A Goldstino at the bottom of the cascade

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based on 1509.03594 (JHEP)
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[see also 1412.6499 (PRD) w/ Argurio, Musso, Porri & Redigolo
and 1310.6897 (JHEP) w/ Argurio, Di Pietro, Porri & Redigolo]

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Motivations

- Understand (super)symmetry breaking in strongly coupled QFTs → Holography it's a powerful tool!
- Here we consider the breaking of supersymmetry. Within top-down models this can also say something on ~~SUSY~~ in String theory and existence of metastable vacua.
- In this talk I focus on 4-dim N=1 theories arising from D-branes at CY singularities (*i.e.* quiver gauge theories):
  - Improve understanding of holographic renormalization for quiver gauge theories.
  - Contribute to ongoing debate about antiD-branes in warped throats (from a complementary perspective).

Preliminaries

- A necessary condition for a SQFT to break SUSY is that conformal invariance is explicitly broken:

$$E_{vac} = \langle T_{00} \rangle \sim \langle T^{\mu}_{\mu} \rangle \text{ at odds with operator identity } T^{\mu}_{\mu} = 0$$

From Lorentz invariance

The SCFT must be deformed by (marginally) relevant, SUSY-preserving operators.

Note: This means that dual backgrounds cannot be AdS!

~~AdS~~

Should depart from AdS-ness... and do it at enough pace!

In QGT there is a sharp departure from AdS-ness: log-divergent, not even AAdS!

Cascading backgrounds

Preliminaries

- ***Recall***: in AdS/CFT a QFT vacuum is described by a given solution of bulk EOM. [A ***necessary*** condition for different solutions to describe vacua of ***same*** QFT is to have same asymptotic.]

- Suppose to have a bulk solution which breaks SUSY. There are two basic questions one should answer:

  - Q1: Is the solution gravitationally (meta)stable?

    YES: then the solution describes an actual QFT vacuum.

  - Q2: Is the bulk mode dual to the goldstino present?

    YES: then SUSY is broken spontaneously in the FT dual.

Preliminaries

- It should be possible to answer these two questions independently:
  - The goldstino appears as a massless pole in supercurrent 2-point function (in IR $\mathbf{S}_\mu = \sigma_\mu \bar{\mathbf{G}}$)

$$\langle S_{\mu\alpha}\, \bar{S}_{\nu\dot{\beta}} \rangle$$

Complicated structure; it depends on the vacuum one is considering!

  - In fact, we don't need it all! Information fully encoded in (quasi local) contact term implied by SUSY Ward identity

$$\langle \partial^\mu S_{\mu\alpha}(x)\, \bar{S}_{\nu\dot{\beta}}(0) \rangle = -2\sigma^\mu_{\alpha\dot{\beta}}\, \langle T_{\mu\nu} \rangle\, \delta^4(x)$$

Upon integration, it relates pole residue to vacuum energy.

WIs depend on UV data. Vacuum stability is IR property.

The basic goal

- This disentanglement should hold also from a holographic dual perspective.
  - ***Goal***: see how much can we learn on structure of QFT vacua ***without*** detailed knowledge of the deep interior.
  - More concretely, we would like to:

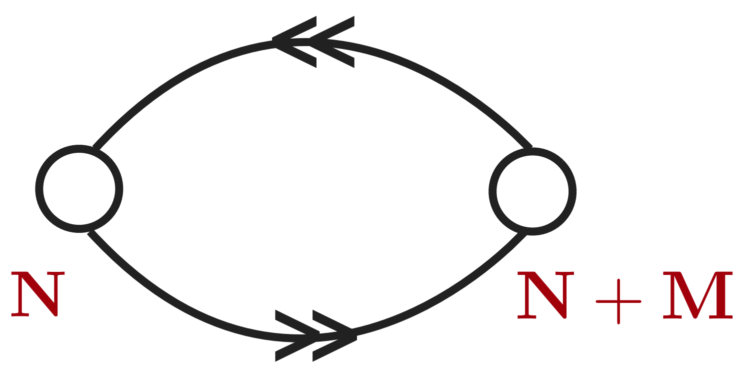
1. Derive the SUSY Ward identities holographycally.
2. See if and when a goldstino mode is present.

