The Turbulent Dynamics of Accretion Disks

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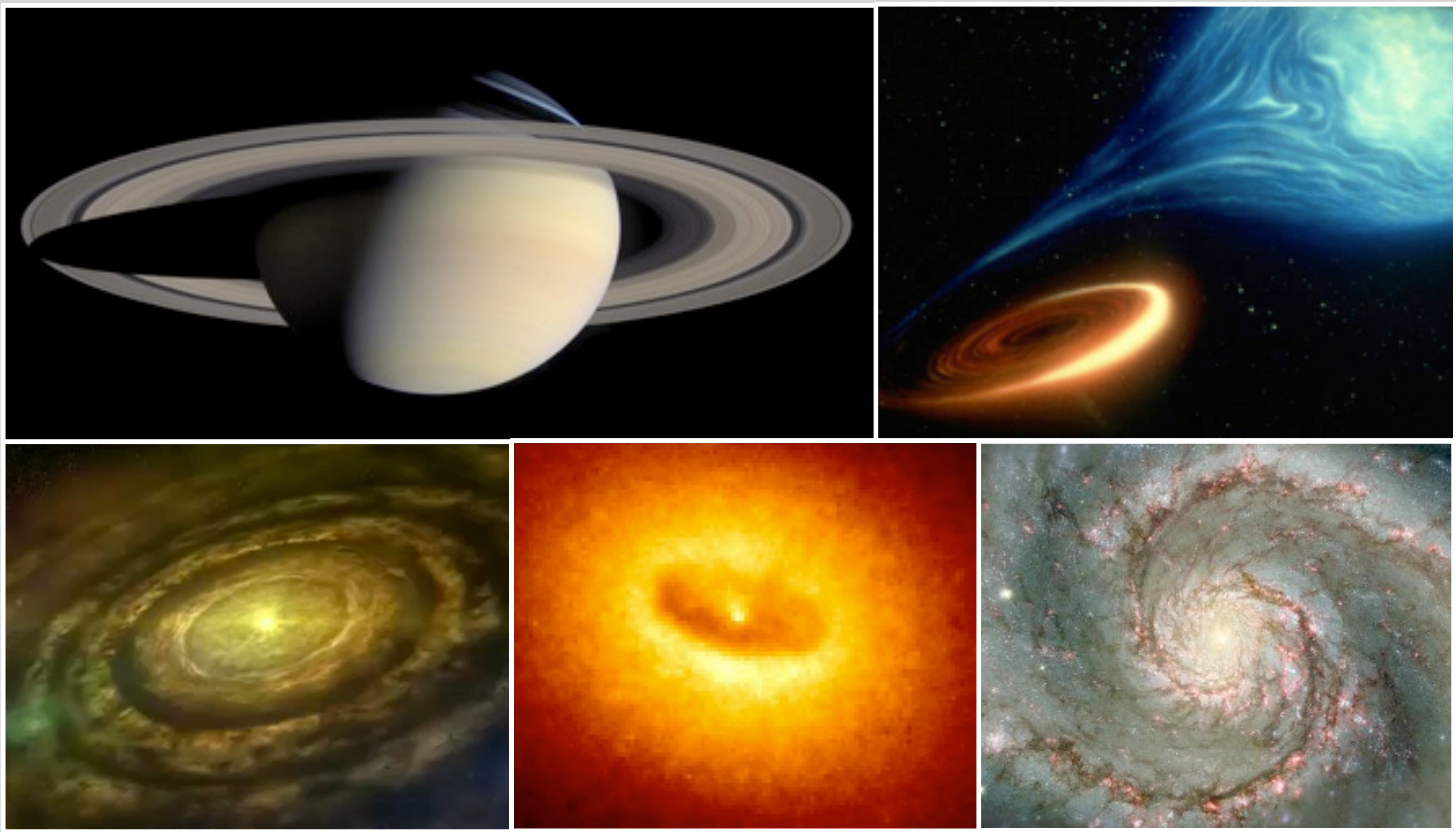
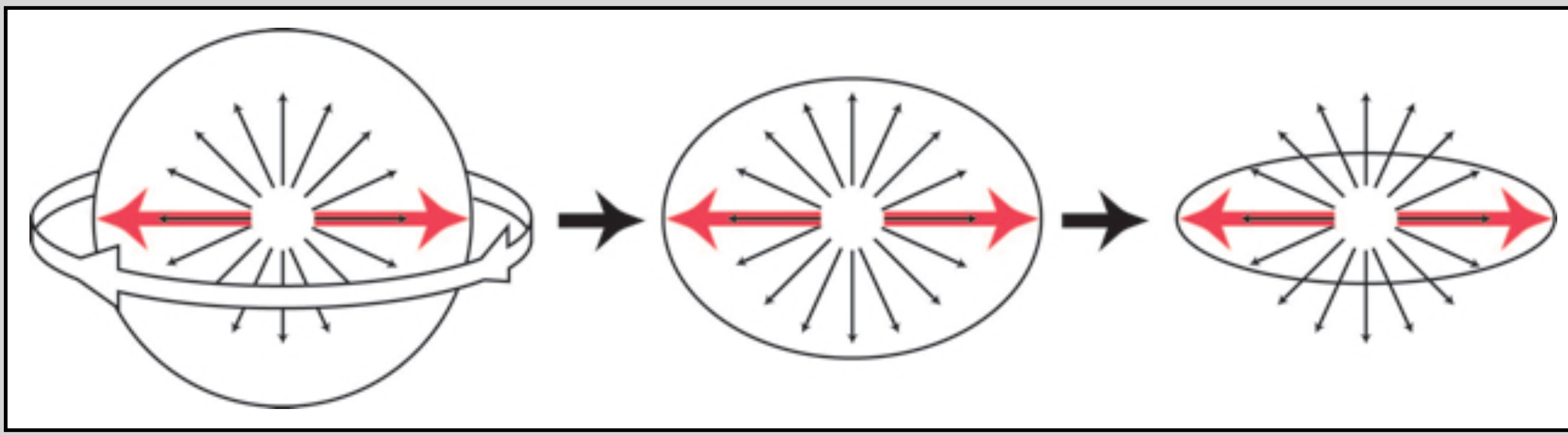
Astrophysical Disks



Angular Momentum Conservation

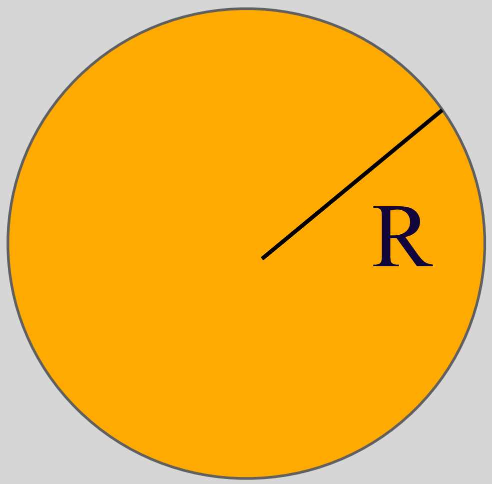
Why Do Disks Form?

Imagine a cloud of gas collapsing due to its own gravity ...

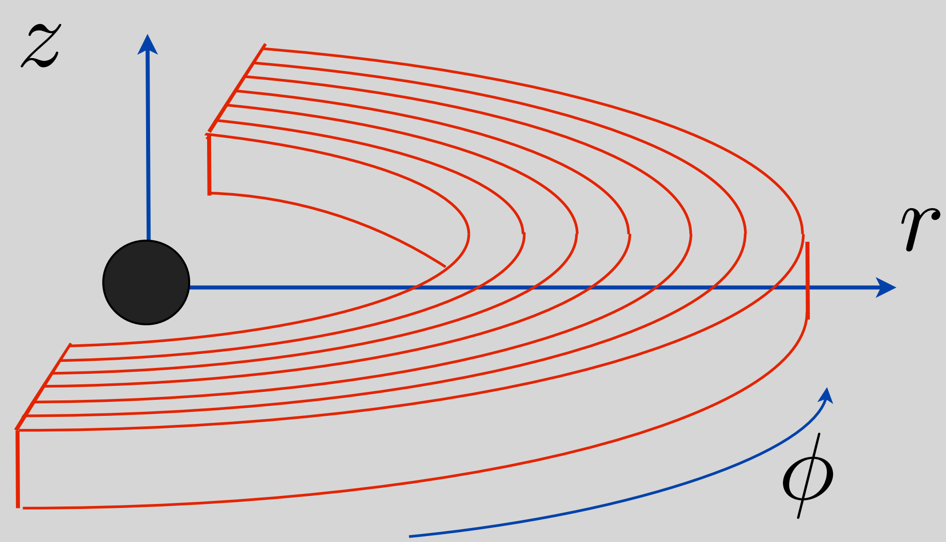


Gas cools down faster than it can get rid of angular momentum.

Stars vs. Disks

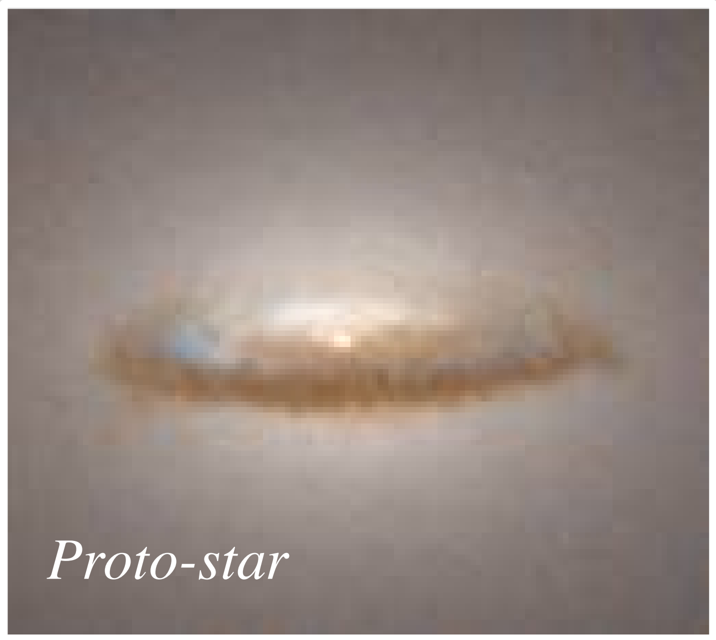


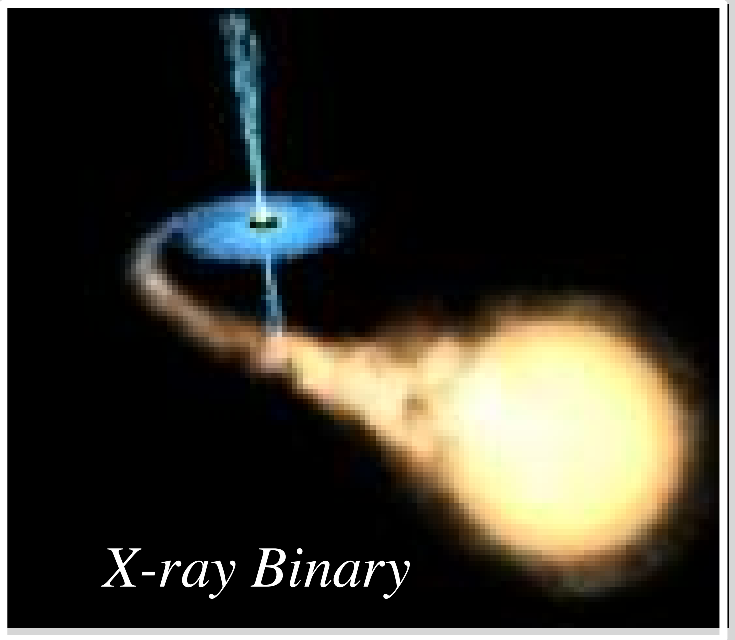
- Mostly spherical
- Gravity balanced by P grad.
- Magnetic fields do not influence stellar structure
- Mostly thermal energy



- Large aspect ratios
- Differential rotation
- Magnetic fields are essential for disk to work
- Non-thermal processes

Why Study These Disks?

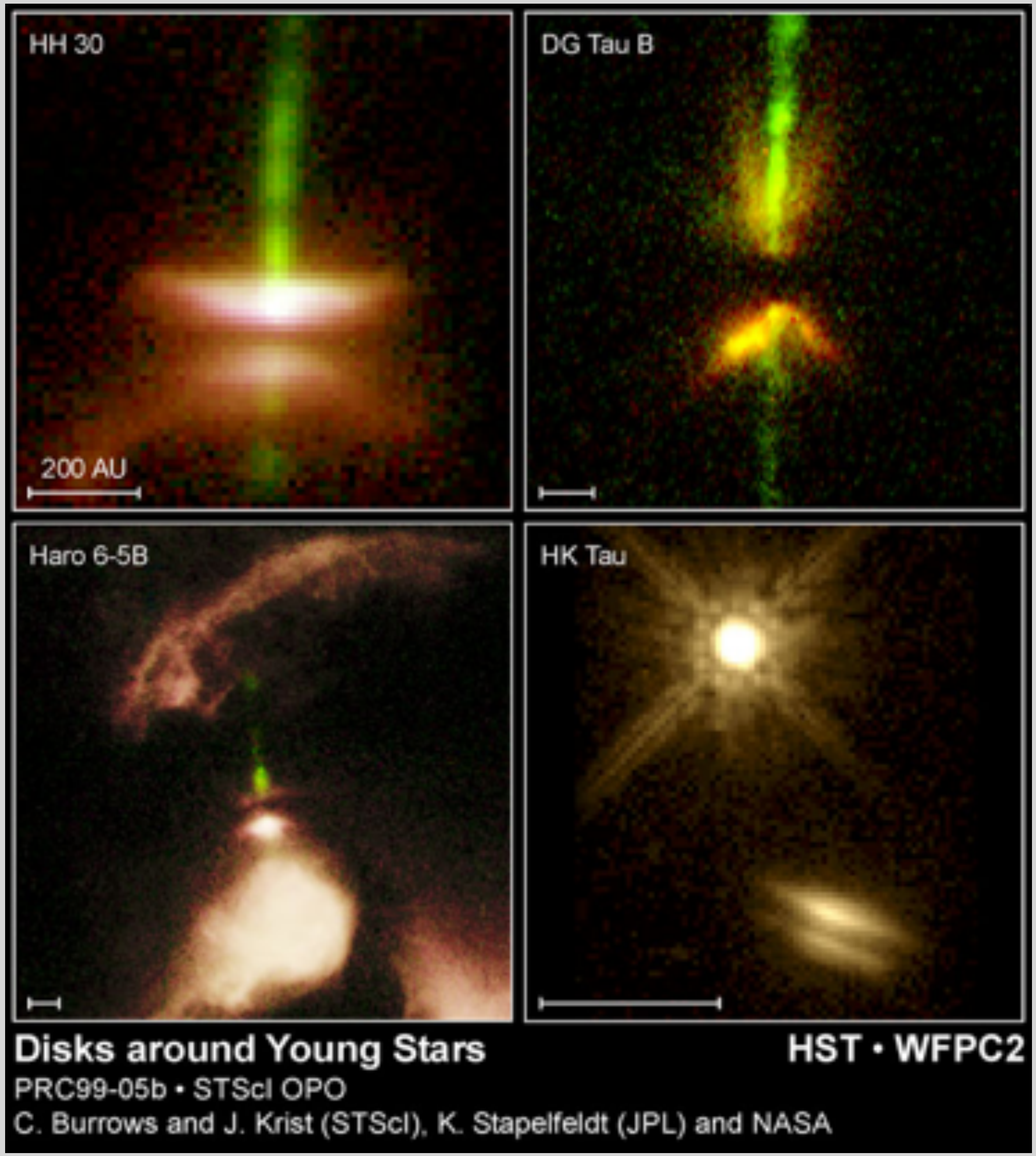




- How do stars and planets form?
- What powers the brightest X-ray sources in the sky?
- Why do Active Galactic Nuclei (Quasars) shine?
- What are the properties of space-time close to a black hole?

