Homes' law in holographic superconductors

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Motivation:

What is Homes' law?

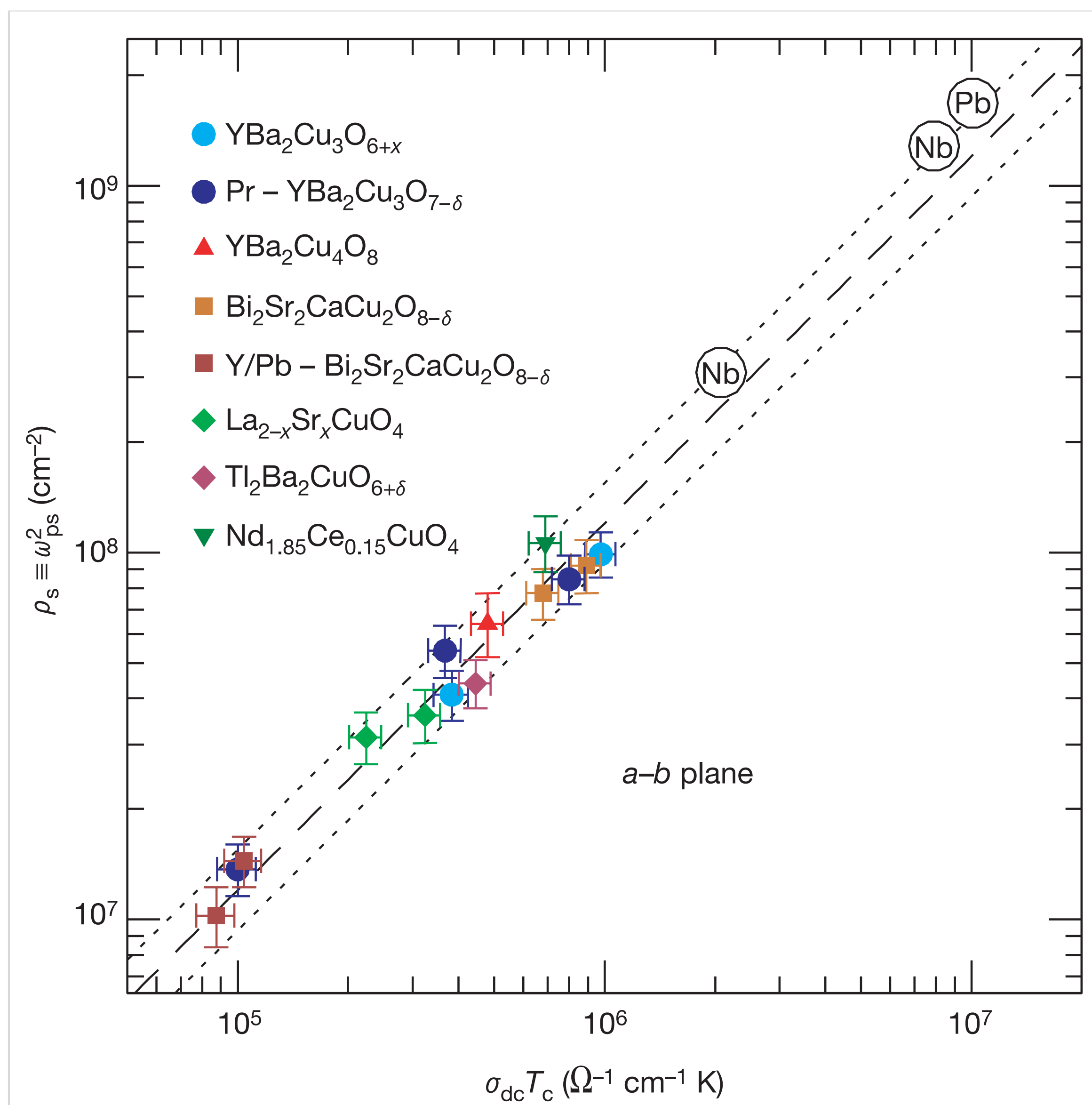
Why is Homes' law interesting?

Homes law

[Homes, et al. 2004 Nature]

A universal scaling relation in high-temperature superconductors

C. C. Homes[1], S. V. Dordevic[1], M. Strongin[1], D. A. Bonn[2], Ruixing Liang[2], W. N. Hardy[2], Seiki Komiya[3], Yoichi Ando[3], G. Yu[4], N. Kaneko[5*], X. Zhao[5], M. Greven[5,6], D. N. Basov[7] & T. Timusk[8]



Superconducting transition temperature

Electric DC conductivity

Superfluid density

- Homes' law: $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$

C is constant regardless of doping level, nature of dopant, crystal structure and type of disorder.

C=4.4: a-b plane high-Tc superconductor,
clean BCS superconductor

C=8.1: c-axis high-Tc superconductor,
dirty BCS superconductor

[Erdmenger, Herwerth, Klug, Meyer, Schalm: 1501.07615]

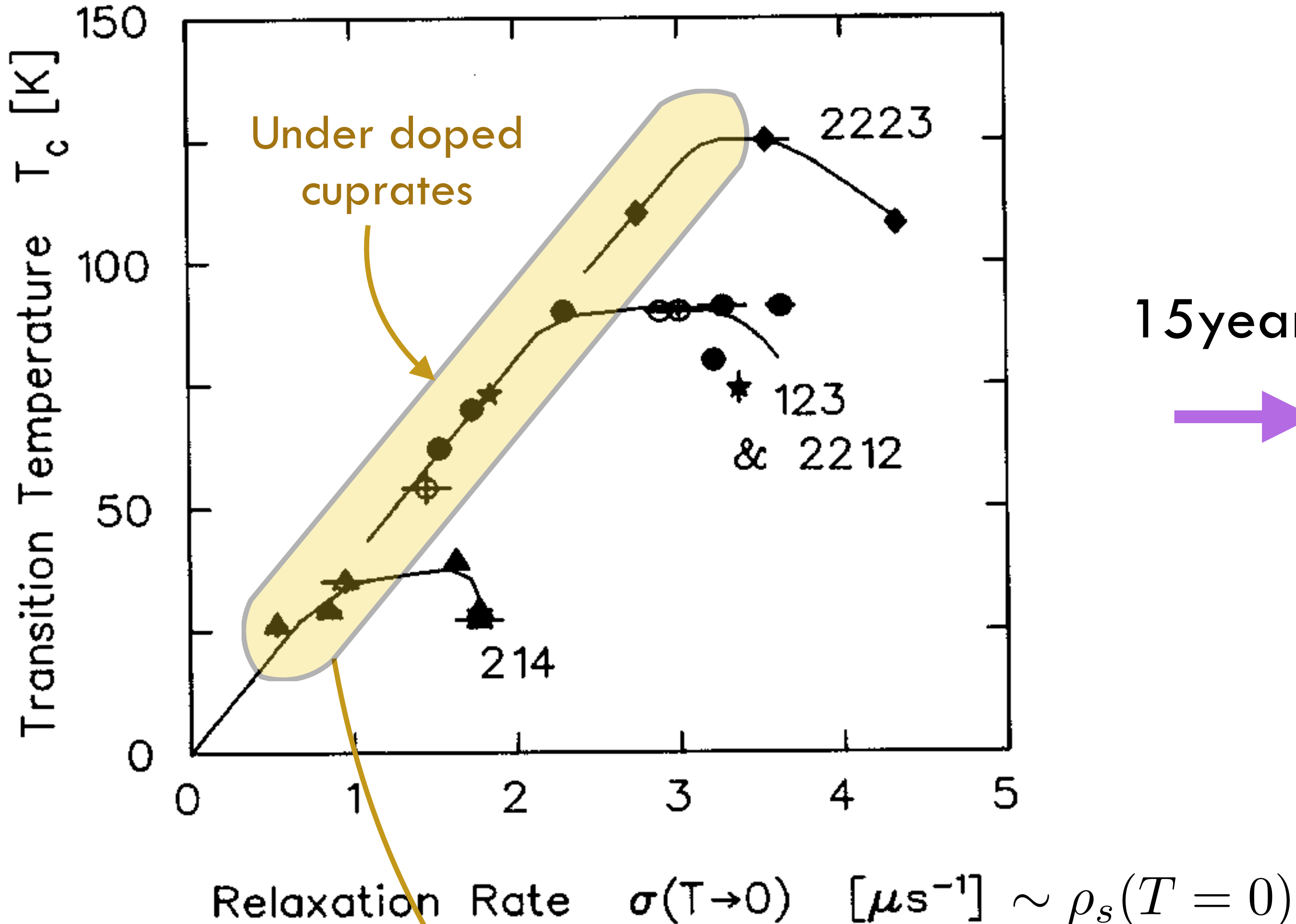
- A clue to the origin of the high Tc superconductivity?
- Universal property of the hairy black holes?

History for finding universality: Uemura's law

1986: Discovery of cuprate, Bednorz, et al. (Z. Phys. B)

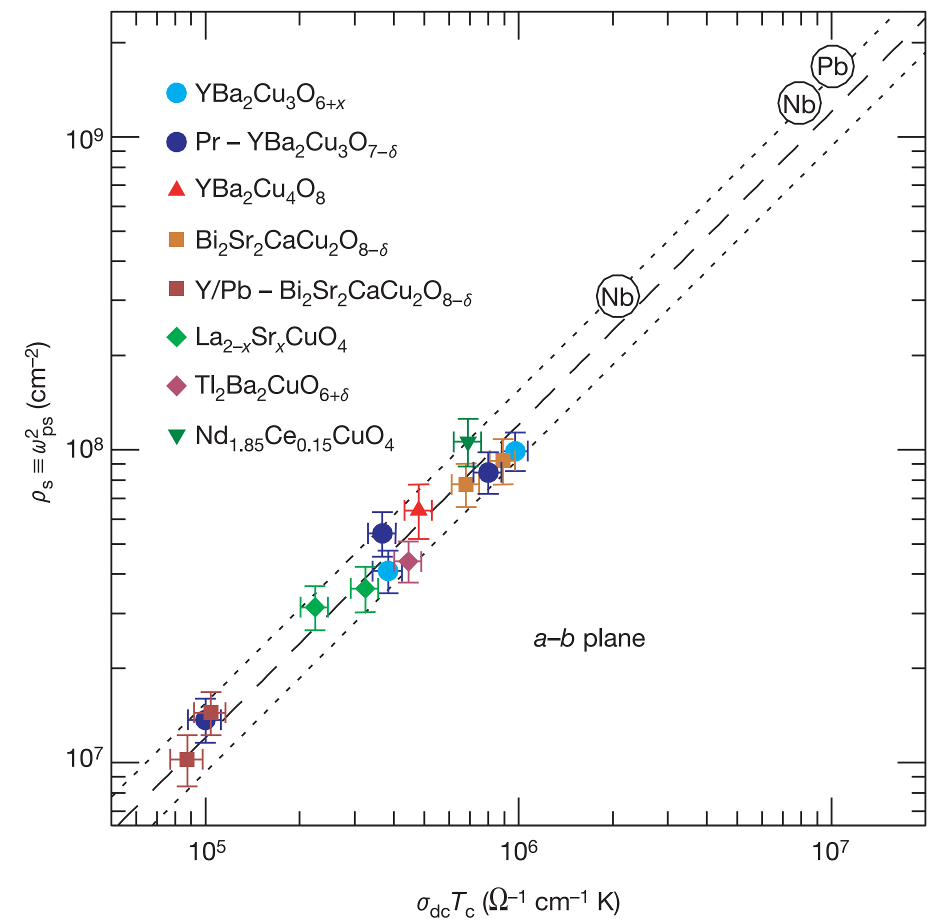
3years

1989: Uemura et al (PRL)



15years

2004: Homes et al (Nature)



Uemura's law

$$\rho_s(T=0) = BT_c$$

Homes' law

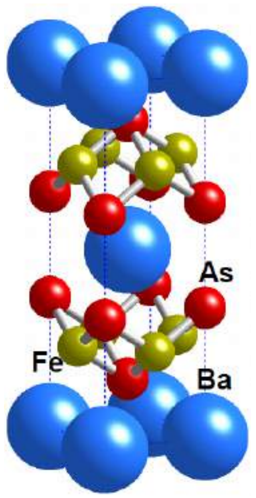
$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

Another universality: resistivity

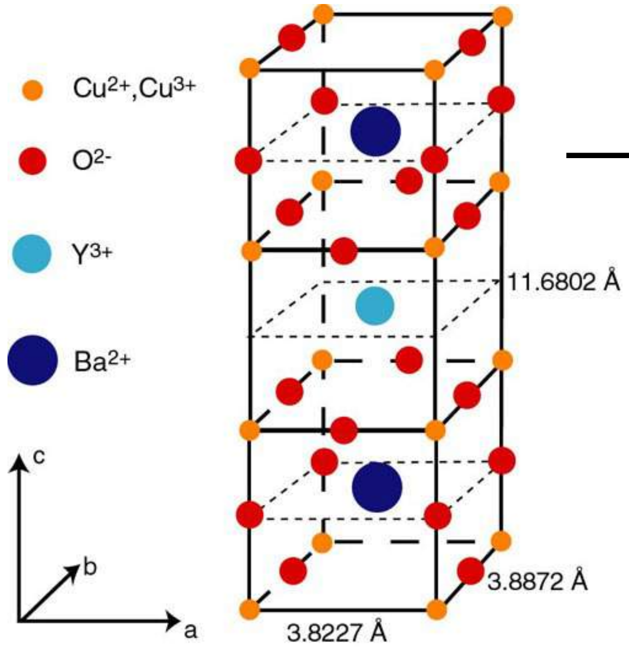
[Sachdev, Keimer: 2011, Physics today]

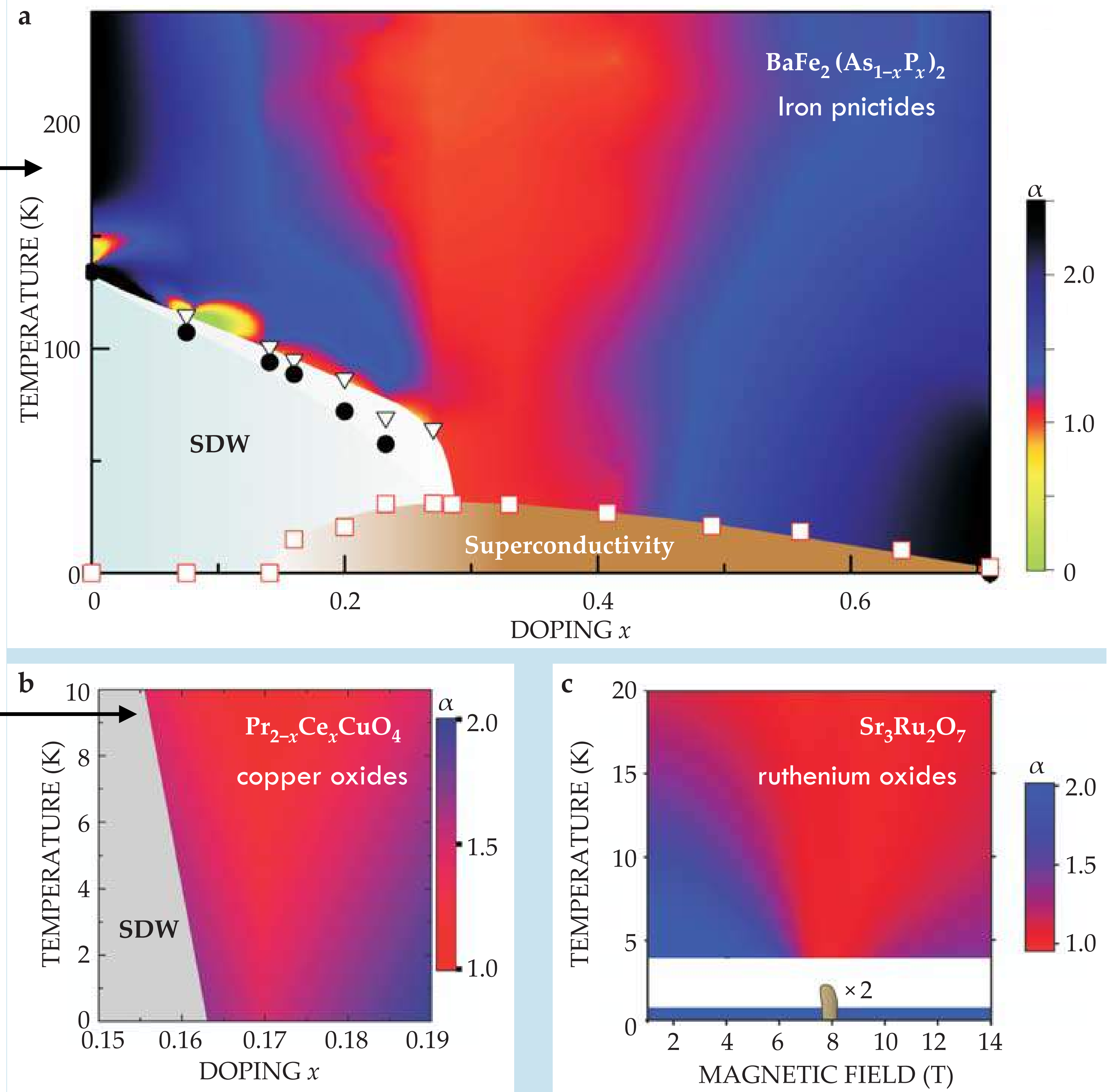
Resistivity: $\rho \sim \rho_0 + AT^\alpha$

