SGS Modeling for Fast MHD Magnetic Reconnection

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Magnetic Reconnection in the Corona, Turbulent Approach

- Corona realm of MHD and reconnection a key process
- Diffusive timescale $t_{diff} = L^2/\eta$
- Resistive MHD (Spitzer 1962) not fast enough for solar flare: $t_{diff} \cong 0.3\text{years},\ t_{obs} \cong 100s$
- Turbulent diffusivity assumed $\Rightarrow$ Model to account for turbulence

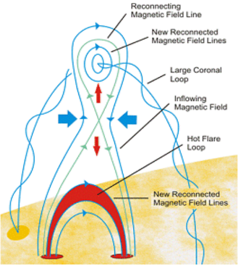
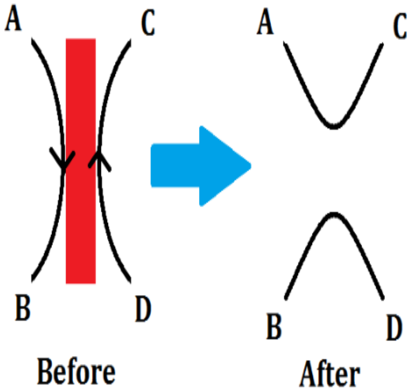


Figure : [http://science.nasa.gov]

Magnetic Reconnection

- Magnetic Reconnection
  - Change in field lines topology and connectivity
  - Magnetic energy converted to heat and kinetic energy
- Reconnection Rate
  - Speed at which magnetic field lines are carried towards the 'X' point
  - Ratio inflow to outflow



Reynolds Averaged Navier-Stokes (RANS) Modeling

Induction equation, resistive MHD

$$\partial_t \boldsymbol{B} = \nabla \times (\boldsymbol{U} \times \boldsymbol{B} - \eta \boldsymbol{J})$$

- Mean field turbulent model

$$f = \overline{f} + f', \; \overline{f} \equiv \langle f \rangle$$

$$\partial_t \overline{\boldsymbol{B}} = \nabla \times (\overline{\boldsymbol{U}} \times \overline{\boldsymbol{B}} + \langle \boldsymbol{u'} \times \boldsymbol{b'} \rangle - \eta \overline{\boldsymbol{J}})$$

- EMF $\langle \boldsymbol{u'} \times \boldsymbol{b'} \rangle$ needs to be modelled
  - $\langle \ldots \rangle$ obeys Reynolds' rules

$\beta - \gamma$ Model

(Yokoi 2013): $\boldsymbol{EMF} = -\beta \boldsymbol{J} + \gamma \boldsymbol{\Omega}$

- $\beta = C_\beta \tau_t K$
- $K = \frac{1}{2} \left\langle \boldsymbol{u}'^2 + \boldsymbol{b}'^2 \right\rangle$
- K: turbulent energy

- $\gamma = C_\gamma \tau_t W$
- $W = \langle \boldsymbol{u}' \cdot \boldsymbol{b}' \rangle$
- W: turbulent cross-helicity

