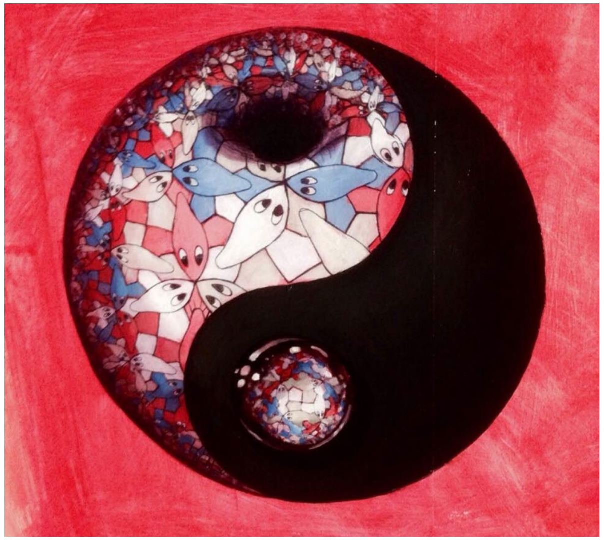
Holographic metals do not Anderson localize

Koenraad Schalm

Institute Lorentz for Theoretical Physics, Leiden University



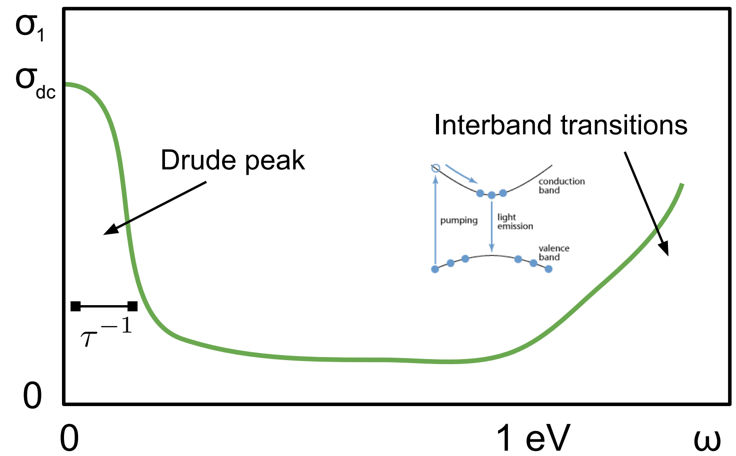
Netherlands Organisation for Scientific Research

"Lattice" effects at low energy

- Broken translations affect transport
  - Drude model

$$\frac{d\vec{p}}{dt} = q\vec{E} - \frac{\vec{p}}{\tau} \qquad \vec{J} = \frac{n_e q}{m}\vec{p} \qquad \vec{J} = \sigma\vec{E}$$

$$\sigma(\omega) \sim \frac{1}{1/\tau - i\omega}$$



- Origin translational symmetry breaking
  - Lattice — distinct — k_L
  - Impurities ("disorder") — "ensemble average" — $\langle\langle \cdot \rangle\rangle = \int d^d k_L \, \langle \cdot \rangle_{k_L}$
- Translational symmetry breaking can be *weak* or *strong*

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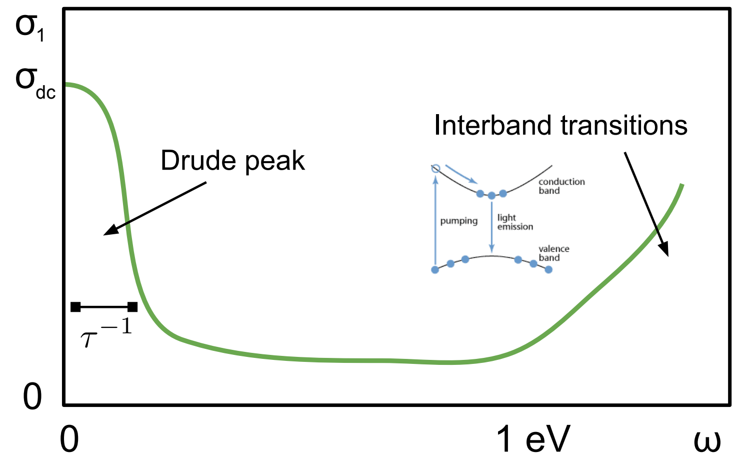

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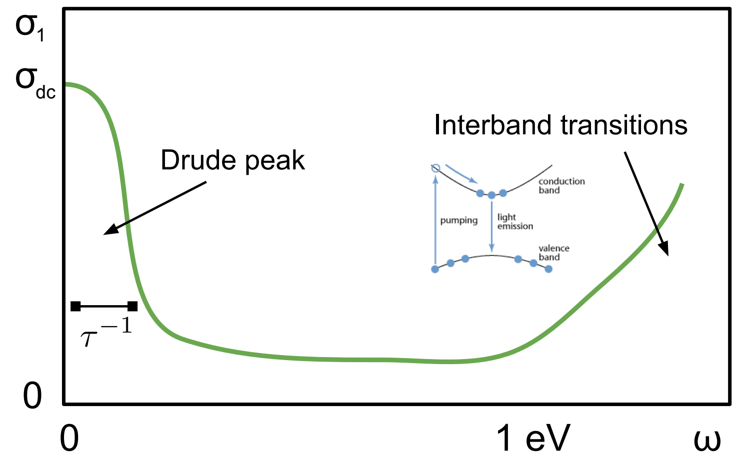

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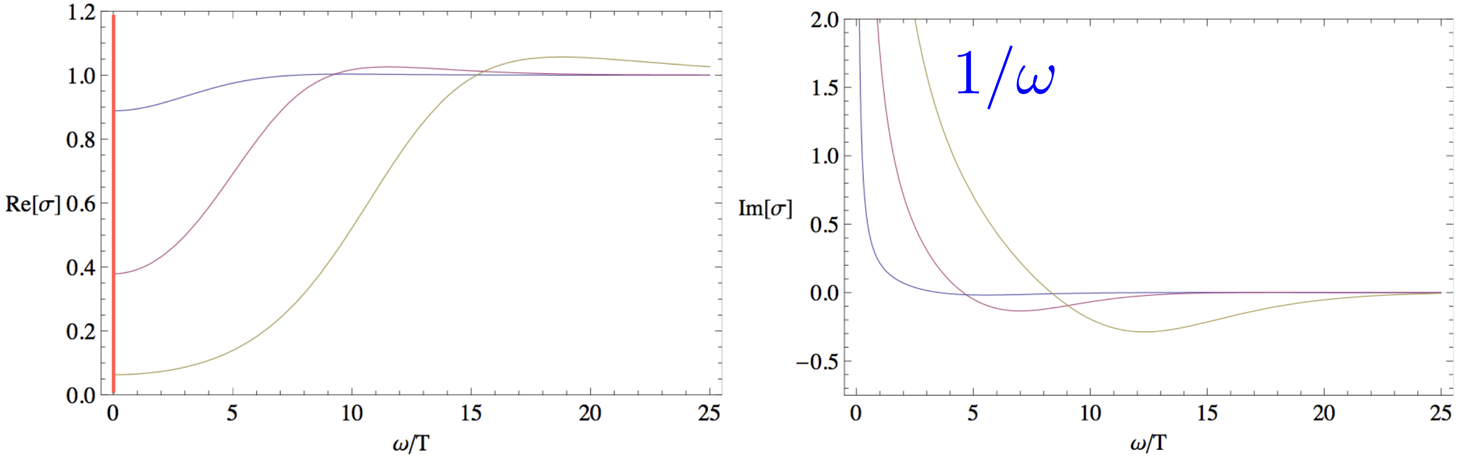
Resistivities and relaxation

- Conductivity of the CFT dual to AdS-RN



Finite DC conductivity = broken translational invariance

Momentum relaxation: (1) lattice or (2) impurities or ...

