Toward a Theory of Plasma Dynamo

Magnetic Fields and Microinstabilities in a Weakly Collisional Plasma

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Rincon, AAS & Cowley, *MNRAS* **447**, L45 (2015) [arXiv:1407.4707]
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]
Mogavero & AAS, *MNRAS* **440**, 3226 (2014) [arXiv:1312.3672]
AAS et al., *PRL* **100**, 081301 (2008) [arXiv:0709.3828]

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Everything is Magnetised…

Picture of your favourite astro object here

AAS et al., *ApJ* **612**, 276 (2004) [astro-ph/0312046]

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Standard Turbulent MHD Dynamo

AAS et al., *ApJ* **612**, 276 (2004) [astro-ph/0312046]

Standard Turbulent MHD Dynamo

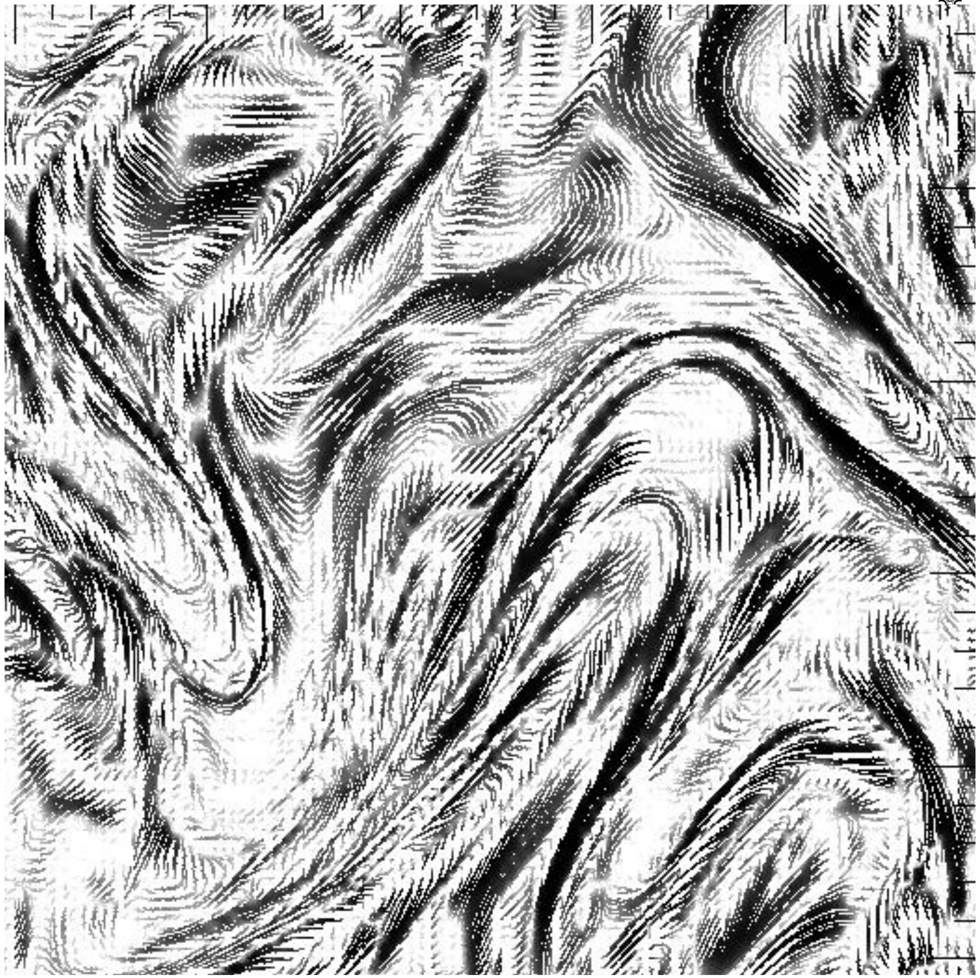
This was the solution of

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B})$$

$$\frac{\mathrm{d}\mathbf{B}}{\mathrm{d}t} \equiv \frac{\partial \mathbf{B}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{B} = \mathbf{B} \cdot \nabla \mathbf{u}$$

$$\frac{\mathrm{d}B}{\mathrm{d}t} = (\mathbf{bb} : \nabla \mathbf{u}) B \equiv \gamma B$$

$$\ln B \sim \int^t \mathrm{d}t' \, (\mathbf{bb} : \nabla \mathbf{u})(t')$$



So, roughly, field in Lagrangian frame accumulates as random walk
(in fact, situation more complex because of need to combat resistivity)

AAS et al., *ApJ* **612**, 276 (2004) [astro-ph/0312046]

Standard Turbulent MHD Dynamo

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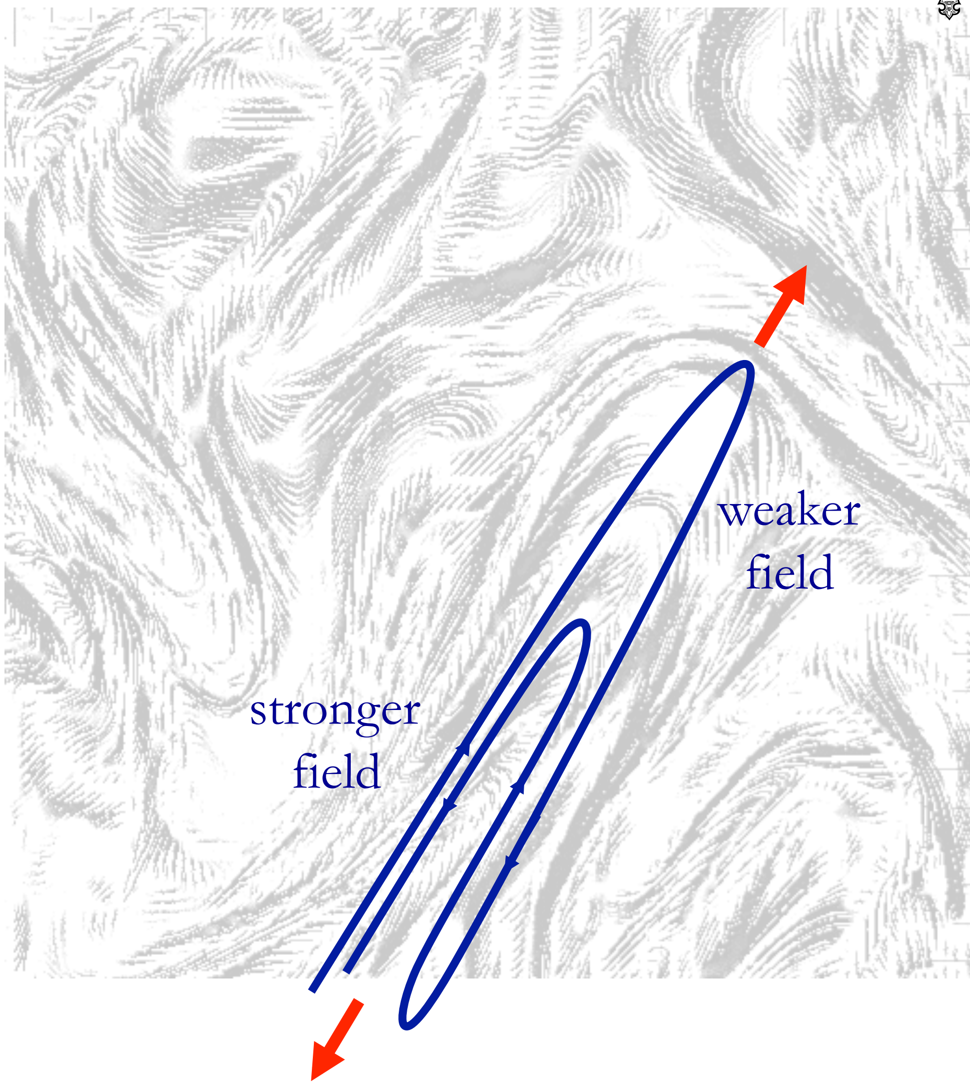

Key effect: a succession of random stretchings (and un-stretchings)

AAS et al., *ApJ* **612**, 276 (2004) [astro-ph/0312046]

Weak Collisions ➔ Pressure Anisotropy

Changing magnetic field causes local pressure anisotropies:

$$\frac{1}{p_\perp}\frac{\mathrm{d}p_\perp}{\mathrm{d}t} = \frac{1}{B}\frac{\mathrm{dB}}{\mathrm{d}t} - \nu\frac{p_\perp - p_\parallel}{p_\perp}$$

conservation of $\mu = v_\perp^2/B$

$$\frac{1}{2p_\parallel}\frac{\mathrm{d}p_\parallel}{\mathrm{d}t} = -\frac{1}{B}\frac{\mathrm{dB}}{\mathrm{d}t} - \nu\frac{p_\parallel - p_\perp}{p_\parallel}$$

conservation of $J = \oint \mathrm{d}\ell\, v_\parallel$

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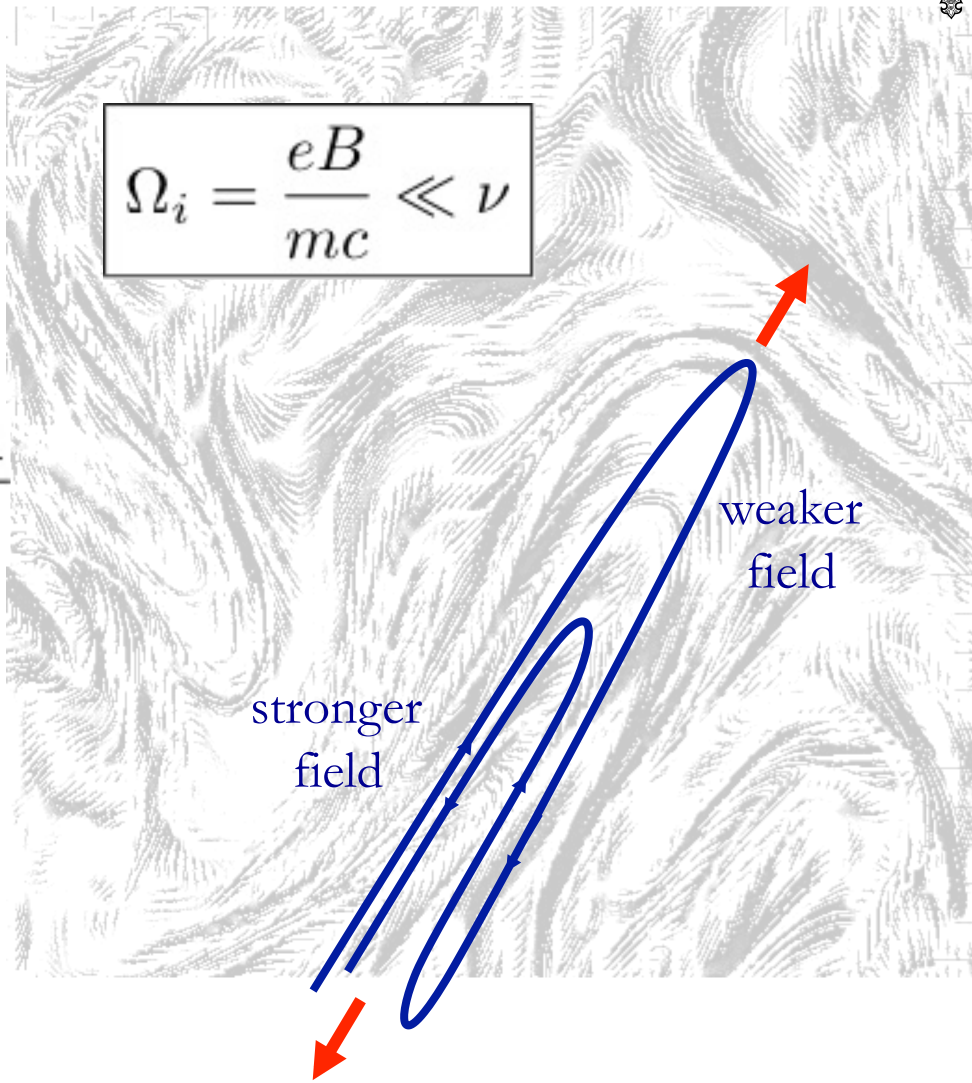

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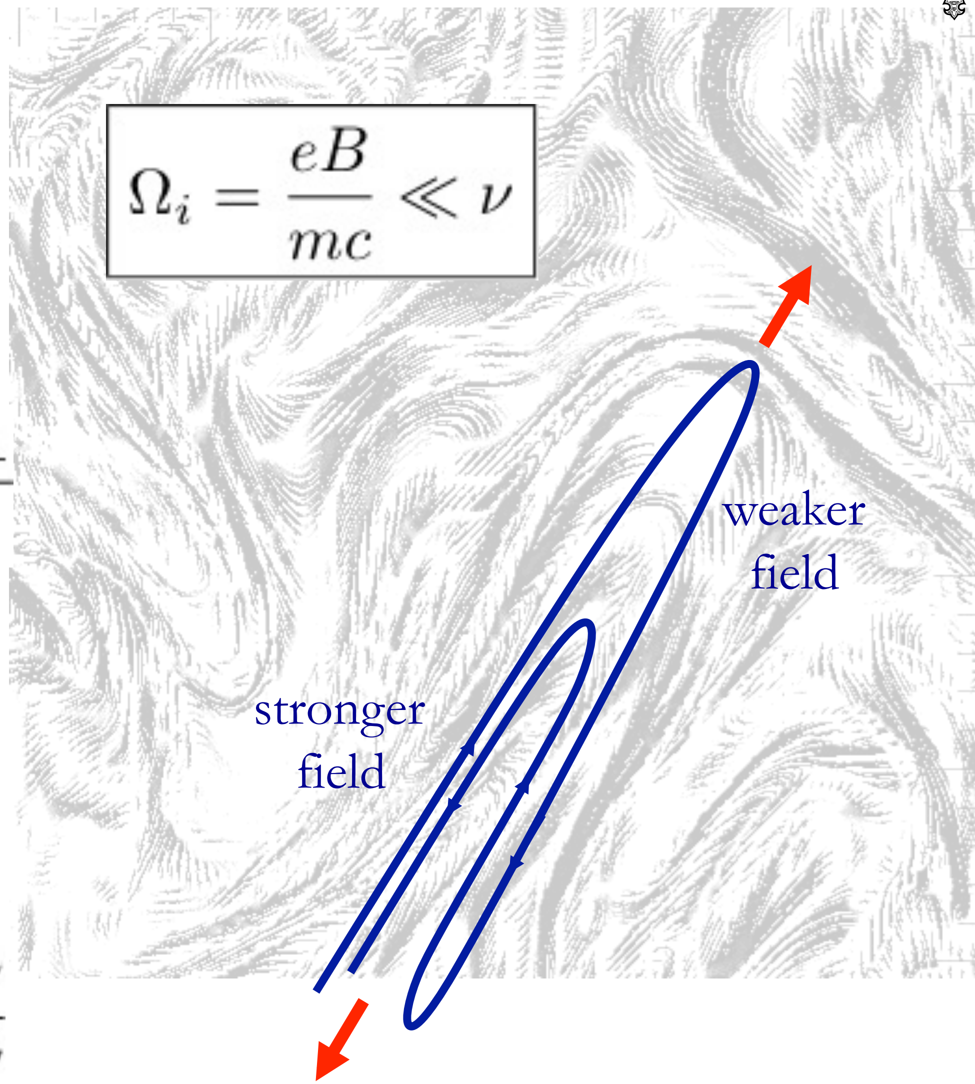
$$\frac{\mathrm{d}B}{\mathrm{d}t} = (\mathbf{bb}:\nabla\mathbf{u})B \equiv \gamma B$$

Typical pressure anisotropy:

$$\Delta \equiv \frac{p_\perp - p_\|}{p} \sim \frac{1}{\nu}\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\gamma}{\nu}$$

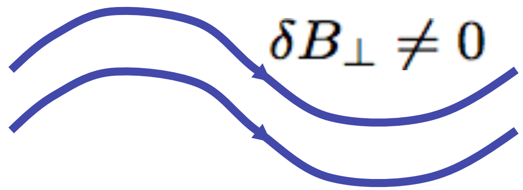
$$\Omega_i = \frac{eB}{mc} \ll \nu$$



Pressure Anisotropy ➔ Microinstabilities

Instabilities are fast, small scale.
They are instantaneous compared
to "fluid" dynamics.

"Plasma beta" $\beta = \dfrac{p}{B^2/8\pi}$



destabilised Alfvén wave

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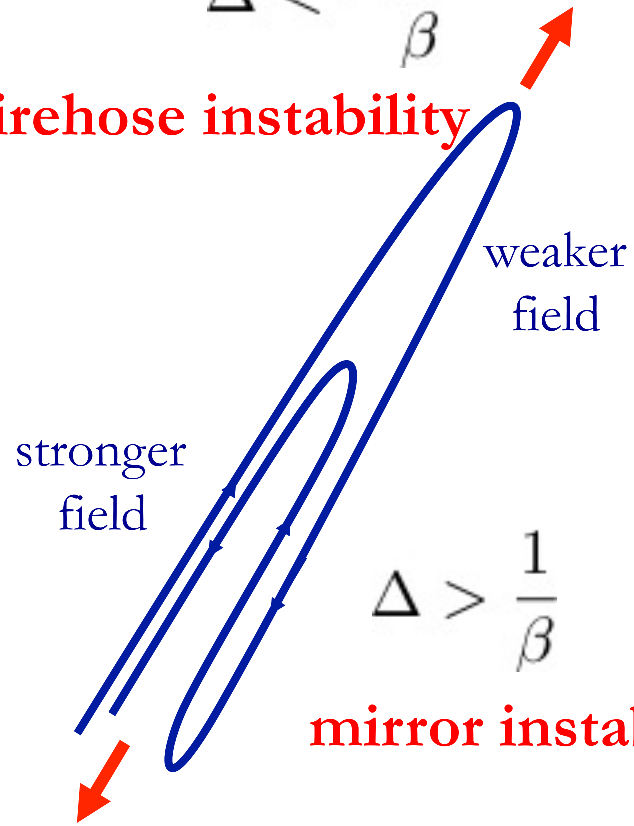
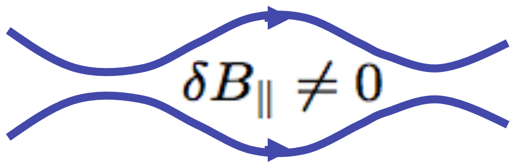
$$\Delta < -\frac{2}{\beta}$$

firehose instability



$$\Delta > \frac{1}{\beta}$$

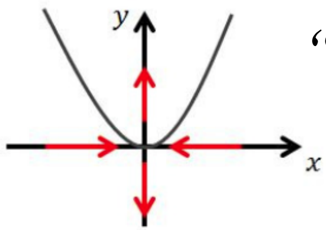
mirror instability



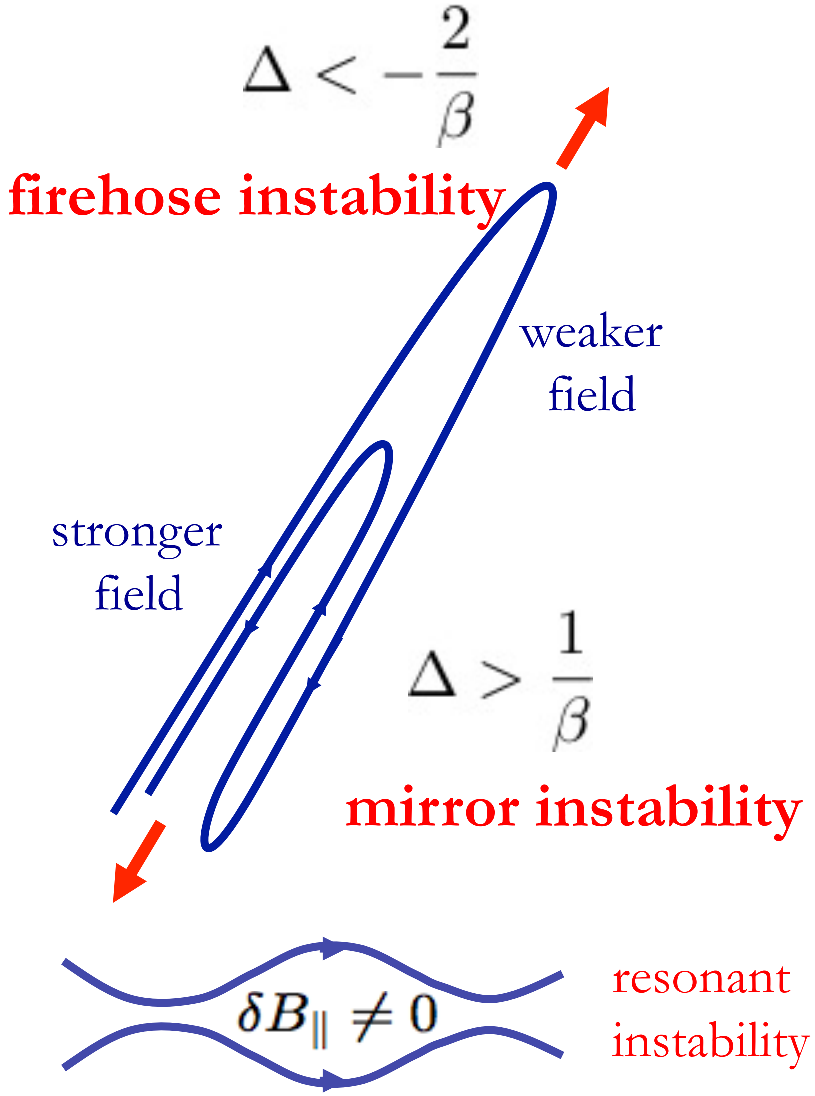
resonant
instability

Pressure Anisotropy ➔ Microinstabilities

Scott Melville:
folding field goes **firehose-unstable**
(in a 1D Braginskii model)



