UNIVERSITY OF ICELAND

Stockholm University

A holographic model of black hole complementarity

Lárus Thorlacius

- based on

D. Lowe & L.T. - arXiv:1605.xxxx
JHEP **1512** (2015) 096
Phys. Lett. B **737** (2014) 320
Phys. Rev. D **88** (2013) 044012

- earlier work

K. Larjo, D. Lowe, & L.T. - Phys. Rev. D **87** (2013) 104018.

D. Lowe & L.T. - Phys. Rev. D **73** (2006) 104027,
Phys. Rev. D **60** (1999) 104012.

Current Themes in Holography: Exact results, applications, extensions and fundamentals
University of Copenhagen, 25 - 29 April 2016

Summary

The information paradox highlights the incompatibility between general relativity (locality + equivalence principle) and quantum physics (unitarity).

Breakdown of local effective field theory, even on "nice" time slices that avoid strong curvature region.

Gauge theory - gravity correspondence implies unitary black hole evolution.

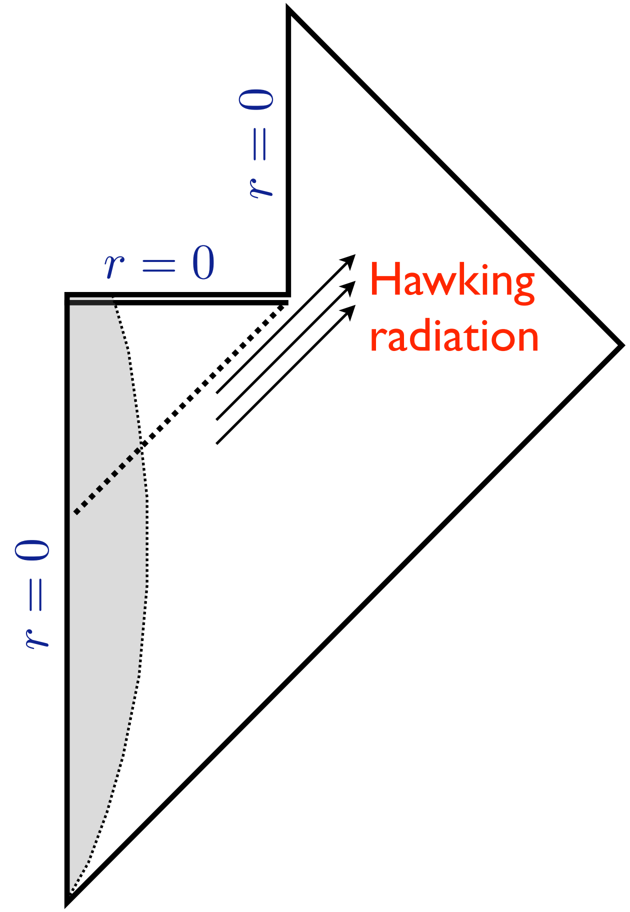
Black hole complementarity provides a "phenomenological" description, which preserves unitarity and the equivalence principle, but requires giving up locality.

Stretched horizon for outside observers can be motivated from QFT in black hole background.

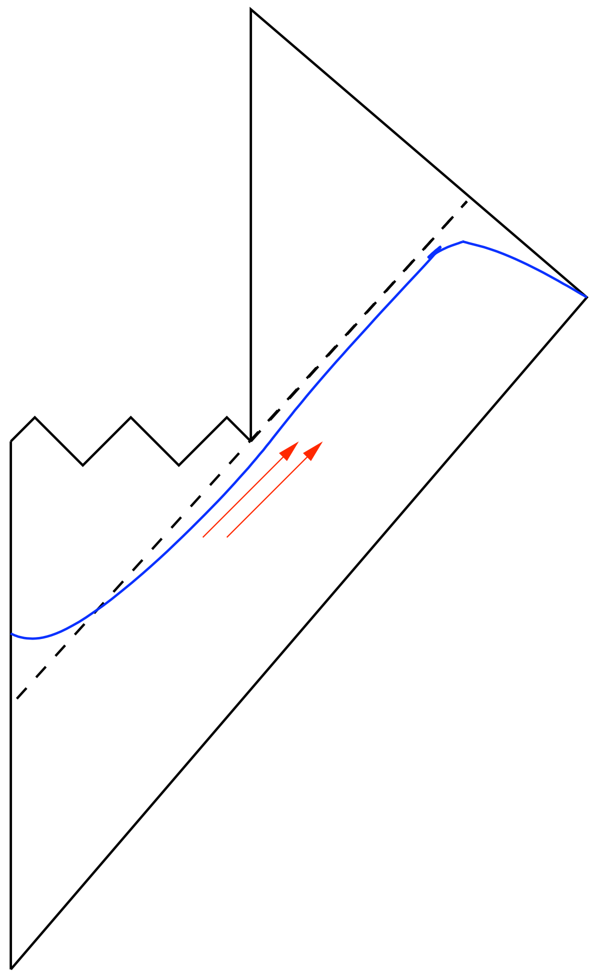
Typical infalling observers do not see drama on their way towards a black hole formed from a generic pure state. (Special pure states, as well as special observers, exist for which this is not true.)