τ_t: turbulent timescale
System closed by evolution equations

MHD Equations, RANS Model

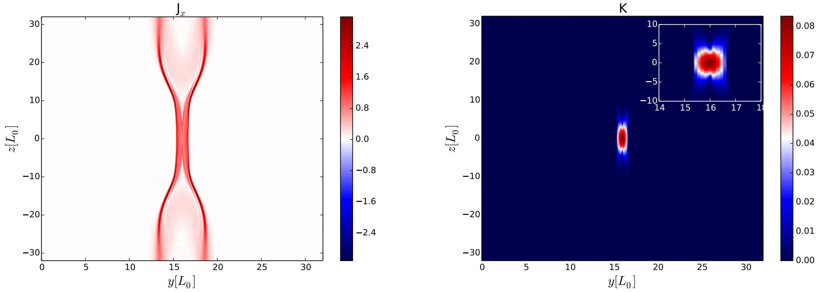
$$
\begin{aligned}
\frac{\partial \rho}{\partial t} &= -\boldsymbol{\nabla}\cdot(\rho \boldsymbol{U}) \\
\frac{\partial \rho \boldsymbol{U}}{\partial t} &= -\nabla\cdot\left[\rho \boldsymbol{U}\otimes\boldsymbol{U} + \frac{1}{2}(p+B^2)\boldsymbol{I} - \boldsymbol{B}\otimes\boldsymbol{B}\right] + \chi\nabla^2(\rho\boldsymbol{U}) \\
\frac{\partial \boldsymbol{B}}{\partial t} &= \nabla\times(\boldsymbol{U}\times\boldsymbol{B}) - (\nabla(\eta+\beta))\times\boldsymbol{J} + (\eta+\beta)\nabla^2\boldsymbol{B} \\
&\quad +\nabla\times(\gamma\sqrt{\rho}\Omega) \\
\frac{\partial h}{\partial t} &= -\nabla\cdot(h\boldsymbol{U}) + \frac{\gamma-1}{\gamma h^{\gamma-1}}(\eta\boldsymbol{J}^2 + \frac{\rho K}{\tau_t}) + \chi\nabla^2 h \\
\frac{\partial K}{\partial t} &= -\boldsymbol{U}\cdot\nabla K + C_\beta\tau_t K\frac{\boldsymbol{J}^2}{\rho} - C_\gamma\tau_t W\frac{\boldsymbol{\Omega}\cdot\boldsymbol{J}}{\sqrt{\rho}} + \frac{\boldsymbol{B}}{\rho}\cdot\nabla W - \frac{K}{\tau_t} \\
\frac{\partial W}{\partial t} &= -\boldsymbol{U}\cdot\nabla W + C_\beta\tau_t K\frac{\boldsymbol{\Omega}\cdot\boldsymbol{J}}{\sqrt{\rho}} - C_\gamma\tau_t W\boldsymbol{\Omega}^2 + \frac{\boldsymbol{B}}{\sqrt{\rho}}\cdot\nabla K - C_W\frac{W}{\tau_t}
\end{aligned}
$$

Current Sheet Equilibrium Tested

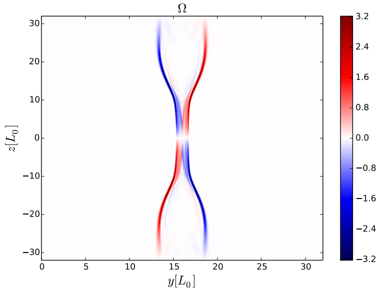
- Harris equilibrium
  - Pressure equilibrium across the current sheet
  - Not realistic for the Solar Corona

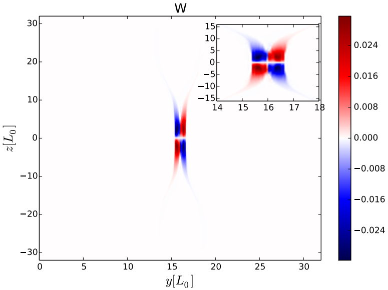
- Force free equilibrium with out of plane guide field b_g
  - No initial Lorentz force: $\boldsymbol{J} \times \boldsymbol{B} = 0$
  - More realistic for the Solar Corona

