Particle Physics from String Theory

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University of Oxford

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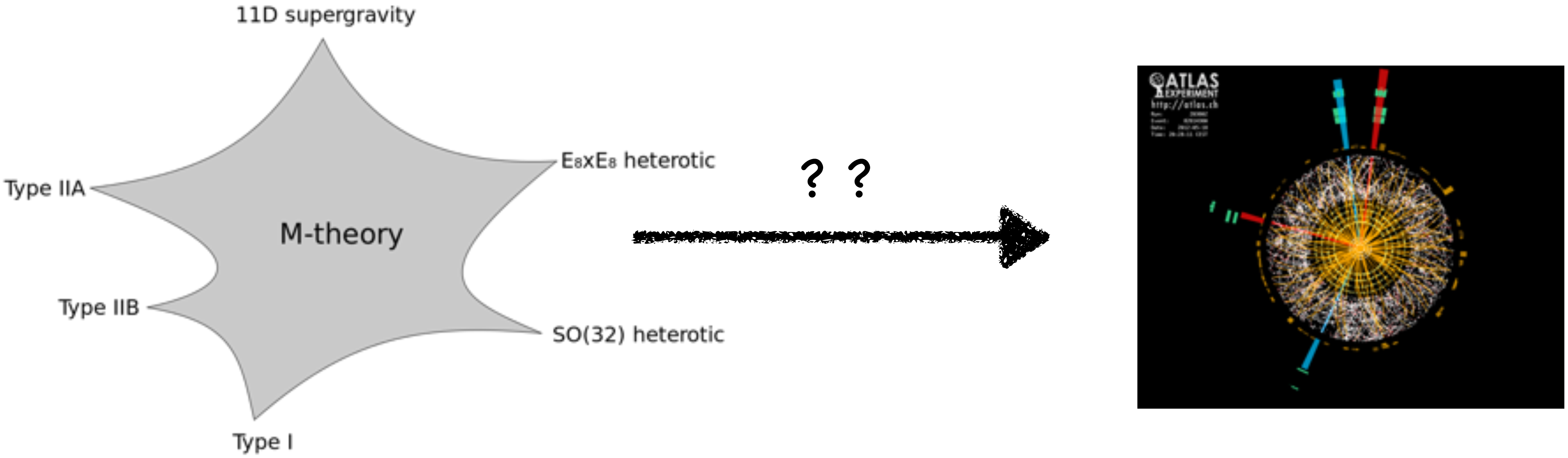

based on:

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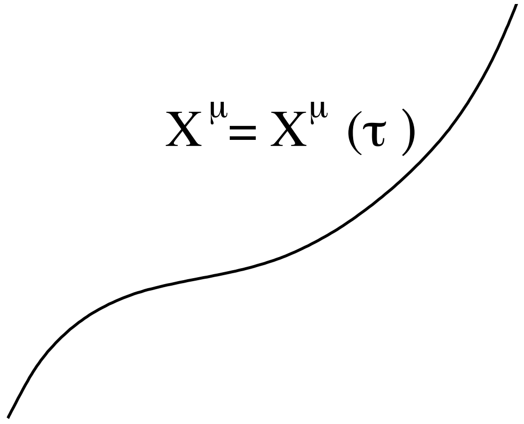
with Lara Anderson, Evgeny Buchbinder, Andrei Constantin, James Gray, Yang-Hui He, Michael Klaput, Seong-Joo Lee, Cyril Matti, Burt Ovrut, Eran Palti, Eirik Svanes

Outline

- Introduction: String- and M-theory
- String theory and particle physics: some general features
- Model building
- The standard model of particle physics from strings
- Conclusion

Introduction: String- and M-theory

Starting point: From worldline



$$S = -m \int d\tau \sqrt{-\frac{dX^\mu}{d\tau}\frac{dX^\nu}{d\tau}\eta_{\mu\nu}}$$

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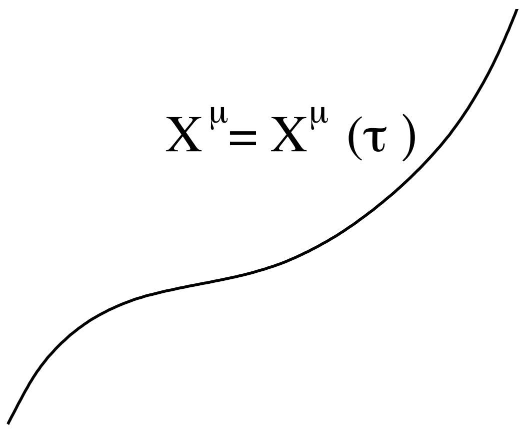

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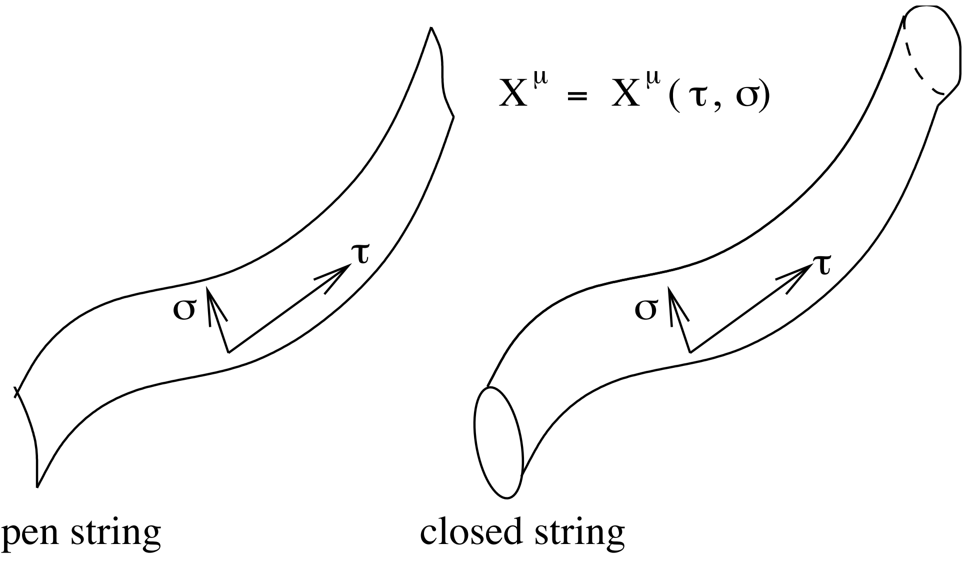
to world sheet



$$S = -\frac{1}{2\pi\alpha'} \int d^2\sigma \sqrt{-\det\left(\frac{dX^\mu}{d\sigma^\alpha}\frac{dX^\nu}{d\sigma^\beta}\eta_{\mu\nu}\right)}$$

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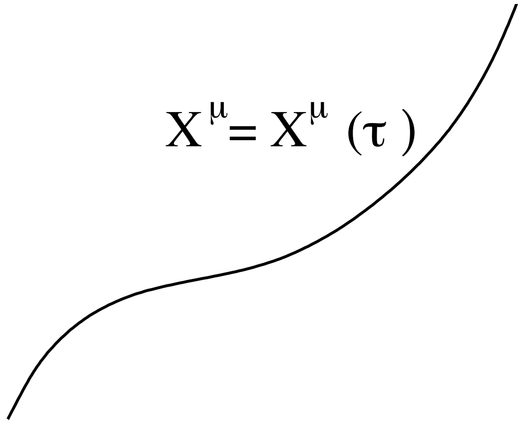

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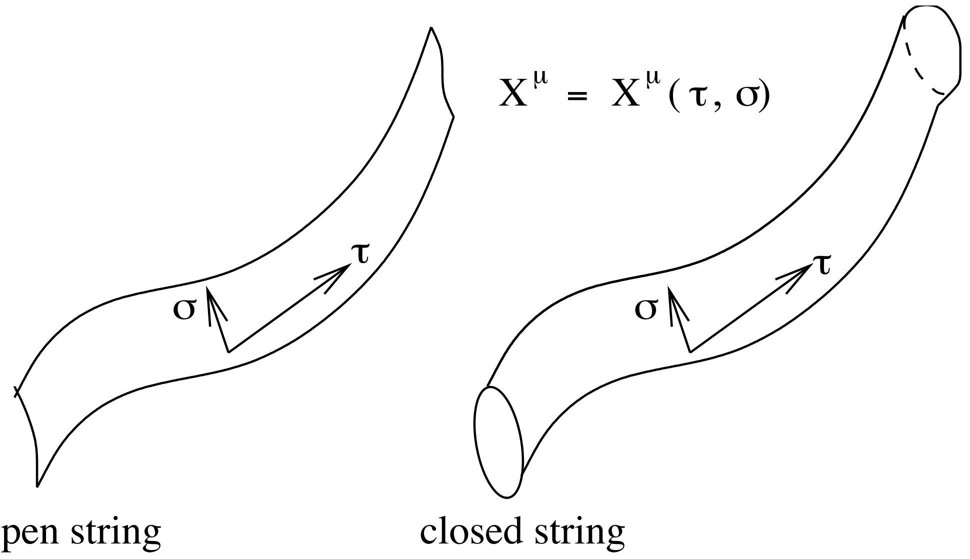

string tension

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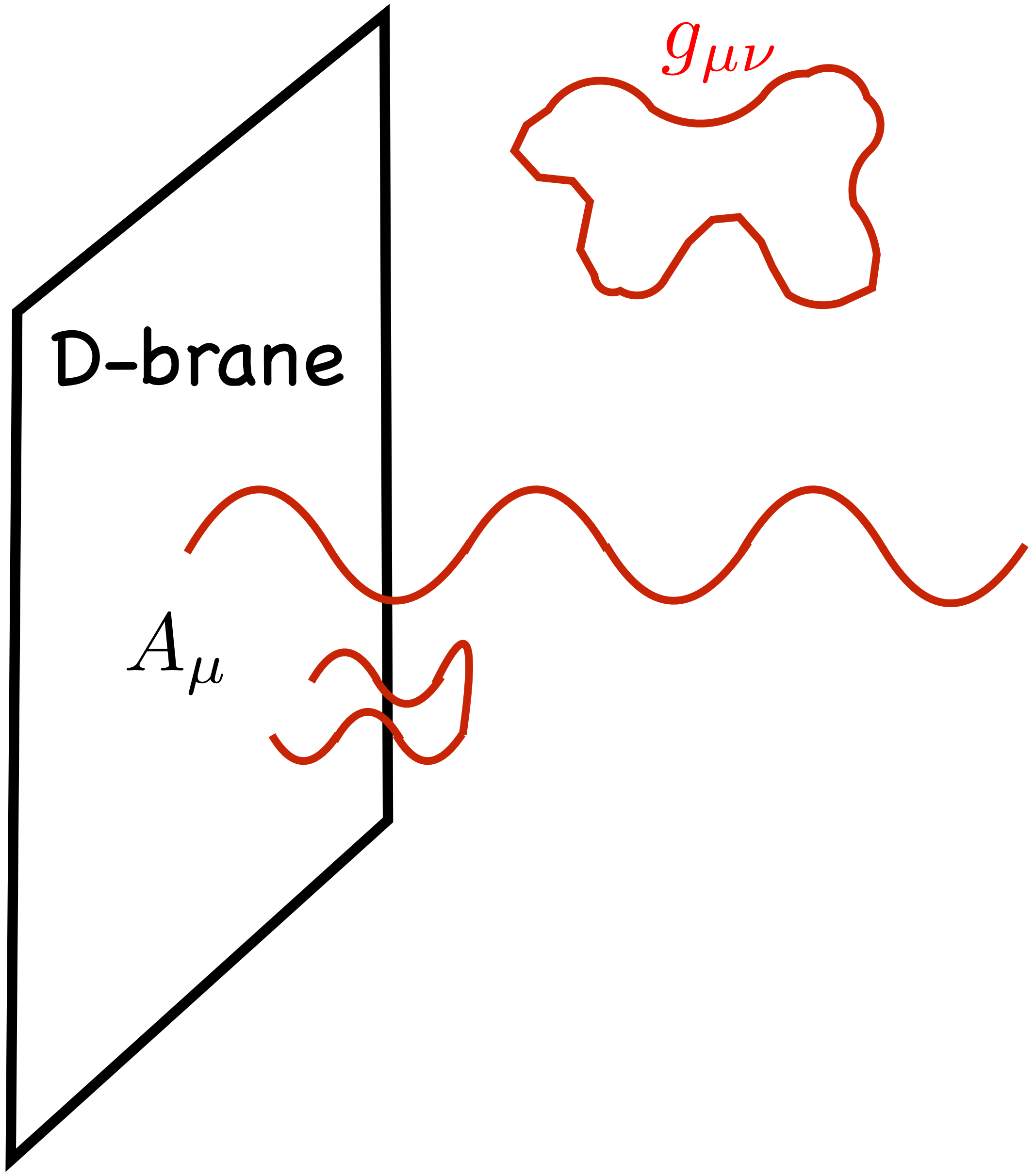
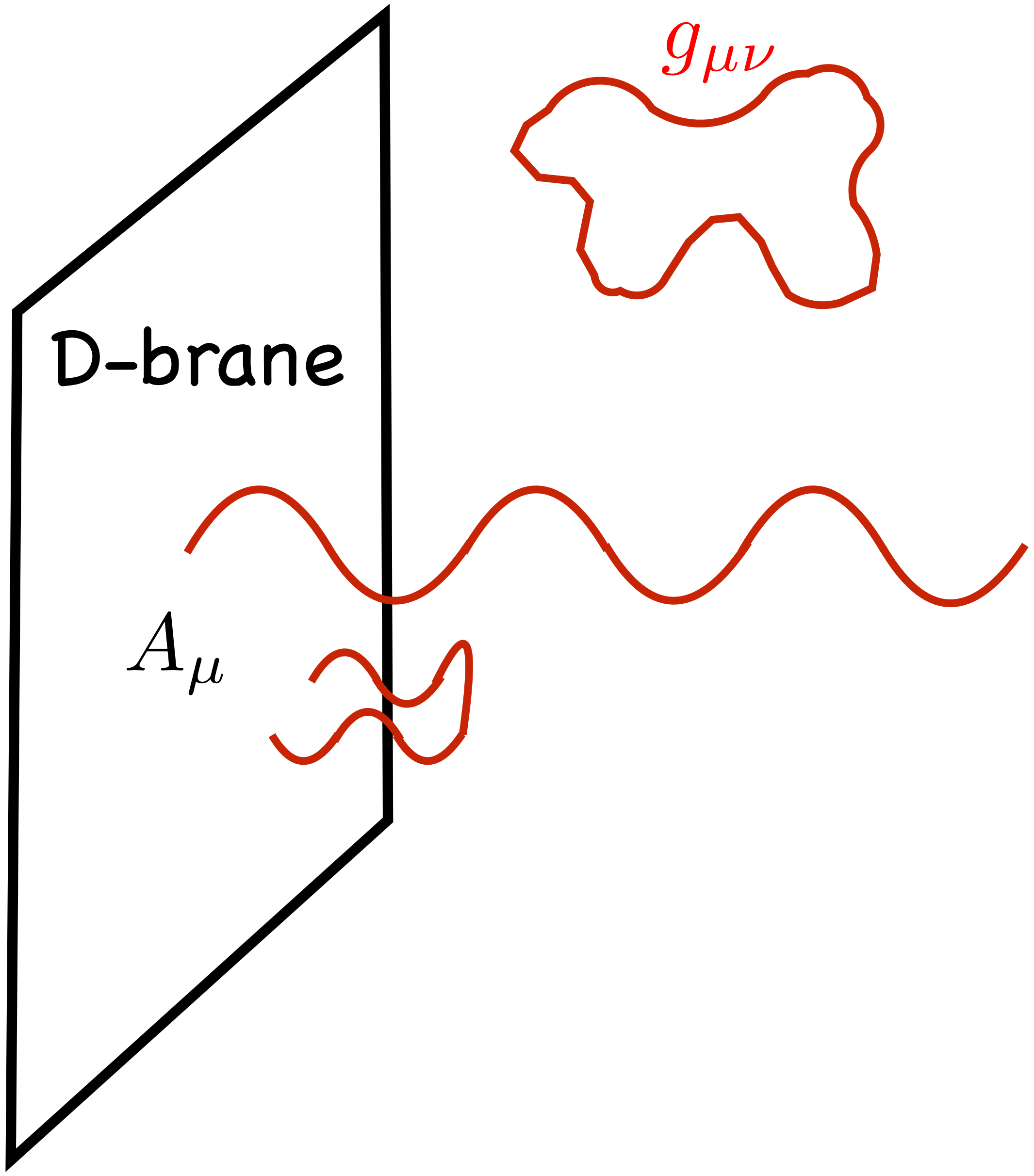
but basically:

spectrum: $\alpha' m^2 = n \in \mathbb{Z} \begin{cases} n = 0 & \rightarrow \quad \text{observed particles} \\ n \neq 0 & \rightarrow \quad \text{supermassive} \end{cases}$

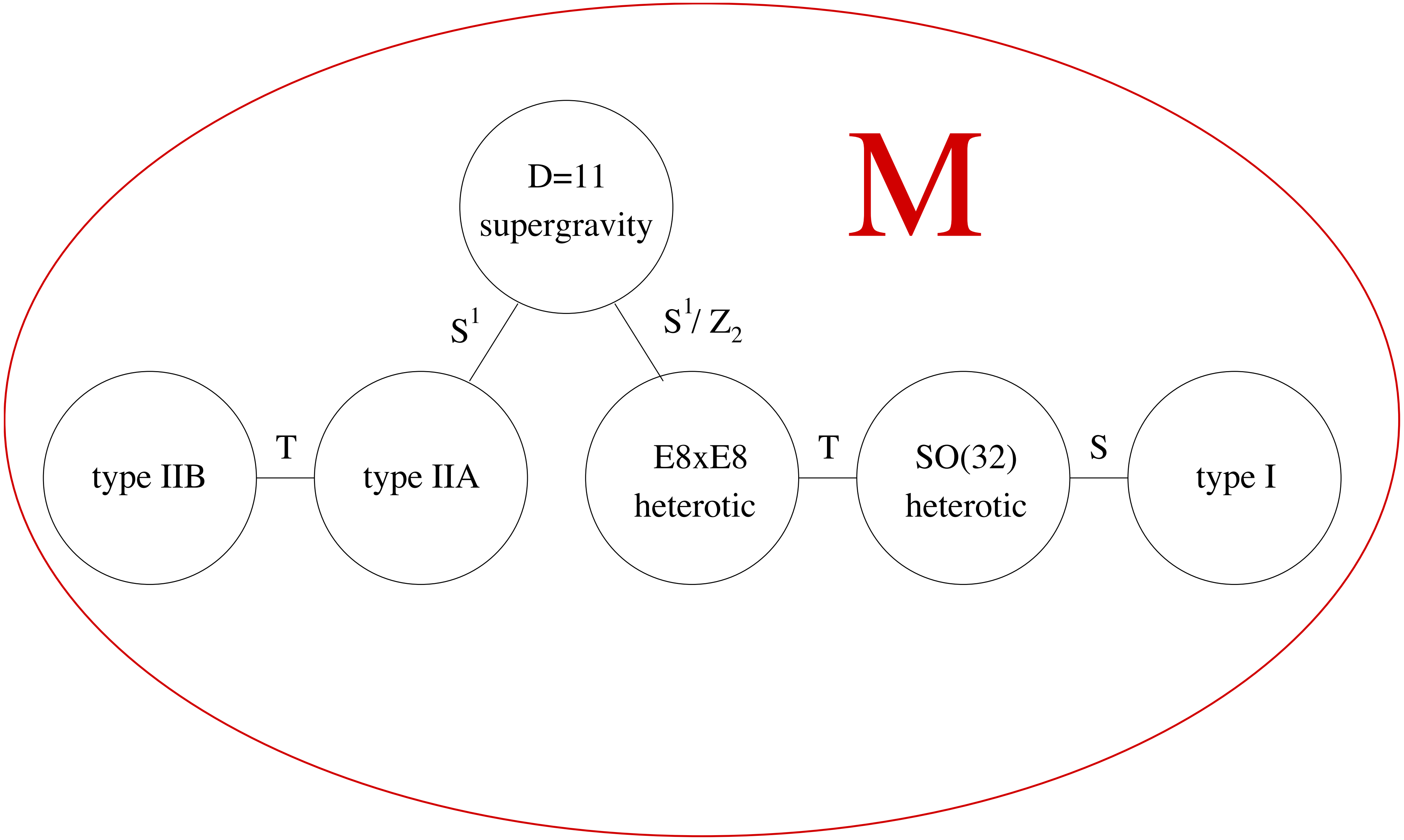
massless modes contain graviton (closed strings)
and gauge fields (open strings)


$g_{\mu\nu}$
D-brane
A_μ



- String theory contains extended objects of all dimensions -> p-branes

- spectrum of these objects leads to relations between string theories -> dualities


M
D=11
supergravity
S^1
S^1/Z_2
type IIB
T
type IIA
E8xE8
heterotic
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SO(32)
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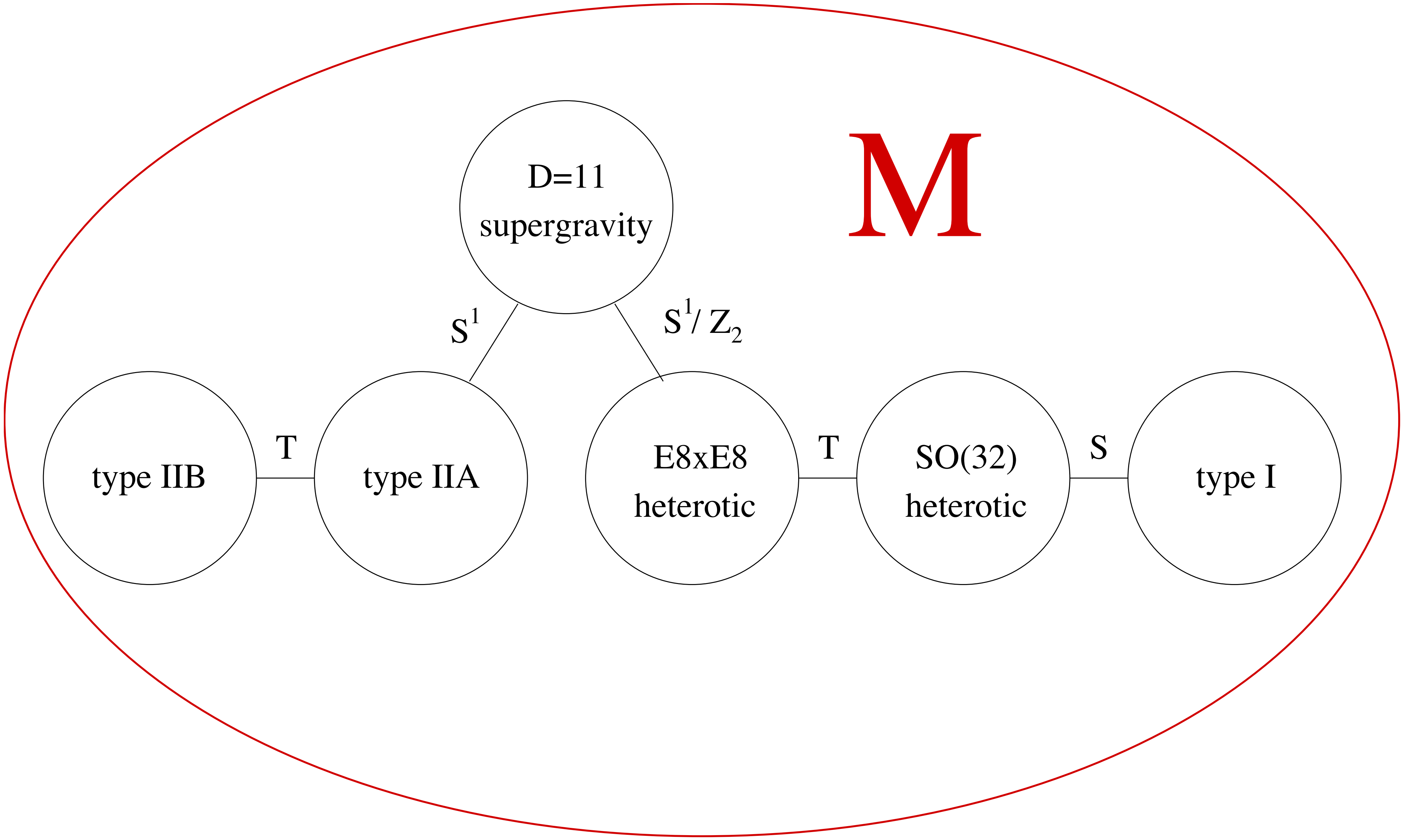