- I will focus on the conifold theory, a prototype for (a large class of) QGT which can accommodate ~~SUSY~~ vacua.
- Underlying ***question***: are cascading theories renormalizable?

~~SUSY~~ in String th. & Holography

- The conifold theory (*i.e.* the KS model) in a nutshell.

[Klebanov-Strassler '00]



Gauge Group	$SU(N+M) \times SU(N)$
Global Symmetries	$SU(2) \times SU(2) \times U(1)_B \times Z_{2M}$
Bifundamental Matter	$A_i, B_k (i,k = 1,2)$
Superpotential	$W = \lambda Tr(A_i B_k A_j B_l)\epsilon^{ij}\epsilon^{kl}$

→ N regular and M fractional D3-branes at the tip of $\mathbf{C(T^{1,1})}$.

- For $\mathbf{M = 0}$ the theory is (super)conformal. [Klebanov-Witten '98]

$$\mathcal{O}_\phi \sim \frac{1}{g_1^2} + \frac{1}{g_2^2} \longleftrightarrow e^{-\phi} \quad , \quad \mathcal{O}_{\tilde{b}} \sim \frac{1}{g_1^2} - \frac{1}{g_2^2} \longleftrightarrow \tilde{b}^\Phi = e^{-\phi} b^\Phi$$

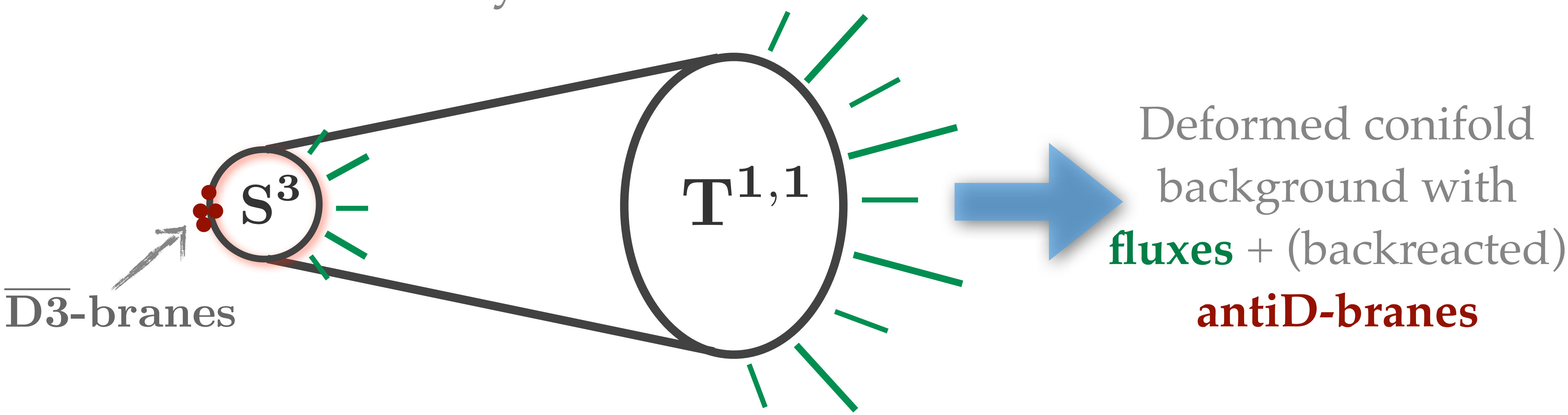
- For $\mathbf{M \neq 0}$ conformal invariance is broken: $\mathcal{O}_{\tilde{b}}$ becomes relevant and triggers an RG-flow → duality cascade.