Iron pnictides



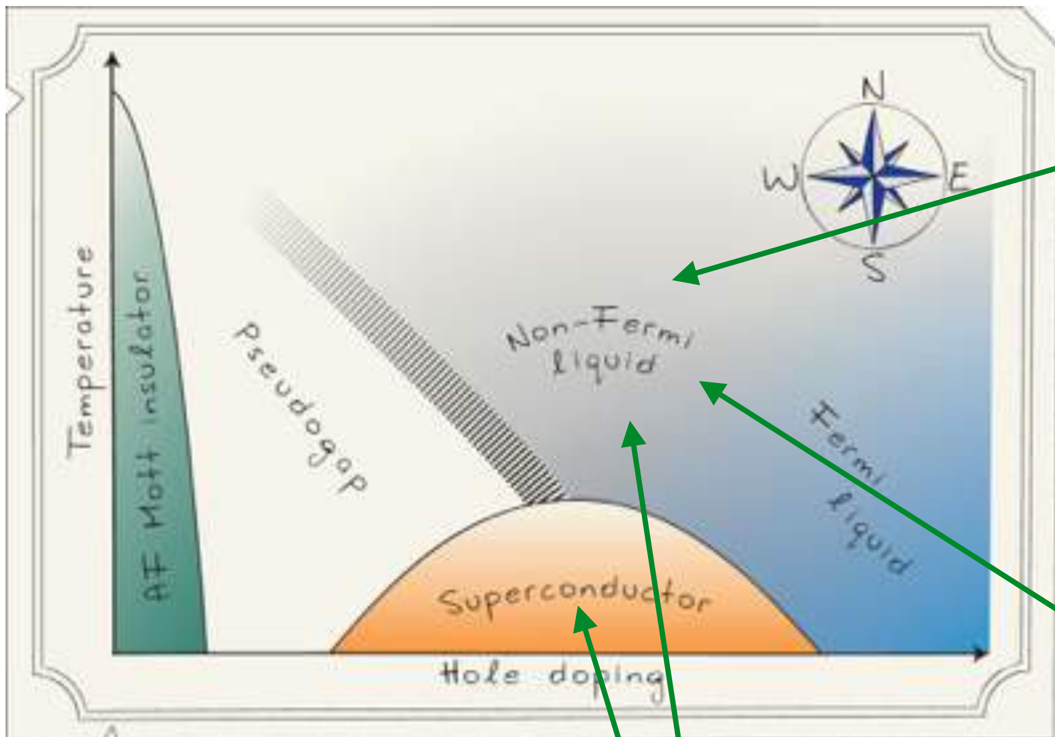
Copper oxides





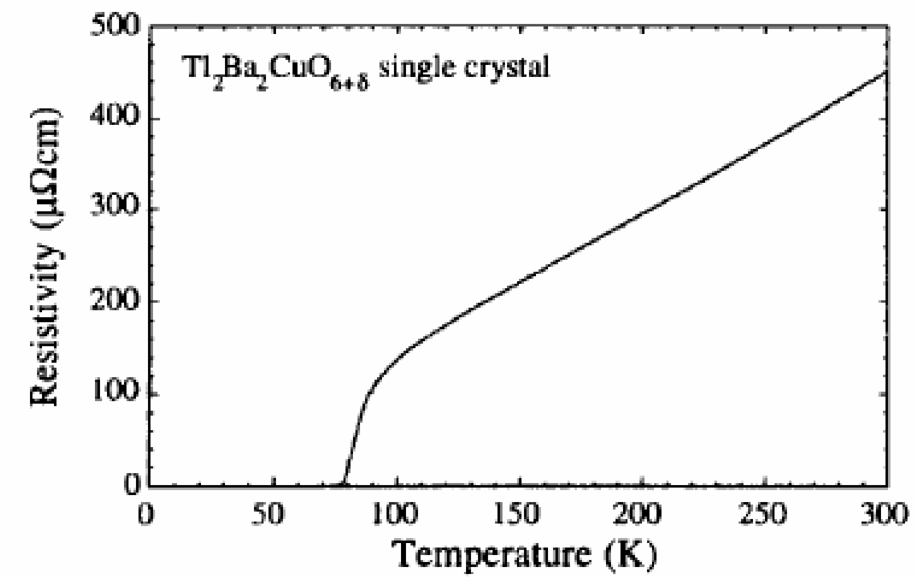
Universal properties in cuprates

Cuprate phase diagram

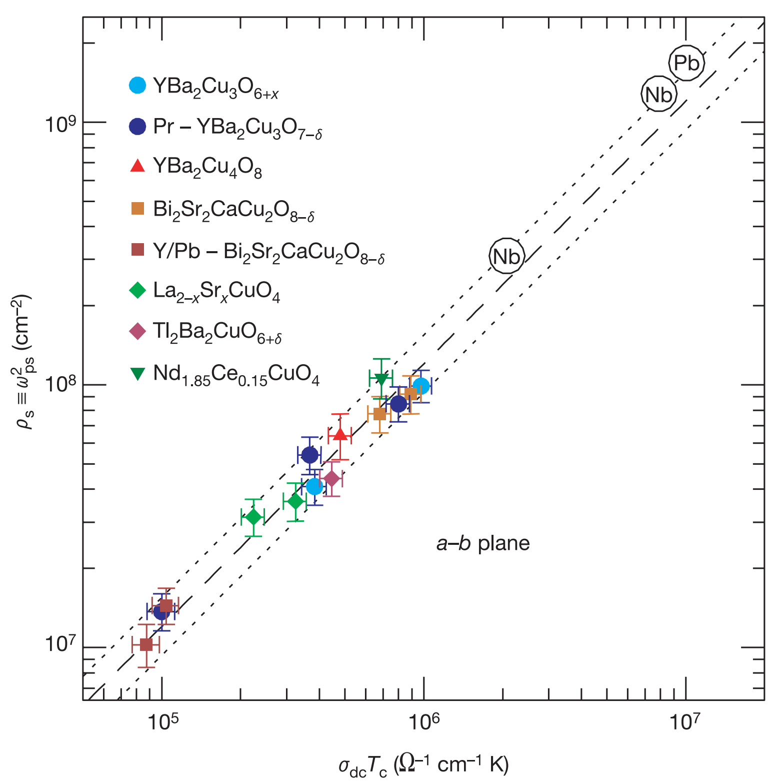


[Peter Wahl: 2012, Nature

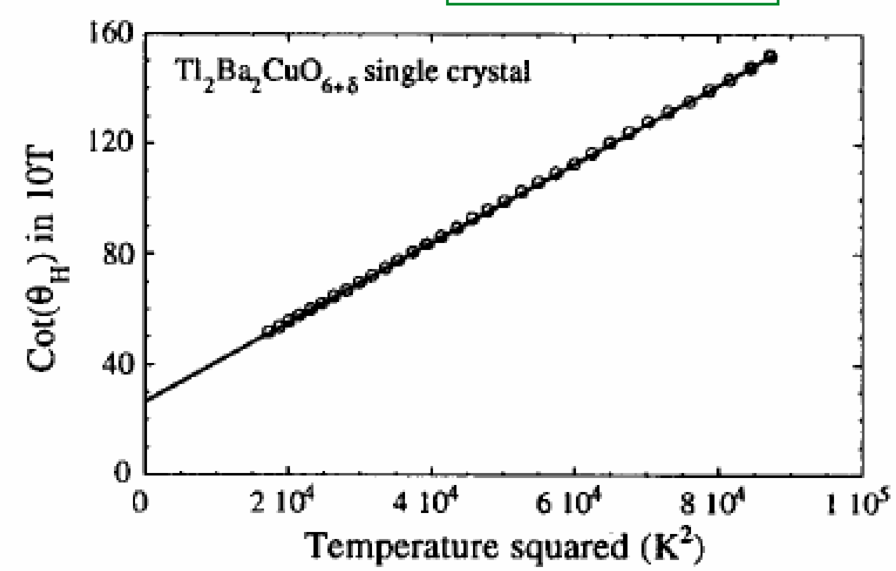
- DC resistivity $\boxed{\rho \sim T}$



- Homes law $\boxed{\rho_s(T=0) = C\sigma_{DC}(T_c)T_c}$



- Hall angle $\boxed{\sigma_{xx}/\sigma_{xy} \sim T^2}$



Mackenzie, 1997

Contents

Homes' law in Holographic context

arXiv.org > hep-th > arXiv:1002.1722

High Energy Physics – Theory

Introduction to Holographic Superconductors

Gary T. Horowitz

8.1 Open problems

We close with a list of open problems[15]. They are roughly ordered in difficulty with the easier problems listed first. (Of course, this is my subjective impression. With the right approach, an apparently difficult problem may become easy!)

1. In the probe limit below the critical temperature, there is an infinite discrete set

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10. The high temperature cuprate superconductors satisfy a simple scaling law relating the superfluid density, the normal state (DC) conductivity and the critical temperature [36]. Can this be given a dual gravitational interpretation?

Homes' law in Holographic context

arXiv.org > hep-th > arXiv:1206.5305

High Energy Physics - Theory

Towards a Holographic Realization of Homes' Law

Johanna Erdmenger, Patrick Kerner, Steffen Muller

arXiv.org > hep-th > arXiv:1501.07615

High Energy Physics - Theory

S-Wave Superconductivity in Anisotropic Holographic Insulators

Johanna Erdmenger, Benedikt Herwerth, Steffen Klug, Rene Meyer, Koenraad Schalm

arXiv.org > hep-th > arXiv:1604.06205

High Energy Physics - Theory

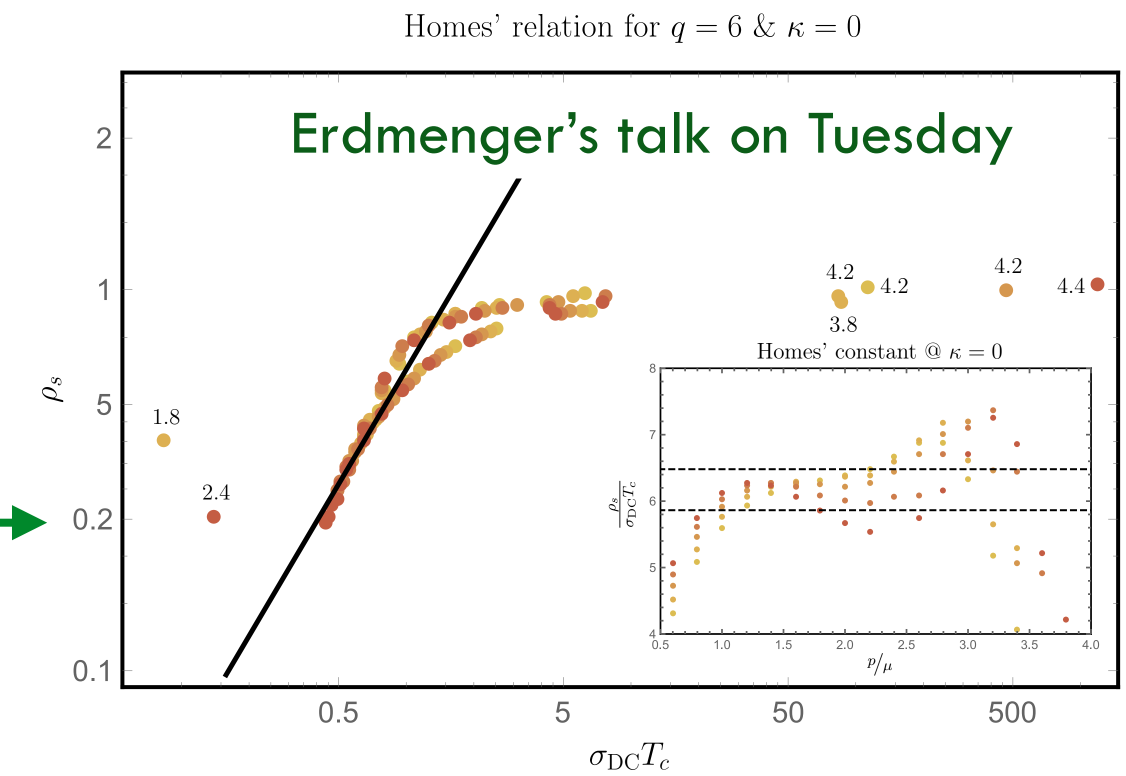
Ward Identity and Homes' Law in a Holographic Superconductor with Momentum Relaxation

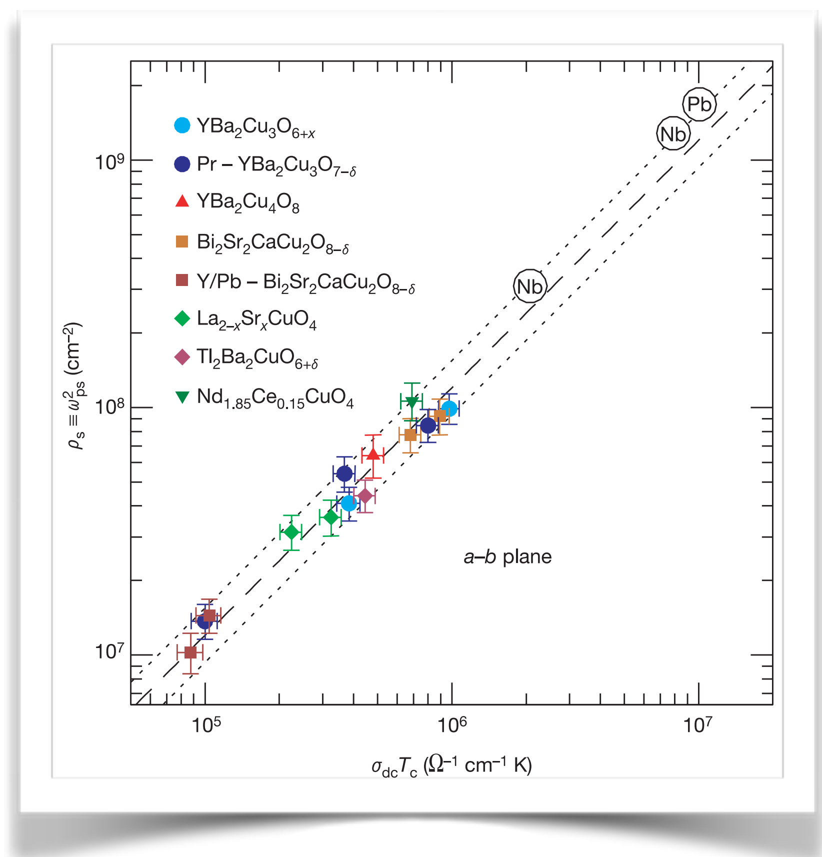
Keun-Young Kim, Kyung Kiu Kim, Miok Park

arXiv:1605.XXXXX

Homes' law in Holographic Superconductor with Q-lattices

Keun-Young Kim and Chao Niu





Homes' law in Holographic context

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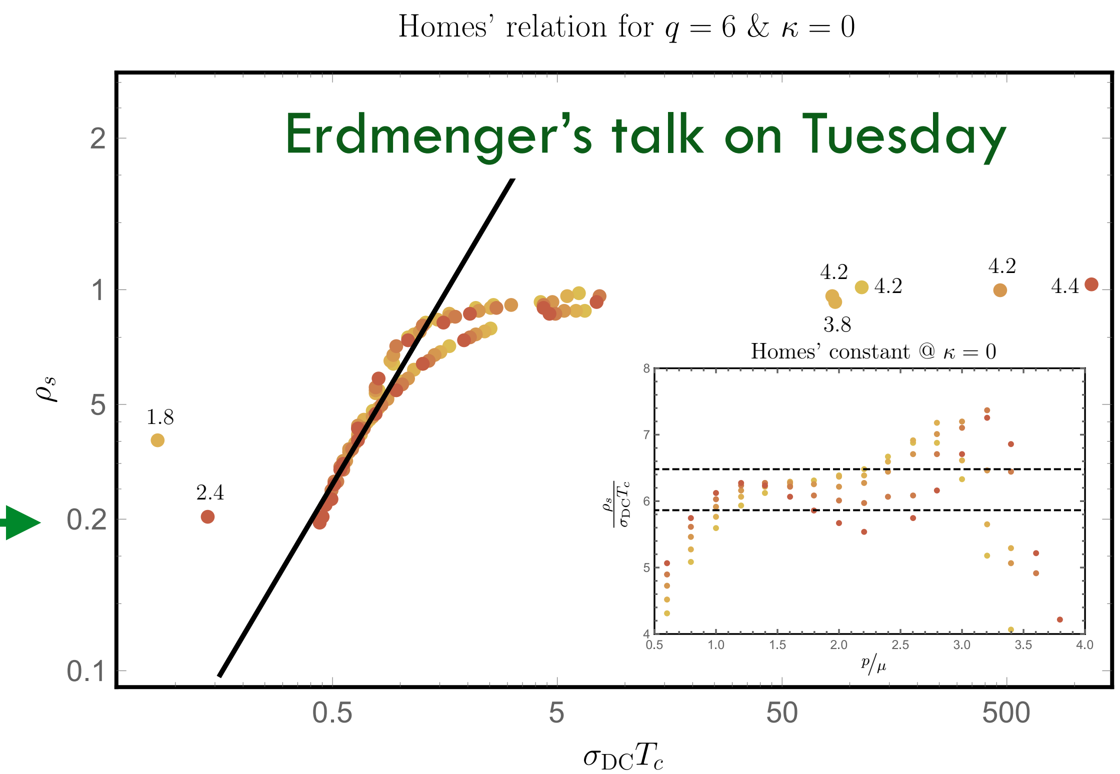

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High Energy Physics – Theory

Ward Identity and Homes' Law in a Holographic Superconductor with Momentum Relaxation

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Homes' law in Holographic Superconductor with Q-lattices

Keun-Young Kim and Chao Niu

This talk

arXiv.org > hep-th > arXiv:1409.8346

High Energy Physics – Theory

Coherent/incoherent metal transition in a holographic model

Keun-Young Kim, Kyung Kiu Kim, Yunseok Seo, Sang-Jin Sin

arXiv.org > hep-th > arXiv:1501.00446

High Energy Physics – Theory

A Simple Holographic Superconductor with Momentum Relaxation

Keun-Young Kim, Kyung Kiu Kim, Miok Park

Goals and method

Goals

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

Holographer's tool box

1. Need a holographic superconductor ~ hairy black hole (0803.3295: Hartnoll, Herzog, Horowitz)
2. Conductivity?

Linear response theory

$$\boxed{\sigma(\omega) = \frac{G^R_{JJ}(\omega)}{i\omega}}$$

Holography

$$G^R$$

Son and Starinets, hep-th/0205051
Herzog and Son, hep-th/0212072
Skenderis and van Rees, 0805.0150

$$\sigma_{DC} = \sigma(\omega = 0)$$

$$\sigma(\omega) \sim i\frac{\rho_s}{\omega}$$

The model and method are well established.
Why is the progress slow?

Original holographic superconductor: HHH

[Hartnoll, Herzog, Horowitz: 0803.3295]

The first holographic superconductor

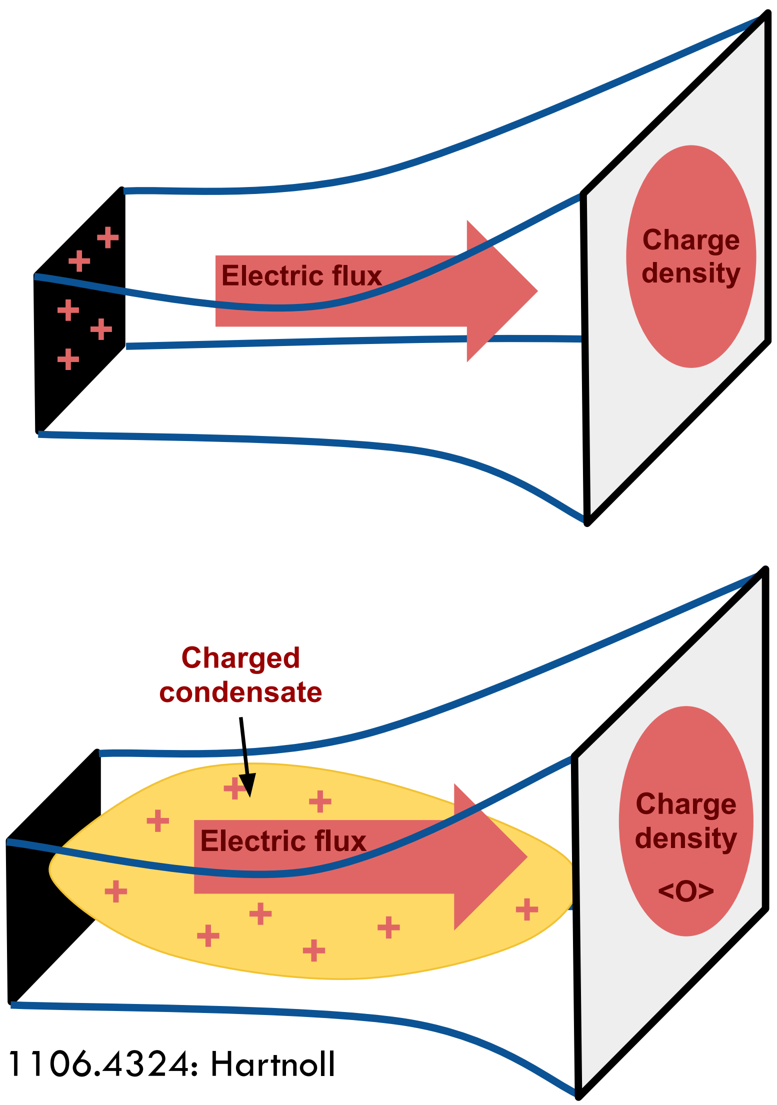
- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

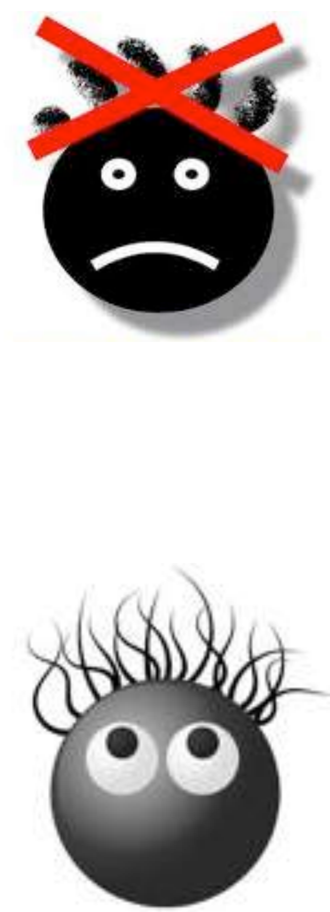
$$S_{HHH} = \int d^4x\sqrt{-g}\left[R+6-\frac{1}{4}F^2-|(\partial-iqA)\Phi|^2-m_\Phi^2\Phi\Phi^*\right]$$