Spatial Distribution: Mean Current density J, turbulent energy K

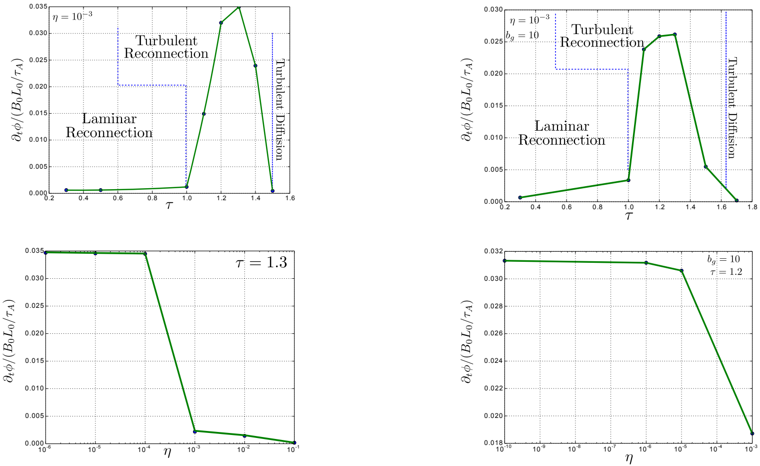


Spatial Distribution: Mean Current vorticity $\boldsymbol{\Omega}$, turbulent cross-helicity W

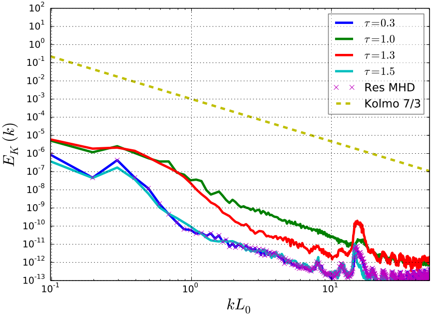


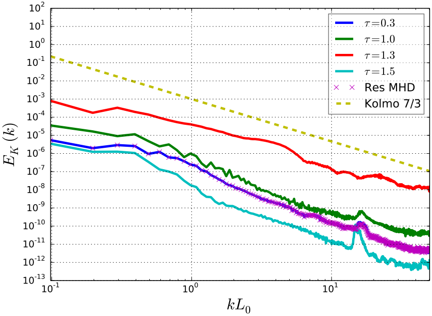


RANS Reconnection Regimes, Harris and Force Free

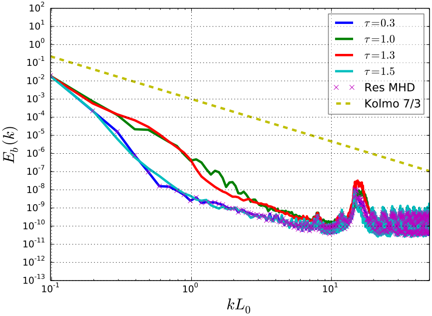


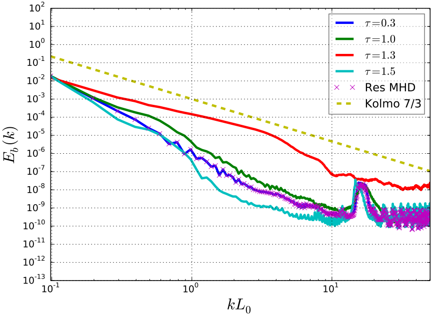
Kinetic Energy Transfer for Different Timescale



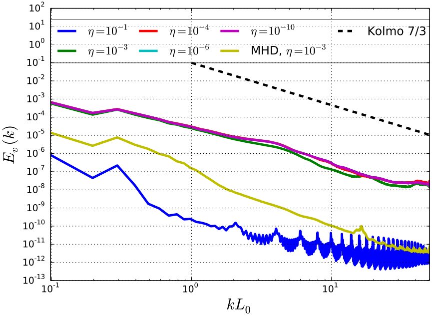


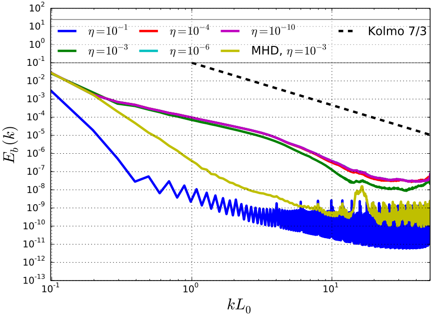
Magnetic Energy Transfer for Different Timescale