Momentum relaxation in holography

- Momentum relaxation most generally

Vegh;
Andrade, Withers;
Donos, Gauntlett; ...

  - AdS/CFT:

$$CFT \text{ isometries} = AdS \text{ diffeomorphisms}$$

  - Simplest way to break AdS diffs = massive gravity

de Rahm
Gabadadze
Tolley

$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left[R - \frac{1}{2}m^2\left((\mathrm{Tr}\mathcal{K})^2 - \mathrm{Tr}\mathcal{K}^2\right) + \ldots\right]$$

specific reference metric

$$\mathcal{K}^\mu_\alpha \mathcal{K}^\alpha_\nu = g^{\mu\alpha} f_{\alpha\nu}$$

  - Resistivity in a charged black hole in massive gravity

$$\tau_{\mathrm{rel.}}^{-1} = \frac{sm^2}{2\pi(\epsilon + P)}$$

Davison;
Blake, Tong

no momentum dependence: disorder!

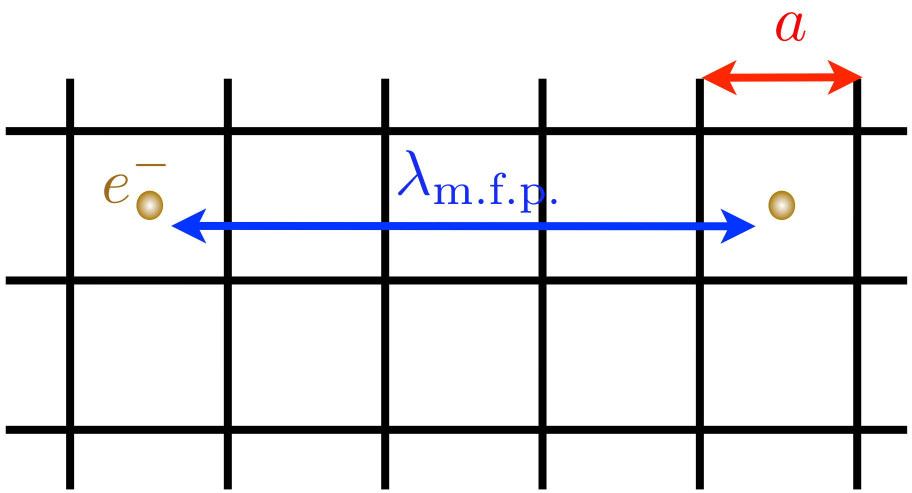
- Resistivity in a charged black hole in massive gravity

$$\tau_{\text{rel.}}^{-1} = \frac{sm^2}{2\pi(\epsilon + P)}$$

  - Controlled by macroscopic properties !

- Ordinary metals
  - Momentum relaxes before collective behavior sets in

$$\tau_{\text{rel.}}^{-1} \sim \text{micro. physics}$$



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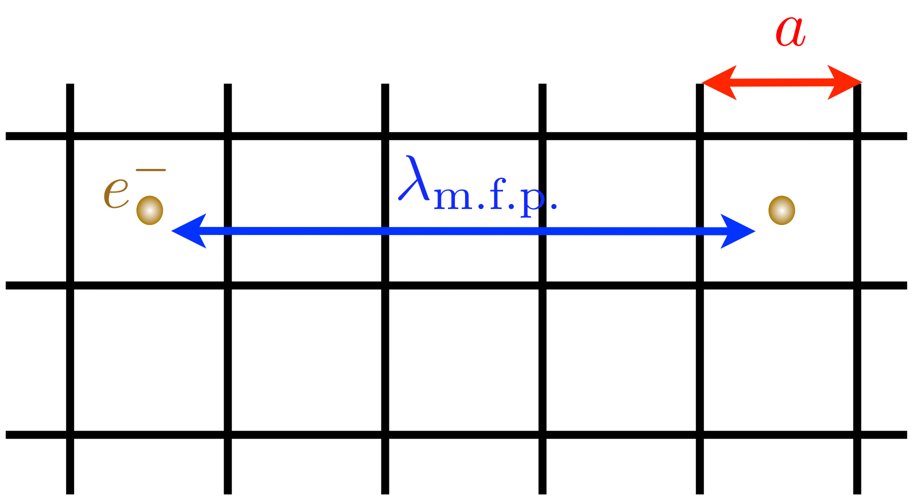


- Strongly correlated metals (no quasiparticles)

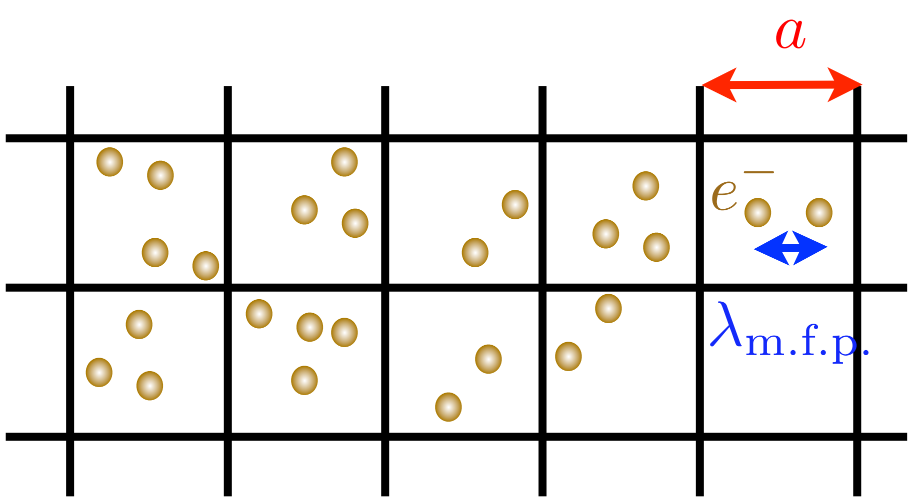
$$\lambda_{\mathrm{m.f.p.}} \ll \mathrm{external\ scales}$$

