Energy based Active-Contour Methods for Scientific Image Segmentation

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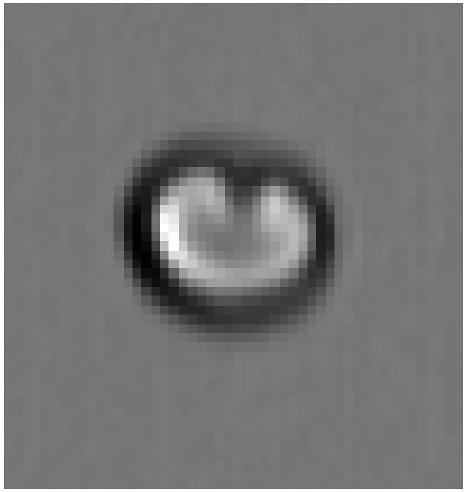
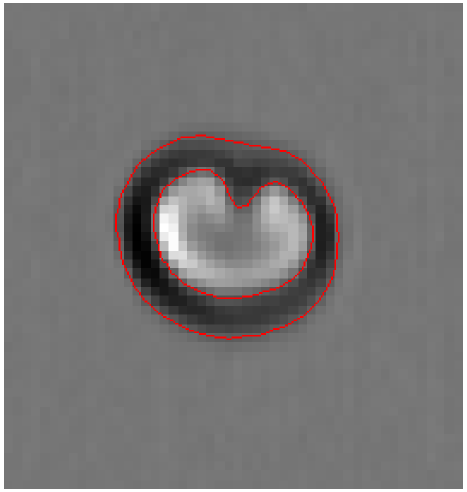
Agenda

- Image Segmentation & Applications
- Energy based Active Contour (Snakes)
  - Using Edge-Detectors
  - Limitations
  - Level-Set Functions (ϕ – formalism)
  - Edge-Free Segmentation (Chan-Vese)
- Statistical Interpretation
- Extension to Multi-phase and Vector Images

Image Segmentation

- Part of image processing workflows
- Identify objects within an image
- Separate objects from background / clutter





* Erythrocyte, Courtesy of Dr. Alexander Barbul, Medicine, TAU

Problems

- Not an easy task at all
- Many models were developed over the years

- No single method that works best in all cases
- Can make it automatically?

Applications

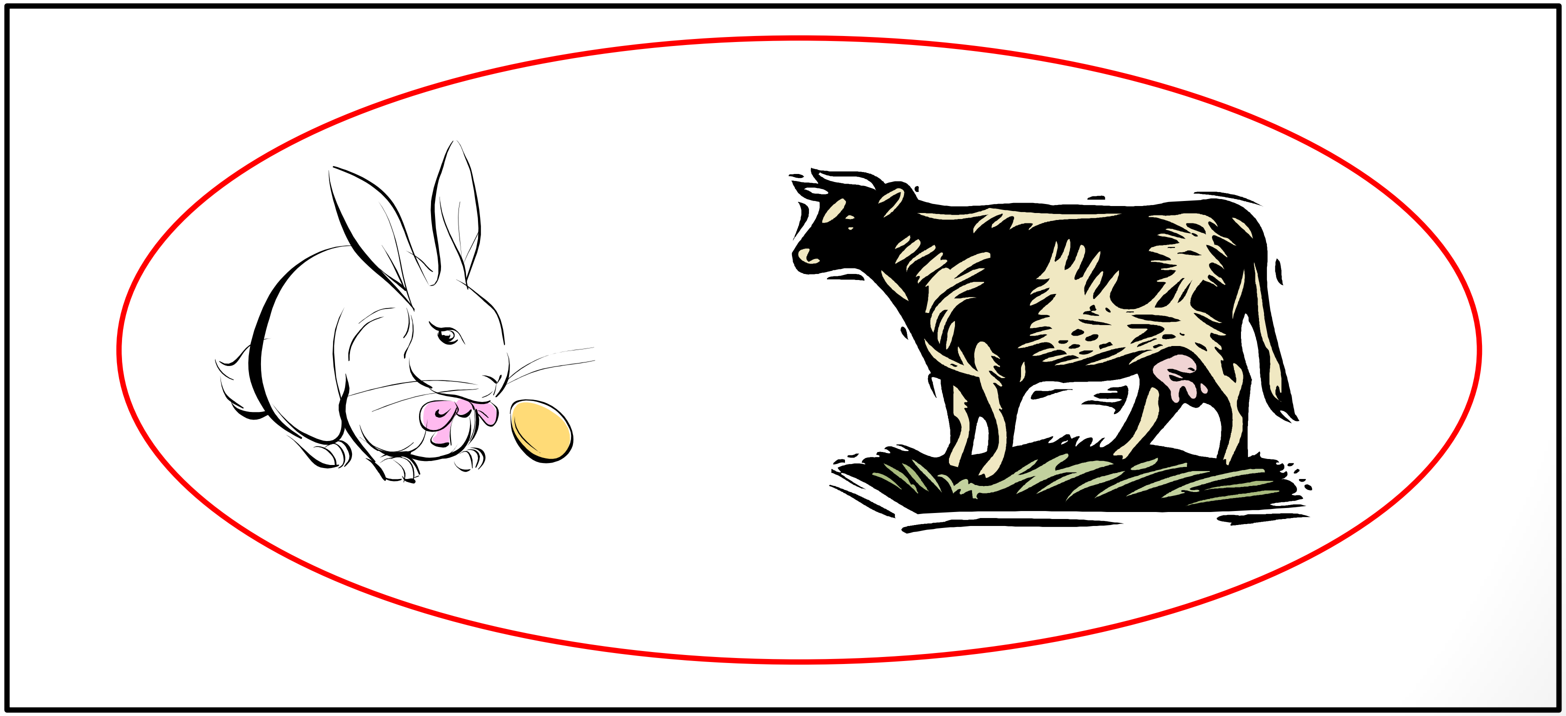
- Geometry processing
- Feature extraction – Object learning & classification
- Computer vision
- Not limited to 2D
- …