~~SUSY~~ in String th. & Holography

- For $\mathbf{N = kM}$ there are both mesonic and baryonic branches of SUSY vacua.
- For $\mathbf{N = kM - p}$ with $\mathbf{p < M}$ the baryonic branch is lifted and only the mesonic branch survives.

[Dymarsky-Klebanov-Seiberg'05]

[Kachru-Pearson-Verlinde '01] argued that there exist ~~SUSY~~ vacua on the would-be baryonic branch!



- ***Question***: is there a goldstino there?

Cascading theories from 5d SUGRA

- Holographic dictionary (and machinery) defined in terms of 5d effective d.o.f. $\rightarrow$ need to compactify type IIB on $\mathbf{T^{1,1}}$. The resulting effective theory is very complicated. However, several simplifications make our life simpler:
  - We need to look to UV asymptotic only, up to the order ~~SUSY~~ deformation appears.

    $\Rightarrow$ need to look for solutions up to order $\mathbf{z^4}$ only.
  - We focus on $\mathbf{SU(2)\times SU(2)}$ invariant sector (and restrict to fields invariant under an extra $\mathbf{U(1)}$ symmetry).

[Cassani-Faedo '10, Liu-Szepietowski '11]

see also [Buchel '05]

- *Note*: Solutions should be domain-wall like (metric+scalars) and have all same asymptotic (~KT like [Klebanov-Tseytlin '00])!

Cascading theories from 5d SUGRA

- The most general solution compatible with UV b.c. is

$$
\left\{
\begin{aligned}
ds^2 &= \frac{1}{z^2}\left(e^{2Y(z)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{2X(z)}dz^2\right)\\
e^{2Y} &= h^{\frac{1}{3}}(z)\, h_2^{\frac{1}{2}}(z)\, h_3^{\frac{1}{2}}(z) \quad , \quad e^{2X} = h^{\frac{4}{3}}(z)\, h_2^{\frac{1}{2}}(z)
\end{aligned}
\right.
$$

$$
\left\{
\begin{aligned}
e^{2U} &= h^{\frac{5}{2}}(z)\, h_2^{\frac{3}{2}}(z) \quad , \quad e^{2V} = h_2^{-\frac{3}{2}}(z)\\
b^\Phi(z) &= -\frac{9}{2}g_s M \ \log\left(z/z_0\right)\\
&\quad + z^4\left[\left(\frac{9\pi N}{4M} + \frac{99}{32}g_s M - \frac{27}{4}g_s M\log\left(z/z_0\right)\right)\mathcal{S} - \frac{9}{8}g_s M\varphi\right] + \mathcal{O}(z^8)\\
\phi(z) &= \log g_s + z^4\left(3\,\mathcal{S}\log\left(z/z_0\right) + \varphi\right) + \mathcal{O}(z^8)
\end{aligned}
\right.
$$

$$
\begin{aligned}
h(z) &= \frac{27\pi}{4g_s}\left(g_s N + \frac{3}{8\pi}(g_s M)^2 - \frac{3}{2\pi}(g_s M)^2\log\left(z/z_0\right)\right)\\
&+ \frac{z^4}{g_s}\left[\left(\frac{54\pi g_s N}{64} + \frac{81}{4}\frac{13}{64}\left(g_s M\right)^2 - \frac{81}{16}\left(g_s M\right)^2\log\left(z/z_0\right)\right)\mathcal{S} - \frac{81}{64}\left(g_s M\right)^2\varphi\right] + \mathcal{O}(z^8)\\
h_2(z) &= 1 + \frac{2}{3}\,\mathcal{S}\,z^4 + \mathcal{O}(z^8) \quad , \quad h_3(z) = 1 + \mathcal{O}(z^8)
\end{aligned}
$$

Two-parameter family of ~~SUSY~~ solutions.

SEE ALSO [DE WOLFE ET AL. '08]

Holographic Ward identities

- Ward identities are relations among correlators of local operators, descending from global symmetries.
- Turn on sources for local operators, gauge the global symmetries and require invariance of the generating functional under local gauge transformations.