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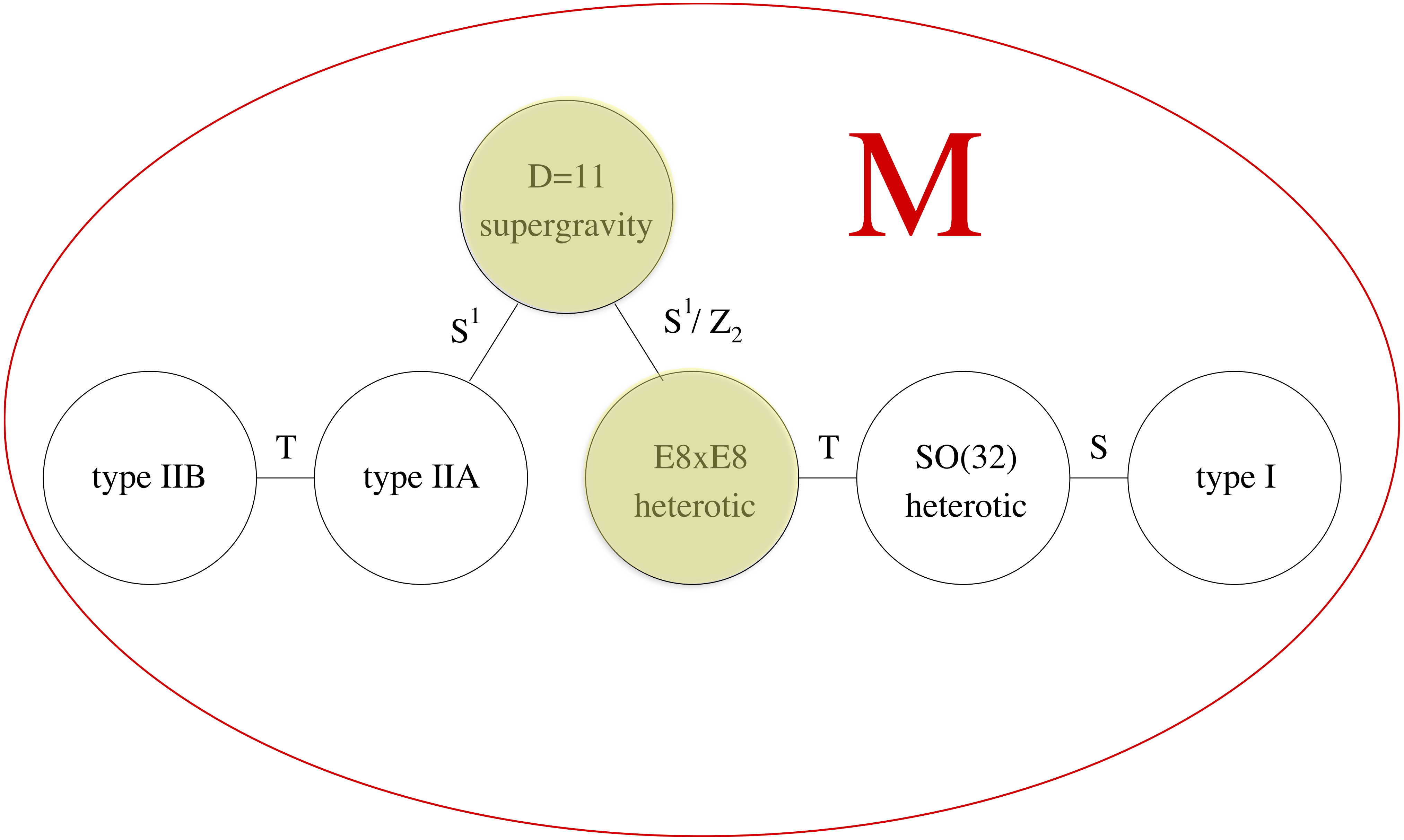
- a ``unique" theory of relativistic extended objects
- certainly the most complicated and richest structure ever in mathematical physics



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Is it relevant to particle physics?

String theory and particle physics: some general features

1) Gauge theories and gravity

Gauge theories and gravity are the main structural features of the established fundamental theories.

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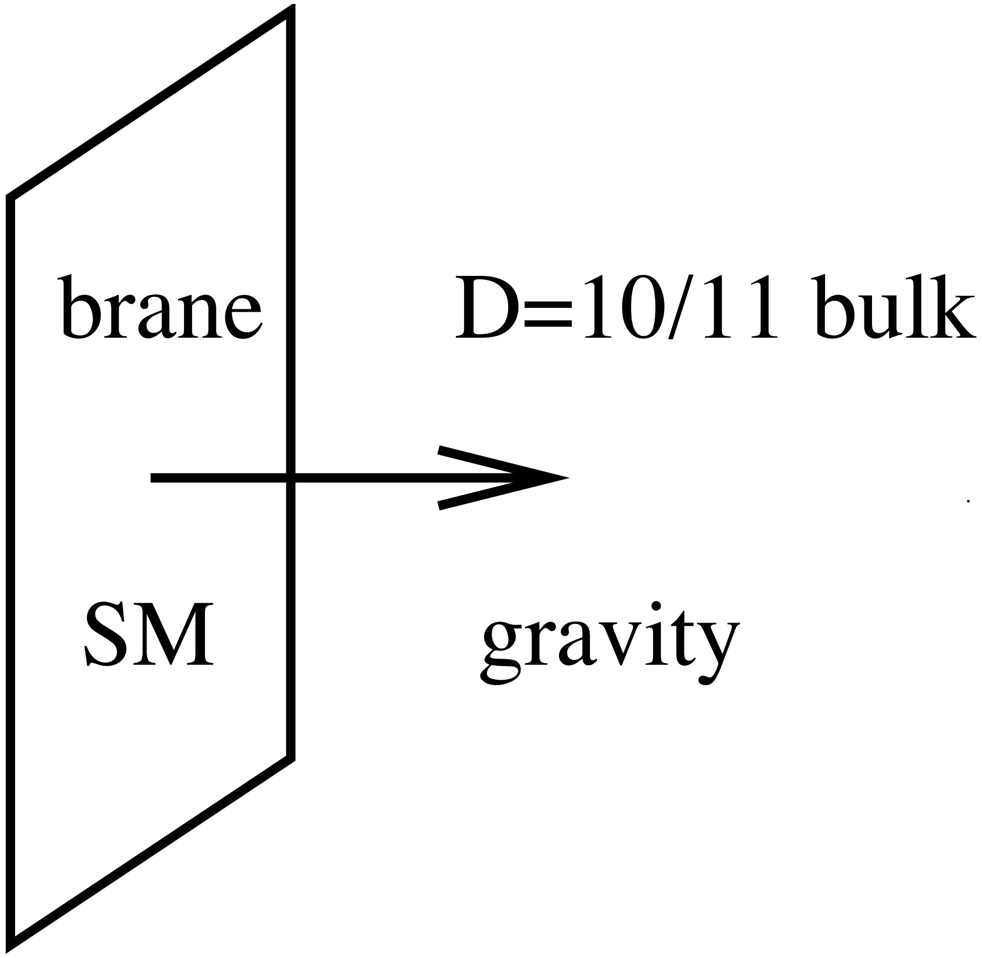

``brane world"

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Unlike field theory, string theory is UV finite, including for processes which involve gravitons.

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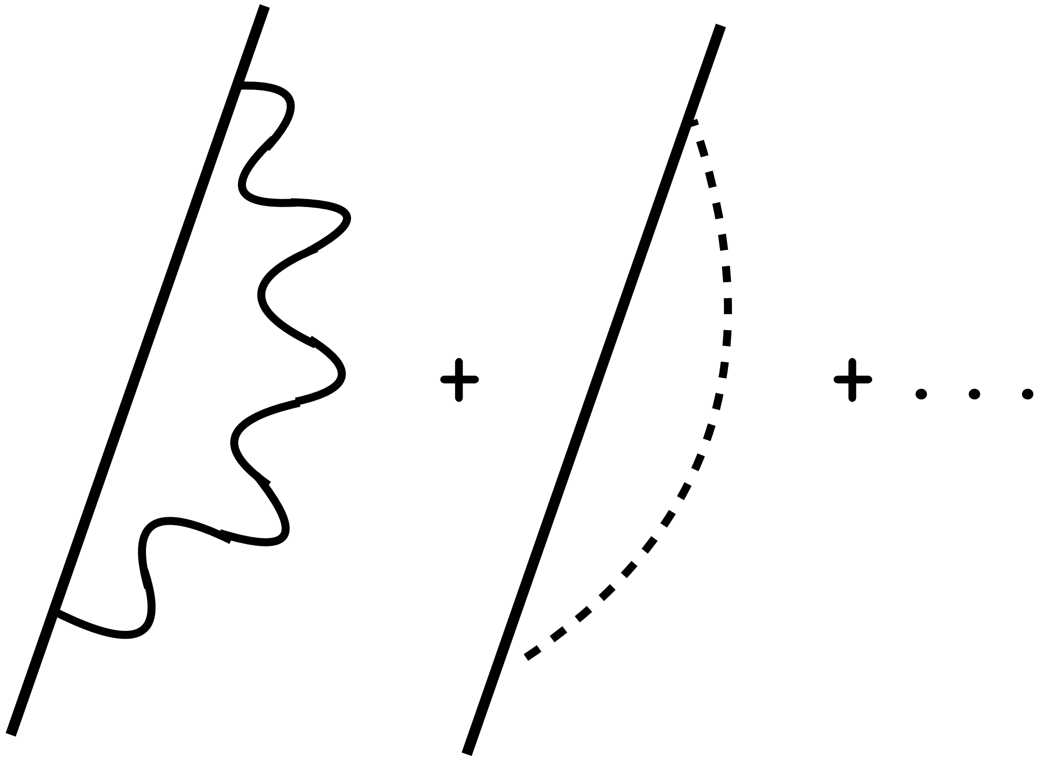

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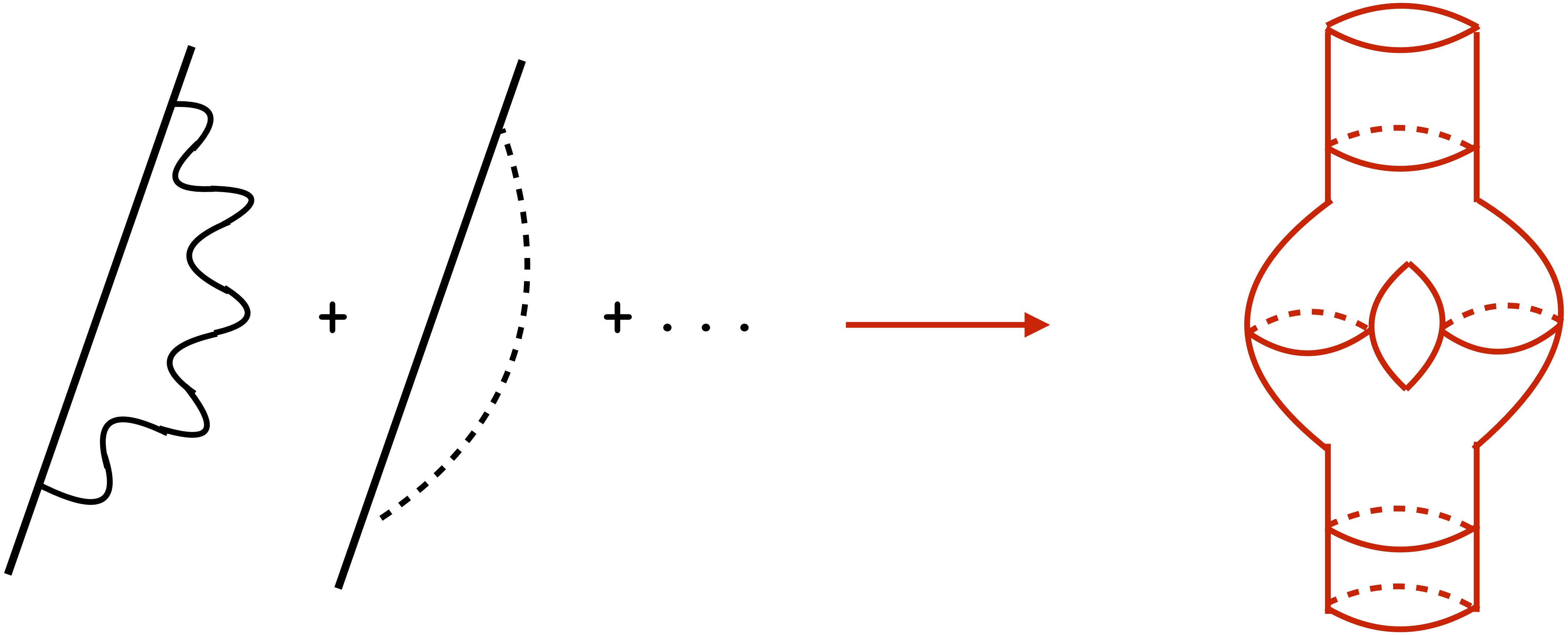

3) family repetition

every matter particle appears in three flavours, a characteristic put puzzling feature of the standard model