  - Hydro sets in when

$$\lambda_{\mathrm{m.f.p.}} \ll \frac{g_{\mathrm{coupling}}}{T}$$

  - Momentum relaxes after collective behavior sets in

$$\tau_{\mathrm{rel.}}^{-1} \sim \mathrm{macro.\ physics}$$



Resistivity and hydrodynamics

- Hydrodynamics is a universal LEET

Davison, Schalm, Zaanen

Andreev, Kivelson, Spivak

$$\rho_{DC} \sim \lim_{\omega \to 0} \int dk k^2 \frac{\mathrm{Im}\langle \mathcal{O}\mathcal{O} \rangle}{\omega}$$

- What choice for the impurity operator $\mathcal{O}$?
- Hydrodynamics: $T_{\mu\nu},\ J_\nu\ +\ \text{"irrelevant" ops}$
- For $\mathcal{O} = T^{00}$

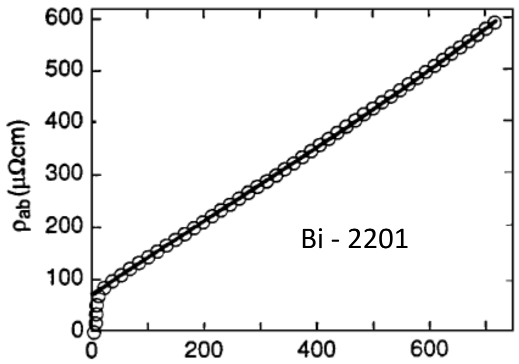
$$\langle T^{00} T^{00} \rangle \sim \frac{1}{\omega^2 - k^2 + i\omega k^2 c_d \frac{\eta}{\epsilon + P} k^2 + \ldots}$$

$$\rho_{DC} \sim \lim_{\omega \to 0} \int dk k (\eta k^2 + \ldots) \sim s(T) \qquad \eta = \frac{1}{4\pi} s$$

- Caveat: theory must be locally quantum critical $z \simeq \infty$

Lucas, Sachdev, Schalm,
Hartnoll, Mahajan, Punk, Sachdev

- Strange metals exhibit a universal linear resistivity



Martin et al, PRB41 (1990) 846

- Follows from $\rho \sim s(T)$ if $s(T) \sim T + \ldots$

- Entropy density at low T (free fermi gas)

Davison, Schalm, Zaanen

$$s(T) \sim T + \dots$$

  - Then $\rho_{DC} \sim s(T) \sim T + \dots$
  - Can be confirmed in a massive gravity model

"Two-charge" AdS-black hole

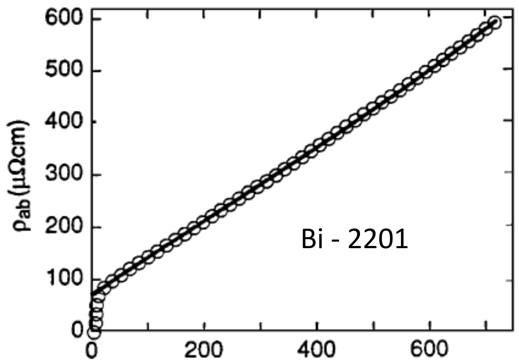
$$S = \frac{1}{2\kappa^2}\int d^4x\sqrt{-g}\left(R - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{3}{2}|\partial_\mu\Phi|^2 + \frac{6}{L^2}\cosh\Phi - \frac{1}{2}m^2(\mathrm{Tr}(\mathcal{K})^2 - \mathrm{Tr}(\mathcal{K}^2))\right)$$

$$s_{BH} \sim T\mu + \dots$$

Resistivity from hydro + disorder

$$\rho_{DC} \sim T + \dots$$

- Strange metals exhibit a universal linear resistivity

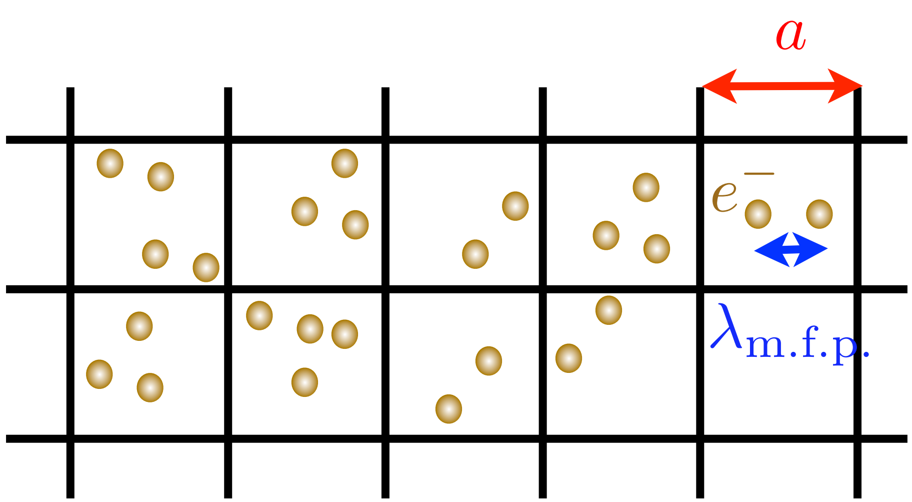


Martin et al, PRB41 (1990) 846

  - Follows from $\rho \sim s(T)$ if $s(T) \sim T + \ldots$

- Caveat: holography has many other "linear resistivity" scenarios

- System collectivizes before momentum relaxes



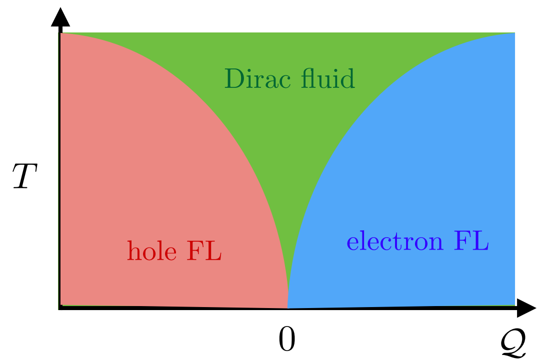
  - Hydrodynamics as universal LEET

$$\mathcal{O} = J^0 \longrightarrow \rho_{DC} \sim \lim_{\omega\to 0}\int dk k^2 \frac{\mathrm{Im}\langle \mathcal{O}\mathcal{O}\rangle}{\omega}$$

$$\mu = \mu_0 + \delta\mu(x) \longrightarrow \begin{pmatrix} J \\ Q \end{pmatrix} = \begin{pmatrix} \sigma & \alpha T \\ \alpha T & T\bar{\kappa} \end{pmatrix} \begin{pmatrix} E \\ -(\nabla T)/T \end{pmatrix}$$

Lucas

$$\epsilon_{a\sigma} = \hbar v_{\mathrm{F}} k$$
$$+$$
$$V_{\mathrm{int}} = \frac{\alpha_{\mathrm{eff}}}{r}$$



- marginally irrelevant $1/r$ Coulomb interactions:

$$\alpha_{\mathrm{eff}} = \frac{\alpha_0}{1+(\alpha_0/4)\log((10^5\ \mathrm{K})/T)}, \quad \alpha_0 \approx \frac{1}{137}\frac{c}{v_{\mathrm{F}}\epsilon_{\mathrm{r}}} \sim 0.5.$$