- Extract properties of observed phenomena:
  - Intensity variation or other statistics
  - Curvature or other geometrical properties

ACTIVE CONTOUR (SNAKES)

Active Contour

- One approach for segmentation
- An initial contour is placed around objects to be segmented



Active Contour (Cont.)

- Contour evolves toward objects' boundaries
- Propagation is stopped when a criteria is met
  - E.g. contour didn't change much from previous iteration

- What forces the contour to move toward objects?
- "Balloon" forces, for example:
  - Low → high gradients
  - Gray-level curvature

- Classical approaches: *Geometric/Geodesic Active Contours*

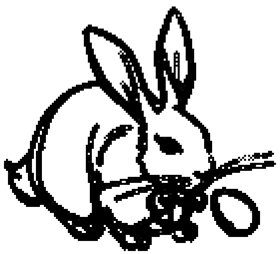
Gradient Forces

- Denote observed image as $I_0(x, y)$
  - I for **i**ntensity / gray-level

- A typical edge-detector:

$$g\left(\left|\vec{\nabla} I_0\right|\right) = \frac{1}{1 + \left|\vec{\nabla} G_\sigma * I_0\right|^p} \qquad p \geq 1$$

- Term in denominator represents a smoother image
- An example for g:




Energy Functional

- Denote $C(s)$ a parametrized curve

- The energy to minimize:

$$E(C) = \int [\alpha|C'(s)|^2 + \beta|C''(s)|^2 + \lambda \cdot g^2]ds$$

- Searching:

$$\inf_C E(C)$$

- Terms:
  1. First two are <u>internal energy</u>, control curve smoothness/curvature/rigidity
  2. Third is <u>external energy</u>, attracting curve toward objects

