Hydrodynamic theory of quantum fluctuating superconductivity

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Current themes in holography
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Acknowledgments

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Infinite vs finite DC conductivity

- If a conserved quantity (momentum) overlaps with the electric current, the electric conductivity at zero frequency diverges

$$\sigma(\omega) = \sigma_Q + \frac{i\rho^2}{(\epsilon + p)\omega}$$

- Recently in gauge/gravity duality: model metallic thermoelectric transport realistically and account for momentum relaxation

$$\sigma(\omega) = \sigma_1 + \frac{\rho^2}{(\epsilon + p)(\Gamma - i\omega)} .$$

- If a symmetry is spontaneously broken (eg superfluid), the Goldstone boson can also source a delta function in the conductivity

$$\sigma(\omega) = \sigma_0 + \frac{i\rho_s}{m^2\omega}$$

- This talk is about resolving this superfluid delta function, why it is interesting and to which physical systems it applies.

Superfluid hydrodynamics

- In a superconductor, the U(1) of electromagnetism is spontaneously broken. The phase of the complex order parameter couples to hydrodynamics via the Josephson relation (gauge invariance):

$$m\partial_t u_\phi = -\nabla\mu\,, \qquad u_\phi = \frac{1}{m}\nabla\phi$$

  with the usual conservation equation for the charge density

$$\partial_t \rho + \nabla\cdot j = 0$$

- Writing down the constitutive relation in the strong disorder limit

$$j = \frac{\rho_s}{m}u_\phi - D\nabla\rho$$

  we obtain the conductivity advertised on the previous slide

$$\sigma(\omega) = \sigma_0 + \frac{i\rho_s}{m^2\omega}\,, \qquad \sigma_0 = D\chi, \qquad \chi = \frac{\partial\rho}{\partial\mu}$$

Vortices in two dimensions

- At finite temperature, vortices can proliferate due to thermal fluctuations and destroy quasi long range order (BKT transition).
- Around a vortex, quantized superfluid velocity circulation

$$\oint_{\text{vortex}} u_\phi = \frac{2\pi n}{m}$$

  The superfluid velocity is no longer a pure gradient

$$u_\phi = \frac{1}{m}\left(\nabla\phi + \epsilon \times \nabla\psi\right)$$

- Mobile vortices will relax the supercurrent, $\partial_t u_\phi \neq 0$, by (un)winding the phase.
- As vortex cores are not superconducting, mobile vortices produce dissipation and regulate the conductivity

$$\sigma = \sigma_0 + \frac{\rho_s}{m^2}\frac{1}{-i\omega + \Omega}$$

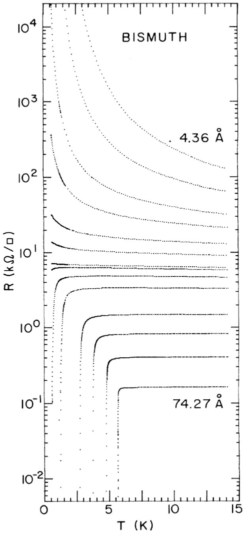
  Classically [Bardeen & Stephen'65]

$$\Omega \sim \frac{n_f}{\sigma_n}$$

Outline of the talk

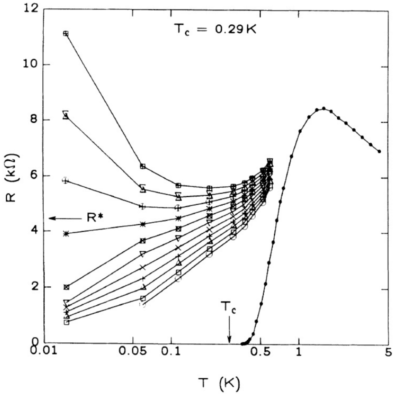
- Can superfluid current relaxation happen at zero temperature? Much debated in Condensed Matter: experimental evidence.
- Memory matrix formulation of quantum superconducting phase disordering.
- With parity, we recover the Bardeen-Stephen result.
- Without parity, alternate dissipation mechanism: emergent Chern-Simons gauge field.

