Solvable Matter on 2D Causal Dynamical Triangulation?

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1. QUANTUM GRAVITY

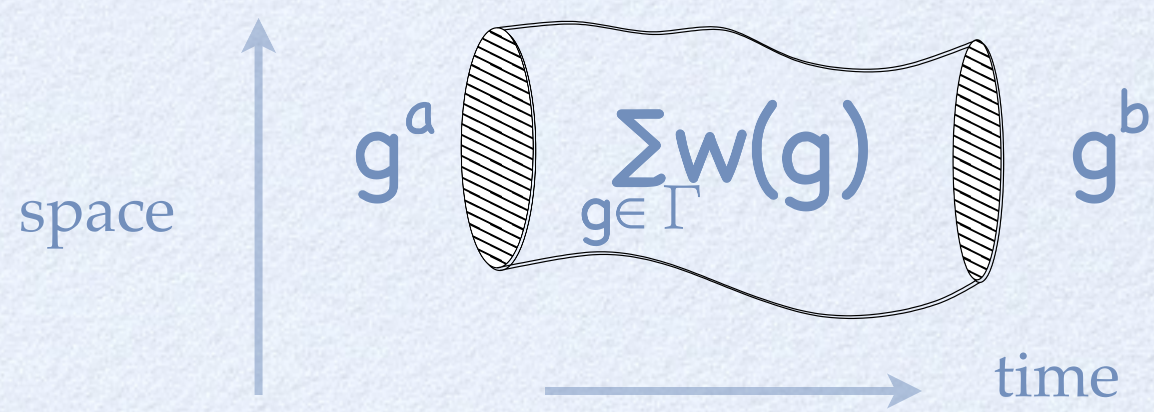
Gravity's dynamical degree of freedom is the metric $g_{\mu\nu}(x,t)$

Classically $g_{\mu\nu}(x,t)$ obeys Einstein's equations:

$$g_{\mu\nu}(x,0) \longrightarrow g_{\mu\nu}(x,t)$$

Quantum mechanics is different:

$$\langle g^b(x),\ t=T \mid g^a(x),\ t=0 \rangle \sim$$



Probability amplitude for evolution from g^a to g^b

How are Γ and w defined ?

Several approaches; some non-stringy ones ....

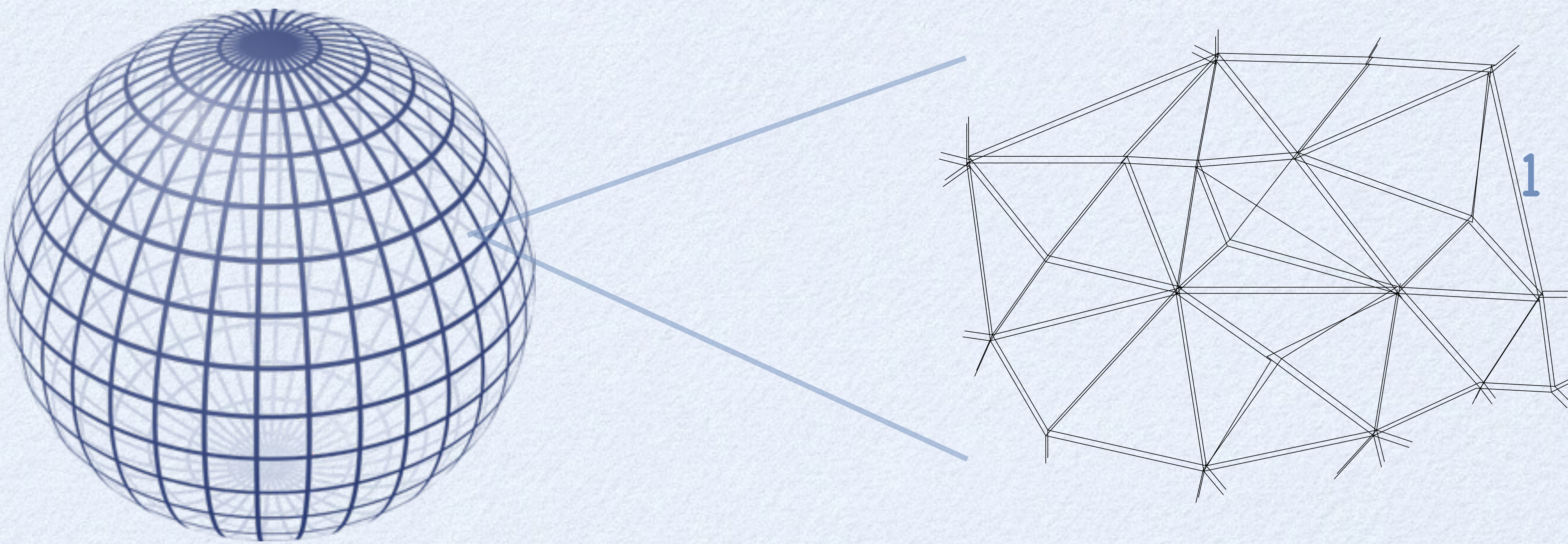
1. Continuum field theoretical -- exact RG looking for non-Gaussian fixed points where QG non-perturbatively renormalizable (asymptotic safety)