$\Phi = 0$
AdS-RN-black brane

$\Phi \neq 0$
Holographic superconductor



1106.4324: Hartnoll

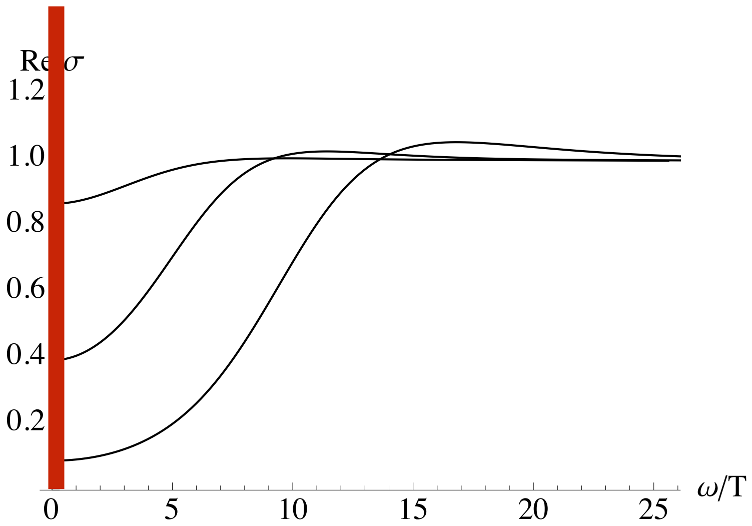


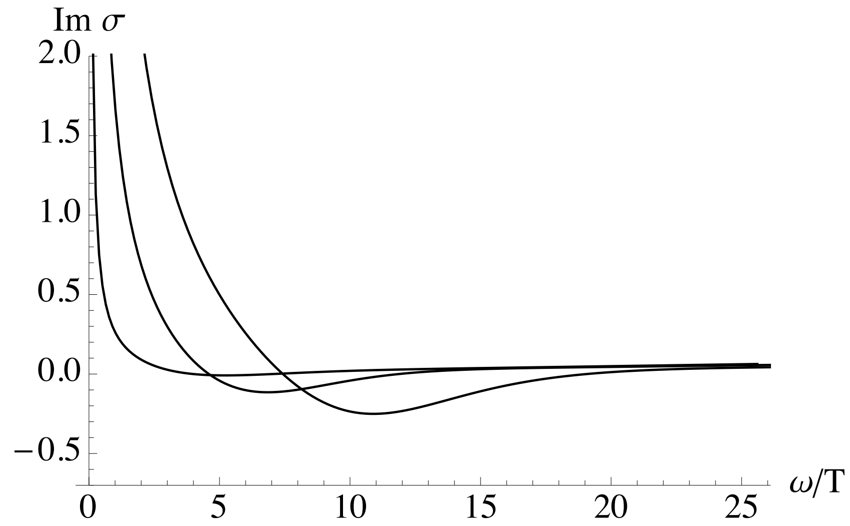
Optical conductivity

Conductivity: normal phase [Hartnoll: 0903.3234]

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$





$$\mathrm{Im}\,\sigma \sim 1/\omega \quad \Leftrightarrow \quad \mathrm{Re}\;\sigma(\omega) \sim \delta(\omega)$$

Kramers-Kronig relation

$$\chi(\omega) = \chi_R(\omega) + i\chi_I(\omega)$$

$$\chi_R(\omega) = \frac{1}{\pi}\mathcal{P}\int \frac{\chi_I(\omega')}{\omega'-\omega}d\omega' \,, \qquad \chi_I(\omega) = -\frac{1}{\pi}\mathcal{P}\int \frac{\chi_R(\omega')}{\omega'-\omega}d\omega'$$

$\boxed{\text{Translation invariance + finite density}}$ Gauntlett' talk on Monday

Holographic superconductor with momentum relaxation

[Horowitz and Santos: 1302.6586]

The first holographic superconductor + momentum relaxation

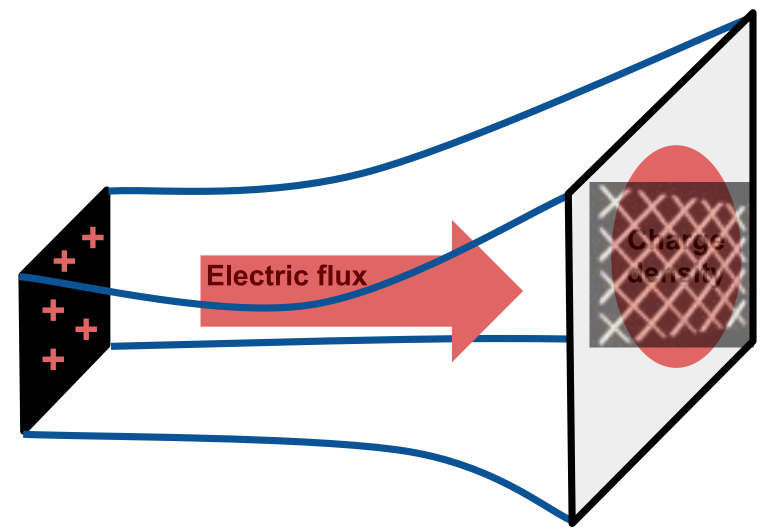
- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

$\Phi = 0$

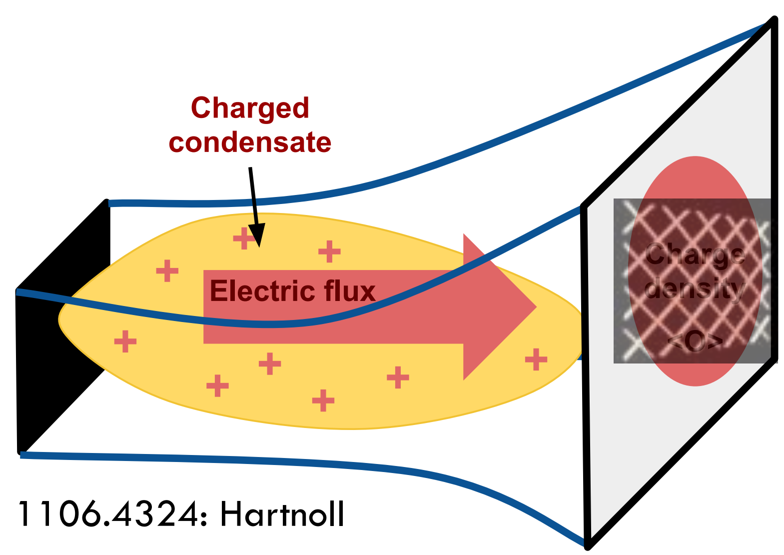
AdS-RN-black brane



$$A_t \sim 1 + A_0\cos(k_0 x)$$

$\Phi \neq 0$

Holographic superconductor



1106.4324: Hartnoll

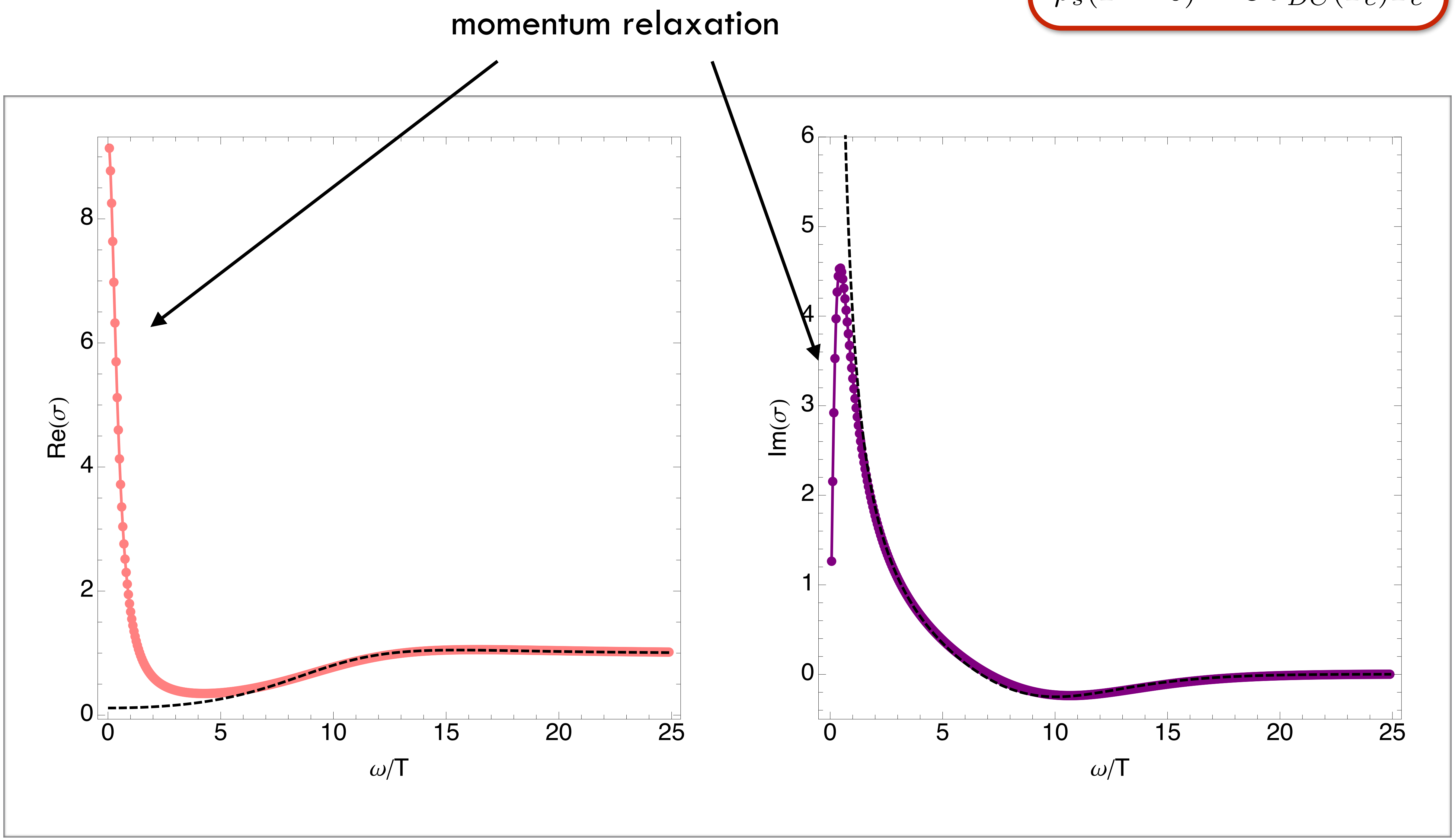
Optical conductivity

[Horowitz, Santos, Tong: 1204.0512, 1209.1098]

Conductivity: normal phase

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$



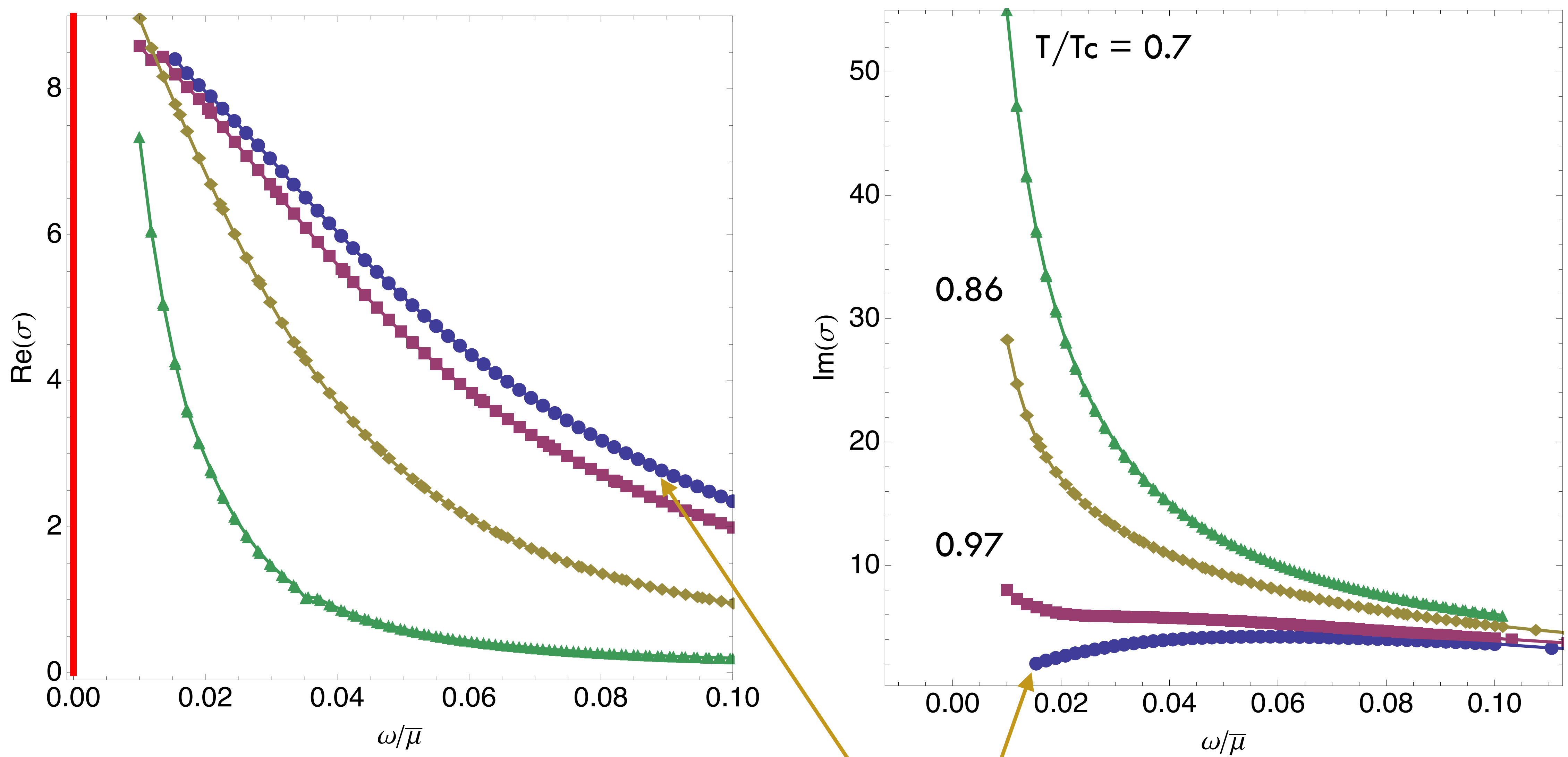
Optical conductivity

Horowitz and Santos (1302.6586)

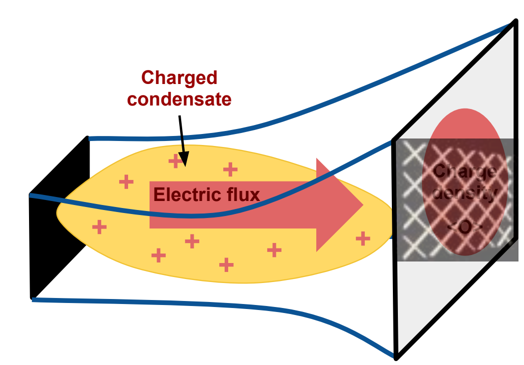
Conductivity: normal and superconducting phase

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$



normal phase



Holographic superconductor with momentum relaxation

$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

Massless scalar [Andrade, Withers: 1311.5157] → [Andrade, Gentle: 1412.6521] [KYK, Kim, Park: 1501.00446]

$$S_{MS} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

$\psi_I = (\beta x, \beta y)$

Q-lattice [Donos, Gauntlett: 1311.3292] → [Ling, Liu, Niu, Wu, Xian: 1410.6761] [Andrade, Gentle: 1412.6521]

$$S_Q = \int d^4x\sqrt{-g}\left[-|\partial\Psi|^2 - m_\Psi^2|\Psi|^2\right]$$

$\Psi = e^{ikx}z\psi(z)$

$\psi(0) = \lambda$

Helical lattice: [Erdmenger, Herwerth, Klug, Meyer, Schalm: 1501.07615] Erdmenger's talk on Tuesday

Massive gravity: [Zeng, Wu: 1404.532] [Baggioli and Goykhman: 1504.05561]

More on momentum relaxation: Gauntlett' talk on Monday

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

We want to check if C is universal (independent of momentum relaxation parameters)

Contents

- Motivations
- Holographic superconductor with momentum relaxation
- Massless scalar model
- Q-lattice model
- Summary and outlook

Model

[KYK, Kim, Park: 1501.00446]

Action

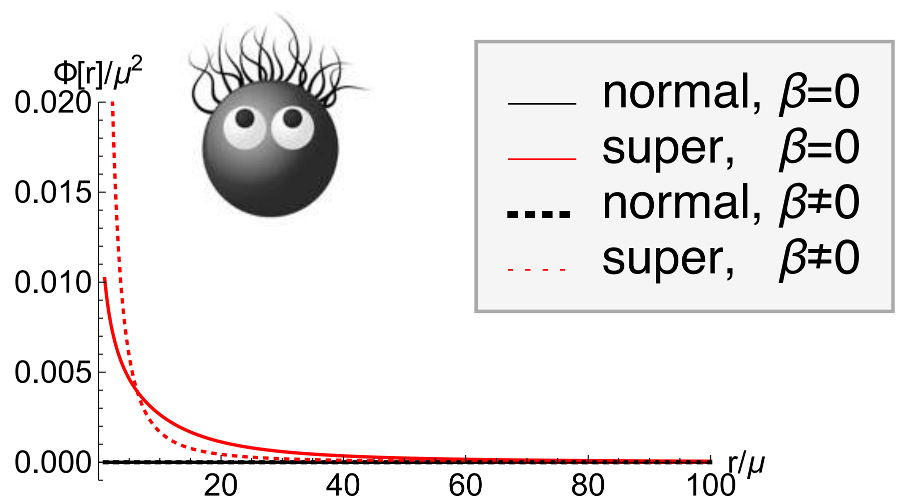
$$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^* - \frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

Ansatz

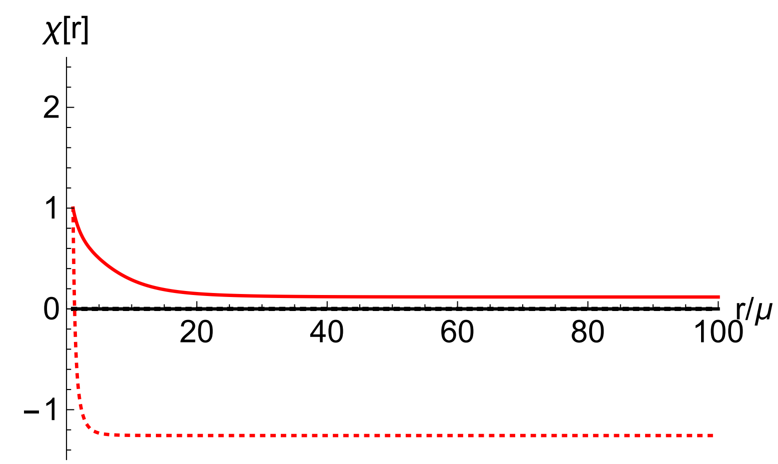
$$A = A_t(r)\mathrm{d}t \qquad \Phi = \Phi(r) \qquad \psi_I = (\beta x, \beta y)$$

$$\mathrm{d}s^2 = -U(r)e^{-\chi(r)}\mathrm{d}t^2 + \frac{\mathrm{d}r^2}{U(r)} + r^2(\mathrm{d}x^2 + \mathrm{d}y^2)$$