Superfluid/insulator transitions in thin superfluid films



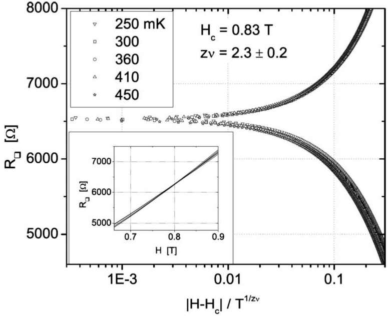
[Haviland et al'89]

2D superfluid films exhibit two different kinds of (quantum) superfluid/insulator phase transition, disorder or magnetic field-driven.

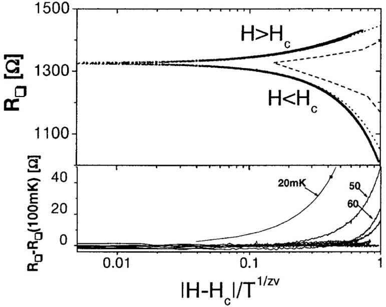


[Hebard & Paalanen'90]

Zero temperature metallic phases



[Mason & Kapitulnik'99]



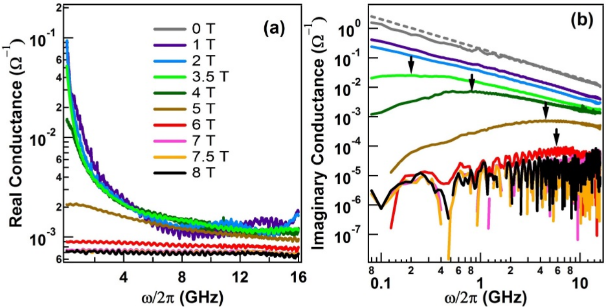
[Steiner-Breznay-Kapitulnik'08]

For strong disorder, quantum critical point separating the superconducting from the insulating phase.

For weak disorder, intervening metallic phase.

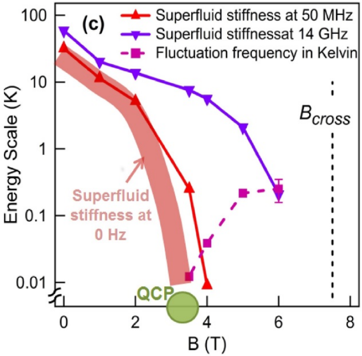
AC measurements: peaks in the conductivity

Zero temperature metallic phase in weakly-disordered films: Sharp Drude-like peaks appear in the real part of the conductivity. Directly motivates superfluid current relaxation.



[Liu-Pan-Wen-Kim-Sambandamurthy-Armitage'13]

The width Ω is B dependent and vanishes when superfluidity is restored: quantum origin. Quantum critical point at the superfluid/metal transition? [Liu-Pan-Wen-Kim-Sambandamurthy-Armitage'13]



Memory matrix description of superfluid current relaxation

- Approach inspired by momentum relaxation in gauge/gravity duality.
- Slow superfluid relaxation: need hierarchy of time scales $\Omega \ll T, \mu$. Separation between slow modes and fast modes.
- Decompose the Hamiltonian as

$$H = H_0 + \epsilon \Delta H \,, \qquad \epsilon \ll 1$$

$$[H_0, J_\phi] = 0 \,, \qquad [\Delta H, J_\phi] \neq 0 \,, \qquad J_\phi = \frac{1}{m} \int d^2x \, u_\phi$$

- This leads to finite DC conductivities

$$\sigma = \frac{\chi^2_{JJ_\phi}}{\chi_{J_\phi J_\phi}} \frac{1}{-i\omega + \Omega}$$

- We obtain a formula for the decay rate [Forster'75]

$$\Omega = \frac{\epsilon^2}{\chi_{J_\phi J_\phi}} \lim_{\omega \to 0} \frac{\mathrm{Im}\, G^R_{i[\Delta H, J_\phi]\, i[\Delta H, J_\phi]}}{\omega}$$

Density-density interaction

- Since ϕ and ρ are canonical variables

$$\pi_\phi = \frac{\partial \mathcal{L}}{\partial \dot{\phi}} = \frac{\partial \mathcal{L}}{\partial \mu} = \rho$$

they obey a Heisenberg uncertainty relation

$$\Delta\phi\Delta\rho \gtrsim \hbar \qquad \Longleftrightarrow \qquad [\phi(x), \rho(y)] = i\delta(x-y)$$

and phase fluctuations are enhanced by Coulomb interactions [Doniach'81].