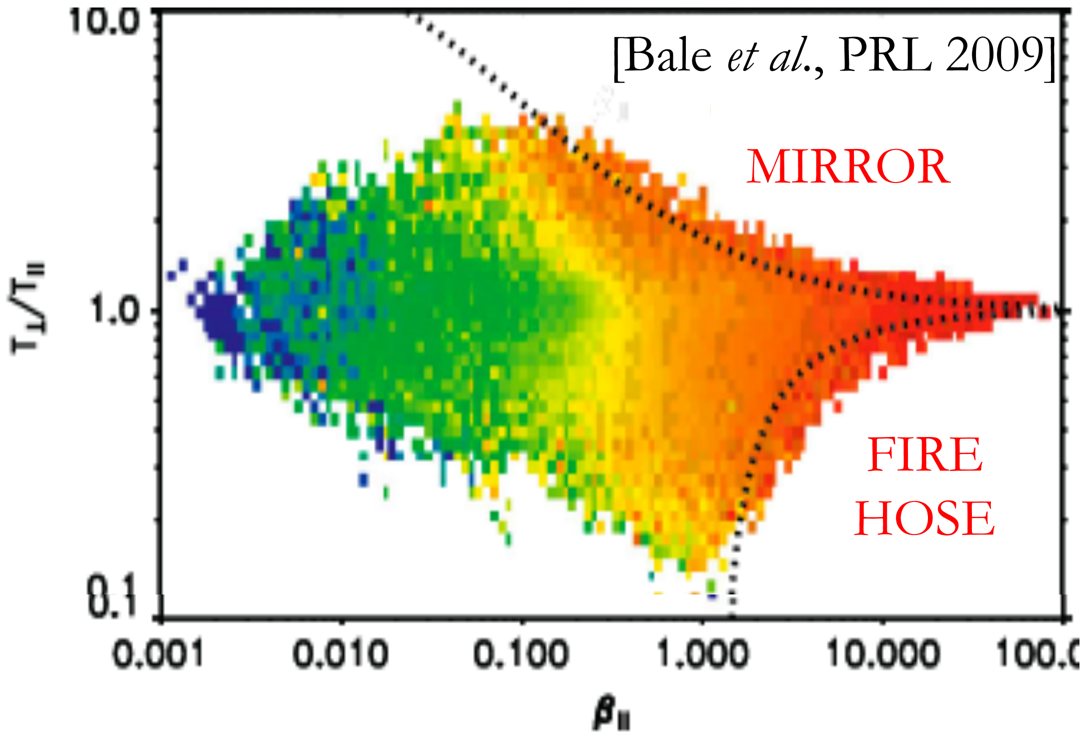
"*Plasma beta*" $\beta = \dfrac{p}{B^2/8\pi}$



Marginal State At All Times?

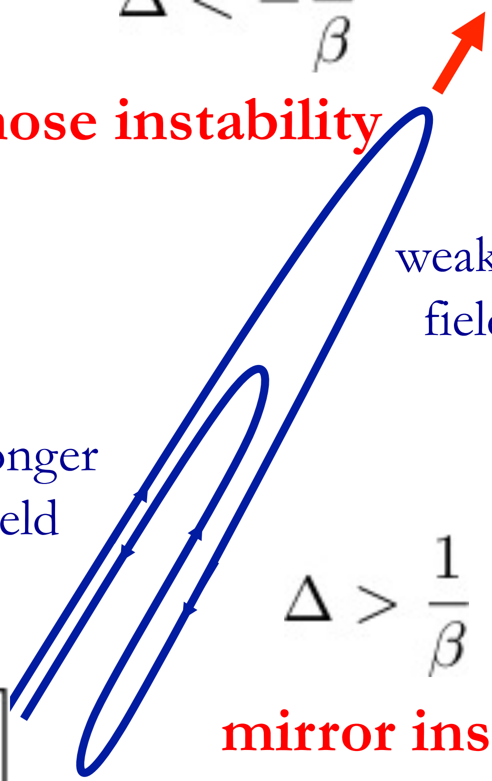
In the **solar wind**:

"Plasma beta" $\beta = \dfrac{p}{B^2/8\pi}$



$$\Delta < -\frac{2}{\beta}$$

firehose instability



$$\Delta > \frac{1}{\beta}$$

mirror instability

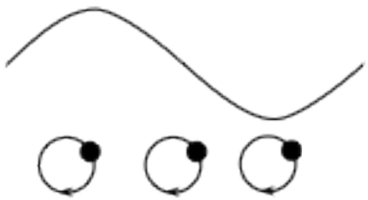
$$\Delta \equiv \frac{p_\perp - p_\parallel}{p} \sim \frac{1}{\nu}\frac{1}{B}\frac{dB}{dt} = \frac{\gamma}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$

How do you evolve the field from small to large while keeping everywhere within marginal stability boundaries?

Effective Closure Dilemma

How do you evolve the field from small to large while keeping everywhere within marginal stability boundaries?

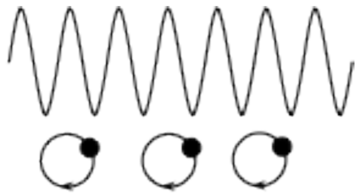
Way to keep const rms B needed for this



Model I: Suppress stretching

$$\Delta \equiv \frac{p_\perp - p_\parallel}{p} \sim \frac{1}{\nu}\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\gamma}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$

Model II: Enhance collisionality



Anomalous scattering of particles by Larmor scale fluctuations needed for this

Dynamo under Model I (suppression of γ)

$$\frac{\mathrm{d}B}{\mathrm{d}t} = (\mathbf{bb} : \nabla\mathbf{u})B \equiv \gamma B \in \nu\left[-\frac{2}{\beta}, \frac{1}{\beta}\right] B$$

Suppose there is enough stirring to keep Δ at the threshold:

$$\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\nu}{\beta} B = \frac{\nu}{8\pi p} B^3$$

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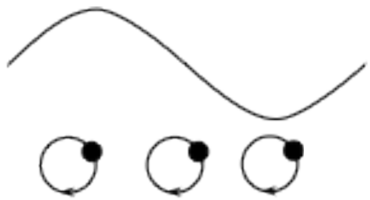

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Mogavero & AAS, *MNRAS* **440**, 3226 (2014) [arXiv:1312.3672]

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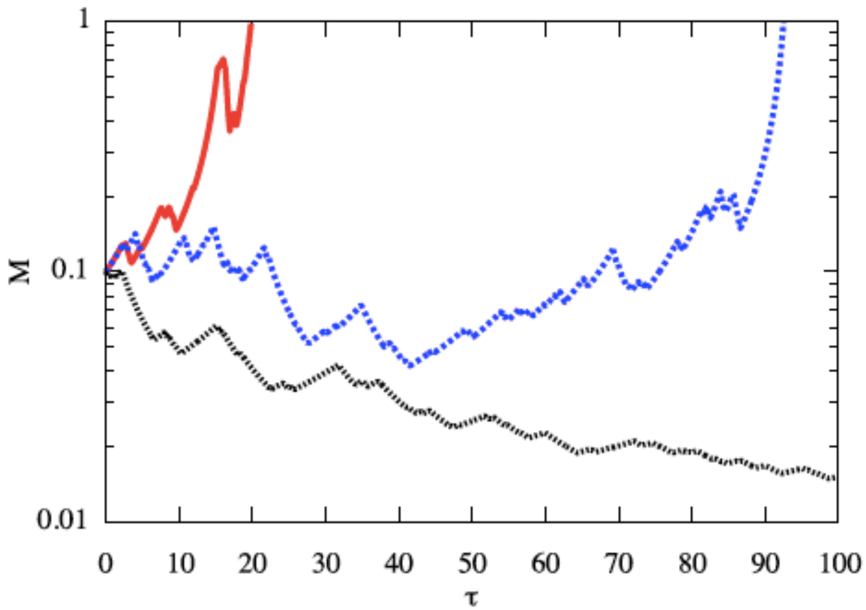
Thus, explosive growth, but takes a long time to explode: $t_c = \frac{\beta_0}{2\nu}$



for modeling details, caveats, complications, validity constraints, see

Mogavero & AAS, *MNRAS* **440**, 3226 (2014) [arXiv:1312.3672]

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For typical ICM parameters,

$$t_{\mathrm{growth}} \sim \frac{\beta_0}{\nu} \sim \beta_0 \times 10\left(\frac{n_e}{0.1\,\mathrm{cm}^{-3}}\right)^{-1}\left(\frac{T}{2\,\mathrm{keV}}\right)^{3/2}\mathrm{yrs}$$

So this can efficiently restore fields from $B \gtrsim 10^{-8}\mathrm{G}$
to current values $B \sim 10^{-5}\mathrm{G}$,
but for growth from a tiny seed, need a different mechanism

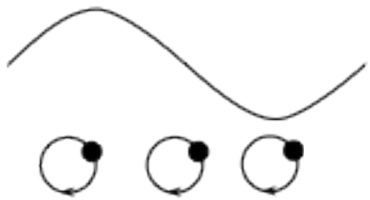
Mogavero & AAS, *MNRAS* **440**, 3226 (2014) [arXiv:1312.3672]

ICM heating under Model I

Viscous heating rate ($= Q_{\rm turb}$ if we ignore energy cascade below $\ell_{\rm visc}$)

$$Q_{\rm visc} = \underbrace{(p_\perp - p_\parallel)}_{p\Delta}\underbrace{\mathbf{bb}:\nabla\mathbf{u}}_{\gamma\sim\nu\Delta} \sim p\Delta\gamma \sim p\nu\Delta^2 \sim \frac{p\nu}{\beta^2}$$

Model I: Suppress stretching



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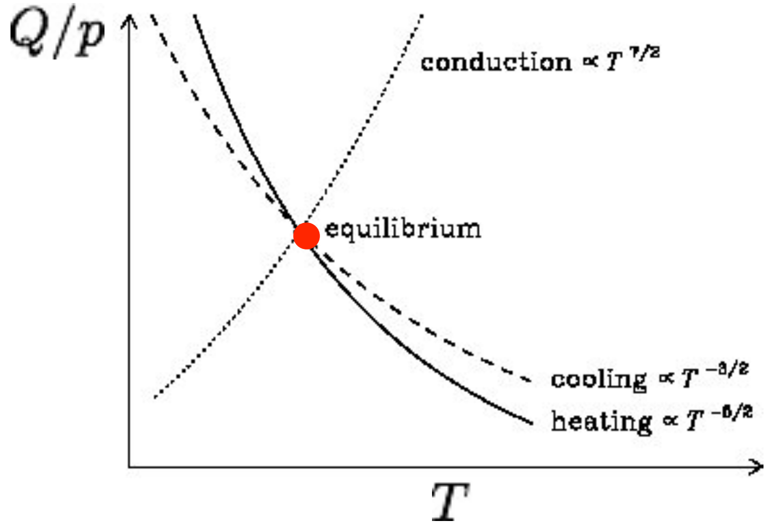
$$\sim 10^{-25}\left(\frac{B}{10\,\mu{\rm G}}\right)^4\left(\frac{T}{2\,{\rm keV}}\right)^{-5/2}\frac{\rm erg}{\rm s\,cm^3}$$

$$Q_{\rm cool} \sim 10^{-25}\left(\frac{n_e}{0.1\,{\rm cm}^{-3}}\right)^2\left(\frac{T}{2\,{\rm keV}}\right)^{1/2}\frac{\rm erg}{\rm s\,cm^3}$$

- Thermally **stable** ICM



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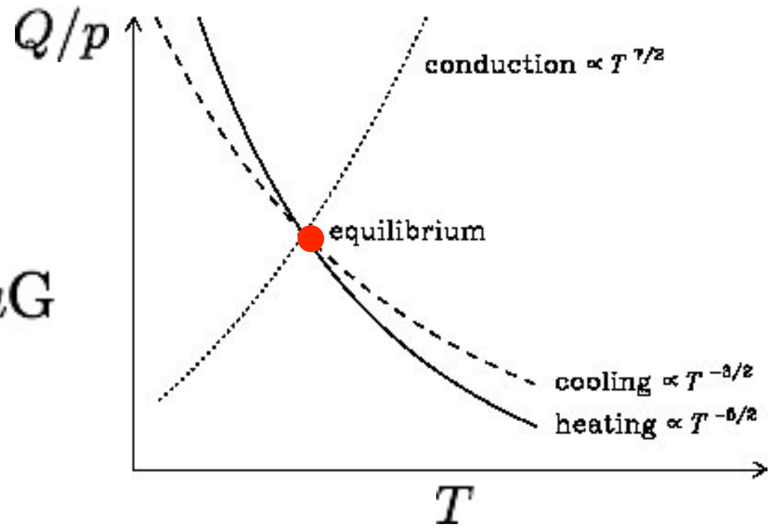
- Thermally **stable** ICM

- If $Q_{\rm visc} \sim Q_{\rm cool}$,

$$B \sim 10\left(\frac{n_e}{0.1\,\mathrm{cm^{-3}}}\right)^{1/2} \left(\frac{T}{2\,\mathrm{keV}}\right)^{3/4} \mu\mathrm{G}$$

- If $\rho u^2/2 \sim B^2/8\pi$,

$$u \sim 10^2\left(\frac{T}{2\,\mathrm{keV}}\right)^{3/4} \frac{\mathrm{km}}{\mathrm{s}}$$



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Dynamo under Model II (enhancement of ν)

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To stay at threshold, need effective collisionality $\nu \sim \gamma\beta$

$$\Delta \equiv \frac{p_\perp - p_\|}{p} \sim \frac{1}{\nu}\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\gamma}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$

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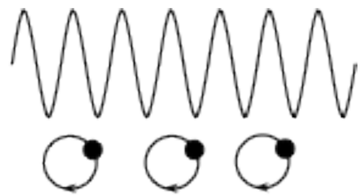

Anomalous scattering of particles by Larmor scale fluctuations needed for this

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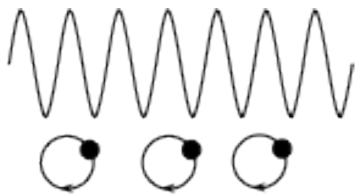

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And viscosity determines maximal rate of strain:

$$\gamma \sim \left(\frac{\varepsilon}{\mu}\right)^{1/2} \sim \left(\frac{\varepsilon\nu}{p}\right)^{1/2} \sim \left(\frac{\varepsilon\gamma\beta}{p}\right)^{1/2} \quad \Rightarrow \quad \gamma \sim \frac{\varepsilon\beta}{p} \sim \frac{\varepsilon}{B^2}$$

$\varepsilon \sim \rho u^3/l$ is Kolmogorov's energy flux

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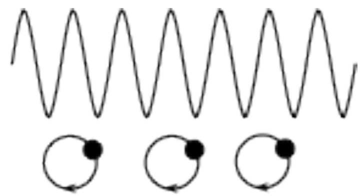

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$$\frac{\mathrm{d}B^2}{\mathrm{d}t} = 2\gamma B^2 \sim \varepsilon$$

$$\Delta \equiv \frac{p_\perp - p_\parallel}{p} \sim \frac{1}{\nu}\frac{1}{B}\frac{\mathrm{d}B}{\mathrm{d}t} = \frac{\gamma}{\nu} \in \left[-\frac{2}{\beta}, \frac{1}{\beta}\right]$$

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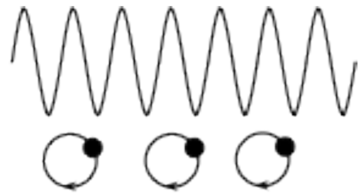

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$$\frac{\mathrm{d}B^2}{\mathrm{d}t} = 2\gamma B^2 \sim \varepsilon \quad \Rightarrow \quad B^2 \sim \varepsilon t$$

Thus, secular growth, but gets to dynamical strength very quickly:

$$t \sim \frac{B_{\mathrm{sat}}^2}{\varepsilon} \sim \frac{u^2}{\varepsilon} \sim \frac{l}{u}$$ one large-scale turnover rate

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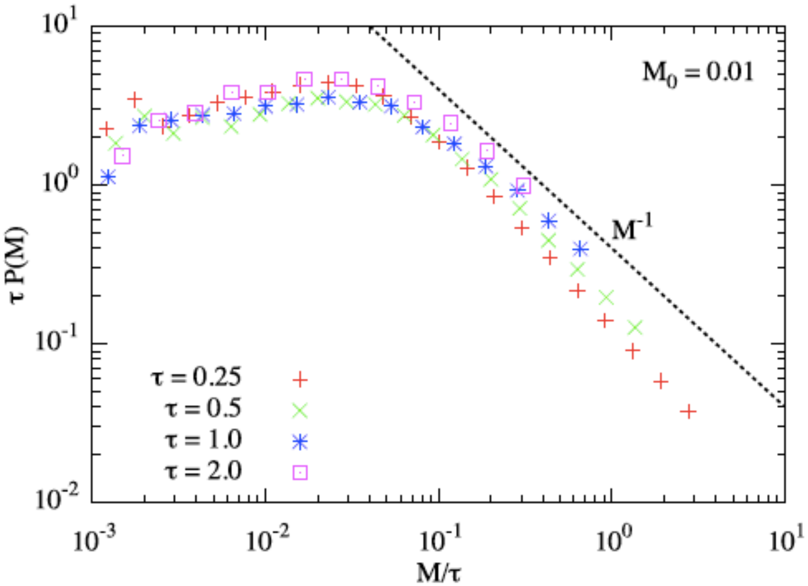

$$M = B^2$$

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Modeling gives extremely intermittent, self-similar field distribution; see
(➔ intermittent viscosity, intermittent rate of strain,
very hard to do right in "real" simulations with this effective closure!)

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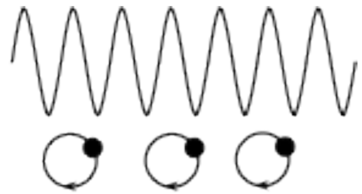

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ICM heating under Model II

$$Q_{\rm visc} = \underbrace{(p_\perp - p_\|)}_{p\Delta}\underbrace{\mathbf{bb}:\nabla\mathbf{u}}_{\gamma} \sim p\Delta\gamma \sim \varepsilon$$

So we learn nothing new: all the turbulent power input, whatever it is, gets viscously dissipated

(in Model I, $Q_{\rm visc} \sim \varepsilon$ as well, but it allows one to fix the temperature profile in terms of other parameters, while in Model II it is hard-wired)

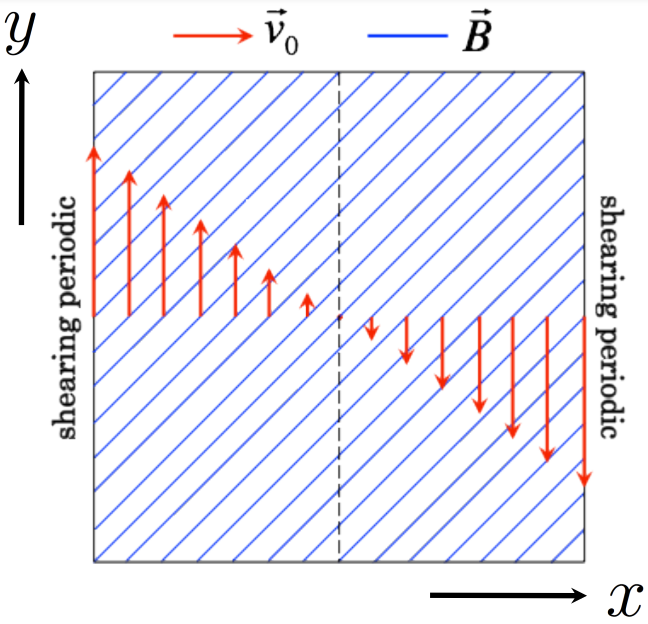
This would mean that whatever determines the thermal stability of the ICM has, under Model II, to do with large-scale energy deposition processes, not with microphysics:

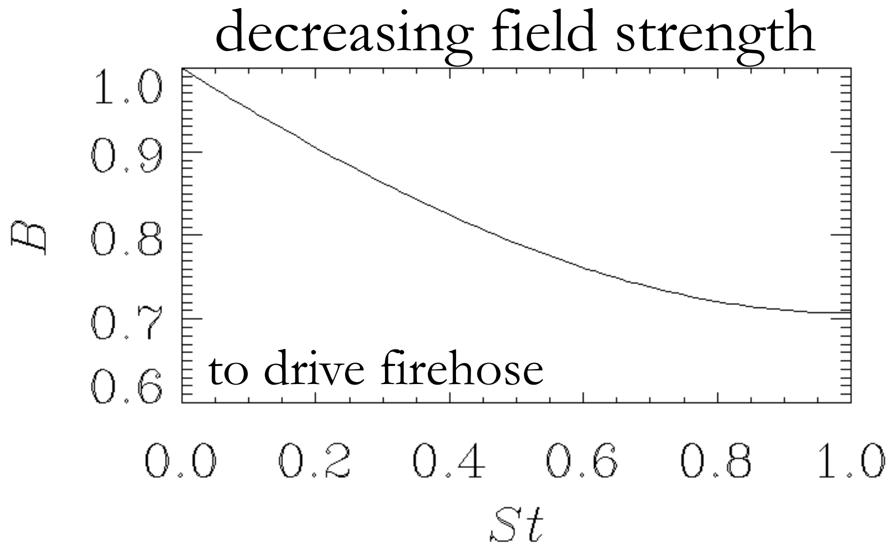
Rejoice all ye believers that microphysics should never matter!