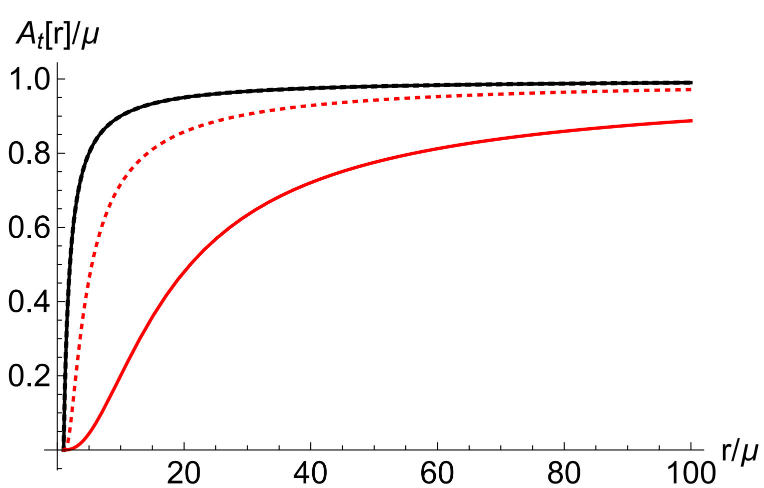
Solutions



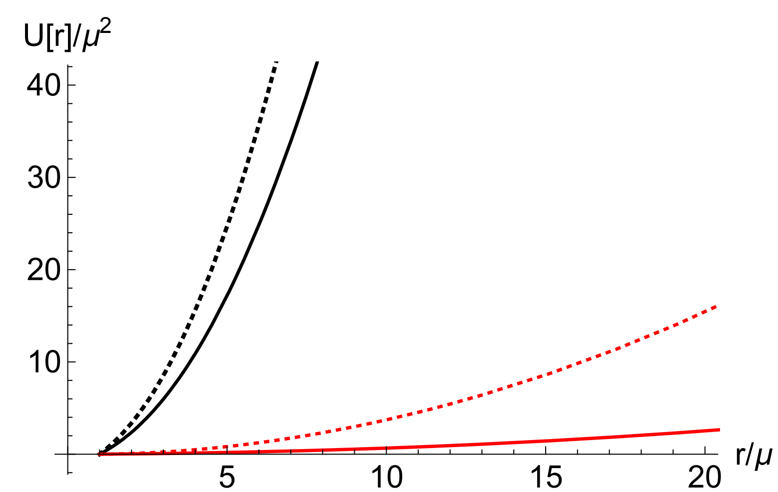
(a) $\Phi(r)/\mu^2$



(b) $\chi(r)$



(c) $A_t(r)/\mu$

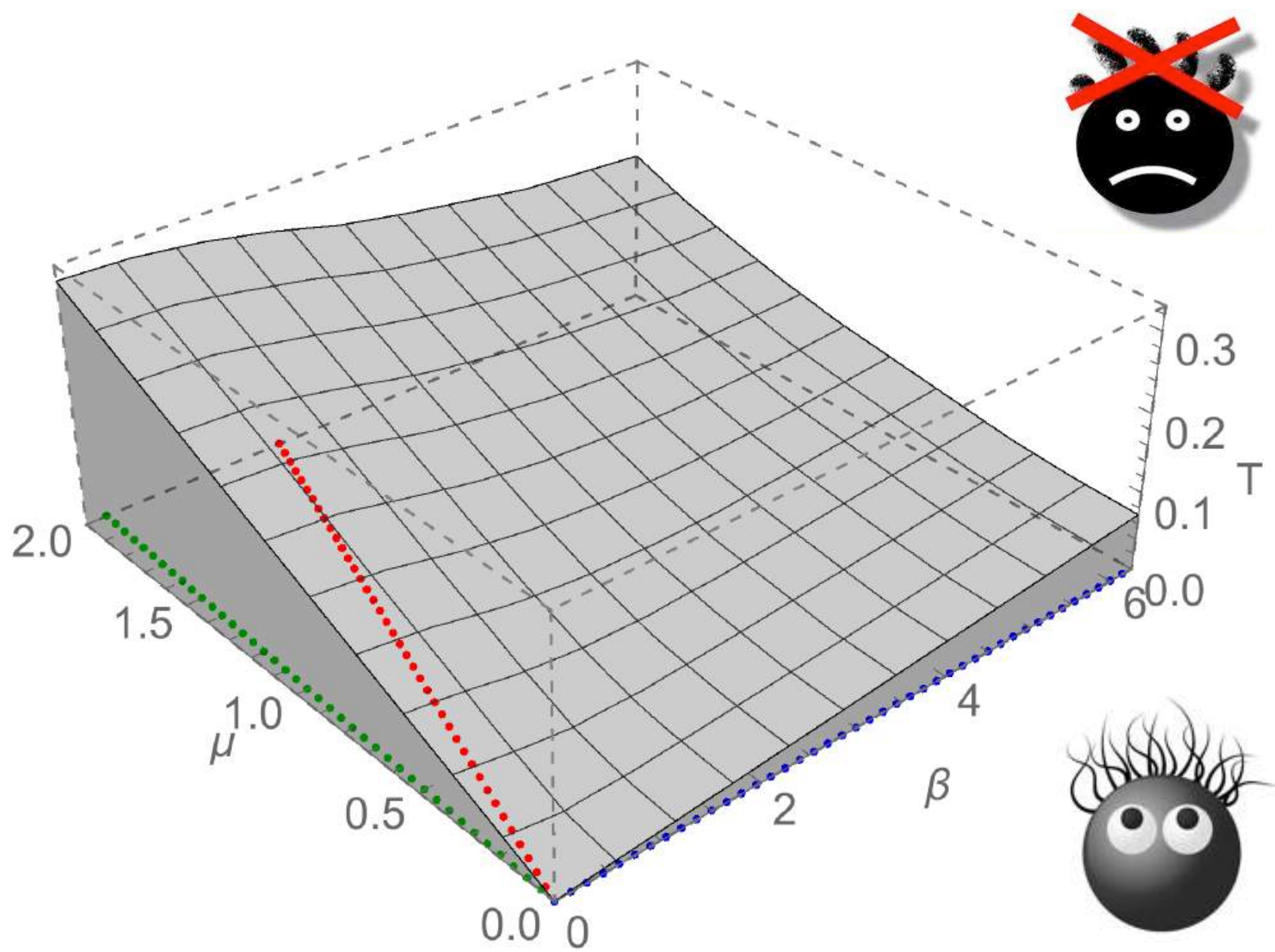


(d) $U(r)/\mu^2$

Critical temperature

[KYK, Kim, Park: 1501.00446]

- **Homes' law** $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$



$\beta/\mu = 1$

- Homes' law

$$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$$

$$\sigma_{DC} = \sigma(\omega = 0)$$

$$\sigma(\omega) \sim i\frac{\rho_s}{\omega}$$

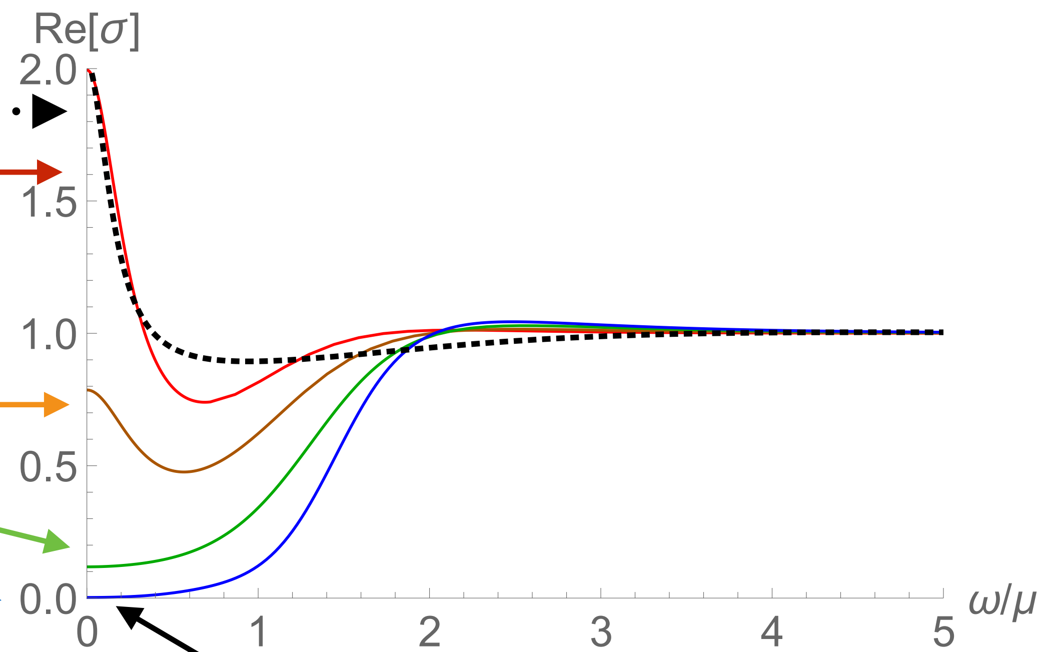
$T/T_c = 3.2$

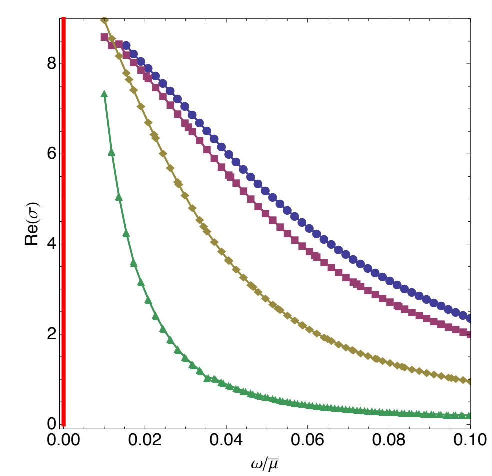
$T/T_c = 1$

$T/T_c = 0.89$

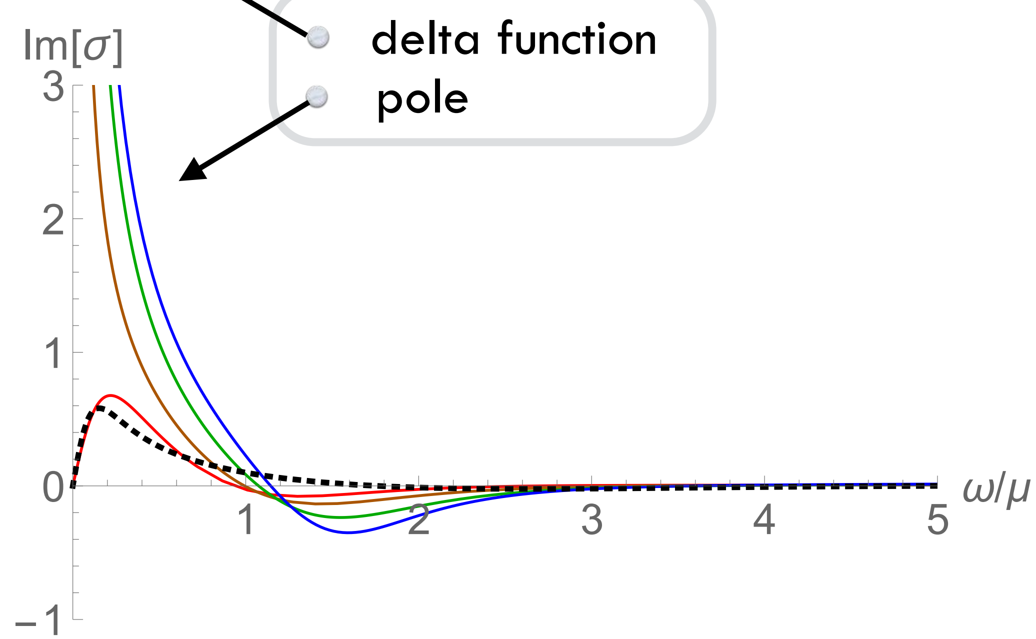
$T/T_c = 0.66$

$T/T_c = 0.27$

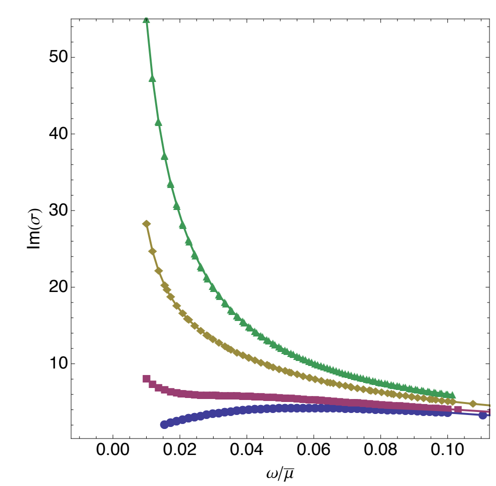




- delta function
- pole



no pole



Conductivities

[KYK, Kim, Park: 1501.00446]

Action

$$S=\int \mathrm{d}^4x\sqrt{-g}\left[R+6-\frac{1}{4}F^2-|(\partial-iqA)\Phi|^2-m_\Phi^2\Phi\Phi^*-\frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right]$$

Background

$$A=A_t(r)\mathrm{d}t \qquad \Phi=\Phi(r) \qquad \psi_I=(\beta x,\beta y)$$

$$\mathrm{d}s^2=-U(r)e^{-\chi(r)}\mathrm{d}t^2+\frac{\mathrm{d}r^2}{U(r)}+r^2(\mathrm{d}x^2+\mathrm{d}y^2)$$

$+$

Fluctuations

$$\delta A_x(t,r)=\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}a_x(\omega,r)$$

$$\delta g_{tx}(t,r)=\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}r^2h_{tx}(\omega,r)$$

$$\delta\psi_1(t,r)=\int_{-\infty}^{\infty}\frac{\mathrm{d}\omega}{2\pi}e^{-i\omega t}\xi(\omega,r)$$

$$h_{tx}=h_{tx}^{(0)}+\frac{1}{r^2}h_{tx}^{(2)}+\frac{1}{r^3}h_{tx}^{(3)}+\cdots,$$

$$a_x=a_x^{(0)}+\frac{1}{r}a_x^{(1)}+\cdots,$$

$$\xi=\xi^{(0)}+\frac{1}{r^2}\xi^{(2)}+\frac{1}{r^3}\xi^{(3)}+\cdots,$$

$$S_{\mathrm{ren}}^{(2)}=\frac{V_2}{2}\int_0^{\infty}\frac{\mathrm{d}\omega}{2\pi}\left(-\rho\bar{a}_x^{(0)}h_{tx}^{(0)}-\epsilon\bar{h}_{tx}^{(0)}h_{tx}^{(0)}+\bar{a}_x^{(0)}a_x^{(1)}-3\bar{h}_{tx}^{(0)}h_{tx}^{(3)}+3\bar{\xi}^{(0)}\xi^{(3)}\right)$$

- Homes' law

$$\rho_s(T=0)=C\sigma_{DC}(T_c)T_c$$

$$\sigma_{DC}=\sigma(\omega=0)$$

$$\sigma(\omega)\sim i\frac{\rho_s}{\omega}$$

Numerical method for multi fields

[Kaminski, Landsteiner, Mas, Shock, Tarrio: 2009]
[KYK, Kim, Sin, Seo: 2014)]

Action

$$S^{(2)}_{\rm ren} = \frac{V_2}{2}\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\left(-\rho \bar{a}^{(0)}_x h^{(0)}_{tx} - \epsilon \bar{h}^{(0)}_{tx} h^{(0)}_{tx} + \bar{a}^{(0)}_x a^{(1)}_x - 3\bar{h}^{(0)}_{tx} h^{(3)}_{tx} + 3\bar{\xi}^{(0)}\xi^{(3)}\right)$$

Structure

$$S^{(2)}_{\rm ren} = \frac{V_2}{2}\int_0^\infty \frac{\mathrm{d}\omega}{(2\pi)}\left[\bar{J}^a \mathbb{A}_{ab} J^b + \bar{J}^a \mathbb{B}_{ab} R^b\right]$$

$$R^a = \mathbb{M}^a_b J^b$$

$$J^a = \begin{pmatrix} a^{(0)}_x \\ h^{(0)}_{tx} \\ \xi^{(0)} \end{pmatrix}, \quad R^a = \begin{pmatrix} a^{(1)}_x \\ h^{(3)}_{tx} \\ \xi^{(3)} \end{pmatrix}, \quad \mathbb{A} = \begin{pmatrix} 0 & -\rho & 0 \\ 0 & -\epsilon & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \mathbb{B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

$$S^{(2)}_{\rm ren} = \frac{V_2}{2}\int_0^\infty \frac{\mathrm{d}\omega}{(2\pi)}\left[\bar{J}^a \overbrace{(\mathbb{A}_{ab} + \mathbb{B}_{ac}\mathbb{M}^c_b)}^{G^R_{ab}} J^b\right]$$

$$\begin{pmatrix} \sigma & \alpha T \\ \bar{\alpha} T & \bar{\kappa} T \end{pmatrix} = \begin{pmatrix} -\frac{iG_{11}}{\omega} & \frac{i(G_{11}\mu - G_{12})}{\omega} \\ \frac{i(G_{11}\mu - G_{21})}{\omega} & -\frac{i(G_{22} + \mu(-G_{12} - G_{21} + G_{11}\mu))}{\omega} \end{pmatrix}$$

How to compute $\mathbb{M}^a_b$

$$\Phi^a_i(r) \to \mathbb{S}^a_i + \cdots + \frac{\mathbb{O}^a_i}{r^{\delta_a}} + \cdots$$

$$\Phi^a(r) = \Phi^a_i(r) c^i \to \mathbb{S}^a_i c^i + \cdots + \frac{\mathbb{O}^a_i c^i}{r^{\delta_a}} + \cdots$$

$$\equiv J^a + \cdots + \frac{R^a}{r^{\delta_a}} + \cdots,$$

$$c^i = (\mathbb{S}^{-1})^i_a J^a \qquad R^a = \mathbb{O}^a_i c^i = \mathbb{O}^a_i (\mathbb{S}^{-1})^i_b J^b$$

$$h_{tx} = h^{(0)}_{tx} + \frac{1}{r^2} h^{(2)}_{tx} + \frac{1}{r^3} h^{(3)}_{tx} + \cdots,$$

$$a_x = a^{(0)}_x + \frac{1}{r} a^{(1)}_x + \cdots,$$

$$\xi = \xi^{(0)} + \frac{1}{r^2}\xi^{(2)} + \frac{1}{r^3}\xi^{(3)} + \cdots,$$

ex) one field case: $\frac{a^{(1)}_x}{a^{(0)}_x}$

Entrance: Four quick Digressions

Digression1: thermoelectric conductivities and DC limits

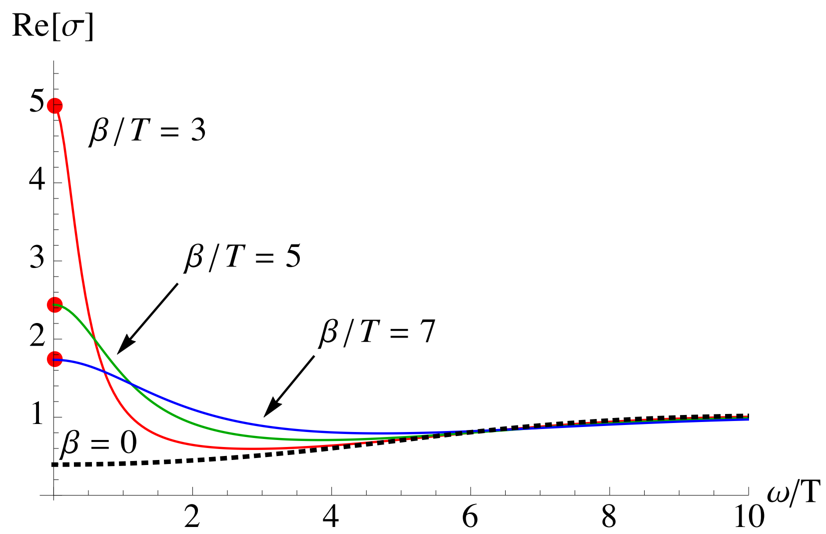
[KYK, Kim, Sin, Seo: 1409.8346]

