Further Lifshitz Reconstruction

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Is Behavior Expected?

Spectral function features in Lifshitz

- At large ω,

$$\text{Im}(\mathcal{G}) \sim \omega^{2\nu}$$

- For small ω,

$$\text{Im}(\mathcal{G}) \neq 0$$

  but is exponentially supressed.
- At $\omega = 0$ there is an essential singularity.

Are these features universal? Are they expected from field theory?

Universality and Field Theory

'Higher Curvature' terms

- Scalar with higher curvature action in WKB approx
- Lifshitz scaling fixes large ω behavior, but not conformal; $\mathcal{G}$ not entirely universal
- higher curvature terms in scalar action provide one way to generate new behavior
- nonzero but exponentially suppressed region in Im$(\mathcal{G})$ *is* universal
- essential singularity can be moved or removed

Field Theory model (K. Sun)

- Quadratic band crossing model, appropriate for e.g. bilayer graphene
- Lifshitz scaling fixes large ω behavior
- Perturbative calculation gives Im$(\mathcal{G}) = 0$ for small ω
- nonperturbative resumming gives nonzero but exponentially suppressed region!
- no essential singularity, but expansion untrustworthy near $\omega = 0$.

Upcoming: Entanglement Reconstruction

What can a subregion of the bulk tell us?

1. ‘Hole’-ography/ Differential entropy
2. Causal Wedge
3. Entanglement Wedge

All of these methods will have some limitations when applied to Lifshitz spacetimes.

'Hole'-ography/Differential Entropy Reconstruction

We can successfully reconstruct lengths of curves of constant radius $r_\star$ satisfying

$$r_\star^{2z-2} \geq \left(\frac{T'(\lambda)}{\xi}\right)^2 z.$$

where $T(\lambda)$ is the time dependence of the curve, and ξ is the length of the (now periodic) x direction.

However such curves are spacelike for smaller r. So what about curves between

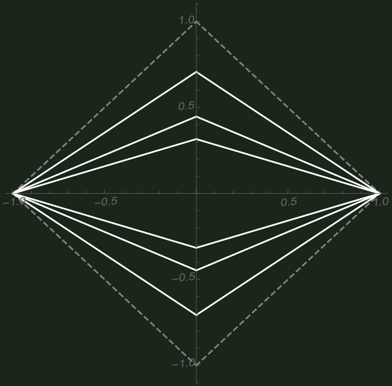
$$\left(\frac{T'(\lambda)}{\xi}\right)^2 z > r_\star^{2z-2} > \left(\frac{T'(\lambda)}{\xi}\right)^2 ?$$

Limitations of Differential Entropy

These curves are not reconstructible because their tangent spacelike geodesics are not boundary anchored!

Causal Wedge degeneration

The causal wedge is one proposal for the bulk region reproducable from a boundary subregion.



The causal wedge is defined as the intersection of the bulk past and bulk future of the boundary causal domain.

As long as we use a cutoff $r = \epsilon$ this object can be defined in Lifshitz.

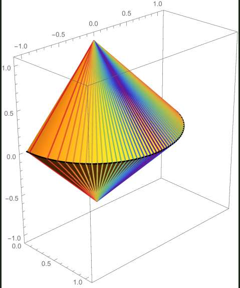
As $\epsilon \to 0$ the wedge flattens (and its depth in the bulk vanishes).

Degeneration of the Causal Wedge

The causal wedge both flattens and shrinks to zero depth upon removal of the cutoff ϵ.

Entanglement Wedge Changes

Entanglement wedges are built from light sheets emanating orthogonally from the spacelike geodesic towards the boundary. But light rays in Lifshitz turn around!



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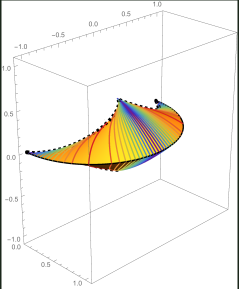
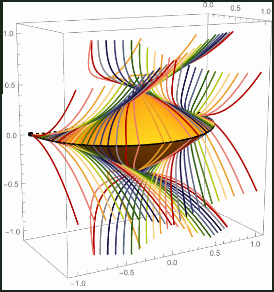


Lifshitz $z = 2$



Lifshitz past caustics.

Entanglement light sheets alone do not close!

Light sheets from a spacelike geodesic in Lifshitz do not reach the boundary, even continuing past caustics!

Completing the Entanglement Wedge

Entanglement Reconstruction in Lifshitz is Different!

What can a subregion of the bulk tell us?

1. ‘Hole’-ography/ Differential entropy: only works for some curves
2. Causal Wedge: only works at a nonzero cutoff
3. Entanglement Wedge: only closes including boundary light sheets.

All of these methods will have some limitations when applied to Lifshitz spacetimes.

Note Entanglement Wedge closure was important for the covariant holographic entropy proposal so its failure here is particularly concerning!

Further Work

Questions

- Consider probe limit:
    1. Change spacetime in IR by high transverse momentum wiggle
    2. probe spacetime boundary by scalar profile
    3. Can the effect of the high-p wiggle be seen before the probe limit is exceeded?
- We work in Poincare-like coordinates (Lifshitz has no global coords), so we shouldn't see an entanglement shadow. Yet we otherwise have similar reconstruction difficulties to Freivogel et. al.– how can this be understood?
- How do our reconstruction difficulties relate back to ones in AdS, e.g. with black holes?
- Is there a better spacetime?

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If these applications are successful, great!

If they fail somehow, we still learn about the nature of holography.