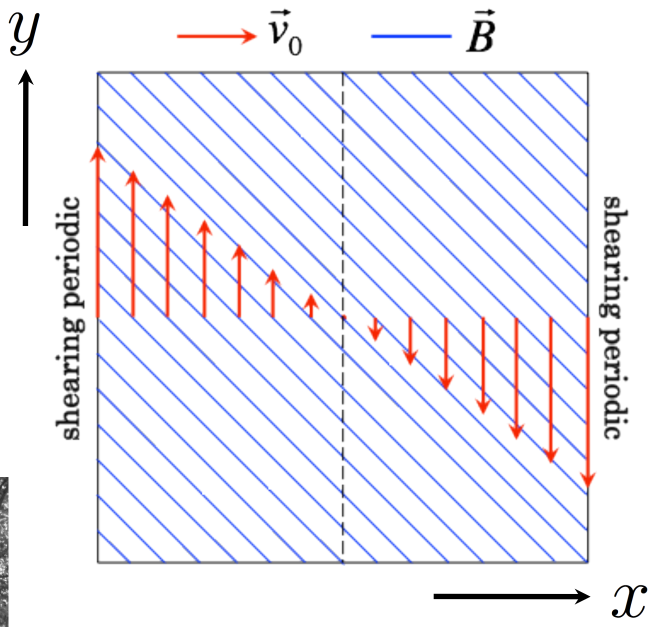
(although you need microphysics to know whether Model II is right)

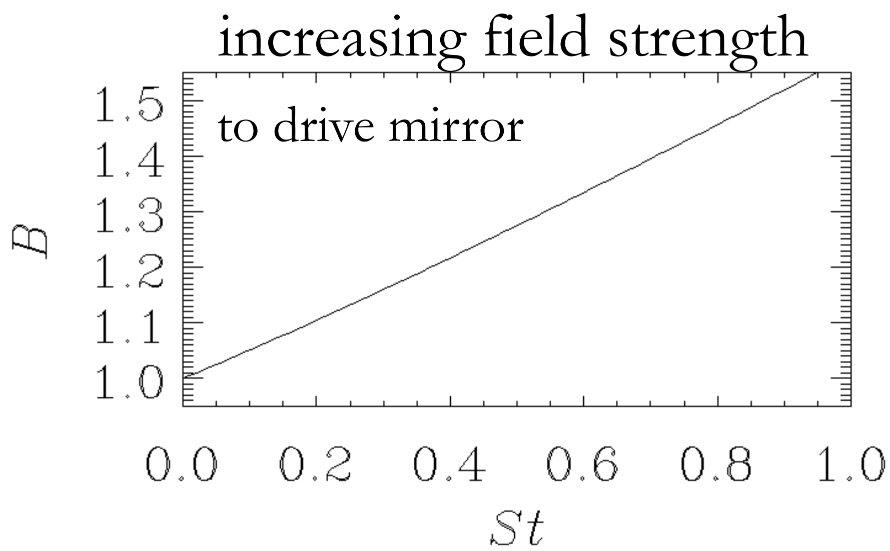
Mogavero & AAS, *MNRAS* **440**, 3226 (2014) [arXiv:1312.3672]

Instabilities in a Box (M. Kunz)









Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Instabilities in a Box (M. Kunz)

Hybrid kinetic system solved by PEGASUS code:

$$\left(\frac{\partial}{\partial t} - Sx\frac{\partial}{\partial y}\right) f_{\mathrm{i}} + \boldsymbol{v}\cdot\boldsymbol{\nabla} f_{\mathrm{i}} + \left[\frac{Ze}{m_{\mathrm{i}}}\left(\boldsymbol{E} + \frac{\boldsymbol{v}}{c}\times\boldsymbol{B}\right) + Sv_x\hat{\boldsymbol{y}}\right]\cdot\frac{\partial f_{\mathrm{i}}}{\partial \boldsymbol{v}} = 0$$

$$\left(\frac{\partial}{\partial t} - Sx\frac{\partial}{\partial y}\right)\boldsymbol{B} = -c\boldsymbol{\nabla}\times\boldsymbol{E} - SB_x\hat{\boldsymbol{y}}$$

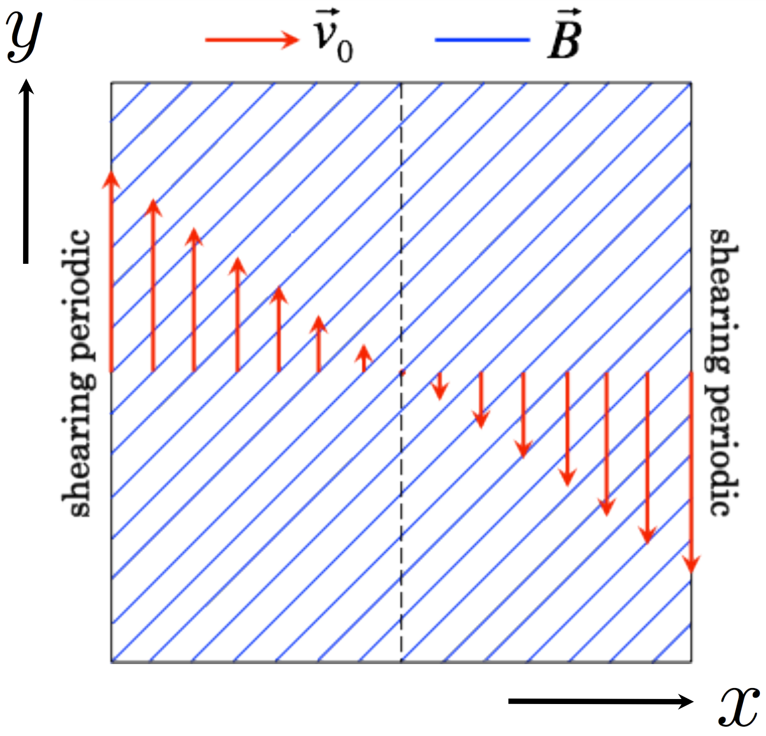
$$\boldsymbol{E} = -\frac{\boldsymbol{u}_{\mathrm{i}}\times\boldsymbol{B}}{c} + \frac{(\boldsymbol{\nabla}\times\boldsymbol{B})\times\boldsymbol{B}}{4\pi Zen_{\mathrm{i}}} - \frac{T_{\mathrm{e}}\boldsymbol{\nabla} n_{\mathrm{i}}}{en_{\mathrm{i}}}$$

…in a shearing sheet $\mathbf{u} = -Sx\hat{\mathbf{y}}$

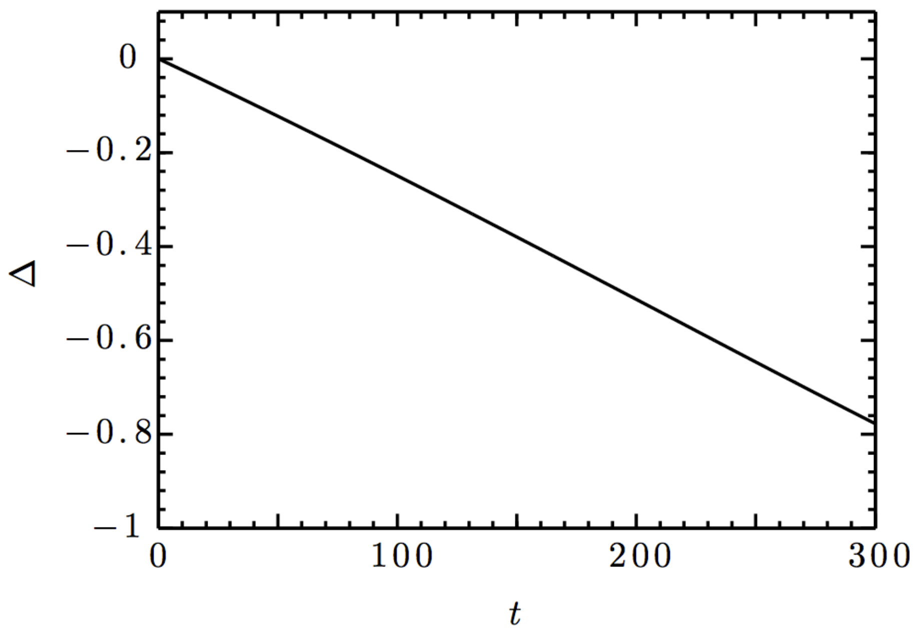
Kunz, Stone & Bai, *JCP* **259**, 154 (2014)

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Firehose Instability (M. Kunz)



$$\frac{\mathrm{d}B}{\mathrm{d}t} < 0 \quad \Rightarrow \quad \Delta = \frac{p_\perp - p_\parallel}{p} < 0$$

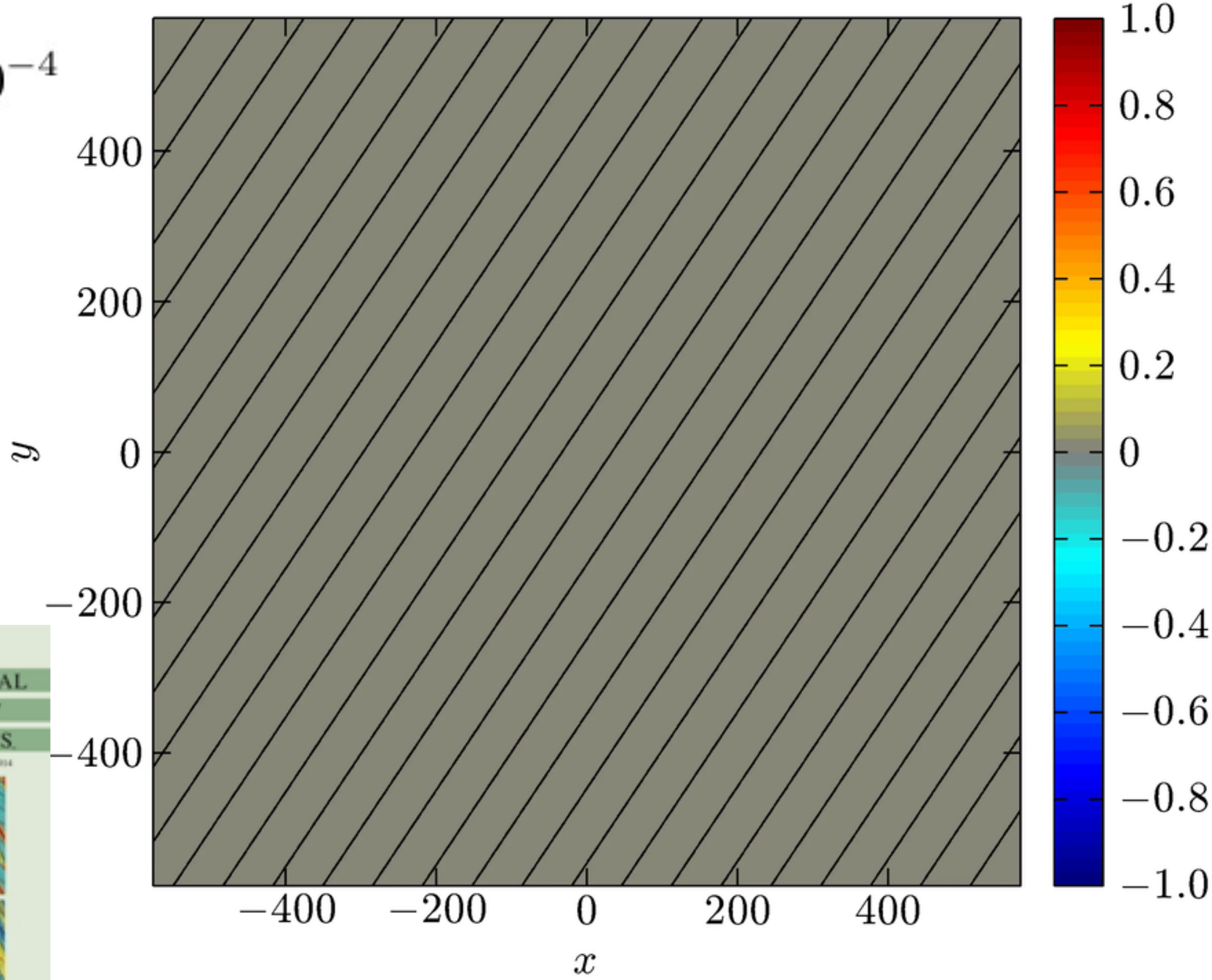


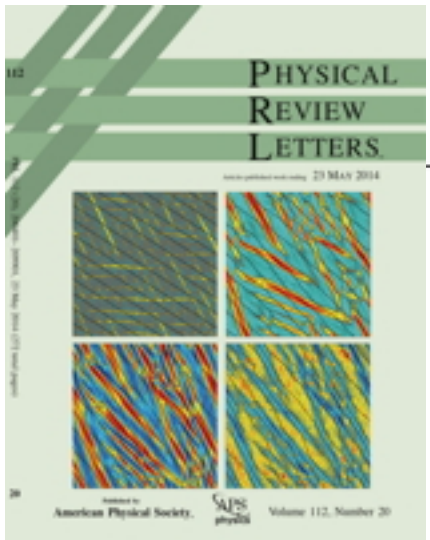
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Firehose Instability (M. Kunz)

$$\frac{S}{\Omega_i} = 3 \times 10^{-4}$$

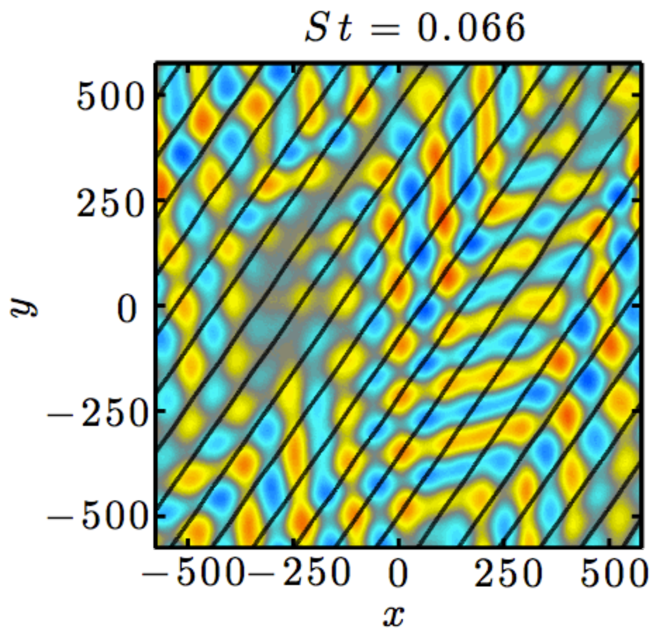
$$\beta_i = 200$$

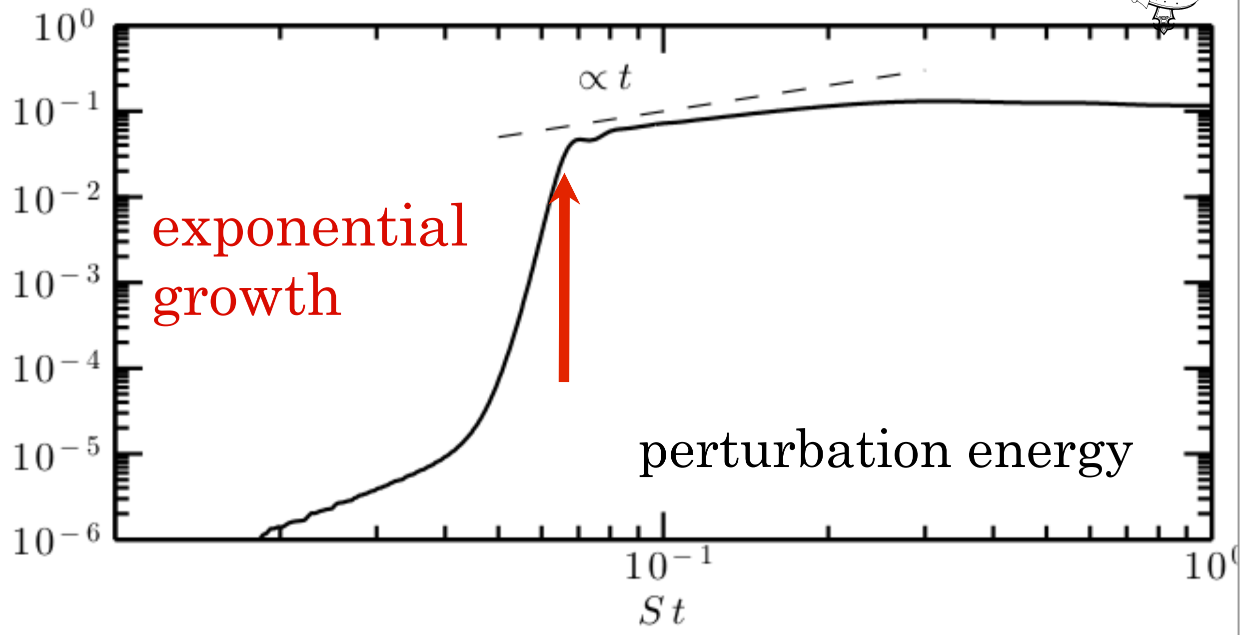


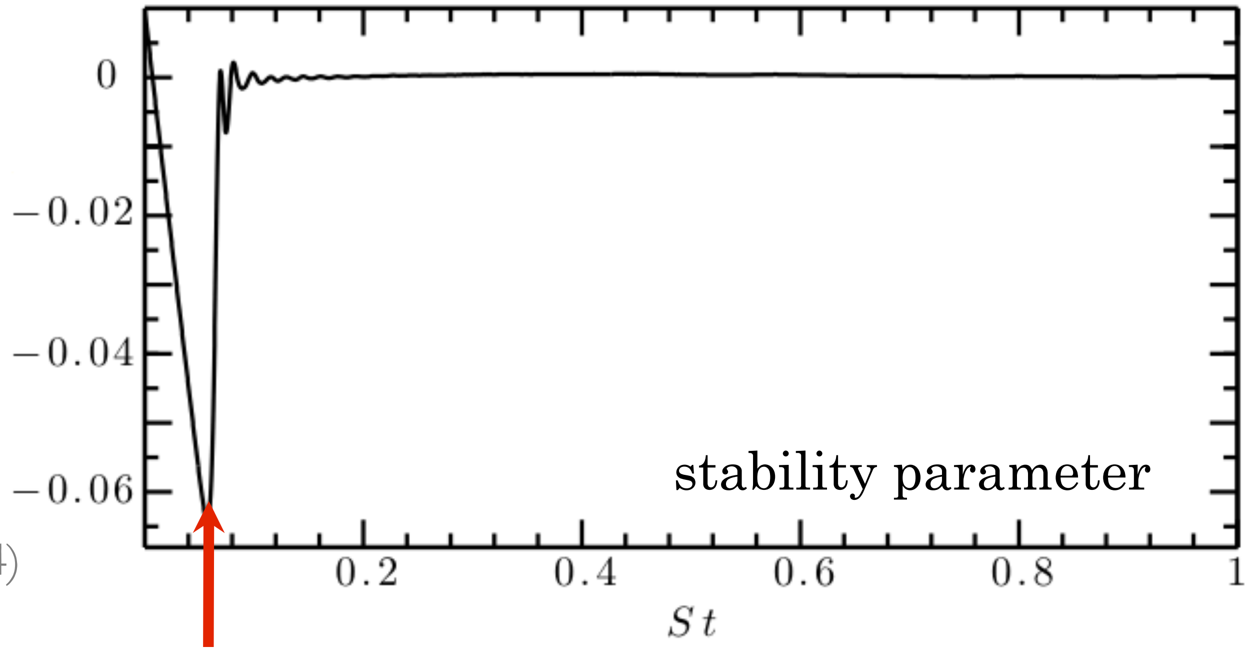


Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Firehose Instability: Linear