2. Discretized -- Causal Dynamical Triangulations Γ is a set of graphs, w(g) defined in terms of graph quantities, we look for a critical point (or line) where a continuum limit can be taken.

Recently much discussed also in context of Horava/ Lifshitz gravity

2. Triangulations & Metrics

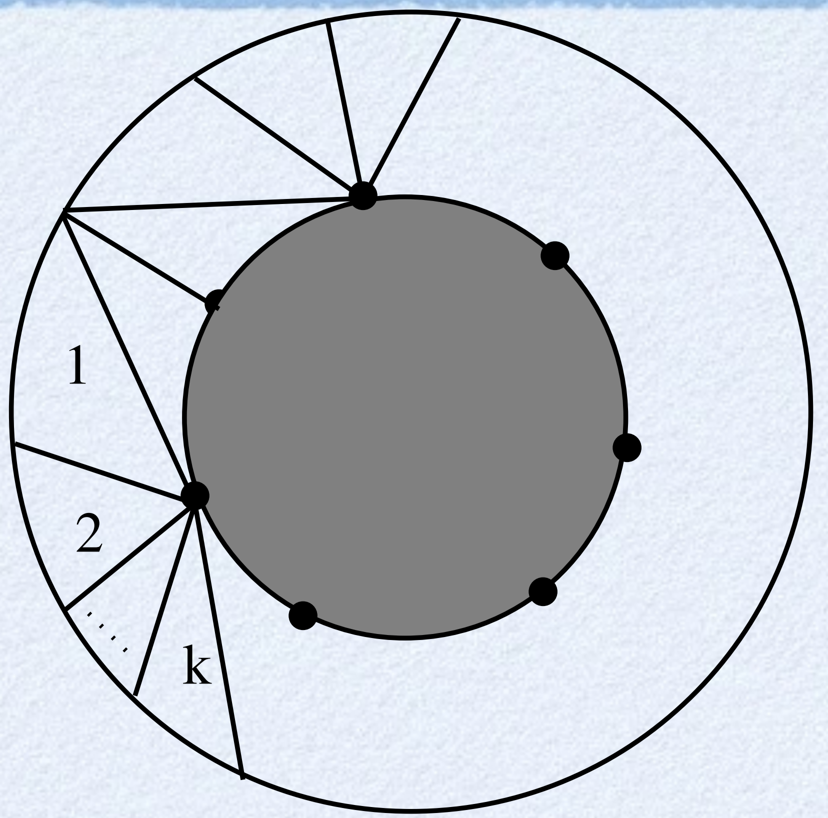
- So how do we do Σ_g ?
- Note $g(x,t)$ ↔ {geodesic distances} ↔ {graph distances}
- Think about 2-dim space with spherical topology