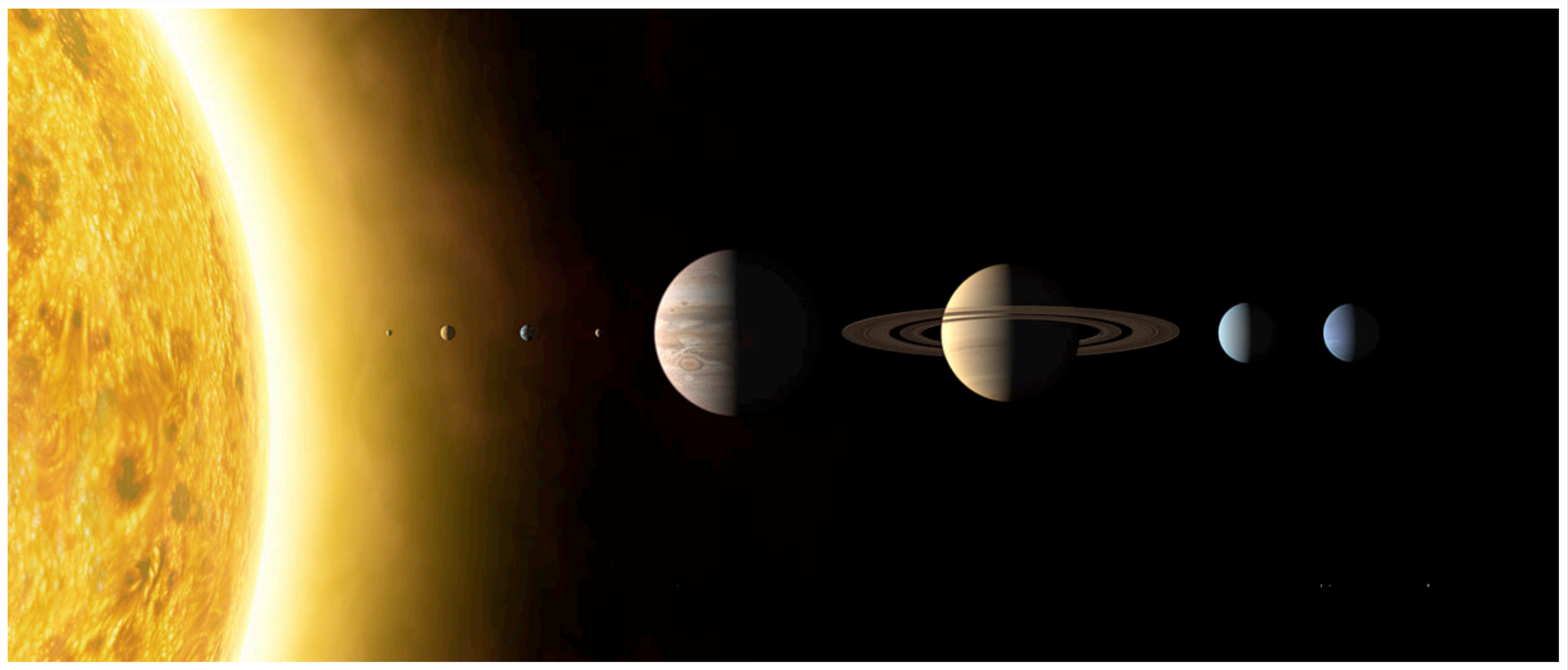
Accretion Disks in the Universe

Protoplanetary Disks



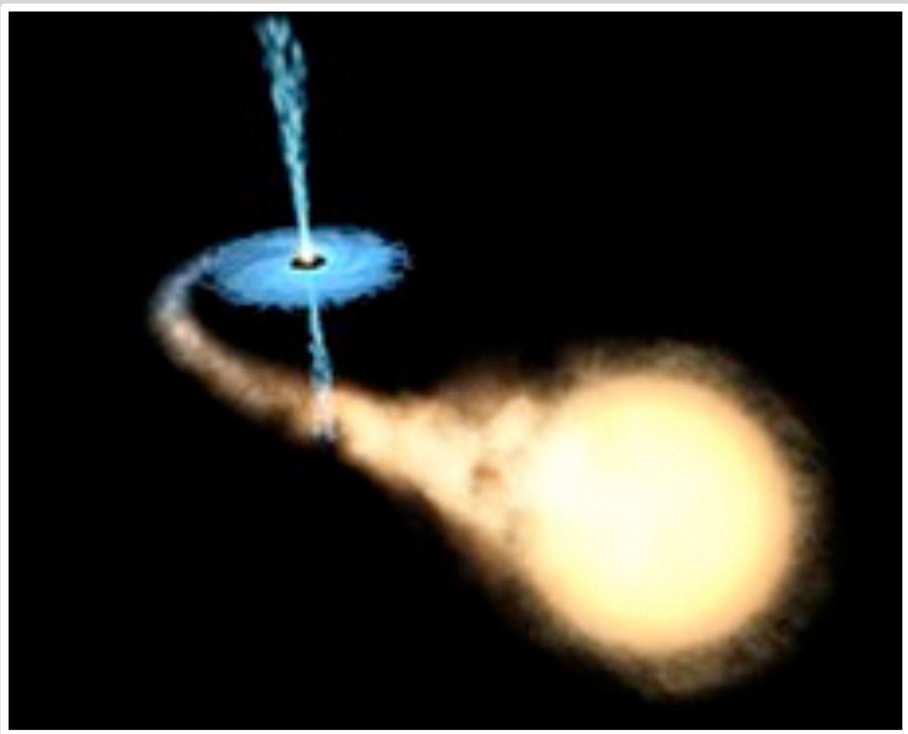
Dawn of a New Era in Astrophysics

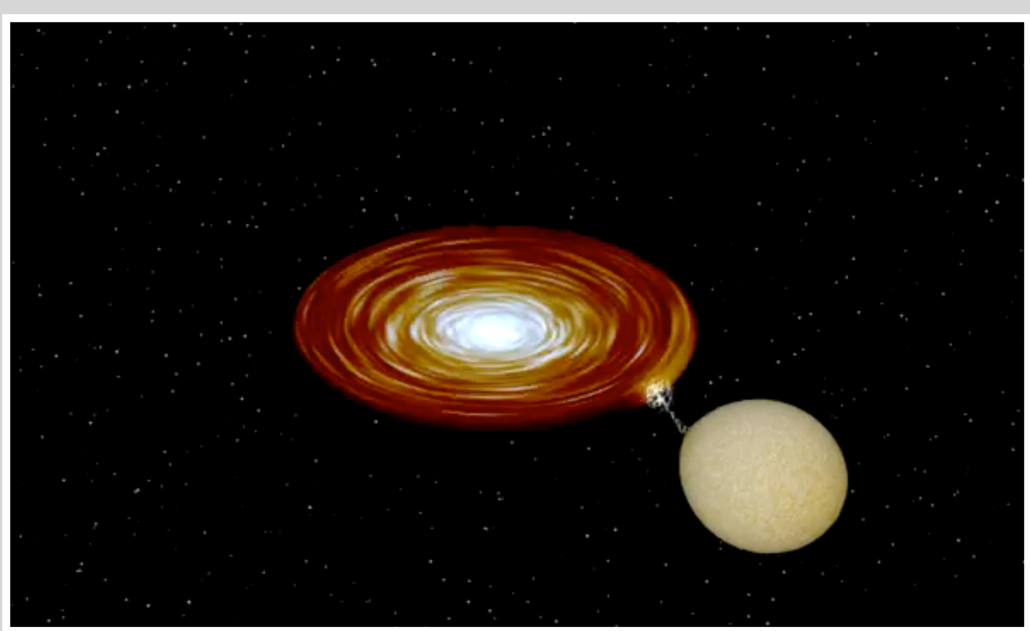
How Nature goes from this... ... to this?



Addressing this questions demands a detailed understanding of how accretion disks work

Accretion Disks in Binary Systems





X-ray Binary Disks

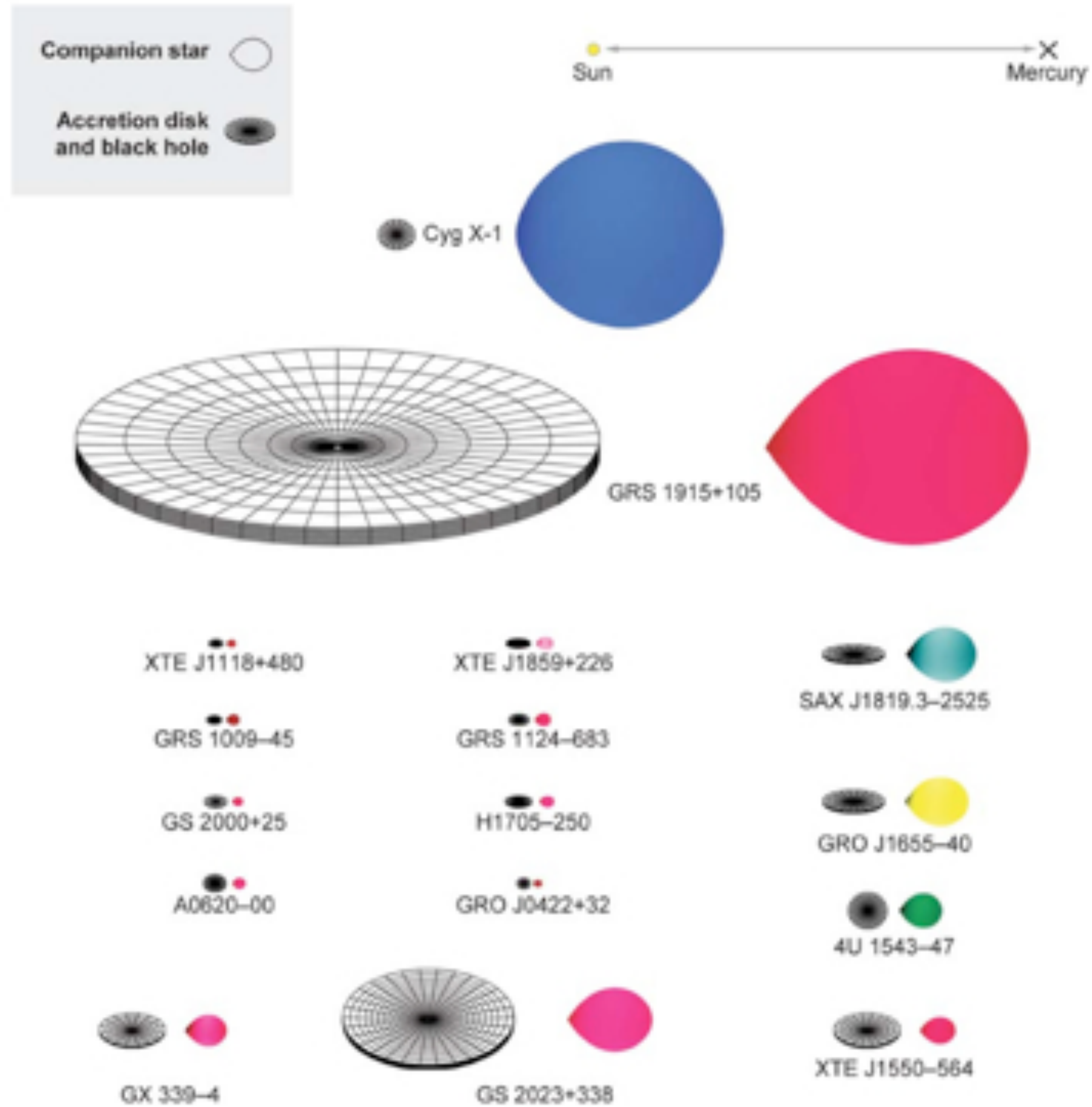
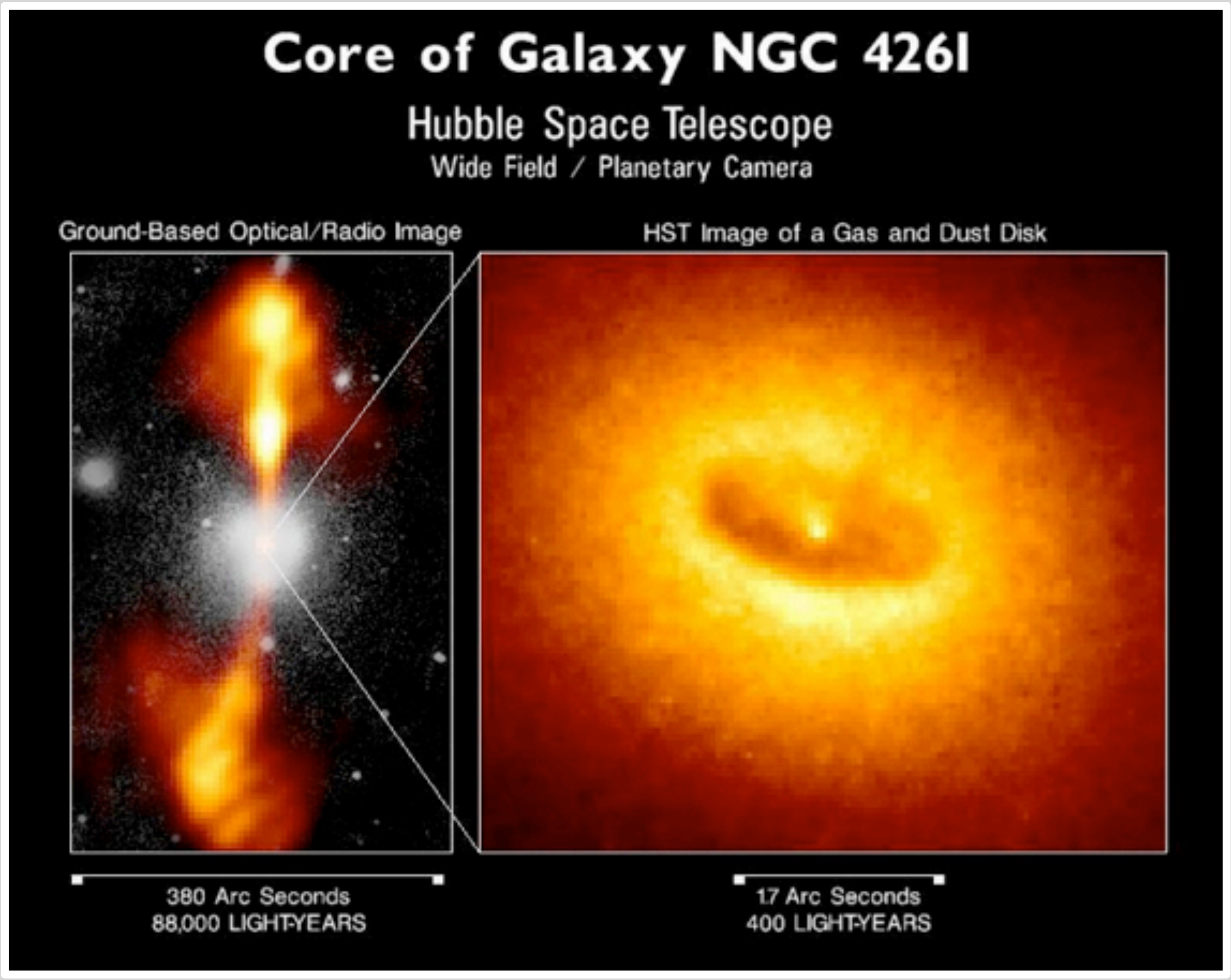