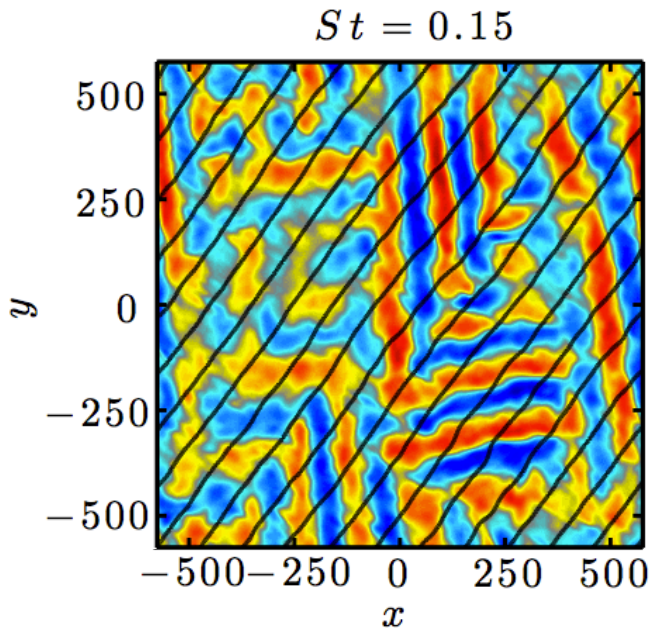


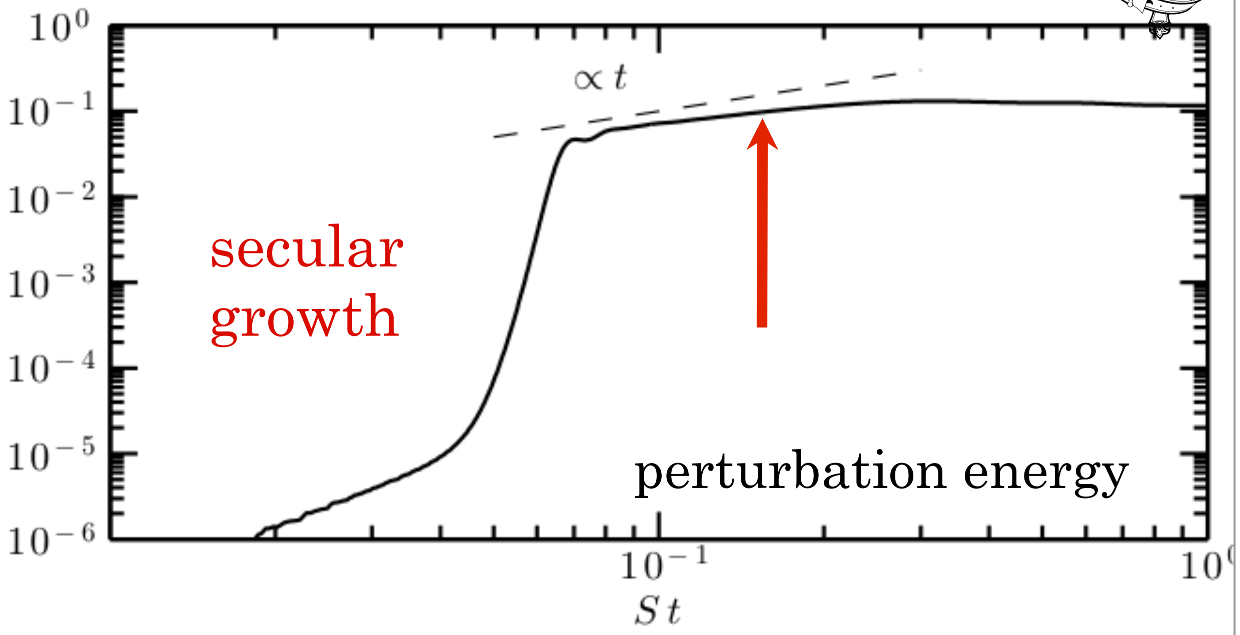
oblique modes

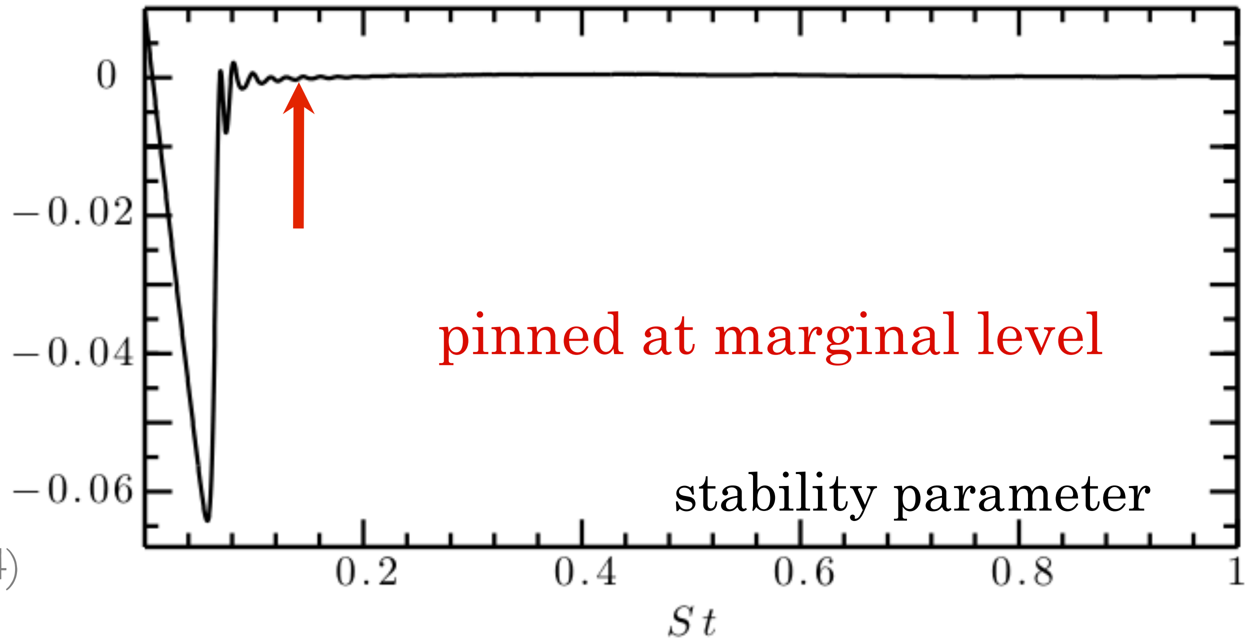
$$k_{\parallel}\rho_i \approx k_{\perp}\rho_i \approx 0.4$$

Kunz et al., *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Firehose Instability: Secular







Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

Firehose Instability: Secular

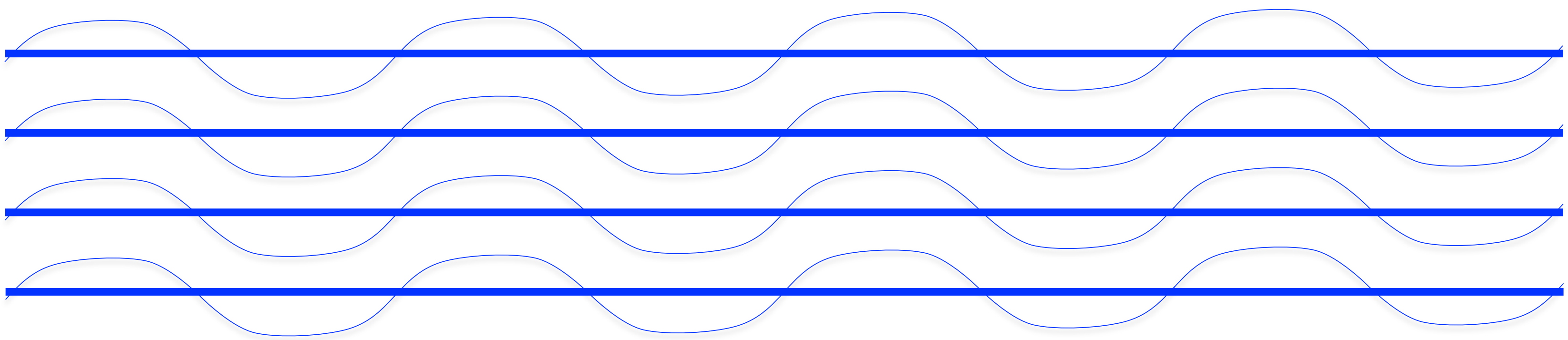
$$\overline{B^2} = \overline{|\mathbf{B}_0 + \delta\mathbf{B}_\perp|^2} = B_0^2 + \overline{|\delta\mathbf{B}_\perp|^2}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \overline{\frac{\mathrm{d}\ln B}{\mathrm{d}t}} = \int^t \mathrm{d}t' \left(-3\left|\frac{\mathrm{d}\ln B_0}{\mathrm{d}t}\right| + \frac{3}{2}\frac{\mathrm{d}}{\mathrm{d}t}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} \right) \to -\frac{2}{\beta}$$

pressure anisotropy driven by shear

pressure anisotropy from firehose

marginal stability



AAS et al., *PRL* **100**, 081301 (2008) [arXiv:0709.3828]

Rosin et al., *MNRAS* **413**, 7 (2011) [arXiv:1002.4017]

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

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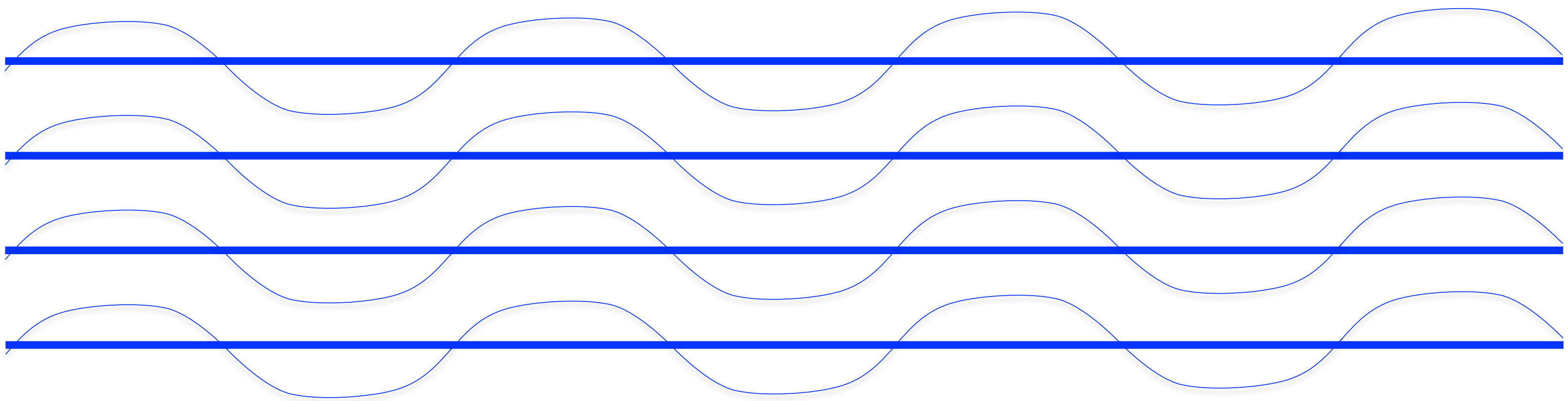

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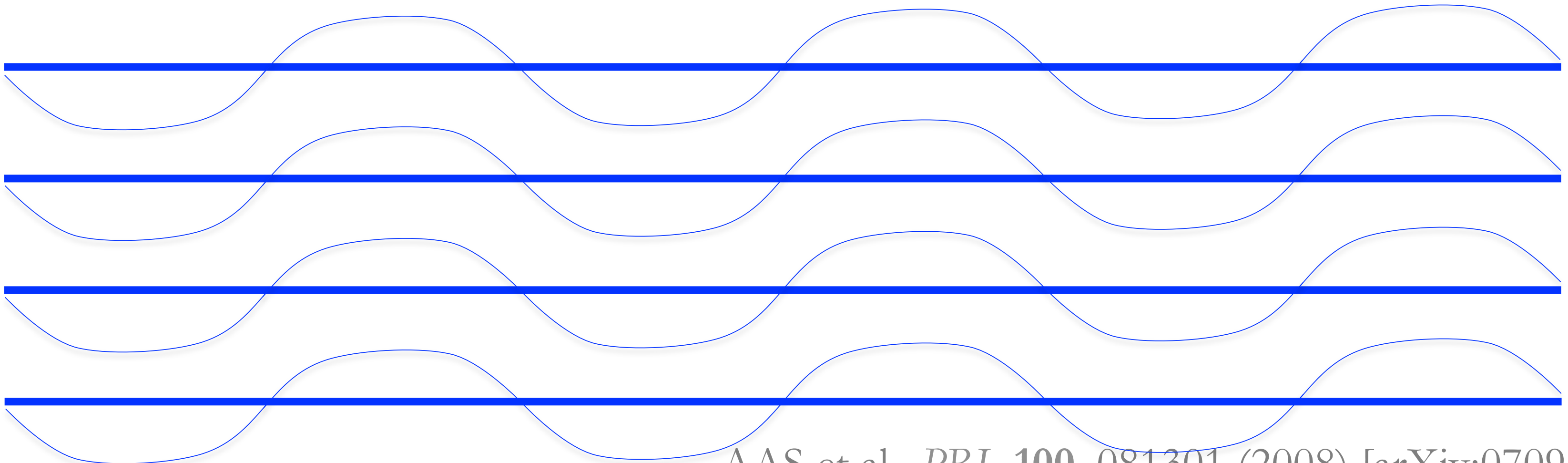

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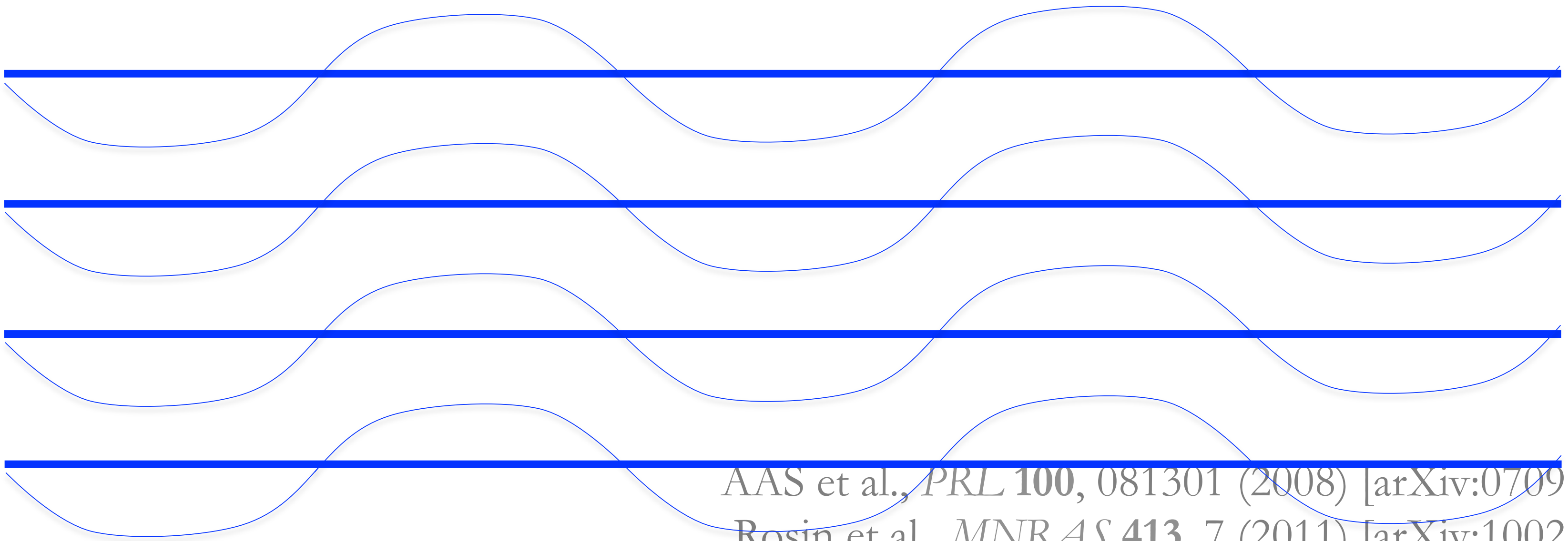
pressure anisotropy driven by shear — pressure anisotropy from firehose — marginal stability



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$$\overline{B^2} = \overline{|\mathbf{B}_0 + \delta\mathbf{B}_\perp|^2} = B_0^2 + \overline{|\delta\mathbf{B}_\perp|^2}$$

$$\Delta = 3\int^t dt' \, \overline{\frac{d\ln B}{dt}} = \int^t dt' \left(-3\left|\frac{d\ln B_0}{dt}\right| + \frac{3}{2}\frac{d}{dt}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} \right) \to -\frac{2}{\beta}$$

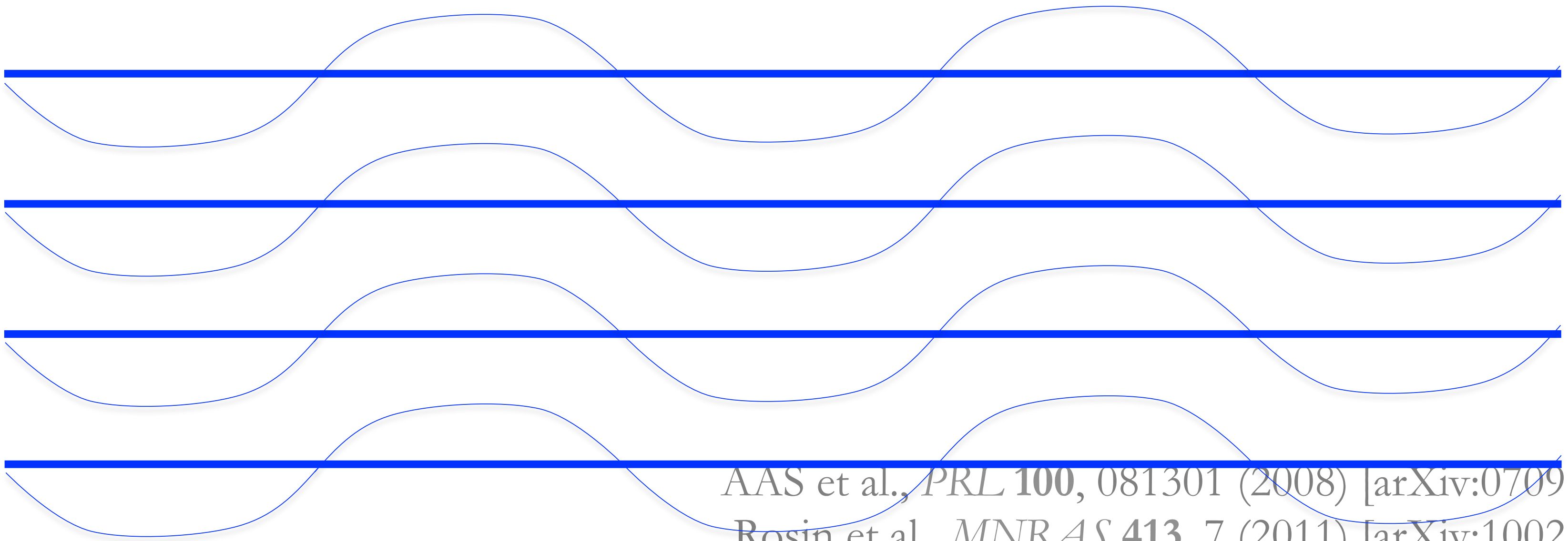
pressure anisotropy driven by shear

pressure anisotropy from firehose

marginal stability

$$\frac{3}{2}\frac{\overline{|\delta\mathbf{B}_\perp|^2}}{B_0^2} = 3S\int^t dt' \, \hat{b}_x(t')\hat{b}_y(t') - \frac{2}{\beta} \sim St$$

secular growth



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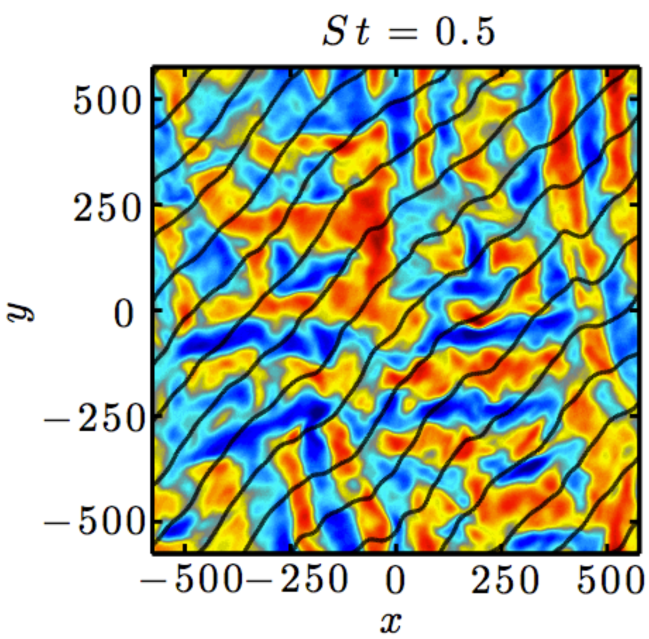
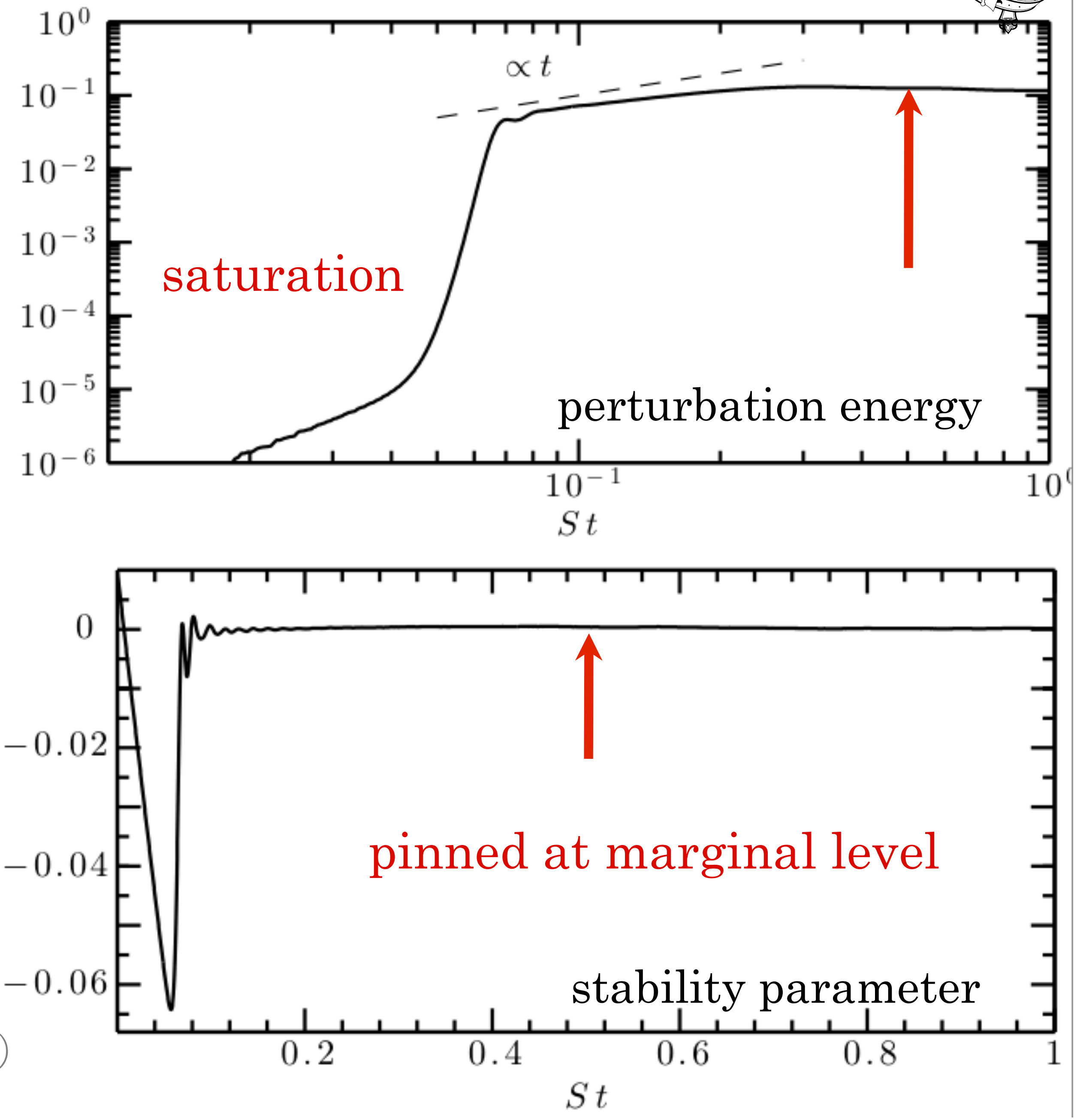
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