Minimization

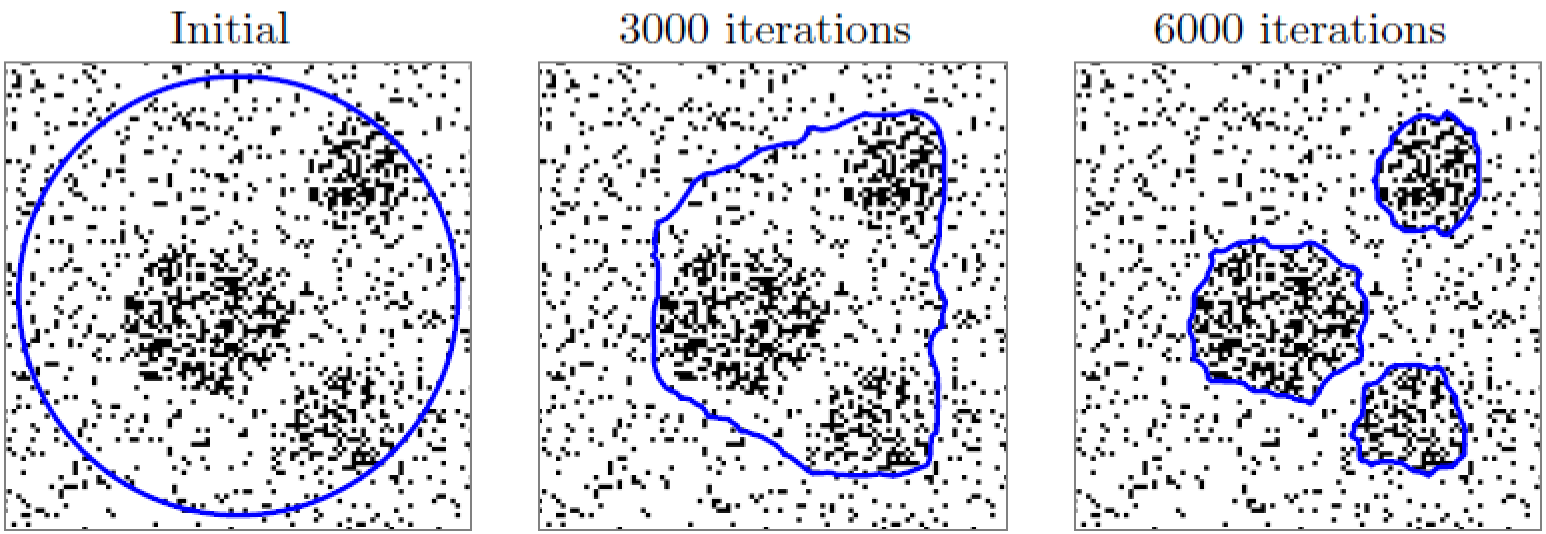
- Using variational calculus and gradient-descent
- Minimization by Euler-Lagrange leads to **curve evolution**:

$$\frac{\mathrm{d}C}{dt} = -\frac{\delta E}{\delta C} = 0$$

- t is an **artificial** "time", just used to advance simulation
- $\mathcal{L}$ is the integrand, s.t. $E = \int \mathcal{L} ds$
- And the variation of E can be transformed to $\mathcal{L}$ by the **chain-rule**

Limitation #1

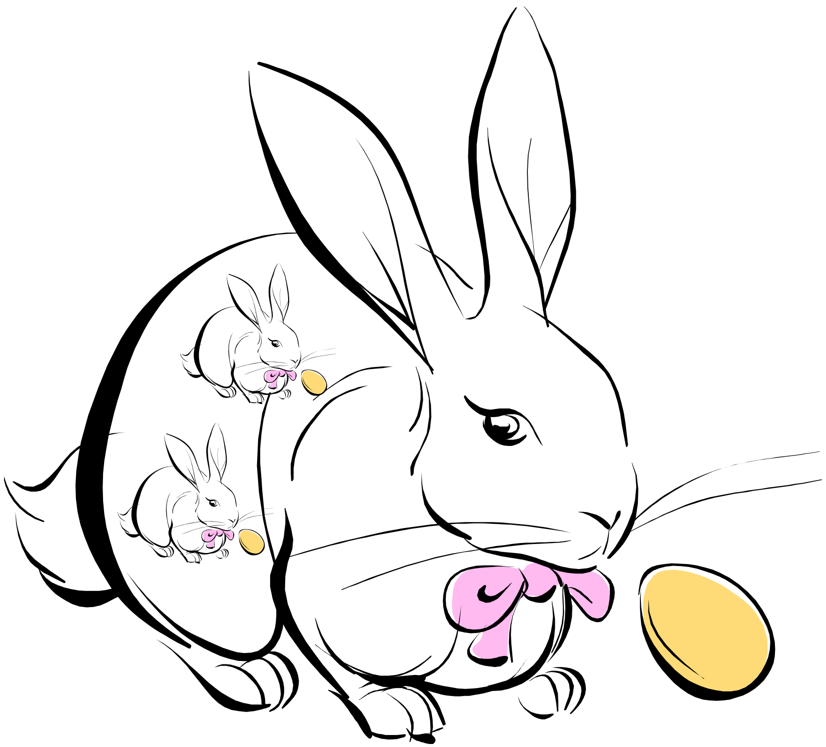
- Gradients are susceptible to noise
  - But, smoothing an image can remove important information
- Example (desired, but most gradient methods **fail**):