Figure 1

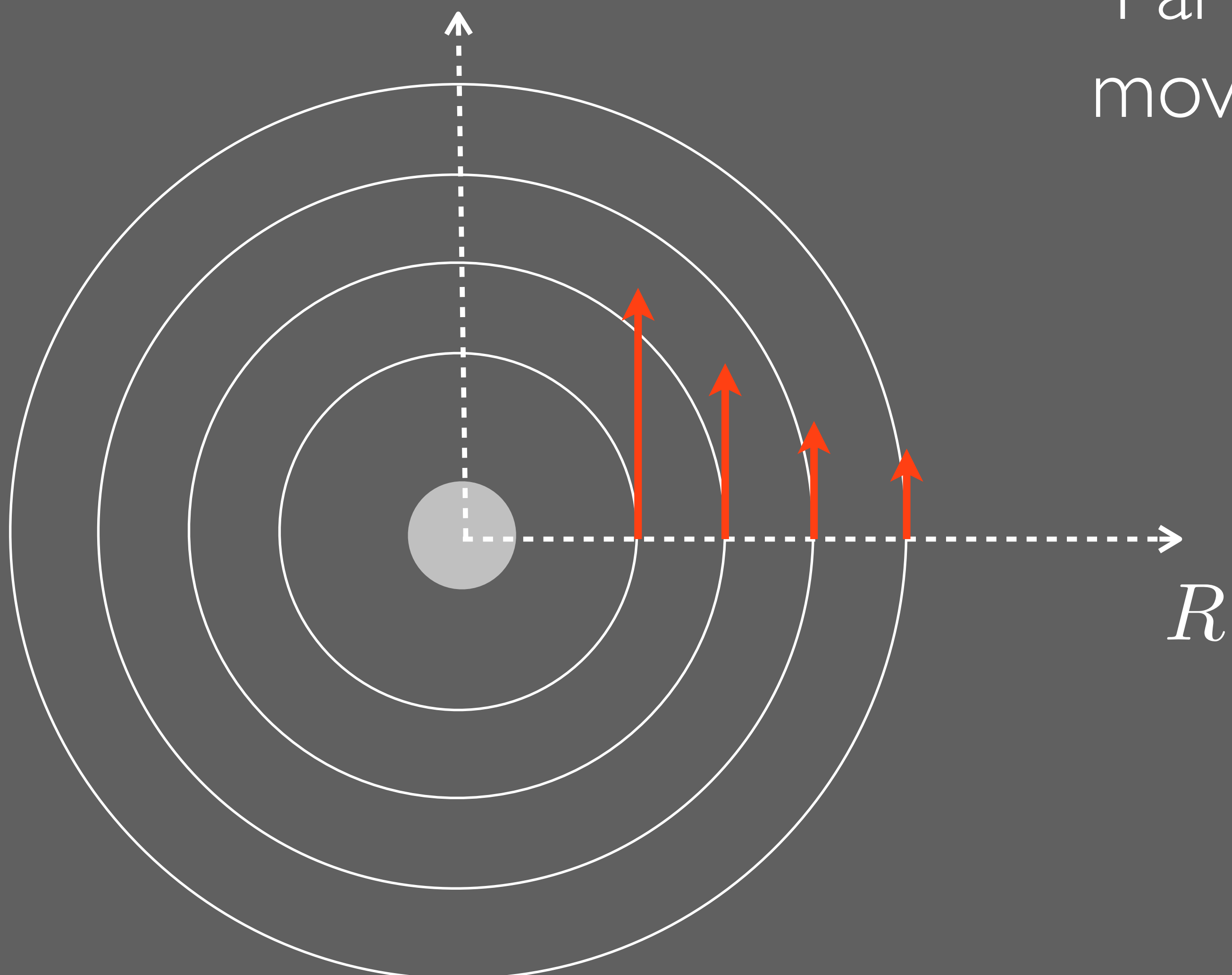
Scale drawings of 16 black-hole binaries in the Milky Way (courtesy of J. Orosz). The Sun–Mercury distance (0.4 AU) is shown at the top. The estimated binary inclination is indicated by the tilt of the accretion disk. The color of the companion star roughly indicates its surface temperature.

Disks in Active Galactic Nuclei



A Crash Course on Accretion Disks

Dynamical Considerations



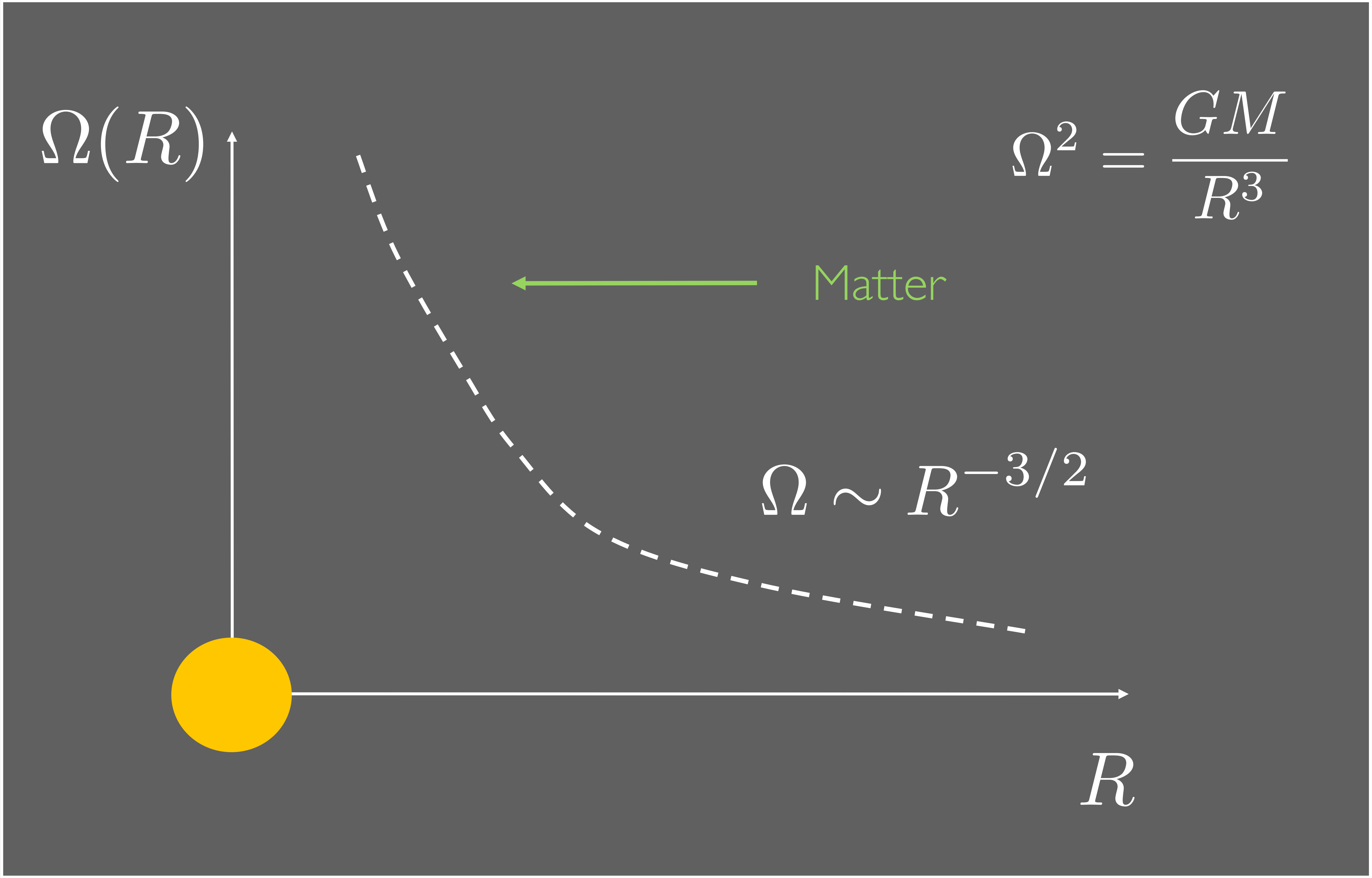
Particles in a central potential move in stable Keplerian orbits

$$v_K = \sqrt{\frac{GM}{R}}$$

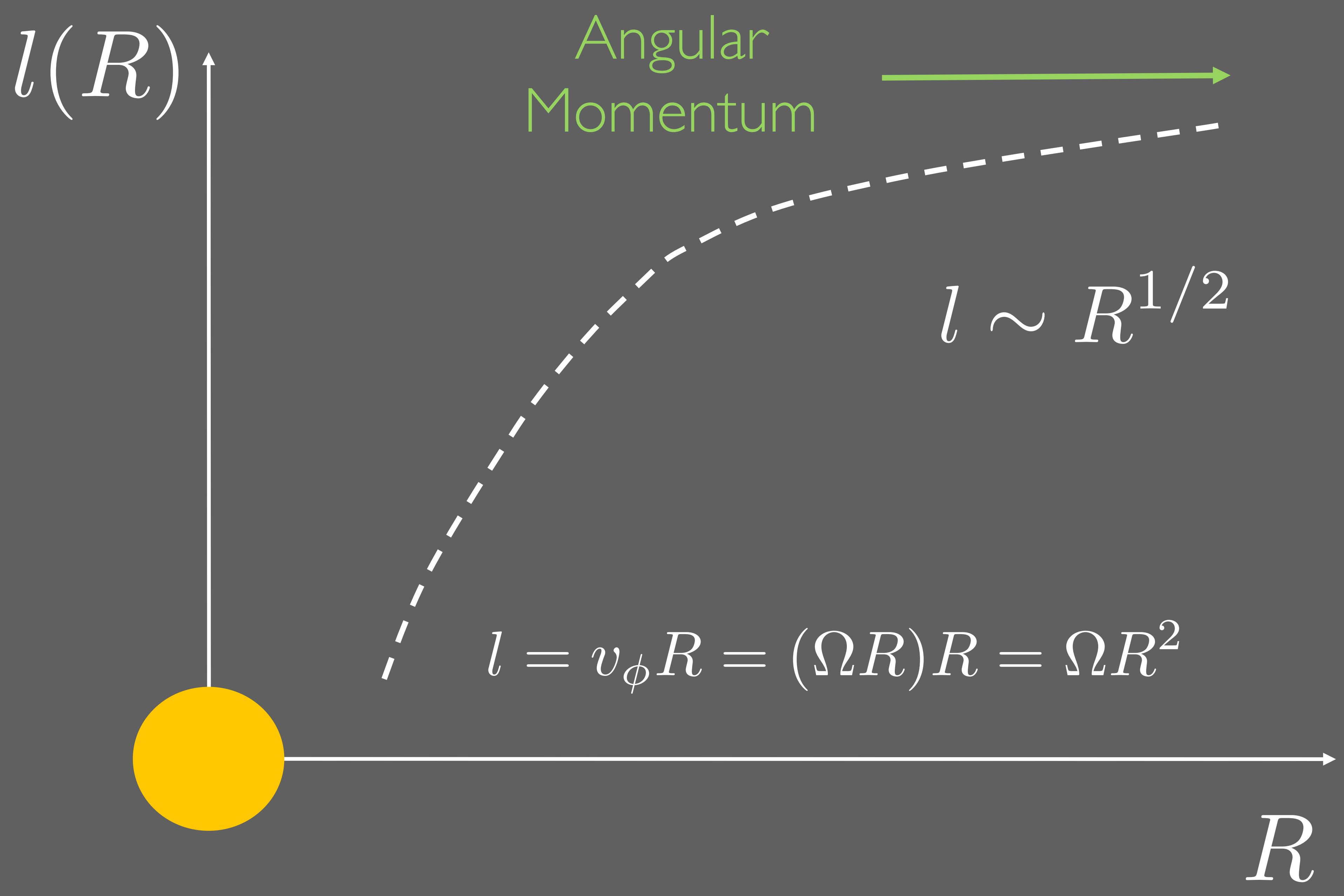
$$\Omega_K = \sqrt{\frac{GM}{R^3}}$$

If the disk were a collection of non-interacting particles there would be no accretion

Keplerian Disks



Angular Momentum Transport



Gas in the disk must lose angular momentum!!!!

Stress and Angular Momentum

$$\frac{\partial \rho}{\partial t} + \nabla.(\rho \mathbf{v}) = 0$$

Mass conservation

$$\frac{\partial l}{\partial t} + \nabla.(l \mathbf{v}) \neq 0$$

Angular momentum is not conserved....

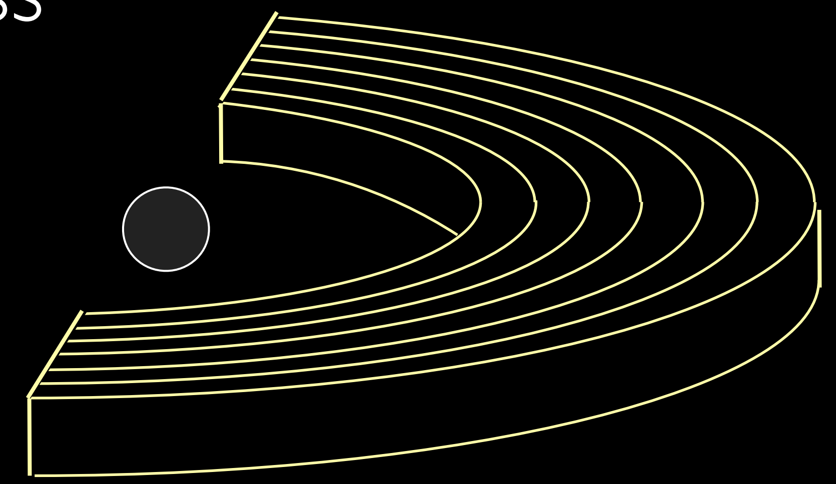
$$\frac{\partial l}{\partial t} + \nabla.(l \mathbf{v}) = -\frac{1}{r}\frac{\partial}{\partial r}(r^2 \bar{T}_{r\phi})$$

If there is no stress, angular momentum for fluid elements in the disk is conserved and matter does not accrete!