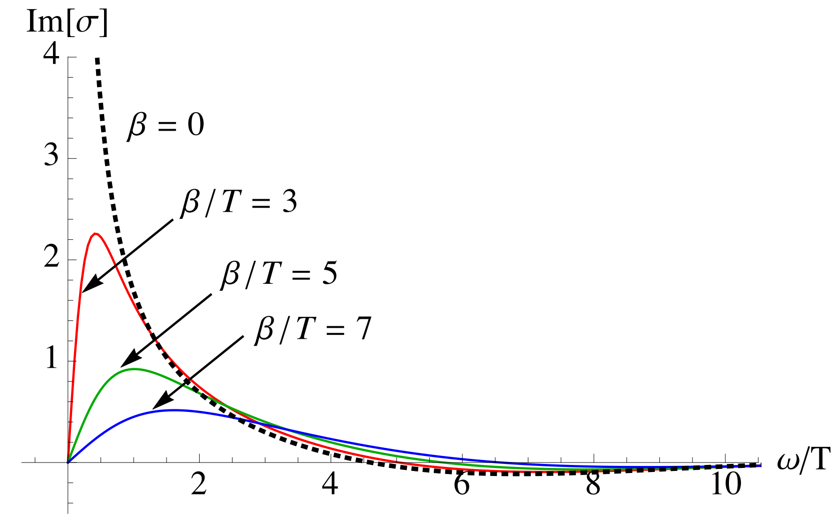
Electric conductivity

$\mu/T = 6$

DC result:
Andrade, Withers
1311.5157

$$\boxed{\sigma = 1 + \frac{\mu^2}{\beta^2}}$$



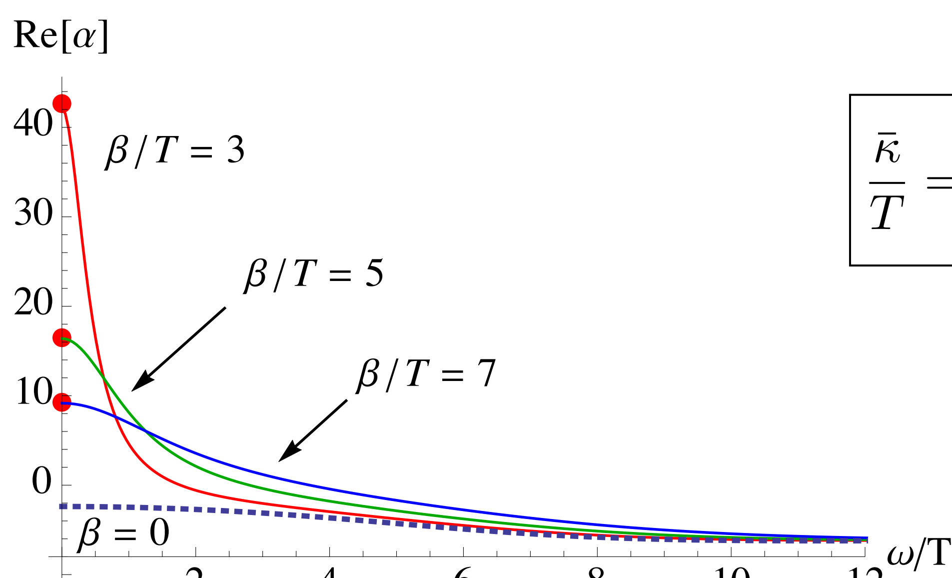


Thermoelectric conductivity

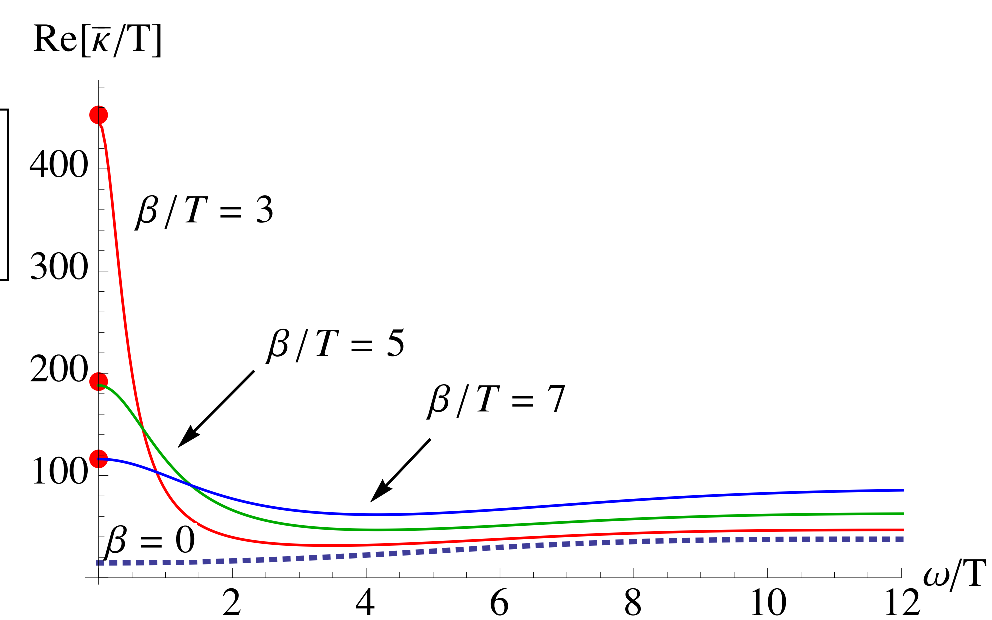
DC results:
Donos and Gauntlett
1406.4742

Thermal conductivity

$$\boxed{\alpha = \frac{4\pi\mu}{\beta^2} r_0}$$



$$\boxed{\frac{\bar{\kappa}}{T} = \frac{(4\pi)^2}{\beta^2} r_0^2}$$

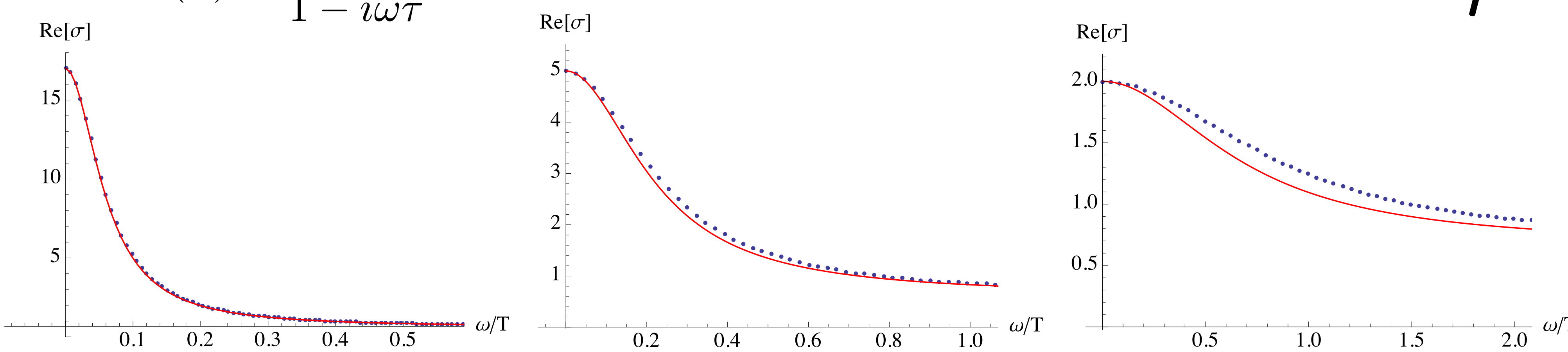


Digression2: coherent vs incoherent

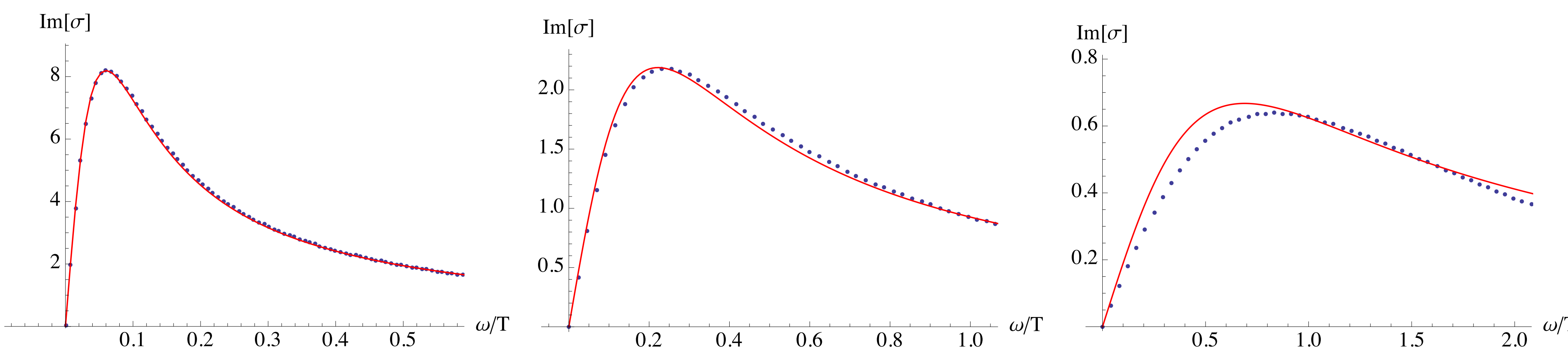
[KYK, Kim, Sin, Seo: 1409.8346]

β Drude → Non-Drude β

$$\sigma(\omega) = \frac{K\tau}{1 - i\omega\tau}$$



(a) Re σ at $\beta/\mu = 1/4$ (b) Re σ at $\beta/\mu = 1/2$ (c) Re σ at $\beta/\mu = 1$



(d) Im σ at $\beta/\mu = 1/4$ (e) Im σ at $\beta/\mu = 1/2$ (f) Im σ at $\beta/\mu = 1$

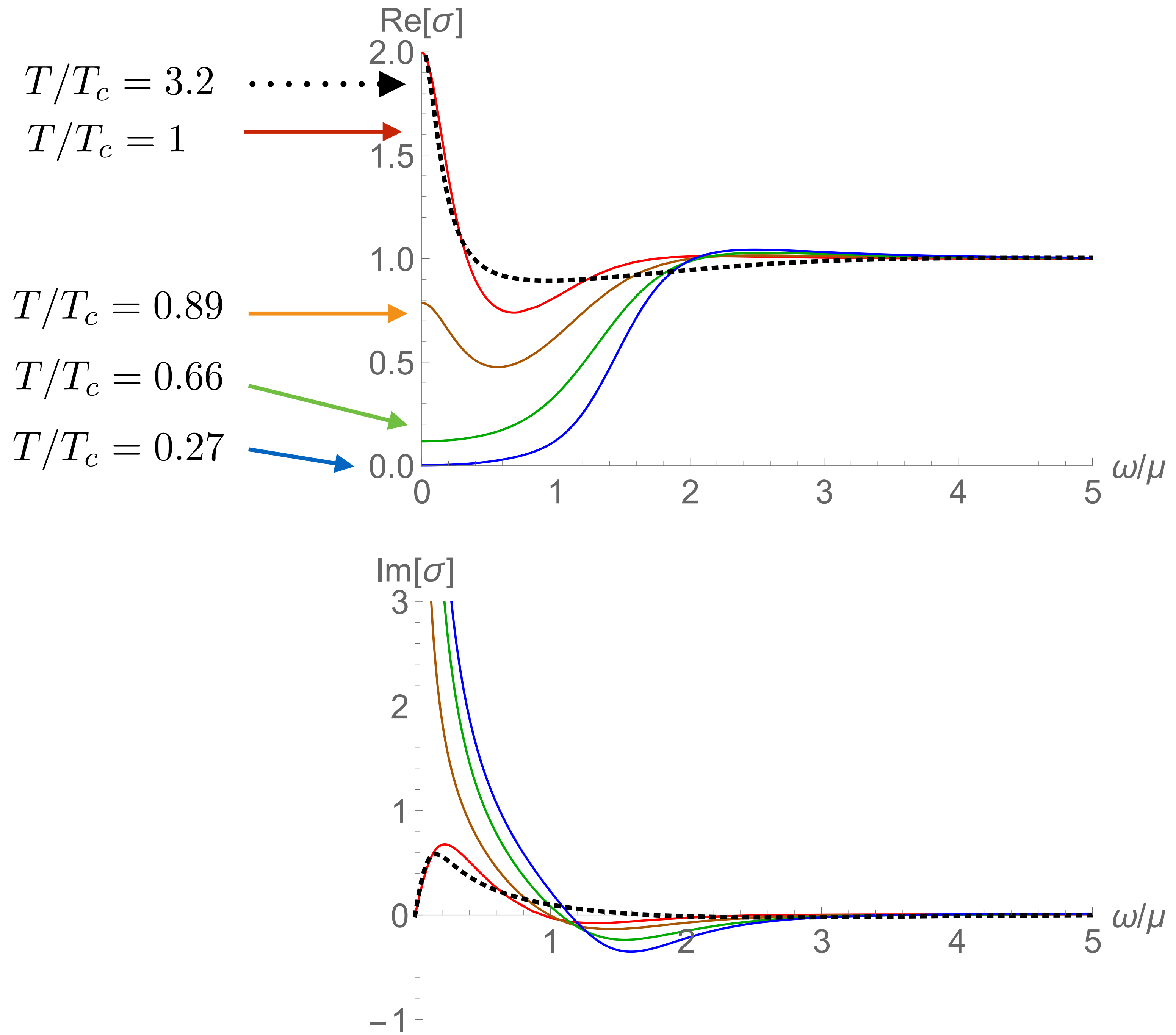
First numerical evidence of coherent/incoherent transition
with [Davison and Gouteraux: 1411.1062]

[Hartnoll: 1405.3651]

Sum rule works!

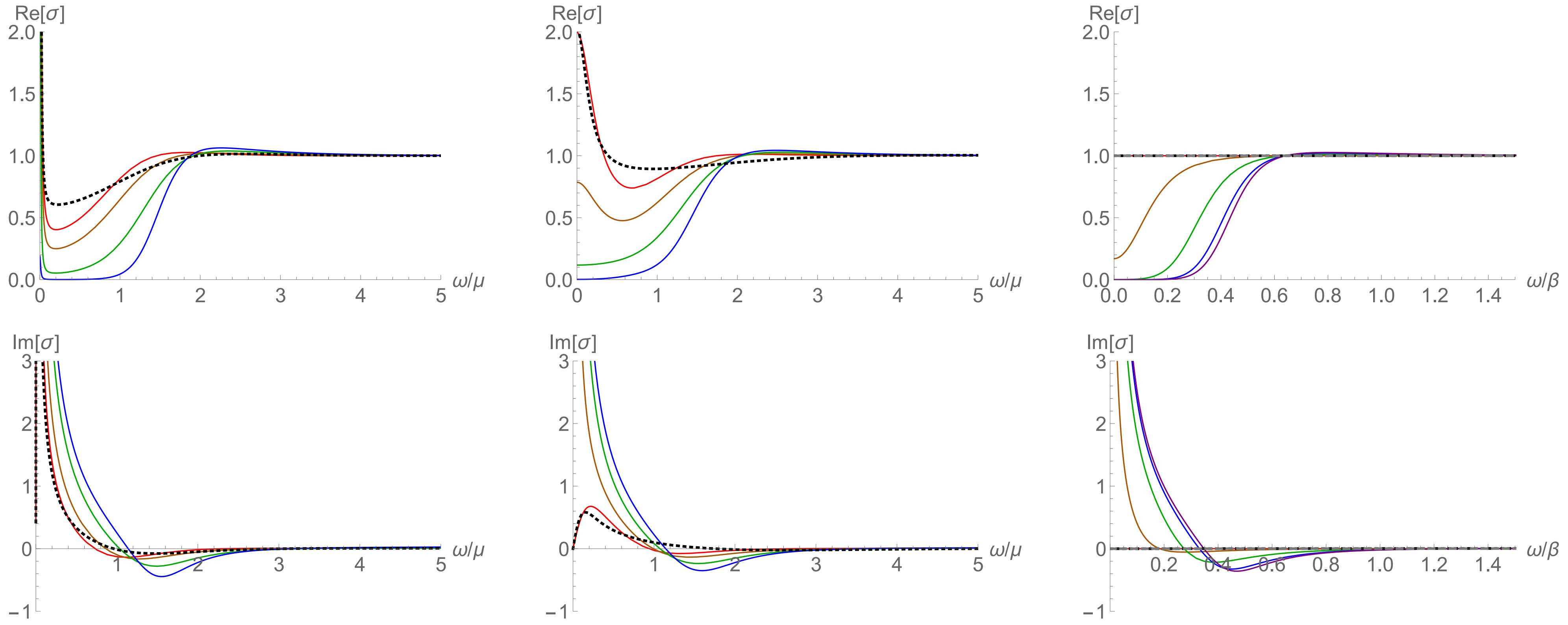
$$\mathrm{FGT} \equiv \int_{0^+}^{\infty} \mathrm{d}\omega \mathrm{Re}[\sigma_n(\omega) - \sigma_s(\omega)] - \frac{\pi}{2} K_s = 0$$

Ferrell-Glover-Tinkham(FGT)



Digression3: FGT sum rule

β

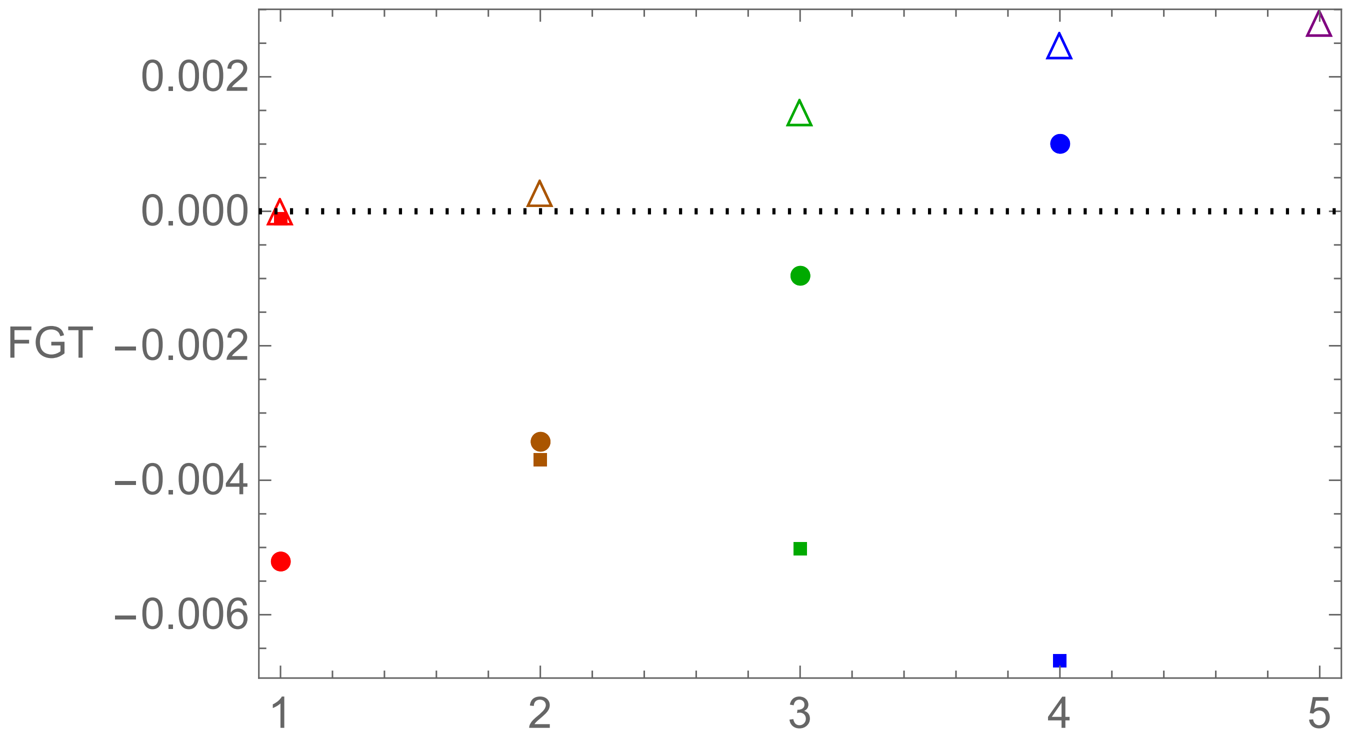


β

Sum rule works!

$$\mathrm{FGT} \equiv \int_{0+}^{\infty} \mathrm{d}\omega \mathrm{Re}[\sigma_n(\omega) - \sigma_s(\omega)] - \frac{\pi}{2} K_s = 0$$