An approximate description of observers in the black hole interior can be given in terms of an effective field theory, defined on a limited set of time slices, such that no drama is seen until near the singularity.

Semi-classical black hole



Effective field theory



- assume that local effective field theory can be applied in regions of weak curvature, away from black hole singularity

- the explicit form of the effective field theory is not needed for information loss argument

- construct a convenient set of Cauchy surfaces

  'nice' time slices

  R.Wald '93

  D.Lowe, J.Polchinski, L.Susskind, LT, J.Uglum '95

- effective field theory Hamiltonian generates evolution of states

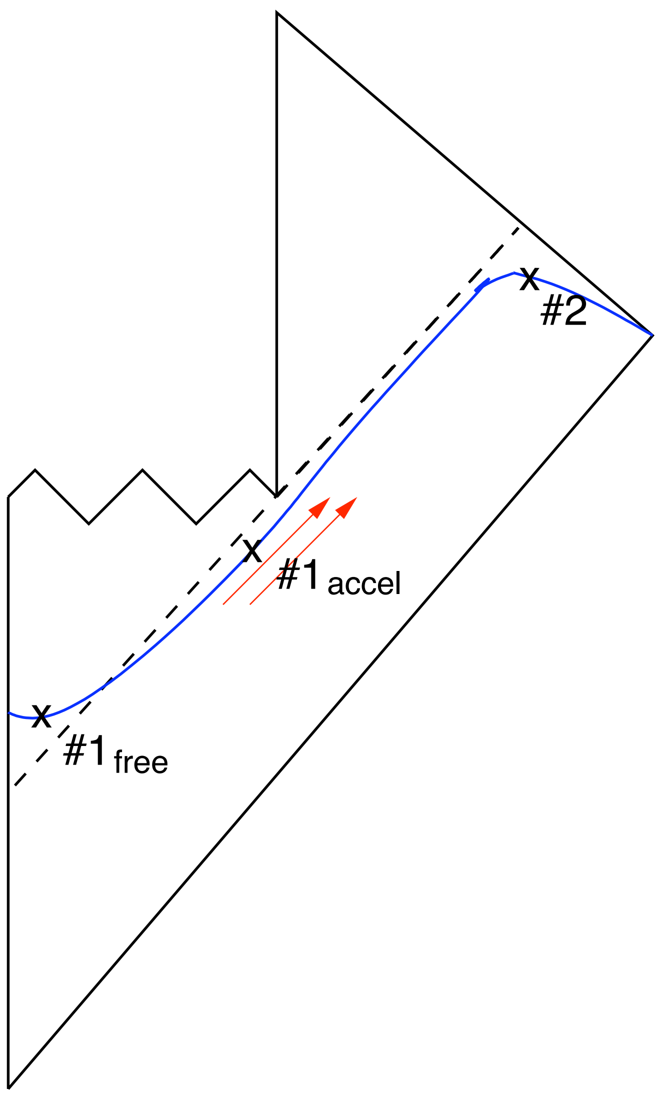
- the Hamiltonian on nice slices is time-dependent

  $\longrightarrow$ Hawking emission

- the nice slices extend into black hole region

  $\longrightarrow$ information loss

Formulation of the paradox



- prepare singlet pair (#1,#2)

- keep #2 outside and send #1 into black hole

- $\#1_{\text{free}}$ and $\#1_{\text{accel}}$ measure spin along z-axis

- #2 measures spin either along z-axis or x-axis

- local qft $\Rightarrow$ independent measurements by $\#1_{\text{free}}$ and $\#1_{\text{accel}}$

- if they disagree $\#1_{\text{accel}}$ discovers that #2 measured along x-axis $\Rightarrow$ acausal signal from #2 to $\#1_{\text{accel}}$

Black hole complementarity

BHC postulates:

L.Susskind, LT, J.Uglum '93

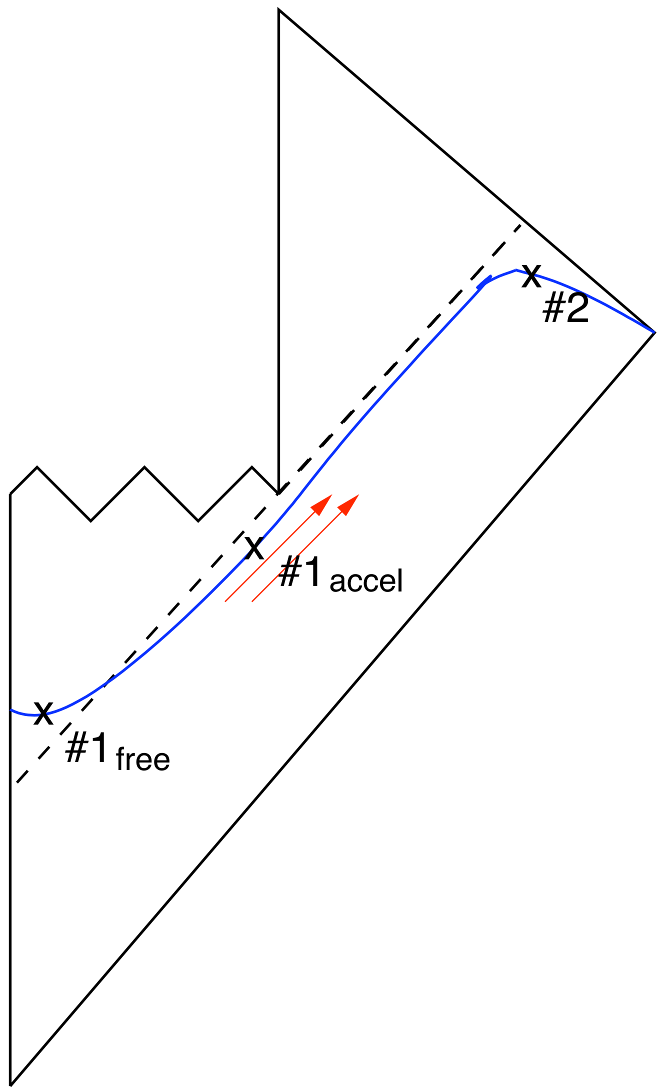
1. Black hole evolution, as viewed by a distant observer, is described by a quantum theory with a unitary S-matrix relating the initial state of the collapsing matter to that of outgoing radiation
2. Outside the stretched horizon of a massive black hole, physics can be described to good approximation by a set of semi-classical field equations
3. To a distant observer, a black hole appears to be a quantum system with discrete energy levels. The dimension of the subspace of states that describe a black hole of mass M is

$$\exp\left(\frac{A}{4}\right) = \exp\left(4\pi M^2\right)$$

4. There is no contradiction between outside observers finding information encoded in Hawking radiation and infalling observers entering a black hole unharmed.

Firewall for infalling observers?

Revisit gedanken experiment



Oaccel must wait before information can be extracted from Hawking radiation

Young BH: $t \sim r_s \, S_{bh}$ D.Page 1993

Old BH: $t \sim r_s \, \log r_s$ P.Hayden & J.Preskill 2007

Ofree has short time for spin measurement

Young BH: $\Delta t \sim e^{-S_{bh}}$

Old BH: $\Delta t \sim r_s^{-1}$

$\longrightarrow$ limited measurement accuracy

Ofar measures state of Hawking radiation to arbitrary accuracy

$\longrightarrow$ projects BH state into eigenstate of Hawking radiation

State of infalling observer is also projected

$\longrightarrow$ observation of Hawking radiation burns infalling observer at horizon D.Lowe, LT '06

A. Almheiri, D. Marolf, J. Polchinski, J. Sully '12

Braunstein, Pirandola, Zyczkowski '12

BHC in a holographic setting

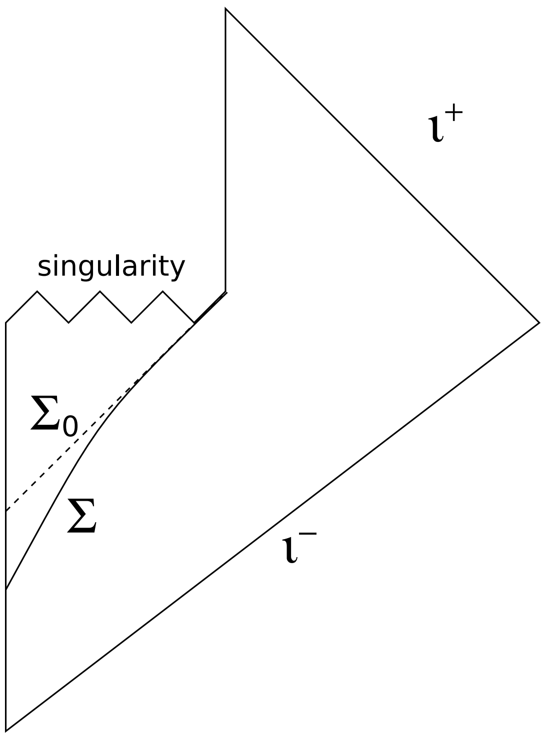
- In a quantum theory, general covariance leads to a conflict between unitarity and locality.
- In holographic models unitarity is preserved at the expense of bulk locality.
- How does the non-locality avoid infecting observations made by low-energy local observers?
- Soft violation of general covariance at finite N in holographic models

  — symmetry is restored in $N \to \infty$ limit.
- Hawking emission is a 1/N effect

  — information paradox cannot be posed in the strict $N \to \infty$ limit.
- The breaking of general covariance is implemented via the holographic reconstruction of the bulk radial direction.
- We model this “holographic regulator” by discretising radial direction.

Modeling the exterior region

D.Lowe & LT '13



The effective field theory of Postulate 2 applies outside stretched horizon Σ

This effective field theory can in principle be obtained from the dual boundary theory.

Model slowly evaporating black hole by static Schwarzschild solution.