The Angular Momentum Problem

A Newtonian Model for the Stress

$$\bar{T}_{r\phi} = -r\Sigma\nu_{\rm mol}\frac{d\Omega}{dr}$$



$$\nu_{\rm mol} = \lambda_{\rm mol}c_{\rm s}$$

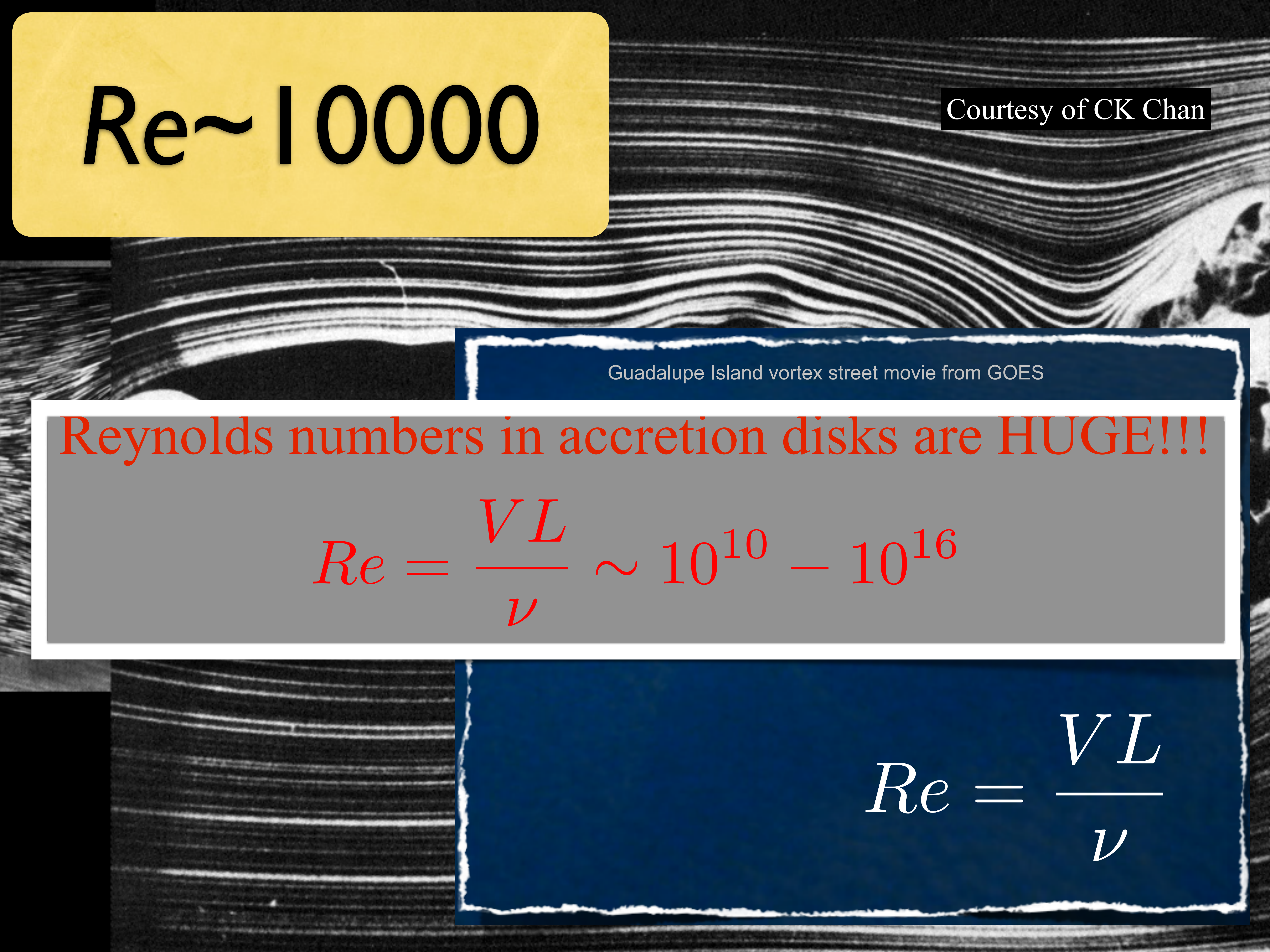
$$\lambda_{\rm mol} \sim 10^{-2}\rm cm$$

$$c_{\rm s} \sim 10^{6}\rm\ cm\ s^{-1}$$

$$t_{\rm mol} \sim \frac{R^2}{\nu_{\rm mol}} \sim 10^{8}\rm yr$$

$$t_{\rm obs} \sim 1\text{ week !!!}$$

Re~10000



Courtesy of CK Chan

Guadalupe Island vortex street movie from GOES

Reynolds numbers in accretion disks are HUGE!!!

$$Re = \frac{VL}{\nu} \sim 10^{10} - 10^{16}$$

$$Re = \frac{VL}{\nu}$$

The Angular Momentum Problem

Proposal: Assume some kind of "turbulent" viscosity

Eddies of size 'H' interacting with turnover velocity '**α** c_s'

$$\bar{T}_{r\phi} = -r\Sigma\nu_{\rm turb}\frac{d\Omega}{dr}$$

$$\nu_{\rm turb} = \lambda_{\rm turb} v_{\rm turb}$$

$$\nu_{\rm turb} \equiv \alpha H c_{\rm s}$$

Shakura & Sunyaev '70s, **α**-model

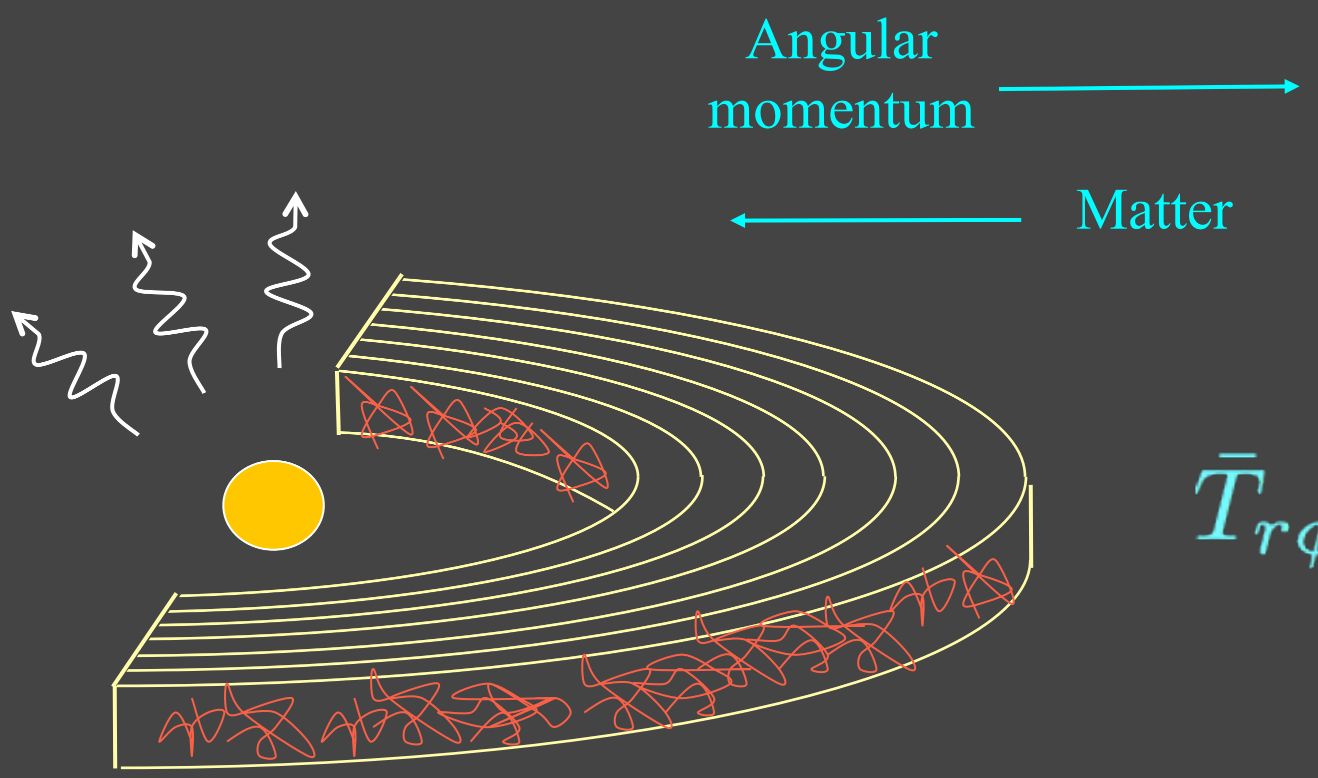
$$\frac{\nu_{\rm turb}}{\nu_{\rm mol}} \sim \frac{\alpha H}{\lambda_{\rm mol}} \sim 10^{10}$$

$$t_{\rm turb} \sim t_{\rm obs} \ !!!$$

With this enhanced stress we can match the fast timescales observed!

Standard Accretion Disk

'viscosity' prescription to remove angular momentum



$$\bar{T}_{r\phi} = \alpha P \left(- \frac{d \ln \Omega}{d \ln r} \right)$$

$$\Sigma(r), P(r), T(r), v_r(r), \ldots$$

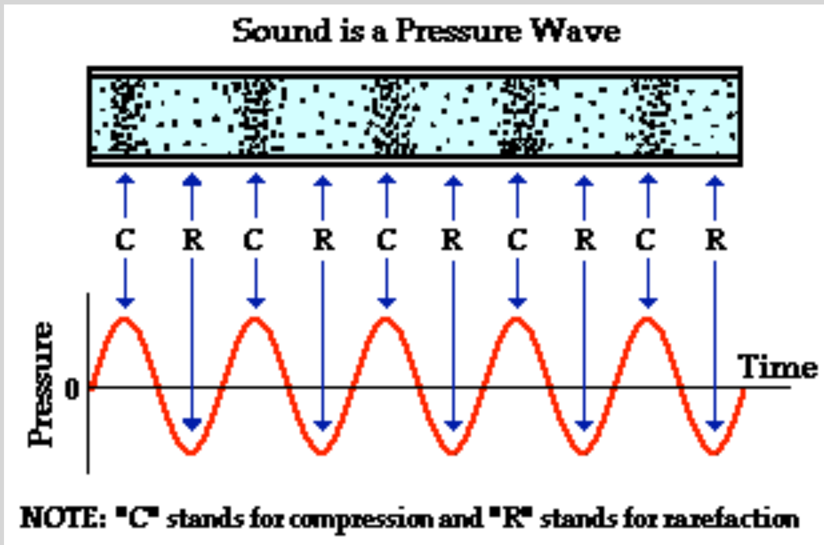
Keplerian hydrodynamic disks are quite resilient...

However, magneto-hydrodynamic disks are prone to a plethora of instabilities.