Ferrell-Glover-Tinkham(FGT)



Digression4: Ward identities

[KYK, Kim, Park: 1604.06205]

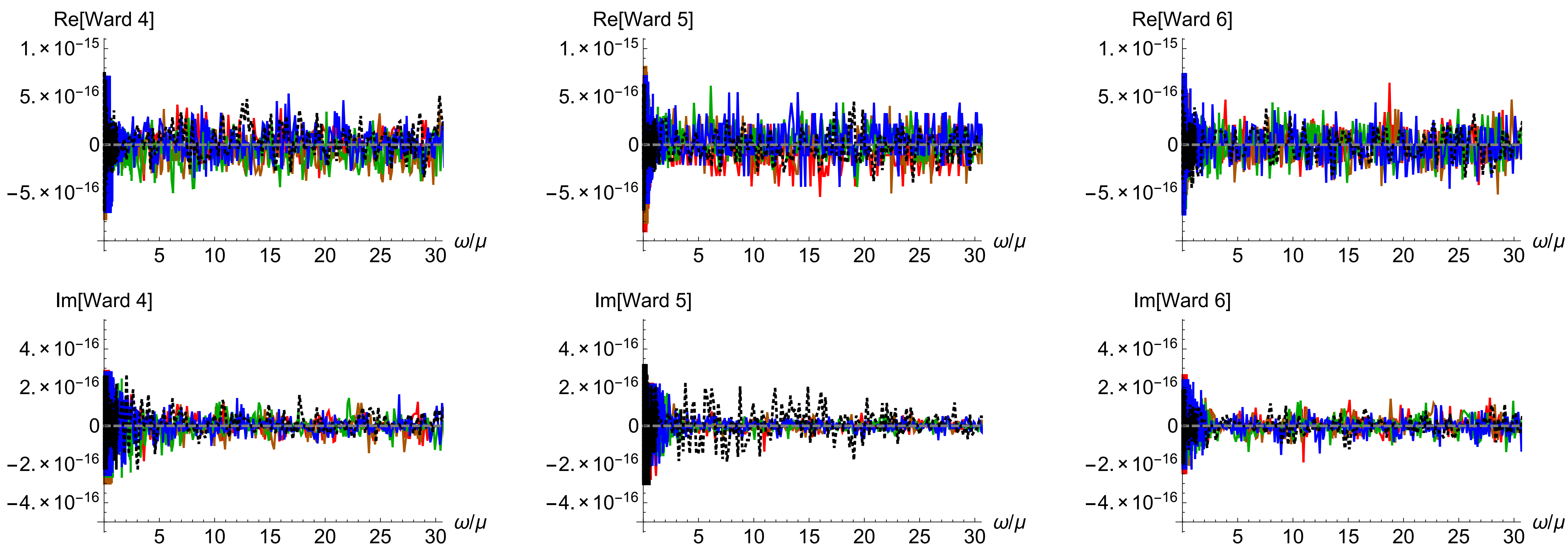
Derivation from field theory

$$\text{Ward 4}: \quad \alpha + \frac{\mu}{T}\sigma - i\frac{n}{\omega T} + \beta\frac{\langle JS\rangle}{\omega^2 T} = 0\,,$$

$$\text{Ward 5}: \quad \bar{\kappa} + 2\mu\alpha + \frac{\mu^2\sigma}{T} - i\frac{\epsilon'}{\omega T} + \beta\frac{\langle QS\rangle}{\omega^2 T} + \beta\frac{\mu\langle JS\rangle}{\omega^2 T} = 0$$

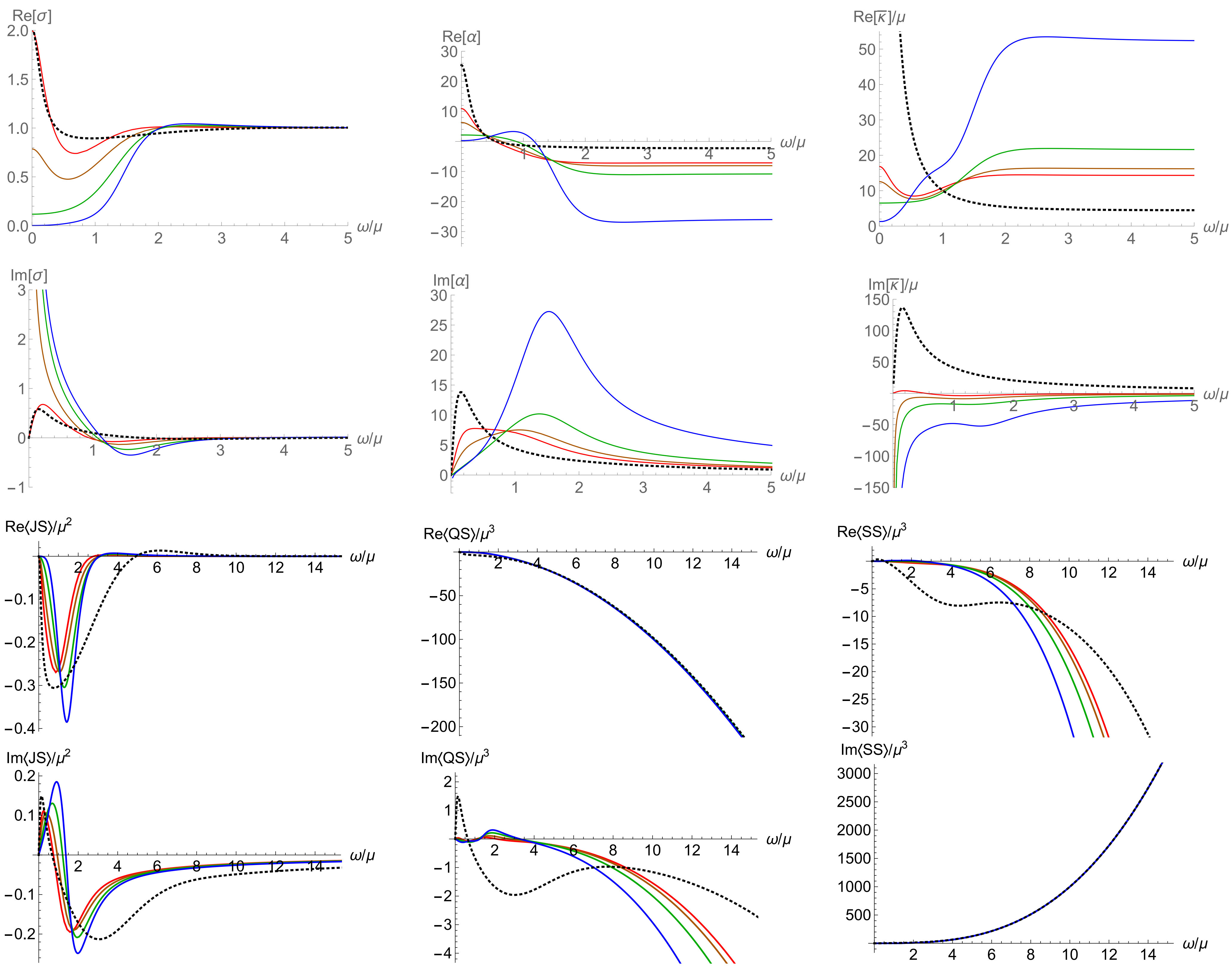
$$\text{Ward 6}: \quad \langle ST\rangle + i\beta\frac{\langle SS\rangle}{\omega} = 0\,,$$

Confirmation by numerical holography



Digression4: Ward identities

[KYK, Kim, Park: 1604.06205]



Exit: Four quick Digressions

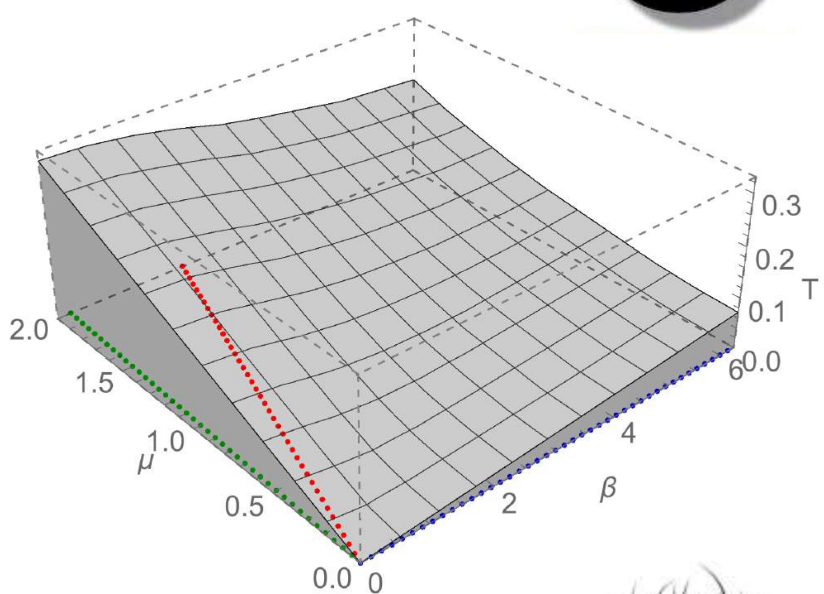
Homes' law and Uemura's law

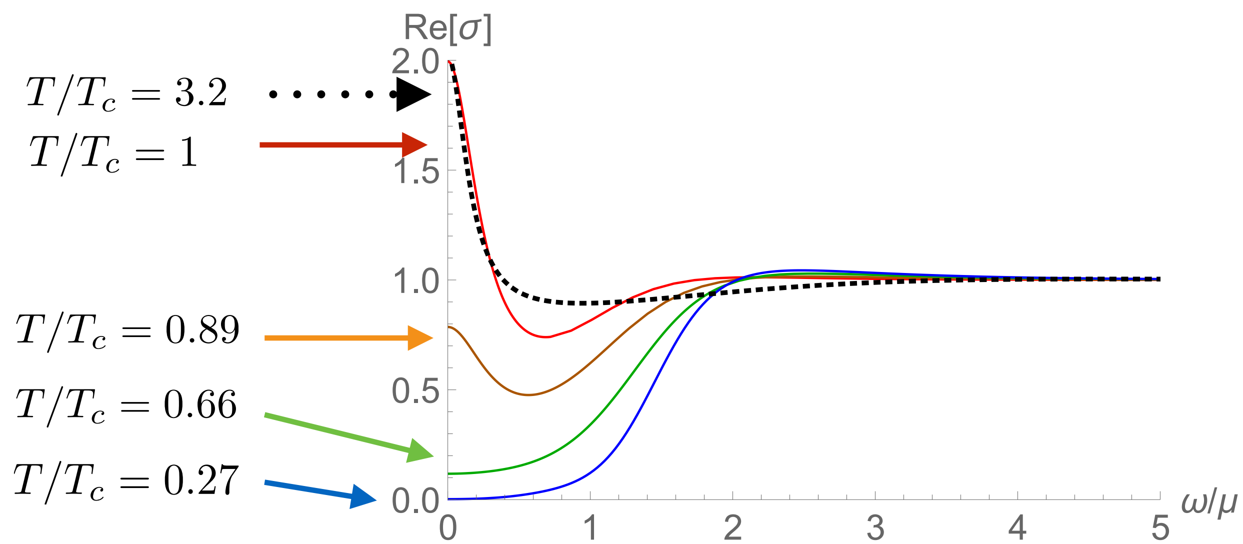
Are we ready?

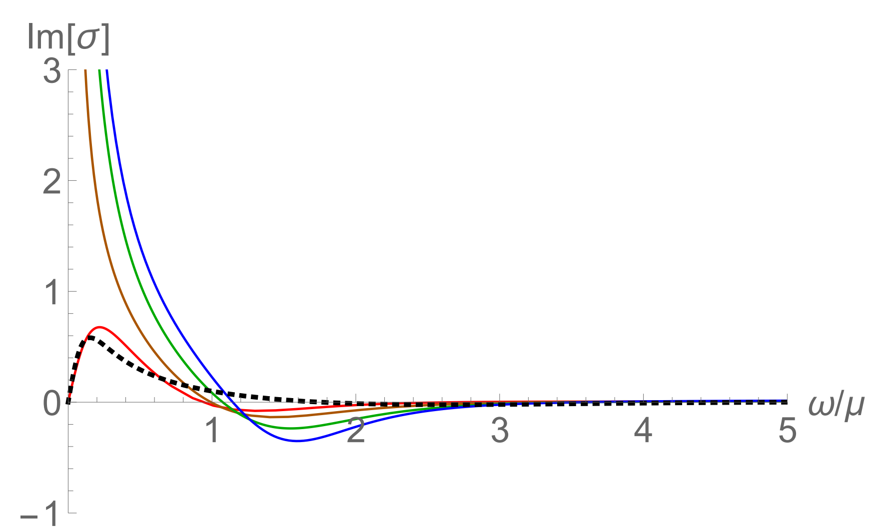
- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

$$\sigma_{DC} = \sigma(\omega = 0)$$

$$\sigma(\omega) \sim i\frac{\rho_s}{\omega}$$

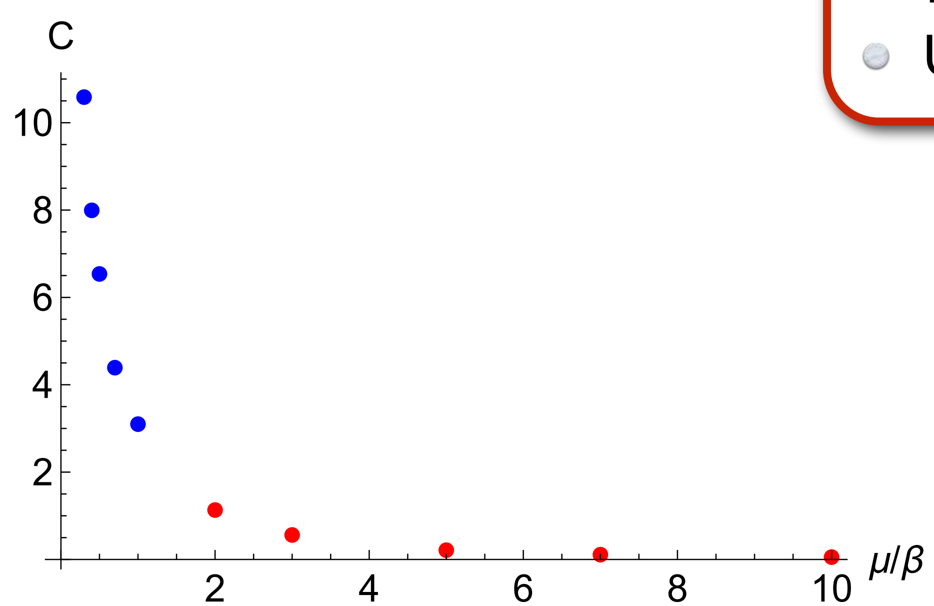






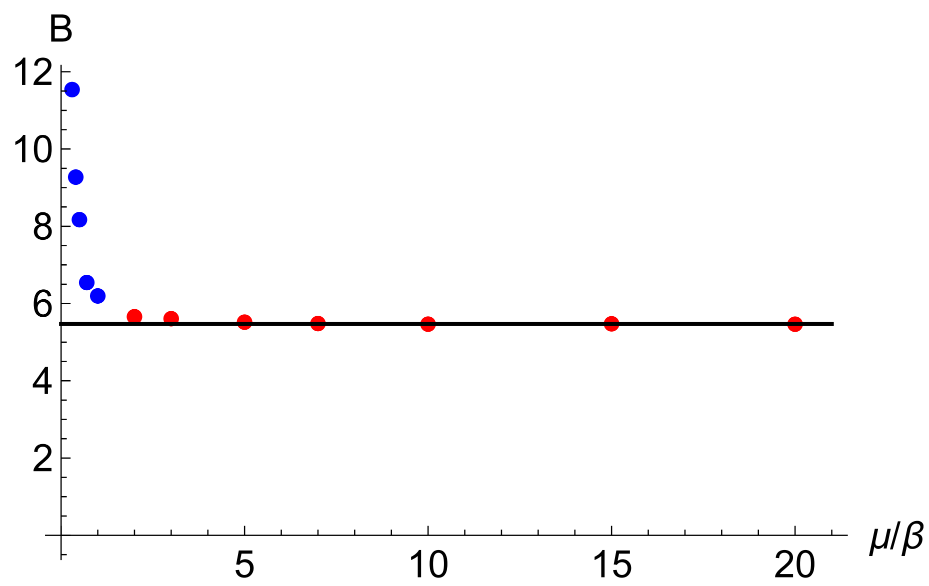
- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