* Pascal Getreuer, Chan-Vese Segmentation, IPOL (2012)

Limitation #2

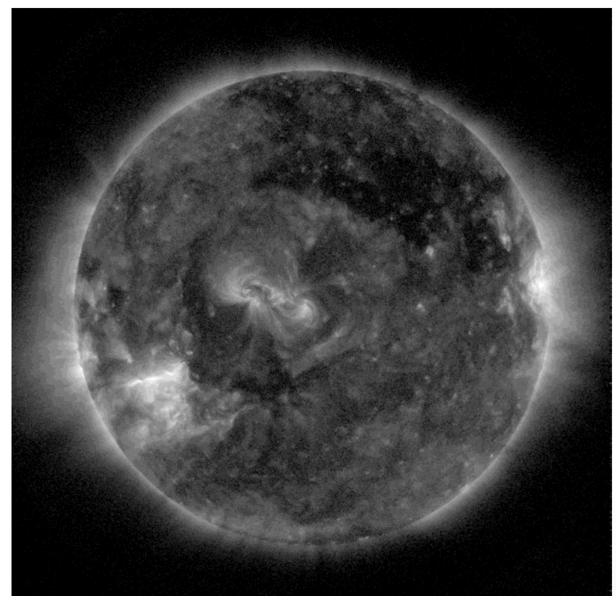
- Hard to detect interior objects
- Initial contour matters



Limitation #3

- Sometimes gradient information is not enough
  - Glow or halo
  - Many realistic scenarios

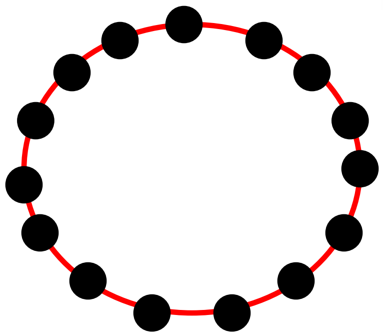
* www.dpic.org



* SDO Fe XVIII 940nm, Nov 09, 2015

Limitations #4 +

- Hard to represent contours in discrete world
- Through anchor points and interpolate?
  - Not always working



- Hard to handle topological changes
- Same for merge/split contour while evolving

- Level-set functions comes to rescue
  - Elegant contour representation
  - Automatic handling of topological changes

LEVEL-SET FUNCTIONS

ϕ - formalism

- Used by Osher & Sethian [1988]
- Solving phase/front evolution in chemical reactions

- ϕ is a level-set function
- Contour is now represented by zero level-set of ϕ
  - $C(s)$ can be obtained by $\phi = 0$

- Requirements on ϕ:
  - Piecewise-continuous
  - At least C^1

Useful Constructs

- With ϕ as level-set function

- Important constructs / terminology:
  - $C \rightarrow \phi = 0$
  - $inside(C) \rightarrow \phi > 0$
  - $outside(C) \rightarrow \phi < 0$

  - H will be the familiar Heaviside function
  - δ is the familiar Dirac delta function

  - Also note (change of variables):

$$\vec{\nabla} H(\phi) = \delta(\phi)\vec{\nabla}\phi$$

Curve Evolution

- Using previous energy functional
- E is minimized as before with respect to ϕ

- In terms of ϕ (instead of C):

$$\frac{\partial \phi}{\partial t} = g\left(\left|\vec{\nabla} I_0\right|\right) \cdot \left|\vec{\nabla}\phi\right| \cdot \mathrm{div}\left(\frac{\vec{\nabla}\phi}{\left|\vec{\nabla}\phi\right|} + \lambda\right)$$

- First term in brackets is the curvature
- λ controls the external energy as before