Sound Waves & Magnetic Waves

P

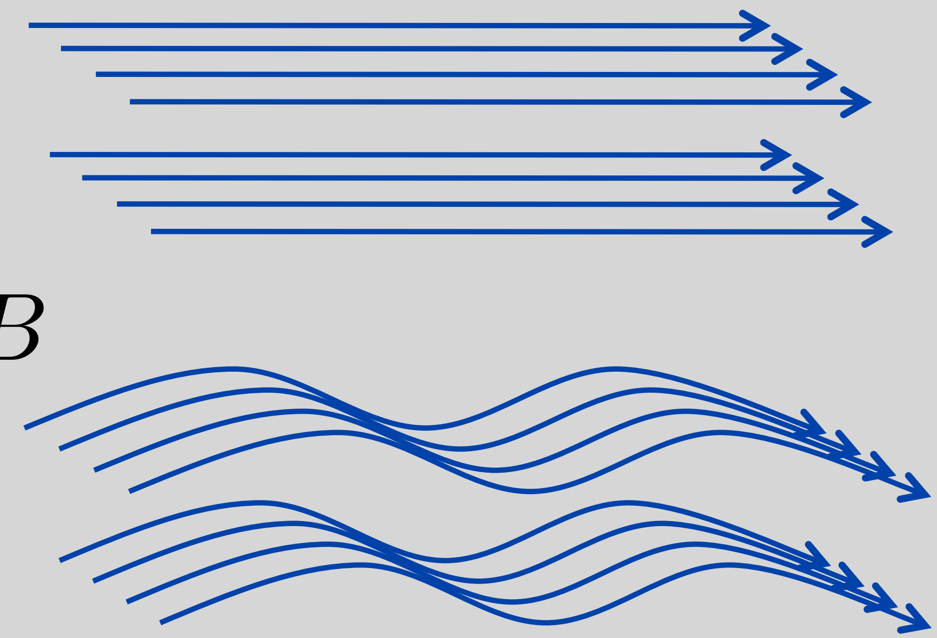
δP



$k \longrightarrow$

Restoring force: pressure gradient

B



δB

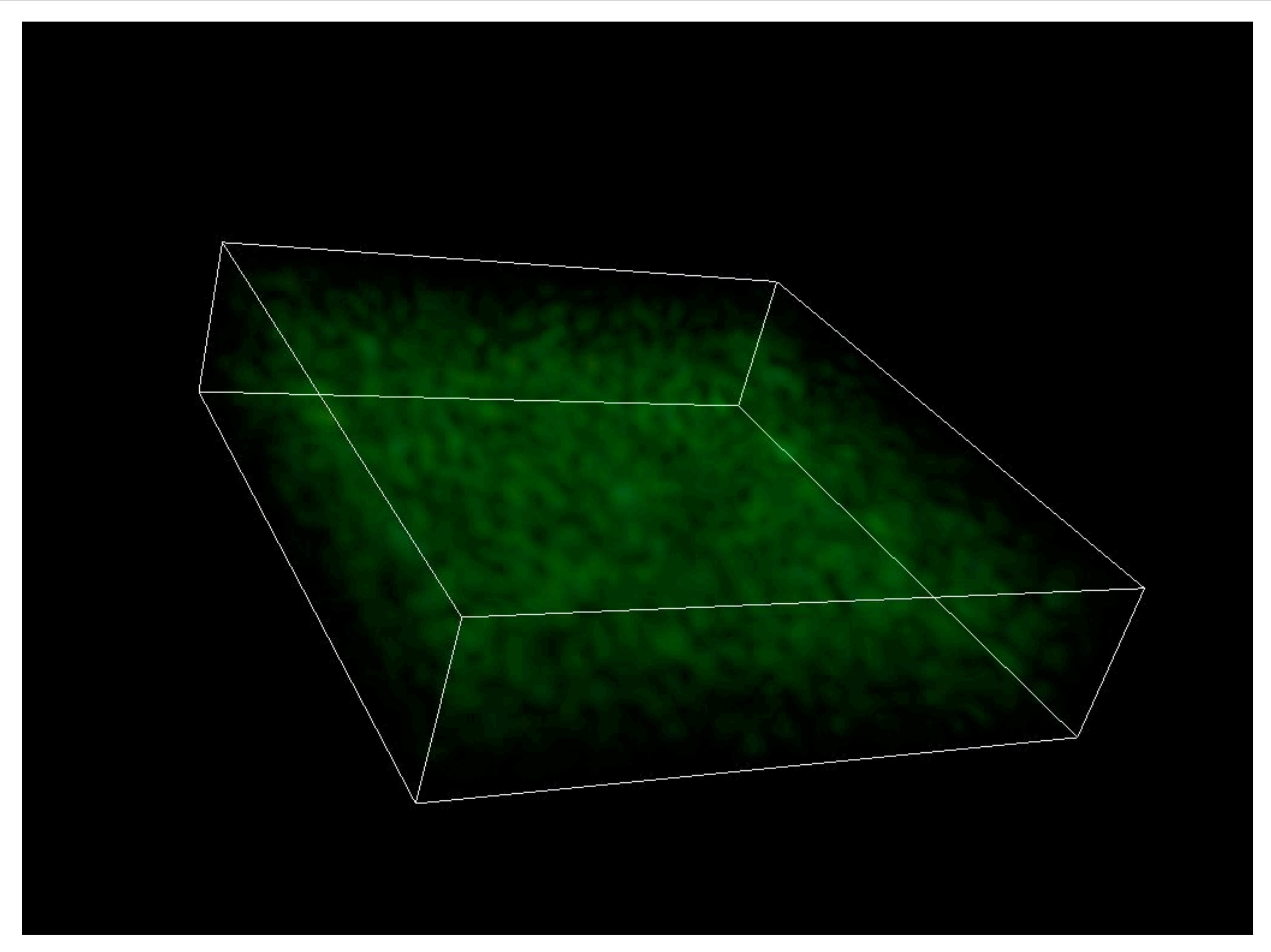
$k \longrightarrow$

Restoring force: magnetic tension

Magnetic waves can become unstable in differential rotation

These instabilities drive the turbulence responsible for accretion

Instabilities in Accretion Disks



courtesy of C.K. Chan

Transport in Turbulent Flow

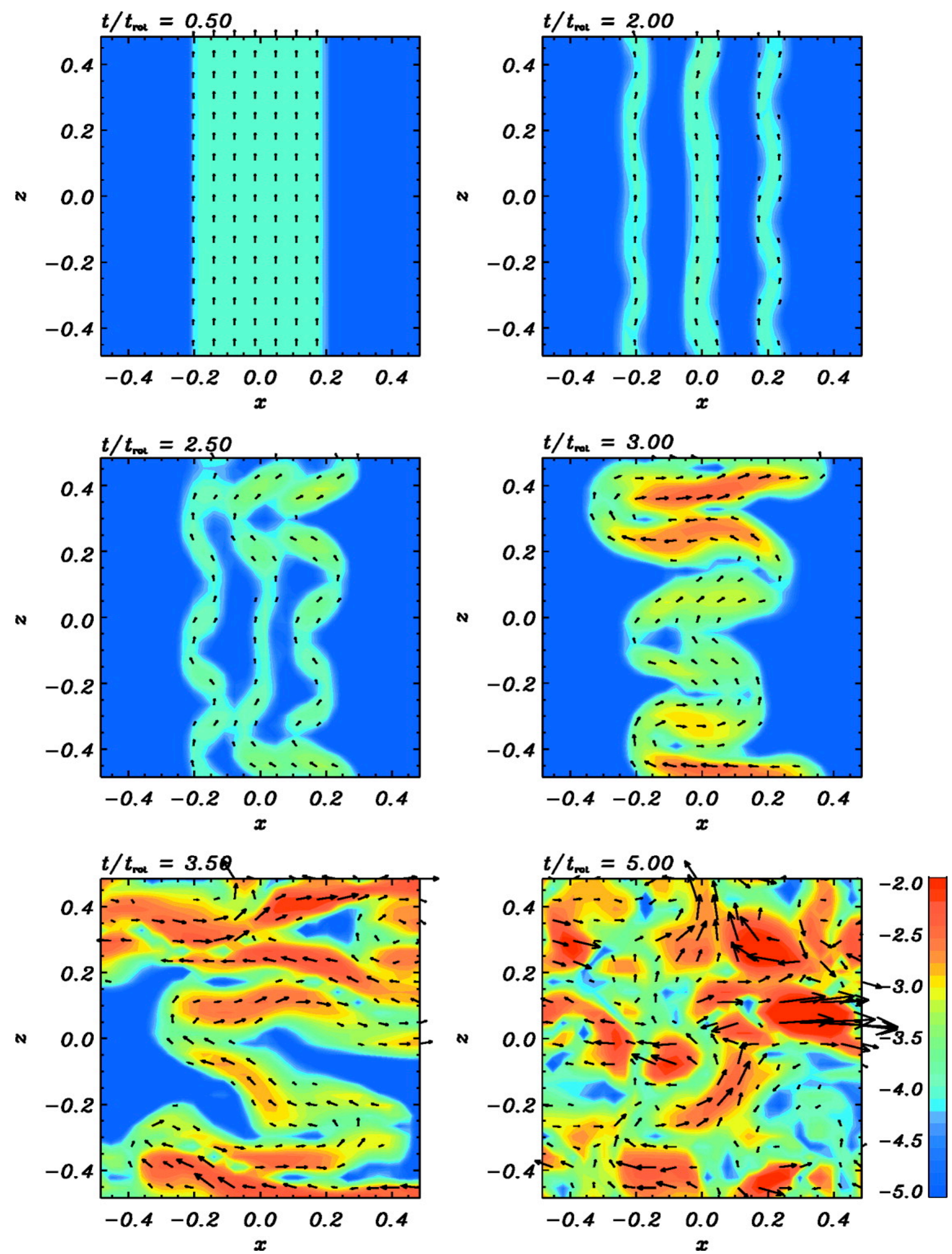
$$\frac{\partial l}{\partial t} + \nabla.(l\mathbf{v}) = -\frac{1}{r}\frac{\partial}{\partial r}(r^2 \bar{T}_{r\phi})$$

$$\bar{T}_{r\phi} = \bar{R}_{r\phi} - \bar{M}_{r\phi}$$

$$\bar{R}_{r\phi} = \langle \rho \, \delta v_r \, \delta v_\phi \rangle$$ Reynolds

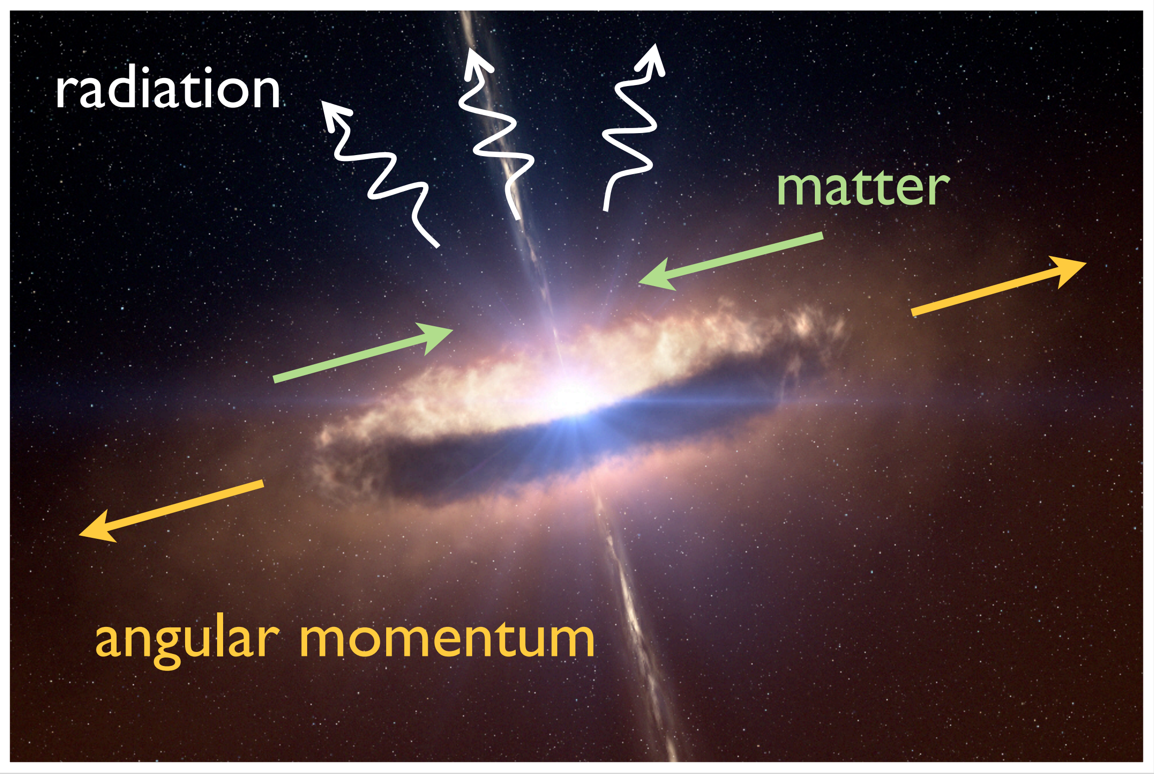
$$\bar{M}_{r\phi} = \langle \delta B_r \, \delta B_\phi \rangle$$ Maxwell

MHD turbulence in disks leads to correlated fluctuations of the PROPER sign!



Sano et al. 2004

Accretion Disk Theory in a Nutshell



Matter must lose most of its angular momentum in order to accrete

Important Milestones

1- Disks are turbulent (1973)
Shakura & Sunyaev **(alpha model)**

2- Magnetic fields key (1991)
Balbus & Hawley **(MRI)**

Efficiency of angular momentum transport intimately related to anisotropic turbulence

Ideal Magnetohydrodynamics

$$\boxed{\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0}$$

continuity

$$\boxed{\frac{\partial \boldsymbol{v}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\, \boldsymbol{v} = \; - \nabla \Phi - \frac{\nabla P}{\rho} + \frac{1}{\rho} \boldsymbol{J} \times \boldsymbol{B}}$$

momentum

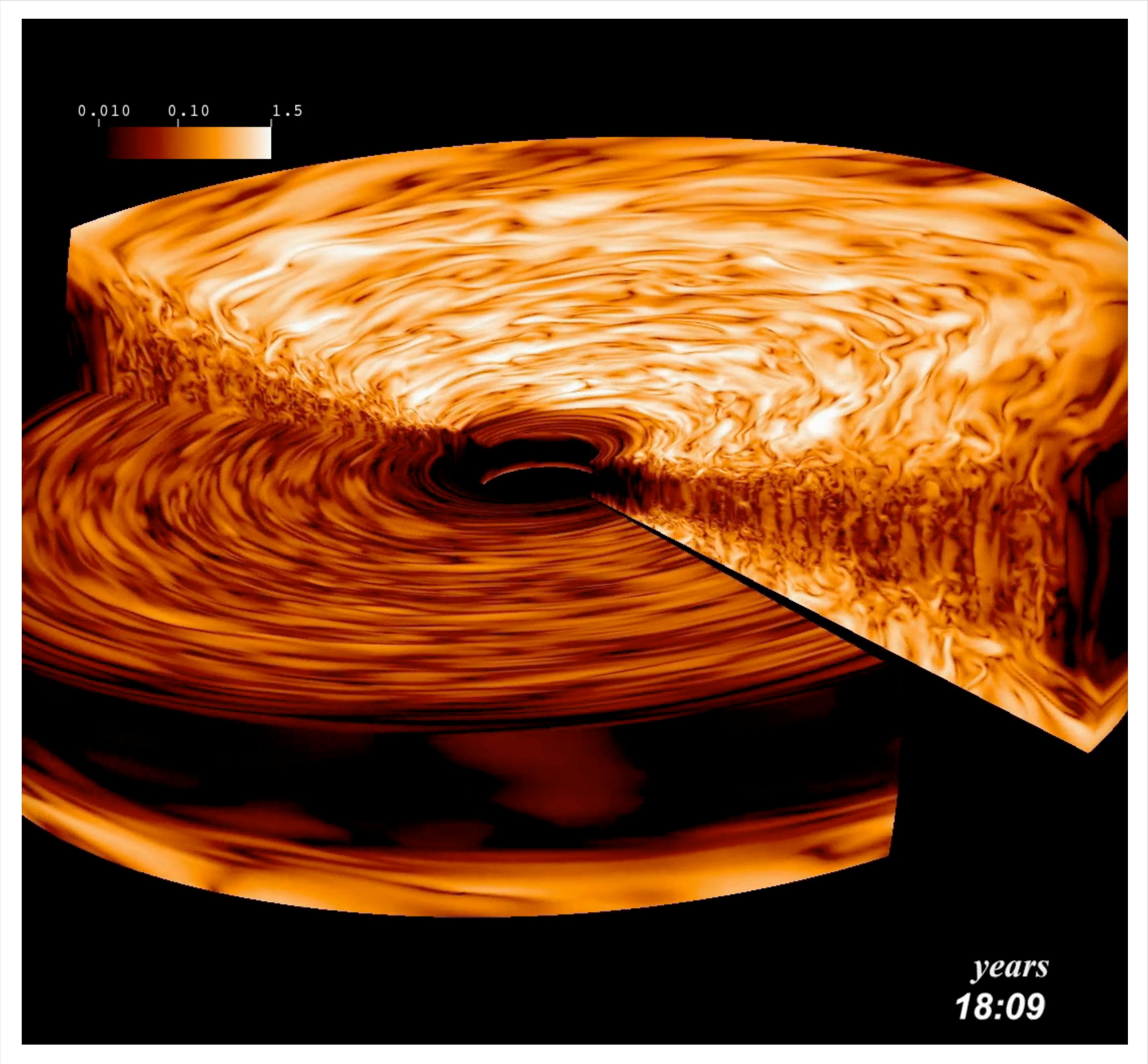
$$\boxed{\frac{\partial \boldsymbol{B}}{\partial t} + (\boldsymbol{v} \cdot \nabla)\, \boldsymbol{B} = (\boldsymbol{B} \cdot \nabla)\, \boldsymbol{v} - \boldsymbol{B}\, (\nabla \cdot \boldsymbol{v})}$$

induction

$$\boxed{\frac{\partial e}{\partial t} + \nabla \cdot (e \boldsymbol{v}) = -P\, (\nabla \cdot \boldsymbol{v})}$$

