delta_i)] \ , \quad \beta = \frac{1}{k_{\mathrm{B}} T},$$

- Gibbs distribution for $H(\Delta_i)$ at temperature T
- time evolve with $H(\Delta)$

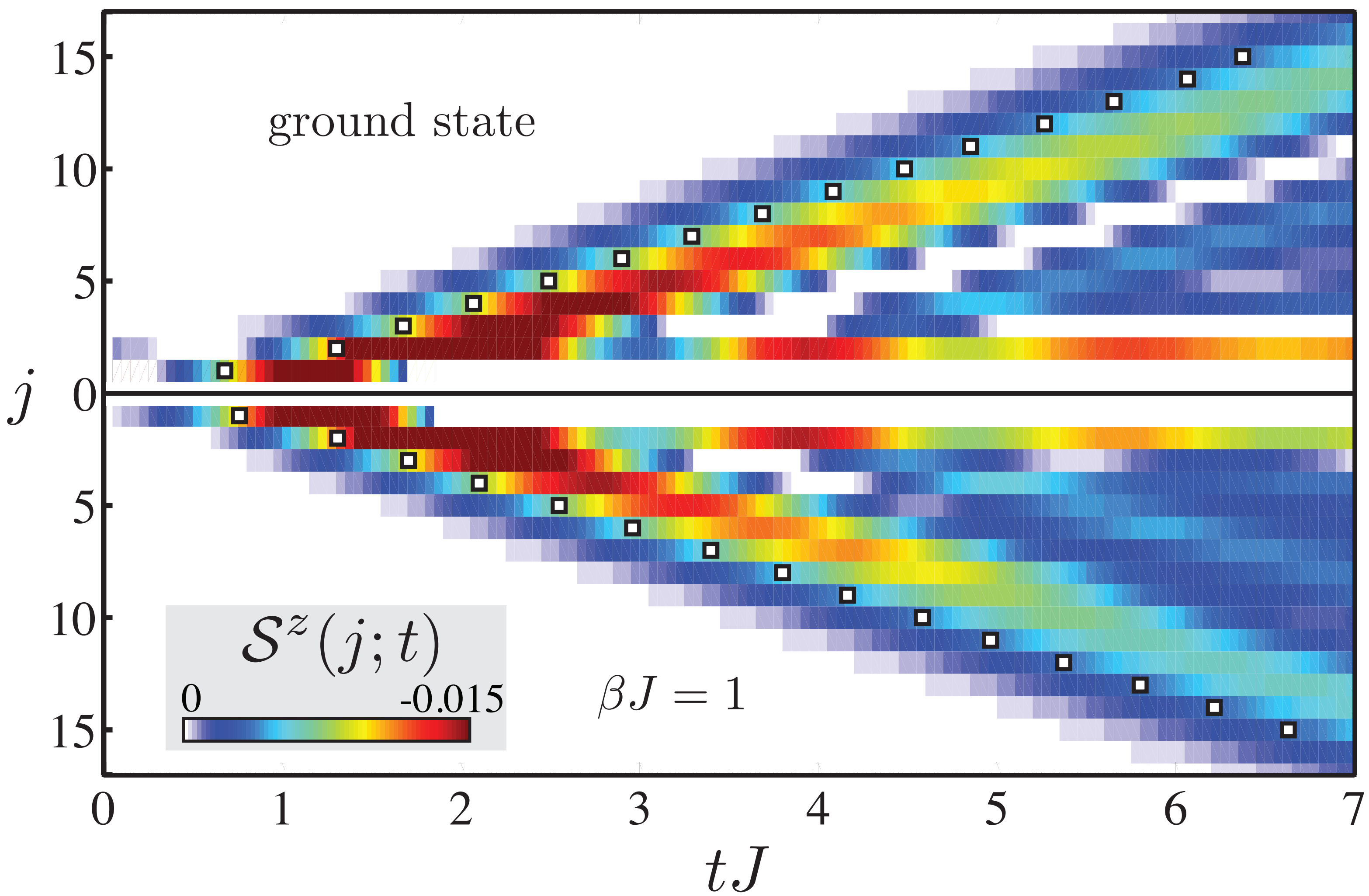
The observable:

$$\mathcal{S}^z(j;t) = \langle S^z_{\frac{L}{2}} S^z_{\frac{L}{2}+j} \rangle - \langle S^z_{\frac{L}{2}} \rangle \langle S^z_{\frac{L}{2}+j} \rangle$$