$$\Sigma_g \leftrightarrow \Sigma_{\text{triangulations}} \;,\; a \to 0$$

3. Causal Triangulation & Tree Bijection

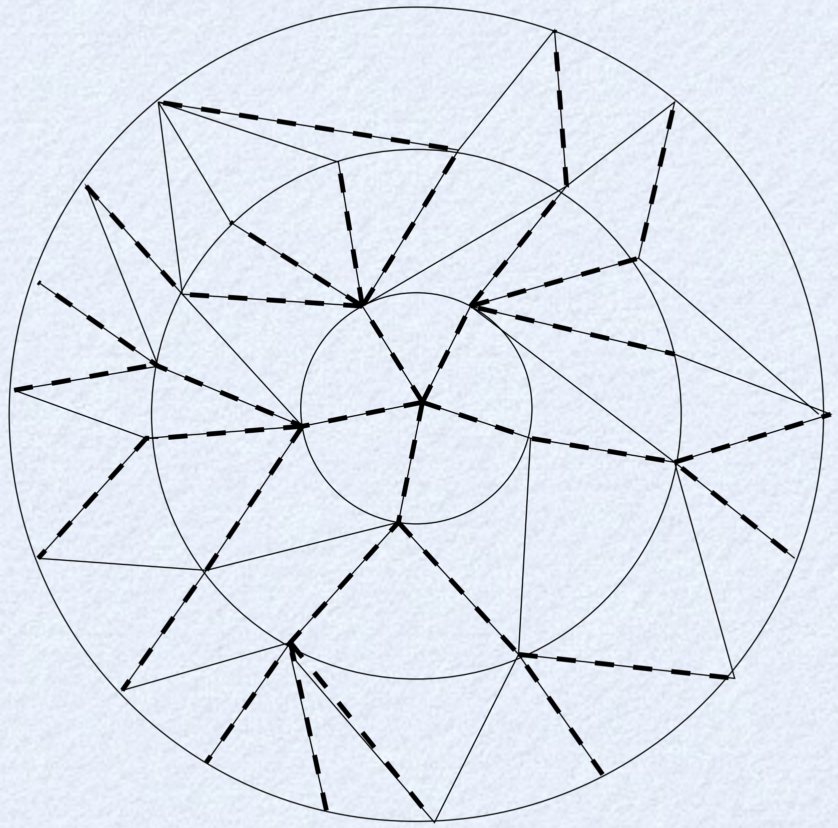
arXiv:0908.3643



$$w_G = \prod_{v \in G} g^{k_v + 1}$$

$$\mathcal{Z}(g) = \sum_G w_G$$

Critical at $g = g_c = 1/2$



at g_c offspring probability

$$p_n = (1/2)^{n+1}$$

so critical Galton Watson

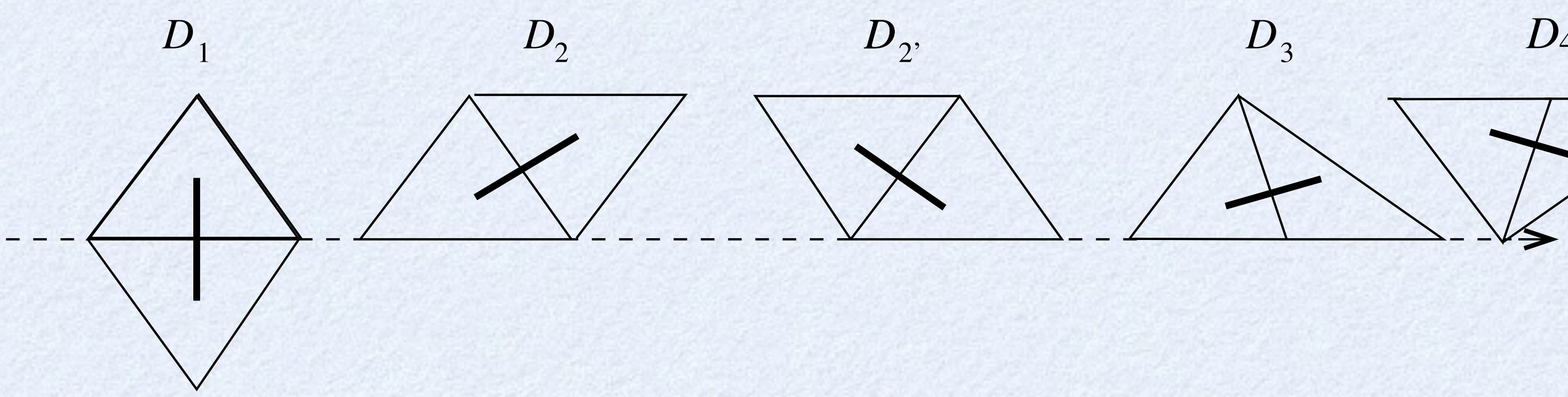
$$\langle n \rangle = 1$$