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2$$

Modeling the black hole interior

L.Susskind '13; D.Lowe, LT '14

#1 : Stretched horizon theory

Black hole interior is encoded in outside dof's

Description of the interior is non-local and employs finite # of dof's

#2 : Local effective field theory (extended inside horizon)

Approximately describes measurements made by a typical observer who falls inside black hole

Applies on a restricted set of time slices with a radial cutoff in place

Fails for an atypical observer who has measured state of BH

The inside view

D.Lowe, LT '14

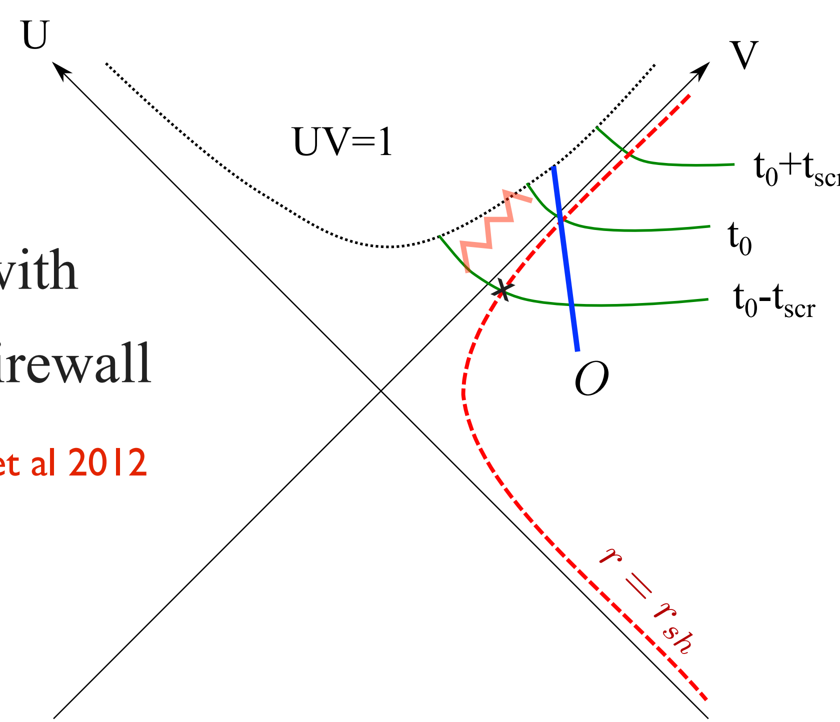
Typical low energy observer O in free fall enters BH at $t = t_0$

O is well described by theory #1 outside stretched horizon

Construct initial state for theory #2 at $t = t_0 - t_{scr}$ with $t_{scr} \sim 4M \log(4M)$

$r > r_{sh}$: Use state from theory #1

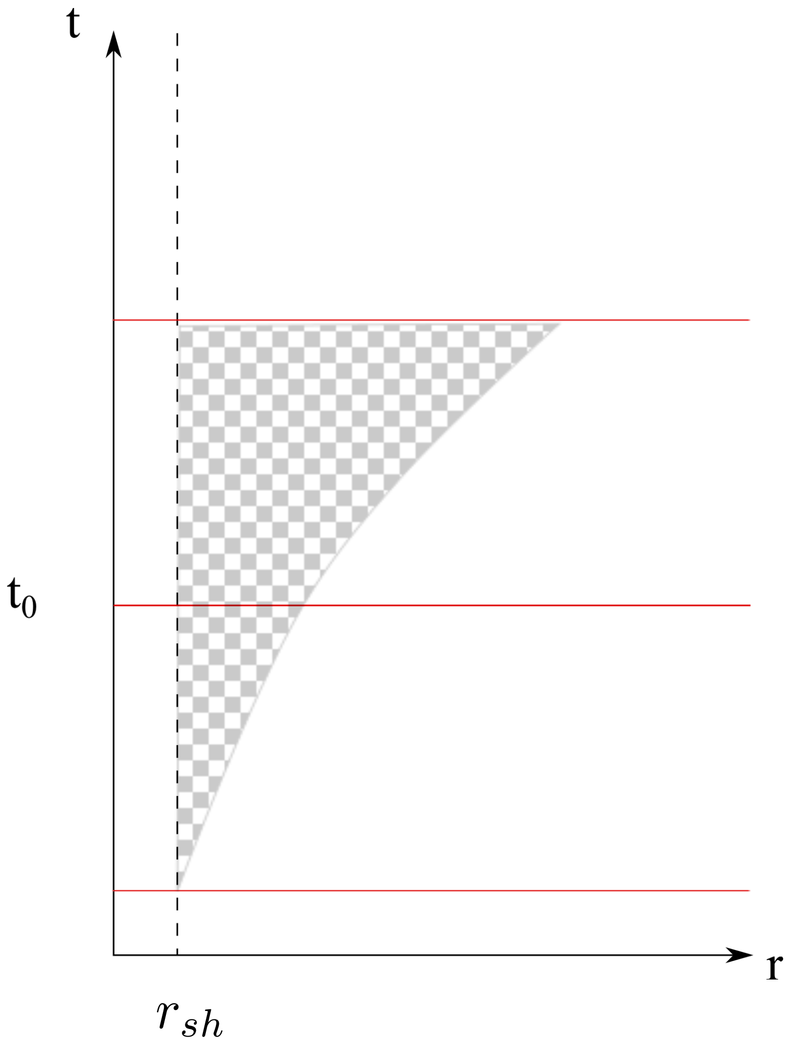
$r < r_{sh}$: Use free-fall vacuum



Outgoing modes at $r > r_{sh}$ are entangled with early Hawking radiation giving rise to a firewall inside black hole Almheiri et al 2012, Braunstein et al 2012