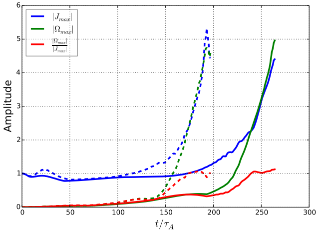


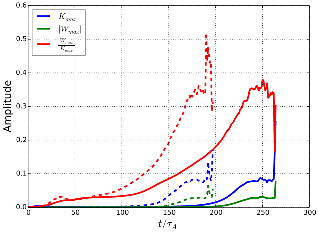
Averaged Energy Transfer in Time for Different Resistivity

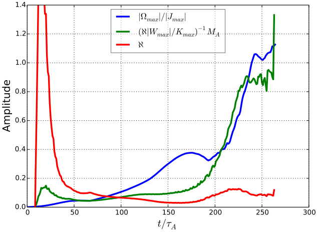




Gradients and turbulence



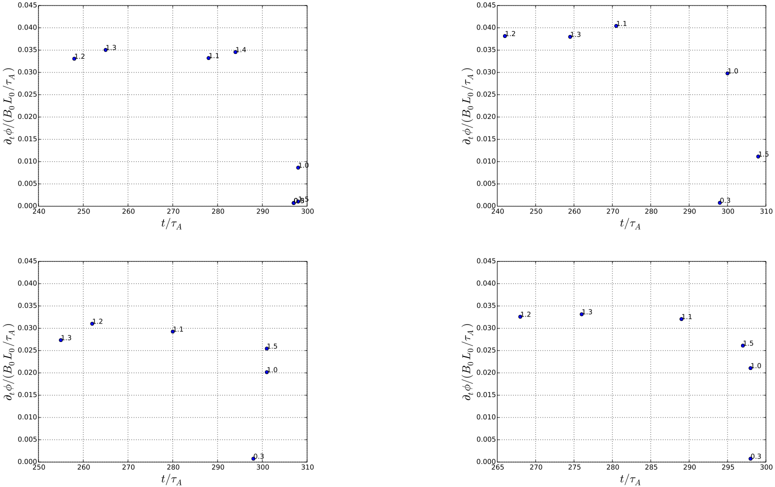




- Mean field inhomogeneities

$$\frac{|\boldsymbol{\Omega}|}{J} \simeq \frac{U}{\delta B}\frac{\Delta}{L} \simeq \left(\aleph\frac{W}{K}\right)^{-1} M_A$$