Example

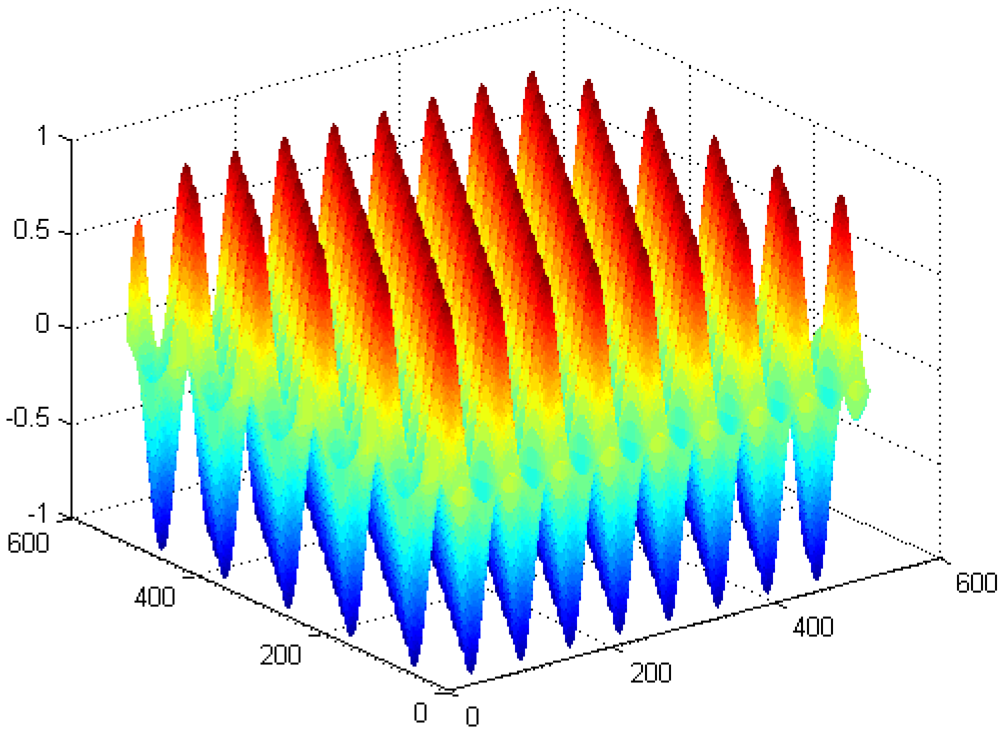
- Any function satisfying previous constraints can serve as ϕ
- A common choice is check-board or egg-pattern (C^{∞}):

$$\phi(x,y) = \sin\left(\frac{\pi \cdot x}{30}\right) \cdot \sin\left(\frac{\pi \cdot y}{40}\right)$$

- Where x, y are in pixel coordinates

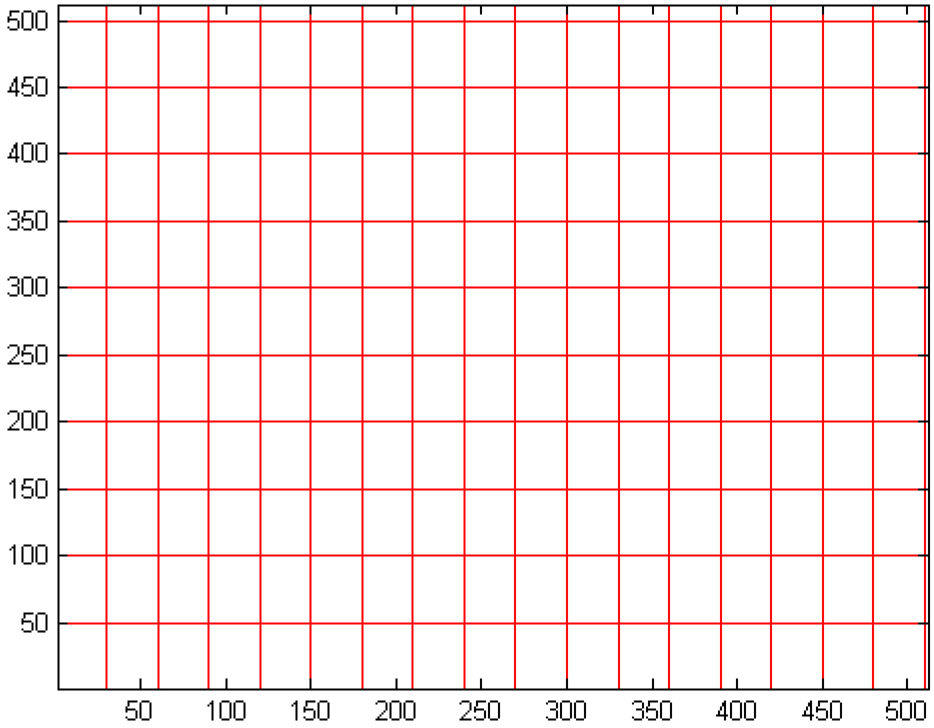
Example (Cont.)