BUT the firewall meets the singularity before $t = t_0$ so O is not affected

Overlap region



Theories #1 and #2 both describe observations in shaded region

Q: Can measurements of Hawking particles emitted during [t_0 - t_{scr} , t_0 + t_{scr}] affect O inside the black hole?

A: O will burn at $t \sim t_0 + t_{scr}$, but that is also when O runs into the singularity.

No drama for infalling observer

Theories #1 and #2 need to have the following properties:

(1) The time required for outside observers to extract quantum information from the black hole (in theory #1) has a lower bound of order the scrambling time.

P. Hayden, J.Preskill '07
D.Lowe, LT in progress

(2) From the viewpoint of an infalling observer, who enters the black hole, any quantum information that entered more than a scrambling time earlier has been erased.

Property (2) holds in infalling lattice model D.Lowe, LT '15

Infalling lattice model

S.Corley & T.Jacobson '97; D.Lowe, LT '15

Coordinate system for infalling observer

$$ds^2 = -dt^2 + v^2(r)dy^2 + r^2 d\Omega^2$$

$$v(r) = -\sqrt{\frac{2M}{r}}$$

$$r(y,t) = 2M\left(1 + \frac{3}{4M}(y-t)\right)^{2/3}$$

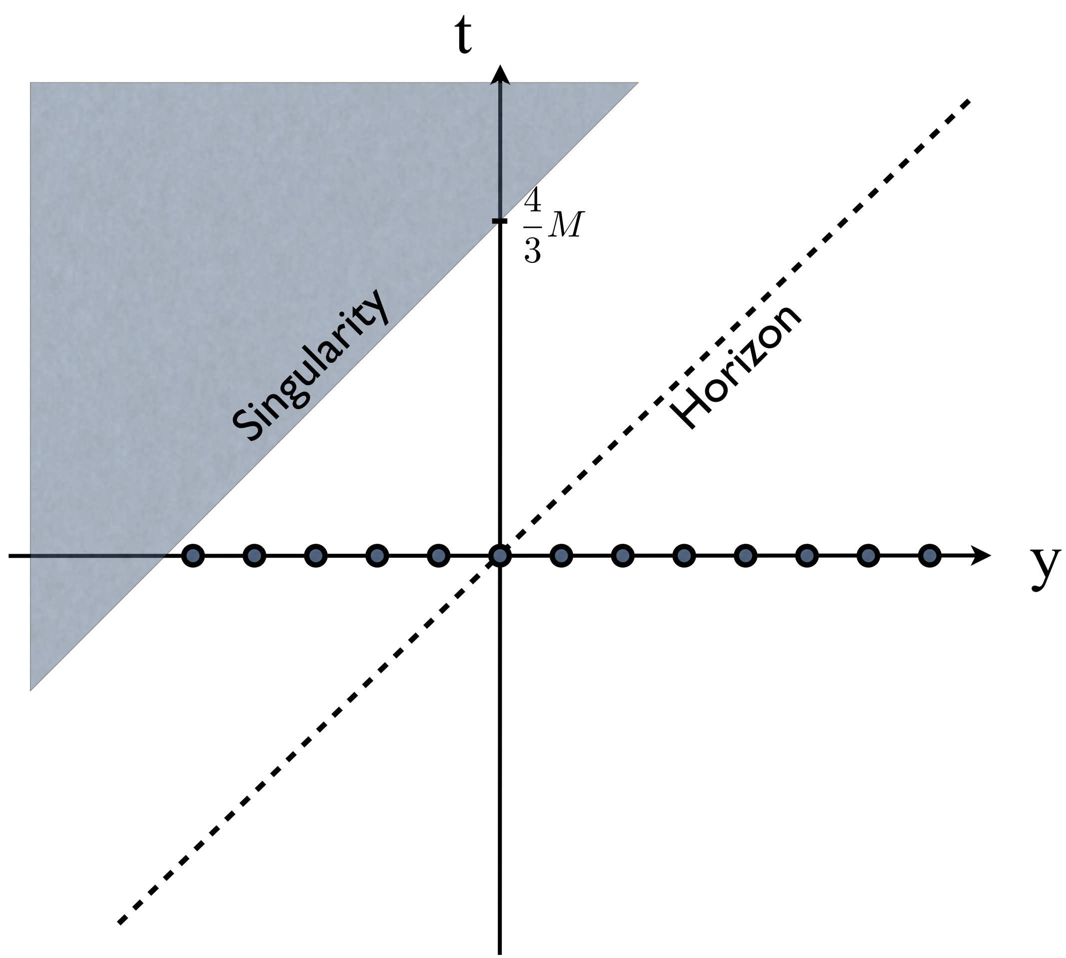
Observer in free fall near horizon: t = proper time, y = constant

Horizon is at $y = t$. Observer enters black hole at $t = y = 0$.