~~Homes' law~~

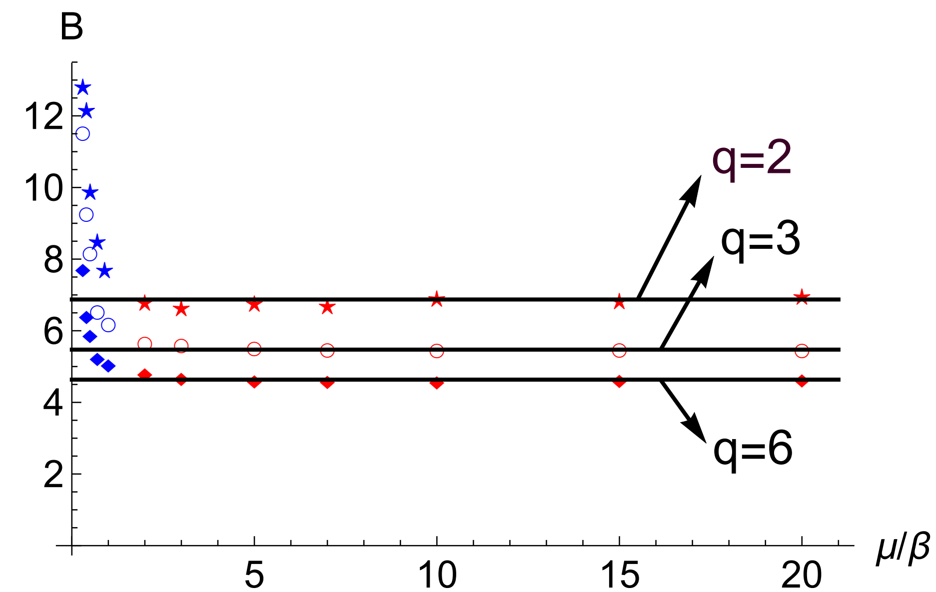


(a) $C(=\tilde{\rho}_s/(\sigma_{DC}\tilde{T}_c))$, $q=3$

Uemura's law



(a) $B(=\tilde{\rho}_s/\tilde{T}_c)$, $q=3$

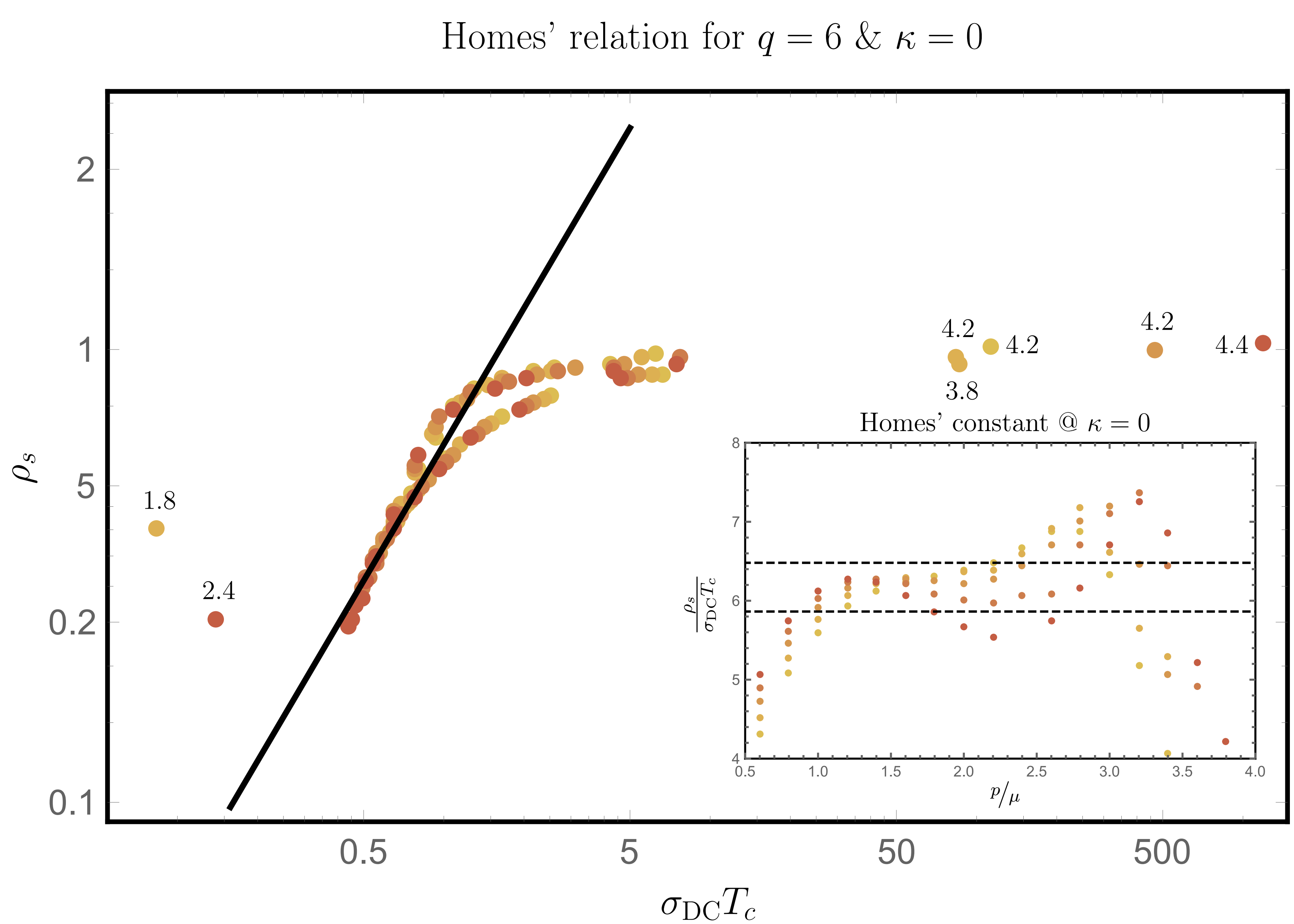


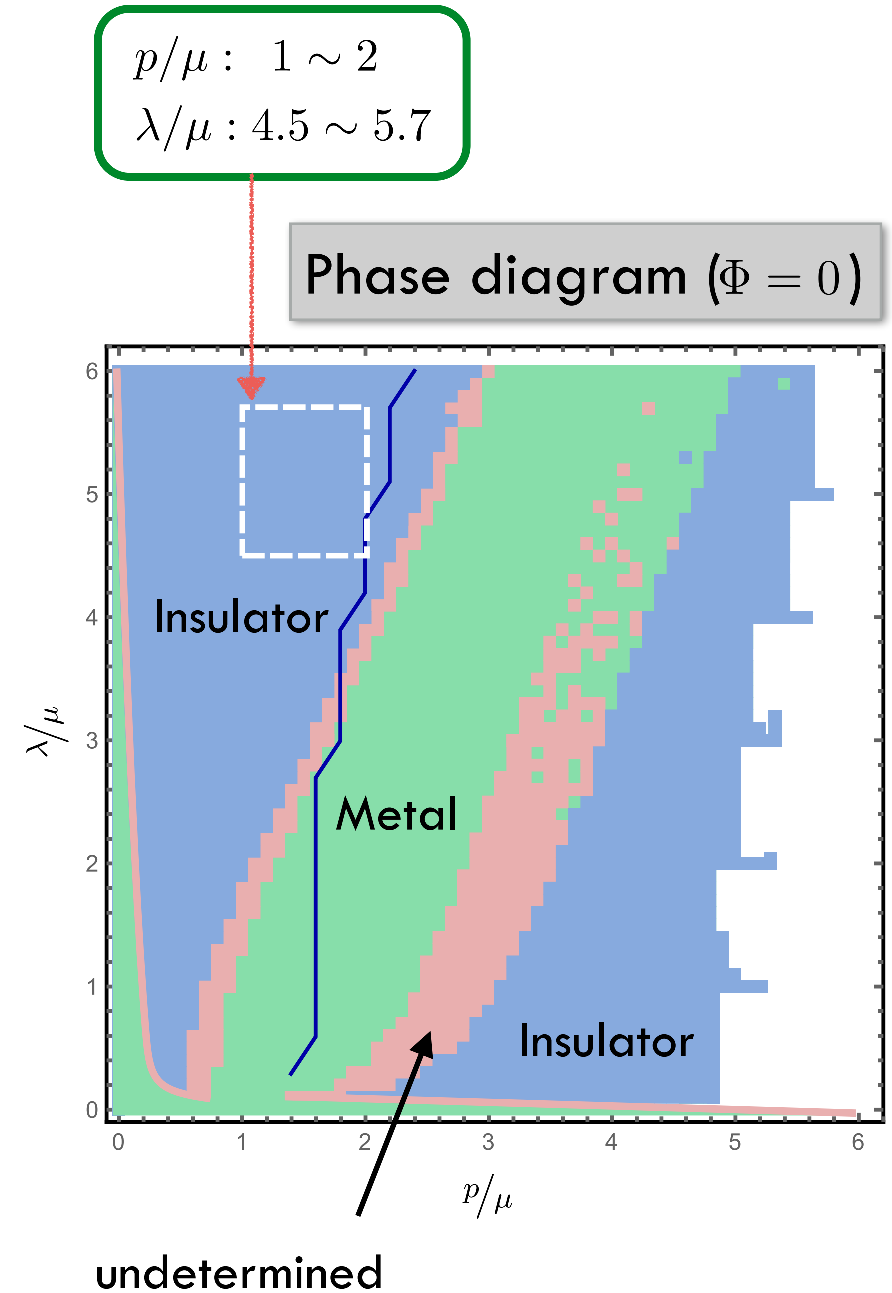
(b) $B(=\tilde{\rho}_s/\tilde{T}_c)$ for $q=2,3,6$

Helical lattice case

[Erdmenger, Herwerth, Klug, Meyer, Schalm: 1501.07615]

Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$





$\lambda/\mu = 4.5,\ 4.8,\ 5.1,\ 5.4,\ 5.7$

Contents

- Motivations
- Holographic superconductor with momentum relaxation
- Massless scalar model
- Q-lattice model
- Summary and outlook

Homes' law: Q-lattice

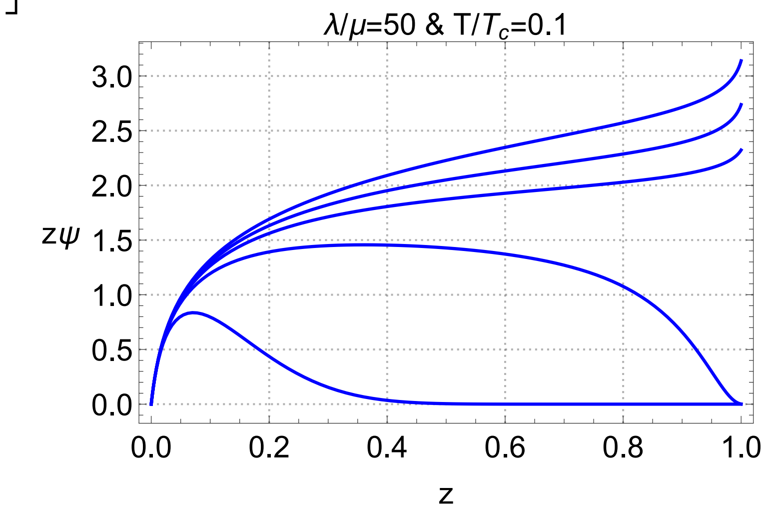
[KYK, Niu: 1605.XXXXX]

Action $$S = \int \mathrm{d}^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^* - |\partial\Psi|^2 - m_\Psi^2|\Psi|^2\right]$$

Ansatz $$\mathrm{d}s^2 = \frac{1}{z^2}\left[-(1-z)U(z)\mathrm{d}t^2 + \frac{\mathrm{d}z^2}{(1-z)U(z)} + V_1(z)\mathrm{d}x^2 + V_2(z)\mathrm{d}y^2\right]$$

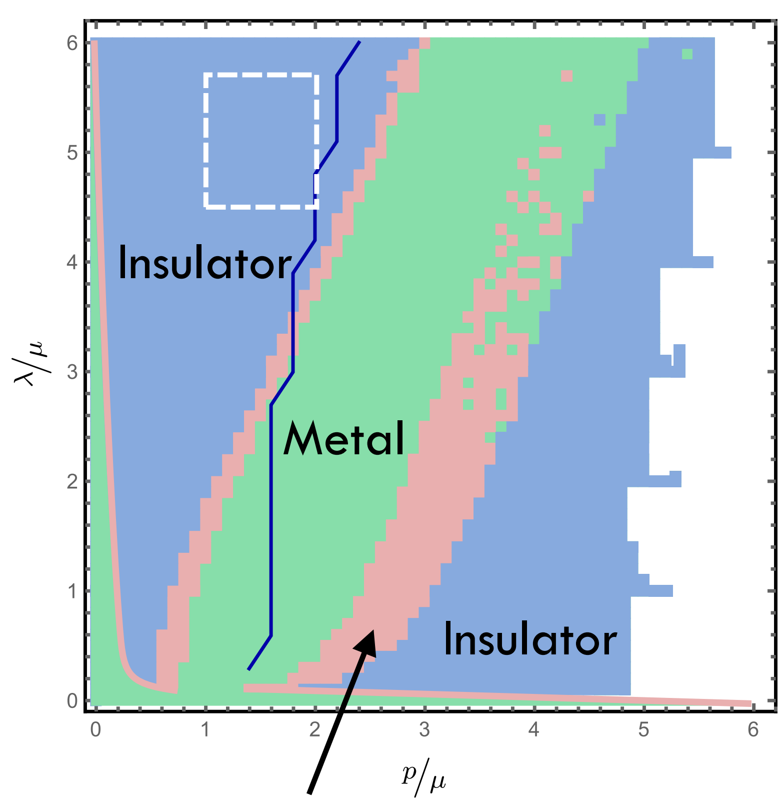
$$A = \mu(1-z)a(z)\mathrm{d}t \qquad \Phi = z\phi(z) \qquad \Psi = e^{ikx}z\psi(z) \quad (\psi(0) = \lambda)$$

Two parameters



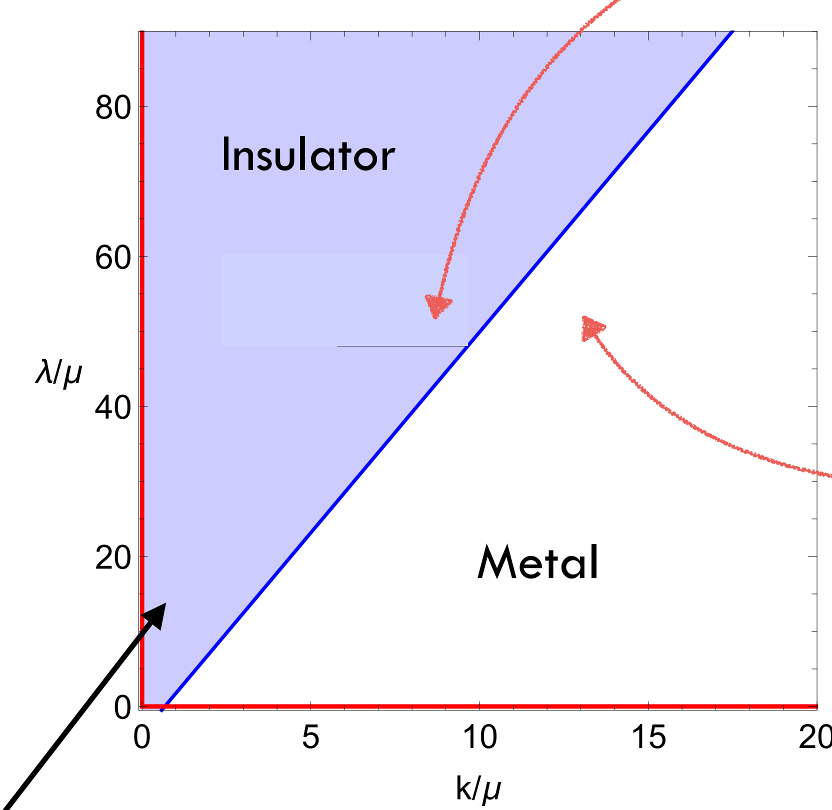
Phase diagram ($\Phi = 0$)

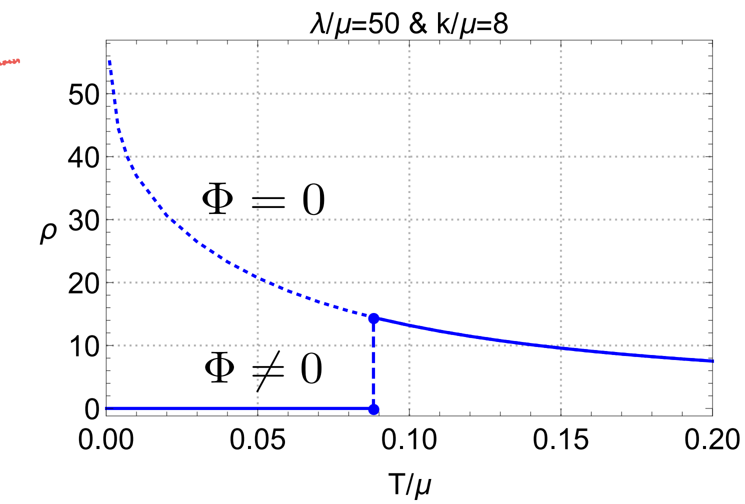
Helical lattice

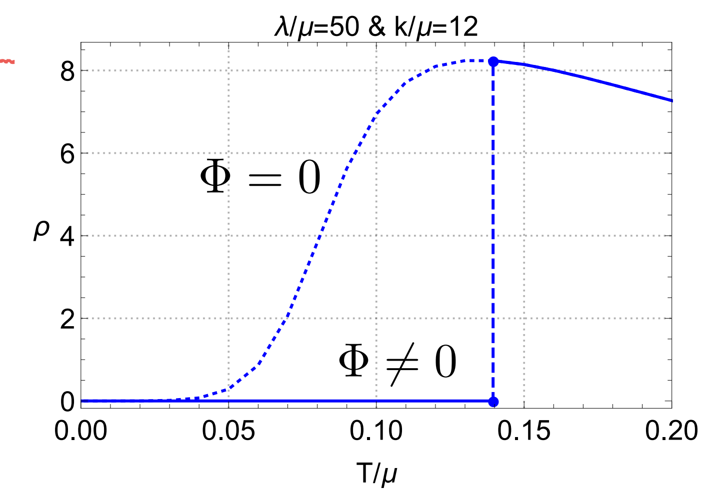


undetermined

Q lattice



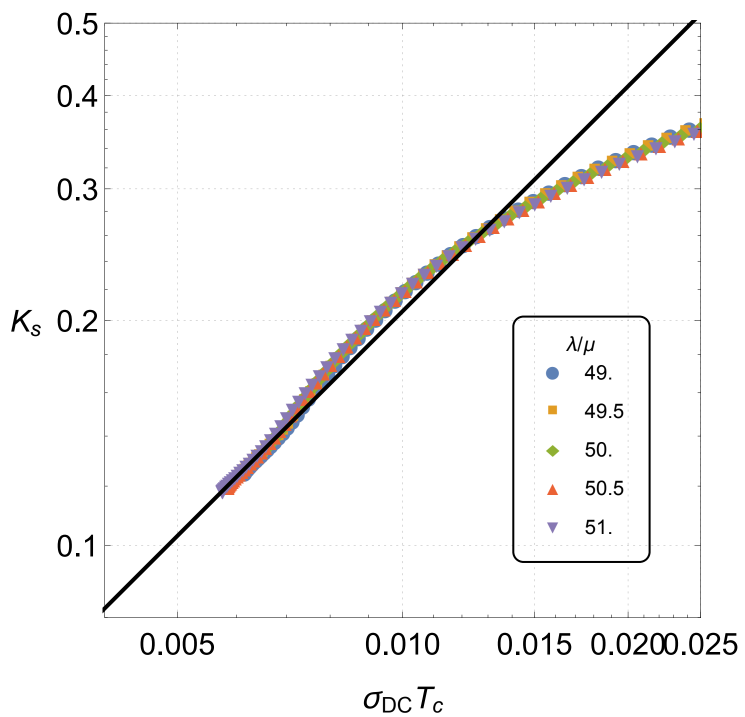




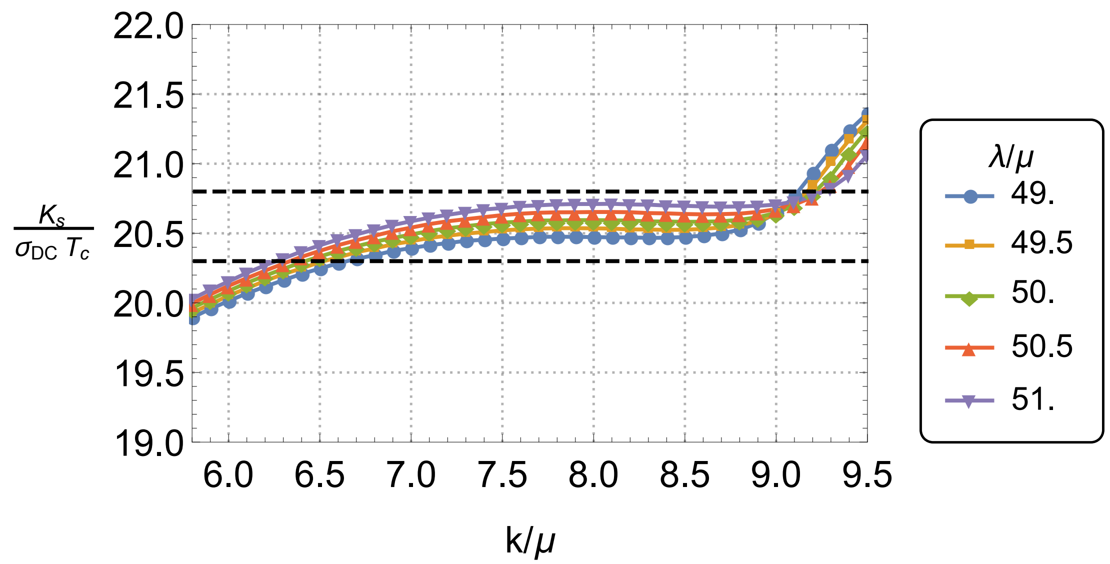
There may be metal regime near k=0: numerical issue

Comparison: Helical lattice and Q-lattice

[KYK, Niu: 1605.XXXXX]