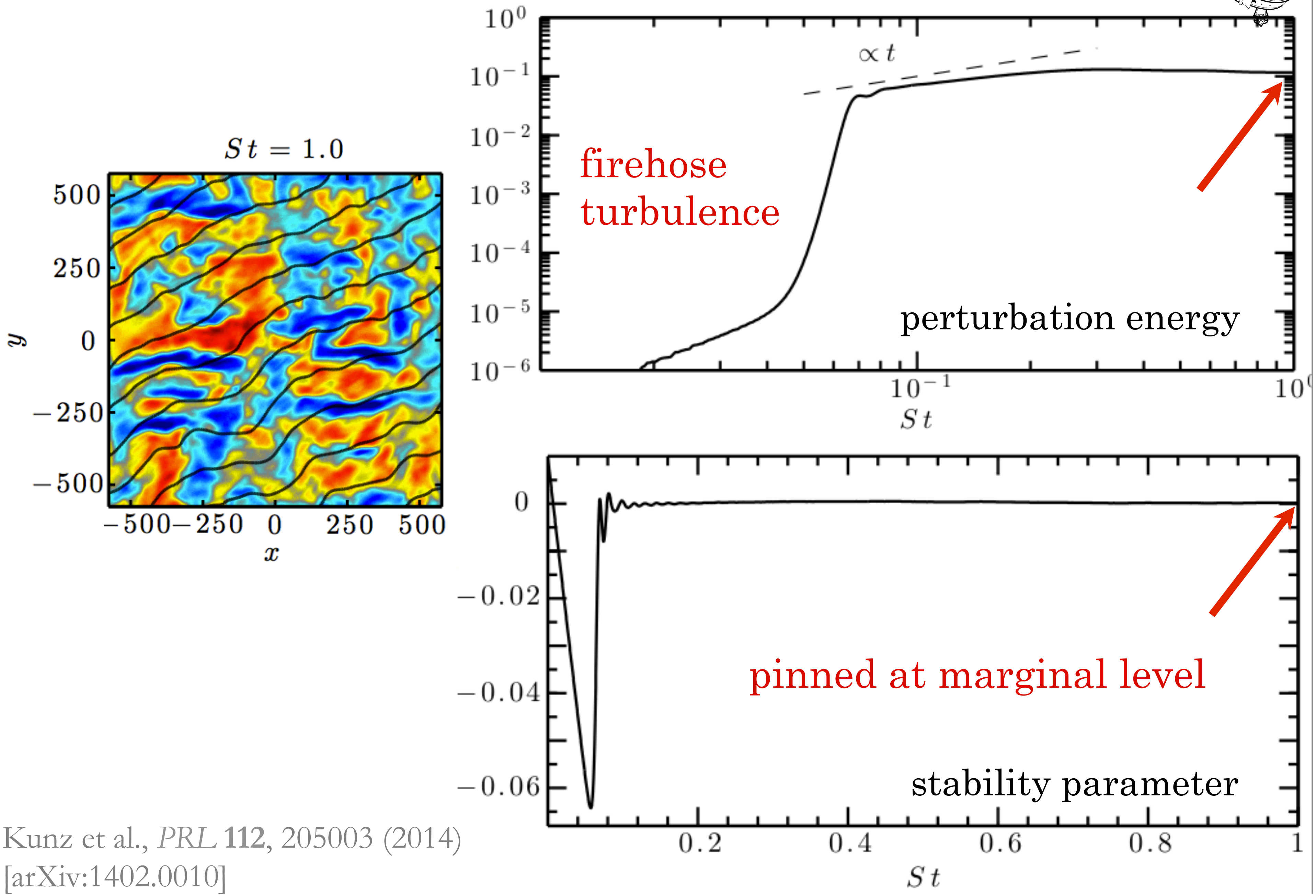
Firehose Instability: Saturated





Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

Firehose Instability: Saturated

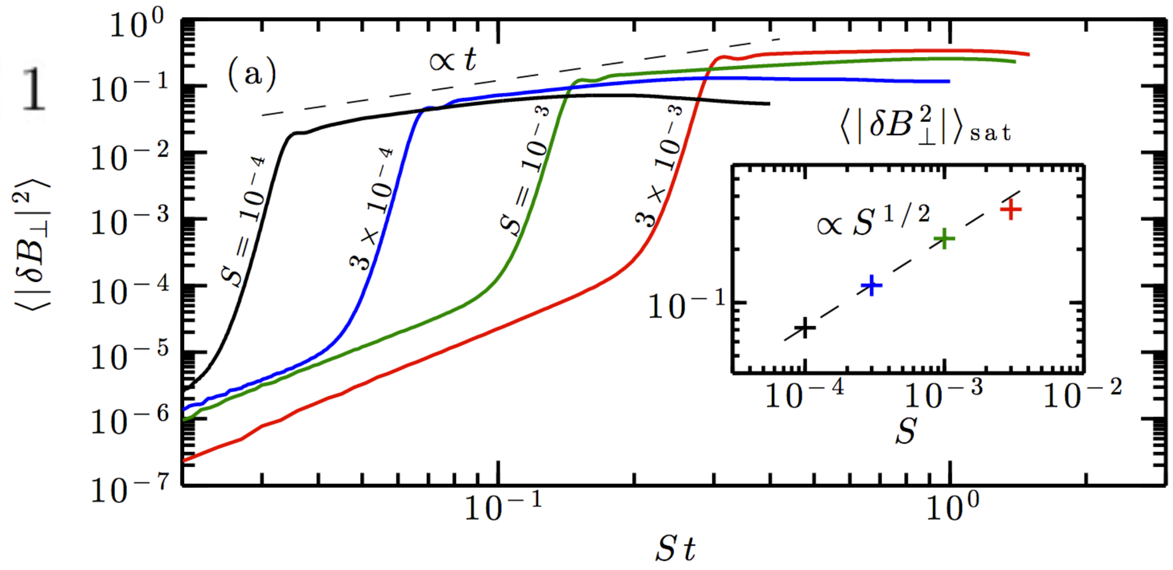


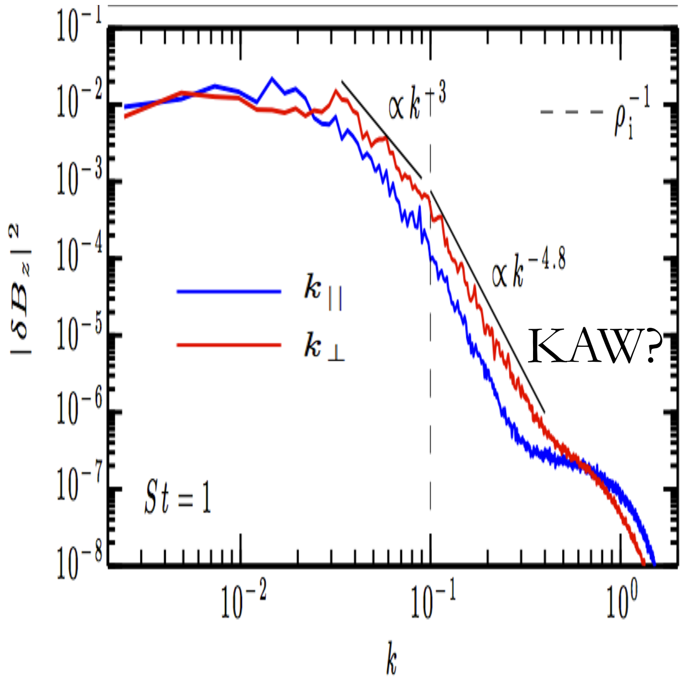
Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

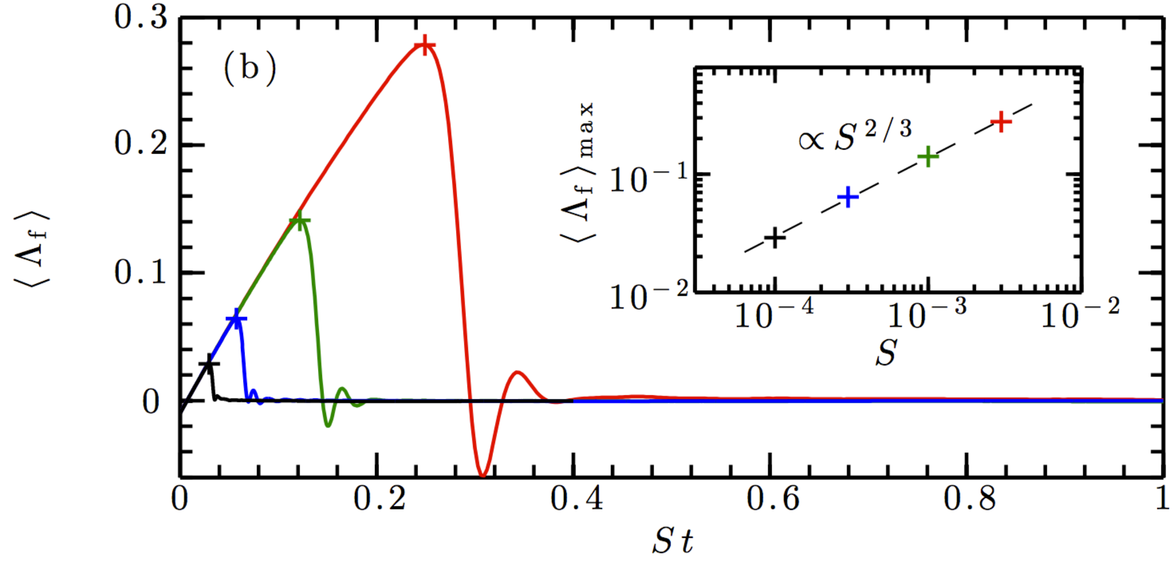
Firehose Saturates at Small Amplitudes

$$\frac{\langle|\delta \mathbf{B}_\perp|^2\rangle}{B_0^2} \propto \left(\frac{S}{\Omega_i}\right)^{1/2} \ll 1$$

small-amplitude
Larmor-scale
firehose turbulence

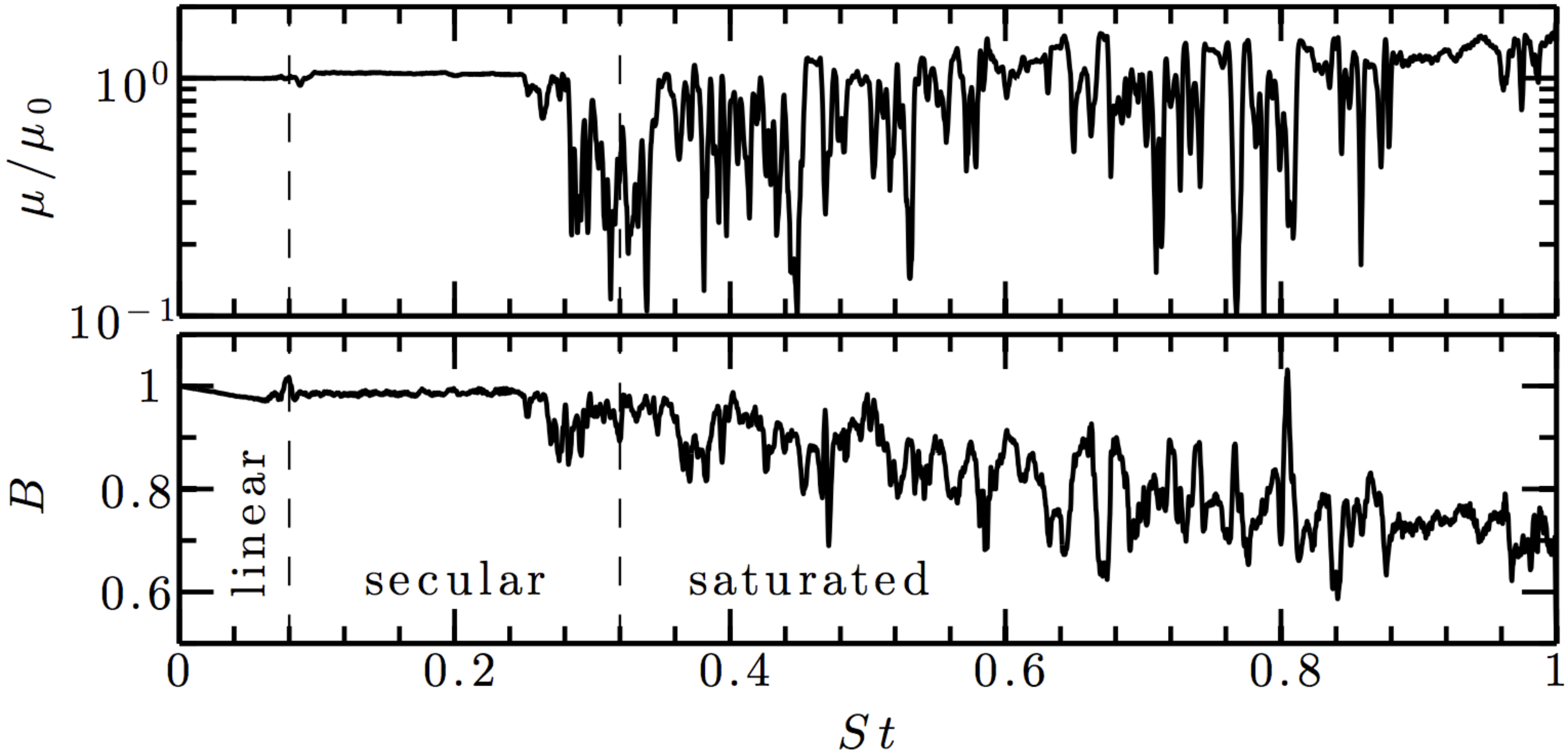






Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Saturated Firehose Scatters Particles



μ conservation is broken at long times, firehose fluctuations scatter particles to maintain pressure anisotropy at marginal level

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Saturated Firehose Scatters Particles

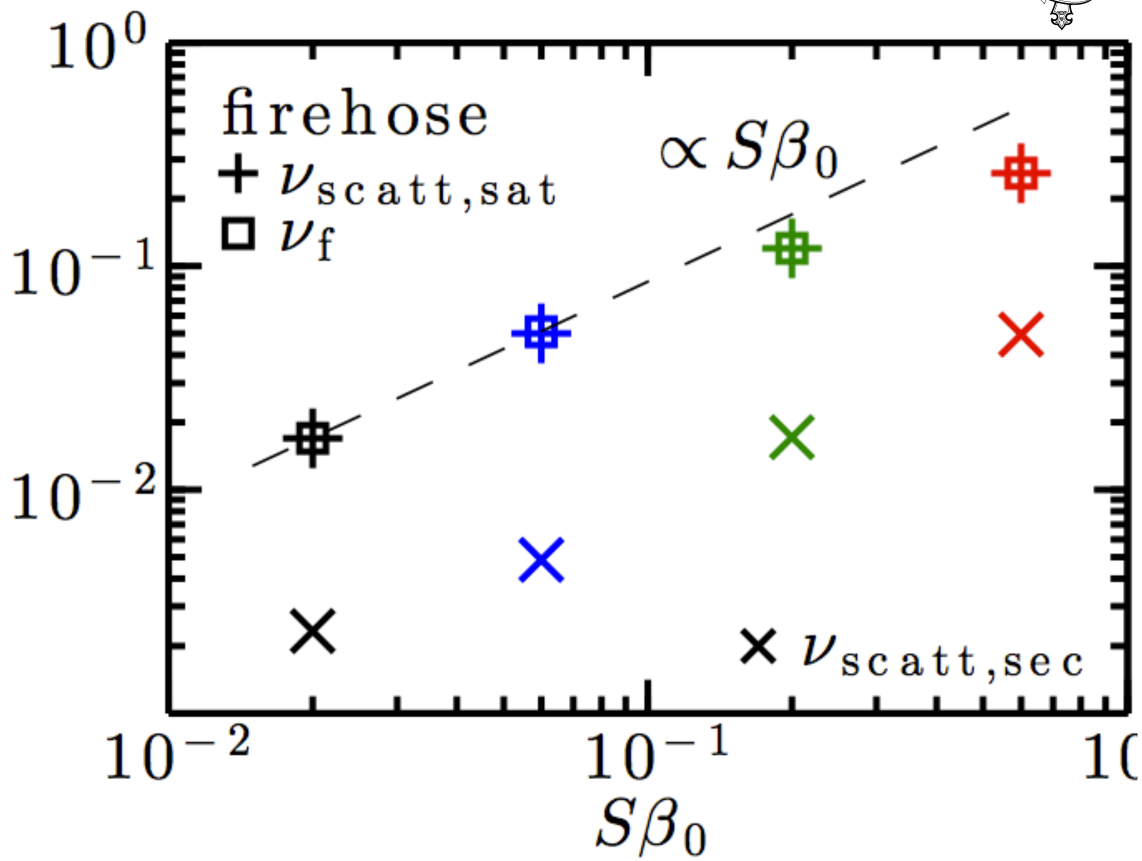
$$\frac{d\Delta}{dt} = 3\frac{d\ln|\langle \mathbf{B}\rangle|}{dt} - \nu_f \Delta$$

$$= 0$$

$$\nu_f = \frac{3}{\Delta}\frac{d\ln|\langle \mathbf{B}\rangle|}{dt}$$

$$= -\frac{3\beta}{2}\frac{d\ln|\langle \mathbf{B}\rangle|}{dt}$$

$$\sim S\beta$$



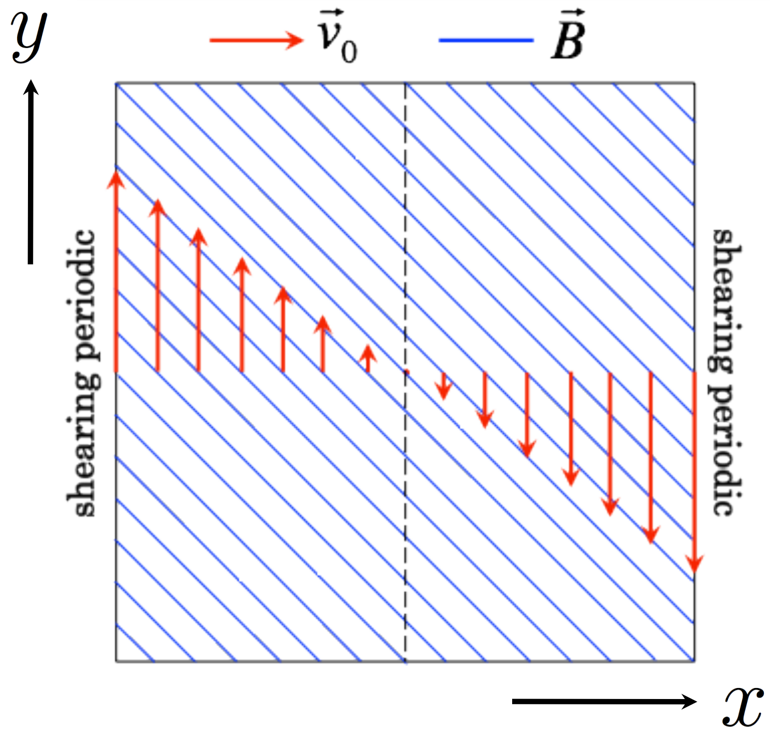
□ effective collisionality required to maintain marginal stability