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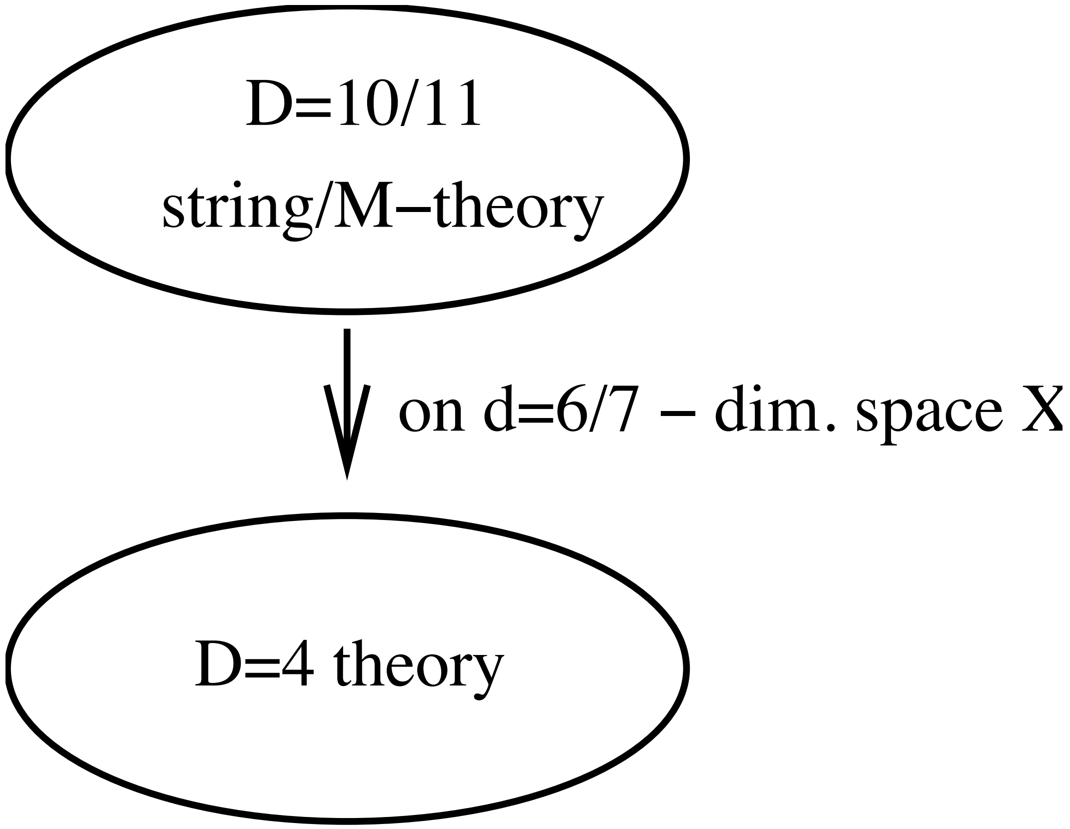
D=10/11

string/M–theory

on d=6/7 – dim. space X

D=4 theory

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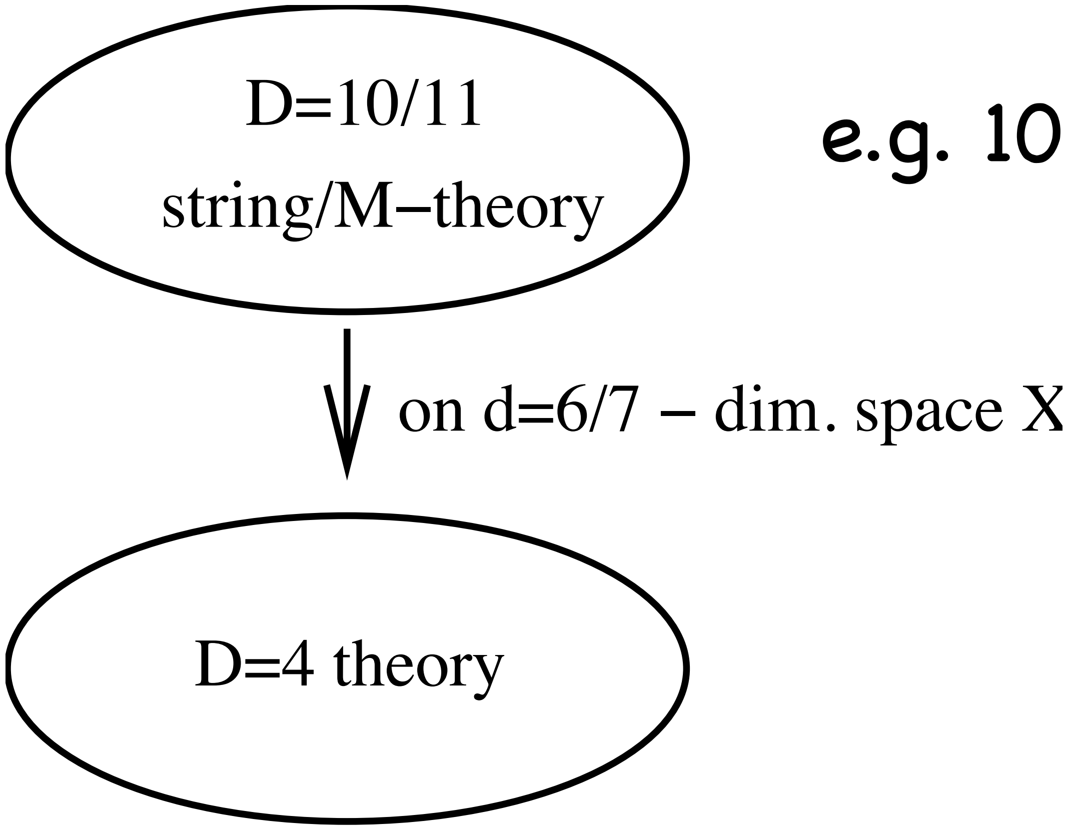

e.g. 10-dim. 2 form B_{IJ} with $\triangle_{10} B = 0$

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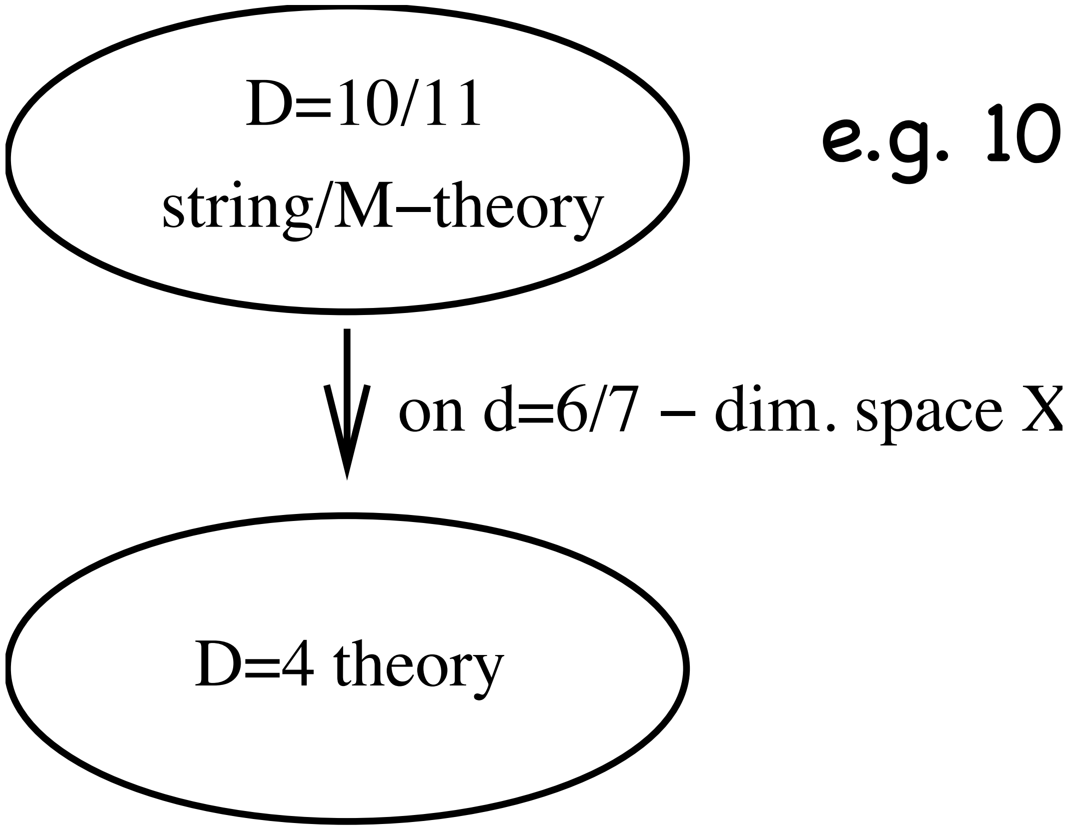

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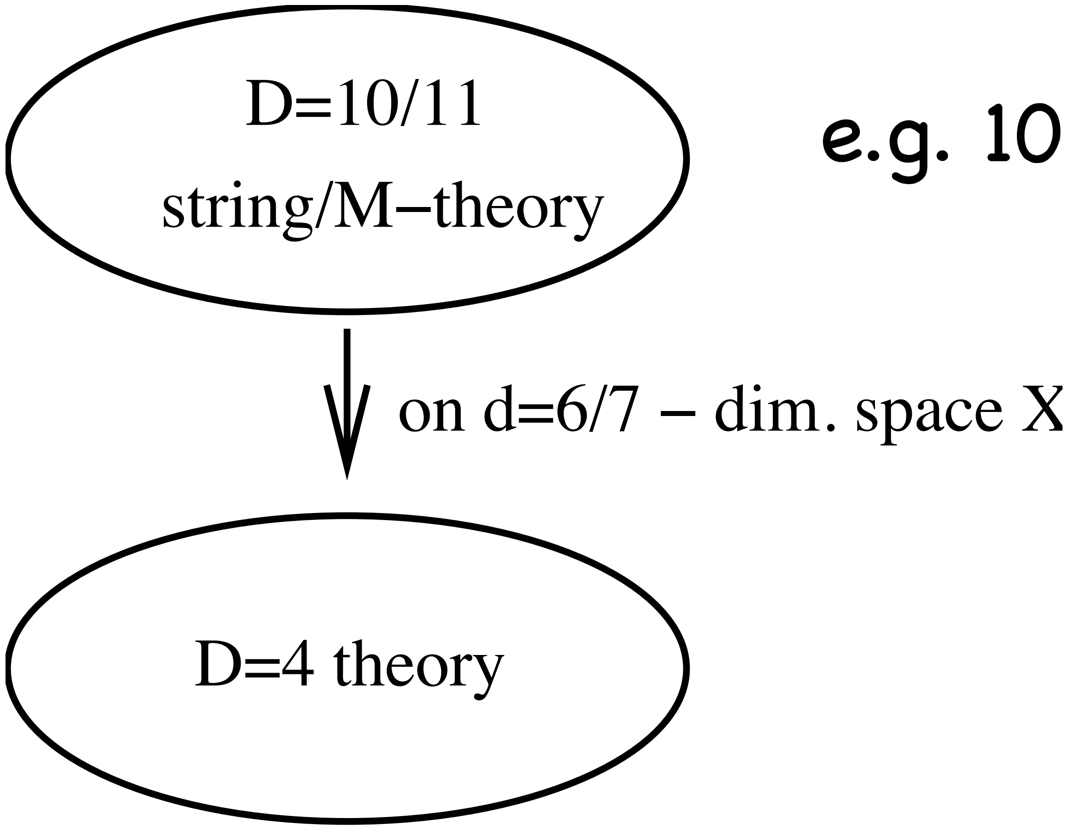

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$$\triangle_4 \phi_4 = 0 \ \Leftrightarrow \ \triangle_X \omega_X = 0$$

$$\Longrightarrow \ \# \phi_4 = b^2(X)$$

4) SU(3)xSU(2)xU(1) representation structure

one standard model family:

$$SU(3)\times SU(2)\times U(1)\ :\ \overset{d}{(\bar{\mathbf{3}},\mathbf{1})_{2/3}}\oplus\overset{L}{(\mathbf{1},\mathbf{2})_{-1}}\oplus\overset{Q}{(\mathbf{3},\mathbf{2})_{-1/3}}\oplus\overset{u}{(\bar{\mathbf{3}},\mathbf{1})_{-4/3}}\oplus\overset{e}{(\mathbf{1},\mathbf{1})_{2}}$$

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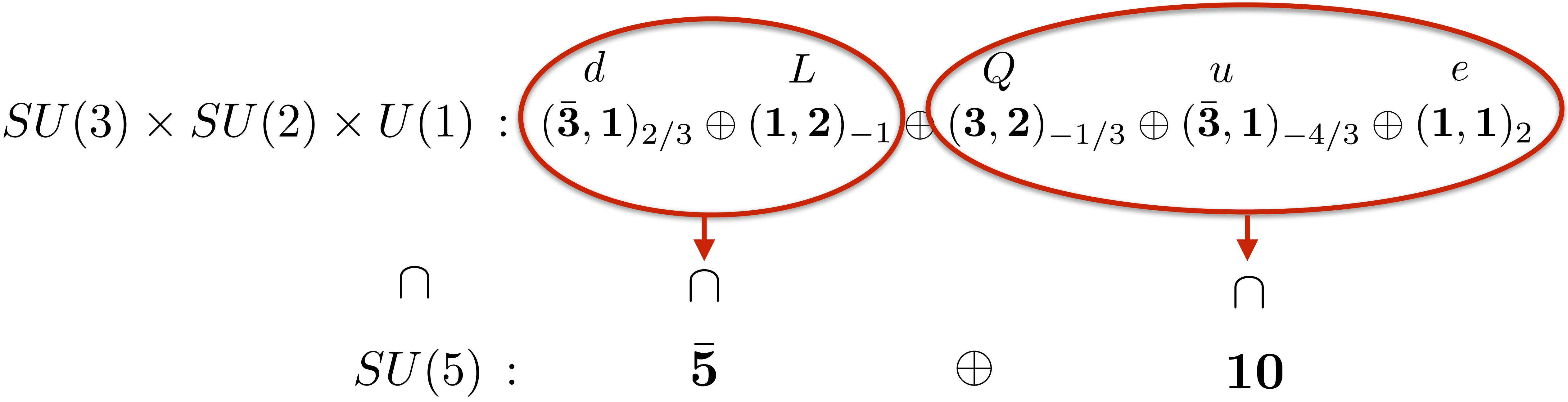

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$$\begin{array}{ccccc} \cap & & \cap & & \cap \\ SU(5)\ : & & \bar{\mathbf{5}} & \oplus & \mathbf{10} \end{array}$$