- A 3D view of ϕ:



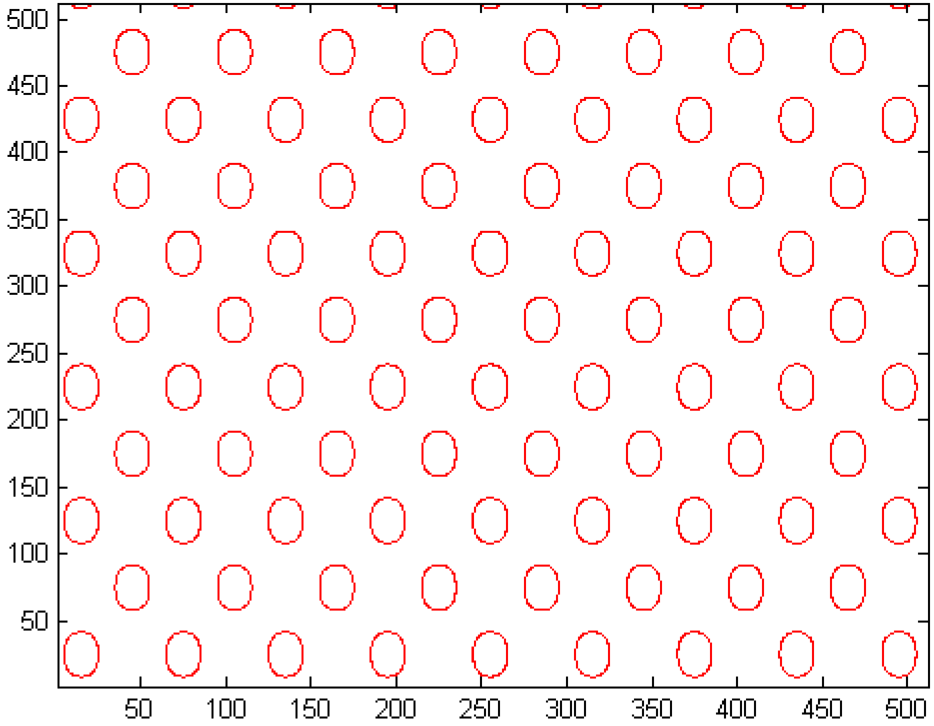
Example (Cont.)

- Zero level-set of ϕ:



Example (Cont.)

- Shifting ϕ to 0.5 level-set



Up Until Now…

- Solved contour representation issues
- Level-set functions are very easy to work with

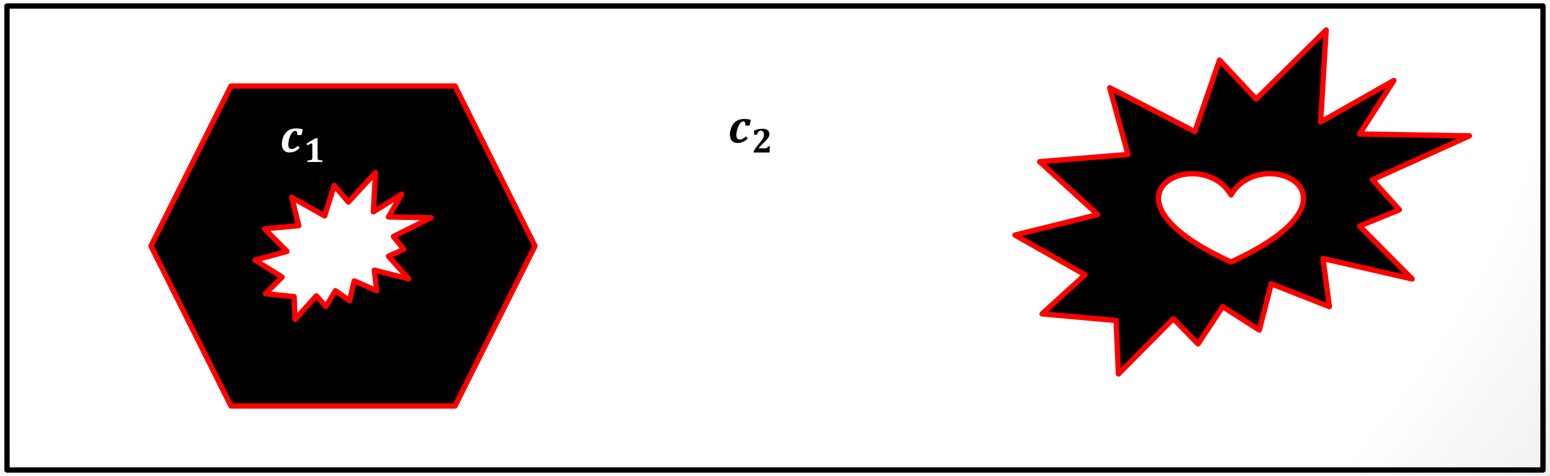
- Would like to overcome gradient limitations