- So pick

$$\Delta H = \frac{\lambda}{2}\int d^2x\, \rho(x)^2 \qquad \Longrightarrow \qquad i[\Delta H, J_\phi] = \frac{\lambda}{m}\int d^2x\, \nabla\rho(x)$$

Short range Coulomb interaction

Dissipation from mobile vortices

- The superfluid current is only defined outside of vortices

$$J_\phi = \frac{1}{m} \int_{T^2 \setminus \{\text{vortex cores}\}} d^2x \, \nabla \phi$$

- This leads to

$$i[\Delta H, J_\phi] = \frac{\lambda}{m} \int_{T^2 \setminus \{\text{v.c.}\}} d^2x \, \nabla \rho(x) = -\frac{\lambda}{m} \int_{\text{v.c.}} d^2x \, \nabla \rho(x)$$

  using single valuedness of the charge density.

- Assuming a diffusive form for the density density retarded Green's function

$$G^R_{\rho\rho} = \frac{k^2 D \chi_{\rho\rho}}{-i\omega + Dk^2} \, , \qquad D = \frac{\sigma_n}{\chi_{\rho\rho}}$$

$$\Omega \sim k^2 \lim_{\omega \to 0} \frac{\mathrm{Im}\, G^R_{\rho\rho}}{\omega} \sim \frac{n_f}{\sigma_n}$$

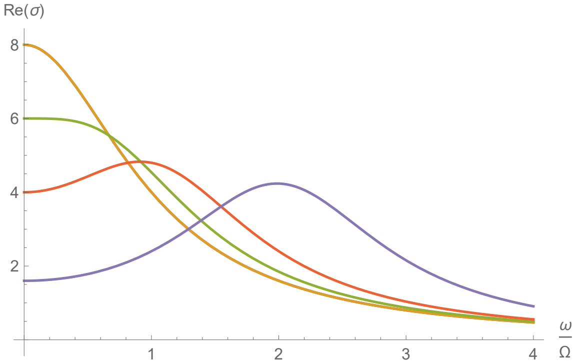
  Exact quantum derivation of Bardeen-Stephen!
  n_f and σ_n are external inputs.

Parity violation

- With magnetic fields, parity is violated.
- This leads to a (super)cyclotron pole in the complex conductivities

$$\omega_\star = \pm\Omega^H - i\Omega$$

  where both Ω and Ω^H are related to memory matrix elements.
- Interestingly, the peak only moves off the vertical axis once $\Omega_H > \Omega/\sqrt{3}$ (large enough parity violation).



Chern-Simons interaction

- Example of parity violation: emergent Chern-Simons gauge field

$$\mathcal{L} = \mathcal{L}_{\mathrm{matter}} + j_\mu (A^\mu + a^\mu) - \frac{1}{2\lambda'} \epsilon^{\mu\nu\rho} a_\mu \partial_\nu a_\rho$$

- Integrating out the gauge field leads to the non-local interaction

$$\Delta H = \frac{\lambda'}{2} \int \frac{d^2 k}{(2\pi)^2} \frac{\rho_{-k} \, (\nabla \times j)^z_k}{k^2} + \text{h.c.}\,,$$

and

$$i[\Delta H, J^i_\phi] = -\frac{\lambda'}{m} \lim_{k \to 0} \epsilon^{ij} j^{T\,j}\,.$$

- Using the eom for the CS gauge field a^μ, $b \sim \lambda' \rho$:
  flow of charge $\Leftrightarrow$ flow of magnetic flux $\Leftrightarrow$ flow of vortices.

- This leads to decay rates Ω, Ω_H set by the conductivity of the normal component σ_0, σ_0^H. No external 'BKT' input required.

Summary

- We have described superfluid current relaxation in a purely quantum framework.
- We have not assumed weakly coupled quasiparticles anywhere.
- When parity is preserved, we recovered the classical result of [Bardeen-Stephen'65] on flux flow resistance and contribution of the vortex cores to the resistivity from short range Coulomb interactions.
- Without parity, a supercyclotron pole appears. We gave an explicit example of parity violating interaction (emergent CS) which determines the conductivities in terms of the normal component of the phase ordered superfluid.
- For more details, arxiv:1602.08171

Outlook

- Back to the experimental data: suggestively, the peak seems to be moving off the real axis for $B \gtrsim 5T$.
- A new fit including Ω and Ω^H leads to $\Omega^H \sim 10^{-6} \ll \Omega$: Hall conductivities are suppressed. Emergent particle-hole symmetry?