\+ measured scattering rate during the saturated phase

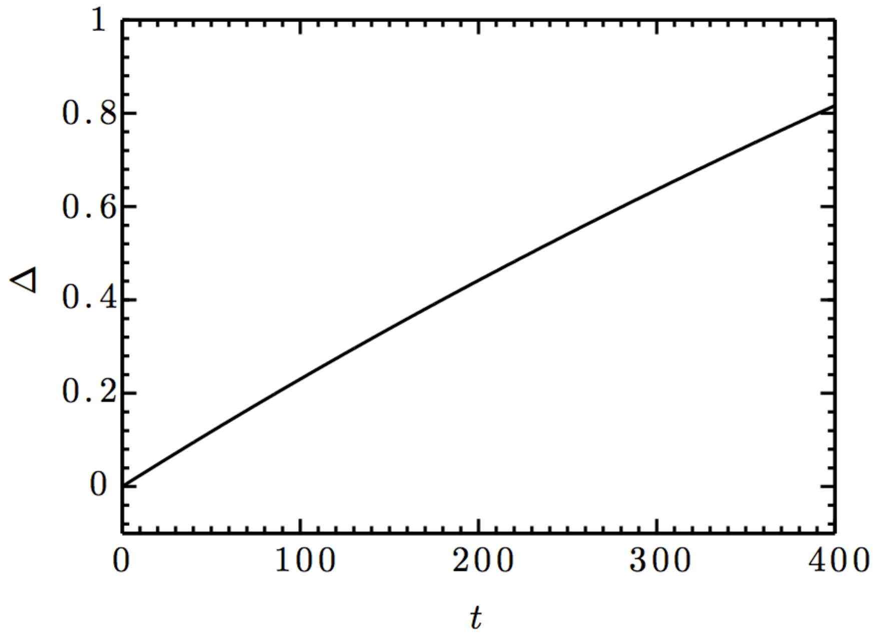
× measured scattering rate during the secular phase

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Mirror Instability (M. Kunz)



$$\frac{\mathrm{d}B}{\mathrm{d}t} > 0 \quad \Rightarrow \quad \Delta = \frac{p_\perp - p_\parallel}{p} > 0$$



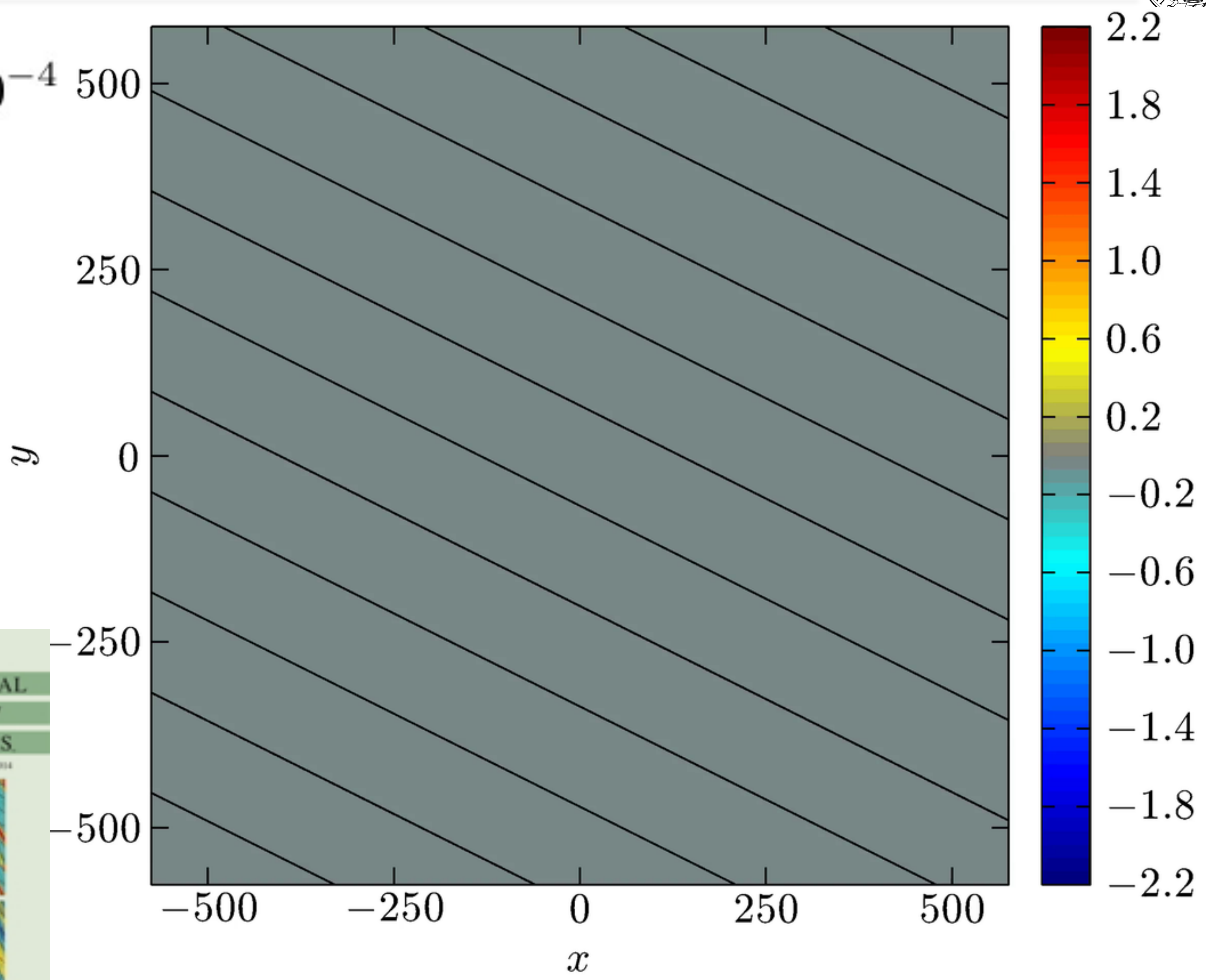
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

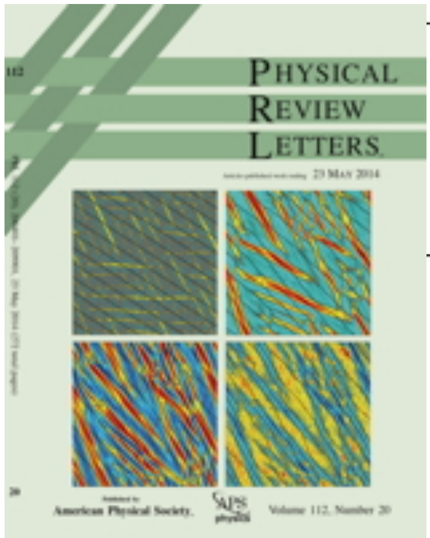
Riquelme, Quataert & Verscharen, arXiv:1402.0014 (2014)

Mirror Instability (M. Kunz)

$$\frac{S}{\Omega_i} = 3 \times 10^{-4}$$

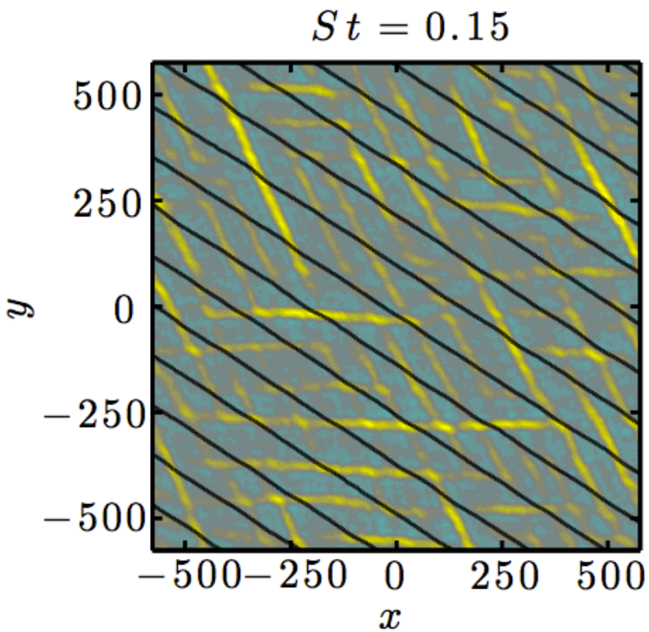
$$\beta_i = 200$$

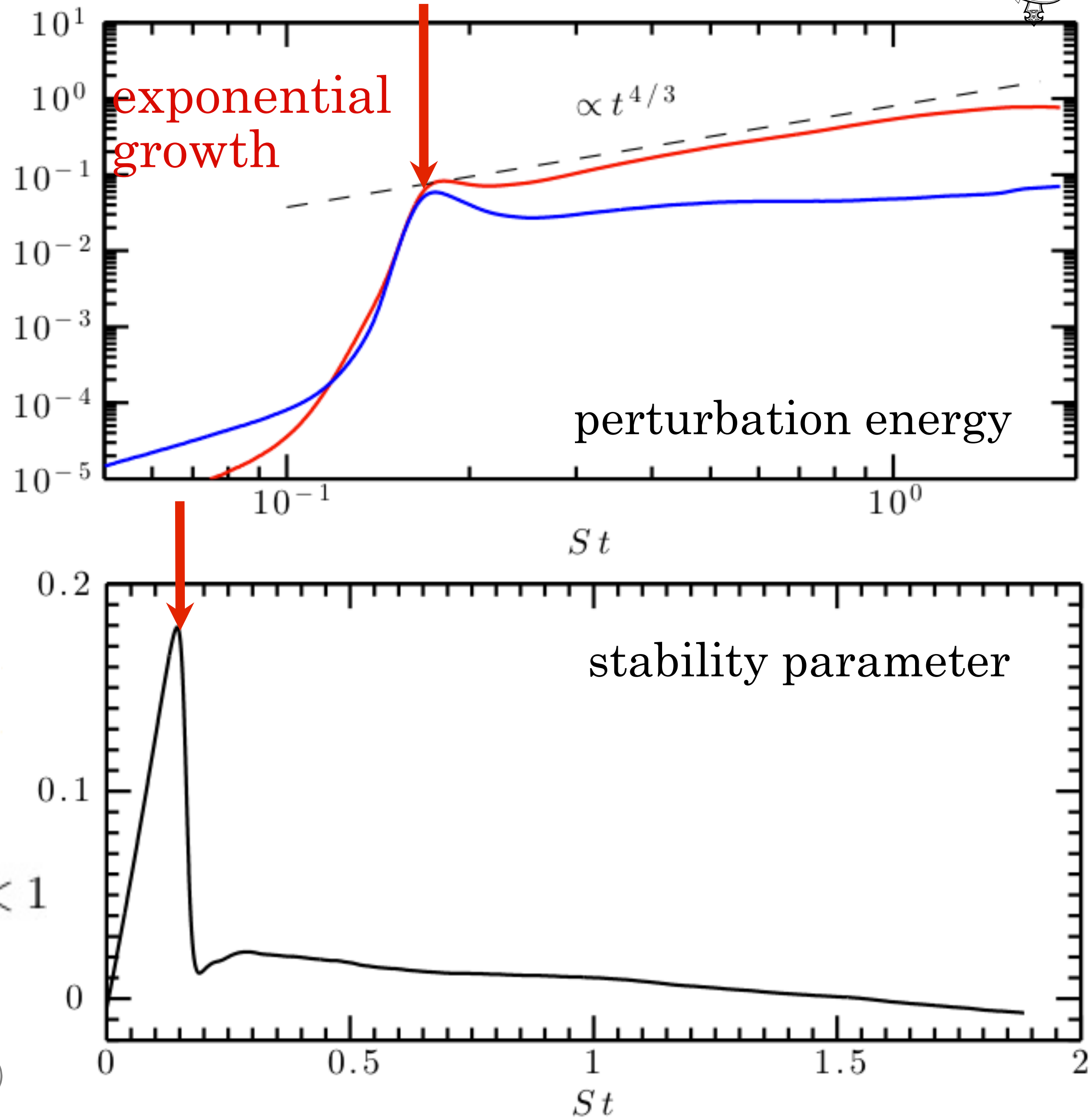




Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Mirror Instability: Linear



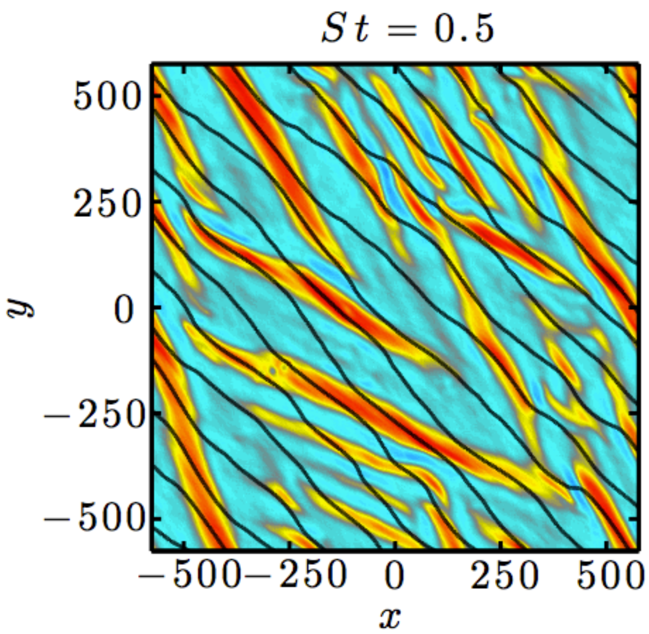


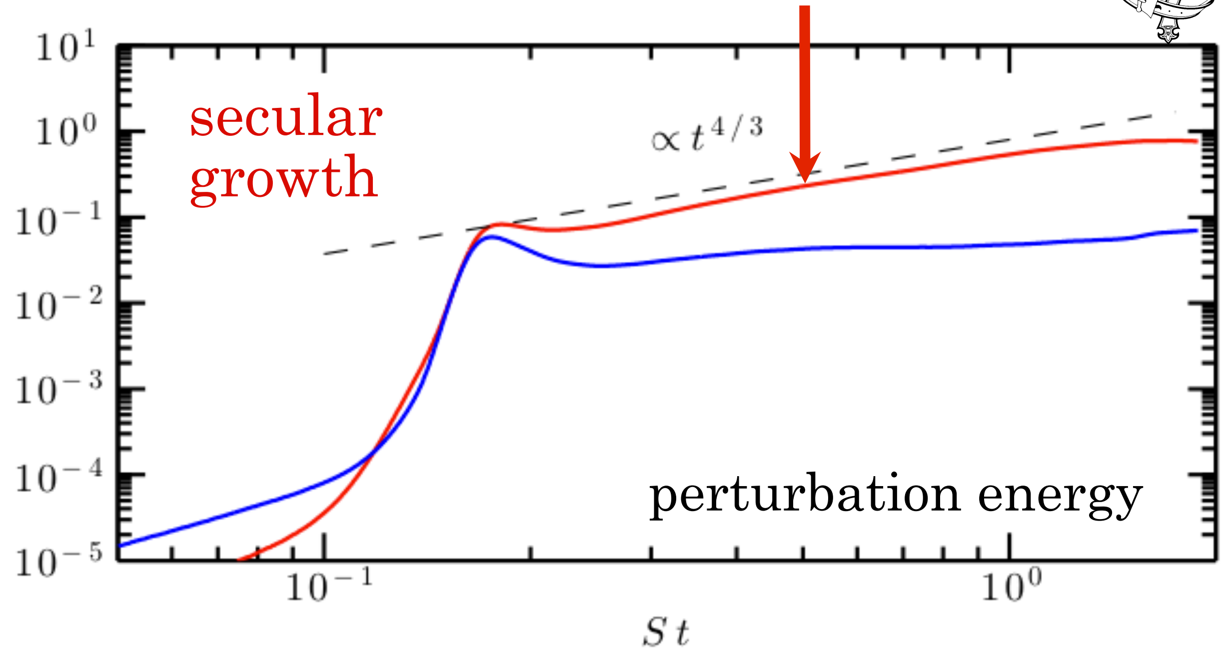
long, oblique modes

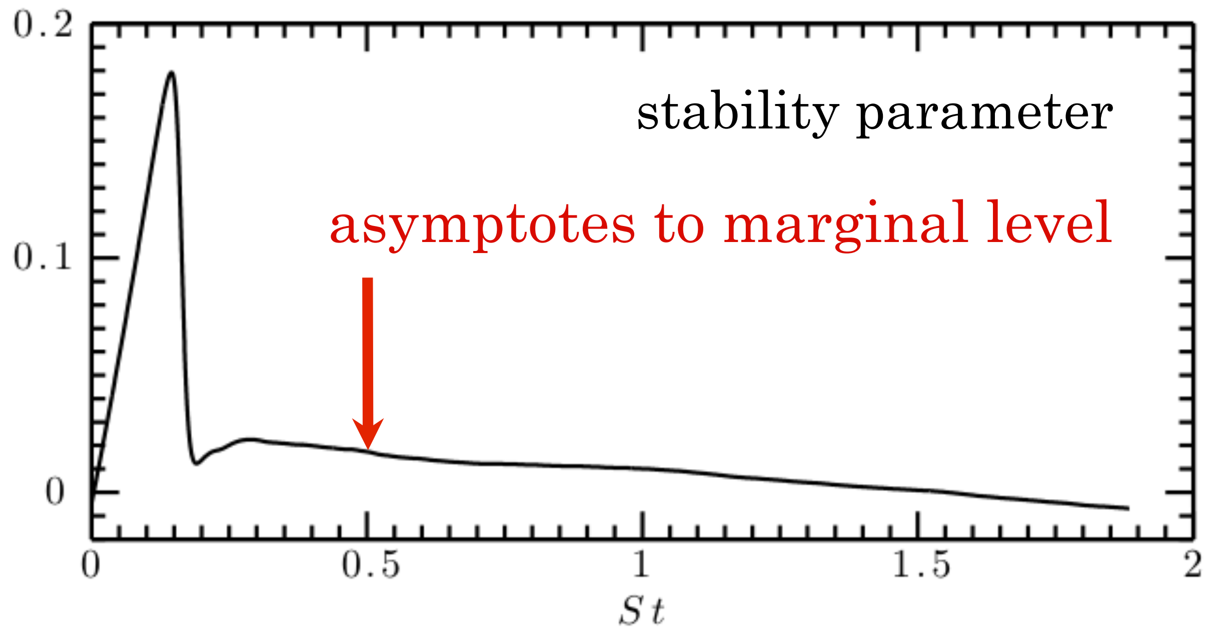
$$k_\parallel \rho_i \sim \left(\Delta - \frac{1}{\beta}\right)^{1/2} k_\perp \rho_i \ll 1$$

Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

Mirror Instability: Secular







Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

Mirror Instability: Secular

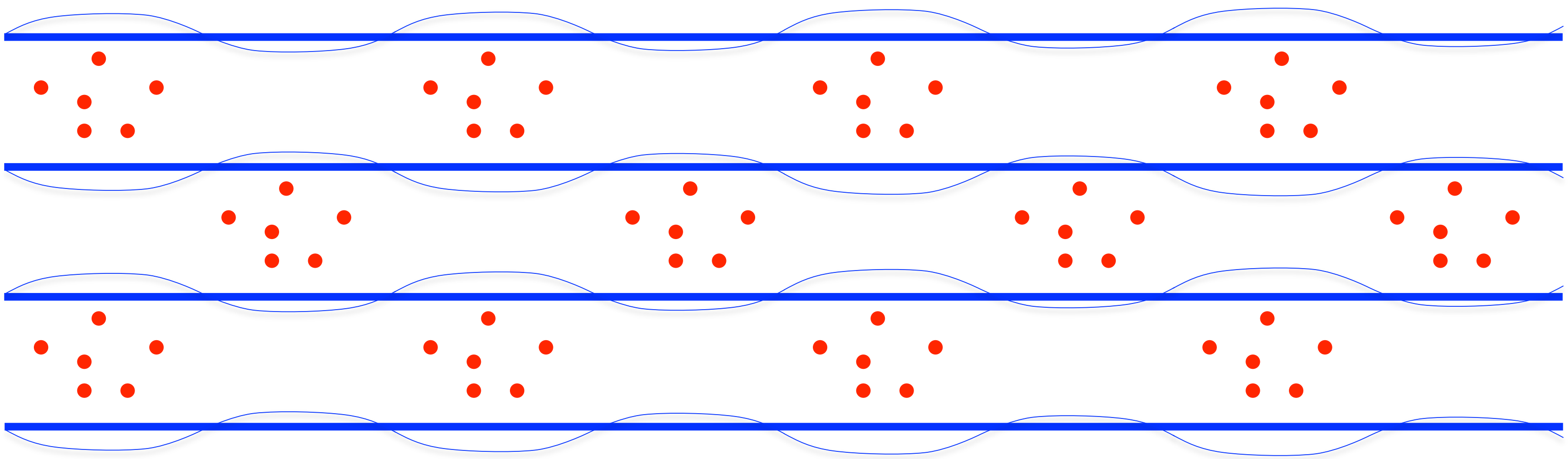
$$\overline{B} = B_0 + \overline{\delta B_\|}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \overline{\frac{\mathrm{d}\ln B}{\mathrm{d}t}} = 3\int^t \mathrm{d}t' \left(\frac{\mathrm{d}\ln B_0}{\mathrm{d}t} + \frac{\mathrm{d}}{\mathrm{d}t}\frac{\overline{\delta B_\|}}{B_0} \right) \to \frac{1}{\beta}$$

pressure anisotropy driven by shear

pressure anisotropy from mirror-trapped particles in holes (fraction $\sim |\delta B_\| / B_0|^{1/2}$)

marginal stability



Rincon, AAS & Cowley, *MNRAS* **447**, L45 (2015) [arXiv:1407.4707]