energy

Global 3D MHD Simulations



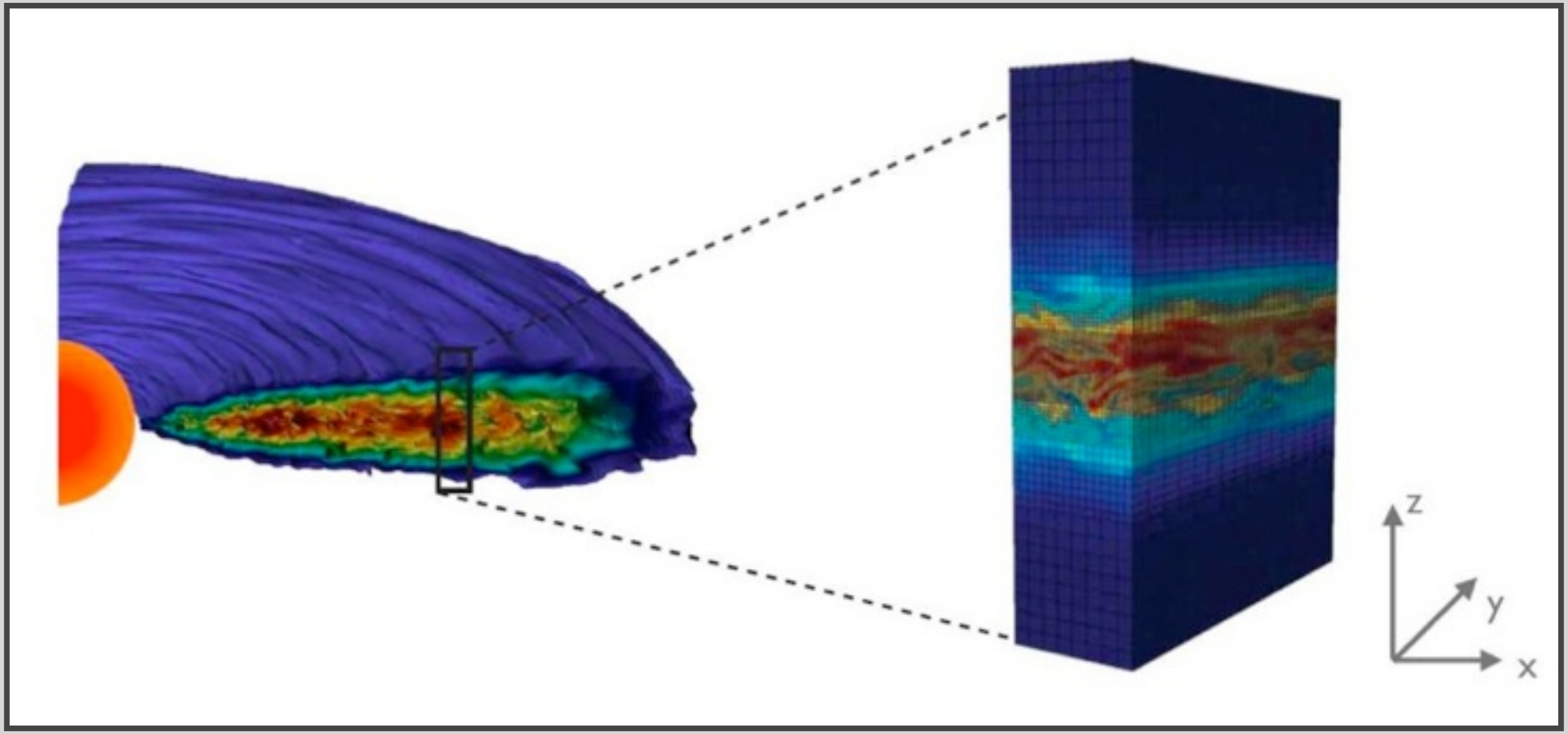
courtesy M. FLock

An effective mean-field theory for magnetohydrodynamic turbulence in disks is highly desirable...

A Current Problem of Interest...

How can we characterize MHD turbulence in disks in a robust way?

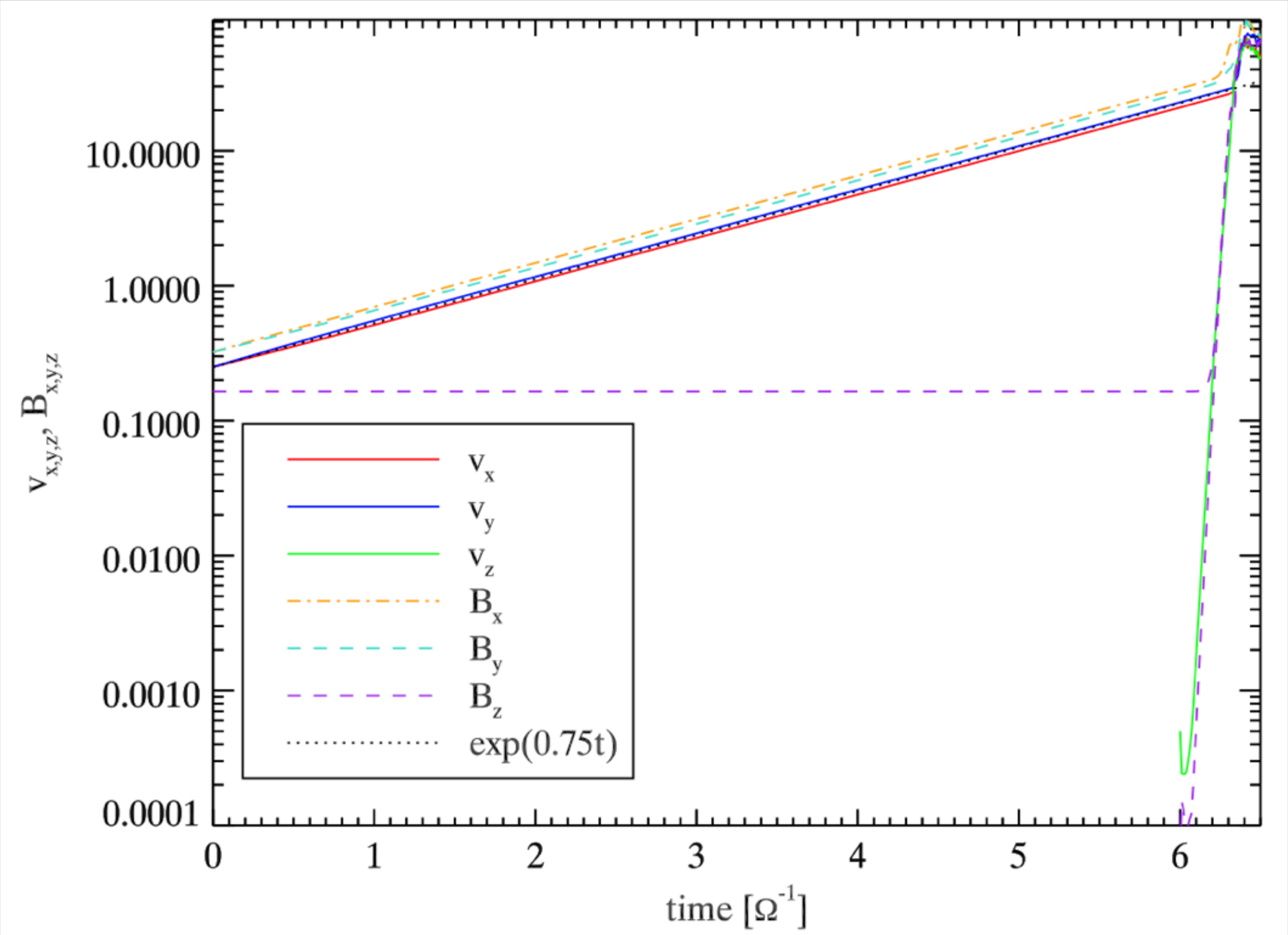
Local Models of Astrophysical Disks



Beckwith, Armitage and Simon, 2011

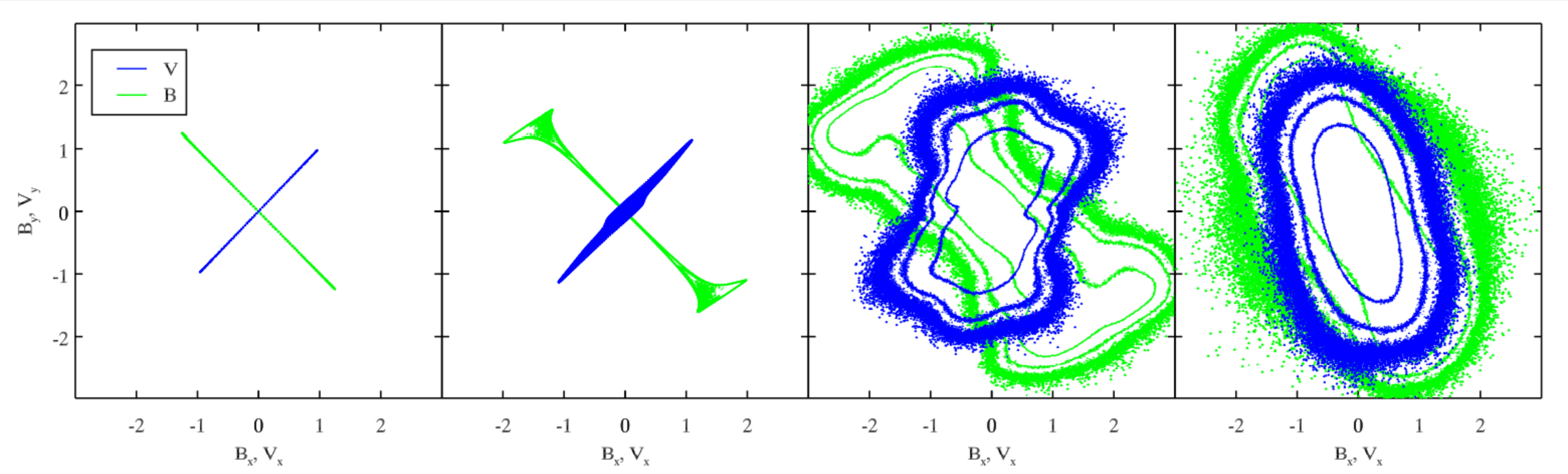
Essence of the local approximation: expand the equations of motion around a point corotating with the disk

Linear Phase of Instability (MRI)



G. Murphy & MEP, 2015

Breakdown of Linear Modes



In late linear regime:
$B_x = B_y > B_z$

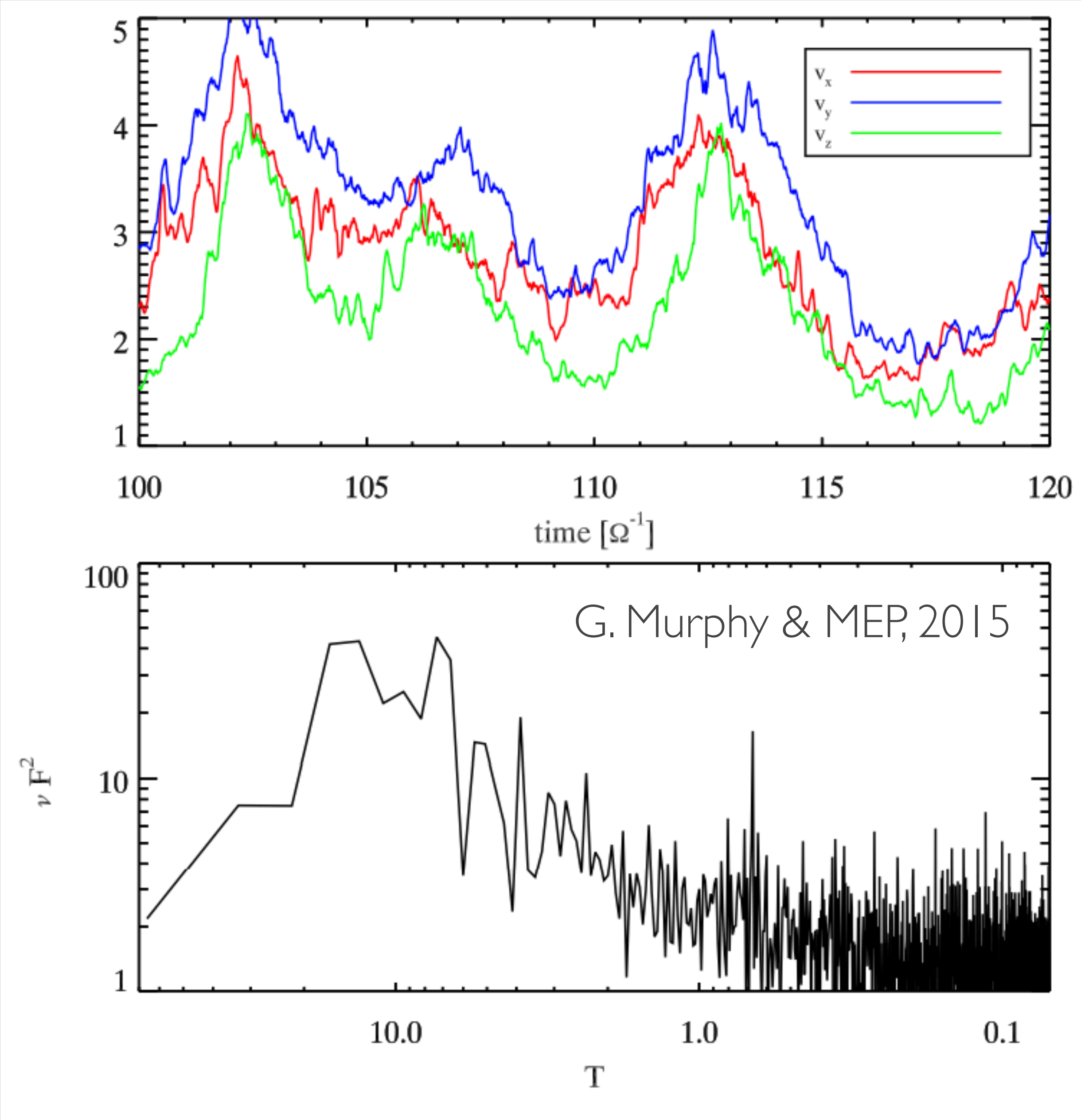
In turbulent regime:
$B_y > B_x, B_z$

What about the fully developed turbulent state?

PSD of Temporal Fluctuations

In the turbulent regime, v_y, v_x grow to be "overtaken" by v_z in a cyclic manner

Time series analysis reveals cyclic behavior



How can we analyze this behavior in a systematic way?