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\cap & \downarrow & & \downarrow & & \\
 & \cap & & \cap & & \nu_R \\
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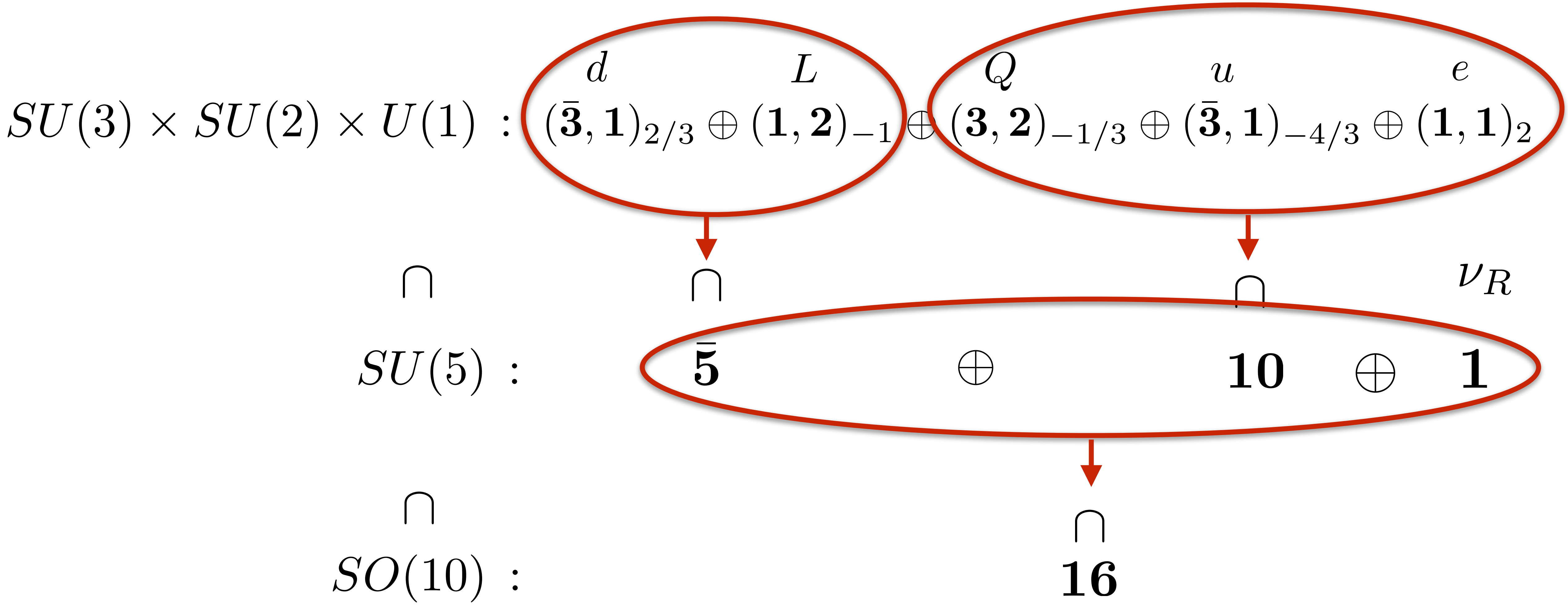

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\cap & \cap & & & & & & \cap & & \nu_R & \\
SU(5)\ : & \bar{\mathbf{5}} & & \oplus & & & & \mathbf{10} & \oplus & \mathbf{1} & \\
\cap & & & & & \cap & & & & & \\
SO(10)\ : & & & & & \mathbf{16} & & & & &
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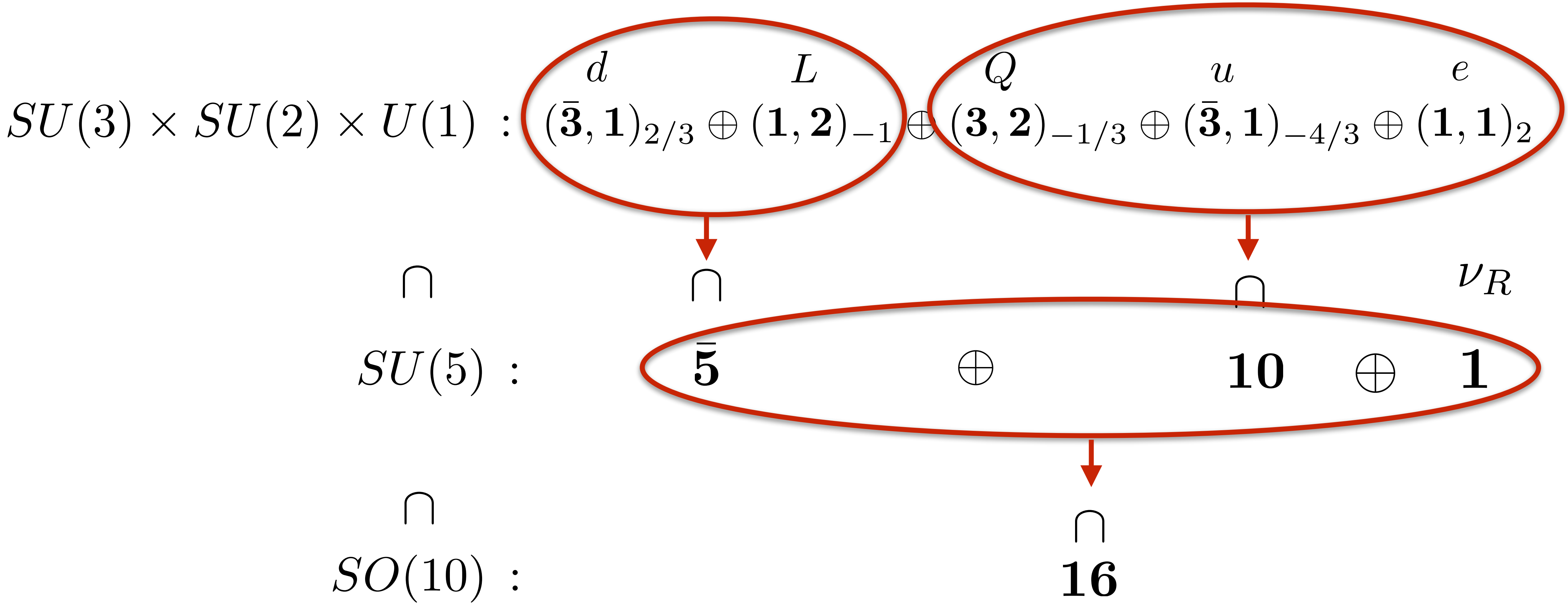

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Which group contains the spinor of SO(10) in its adjoint?

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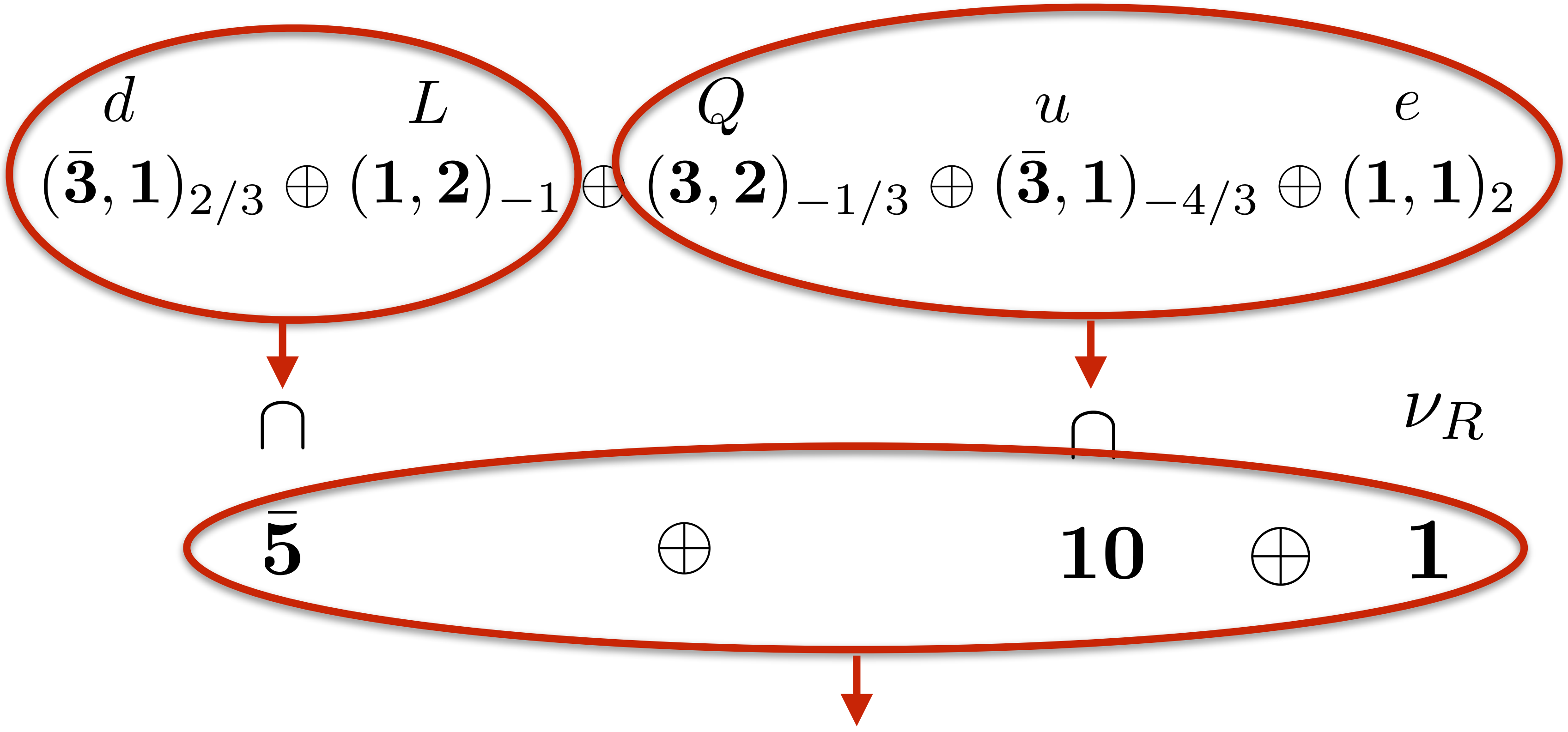

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$$\begin{array}{rcc}
\cap & & \cap \qquad\qquad \cap \\
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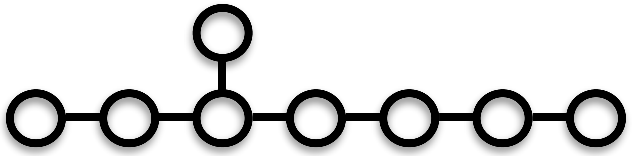
\cap & & \cap \\
SO(10) & : & \mathbf{16} \\
\cap & & \cap \\
E_6 & : & \mathbf{78} \\
\cap & & \cap \\
E_7 & : & \mathbf{133} \\
\cap & & \cap \\
E_8 & : & \mathbf{248}
\end{array}$$

Which group contains the spinor of SO(10) in its adjoint?

E_8



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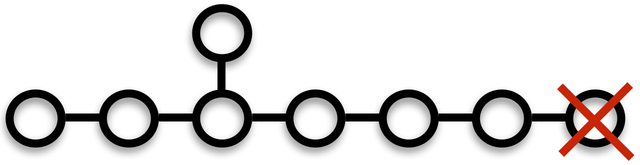
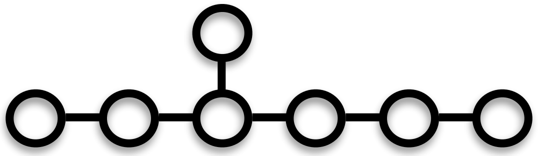


E_7



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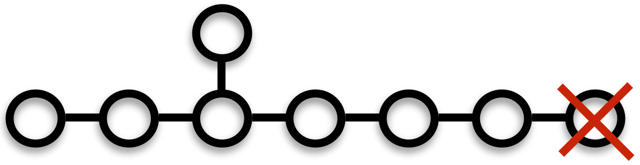

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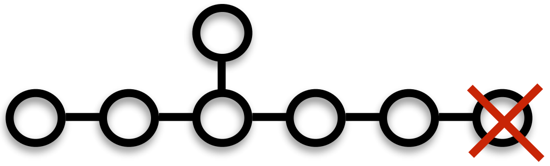
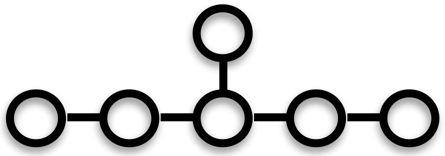


E_6



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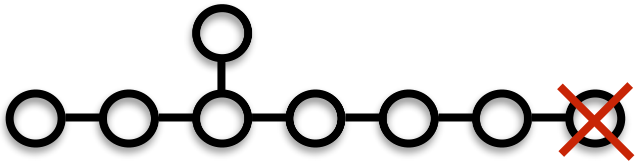

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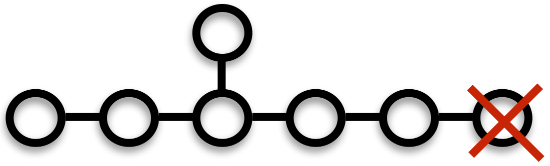