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Mirror Instability: Secular

$$\overline{B} = B_0 + \overline{\delta B_{\|}}$$

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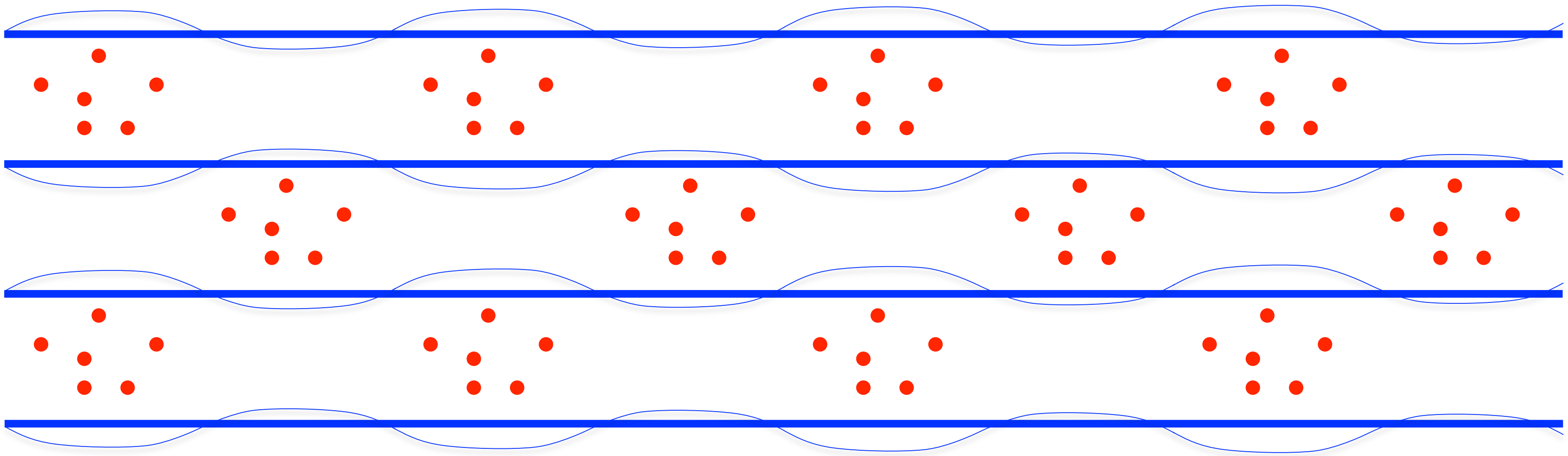

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Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

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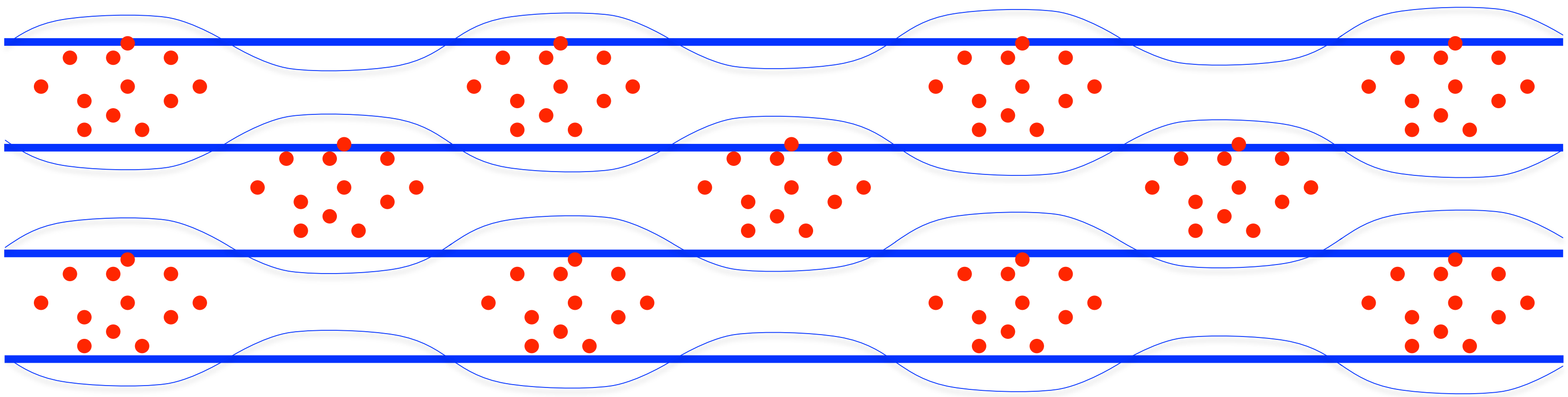

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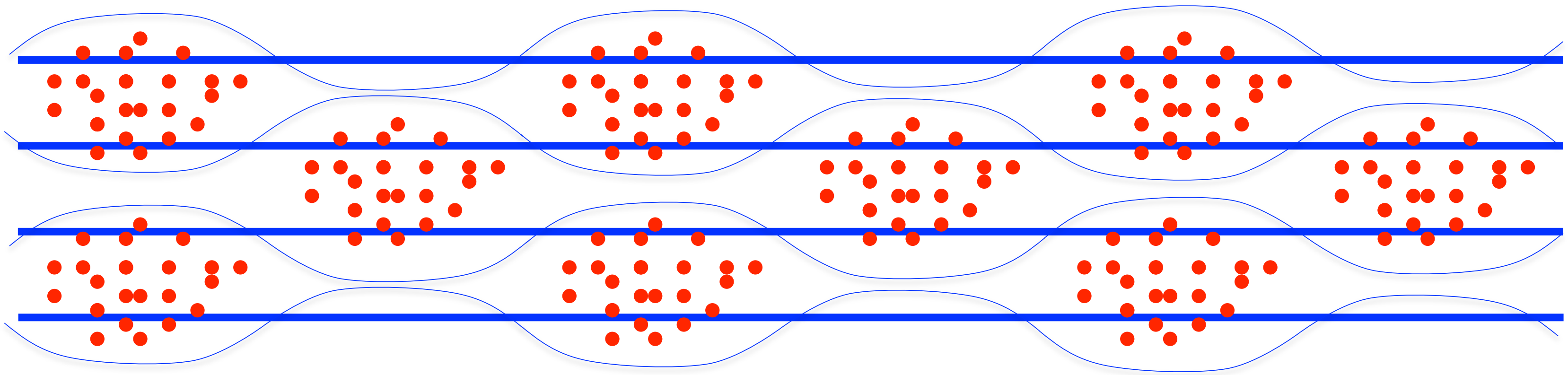
pressure anisotropy from mirror-trapped particles in holes (fraction $\sim |\delta B_\parallel / B_0|^{1/2}$)

marginal stability



Rincon, AAS & Cowley, *MNRAS* **447**, L45 (2015) [arXiv:1407.4707]
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

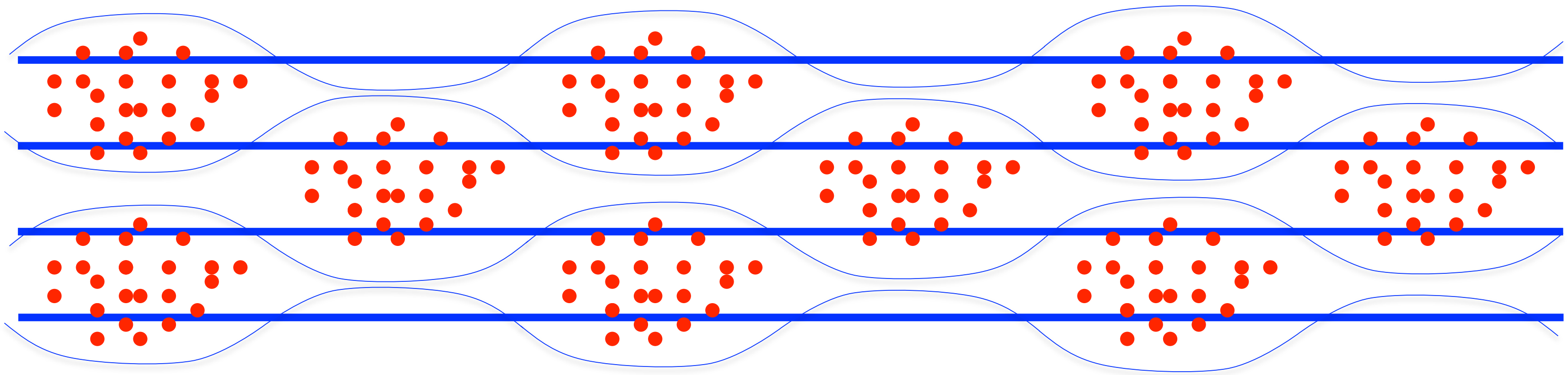
Mirror Instability: Secular

$$\overline{B} = B_0 + \overline{\delta B_\parallel}$$

$$\Delta = 3\int^t \mathrm{d}t' \, \overline{\frac{\mathrm{d}\ln B}{\mathrm{d}t}} \sim 3\int^t \mathrm{d}t' \left(\frac{\mathrm{d}\ln B_0}{\mathrm{d}t} - \frac{\mathrm{d}}{\mathrm{d}t}\overline{\left|\frac{\delta B_\parallel}{B_0}\right|^{3/2}} \right) \to \frac{1}{\beta}$$

$$\overline{\left|\frac{\delta B_\parallel}{B_0}\right|^{3/2}} = S\int^t \mathrm{d}t' \, \hat{b}_x(t')\hat{b}_y(t') - \frac{1}{\beta} \quad \Rightarrow \quad \frac{\delta B_\parallel^2}{B_0^2} \sim (St)^{4/3}$$

secular growth



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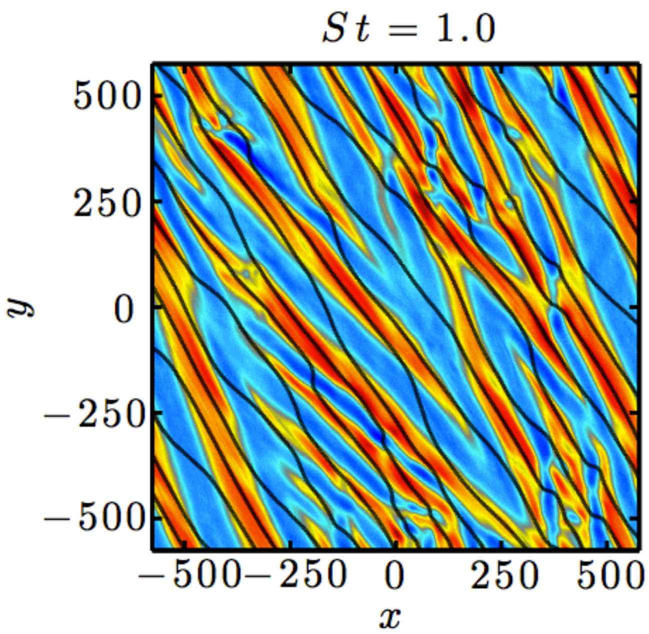
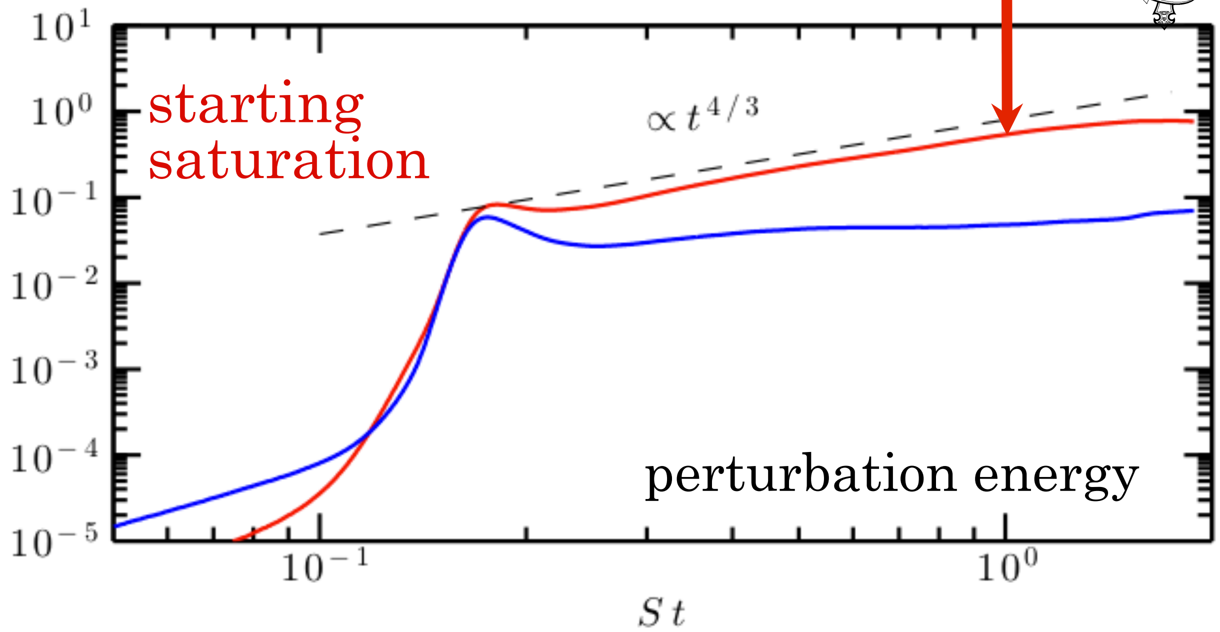
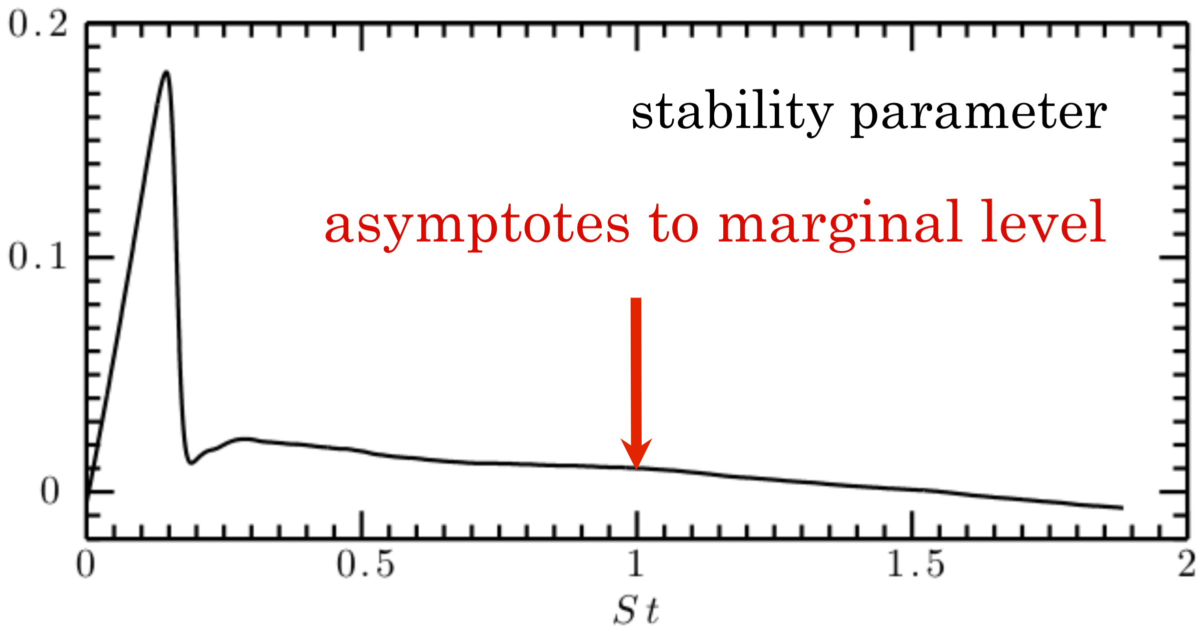
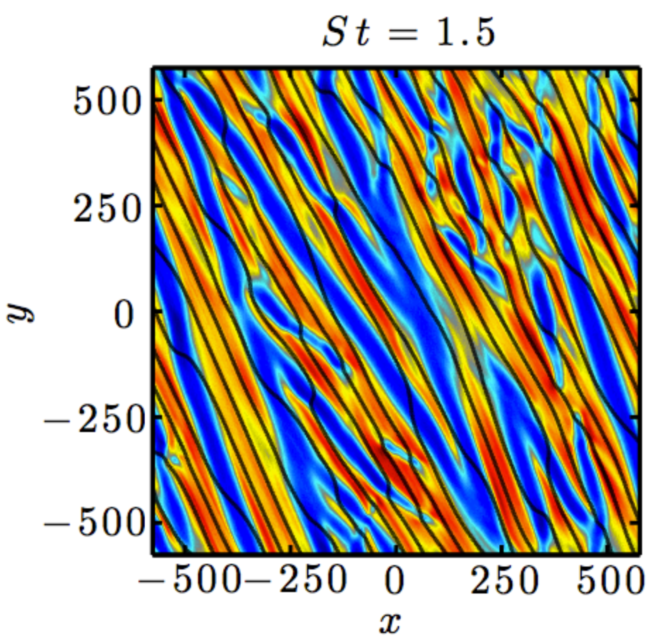


