

AdS₄ Black Holes and 3d Gauge Theories

Alberto Zaffaroni

Università di Milano-Bicocca

Current Themes in Holography
Niels Bohr Institute, April 2016

F. Benini-AZ; arXiv 1504.03698

F. Benini-K.Hristov-AZ; arXiv 1511.04085 and to appear

S. M. Hosseini-AZ; arXiv 1604.03122

Introduction

In this talk I want to relate two quantities

Introduction

In this talk I want to relate two quantities

- the entropy of a supersymmetric AdS_4 black holes in M theory

Introduction

In this talk I want to relate two quantities

- the entropy of a supersymmetric AdS_4 black holes in M theory
- a field theory computation for a partition function of the dual CFT_3

Introduction

In this talk I want to relate two quantities

- the entropy of a supersymmetric AdS_4 black holes in M theory
- a field theory computation for a partition function of the dual CFT_3

The computation uses recent localization techniques that allow to evaluate exact quantities in supersymmetric gauge theories.

Introduction

One of the success of string theory is the microscopic counting of micro-states for a class of asymptotically flat black holes entropy [Vafa-Strominger'96]

- The black holes are realized by putting together D-branes, extended objects that have gauge theories on the world-volume
- The entropy is obtained by counting states in the corresponding gauge theory

Introduction

No similar result for AdS black holes. But AdS should be simpler and related to holography:

- A gravity theory in AdS_{d+1} is the dual description of a CFT_d

The entropy should be related to the counting of states in the dual CFT. People tried for AdS_5 black holes (states in N=4 SYM). Still an open problem.

Prelude

Objects of interest

AdS₄ black holes

We are interested in **BPS** asymptotically AdS₄ static black holes

$$ds^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\mathcal{K}(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 ds_{S^2}^2$$

Some arise in a truncation of M theory on AdS₄ × S⁷

- vacua of a $N = 2$ gauged supergravity with 3 vector multiplets; one vector is the graviphoton.
- four abelian vectors $U(1)^4 \subset SO(8)$ that come from the reduction on S⁷.

[Cacciatori, Klemm; Gnechchi, Dall'agata; Hristov, Vandoren]

AdS₄ black holes

We are interested in **BPS** asymptotically AdS₄ static black holes

$$ds^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\mathcal{K}(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 ds_{S^2}^2$$

$$F = -2i\sqrt{X^0 X^1 X^2 X^3}$$

$$e^{-\mathcal{K}(X)} = i(\bar{X}^\Lambda F_\Lambda - X^\Lambda \bar{F}_\Lambda) = \sqrt{16X^0 X^1 X^2 X^3}$$

$$X^i = \frac{1}{4} - \frac{\beta_i}{r}, \quad X^0 = \frac{1}{4} + \frac{\beta_1 + \beta_2 + \beta_3}{r}$$

with arbitrary parameters $\beta_1, \beta_2, \beta_3$.

AdS₄ black holes

We are interested in **BPS** asymptotically AdS₄ static black holes

$$ds^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\mathcal{K}(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 ds_{S^2}^2$$

The parameters are related to the magnetic charges supporting the black hole

$$\mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_3, \mathbf{n}_4, \quad \mathbf{n}_i = \frac{1}{2\pi} \int_{S^2} F^{(i)}, \quad \sum \mathbf{n}_i = 2$$

by

$$\mathbf{n}_1 = 8(-\beta_1^2 + \beta_2^2 + \beta_3^2 + \beta_2\beta_3),$$

$$\mathbf{n}_2 = 8(-\beta_2^2 + \beta_1^2 + \beta_3^2 + \beta_1\beta_3),$$

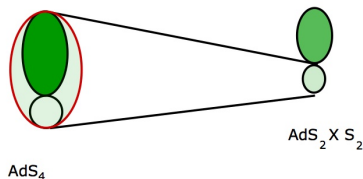
$$\mathbf{n}_3 = 8(-\beta_3^2 + \beta_1^2 + \beta_2^2 + \beta_1\beta_2).$$