Compute time evolution numerically using METTS (Minimally Entangled Typical Thermal States) MPS methods. White '09

Results for quenches $\Delta_i = 4 \longrightarrow \Delta = \cos(\pi/4)$



$$\rho(t=0) = |\mathrm{GS}_{\Delta_i}\rangle\langle\mathrm{GS}_{\Delta_i}|$$

$$\rho(t=0) = \frac{1}{Z}e^{-\beta H(\Delta_i)}$$

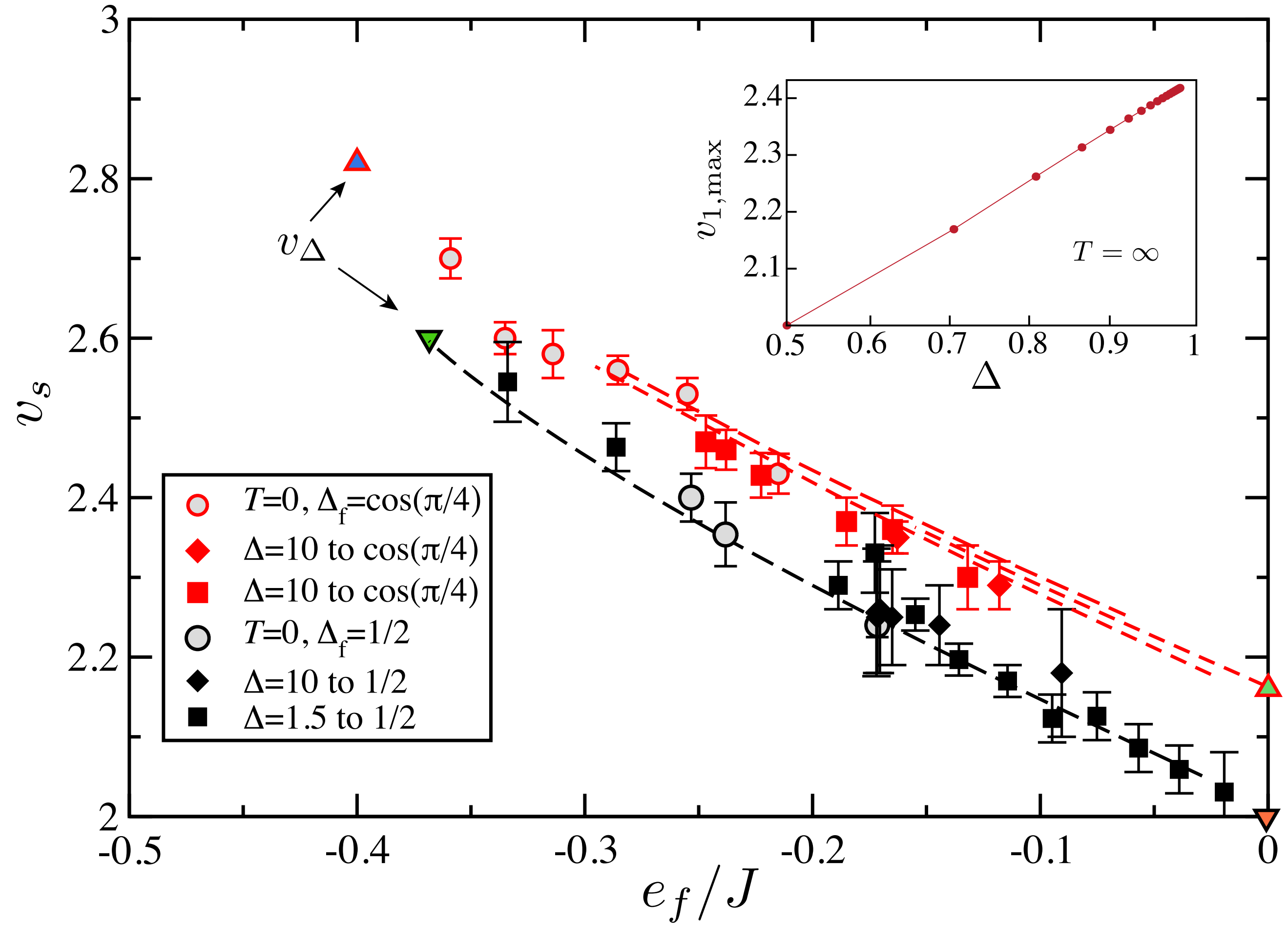
Nice light-cone effect.