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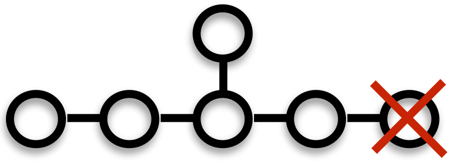
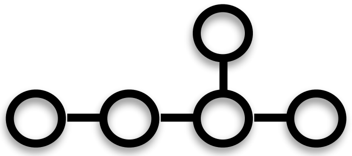


$E_5 = SO(10)$



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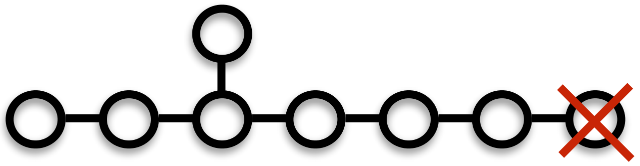

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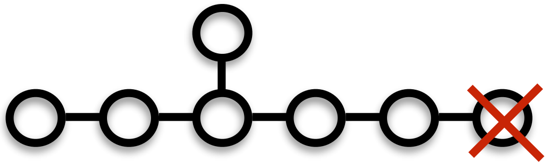

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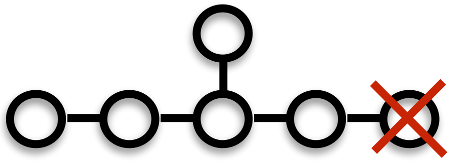

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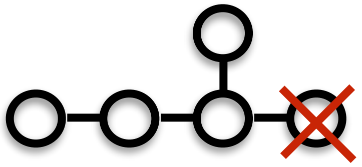
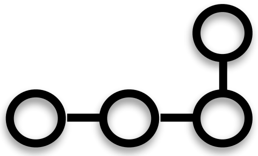
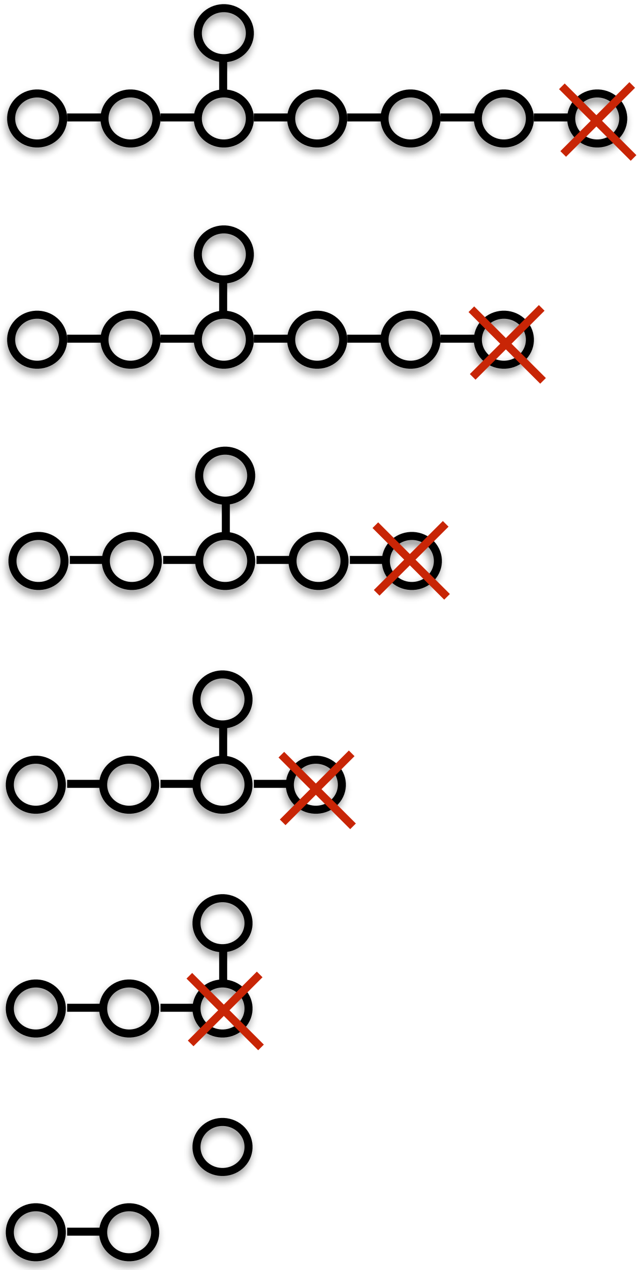


$E_4 = SU(5)$





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E_7

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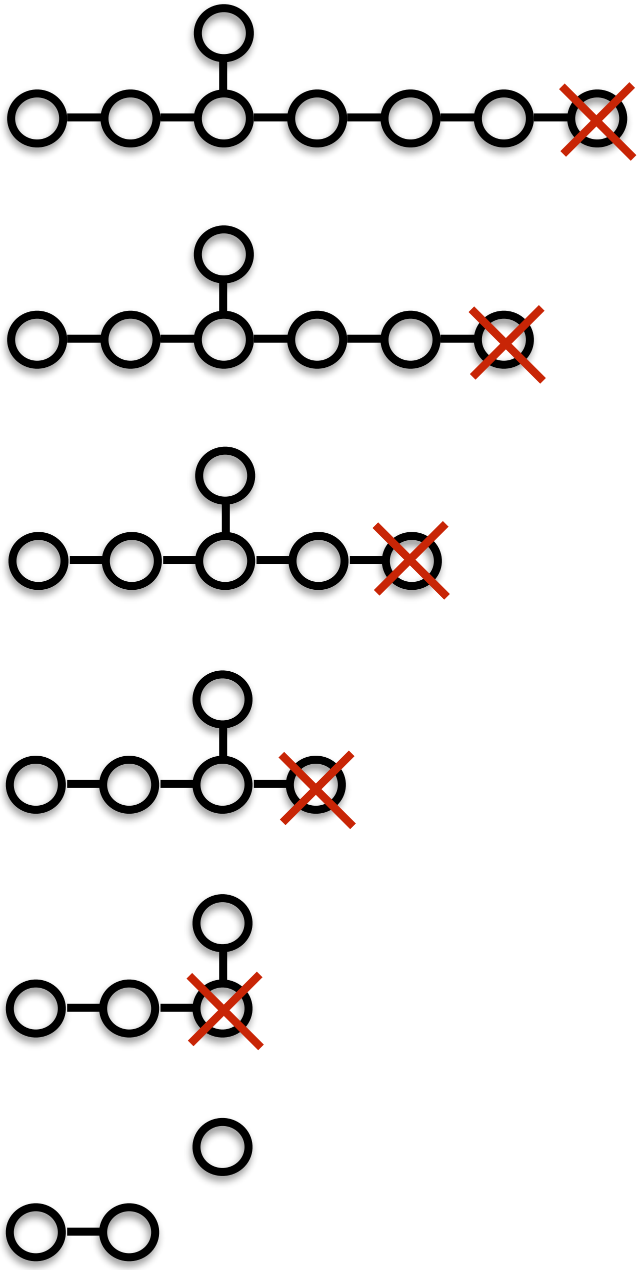

Exceptional gauge groups and E_8 in particular are prevalent in string theory -> representation structure of known particles can be accounted for.

5) The Higgs multiplet

$$\begin{array}{cc} & H \\ SU(3)\times SU(2)\times U(1)\ : & (\mathbf{1},\mathbf{2})_{-1} \end{array}$$

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How can the Higgs be reconciled with unification?
-> heavy mass to triplet, “doublet-triplet” splitting

5) The Higgs multiplet

$$\begin{array}{ccc} SU(3)\times SU(2)\times U(1) & : & \overset{H}{(\mathbf{1},\mathbf{2})_{-1}} \oplus \overset{T}{(\bar{\mathbf{3}},\mathbf{1})_{2/3}} \\ \cap & & \cap \\ SU(5) & : & \bar{\mathbf{5}} \end{array}$$

How can the Higgs be reconciled with unification?
-> heavy mass to triplet, "doublet-triplet" splitting

Works nicely in string theory: topological reason for light doublet and heavy triplet

What is the main problem?

Large degeneracy of vacua through choice in compactification:

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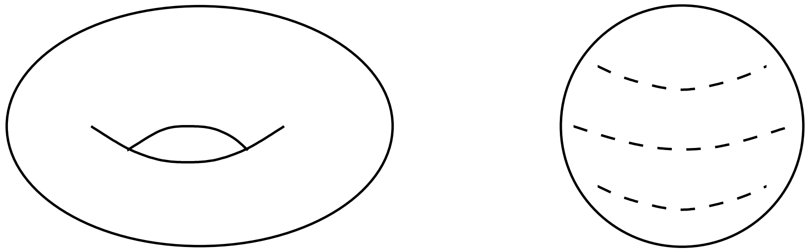

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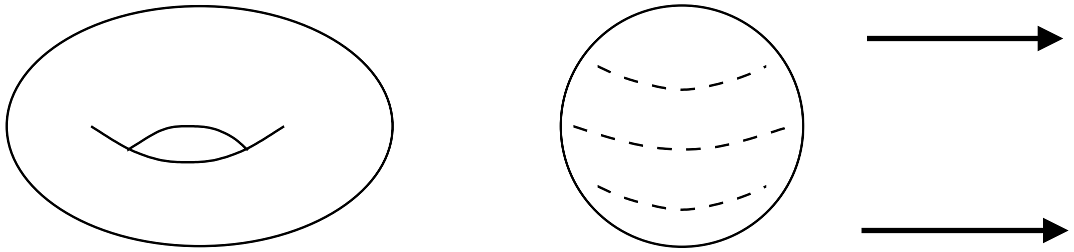

structure of 4d theory

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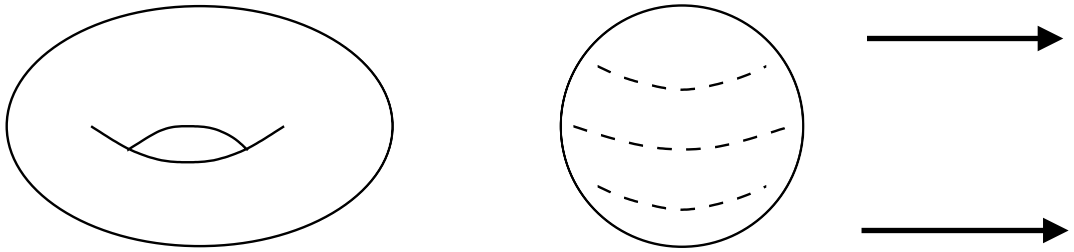

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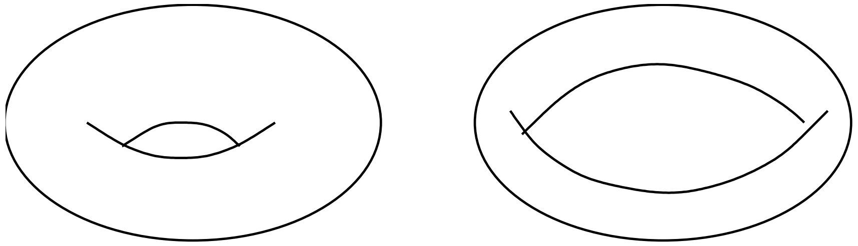

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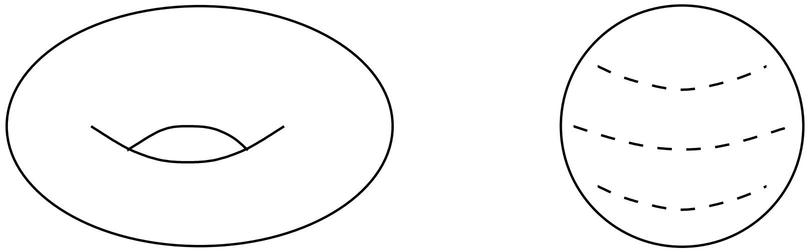

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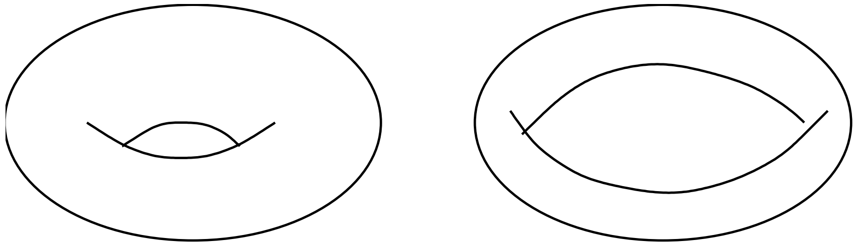

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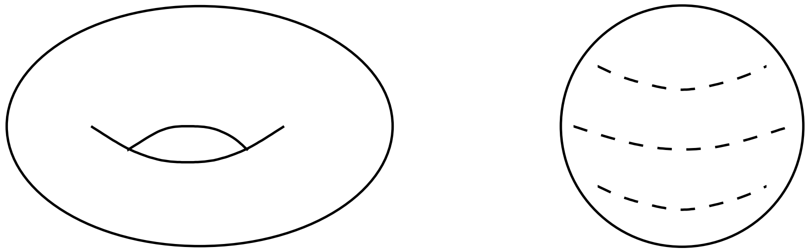

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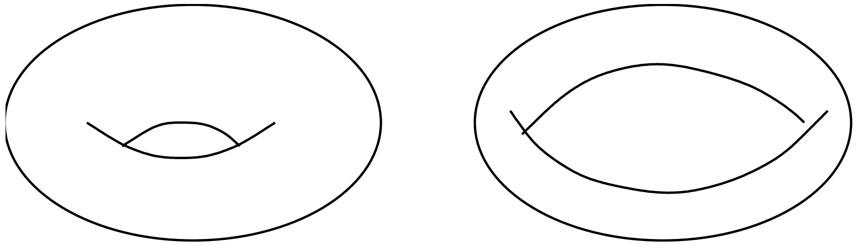

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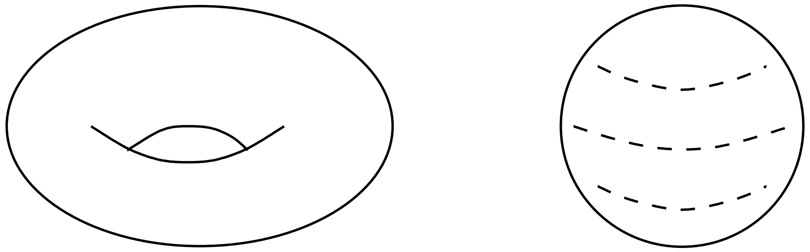

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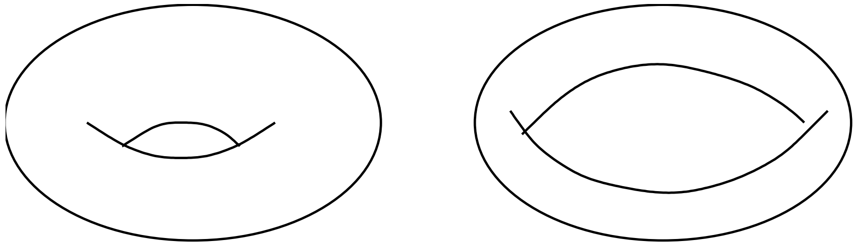

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How do we find the ``right” vacuum?