→ Get relations between 1pt-functions (at finite source!) and in turn, upon differentiation, the WIs.

- ***Holography*** naturally adapted to this procedure:
    - global symmetries on the boundary ⟷ gauge symmetries in the bulk
    - bulk fields contain arbitrary sources for local operators, and transform under local symmetries in the bulk.

Holographic Ward identities

- *Note*: work at a finite cut-off, identify sources with induced fields at cut-off shell & remove the cut-off at the end, only.
- Renormalized 1-point functions in the presence of sources

$$\langle T^{\mu\nu}\rangle = \frac{2}{\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial\tilde{\gamma}_{\mu\nu}} \qquad \langle\overline{S}^{-\mu}\rangle = \frac{-2i}{\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial\tilde{\Psi}^{+}_{\mu}}$$

$$\langle\mathcal{O}_{\phi}\rangle = \frac{1}{2\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial\phi} \qquad \langle\overline{\mathcal{O}}^{+}_{\zeta_\phi}\rangle = \frac{1}{\sqrt{-\tilde{\gamma}}}\frac{i}{\sqrt{2}}\frac{\partial S_{ren}}{\partial\zeta^{-}_{\phi}}$$

$$\langle\mathcal{O}_{\tilde{b}}\rangle = \frac{1}{2\sqrt{-\tilde{\gamma}}}\frac{\partial S_{ren}}{\partial\tilde{b}^{\Phi}} \qquad \langle\overline{\mathcal{O}}^{+}_{\tilde{\zeta}_b}\rangle = \frac{1}{\sqrt{-\tilde{\gamma}}}\frac{i}{\sqrt{2}}\frac{\partial S_{ren}}{\partial\tilde{\zeta}^{-}_{b}}$$

$\mathcal{S}_{\mathbf{ren}} = \mathcal{S}_{\mathbf{reg}} + \mathcal{S}_{\mathbf{ct}}$: renormalized action (at finite cut-off!)

Explicit expression of counter-terms not needed to derive Ward identities!

Holographic Ward identities

- *Recipe*: 1. Fix bulk gauge redundancy by choosing a gauge, 2. Study transformations of the sources under residual local symmetries preserving the gauge.
- Gauge fixing condition (Fefferman-Graham gauge):

$$ds^2 = dr^2 + \gamma_{\mu\nu} dx^\mu dx^\nu \quad , \quad \Psi_r = 0 \qquad (dr = -e^{X(z)} dz/z\ )$$

- Bulk diff preserving this gauge:

$$\dot{\xi}^r = 0\ ,\ \dot{\xi}^\mu + \gamma^{\mu\nu}(r,x)\partial_\nu \xi^r = 0$$

Solution: $\xi^r = \sigma(x)\ ,\ \xi^\mu = \xi_0^\mu(x) - \int^r dr' \gamma^{\mu\nu}(r',x)\partial_\nu \sigma(x)$

- Bulk SUSY transformations preserving this gauge:

$$(\nabla_r + \frac{1}{6}\mathcal{W}\,\Gamma_r)\epsilon = 0$$

Solution: $\begin{cases} \epsilon^+(z,x) = z^{-1/2} h(z)^{1/12} \epsilon_0^+(x) + \mathcal{O}(z^4) \\ \epsilon^-(z,x) = z^{1/2} h(z)^{-1/12} \epsilon_0^-(x) + \mathcal{O}(z^4) \end{cases}$