Curvature singularity is at $y = t - 4M/3$.

Lattice model: Discretise y coordinate

Infalling lattice (continued)



Infalling lattice (continued)

Lattice action: $$S = \frac{1}{2}\sum_y \int dt \left(|v(r(y,t))| \left(\frac{\partial \phi}{\partial t}\right)^2 - \frac{2\,(D_y\phi)^2}{|v(r(y+1,t)) + v(r(y,t))|} \right)$$

Killing symmetry: $$(y,t) \to (y+1, t+1)$$

Mode functions: $$\phi(y,t) = e^{-i\omega t} e^{ik(r)(y-t)}$$

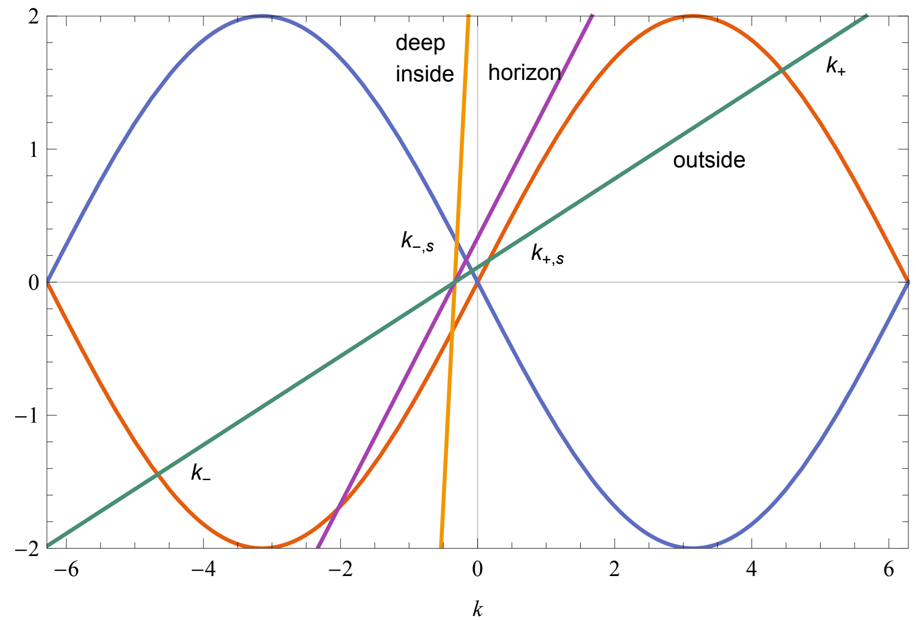
Free fall frequency: $$\omega_{ff} = \omega + k$$

Dispersion relation: $$|v(r)|(\omega + k) = \pm 2 \sin(k/2)$$

Group velocity: $$v_g = \frac{d\omega}{dk} = \pm \frac{\cos(k/2)}{|v|} - 1$$

Dispersion relation

$$|v(r)|(\omega + k) = \pm 2\sin(k/2) \qquad v(r) = -\sqrt{\frac{2M}{r}}$$



S.Corley & T.Jacobson '97:

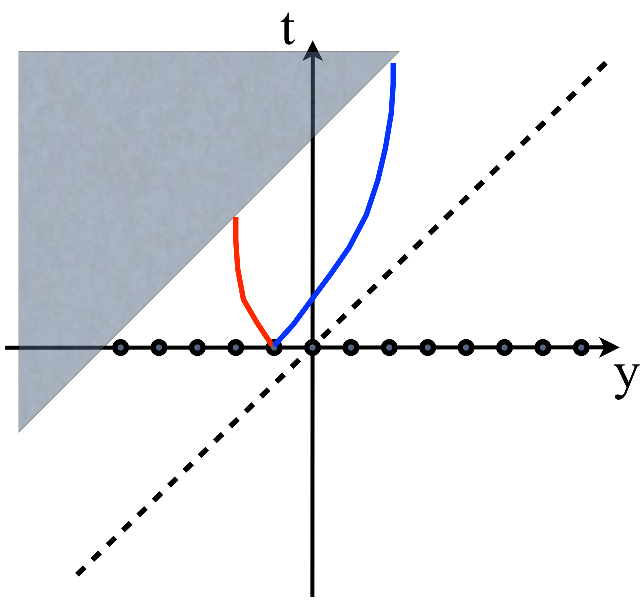
Free-fall vacuum initial state at $t = 0$ gives rise to outgoing thermal flux far outside the black hole.

Follows from WKB analysis of wavepackets outside black hole.