- moduli: presumably fixed dynamically
- topology: currently, we can only explore the possible choices

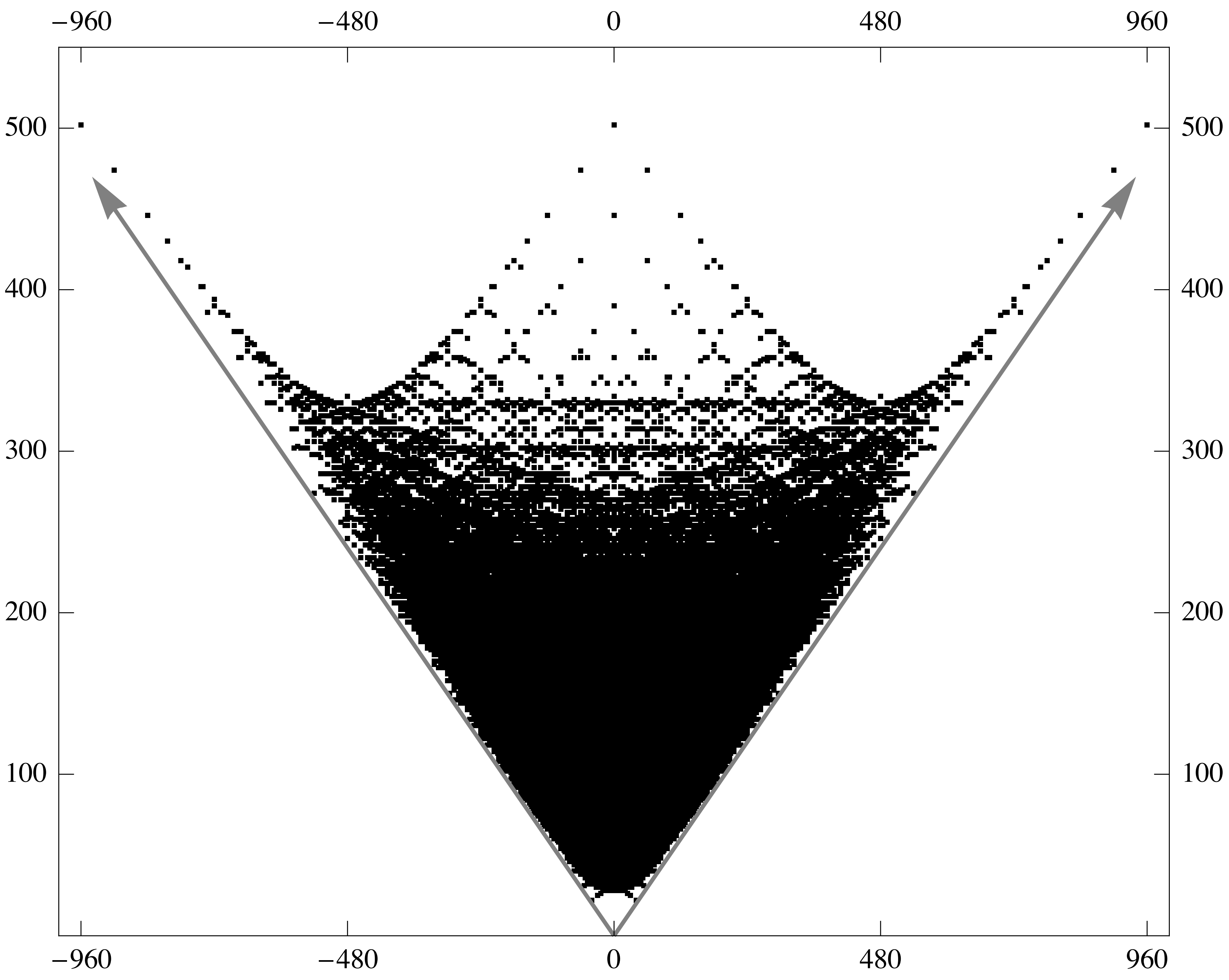
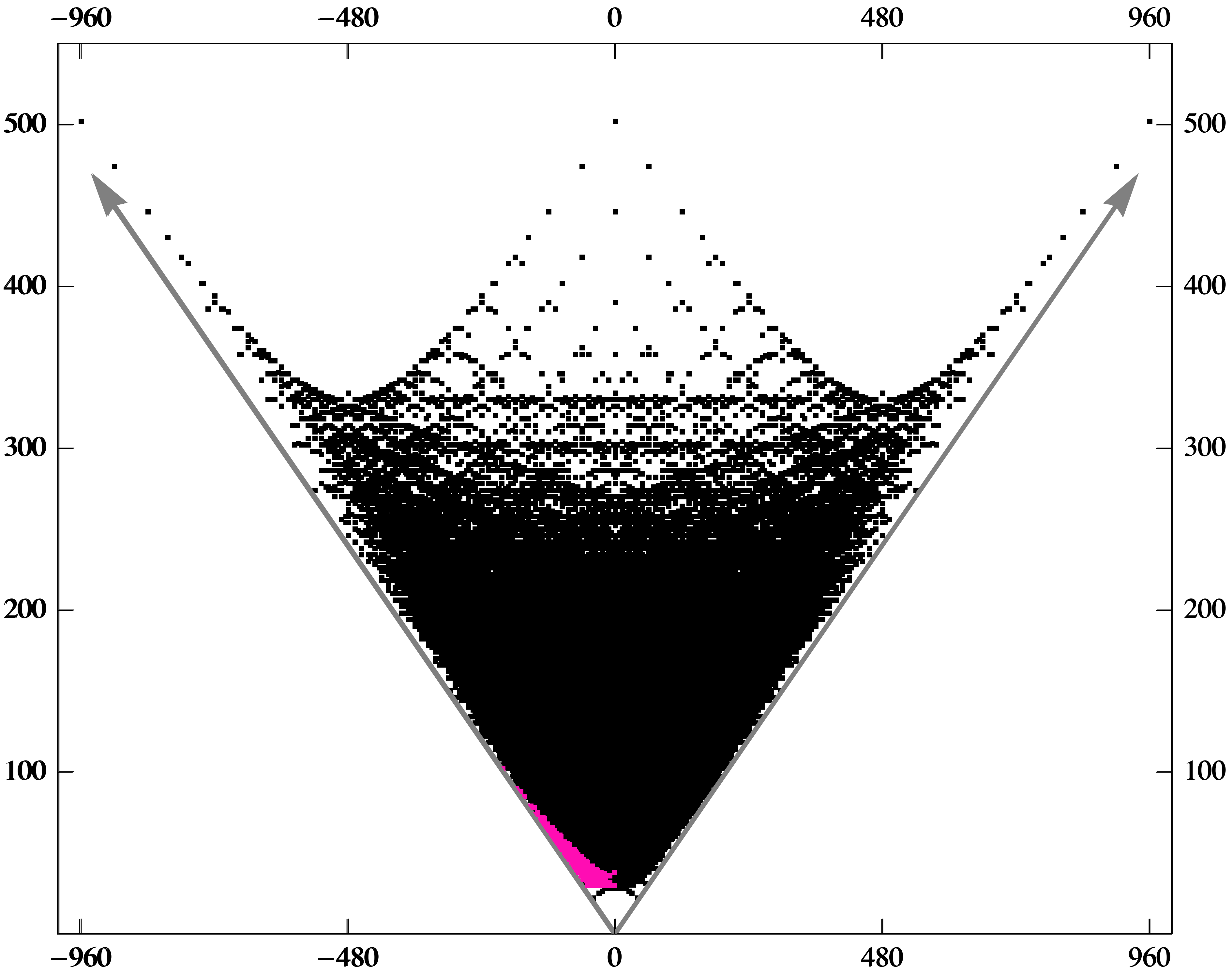


Figure 1: A plot of the Hodge numbers of the Kreuzer–Skarke list. $\chi = 2(h^{11} - h^{21})$ is plotted horizontally and $h^{11} + h^{21}$ is plotted vertically. The oblique axes bound the region $h^{11} \geq 0$, $h^{21} \geq 0$.

(from P. Candelas et al., 0706.3134)



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Model building

. . . in the context of the $E_8 \times E_8$ heterotic string:

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6d manifold

X

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6d manifold		vector bundle
X	$\longleftarrow$	V
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$$R_{ab} = R_{\bar{a}\bar{b}} = 0$$

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-> heterotic vacuum determined by a pair (X, V)

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structure group

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First pass:

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In practice use structure group $S(U(1)^5) \subset SU(5)$ so that

$$V = \bigoplus_{a=1}^{5} L_a\,, \quad L_a = \mathcal{O}_X(\mathbf{k}_a)$$

is a sum of five line bundles, specified by integer vectors $\mathbf{k}_a$.

. . . results in GUT models with gauge group

$$SU(5) \times S(U(1)^5) \longleftarrow$$

typically
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$\mathbf{5}_{-\mathbf{e}_a-\mathbf{e}_b}$	$-\mathbf{e}_a-\mathbf{e}_b$	$L_a^*\otimes L_b^*$	$\wedge^2 V^*$
$\mathbf{1}_{\mathbf{e}_a-\mathbf{e}_b}$	$\mathbf{e}_a-\mathbf{e}_b$	$L_a\otimes L_b^*$	$V\otimes V^*$
$\mathbf{1}_{-\mathbf{e}_a+\mathbf{e}_b}$	$-\mathbf{e}_a+\mathbf{e}_b$	$L_a^*\otimes L_b$	

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families and mirror families

$\longleftarrow = 3|\Gamma|$

$\longleftarrow = 0$

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Can lead to standard models after taking Γ- quotient and including Wilson line.