AdS₄ black holes

We are interested in **BPS** asymptotically AdS₄ static black holes

$$ds^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\mathcal{K}(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 ds_{S^2}^2$$

Asymptotic to AdS₄ for $r \gg 1$ and with horizon AdS₂ × S² at some $r = r_h$



$$ds^2 \sim \frac{dr^2}{r^2} + r^2(-dt^2 + d\theta^2 + \sin^2 \theta d\phi^2)$$

$$ds^2 \sim -(r - r_h)^2 dt^2 + \frac{dr^2}{(r - r_h)^2} + (d\theta^2 + \sin^2 \theta d\phi^2)$$

AdS₄ black holes

We are interested in **BPS** asymptotically AdS₄ static black holes

$$ds^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\mathcal{K}(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 ds_{S^2}^2$$

The entropy is the area of S^2

$$S = -\frac{\pi}{G_4} \sqrt{X_1(r_h) X_2(r_h) X_3(r_h) X_4(r_h)} \sum_a \frac{n_a}{X_a(r_h)}$$

for example, for $n_1 = n_2 = n_3$

$$\sqrt{-1 + 6n_1 - 6n_1^2 + (-1 + 2n_1)^{3/2} \sqrt{-1 + 6n_1}}$$

AdS₄ black holes

We are interested in **BPS** asymptotically AdS₄ static black holes

$$ds^2 = e^{\mathcal{K}(X)} \left(gr + \frac{c}{2gr} \right)^2 dt^2 - \frac{e^{-\mathcal{K}(X)} dr^2}{\left(gr + \frac{c}{2gr} \right)^2} - e^{-\mathcal{K}(X)} r^2 ds_{S^2}^2$$

two real supercharges preserved

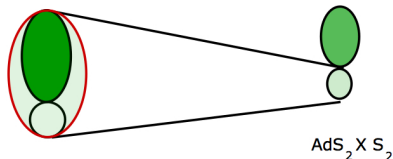
supersymmetry is preserved by a twist

$$(\nabla_\mu - iA_\mu)\epsilon = \partial_\mu\epsilon \quad \implies \quad \epsilon = \text{const}$$

AdS₄ black holes and holography

AdS black holes are dual to a topologically twisted CFT on $S^2 \times S^1$ with background magnetic fluxes for the global symmetries

$$ds_{d+1}^2 = \frac{dr^2}{r^2} + (r^2 ds_{M_d}^2 + O(r)) \quad A = A_{M_d} + O(1/r)$$



AdS₄

AdS₂ × S₂

Entropy of black holes
Counting of microstates

Partition function of twisted

3d CFT on $S^2 \times S^1$

QM fixed point

[In one dimension more: Benini-Bobev]

Part I

The index for topologically twisted theories in 3d

The topological twist

Consider an $\mathcal{N} = 2$ gauge theory on $S^2 \times S^1$

$$ds^2 = R^2(d\theta^2 + \sin^2 \theta d\varphi^2) + \beta^2 dt^2$$

with a magnetic background for the R- and flavor symmetries:

$$A^R = -\frac{1}{2} \cos \theta d\varphi = -\frac{1}{2} \omega^{12}, \quad A^F = -\frac{n^F}{2} \cos \theta d\varphi = -\frac{n^F}{2} \omega^{12}$$

In particular A^R is equal to the spin connection so that

$$D_\mu \epsilon = \partial_\mu \epsilon + \frac{1}{4} \omega_\mu^{ab} \gamma_{ab} \epsilon - i A_\mu^R \epsilon = 0 \quad \implies \quad \epsilon = \text{const}$$

This is just a topological twist. [Witten '88]

The background

Supersymmetry can be preserved by turning on supersymmetric backgrounds for the flavor symmetry multiplets (A_μ^F, σ^F, D^F) :

$$u^F = A_t^F + i\sigma^F, \quad \mathfrak{n}^F = \int_{S^2} F^F = iD^F$$

and the path integral, which can be exactly computed by localization, becomes a function of a set of magnetic charges \mathfrak{n}^F and chemical potentials u^F .