$\epsilon = \epsilon^+ + \epsilon^-$

P-T superpotential

Holographic Ward identities

- $\sigma(\mathbf{x})$ parametrizes Weyl transformations: $\delta_\sigma \mathcal{S}_{\mathbf{ren}} = \mathbf{0}$

$$\langle T^{\mu}_{\mu}\rangle + 9M\langle\mathcal{O}_{\tilde{b}}\rangle + \left[\frac{i}{4}\langle\overline{S}^{-\mu}\rangle\tilde{\Psi}^{+}_{\mu} + \frac{i}{\sqrt{2}}\langle\overline{\mathcal{O}}^{+}_{\zeta^{\phi}}\rangle\zeta^{-}_{\phi} + \frac{i}{\sqrt{2}}\langle\overline{\mathcal{O}}^{+}_{\tilde{\zeta}^{b}}\rangle\tilde{\zeta}^{-}_{b} + \text{h.c.}\right] = 0$$

- $\epsilon^{-}(\mathbf{x})$ parametrizes superWeyl transformation: $\delta_{\epsilon^-}\mathcal{S}_{\mathbf{ren}} = \mathbf{0}$

$$\frac{i}{2}\langle\overline{S}^{-\mu}\tilde{\Gamma}_{\mu}\rangle = \frac{9M}{\sqrt{2}}\,\langle\overline{\mathcal{O}}^{+}_{\tilde{\zeta}_b}\rangle$$

- $\epsilon^{+}(\mathbf{x})$ parametrizes SUSY transformations: $\delta_{\epsilon^+}\mathcal{S}_{\mathbf{ren}} = \mathbf{0}$

$$\frac{i}{2}e^{-\frac{2}{15}U}\langle\partial_{\mu}\overline{S}^{-\mu}\rangle = -\frac{1}{2}\langle T^{\mu\nu}\rangle\overline{\tilde{\Psi}}^{+}_{\mu}\tilde{\Gamma}_{\nu} + i\langle\mathcal{O}_{\phi}\rangle\overline{\zeta}^{-}_{\phi} + i\langle\mathcal{O}_{\tilde{b}}\rangle\overline{\tilde{\zeta}}^{-}_{b}$$

- Ward identities can be obtained differentiating the above relations wrt covariant sources, then removing the cut-off (and set sources to 0, when needed).

at the cut-off w/ non-zero sources!

Holographic Ward identities

- From ϵ^+ identity, further differentiating wrt gravitino/hyperini we get:

$$\langle \partial^\mu S_{\mu\alpha}(x)\ \bar{S}_{\nu\dot\beta}(0)\rangle = -2\,\sigma^\mu_{\alpha\dot\beta}\langle T_{\mu\nu}\rangle\ \delta^4(x)$$

$$\langle \partial^\mu S^\alpha_\mu(x)\ \mathcal{O}_{\zeta_\phi\alpha}(0)\rangle = -\sqrt{2}\ \langle \mathcal{O}_\phi\rangle\ \delta^4(x)$$

$$\langle \partial^\mu S^\alpha_\mu(x)\ \mathcal{O}_{\tilde\zeta_b\alpha}(0)\rangle = -\sqrt{2}\ \langle \mathcal{O}_{\tilde b}\rangle\ \delta^4(x)$$

Higher-components operators VEVs

→ break SUSY!

- From σ and ϵ^- we get the trace operator identities:

$$\langle T^\mu_\mu\rangle = -9M\langle \mathcal{O}_{\tilde b}\rangle \quad , \quad \langle \sigma^\mu_{\alpha\dot\beta}\bar{S}^{\dot\beta}_\mu\rangle = -9\sqrt{2}M\ \langle \mathcal{O}_{\tilde\zeta_b\,\alpha}\rangle$$

All relations are in perfect agreement with QFT expectations:

$$\Delta(\mathcal{O}_{\tilde b}) < 4\ ,\ \Delta(\mathcal{O}_\phi) = 4\ \text{ and } T^\mu_\mu = -(1/2)\sum \beta_i\, O_i$$

→ Goldstino eigenstate: $\mathbf{G} \sim \langle \mathcal{O}_{\tilde b}\rangle\, \mathcal{O}_{\tilde\zeta_b}$!