The standard model of particle physics from string theory

An example:

CY data:

- **Cicy 7862, Symmetry 3**

$$X = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}$$

$\eta(X) = -128 \quad h^{1,1}(X) = 4 \quad h^{2,1}(X) = 68 \quad c_2(TX) = \{24, 24, 24, 24\}$

$\kappa = 12\, t_1\, t_2\, t_3 + 12\, t_1\, t_2\, t_4 + 12\, t_1\, t_3\, t_4 + 12\, t_2\, t_3\, t_4$

symmetry: 3   order: 4

Abelian: True   block diagonal: True   factors: {2, 2}

Action on coordinates:
$$\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \right\}$$

Action on polynomials: $\{(\,1\,), (\,1\,)\}$

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$X = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \longleftarrow$ CY: tetra-quadric in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$

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- **Cicy 7862, Symmetry 3**

$$X = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix} \longleftarrow \text{CY: tetra-quadric in } \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$$

$\eta(X) = -128 \quad h^{1,1}(X) = 4 \quad h^{2,1}(X) = 68 \quad c_2(TX) = \{24, 24, 24, 24\}$ ⟵ topological data

$\kappa = 12\, t_1\, t_2\, t_3 + 12\, t_1\, t_2\, t_4 + 12\, t_1\, t_3\, t_4 + 12\, t_2\, t_3\, t_4$

symmetry: 3 order: 4

Abelian: True block diagonal: True factors: {2, 2}

Action on coordinates: $\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \right\}$

Action on polynomials: $\{(\ 1\ ),\ (\ 1\ )\}$

The standard model of particle physics from string theory

An example:

CY data:

- **Cicy 7862, Symmetry 3**

$$X = \begin{pmatrix} 2 \\ 2 \\ 2 \\ 2 \end{pmatrix}$$ ← CY: tetra-quadric in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$

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$\kappa = 12\,t_1\,t_2\,t_3 + 12\,t_1\,t_2\,t_4 + 12\,t_1\,t_3\,t_4 + 12\,t_2\,t_3\,t_4$ ← volume

symmetry: 3 order: 4

Abelian: True block diagonal: True factors: {2, 2}

Action on coordinates: $$\left\{\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}\right\}$$

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CY: tetra-quadric in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$

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topological data

$\kappa = 12\, t_1\, t_2\, t_3 + 12\, t_1\, t_2\, t_4 + 12\, t_1\, t_3\, t_4 + 12\, t_2\, t_3\, t_4$

volume

symmetry: 3   order: 4

Abelian: True   block diagonal: True   factors: {2, 2}

Action on coordinates: $\left\{ \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{pmatrix}, \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \right\}$

Action on polynomials: $\{(\ 1\ ), (\ 1\ )\}$

$\mathbb{Z}_2 \times \mathbb{Z}_2$ generators

bundle data:

Basic properties

standard model? True   massless U(1): 1   number of $5\,\overline{5}$ pairs: 3 $c_2(V) = \{24, 8, 20, 12\}$

$$V:\ (k^i{}_a) = \begin{pmatrix} -1 & -1 & 0 & 1 & 1 \\ 0 & -3 & 1 & 1 & 1 \\ 0 & 2 & -1 & -1 & 0 \\ 1 & 2 & 0 & -1 & -2 \end{pmatrix}$$

Cohomology of V:

$L_2 = \{-1, -3, 2, 2\}$ $h[L_2] = \{0, 8, 0, 0\}$ $h[L_2,R] = \{\{0, 0, 0, 0\}, \{2, 2, 2, 2\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

$L_5 = \{1, 1, 0, -2\}$ $h[L_5] = \{0, 4, 0, 0\}$ $h[L_5,R] = \{\{0, 0, 0, 0\}, \{1, 1, 1, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

$L_2\times L_4 = \{0, -2, 1, 1\}$ $h[L_2\times L_4] = \{0, 4, 0, 0\}$ $h[L_2\times L_4,R] = \{\{0, 0, 0, 0\}, \{1, 1, 1, 1\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

$L_2\times L_5 = \{0, -2, 2, 0\}$ $h[L_2\times L_5] = \{0, 3, 3, 0\}$ $h[L_2\times L_5,R] = \{\{0, 0, 0, 0\}, \{0, 1, 1, 1\}, \{0, 1, 1, 1\}, \{0, 0, 0, 0\}\}$

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$L_1\times L_2{}^* = \{0, 3, -2, -1\}$ $h[L_1\times L_2{}^*] = \{0, 0, 12, 0\}$ $h[L_1\times L_2{}^*,R] = \{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{3, 3, 3, 3\}, \{0, 0, 0, 0\}\}$

$L_1\times L_5{}^* = \{-2, -1, 0, 3\}$ $h[L_1\times L_5{}^*] = \{0, 0, 12, 0\}$ $h[L_1\times L_5{}^*,R] = \{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{3, 3, 3, 3\}, \{0, 0, 0, 0\}\}$

$L_2\times L_3{}^* = \{-1, -4, 3, 2\}$ $h[L_2\times L_3{}^*] = \{0, 20, 0, 0\}$ $h[L_2\times L_3{}^*,R] = \{\{0, 0, 0, 0\}, \{5, 5, 5, 5\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

$L_2\times L_4{}^* = \{-2, -4, 3, 3\}$ $h[L_2\times L_4{}^*] = \{0, 12, 0, 0\}$ $h[L_2\times L_4{}^*,R] = \{\{0, 0, 0, 0\}, \{3, 3, 3, 3\}, \{0, 0, 0, 0\}, \{0, 0, 0, 0\}\}$

$L_3\times L_5{}^* = \{-1, 0, -1, 2\}$ $h[L_3\times L_5{}^*] = \{0, 0, 4, 0\}$ $h[L_3\times L_5{}^*,R] = \{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{1, 1, 1, 1\}, \{0, 0, 0, 0\}\}$

Wilson line: {{0, 0}, {0, 1}}   Equivariant structure: {{0, 0}, {0, 0}, {0, 0}, {0, 0}, {0, 0}}   Higgs pairs: 1

Downstairs spectrum: $\{2\,10_2, 10_5, \overline{5}_{2,4}, 2\,\overline{5}_{4,5}, H_{2,5}, \overline{H}_{2,5}, 3\,S_{2,1}, 3\,S_{5,1}, 5\,S_{2,3}, 3\,S_{2,4}, S_{5,3}\}$ Phys. Higgs: $\{H_{2,5}, \overline{H}_{2,5}\}$

Transfer format: {{6, 1, 1, 4, 6, 5, 9, 9, 8, 10, 1, 7, 17}, {6, 6, −1, −1, −1, −1}}

$\mathrm{rk}(Y^{(u)}) = \{2, 2\}$ $\mathrm{rk}(Y^{(d)})) = \{0, 0\}$ dim. 4 operators absent: {True, True}   dim. 5 operators absent: {True, True}

bundle data:

Basic properties

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integer matrix defining line bundle sum

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$L_1\times L_2^* = \{0, 3, -2, -1\}$ $h[L_1\times L_2^*] = \{0, 0, 12, 0\}$ $h[L_1\times L_2^*,R] = \{\{0, 0, 0, 0\}, \{0, 0, 0, 0\}, \{3, 3, 3, 3\}, \{0, 0, 0, 0\}\}$

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$\mathrm{rk}(Y^{(u)}) = \{2, 2\}$ $\mathrm{rk}(Y^{(d)})) = \{0, 0\}$ dim. 4 operators absent: {True, True} dim. 5 operators absent: {True, True}

spectrum: $\mathbf{10}_2,\ \mathbf{10}_2,\ \mathbf{10}_5,\ \overline{\mathbf{5}}_{2,4},\ \overline{\mathbf{5}}_{4,5},\ \overline{\mathbf{5}}_{4,5}\,,\ H_{2,5},\ \bar{H}_{2,5}$

$3\,\mathbf{1}_{2,1},\ 3\,\mathbf{1}_{5,1},\ 5\mathbf{1}_{2,3},\ 3\,\mathbf{1}_{2,4},\ \mathbf{1}_{5,3}$

allowed operators:

■ Operators

basic superpotential terms:

$$\overline{H}10^p10^q:\ Y^{(u)} = \begin{pmatrix} (0) & (0) & (1) \\ (0) & (0) & (1) \\ (1) & (1) & (0) \end{pmatrix}$$

$$H\overline{5}^p10^q:\ Y^{(d)} = \begin{pmatrix} (0) & (0) & (0) \\ (0) & (0) & (0) \\ (0) & (0) & (0) \end{pmatrix}$$

$$H\overline{H}:\ \mu = \{1\}$$

$$W_{sing} = \{0\}$$

R–parity violating terms in superpotential:

$$\overline{H}L^p:\ \rho = \begin{pmatrix} 0 \\ S_{2,4} \\ S_{2,4} \end{pmatrix}$$

$$10^p\overline{5}^q\overline{5}^r:\ \lambda = \{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}$$

Dimension 5 operators in superpotential:

$$\overline{5}^p10^q10^r10^s:\ \lambda' = \{\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\},$$

$$\{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}, \{\{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}, \{\{0\}, \{0\}, \{0\}\}\}\}, \{\{\{\{0\}, \{0\}, \{0\}\}, \{\{0\},$$

D–terms:

$$\text{FI–terms: } k^i{}_a\kappa_i = \begin{pmatrix} 4\,t_1\,t_2 + 4\,t_1\,t_3 - 4\,t_2\,t_4 - 4\,t_3\,t_4 \\ 16\,t_1\,t_2 - 4\,t_1\,t_3 + 4\,t_2\,t_3 - 4\,t_1\,t_4 + 4\,t_2\,t_4 - 16\,t_3\,t_4 \\ -4\,t_1\,t_2 + 4\,t_1\,t_3 - 4\,t_2\,t_4 + 4\,t_3\,t_4 \\ -8\,t_1\,t_2 + 8\,t_3\,t_4 \\ -8\,t_1\,t_2 - 4\,t_1\,t_3 - 4\,t_2\,t_3 + 4\,t_1\,t_4 + 4\,t_2\,t_4 + 8\,t_3\,t_4 \end{pmatrix}$$

$$\text{singlet D–terms: } q_{\alpha a}S^{\alpha}\overline{S}^{\overline{\beta}} = \begin{pmatrix} -S_{2,1}\,S^{\dagger}{}_{2,1} - S_{5,1}\,S^{\dagger}{}_{5,1} \\ S_{2,1}\,S^{\dagger}{}_{2,1} + S_{2,3}\,S^{\dagger}{}_{2,3} + S_{2,4}\,S^{\dagger}{}_{2,4} \\ -S_{2,3}\,S^{\dagger}{}_{2,3} - S_{5,3}\,S^{\dagger}{}_{5,3} \\ -S_{2,4}\,S^{\dagger}{}_{2,4} \\ S_{5,1}\,S^{\dagger}{}_{5,1} + S_{5,3}\,S^{\dagger}{}_{5,3} \end{pmatrix}$$

allowed operators:

■ Operators

basic superpotential terms:

$$\overline{H}10^{p}10^{q}:\ Y^{(u)}=\begin{pmatrix}(0) & (0) & (1)\\ (0) & (0) & (1)\\ (1) & (1) & (0)\end{pmatrix}$$ ⟵ rank 2

$$H\overline{5}^{p}10^{q}:\ Y^{(d)}=\begin{pmatrix}(0) & (0) & (0)\\ (0) & (0) & (0)\\ (0) & (0) & (0)\end{pmatrix}$$

$$H\overline{H}:\ \mu=\{1\}$$

$$W_{\text{sing}}=\{0\}$$

R–parity violating terms in superpotential:

$$\overline{H}L^{p}:\ \rho=\begin{pmatrix}0\\ S_{2,4}\\ S_{2,4}\end{pmatrix}$$

$$10^{p}\overline{5}^{q}\overline{5}^{r}:\ \lambda=\{\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\}\}$$

Dimension 5 operators in superpotential:

$$\overline{5}^{p}10^{q}10^{r}10^{s}:\ \lambda'=\{\{\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\}\},$$

$$\{\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\}\},\{\{\{\{0\},\{0\},\{0\}\},\{\{0\},$$

D–terms:

$$\text{FI–terms: }k'^{i}_{a}\kappa_{i}=\begin{pmatrix}4t_1t_2+4t_1t_3-4t_2t_4-4t_3t_4\\ 16t_1t_2-4t_1t_3+4t_2t_3-4t_1t_4+4t_2t_4-16t_3t_4\\ -4t_1t_2+4t_1t_3-4t_2t_4+4t_3t_4\\ -8t_1t_2+8t_3t_4\\ -8t_1t_2-4t_1t_3-4t_2t_3+4t_1t_4+4t_2t_4+8t_3t_4\end{pmatrix}$$

$$\text{singlet D–terms: }q_{\alpha a}S^{\alpha}\overline{S}^{\overline{\beta}}=\begin{pmatrix}-S_{2,1}S^{\dagger}{}_{2,1}-S_{5,1}S^{\dagger}{}_{5,1}\\ S_{2,1}S^{\dagger}{}_{2,1}+S_{2,3}S^{\dagger}{}_{2,3}+S_{2,4}S^{\dagger}{}_{2,4}\\ -S_{2,3}S^{\dagger}{}_{2,3}-S_{5,3}S^{\dagger}{}_{5,3}\\ -S_{2,4}S^{\dagger}{}_{2,4}\\ S_{5,1}S^{\dagger}{}_{5,1}+S_{5,3}S^{\dagger}{}_{5,3}\end{pmatrix}$$

allowed operators:

- **Operators**

basic superpotential terms:

$$\overline{H}10^p10^q:\ Y^{(u)}=\begin{pmatrix}(0)&(0)&(1)\\(0)&(0)&(1)\\(1)&(1)&(0)\end{pmatrix} \quad \longleftarrow \text{ rank } 2$$

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Dimension 5 operators in superpotential:

$\overline{5}^p10^q10^r10^s$: $\lambda'=\{\{\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\},\{\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\},\{\{0\},\{0\},\{0\}\}\}\},$
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An exhaustive scan over favourable Cicys:

Aim: Find all viable line bundle SU(5) GUT models (and later all standard models) on favourable Cicys with freely-acting symmetries.

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Requires scanning over $\sim 10^{40}$ bundles (k_a^i)

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Roughly, a factor 10 more models per CY for each additional Kahler parameter!

Conclusion

- String theory has all generic ingredients to account for observed particle physics.

- Detailed model building now allows construction of models with the correct spectrum.

- Finer details, such as the values of Yukawa couplings, are within reach but a fully realistic model has yet to be found.

- Possible string physics beyond the standard model includes supersymmetry, additional U(1) gauge symmetries, axions, SM singlets, . . . Details depend on model.

Open problems:

- What is the number of string standard models?
- Details of moduli stabilisation and supersymmetry breaking.
- Many hard mathematical problems related to computation of couplings for CY compactifications.
- How to go beyond CY manifolds: G2 manifolds, G-structure manifolds, non-geometric compactifications, . . .
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Is the choice of topology arbitrary or will string theory provide a mechanism to select a specific topology?

Thanks