[Benini-AZ; arXiv 1504.03698]

A topologically twisted index

The path integral can be re-interpreted as a **twisted index**: a trace over the Hilbert space \mathcal{H} of states on a sphere in the presence of a magnetic background for the R and the global symmetries,

$$\mathrm{Tr}_{\mathcal{H}} \left((-1)^F e^{iJ_F A^F} e^{-\beta H} \right)$$

$$Q^2 = H - \sigma^F J_F$$

holomorphic in u^F

where J_F is the generator of the global symmetry.

The partition function

The path integral on $S^2 \times S^1$ reduces as usual, by localization, to a matrix model depending on few zero modes of the gauge multiplet $V = (A_\mu, \sigma, \lambda, \lambda^\dagger, D)$

- A magnetic flux on S^2 , $\mathfrak{m} = \frac{1}{2\pi} \int_{S^2} F$ in the co-root lattice
- A Wilson line A_t along S^1
- The vacuum expectation value σ of the real scalar

The path integral reduces to an r -dimensional contour integral of a meromorphic form

$$\frac{1}{|W|} \sum_{\mathfrak{m} \in \Gamma_{\mathfrak{h}}} \oint_{\mathcal{C}} Z_{\text{int}}(u, \mathfrak{m}) \quad u = A_t + i\sigma$$

The partition function

- In each sector with gauge flux m we have a meromorphic form

$$Z_{\text{int}}(u, m) = Z_{\text{class}} Z_{1\text{-loop}}$$

$$Z_{\text{class}}^{\text{CS}} = x^{km}$$

$$x = e^{iu}$$

$$Z_{1\text{-loop}}^{\text{chiral}} = \prod_{\rho \in \mathfrak{R}} \left[\frac{x^{\rho/2}}{1 - x^{\rho}} \right]^{\rho(m) - q + 1}$$

$q = R$ charge

$$Z_{1\text{-loop}}^{\text{gauge}} = \prod_{\alpha \in G} (1 - x^{\alpha}) (i du)^r$$

- Supersymmetric localization selects a particular contour of integration C and picks some of the residues of the form $Z_{\text{int}}(u, m)$.

[Jeffrey-Kirwan residue - similar to Benini, Eager, Hori, Tachikawa '13; Hori, Kim, Yi '14]

A Simple Example: SQED

The theory has gauge group $U(1)$ and two chiral Q and \tilde{Q}

$$Z = \sum_{m \in \mathbb{Z}} \int \frac{dx}{2\pi i x} \left(\frac{x^{\frac{1}{2}} y^{\frac{1}{2}}}{1 - xy} \right)^{m+n} \left(\frac{x^{-\frac{1}{2}} y^{\frac{1}{2}}}{1 - x^{-1}y} \right)^{-m+n}$$

| | $U(1)_E$ | $U(1)_A$ | $U(1)_R$ |
|-------------|----------|----------|----------|
| Q | 1 | 1 | 1 |
| \tilde{Q} | -1 | 1 | 1 |

Consistent with duality with three chirals with superpotential XYZ

$$Z = \left(\frac{y}{1 - y^2} \right)^{2n-1} \left(\frac{y^{-\frac{1}{2}}}{1 - y^{-1}} \right)^{-n+1} \left(\frac{y^{-\frac{1}{2}}}{1 - y^{-1}} \right)^{-n+1}$$

Dualities and generalizations

Many generalizations

- We can add refinement for angular momentum on S^2 .
- We can consider higher genus $S^2 \rightarrow \Sigma$

Dualities and generalizations

Many generalizations

- We can add refinement for angular momentum on S^2 .
- We can consider higher genus $S^2 \rightarrow \Sigma$

We can go up and down in dimension and compute

- amplitudes in gauged linear sigma models for $(2,2)$ theories in 2d on S^2 [also Cremonesi-Closset-Park '15]
- an elliptically generalized twisted index for $\mathcal{N} = 1$ theory on $S^2 \times T^2$ we are computing [also Closset-Shamir '13;Nishioka-Yaakov '14;Yoshida-Honda '15]

Dualities and generalizations

Many generalizations

- We can add refinement for angular momentum on S^2 .
- We can consider higher genus $S^2 \rightarrow \Sigma$