Interior wavepacket trajectories

D.Lowe, LT '15

Left- and right-moving wavepackets start at $t = 0$, $y = -1$

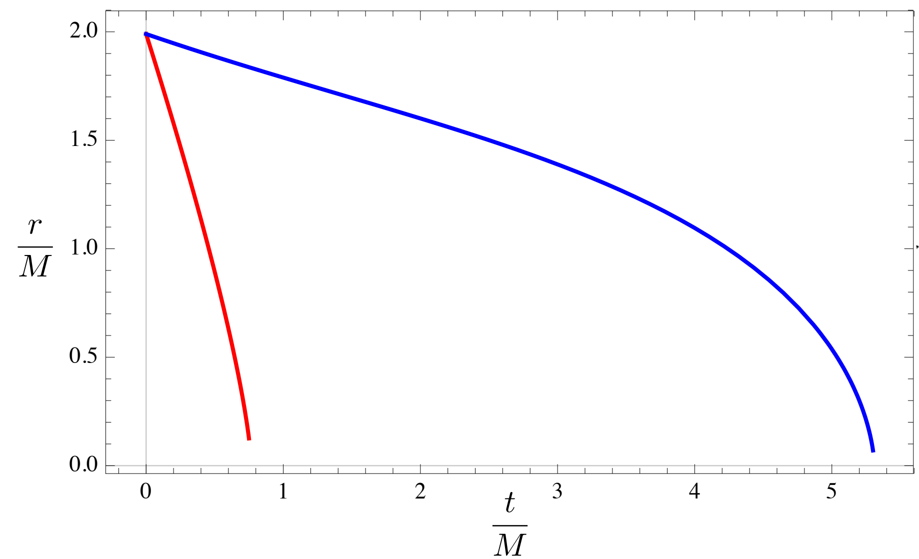


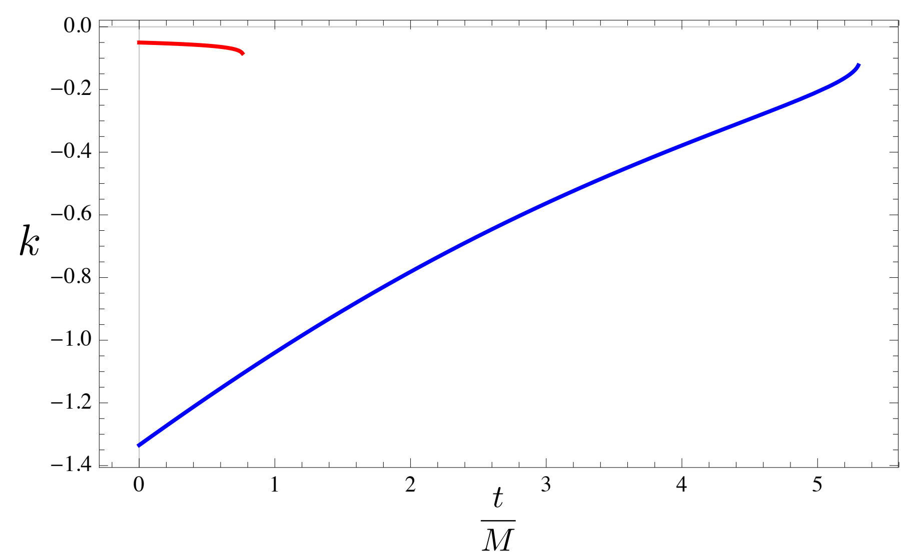
The wavepackets reach the singularity at different times:

Left: $t < 4M/3$

Right: $t < t_{scr} \equiv 4M \log(4M)$

WKB approximation





All information about interior quantum state at $t = 0$ is erased by $t = t_{scr}$

Breakdown of bulk description

We want to model a laboratory that falls into a black hole.

Early on the lab is well described by the bulk effective Hamiltonian of theory #2.

The lab has a complementary description in terms of theory #1 and must eventually decohere with respect to the exact Hamiltonian.

This will appear highly non-local from the interior viewpoint.

In a toy model we find that the decoherence time matches the scrambling time, which is also when lab approaches the singularity.

Results support the idea that singularity approach is complementary to decoherence of the infalling state.

Toy model for theory #1

D. Lowe and LT, in progress.

N. Lashkari et al., JHEP **1304** (2013) 022.

A toy model that exhibits fast scrambling is discussed in [10]. This is a spin model with a non-local pairwise interaction. There are N distinct sites with the Hilbert space of tensor product form $\mathcal{H} = \mathcal{H}_1 \otimes \cdots \otimes \mathcal{H}_N$. The sites interact via a pairwise Hamiltonian $H = \sum_{\langle x,y\rangle} H_{\langle x,y\rangle}$ summing over unordered pairs of sites. The Hamiltonian may therefore be associated with a graph $G = (V, E)$ with N vertices V, and edges E corresponding to the non-zero $H_{\langle x,y\rangle}$.

In order to have fast scrambling, the degree of the vertices D should be of order the size of the system. We shall then set $D = N - 1$. To have a sensible limit for large N, we take the pairwise interactions to be bounded $|H_{\langle x,y\rangle}| < c/D$, for some constant c.