Invariant Tensor Analysis

Introduced by Lumley in1977 to study hydro. turbulence

Stress tensors are real and symmetric they can always be diagonalized

Stress dynamics can be characterized using invariant quantities

$$\mathcal{R}_{ij} = \frac{R_{ij}}{\mathrm{Tr}\,(R_{ij})} - \frac{1}{3}\delta_{ij}$$

$$\mathcal{M}_{ij} = \frac{M_{ij}}{\mathrm{Tr}\,(M_{ij})} - \frac{1}{3}\delta_{ij}$$

$$\chi_1^{\mathcal{R}} = \lambda_1 + \lambda_2 + \lambda_3$$

$$\chi_2^{\mathcal{R}} = -(\lambda_1^2 + \lambda_2^2 + \lambda_3^2)$$

$$\chi_3^{\mathcal{R}} = \lambda_1\,\lambda_2\,\lambda_3$$

λ_i : eigenvalues

Graphic Representation

We can plot tensors as glyphs to obtain a graphic representation

Principal axes of the ellipsoid are given by the eigenvectors

This analysis can be carried out at any point and time in the turbulent flow

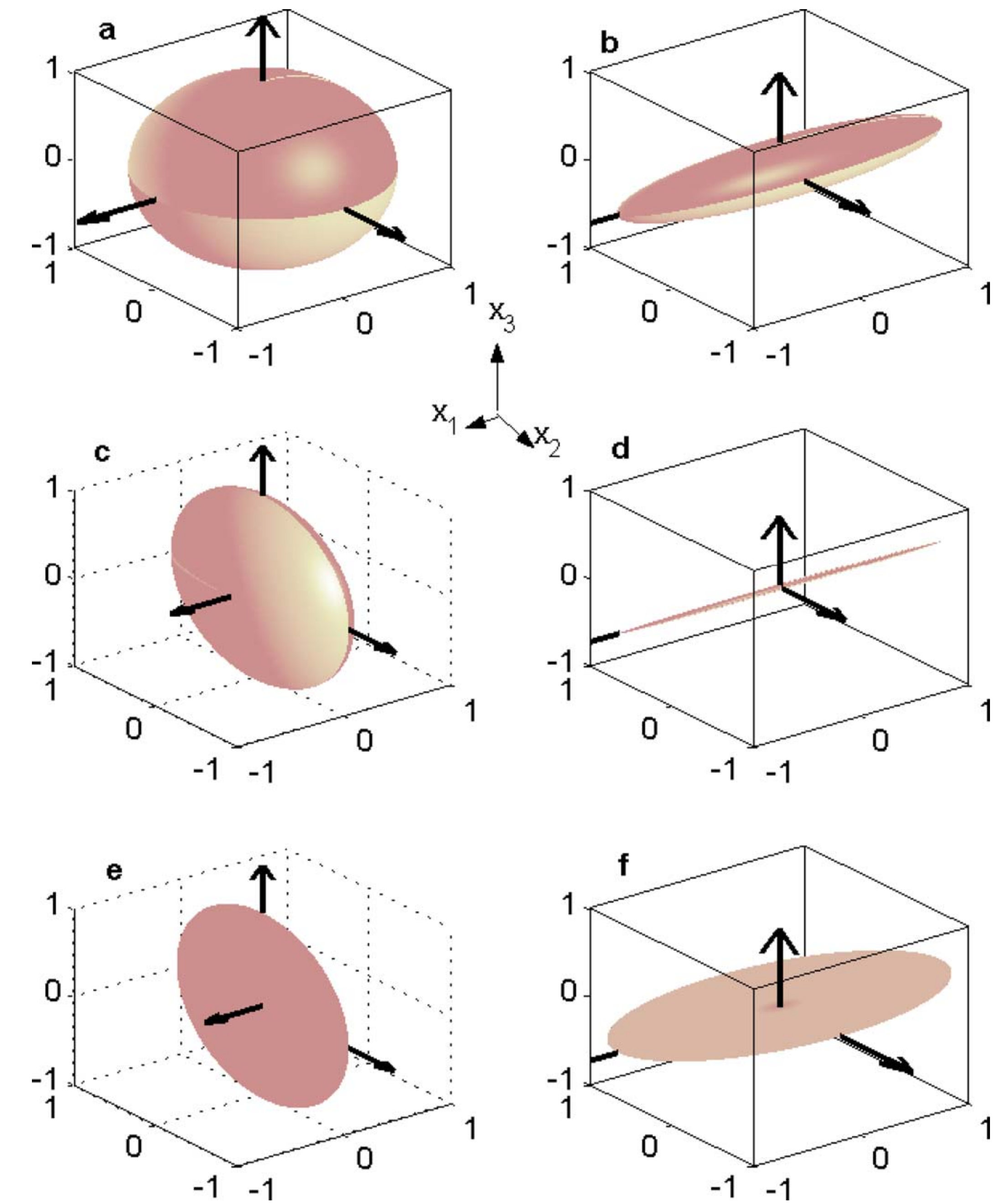
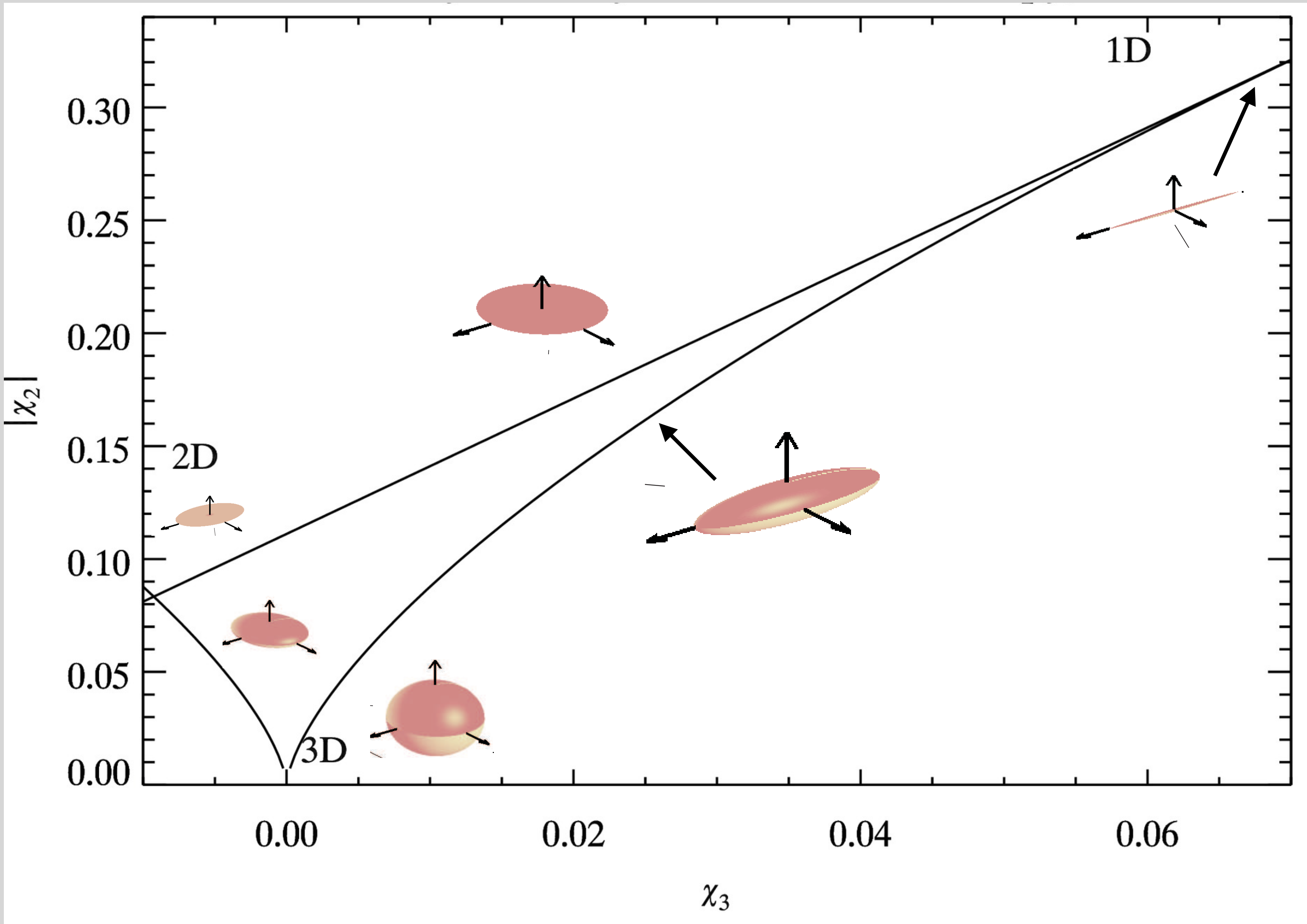


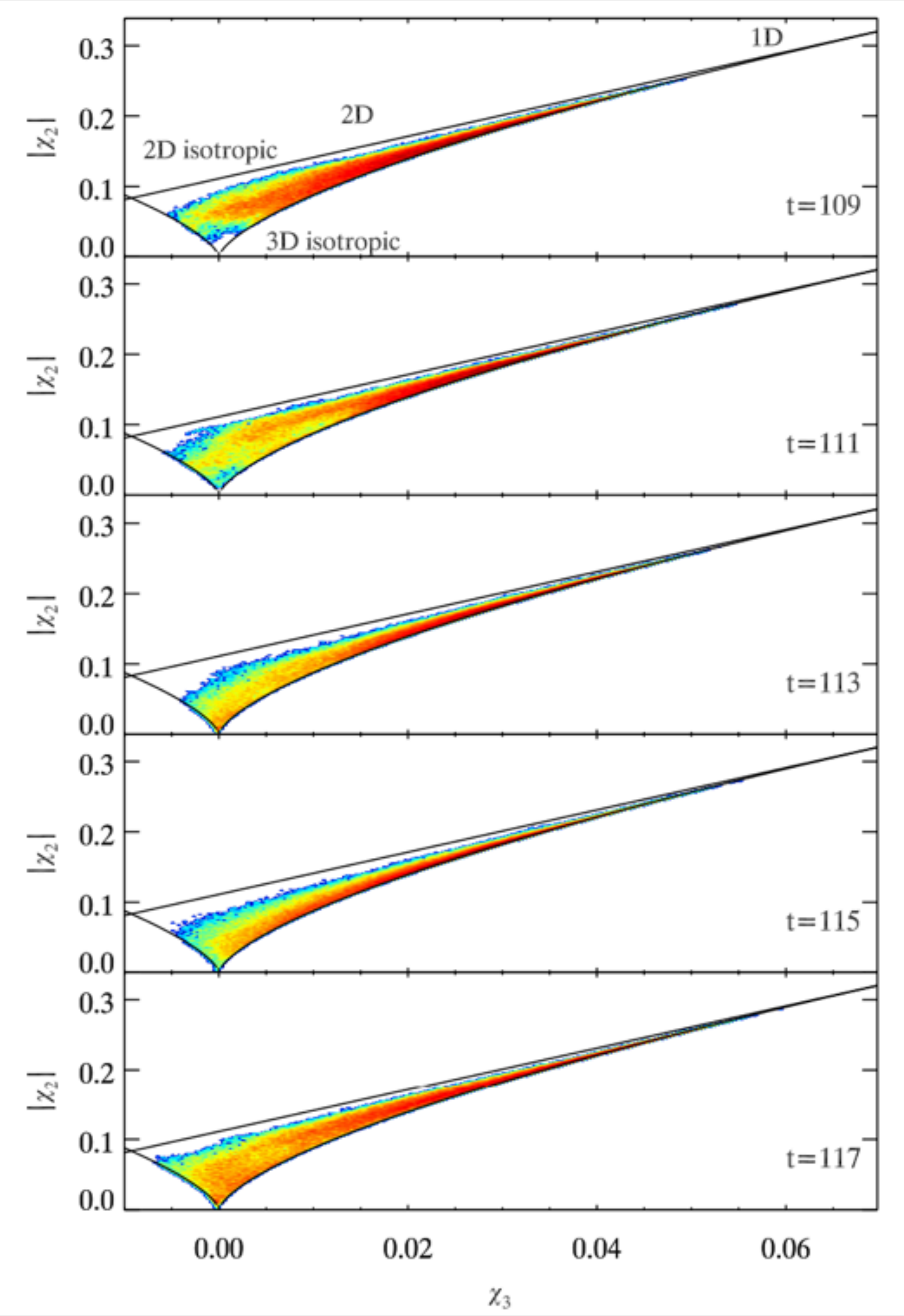
FIG. 3. Illustration of the ellipsoid shapes formed by the Reynolds stress tensor in different regions of the flow.