We can go up and down in dimension and compute

- amplitudes in gauged linear sigma models for (2,2) theories in 2d on S^2 [also Cremonesi-Closset-Park '15]
- an elliptically generalized twisted index for $\mathcal{N} = 1$ theory on $S^2 \times T^2$ we are computing [also Closset-Shamir '13; Nishioka-Yaakov '14; Yoshida-Honda '15]

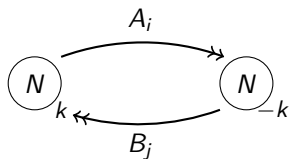
The index adds to and complete the list of existing tools (superconformal indices, sphere partition functions) for testing dualities: **Aharony; Gaiotto-Kutasov in 3d; Seiberg in 4d, ...**

Part II

Comparison with the black hole entropy

Going back to the black hole

The dual field theory to $\text{AdS}_4 \times S^7$ is known: is the ABJM theory with gauge group $U(N) \times U(N)$

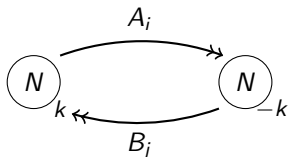


with quartic superpotential

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

Going back to the black hole

Black hole supported by magnetic charges: is the ABJM theory with gauge group $U(N) \times U(N)$



with quartic superpotential

$$W = A_1 B_1 A_2 B_2 - A_1 B_2 A_2 B_1$$

defined on twisted $S^2 \times \mathbb{R}$ with magnetic fluxes n_i for the $U(1)$ /global symmetries

$$U(1)^4 \subset SU(2)_A \times SU(2)_B \times U(1)_B \times U(1)_R \subset SO(8)$$

The dual field theory

The ABJM twisted index is

$$\begin{aligned}
 Z = & \frac{1}{(N!)^2} \sum_{\mathbf{m}, \tilde{\mathbf{m}} \in \mathbb{Z}^N} \int \prod_{i=1}^N \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} x_i^{k m_i} \tilde{x}_i^{-k \tilde{m}_i} \times \prod_{i \neq j} \left(1 - \frac{x_i}{x_j}\right) \left(1 - \frac{\tilde{x}_i}{\tilde{x}_j}\right) \times \\
 & \times \prod_{i,j=1}^N \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}} y_1}{1 - \frac{x_i}{\tilde{x}_j} y_1} \right)^{m_i - \tilde{m}_j - n_1 + 1} \left(\frac{\sqrt{\frac{x_i}{\tilde{x}_j}} y_2}{1 - \frac{x_i}{\tilde{x}_j} y_2} \right)^{m_i - \tilde{m}_j - n_2 + 1} \\
 & \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}} y_3}{1 - \frac{\tilde{x}_j}{x_i} y_3} \right)^{\tilde{m}_j - m_i - n_3 + 1} \left(\frac{\sqrt{\frac{\tilde{x}_j}{x_i}} y_4}{1 - \frac{\tilde{x}_j}{x_i} y_4} \right)^{\tilde{m}_j - m_i - n_4 + 1}
 \end{aligned}$$

where $\mathbf{m}, \tilde{\mathbf{m}}$ are the gauge magnetic fluxes and y_i are fugacities for the three independent $U(1)$ global symmetries ($\prod_i y_i = 1$)

The dual field theory

Strategy:

- Re-sum geometric series in $\mathfrak{m}, \tilde{\mathfrak{m}}$.

$$Z = \int \frac{dx_i}{2\pi i x_i} \frac{d\tilde{x}_i}{2\pi i \tilde{x}_i} \frac{f(x_i, \tilde{x}_i)}{\prod_{j=1}^N (e^{iB_j} - 1) \prod_{j=1}^N (e^{i\tilde{B}_j} - 1)}$$

- Step 1: find the zeros of denominator $e^{iB_j} = e^{i\tilde{B}_j} = 1$ at large N
- Step 2: evaluate the residues at large N

$$Z \sim \sum_I \frac{f(x_i^{(0)}, \tilde{x}_i^{(0)})}{\det \mathbb{B}}$$

[Benini-Hristov-AZ; arXiv 1511.04085]