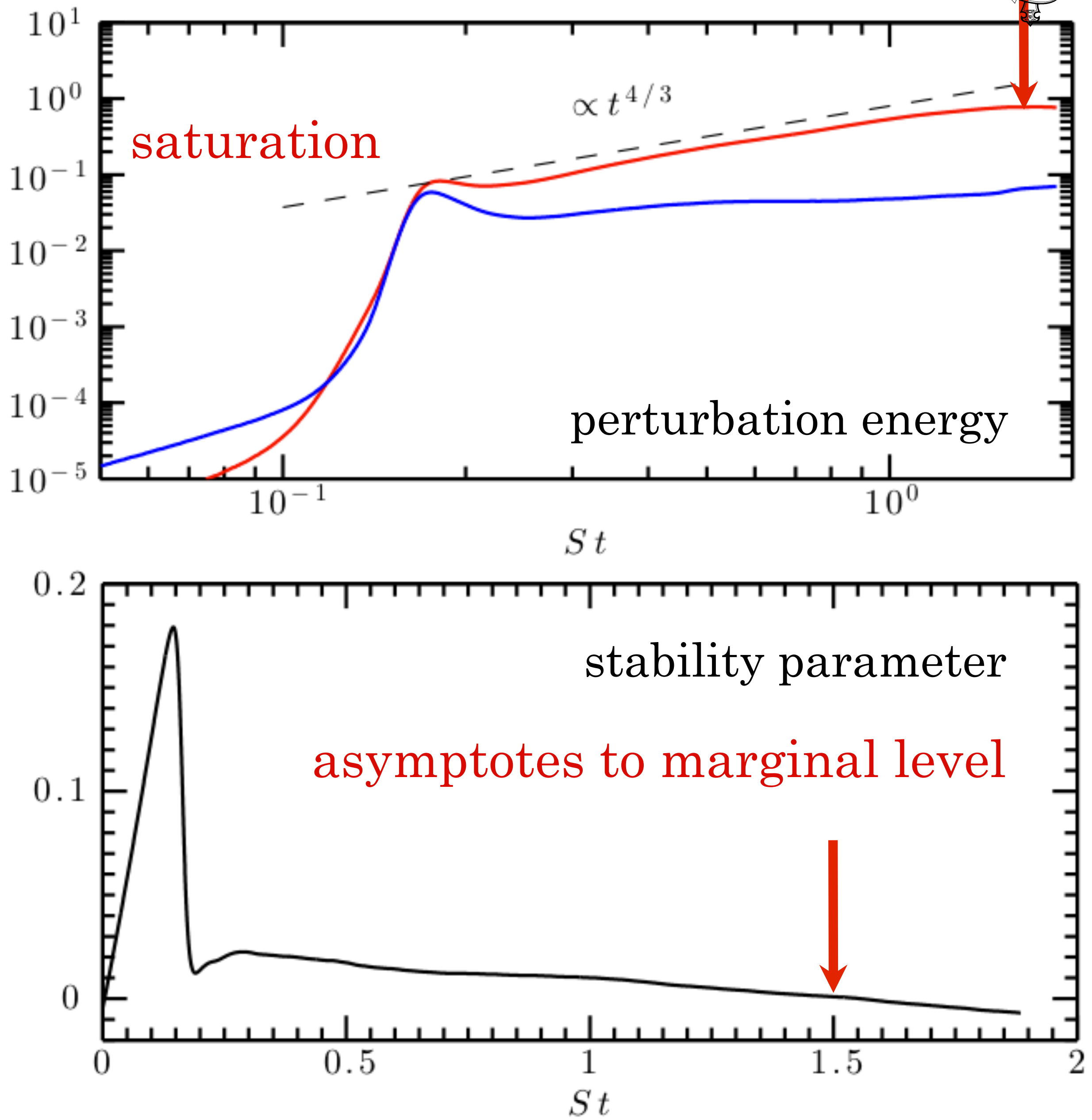




Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

Mirror Instability: Saturated



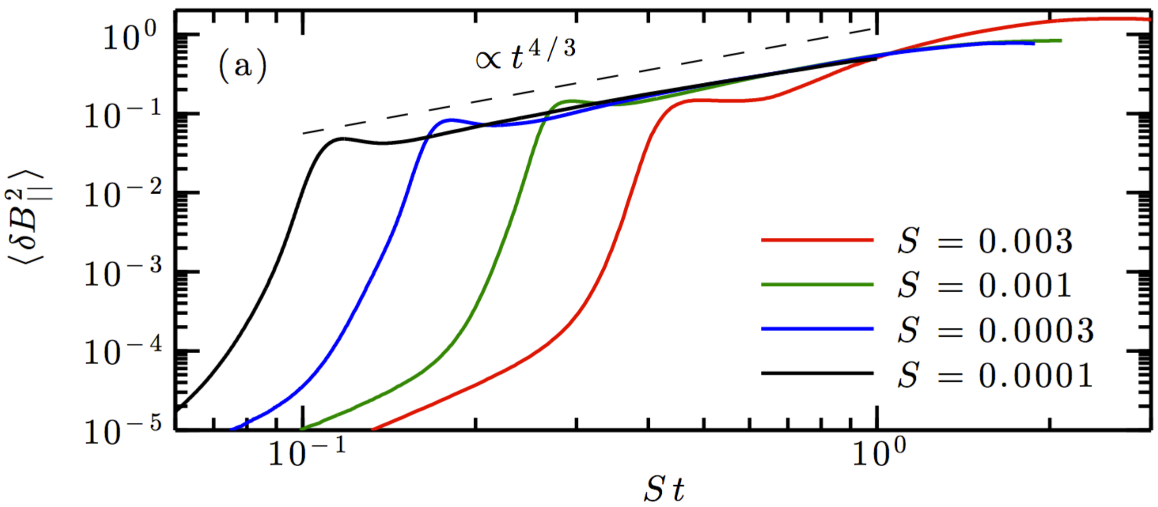


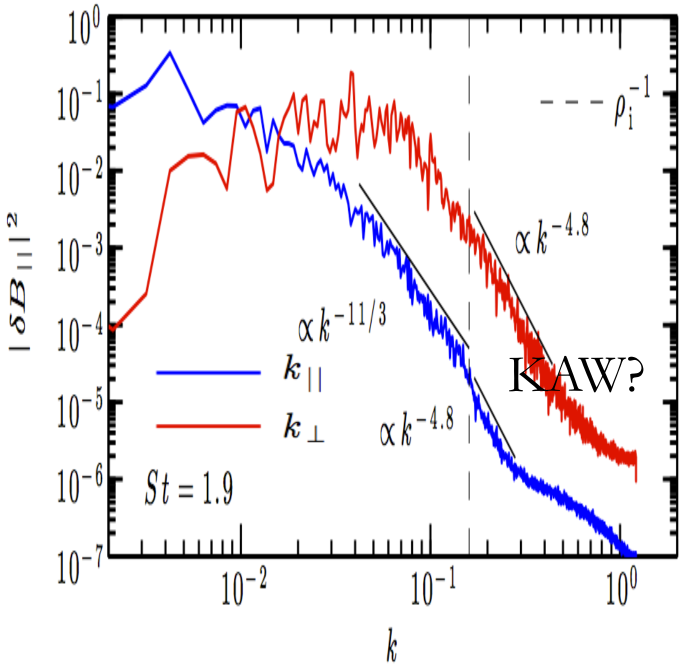
Kunz et al., *PRL* **112**, 205003 (2014)
[arXiv:1402.0010]

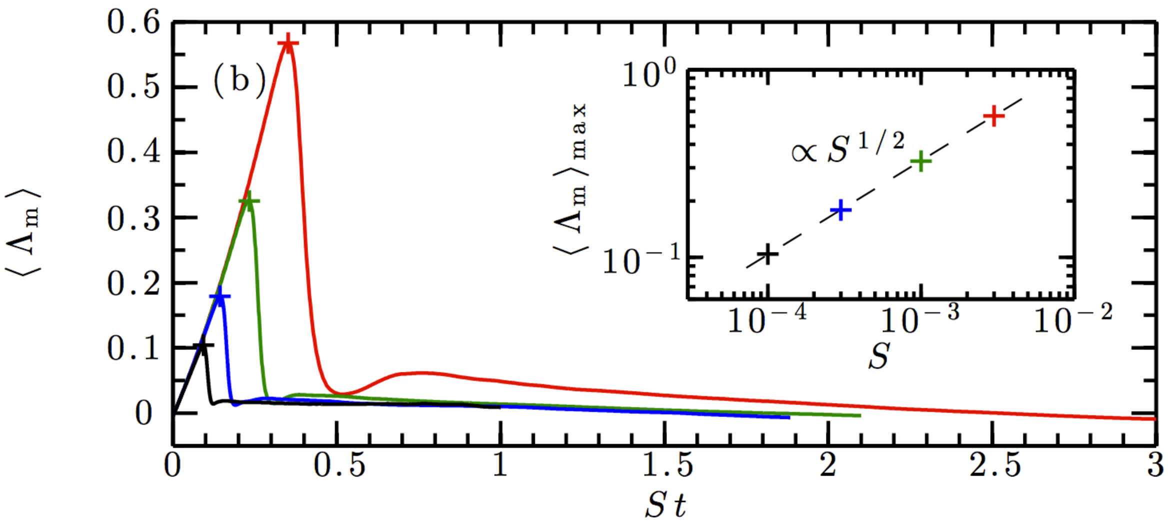
Mirror Saturates at Order-Unity Amplitudes

$$\frac{\langle \delta \mathbf{B}_{\|}^2 \rangle}{B_0^2} \sim 1$$

order-unity-amplitude
(independent of S)
long-parallel-scale
mirror turbulence

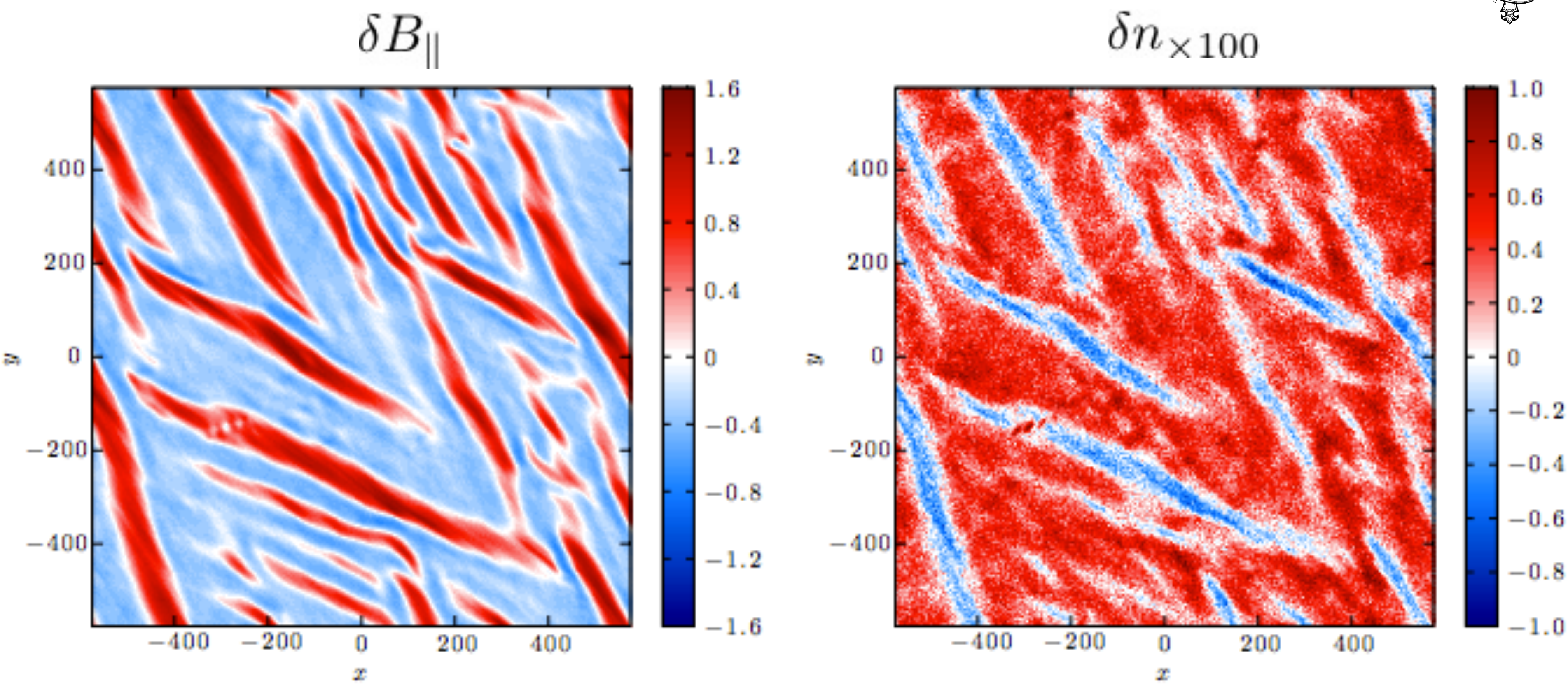






Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

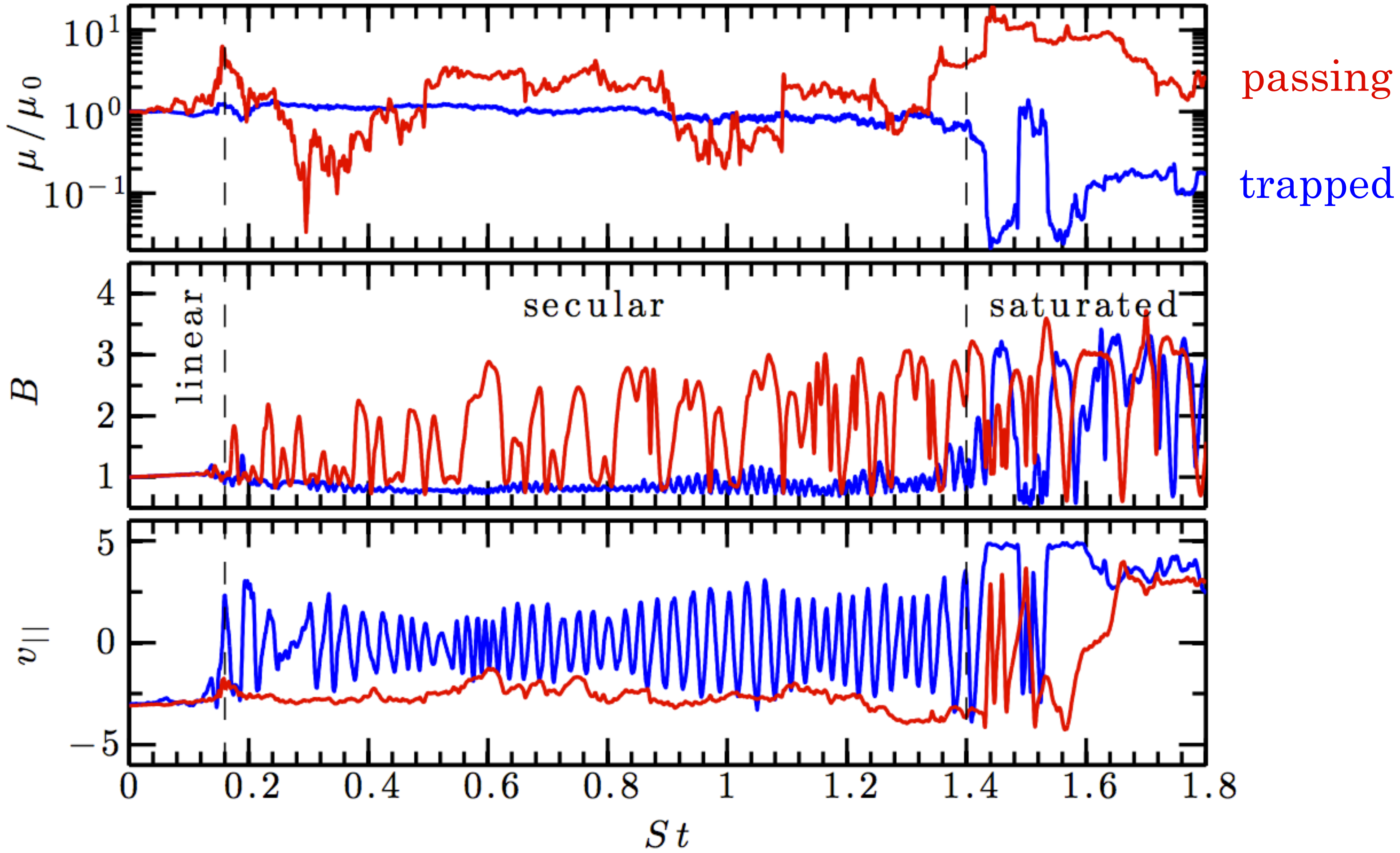
Mirror Instability: Trapped Particles



pressure anisotropy is regulated by trapped particles in magnetic mirrors, where field strength stays constant on average…

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Secular Mirror Doesn't Scatter Particles



pressure anisotropy is regulated by trapped particles in magnetic mirrors, where field strength stays constant on average…

no particle scattering until (late) saturation (off mirror edges)

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Secular Mirror Doesn't Scatter Particles

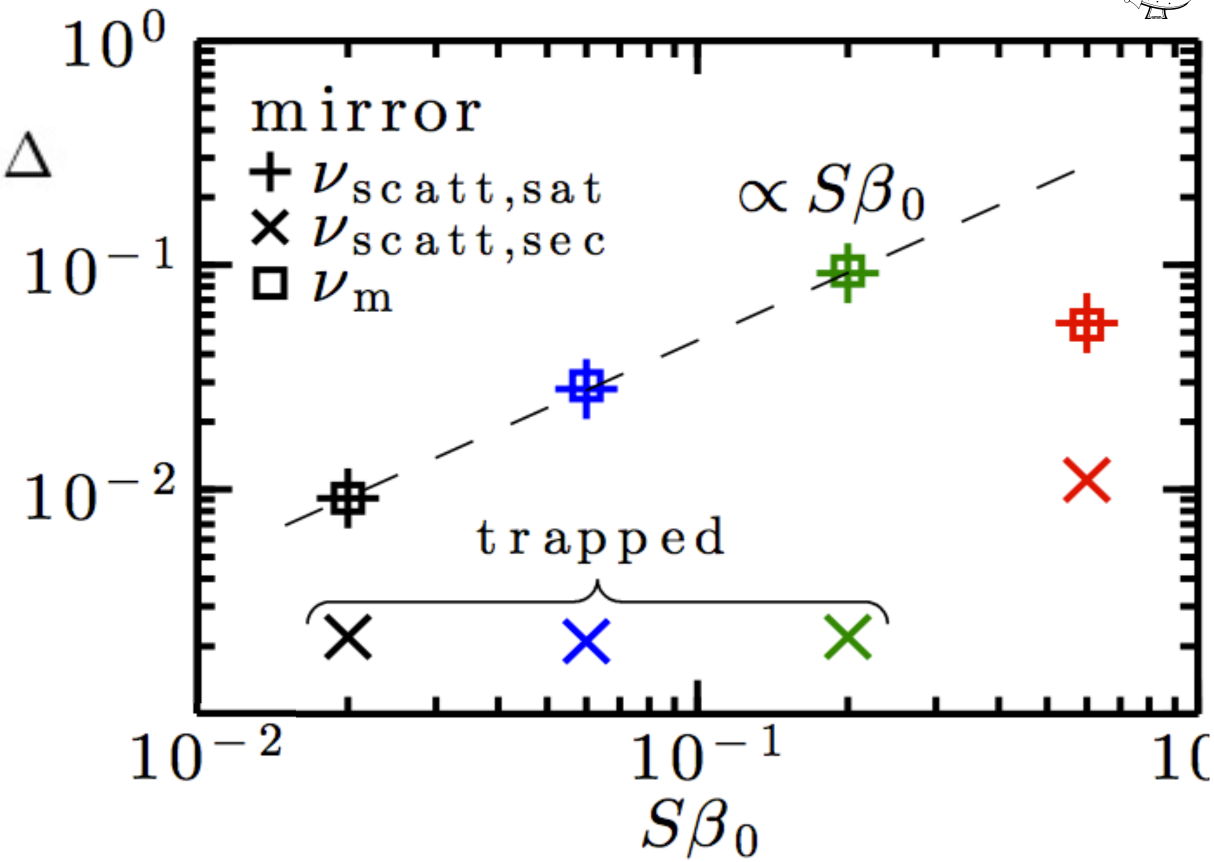
$$\frac{d\Delta}{dt} = 3\frac{d\ln|\langle \mathbf{B}\rangle|}{dt} - \nu_f \Delta$$

$$= 0$$

$$\nu_f = \frac{3}{\Delta}\frac{d\ln|\langle \mathbf{B}\rangle|}{dt}$$

$$= 3\beta \frac{d\ln|\langle \mathbf{B}\rangle|}{dt}$$

$$\sim S\beta$$



□ effective collisionality required to maintain marginal stability

+ measured scattering rate during the saturated phase

× measured scattering rate during the secular phase

Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

Conclusions So Far

- *Very different scenarios for plasma dynamo depending on whether nonlinear firehose and mirror fluctuations regulate pressure anisotropy by scattering particles or by adjusting rate of change of the magnetic field:*
  - **No scattering** ➔explosive growth, but long time to get going

    $t \sim \beta_0 / 2\nu$ scales with collision time and initial field
  - **Efficient scattering** ➔secular growth, but very fast

    $t \sim l/u$ one large-scale turnover time
- Driven **firehose** saturates at low amplitudes, scatters particles
- Driven **mirror** grows to $\delta B/B \sim 1$ without doing much scattering (marginal state achieved via trapped population in mirrors)
- [Both instabilities have a sub-Larmor tail, which appears to be KAW turbulence with the usual spectrum]
- Plasma Dynamo: the race is on

Mogavero & AAS, *MNRAS* **440**, 3226 (2014) [arXiv:1312.3672]
Kunz, AAS & Stone, *PRL* **112**, 205003 (2014) [arXiv:1402.0010]

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A 19th Century Programme…

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When dining, I had often observed that some particular dishes retained their Heat much longer than others; and that apple-pies, and apples and almonds mixed, - (a dish in great repute in England) - remained hot a surprising length of time. Much struck with this extraordinary quality of retaining Heat, which apples appear to possess, it frequently recurred to my recollection; and I never burnt my mouth with them, or saw others meet with the same misfortune, without endeavouring, but in vain, to find out some way of accounting, in a satisfactory manner, for this surprising matter.

Count Rumford, 1799

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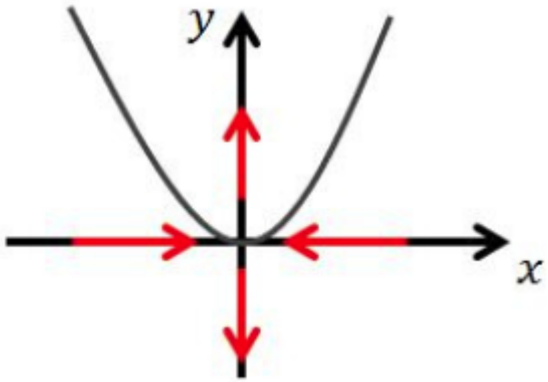
Effects of Magnetic Field

Initially parabolic magnetic field line subject to Braginskii viscosity (by Scott Melville)



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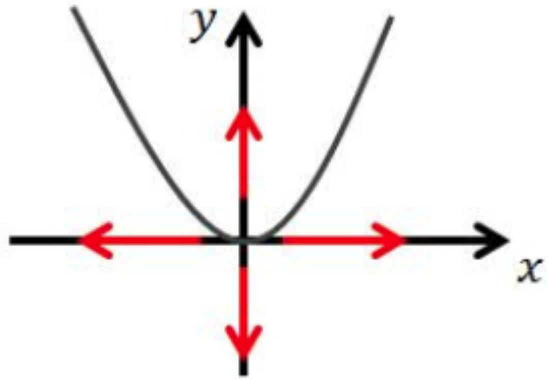

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