Lumley Triangle

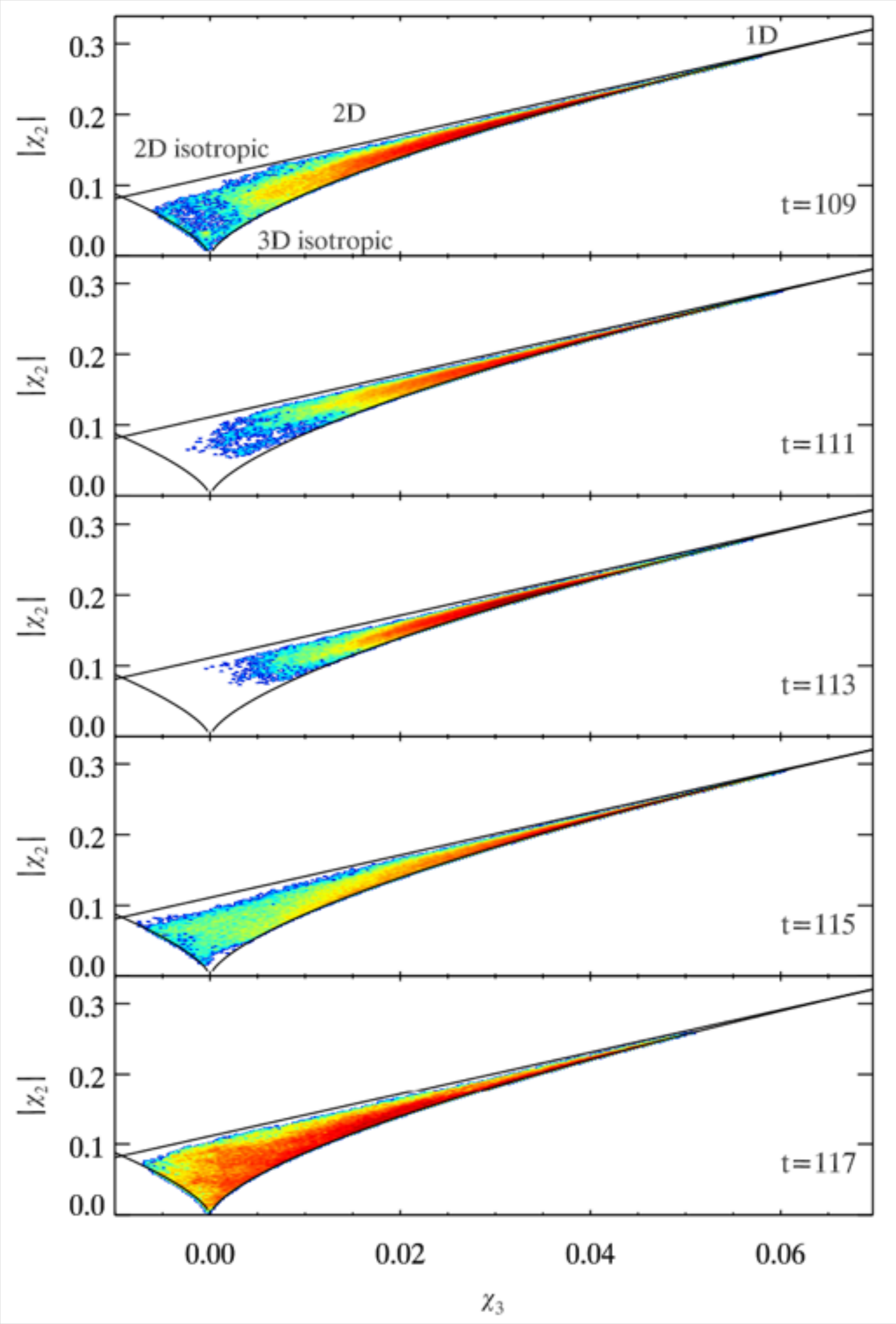


We can quantify anisotropy level, temporal variation and spatial distribution

Temporal Fluctuations in Anisotropy



Reynolds Stress



Maxwell Stress

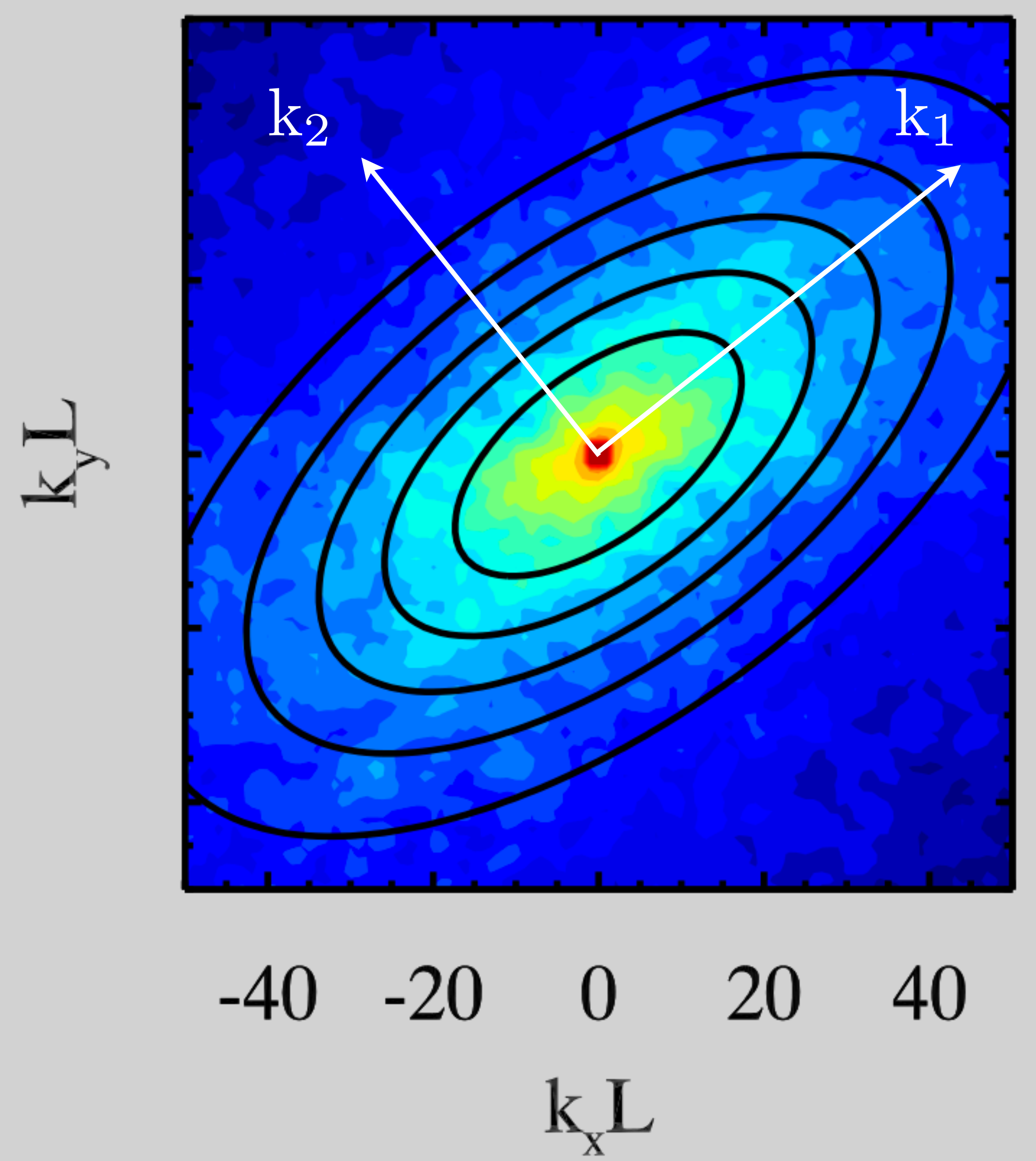
G. Murphy & MEP, 2015

Averages in Fourier Space

Anisotropic nature of MHD disk turbulence has long been known

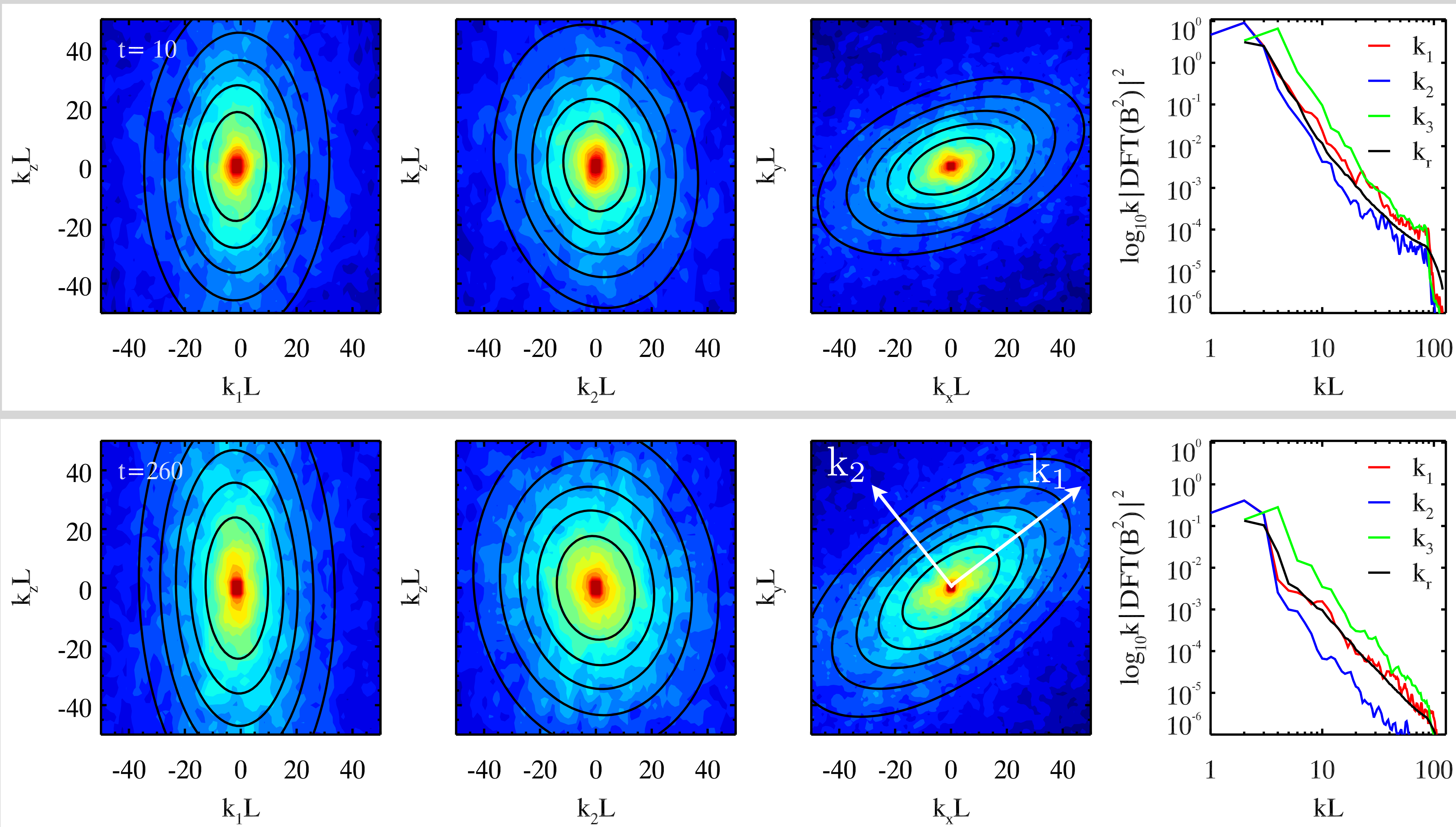
Usually illustrated using 2D cuts along **cartesian** axis in Fourier space

Nevertheless we tend to collapse all this information by taking spherical averages



(Hawley et al 1995, Workman & Armitage 2008, Fromang 2010, Lesur & Longaretti 2011)

Spectral Distribution of Anisotropy



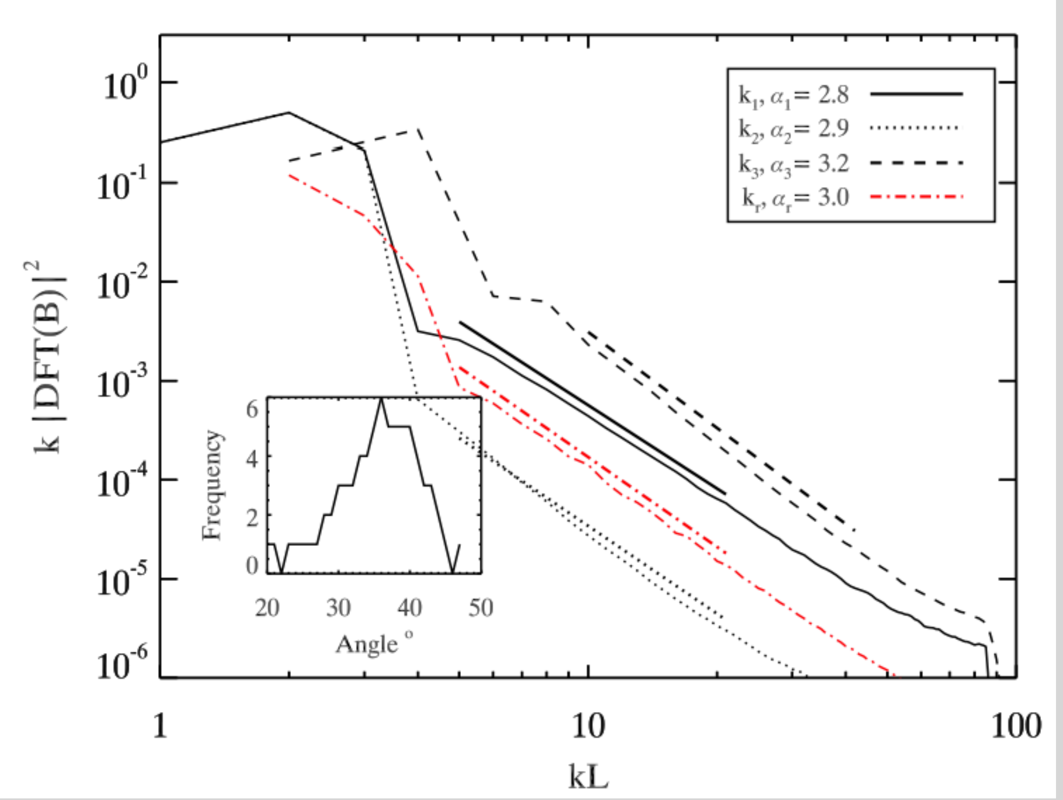
G. Murphy & MEP, 2015

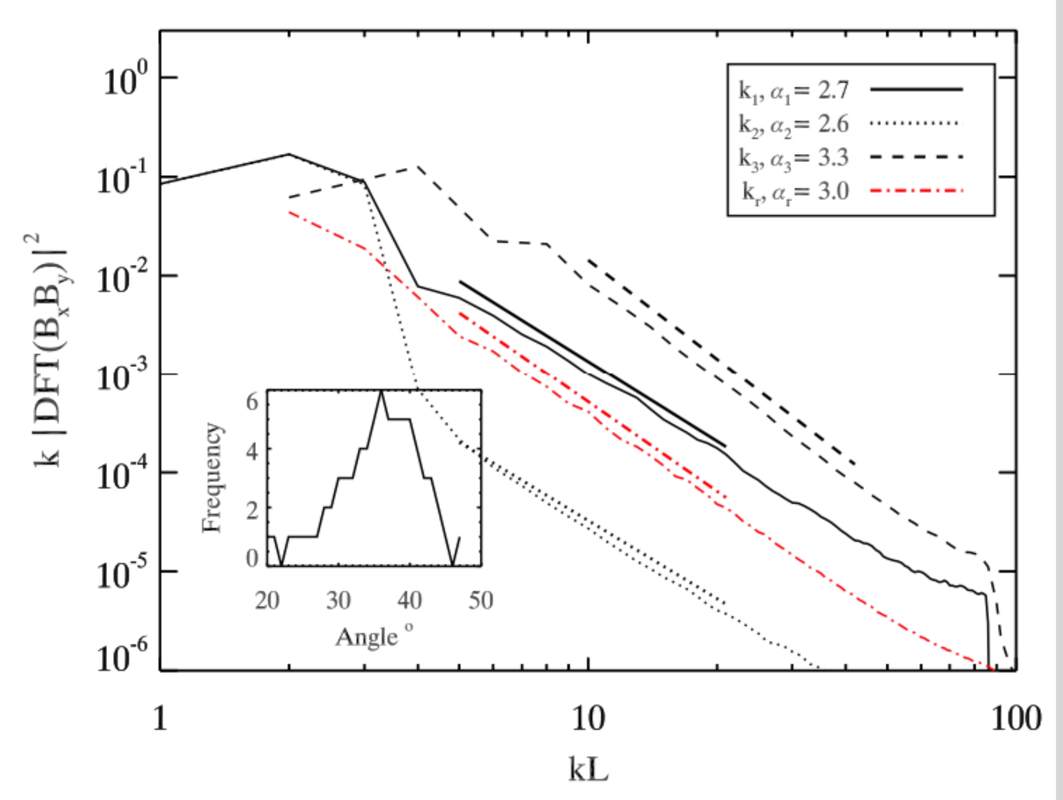
PSD of Temporal Fluctuations

Power along each independent direction can differ significantly wrt spherical average

Direction of maximum power wobbles around well defined angle ~ 35 degs.

Recent ideas about transverse cascade in 2D shear flows (Mamatsashvili et al. 2014)





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