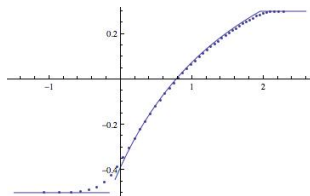
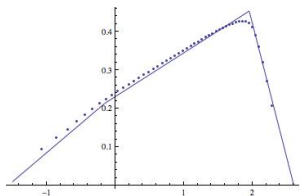
The large N limit

Step 1: solve the large N Limit of algebraic equations giving the positions of poles

$$1 = x_i^k \prod_{j=1}^N \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})} = \tilde{x}_j^k \prod_{i=1}^N \frac{(1 - y_3 \frac{\tilde{x}_j}{x_i})(1 - y_4 \frac{\tilde{x}_j}{x_i})}{(1 - y_1^{-1} \frac{\tilde{x}_j}{x_i})(1 - y_2^{-1} \frac{\tilde{x}_j}{x_i})}$$

with an ansatz

$$\log x_i = i\sqrt{N}t_i + v_i, \quad \log \tilde{x}_j = i\sqrt{N}t_j + \tilde{v}_j$$



The large N limit

- We dubbed this set of equations *Bethe Ansatz Equations* in analogy with similar expressions in integrability business [Nekrasov-Shatashvili]

$$e^{iB_i} = e^{i\tilde{B}_i} = 1$$

They can be derived by a BA potential $\mathcal{V}_{BA}(y)$.

The large N limit

- We dubbed this set of equations *Bethe Ansatz Equations* in analogy with similar expressions in integrability business [Nekrasov-Shatashvili]

$$e^{iB_i} = e^{i\tilde{B}_i} = 1$$

They can be derived by a BA potential $\mathcal{V}_{BA}(y)$.

- In the large N limit, these *auxiliary* BAE are the same appearing in a different localization problem: the path integral on S^3 [Hosseini-AZ; arXiv 1604.03122]

$$\mathcal{V}_{BA}(\Delta) = Z_{S^3}(\Delta) \quad y_i = e^{i\Delta_i}$$

The same holds for other 3d quivers dual to M theory backgrounds $\text{AdS}_4 \times Y_7$ ($N^{3/2}$) and massive type IIA ones ($N^{5/3}$).

The large N limit

Step 2: plug into the partition function. It is crucial to keep into account exponentially small corrections in tail regions where $y_i x_i / \tilde{x}_i = 1$

$$\log Z = N^{3/2}(\text{finite}) + N \log(1 - y_i x_i / \tilde{x}_i) \qquad y_i x_i / \tilde{x}_i = 1 + e^{-N^{1/2} Y_i}$$

The large N limit

Step 2: plug into the partition function. It is crucial to keep into account exponentially small corrections in tail regions where $y_i x_i / \tilde{x}_i = 1$

$$\log Z = N^{3/2}(\text{finite}) + N \log(1 - y_i x_i / \tilde{x}_i) \qquad y_i x_i / \tilde{x}_i = 1 + e^{-N^{1/2} Y_i}$$

One can by-pass it by using a general simple formula [\[Hosseini-AZ; arXiv 1604.03122\]](#)

$$\log Z = - \sum_I n_I \frac{\partial \mathcal{V}_{BA}}{\partial \Delta_I}$$

The main result

The index is obtained from $\mathcal{V}_{BA} \sim \sqrt{\Delta_1 \Delta_2 \Delta_3 \Delta_4}$:

$$\mathbb{R}e \log Z = -\frac{1}{3} N^{3/2} \sqrt{2k \Delta_1 \Delta_2 \Delta_3 \Delta_4} \sum_a \frac{n_a}{\Delta_a} \quad y_i = e^{i\Delta_i}$$

This function can be extremized with respect to the Δ_i and

$$\mathbb{R}e \log Z|_{crit}(\mathbf{n}_i) = \text{BH Entropy}(\mathbf{n}_i)$$

$$\Delta_i|_{crit} \sim X^i(r_h)$$

[Benini-Hristov-AZ; arXiv 1511.04085]