Maximum Reconnection Rate



Dimensional Analysis of the Reconnection Rate

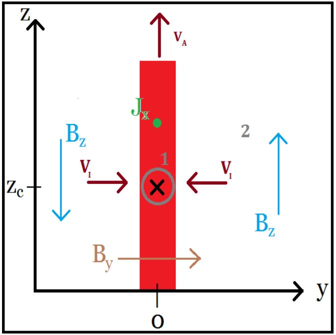


Figure : Current Sheet

- Resistive MHD

$$M_0^2 = \left(\frac{V_i}{V_o}\right)^2 = \eta$$

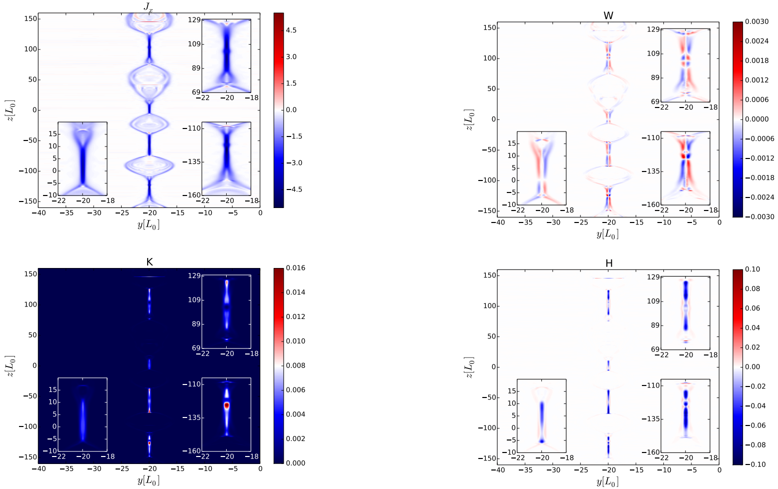
- RANS without guide field

$$M^2 = \eta + \beta\left(1 + \frac{|\gamma|}{\beta}\eta^{3/2}\right)$$

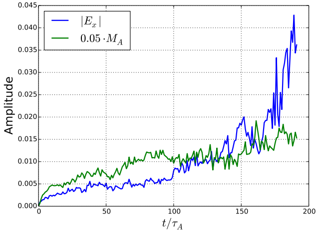
- RANS with guide field

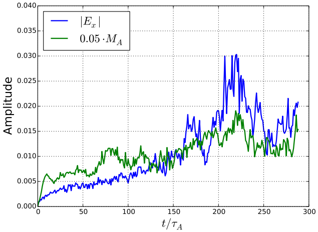
$$M^2 = \eta + \beta\left(1 + \frac{|\gamma|}{\beta}\eta^{3/2}\right) - \alpha\sqrt{\eta}$$

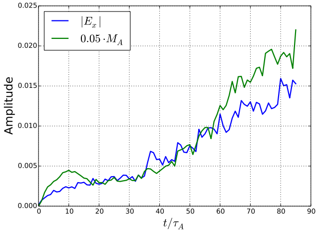
Turbulent Magnetic Helicity $\alpha = \boldsymbol{J} \cdot \boldsymbol{B}$



Reconnection rates







- Reconnection rate from the model

$$M_A = \frac{|\mathbf{\Omega}|}{|\mathbf{J}|}\left(\frac{\aleph |W|}{K}\right)$$

Solar Wind Test

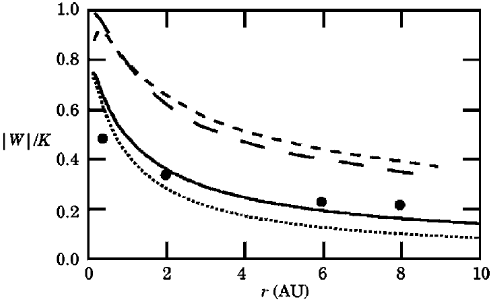


Figure : Yokoi and Hamba (PoP 2007)

(—) RANS Model

(· · ·) RANS with enhance strain

(•) (Helios, Voyager) spacecraft observations D.A.Roberts et al. (1987)

(-) Zhou and Matthaeus (1990)

(- -) Tu and Marsch (1993)

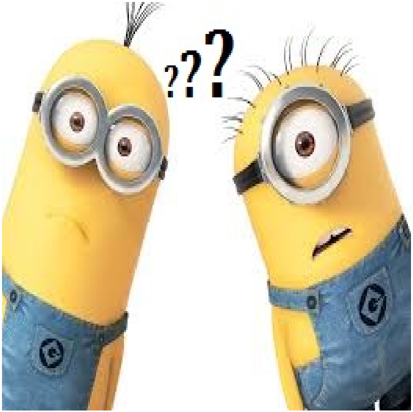
Conclusions

- Turbulence Regimes
  - Controlled by τ
  - τ related to the amount of energy transfered to smaller scales
- Small η enhance the process
- (Widmer et al.) submitted to PoP
- **Open Questions**
  1. Relation between gradients of the system, turbulence and reconnection
  2. Relation between guide field effects and magnetic helicity

Outlook

1. More investigation of the relation between gradients of the system, turbulence and reconnection (upcoming paper)
2. Characterization of guide field effects by (magnetic) Helicity (upcoming paper)
3. Equation for the turbulence timescale
4. 3D simulations and α related term

Questions



Acknowledgment

Thank you for listening