Observation: Light-cone velocity **depends on initial state** (and not just final Hamiltonian):

Question: what properties of the initial state determine v?

Observation: good data collapse if we plot velocity as a function of final state energy

$$e_f = \frac{\mathrm{Tr}[H(\Delta_f)\rho(t=0)]}{L}.$$



How to understand these findings?

Well, the XXZ chain is integrable ...

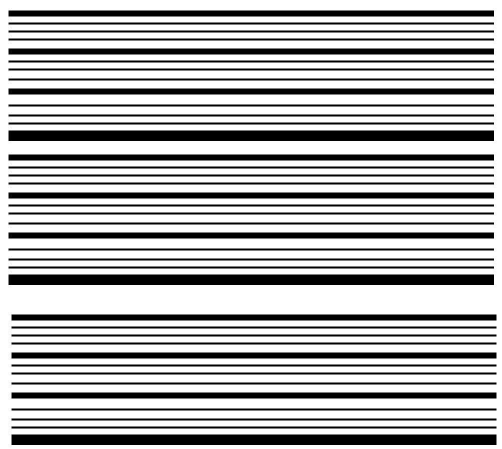
$[I_m, I_n]=0$, $I_1=H(h)$, $n=1,2,3,....$

$$e_n = \lim_{L\to\infty} \frac{\langle\Psi(t)|I_n|\Psi(t)\rangle}{L}$$ are **fixed**

maximize entropy with fixed e_n $\rightarrow$ macrostate ρ

$\rightarrow$ microstate ("representative state") $|\Phi_s\rangle$ Caux& Essler '13

Main idea: light cone velocity determined by “excitations” over the representative state.



want
“excitation spectrum”
around **this** state

This sounds crazy, but let’s look at a case we know:

Free theories: (think of a Fermi gas)

- single species, dispersion ε (p) $H = \sum_p \epsilon(p)\hat{n}(p)$
- can describe $|\Phi_s\rangle$ by densities of particles and holes $\rho^{p,h}(k)$

$$\rho^p(k) = \frac{1}{2\pi} - \rho^h(k) = \frac{\langle \Phi_s|\hat{n}(k)|\Phi_s\rangle}{2\pi}$$

- specific "representative state" in large, finite volume

$$|\Phi_s\rangle_L = \prod_j c^\dagger(k_j)|0\rangle \ , \quad \rho^p(k_j) = \frac{1}{L(k_{j+1}-k_j)} \ , \quad k_j = \frac{2\pi n_j}{L}$$

- Excitations= particles and holes with **energies ± ε (p)**
- velocity= $\max_p|\varepsilon'(p)|$

"No dressing in free theories"

Integrable theories like XXZ:

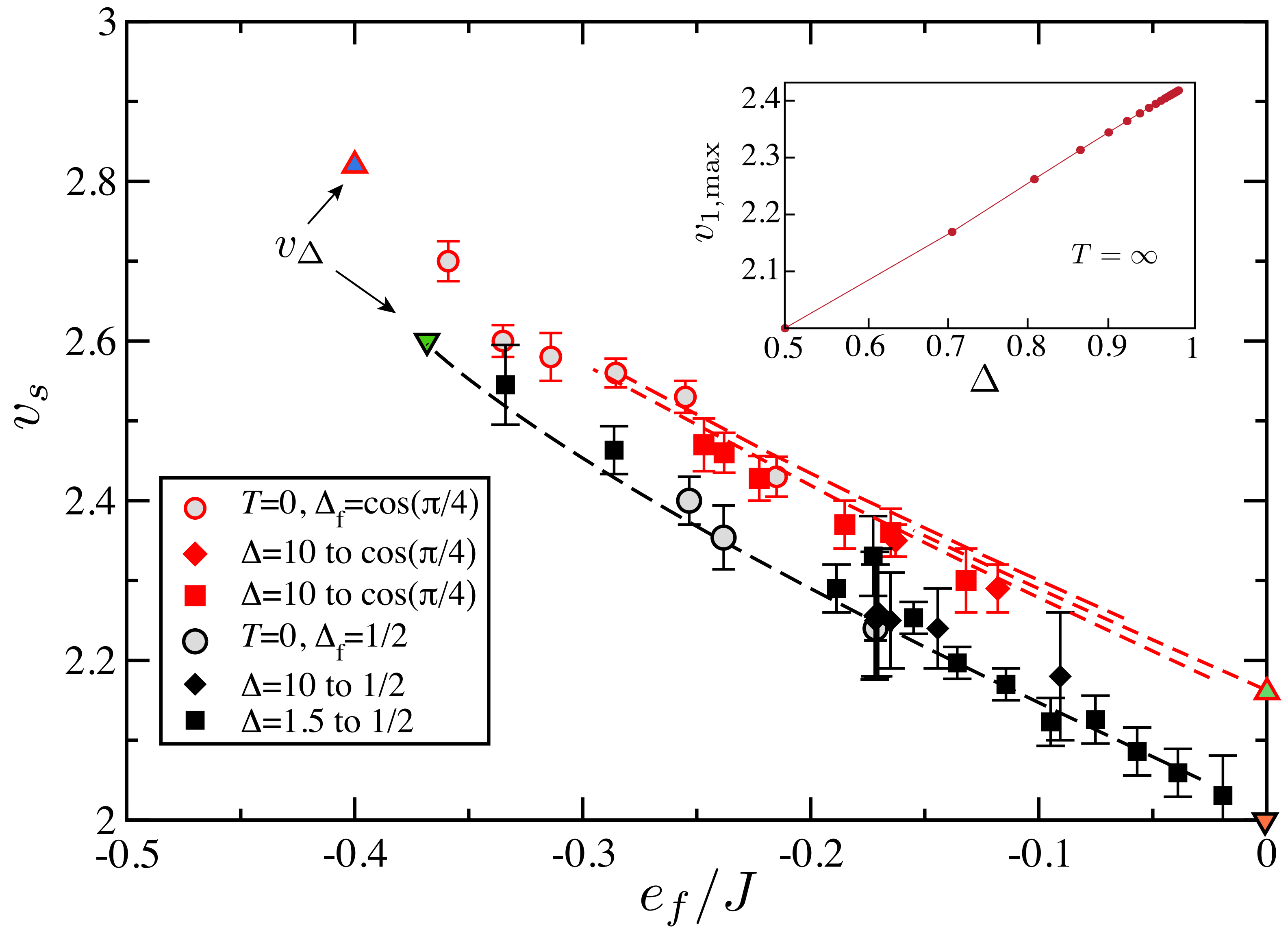
Works analogously to the free case

(but **much** more complicated, e.g. energies given in terms of infinite set of coupled nonlinear integral equations etc)

Crucial point: states in the “middle of the spectrum” can still be understood in terms of **stable** “excitations” due to integrability.

Comparison to numerical results

quenches $\left\{ \begin{array}{l} \Delta_i \longrightarrow \Delta = \cos(\pi/4) \\ \Delta_i \longrightarrow \Delta = 1/2 \end{array} \right\}$ for several T_i



→ consistent

Summary

- Non-equilibrium dynamics in isolated many-particle systems gives access to unexplored, interesting aspects of quantum theory.
- Rich structure in light-cones after both local & global quenches.

Outline

A. Isolated many-particle quantum systems out of equilibrium.

B. Theoretical protocols: local vs global "quantum quenches".

C. Spreading of correlations & "approximate causality".

D. Structure of the "light-cone" after global quenches.