The main result

Compare the field theory formula

$$\mathbb{R}e \log Z = -\frac{1}{3} N^{3/2} \sqrt{2k\Delta_1\Delta_2\Delta_3\Delta_4} \sum_a \frac{n_a}{\Delta_a}$$

with the gravity one

$$S = -\frac{\pi}{G_4} \sqrt{X_1(r_h)X_2(r_h)X_3(r_h)X_4(r_h)} \sum_a \frac{n_a}{X_a(r_h)}$$

Part III

Interpretation and Conclusions

A. Statistical ensemble

Δ_a can be seen as chemical potential in a macro-canonical ensemble defined by the supersymmetric index

$$Z = \text{Tr}_{\mathcal{H}} (-1)^F e^{i\Delta_a J_a} e^{-\beta H}$$

so that the extremization can be rephrased as the statement that the black hole is electrically neutral

$$\frac{\partial}{\partial \Delta} \log Z \sim \langle J \rangle = 0$$

- The result can be extended to dyonic black holes again with perfect matching [Benini-Hristov-AZ to appear].
- Similarities with Sen's entropy formalism based on AdS_2 .

B. Attractor mechanism

The BPS equations at the horizon imply that the gauge supergravity quantity

$$\mathcal{R} = (F_\Lambda \rho^\Lambda - X^\Lambda q_\Lambda) , \quad F_\Lambda = \frac{\partial \mathcal{F}}{\partial X^\Lambda}$$

with (q, ρ) electric and magnetic charges, is extremized with respect to the scalar fields at the horizon and

$$S = |\mathcal{R}|$$

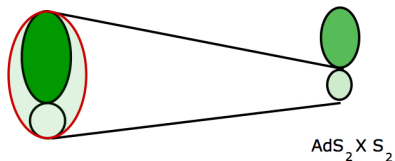
For us $\rho^\Lambda = n^\Lambda$ and $q_\Lambda = 0$ and, under $X^\Lambda \rightarrow \Delta^\Lambda$

$$\mathcal{F} = 2i\sqrt{X^0 X^1 X^2 X^3} \equiv \mathcal{V}_{BA}(\Delta)$$

$$\mathcal{R} = - \sum \frac{n_\Lambda}{X^\Lambda} \sqrt{X^0 X^1 X^2 X^3} \equiv \log Z(\Delta)$$

C. The IR superconformal QM

Recall the cartoon



Entropy of black holes
Counting of microstates

Partition function of twisted
3d CFT on $S_2 \times S_1$

QM fixed point

R-symmetry mixing

The extremization reflects exactly what's going on in the bulk. The graviphoton field strength depends on r

$$T_{\mu\nu} = e^{K/2} X^\Lambda F_{\Lambda,\mu\nu}$$

suggesting that the R-symmetry is different in the IR and indeed

$$\Delta_i|_{crit} \sim X^i(r_h)$$

R-symmetry extremization

Some QFT extremization is at work? symmetry enhancement at the horizon AdS₂

$$\text{QM}_1 \rightarrow \text{CFT}_1$$

The twisted index depends on Δ_i because we are computing the trace

$$\text{Tr}_{\mathcal{H}}(-1)^F e^{i\Delta_i J_i} \equiv \text{Tr}_{\mathcal{H}}(-1)^R$$

where $R = F + \Delta_i J_i$ is a possible R-symmetry of the system.

- R is the exact R-symmetry at the superconformal point
- Natural thing to extremize: in even dimensions central charges are extremized, in odd partition functions...

Conclusions

The main message of this talk is that you can related the entropy of a class of AdS_4 black holes to a microscopic counting of states.

- first time for AdS black holes in four dimensions

Conclusions

The main message of this talk is that you can related the entropy of a class of AdS_4 black holes to a microscopic counting of states.

- first time for AdS black holes in four dimensions

But don't forget that we also gave a general formula for the topologically twisted path integral of 2d $(2,2)$, 3d $\mathcal{N} = 2$ and 4d $\mathcal{N} = 1$ theories.

- Efficient quantum field theory tools for testing dualities.
- With many field theory questions/generalizations