- thermo/hydro nearly that of relativistic theory
- $\alpha_{\mathrm{eff}} \sim 0.3$ at $T = 100$ K

e.g. [Sheehy, Schmalian, *Physical Review Letters* **99** 226803 (2007)]
[Müller, Fritz, Sachdev, *Physical Review* **B78** 115406 (2008)]

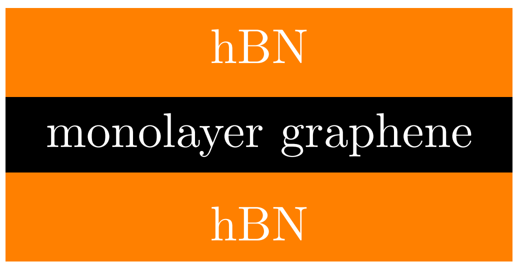
Crossno, Kim et al.
Lucas, Crossno, Fong, Kim, Sachdev

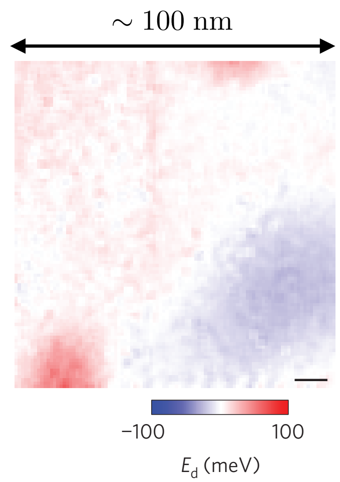
Slides from A. Lucas

Dirac Fluid in Graphene

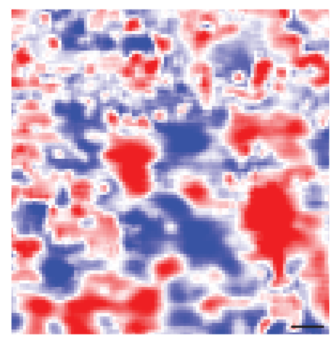
Graphene: an Ideal Experimental Platform

- fabricating ultra pure monolayer graphene:
  [Dean *et al*, *Nature Nanotechnology* **5** 722 (2010)]





hBN
$\Delta\mu \sim 30$ K



SiO_2
$\Delta\mu \sim 300$ K

- weak disorder: charge puddles
  [Xue *et al*, *Nature Materials* **10** 282 (2011)]

Crossno, Kim et al.
Lucas, Crossno, Fong, Kim, Sachdev

Slides from A. Lucas

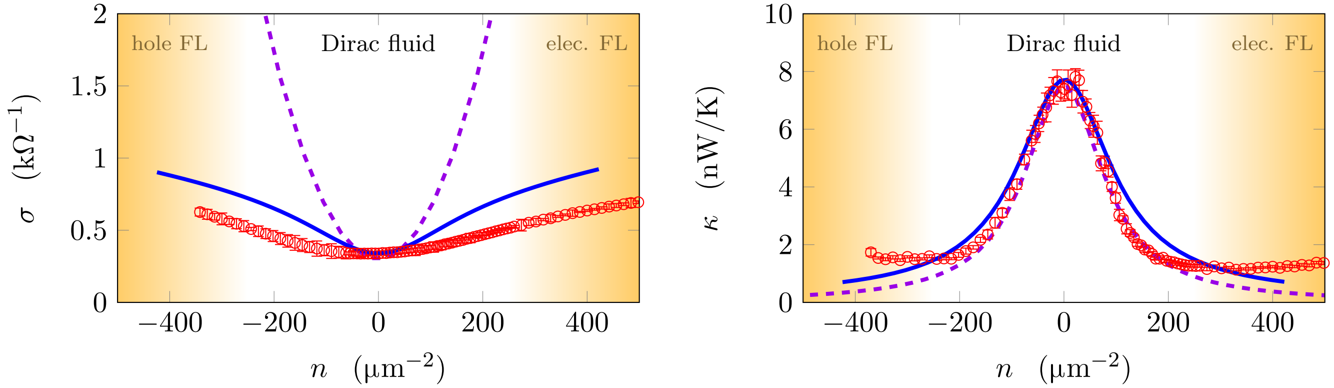
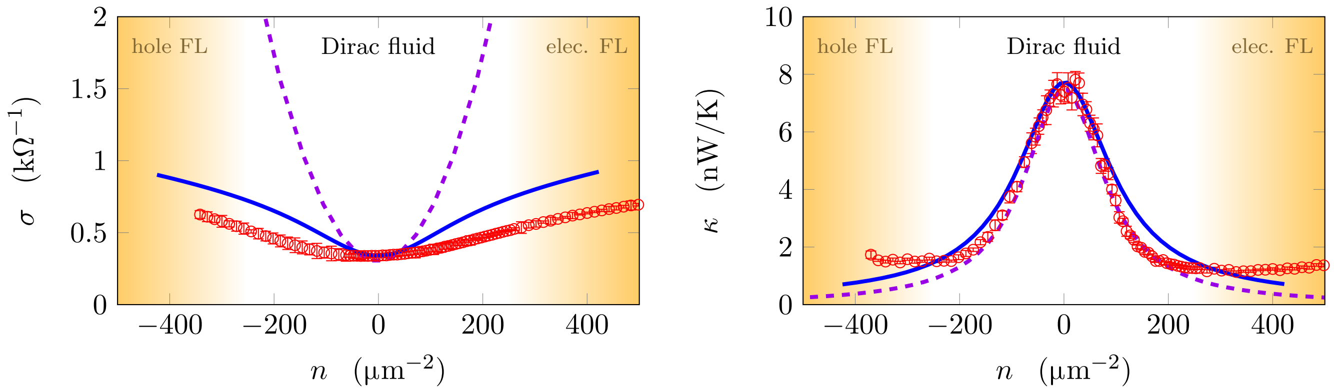


Figure 1: A comparison of our hydrodynamic theory of transport with the experimental results of [33] in clean samples of graphene at $T = 75$ K. We study the electrical and thermal conductances at various charge densities n near the charge neutrality point. Experimental data is shown as circular red data markers, and numerical results of our theory, averaged over 30 disorder realizations, are shown as the solid blue line. Our theory assumes the equations of state described in (27) with the parameters $C_0 \approx 11$, $C_2 \approx 9$, $C_4 \approx 200$, $\eta_0 \approx 110$, $\sigma_0 \approx 1.7$, and (28) with $u_0 \approx 0.13$. The yellow shaded region shows where Fermi liquid behavior is observed and the Wiedemann-Franz law is restored, and our hydrodynamic theory is not valid in or near this regime. We also show the predictions of (2) as dashed purple lines, and have chosen the 3 parameter fit to be optimized for $\kappa(n)$.