Towards edge-free segmentation

CHAN-VESE SEGMENTATION

Two-Phase Model

- Chan & Vese [2001] considered segmentation as minimal partitioning problem
- Limit discussion to 2-phase model, aka:
  - c_1 - gray color of objects to detect
  - c_2 - gray color of background
- **Don't** be mislead by the simplicity



Two-Phase Model (Cont.)

- The functional:

$$E(c_1, c_2, C) = \mu \cdot [Length(C)]^p + \nu \cdot Area\big(inside(C)\big) + \lambda_1 \int_{inside(C)} |I_0 - c_1|^2 dxdy + \lambda_2 \int_{outside(C)} |I_0 - c_2|^2 dxdy$$

- $\mu, \nu, \lambda_1, \lambda_2$ are external constants, controlling term significance

Two-Phase Model (Cont.)

- Minimization problem is:

$$\inf_{c_1, c_2, C} E(c_1, c_2, C)$$

- Existence was proved by Mumford & Shah [1989]

- Clearly, minimal E is obtained only when contour surrounds relevant objects (correct gray levels)
- In any other option energy is higher -> not minimum

Chan-Vese Functional

- With level-set the functional becomes:

$$E(c_1, c_2, \phi) = \int \begin{bmatrix} \mu \cdot \delta\left|\vec{\nabla}\phi\right| + \\ \nu \cdot H + \\ \lambda_1 \cdot |I_0 - c_1|^2 \cdot H + \\ \lambda_2 \cdot |I_0 - c_2|^2 \cdot (1 - H) \end{bmatrix} dxdy$$

- Can analytically minimize E with respect to c_1, c_2
- They simply denote the average intensity levels in/outside C
- Same as the interpretation we gave before

Curve Evolution Equation

- The gradient-descent equation for ϕ:

$$\frac{d\phi}{dt} = -\frac{\delta E}{\delta \phi}$$

- Finally:

$$\frac{\partial \phi}{\partial t} = \delta(\phi)\left[\mu \operatorname{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - \nu - \lambda_1 (u_0 - c_1)^2 + \lambda_2 (u_0 - c_2)^2\right]$$

- Note the $\delta(\phi)$ term (zero level-set revisited)

Curve Evolution (Cont.)

- After each dt:
  - c_1, c_2 are estimated and fed to next iteration

- Considerations:
  - Numerical implementations use (semi-)explicit scheme
  - Watch for stability issues
  - δ is usually omitted following **Rosen's gradient projection** method
  - Other regularizations exist as well, same for H

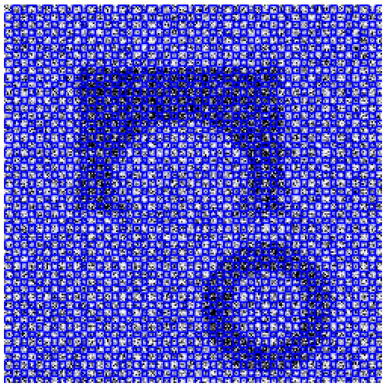
Advantages

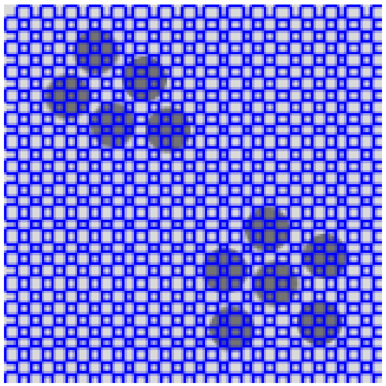
- Very robust method
- Fast convergence
- **No** strict dependence on **initial** contour!
- Easy handling geometric/topological variations