$$\mu(\infty \text{ CDT}) \Leftrightarrow \mu(\text{URT})$$

Uniform RT is a particular GRT

3. Dimers on CDT arXiv:1201.4322 Atkin & Zohren

Introduce dimers - possible configurations



Two dimers may not share a triangle

$$w_G = \prod_{v \in G} g^{k_v + 1}$$

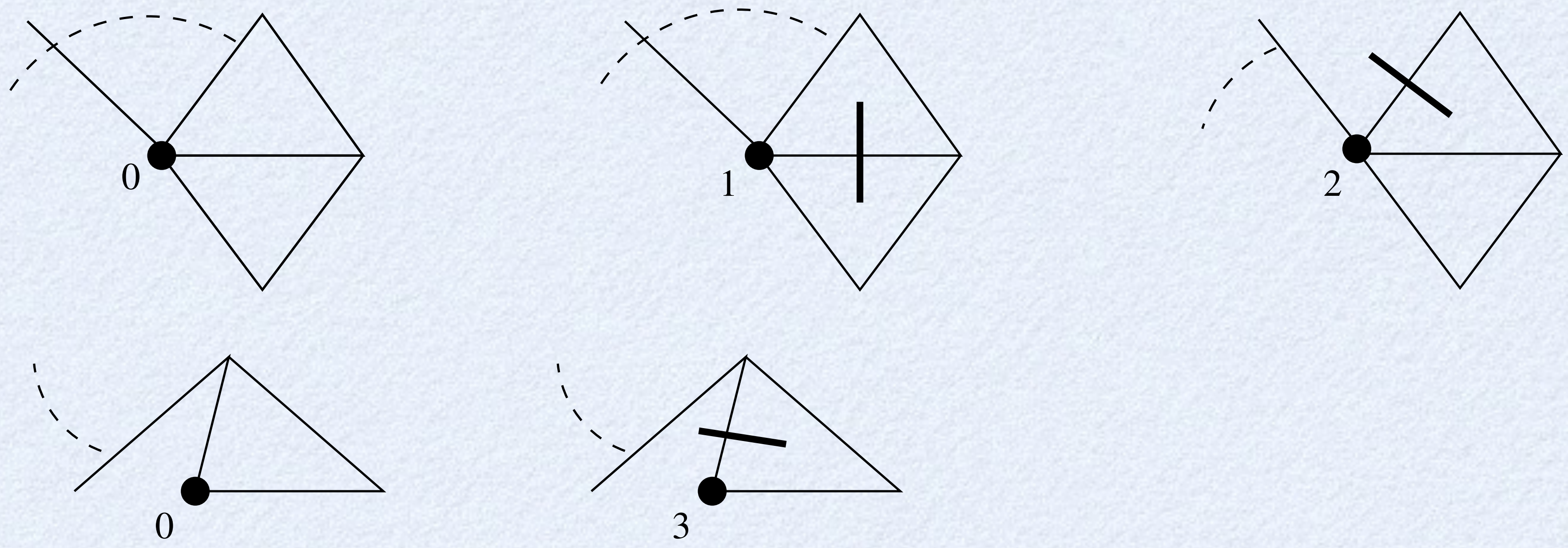
$$Z(\{\xi\};g) = \sum_{G,D} w_G \, \xi_1^{D_1} \xi_2^{D_2} \xi_{2'}^{D_{2'}} \xi_3^{D_3} \xi_4^{D_4}$$