Crossno, Kim et al.
Lucas, Crossno, Fong, Kim, Sachdev



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Observation of the Dirac fluid and the breakdown of the Wiedemann-Franz law in graphene

Jesse Crossno,[1,2] Jing K. Shi,[1] Ke Wang,[1] Xiaomeng Liu,[1] Achim Harzheim,[1] Andrew Lucas,[1] Subir Sachdev,[1,3] Philip Kim,[1,2,*] Takashi Taniguchi,[4] Kenji Watanabe,[4] Thomas A. Ohki,[5] and Kin Chung Fong[5,†]

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[2] *John A. Paulson School of Engineering and Applied Sciences, Harvard University, Cambridge, MA 02138, USA*
[3] *Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada*
[4] *National Institute for Materials Science, Namiki 1-1, Tsukuba, Ibaraki 305-0044, Japan*
[5] *Raytheon BBN Technologies, Quantum Information Processing Group, Cambridge, Massachusetts 02138, USA*
(Dated: September 17, 2015)

Crossno, Kim et al.
Lucas, Crossno, Fong, Kim, Sachdev

Evidence for hydrodynamic electron flow in $PdCoO_2$

Philip J. W. Moll,[1,2,3] Pallavi Kushwaha,[3] Nabhanila Nandi,[3]
Burkhard Schmidt,[3] Andrew P. Mackenzie,[3,4*]

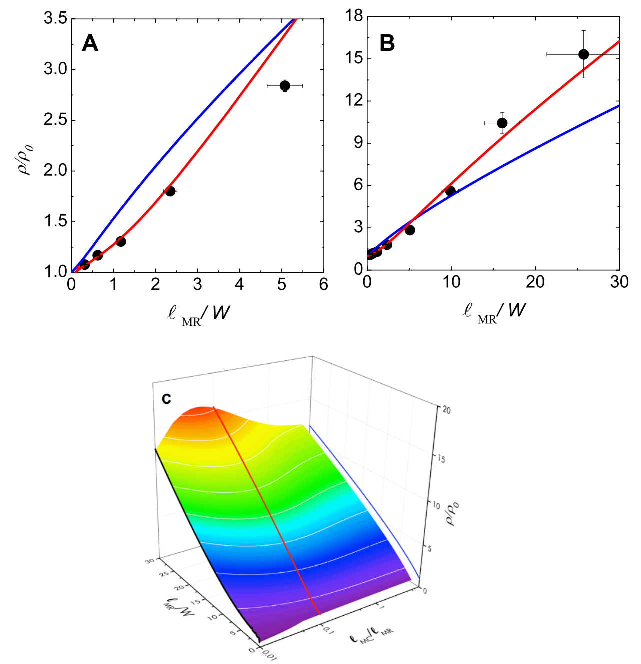

Fig. 4. Hydrodynamic effect on transport. (A, B) The measured resistivity of $PdCoO_2$ channels normalised to that of the widest channel (ρ_0), plotted against the inverse channel width $1/W$ multiplied by the bulk momentum- relaxing mean free path ℓ_{MR} (closed black circles). Blue solid line: prediction of a standard Boltzmann theory including boundary scattering but neglecting momentum-conserving collisions (Red line:prediction of a model that includes the effects of momentum-conserving scattering (see text). In (C) we show the predictions of the hydrodynamic theory over a wide range of parameter space.

Negative local resistance due to viscous electron backflow in graphene

D. A. Bandurin[1], I. Torre[2,3], R. Krishna Kumar[1,4], M. Ben Shalom[1,5], A. Tomadin[6], A. Principi[7], G. H. Autor
E. Khestanova[1,5], K. S. Novoselov[5], I. V. Grigorieva[1], L. A. Ponomarenko[1,4], A. K. Geim[1], M. Polini[3]

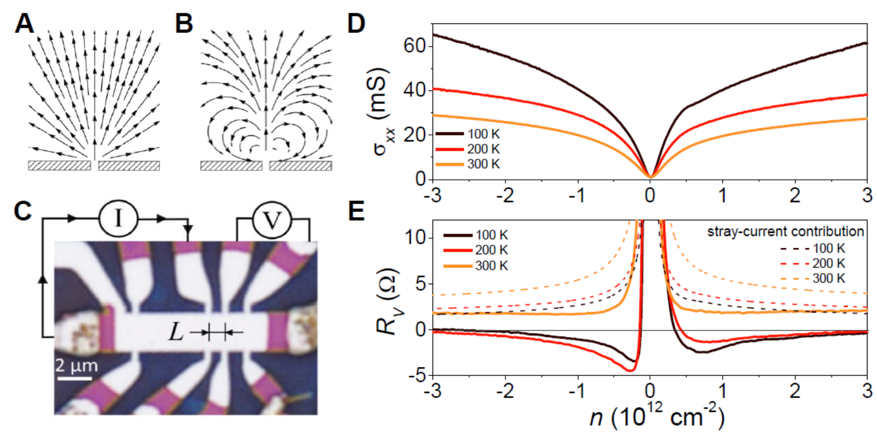

Fig. 1. Viscous backflow in doped graphene. (**A**,**B**) Calculated steady-state distribution of current injected through a narrow slit for a classical conducting medium with zero ν (A) and a viscous Fermi liquid (B). (**C**) Optical micrograph of one of our SLG devices. The schematic explains the measurement geometry for vicinity resistance. (**D**,**E**) Longitudinal conductivity σ_{xx} and R_V as a function of n induced by applying gate voltage. $I = 0.3$ µA; $L = 1$ µm. The dashed curves in (E) show the contribution expected from classical stray currents in this geometry (*18*).

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Davison, Schalm, Zaanen

Andreev, Kivelson, Spivak

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- Caveat: theory must be locally quantum critical $z \simeq \infty$

Lucas, Sachdev, Schalm,
Hartnoll, Mahajan, Punk, Sachdev

Puzzle

- Local vs hyperscaling violating quantum critical
  - emergent dynamical critical exponent

$$t \to \Lambda t \quad , \qquad x \to \Lambda^{1/z} x$$

Locally quantum critical $z = \infty$

Hyperscaling violating quantum critical $z = \text{finite}$

$$s \sim T^{(d-\theta)/z}$$

- Previous model: locally quantum critical

$$\rho_{DC} \sim \lim_{\omega \to 0} \int dk k^2 \frac{\mathrm{Im}\langle \mathcal{O}\mathcal{O} \rangle}{\omega}$$

no T dependence

What if hyperscaling quantum critical

- Resistivity in hyperscaling violating quantum critical theories from disorder

    - Holographic model

$$S = \frac{1}{2\kappa^2}\int d^{d+2}x\sqrt{-g}\left(R - \frac{Z(\Phi)}{4}F_{\mu\nu}F^{\mu\nu} - 2|\partial_\mu\Phi|^2 - V(\Phi)\right)$$

$$ds^2 = \frac{L^2}{r^2}\left[r^{2\theta/(d-\theta)}dr^2 - r^{-2d(z-1)/(d-\theta)}dt^2 + dx^2\right]$$

$$s \sim T^{(d-\theta)/z}$$

    - Assume the presence of a relevant operator in addition to hydro

tuned to the critical point

$$S_\phi = \int d^{d+2}x\left(-\frac{1}{2}|\partial_\mu\phi|^2 - \frac{B(\Phi)}{2}\phi^2\right)$$

$$B(\Phi) = \frac{B_0}{L^2}r^{-2\theta/(d-\theta)}$$

$$G_{\phi\phi}(\mathbf{k},\omega=0) \sim k^{(1-\theta/d)(\nu_+(B_0)-\nu_-(B_0))}$$