Goldstino & antiD-branes

- Finally, evaluating the bosonic 1pt-func on our solutions (explicit expression for (bosonic) $\mathcal{S}_{\mathbf{ren}}$ is known!) we get

$$\langle T^{\mu}_{\mu} \rangle = -12\,\mathcal{S} \quad , \quad \langle \mathcal{O}_{\phi} \rangle = \frac{(3\mathcal{S} + 4\varphi)}{2} \quad , \quad \langle \mathcal{O}_{\tilde{b}} \rangle = \frac{4}{3\,M}\mathcal{S}$$

SEE ALSO [AHARONY ET AL. '05, DE WOLFE ET AL. '08]

- SUSY case $\mathcal{S} = \varphi = \mathbf{0}$: 0 vacuum energy, no goldstino.
- Branch $\mathcal{S} = \mathbf{0}\,,\ \varphi \neq \mathbf{0}$: 0 vacuum energy, no goldstino but ~~SUSY~~!? Not a vacuum of KS theory: explicit breaking!
- Branch $\mathcal{S} \neq \mathbf{0}\,,\ \varphi = \mathbf{0}$: $\neq 0$ vacuum energy, goldstino mode, ~~SUSY~~ VEV, trace identity fulfilled. Spontaneous breaking!
- *Note*: these conclusions can be reached by looking just at the asymptotic solution. One does not need the full solution!

Goldstino & antiD-branes

- ***Note***: in the ***asymptotic*** solution $\mathcal{S}$ and φ are integration constants. In a full solution, they get fixed by IR b.c.

$$\mathcal{S} = \mathcal{S}(k, M, p, \epsilon, \dots)\,,\ \varphi = \varphi(k, M, p, \epsilon, \dots)$$

  - Solution w/ $\mathcal{S} = \varphi = \mathbf{0}$ is known: KS geometry for $\mathbf{p} = \mathbf{0}$ or KS with $\mathbf{M} - \mathbf{p}$ mobile D3s for $\mathbf{p} \neq \mathbf{0}$.

  - Solution w/ $\mathcal{S} = \mathbf{0}\,,\ \varphi \neq \mathbf{0}$ also known [KUPERSTEIN ET AL. '14] . Our analysis shows that no matter how φ depends on the IR data, it does not describe a KS theory vacuum.

  - Solution w/ $\mathcal{S} \neq \mathbf{0}\,,\ \varphi = \mathbf{0}$ corresponds to ~~SUSY~~ vacua of KS theory. IR analysis [DE WOLFE ET AL. '08, BENA ET AL. '11] shows that the mode $\mathcal{S}$ is *sourced* by antiD3, $\mathcal{S} \sim \mathbf{p}\, \mathbf{e}^{-\frac{8\pi \mathbf{k}}{3 \mathbf{g_s} \mathbf{M}}}$ $\rightarrow$ if stable antiD configuration exists, it is a ~~SUSY~~ KS-theory vacuum!

Summary

- Holographic ***derivation*** of Ward identities for cascading backgrounds (providing further evidence that such theories can be consistently renormalized [KRASNITZ '02, AHARONY ET AL. '05]).

Note: insensitive to IR, but give constraints on solutions that correspond to given vacua (e.g. ~~SUSY~~ vacua in KS theory).

- Provided a ***necessary*** consistency check for the existence of metastable vacua in the KS cascading gauge theory.

- Our results give further evidence for 1-1 ***correspondence*** between spontaneous ~~SUSY~~ in (a class of) quiver gauge theories and antiD-brane states in warped throats.

Outlook

- On with the program of HR for the conifold theory (more generally, for cascading backgrounds) → do a systematic derivation of fermionic counter-terms.

- Extension of present analysis to the full background (including conifold deformation parameter)... but we don't expect any dramatic ***qualitative*** change.

- Working at finite cut-off in terms of induced fields looks promising for systematics of HR in generic set-ups. E.g., derive SUSY-preserving counter-terms for SQFT on curved manifolds.

Thank you!