Z fn of $\xi_2 + \xi_{2'}$ so set $\xi_{2'} = 0$

A & Z: $\xi_3 = \xi_4 = 0$ allows bijection to labelled trees

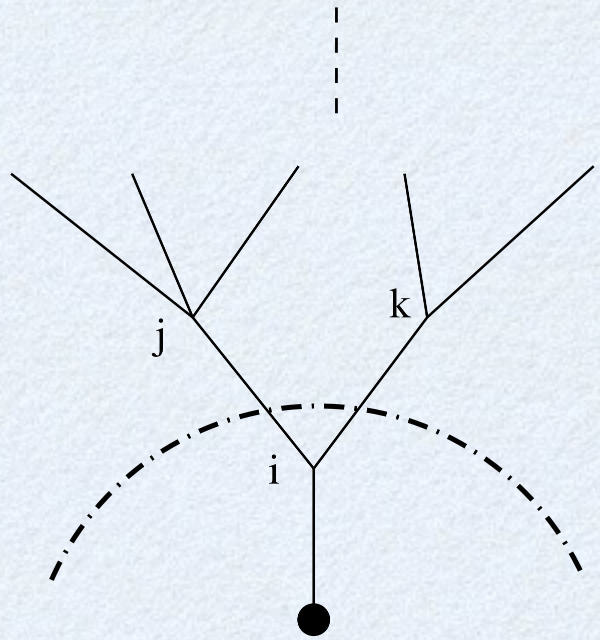
4. Labelled trees

A, D & JW: $\xi_4 = 0$ allows bijection to labelled trees



Label allocation L survives bijection —
— trees τ inherit a set of labelling rules from dimer rule

τ can be decomposed



$$W_i(\{\xi\};g) = \sum_{\tau,L;i} g^{\tau}\, \xi_1^{L_1}\, \xi_2^{L_2}\, \xi_3^{L_3}$$