Random field disorder

- The relevant operator is statistically averaged
  - Construct an ensemble of solutions for $\phi(\mathbf{k}) \neq 0$ with

Lucas, Sachdev, KS

$$\langle \phi(\mathbf{k}) \rangle = 0$$
$$\langle \phi(\mathbf{k})\phi(\mathbf{k}') \rangle = \epsilon^2 \delta(\mathbf{k} + \mathbf{k}')$$

  - Resistivity

$$\rho_{DC} \sim s(T) \int_0^{T^{1/z}} d^d\mathbf{k}\, k^2 \phi(\mathbf{k}, r_h)^2$$

Holography: Perturbation theory breaks down

$$\rho_{\mathrm{DC}} \sim \epsilon^2 T^{2(1+\Delta-z)/z}$$

$$T^{(z-\Delta+(d-\theta)/2)/z} \lesssim \epsilon$$

Memory Matrix

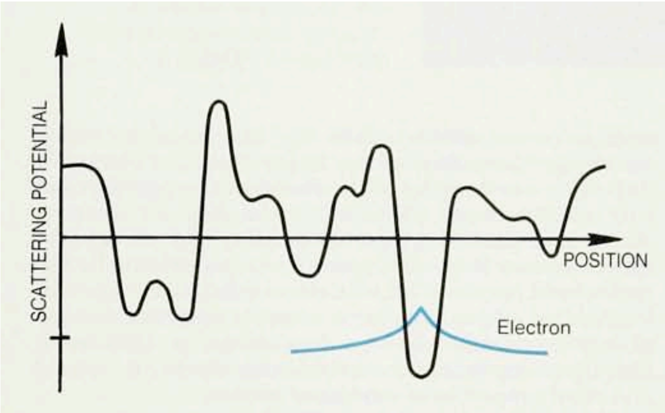
$$\rho_{\mathrm{DC}} \sim \epsilon^2 \int_0^{T^{1/z}} d^d\mathbf{k}\, k^2 \lim_{\omega\to 0} \mathrm{Im} \frac{G^R_{\mathcal{O}\mathcal{O}}(\omega, \mathbf{k})}{\omega}$$

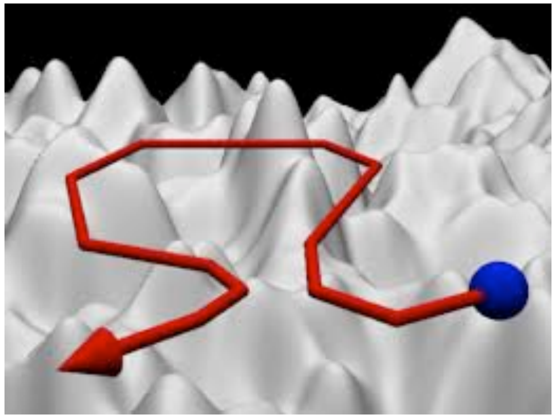
from weak to strong

Disorder and localization

- Strong disorder
  - Anderson: disorder can localize charged excitations

$$\text{free electron:} \quad \frac{\hat{p}^2}{2m}\Psi = E\Psi$$





$$\text{localized electron:} \quad \frac{k\hat{x}^2}{2}\Psi = E\Psi$$

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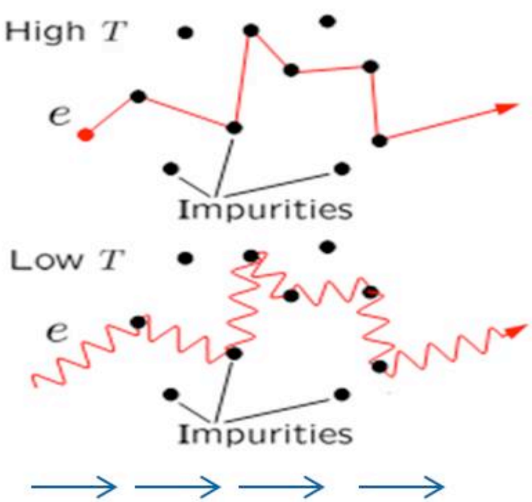
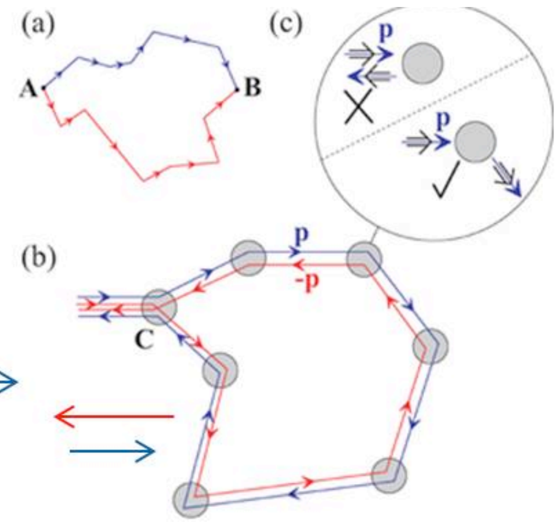


Einstein relation $\sigma = e^2 D\rho(E_F)$

$D = \frac{1}{2} v_F^2 \tau$

Anderson Localization

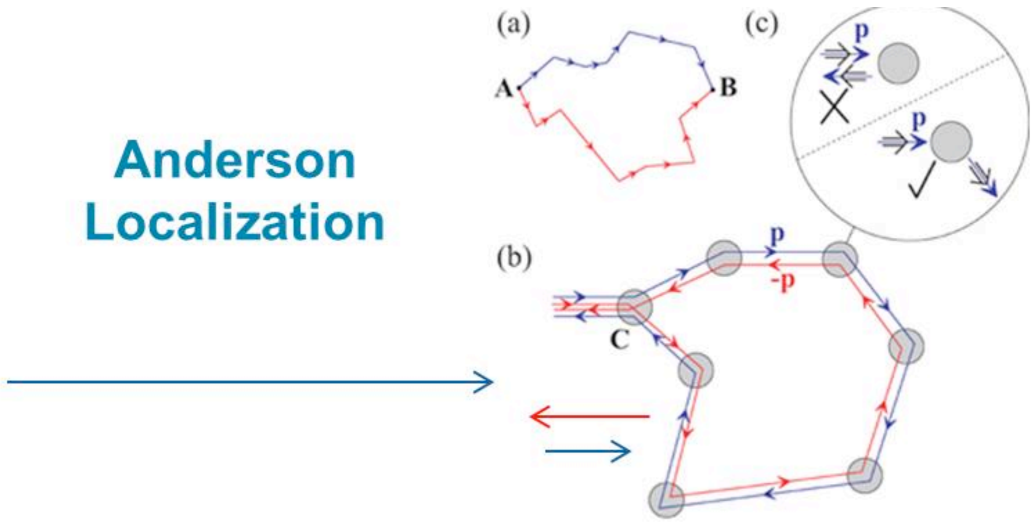


opposing interfering trajectories

constructive interference of backscattering

- Strong disorder
    - Anderson: disorder can localize charged excitations

Anderson Localization



opposing interfering trajectories

constructive interference of backscattering

Strong disorder

- Strong disorder in weakly interacting systems
  - Anderson: disorder can localize charged excitations