We want to use this model to study the evolution of infalling degrees of freedom. We conjecture that evolution with respect to the bulk effective Hamiltonian of theory #2 is dual to mean field evolution in the holographic model.

Decoherence

We suppose the Hilbert space factors as

$$\mathcal{H} = \mathcal{H}_S \otimes \mathcal{H}_{S^c}$$

Consider a pure state in $\mathcal{H}_S$

$$|\Psi\rangle = (|\psi_1\rangle_S + |\psi_2\rangle_S) \otimes |\chi\rangle_{S^C}$$

Under time evolution this becomes

$$|\Psi'\rangle = e^{-iHt}|\Psi\rangle = \sum_i c_{1i}|e_i\rangle \otimes |f_{1i}\rangle + c_{2i}|e_i\rangle \otimes |f_{2i}\rangle$$

where the e_i are some basis of $\mathcal{H}_S$. If there is decoherence, then it is a good approximation to assume $|f_{1i}\rangle$ is orthogonal to $|f_{2j}\rangle$ for *any* i and j. For example, this will typically occur if the Hamiltonian is local in position space and $|f_{1i}\rangle$ and $|f_{2i}\rangle$ are position eigenstates. We will adopt the notation

$$\Psi_S = \mathrm{Tr}_{S^c}|\Psi\rangle\langle\Psi|$$

to denote the partial trace over the complement of S. If there is decoherence, then to a good approximation

$$\Psi'_S \approx \sum_i \left(|c_{1i}|^2 + |c_{2i}|^2\right)|e_i\rangle\langle e_i|$$

which means the probabilities add, without cross terms.

von Neumann entropy and trace distance

We use the von Neumann entropy $H = -\mathrm{Tr}_s \Psi'_S \log \Psi'_S$

to quantify the purity of the reduced density matrix. This vanishes for a pure state. For the maximally mixed state $\Psi'_S = \mathbb{1}/n$, with n is the dimension of the Hilbert subspace S, $H = \log n$.

We can then formulate the decoherence time t_d in the following way. Assume at time $t = 0$ Ψ_S is in a pure state. Then define the decoherence time t_d as the time when

$$H(\Psi_S(t_d)) = \delta \log n$$

for some choice of $\delta < 1$.

In the following we will mostly be interested in studying finite dimensional spin systems.

In this class of models, we can reformulate the condition as a condition on the trace distance

$$\|\Psi_S - \Phi_S\|_1 = \mathrm{Tr}_S \sqrt{(\Psi_S - \Phi_S)^\dagger (\Psi_S - \Phi_S)}$$

$|H(\Psi_1) - H(\Psi_2)| \leq \|\Psi_1 - \Psi_2\|_1 \log n$ M.Fannes, Comm. Math. Pays **31** no 4 (1973) 291.

Therefore the definition of the decoherence time can be reformulated as

$\|\Psi_S(t_d) - \Psi_S(0)\|_1 = \delta$ for some fixed constant $\delta < 1$.

Mean field evolution

Begin with an initial pure state of product form

$$|\Psi(0)\rangle = |\psi_1\rangle_{\mathcal{H}_1} \otimes \cdots \otimes |\psi_N\rangle_{\mathcal{H}_N}$$

Then one may build a state dependent mean field Hamiltonian

$$H^{MF} = \sum_x H_x^{MF}(t)$$
$$H_x^{MF} = \sum_y \mathrm{tr}_y \left(H_{\langle x,y\rangle} \Psi_y^{MF}(t) \right)$$

where Ψ^{MF} evolves according to H^{MF} starting from the same initial state $|\Psi(0)\rangle$. A key point is that with these definitions, and choice of initial state, the mean field Hamiltonian never generates entanglement between different sites, remains in the same product form as the initial state.

It is important to note that not all states yield sensible mean field evolutions. Moreover, the mean field Hamiltonian depends on the state. We conjecture that states close to smooth bulk spacetimes do have useful mean field descriptions, and that the mean field evolution is dual to the usual time evolution with respect to the bulk Hamiltonian.

One then wishes to calculate the timescale for which the trace norm distance between $\Psi_x(t)$ and $\Psi_x^{MF}(t)$ is small.

This problem was solved in [10] via careful application of Lieb-Robinson bounds

$$t \sim \log N$$

Summary

The information paradox highlights the incompatibility between general relativity (locality + equivalence principle) and quantum physics (unitarity).

Breakdown of local effective field theory, even on "nice" time slices that avoid strong curvature region.

Gauge theory - gravity correspondence implies unitary black hole evolution.

Black hole complementarity provides a "phenomenological" description, which preserves unitarity and the equivalence principle, but requires giving up locality.

Stretched horizon for outside observers can be motivated from QFT in black hole background.

Typical infalling observers do not see drama on their way towards a black hole formed from a generic pure state. (Special pure states, as well as special observers, exist for which this is not true.)

An approximate description of observers in the black hole interior can be given in terms of an effective field theory, defined on a limited set of time slices, such that no drama is seen until near the singularity.