$$= F_i(\{W\};\{\xi\};g)$$

these are geometric series so F are rational fns of W

and

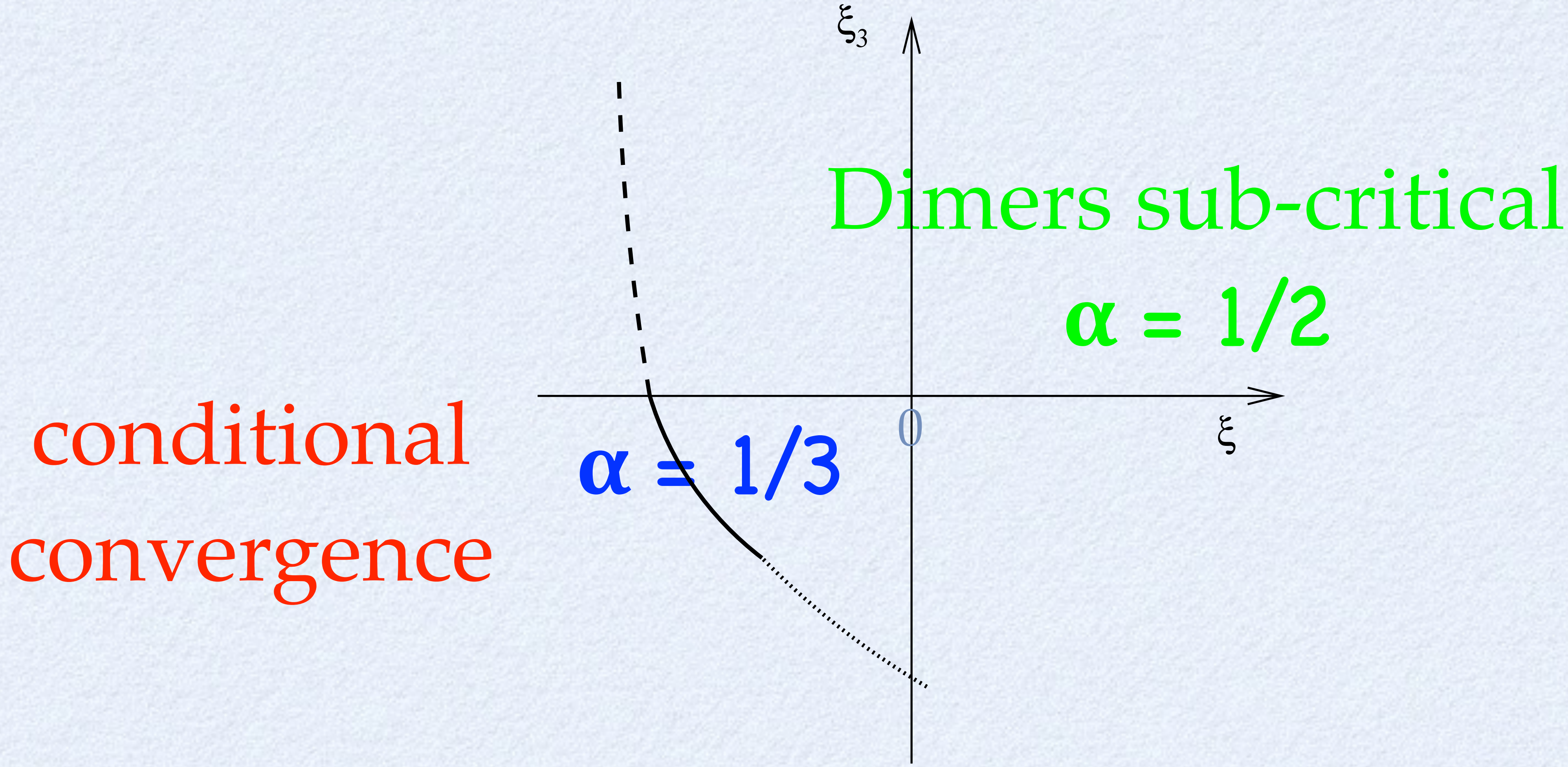
$$Z = g^{-1}\, W_0 - 1$$

and W_0 satisfies a cubic — we are in business

5.Phase diagram

find $W_0 \sim W_{0c}(\{\xi\}) - A(\{\xi\}) \, (g_c(\{\xi\}) - g)^{\alpha}$

$g_c(\{\xi\})$ is free energy; set $\xi = \xi_1 = \xi_2$

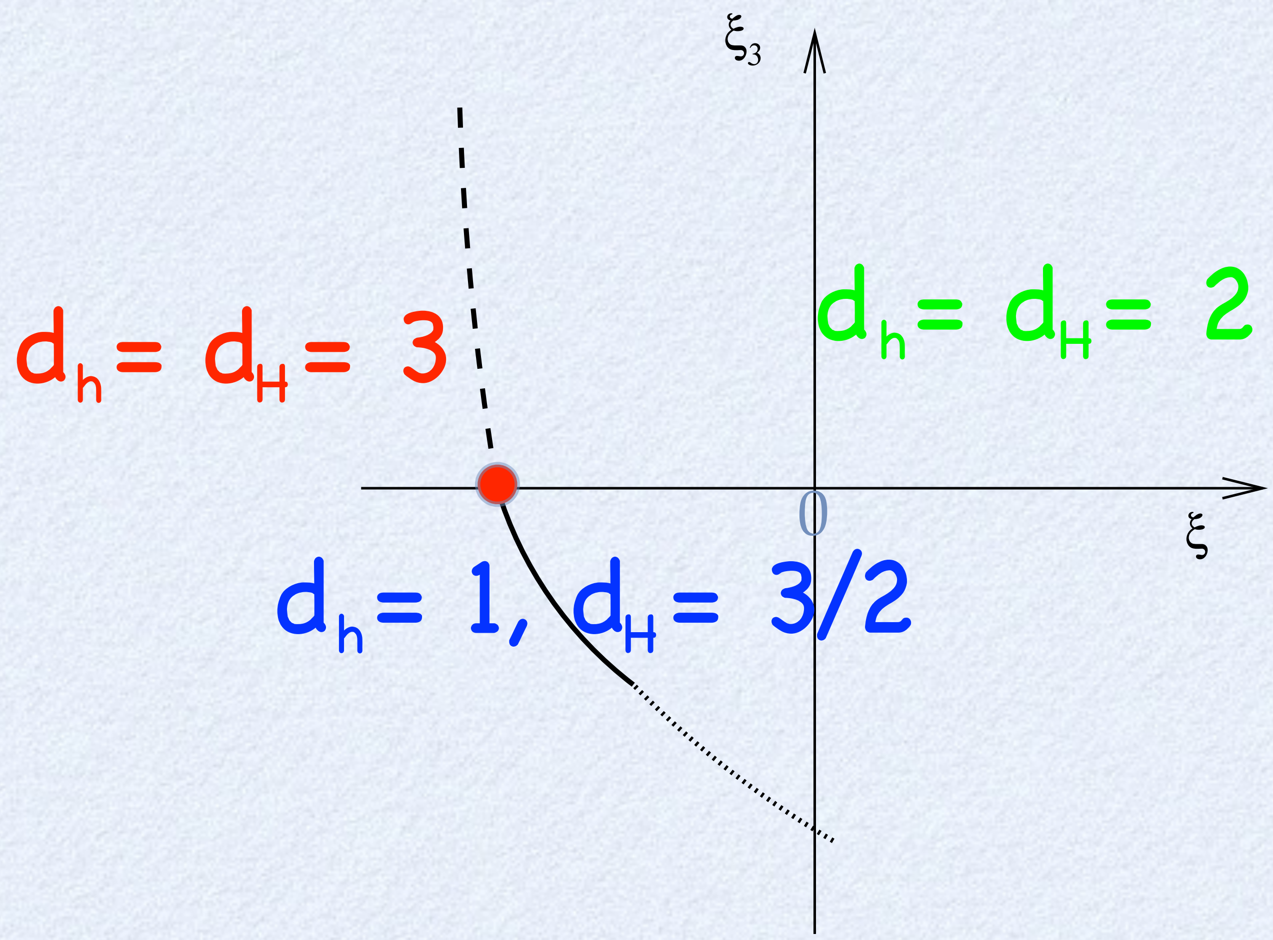


6. Geometric characterization

Hausdorf dimension:

finite tree $N \sim R^{d_H}$

infinite tree $B_R \sim R^{d_h}$



9. Concluding Remarks

1. Away from dimer criticality the continuum theory is just (Horava-Lifshitz) 2d gravity

2. The segment of critical line $d_h = 1,\ d_H = 3/2$ is probably critical dimer CFT ($c=-22/5$) coupled to H-L 2d gravity

3. The AZ point $d_h = d_H = 3$ is special, but not that special — it is still a one-parameter family. Presumably it describes a CFT coupled to H-L gravity but we do not know which CFT

4. Systems with negative weights are intricate and can easily confound out intuition