$\text{relevant: } d = 1, 2$ *groundstate is always an insulator*

$\text{marginal: } d = 3$

- Strong disorder in strongly interacting systems/many-body-theory
  - Many-body-localization

Basko, Aleiner, Altschuler

Connected to quantum entanglement

$$S_{\text{ent};t=0\ |A\rangle\otimes|B\rangle} \sim \log(t)$$

Failure to thermalize

(No eigenstate thermalization; no quantum chaos; "do not decohere"... quantum computer)

A lot of work in 1+1 dimensions

Ideal playground for holography

- Generic holography (for CMT)

$$S_{\mathrm{bulk}} = \int \mathrm{d}^4x\,\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{F^2}{4e^2}\right)$$

- Arbitrary disorder

$$\mathrm{d}s^2 = -U(r)V(r,\mathbf{x})\mathrm{d}t^2 + \frac{1}{U(r)}W(r,\mathbf{x})\mathrm{d}r^2 + G_{ij}(r,\mathbf{x})\mathrm{d}x_i\mathrm{d}x_j$$

$$A = \Phi(r,\mathbf{x})\mathrm{d}t \qquad\qquad \phi(\mathbf{x}) = \Phi(\infty,\mathbf{x})$$

- Observables are disorder averages

$$\mathbb{E}[f] = L^{-2}\int \mathrm{d}^2\mathbf{x}\; f$$

$$\mathbb{E}\left[\langle j^i\rangle\right] \equiv J^i = \sigma^{ij}E_j$$

- Generic holography (for CMT)

$$S_{\text{bulk}} = \int \mathrm{d}^4x\,\sqrt{-g}\left(R + \frac{6}{L^2} - \frac{F^2}{4e^2}\right)$$

- Weak (mean-field) disorder (massive gravity)

$$\sigma = \underbrace{\frac{1}{e^2}}_{\text{ccs/pair}} + \underbrace{\frac{4\pi Q_0^2}{\mathcal{S}_0 m^2}}_{\text{incoherent/diss}}$$

  - Only reliable for $m^2 \ll 1$

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  - Only reliable for $m^2 \ll 1$
  - Can this become an insulator for strong disorder?

- Note: dilaton-driven insulators are not disorder-driven

Baggioli, Pujolas
Gouteraux, Kiritsis, Li
Fadafan

$$S = \frac{1}{2\kappa^2}\int d^{d+2}x\sqrt{-g}\left(R - \frac{Z(\Phi)}{4}F_{\mu\nu}F^{\mu\nu} - 2|\partial_\mu\Phi|^2 - V(\Phi)\right)$$

  - Metal-Insulator transition

$$\text{IR: } Z(\Phi) \to 0$$

due to backreaction from disorder...

This just gaps out charged d.o.f. analogous to soft/hard wall

$$S = \frac{1}{2\kappa^2}\int d^{d+2}x\sqrt{-g}e^{-2\Phi}\left(R - \ldots\right)$$

  - Wall (confinement)

$$\text{IR: } \Phi \to \infty$$

- In this generic holographic disordered system

Grozdanov, Lucas, KS, Sachdev

$$\sigma \geq \frac{1}{e^2} = 1$$

- In this generic holographic disordered system

$$\sigma \geq \frac{1}{e^2} = 1$$

  - Proof follows from Navier-Stokes on the horizon

Donos, Gauntlett

$$U \approx 4\pi T r + \dots$$
$$\Phi = r S(\mathbf{x}) \mathcal{Q}(\mathbf{x}) + \dots,$$
$$G_{ij} = \gamma_{ij}(\mathbf{x}) + \dots$$

$$\nabla_i \left(T\mathcal{S}v^i\right) = \nabla_i \left(\mathcal{Q}v^i + \sigma_{\mathrm{Q}} \left(E^i - \partial^i \mu\right)\right) = 0,$$
$$\mathcal{Q}(E_j - \partial_j \mu) + \mathcal{S}(T\zeta_j - \partial_j \Theta) + 2\eta \nabla^i \nabla_{(i} v_{j)} = 0$$

  - Disorder-averaged currents

$$J^i = \mathbb{E}\left[\sqrt{\gamma}\mathcal{Q}v^i + \sigma_{\mathrm{Q}}\sqrt{\gamma}\gamma^{ij}\left(E_j - \partial_j \mu\right)\right],$$
$$Q^i = \mathbb{E}\left[\sqrt{\gamma}T\mathcal{S}v^i\right].$$

- Charge-less black holes

$$J^i = \mathbb{E}\left[\sqrt{\gamma}\gamma^{ij}\left(E_j - \partial_j \mu\right)\right],$$

$$\sigma^{ik} = \mathbb{E}\left[\sqrt{\gamma}\gamma^{ij}\left(\delta_j^k - \left(\mathsf{E}^k, \partial_j \mu\right)\right)\right] \qquad \left(\mathsf{E}^j, E_i\right) = \delta_j^i$$

- Conservation equation

$$\partial_i\left(\sqrt{\gamma}\gamma^{ij}\left(E_j - \partial_j \mu\right)\right) = 0$$

$$\det(\sigma^{ij}) = 1 \qquad \overset{iso}{\Rightarrow} \qquad \sigma = 1$$

- Charged black holes: variational argument:
    - Consider vector fields

$$\nabla_i \mathcal{V}^i = \nabla_i \mathcal{J}^i = 0, \quad \bar{\mathcal{J}}^i = \mathbb{E}\left[\sqrt{\gamma}\mathcal{J}^i\right], \quad \mathbb{E}\left[\sqrt{\gamma}\mathcal{V}^i\right] = 0$$

Idea: these are *forcing currents*
The power lost is then always positive semidefinite

$$\frac{\bar{\mathcal{J}}^2}{\sigma} \leq \frac{\bar{\mathcal{J}}^2 T\bar{\kappa}}{T\sigma\bar{\kappa} - T^2\alpha^2} \leq \mathbb{E}\left[2\nabla^{(i}\mathcal{V}^{j)}\nabla_{(i}\mathcal{V}_{j)}\sqrt{\gamma} + \left(\mathcal{J}^i - \mathcal{Q}\mathcal{V}^i\right)\left(\mathcal{J}_i - \mathcal{Q}\mathcal{V}_i\right)\sqrt{\gamma}\right]$$

Proof follows by choosing

solution for chargeless BH

$$\bar{\mathcal{J}}^2 = \mathbb{E}\left[\sqrt{\gamma}\tilde{\mathcal{J}}^i\tilde{\mathcal{J}}_i\right], \quad \tilde{\mathcal{J}}^i = E^i - \partial^i\tilde{\mu}$$