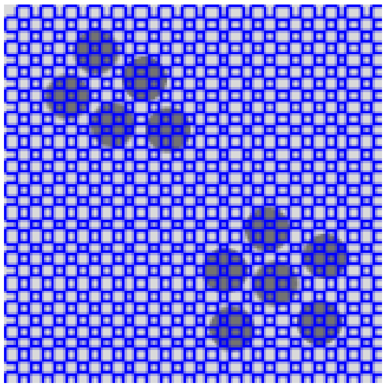
- And many more…

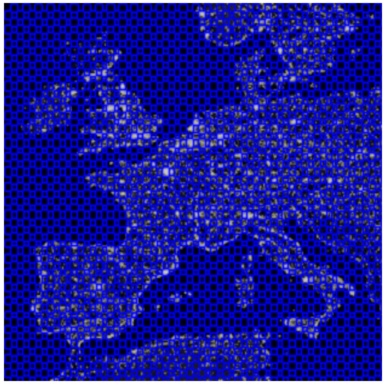
- But, coefficients need be fit based on experience and application
- Not that hard to figure or compute semi-automatically

Few Examples









* Pascal Getreuer, Chan-Vese Segmentation, IPOL (2012) (See companion demo website)

STATISTICAL INTERPRETATION

Introducing Priors

- Cremers, Rousson & Deriche [2007] formulated Chan-Vese segmentation using statistical inference
- What do λ_1, λ_2 mean?

- Giving statistical interpretation in image processing or computer vision is now a very common approach
- Sometimes interpreting previous results by statistical means

- Helps overcome ill-posed/defined problems
  - When vision problems are partially known or easily formulated
  - Replace ‘vision’ with ‘physics’

Priors Derivation

- First, let $P(\Omega)$ express an optimal partition of I_0
- Our goal is now to maximize the MAP:
$$p(P(\Omega)|I_0)$$
- Following simple Bayesian inference:
$$p(P(\Omega)|I_0) \propto p(I_0|P(\Omega)) \cdot p\big(P(\Omega)\big)$$
- Recall that $p(I_0) = 1$ by definition
- This separates image partitioning (1st) from geometrical properties of partition (2nd)

Geometry Prior

- An example for such a prior:

$$p\big(\mathrm{P}(\Omega)\big) \propto e^{-\mu|C|} = e^{-\int \mu|\vec{\nabla} H|}$$

- This favors contours of shorter length

- Or:

$$p\big(\mathrm{P}(\Omega)\big) \propto e^{-\int \nu H}$$

- Which favors regions of smaller area

- Which a priori knowledge is more of less likely?

Image Prior

- Let us assume that different regions are spatially disconnected
- In addition, pixels within each region are i.i.d

- Assume that pixels are drawn from a Gaussian distribution:

$$p(I_0, c_i, \sigma_i) = \frac{1}{\sqrt{2\pi\sigma_i^2}} \exp\left(-\frac{(I_0 - c_i)^2}{2\sigma_i^2}\right)$$

- After plugging this in E and minimizing it analytically with respect to σ_i:

$$\lambda_i = 1/2\sigma_i^2$$

- This can be re-evaluated in each optimization iteration

EXTENSIONS

Advanced Segmentation

- Chan-Vese extended their model to support multi-phase images (multi gray levels)
  - Very elegant, need only $\log_2 N$ level-set functions for N phases

- Also possible to segment vector (color) valued images

Advanced Segmentation (Cont.)

- Probabilistic formulation can be used to identify:
  - Textures
  - Shapes!
  - Motion
  - All other applicable prior knowledge

- Image need not only be considered as intensity
- Each “pixel” can contain complex features (tensor, spatio-temporal info and so)

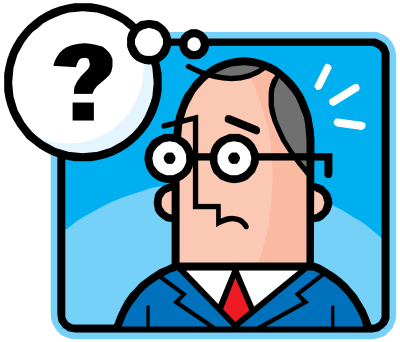
Final Remarks

- Segmentation helps extract geometry and statistical information
- Still an active research (and very rich) domain
- Solutions evolve with increasing complexity of needs

- ϕ formalism is very elegant and powerful
- Apply many differential geometry operators and concepts

- Statistical approaches can be used to insert prior knowledge into physical observations
- Or study statistical properties of observed objects

Questions?



Thank You ☺