- Generic holographic disordered system has no disorder-driven insulating phase

$$\sigma \geq \frac{1}{e^2} = 1$$

  - Note: this is not a 1/N artifact. It is a strong coupling phenomenon.
  - Can prove a similar bound for thermal conductivity. Grozdanov, Lucas, KS

$$\kappa \geq 16\pi^3 \left(\frac{1}{1 - \frac{1}{2} V_{\min}} \right) \frac{T}{s} \quad d = 1$$

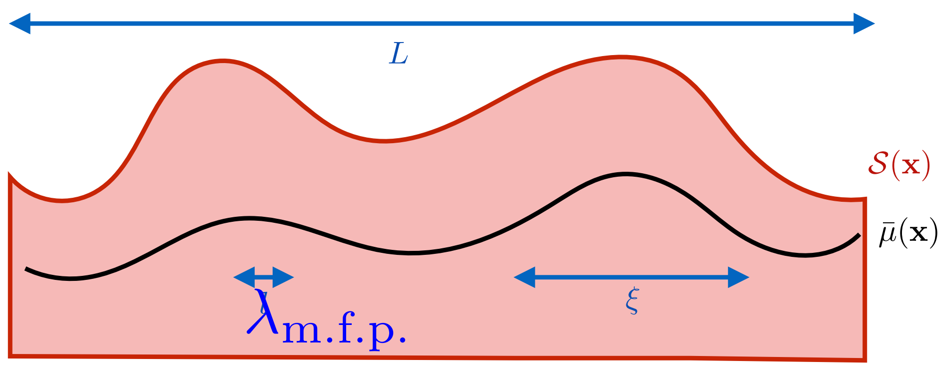
$$\kappa \geq \frac{4\pi^2}{3} \left(\frac{1}{1 - \frac{1}{6} V_{\min}} \right) T \quad d = 2$$

Bound follows from the fact that any Area a distance R from the horizon obeys

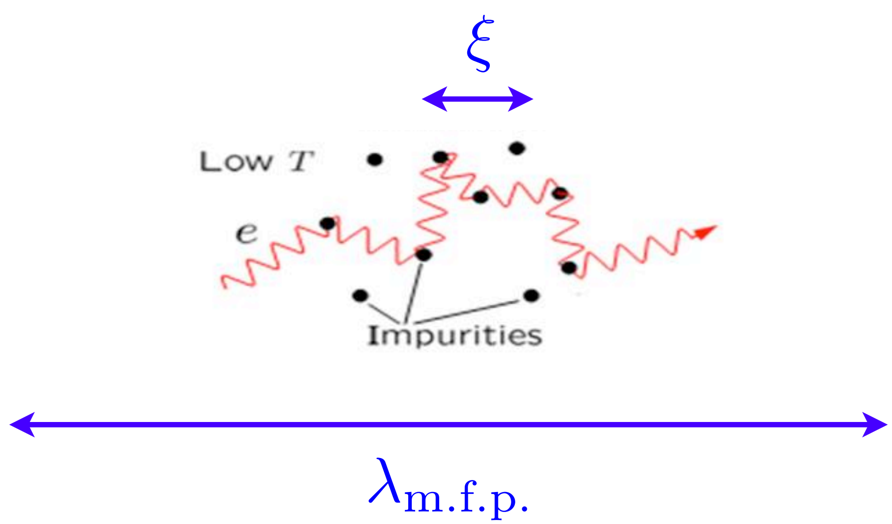
$$A_R \geq A_{\text{hor}}$$

Absence of localization in holography

- Classical gravity is infinitely strongly coupled system
  - Hydrodynamics "always" applies
  - No possibility for "random interference".



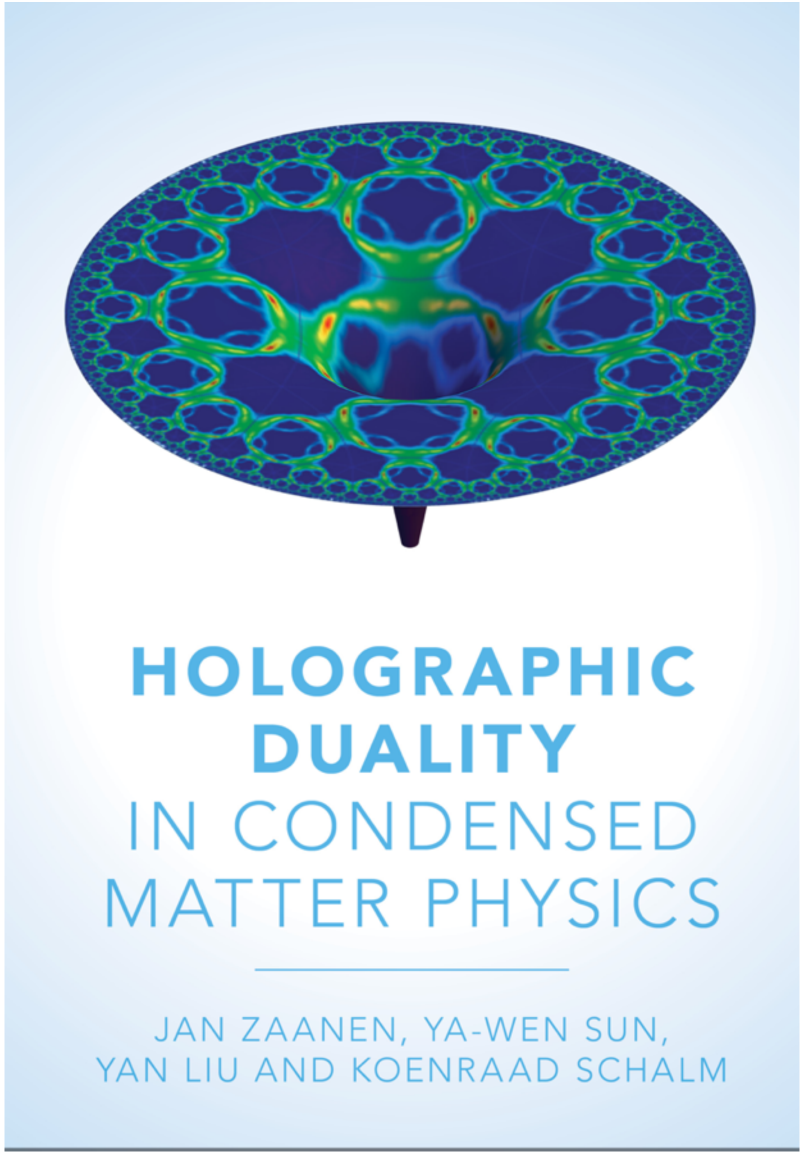
- A metal is a weakly coupled system
  - Wave interference



- Disorder does not localize in ultra-strongly coupled systems

- For localization in holography one has to go beyond the classical approximation.

Fall 2015


HOLOGRAPHIC
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IN CONDENSED
MATTER PHYSICS
JAN ZAANEN, YA-WEN SUN,
YAN LIU AND KOENRAAD SCHALM

Thank you