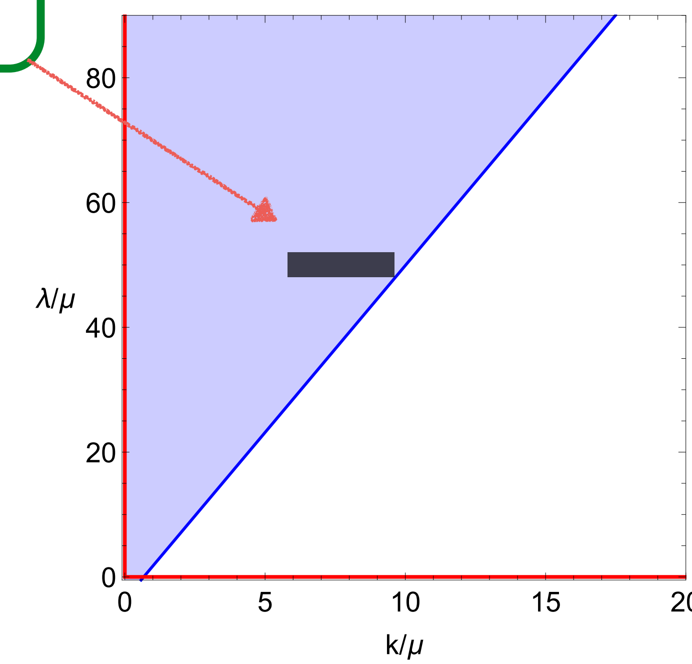


$\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$



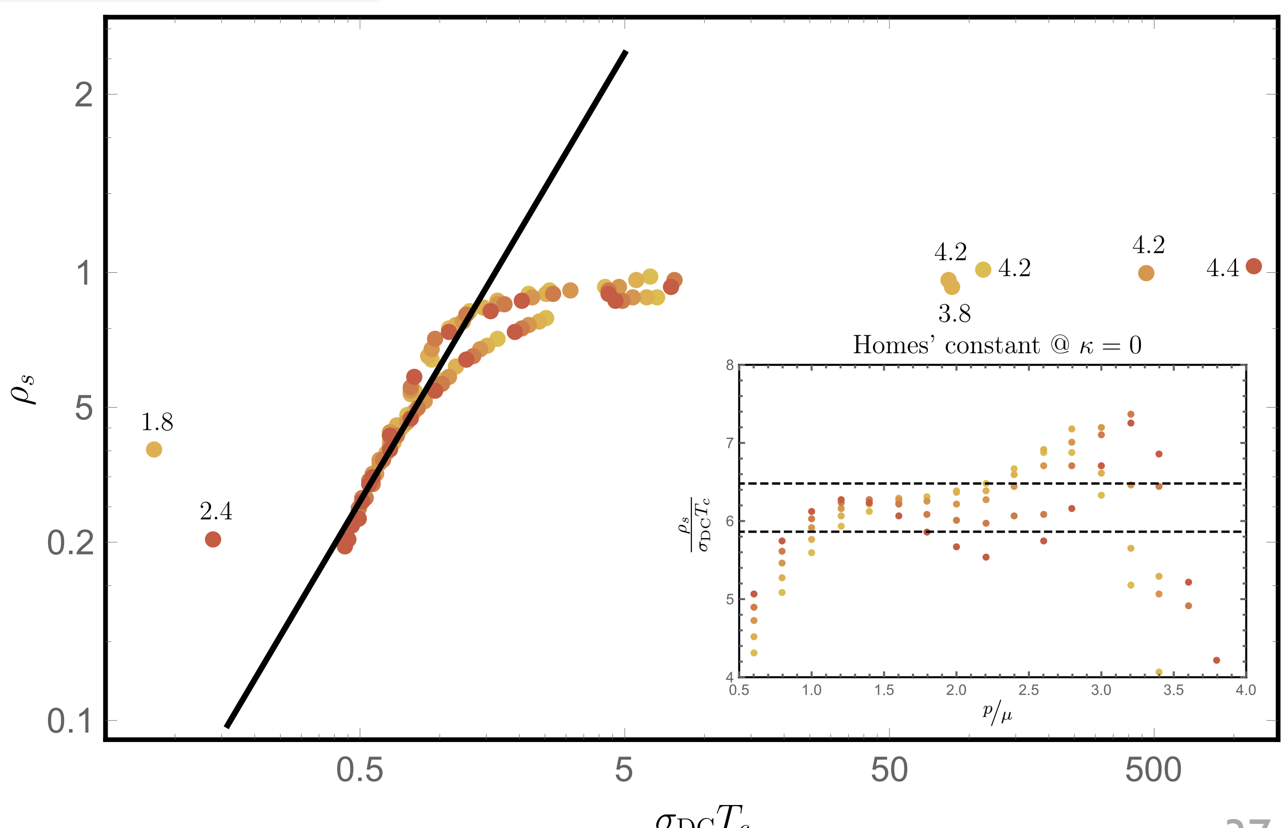
$k/\mu: \ 7 \sim 9$

$\lambda/\mu: 49 \sim 51$



Helical lattice

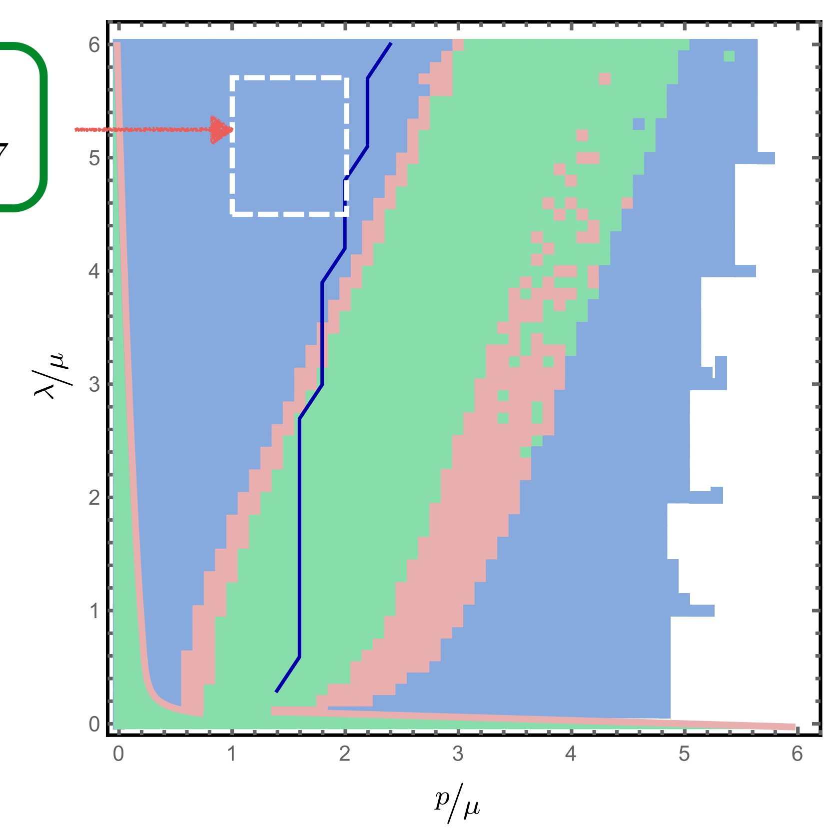
Homes' relation for $q = 6$ & $\kappa = 0$



$p/\mu: \ 1 \sim 2$

$\lambda/\mu: 4.5 \sim 5.7$

$\kappa = 0$



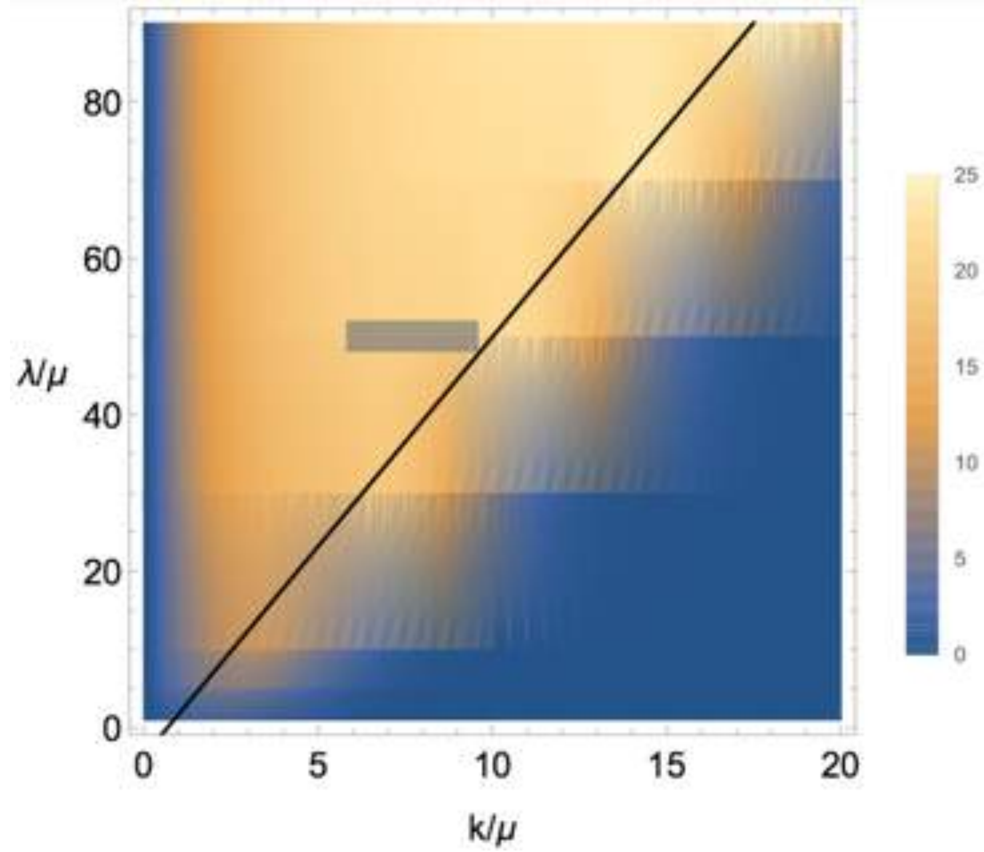
Comparison: Helical lattice and Q-lattice

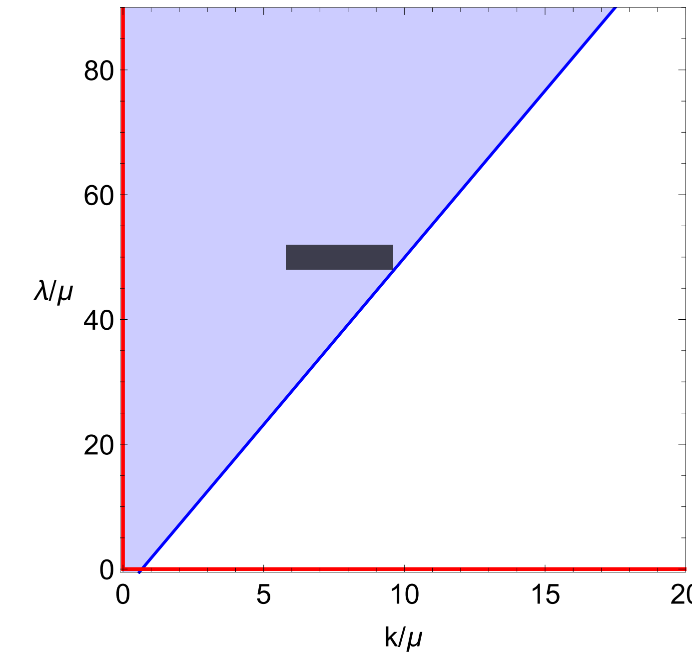
[KYK, Niu: 1605.XXXXX]

Preliminary data

[KYK, Niu: in progress]

Homes' law



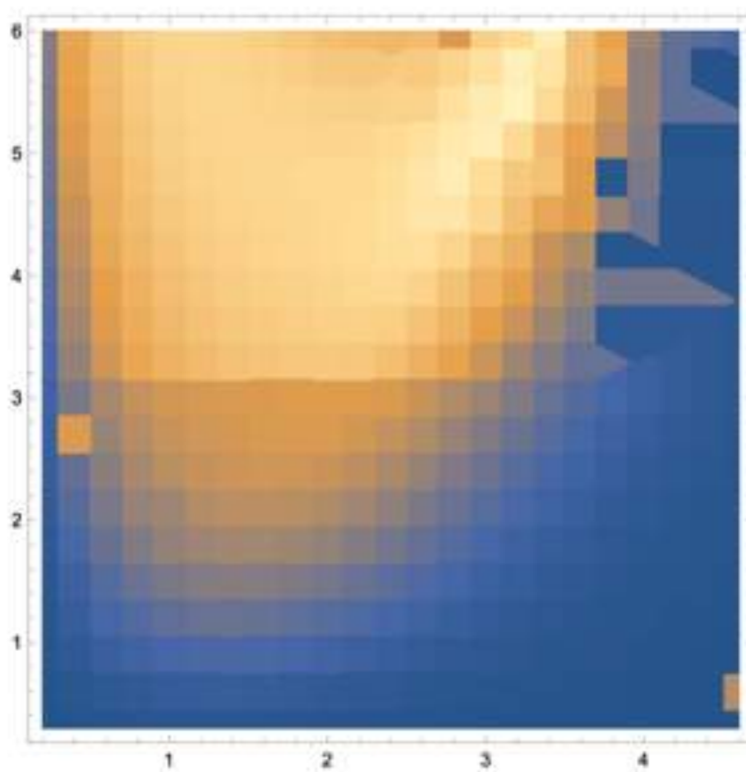


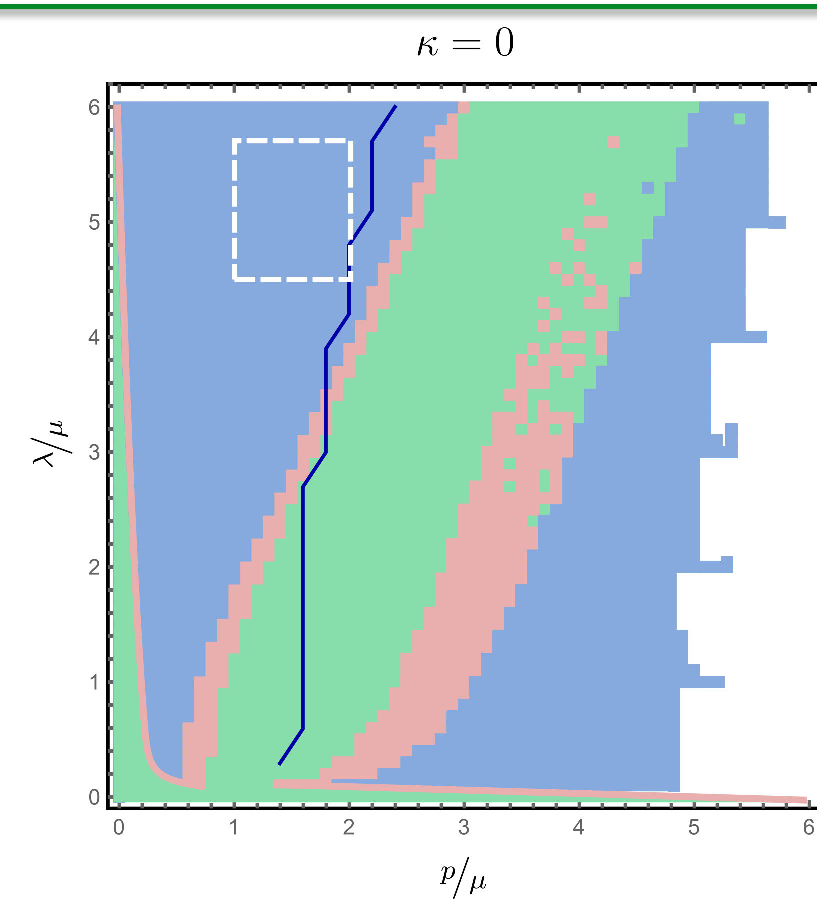
Helical lattice

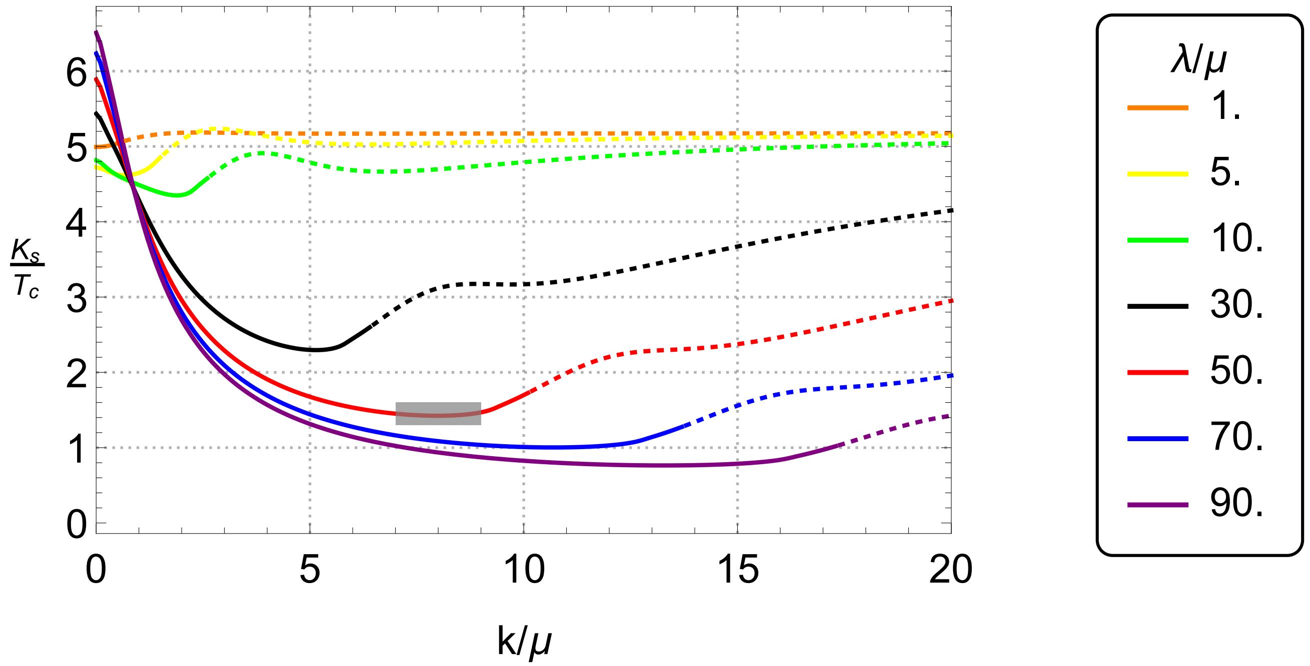
Preliminary

[Erdmenger, Meyer, Schalm, Shock: in

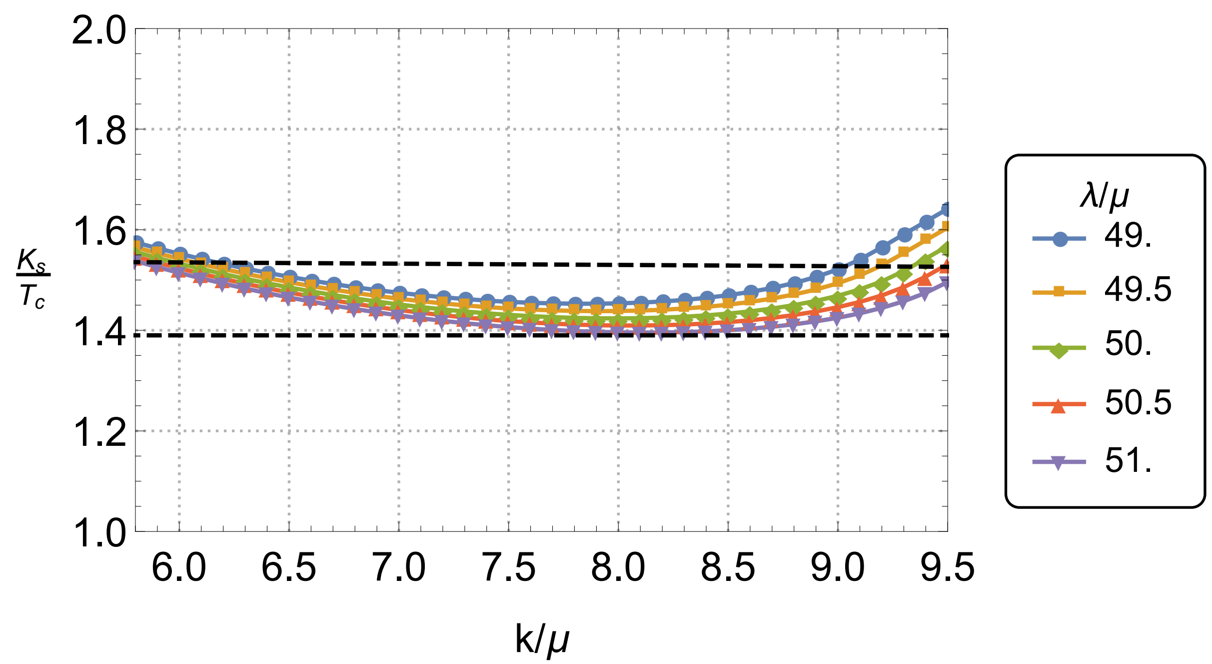
Erdmenger's Tuesday talk

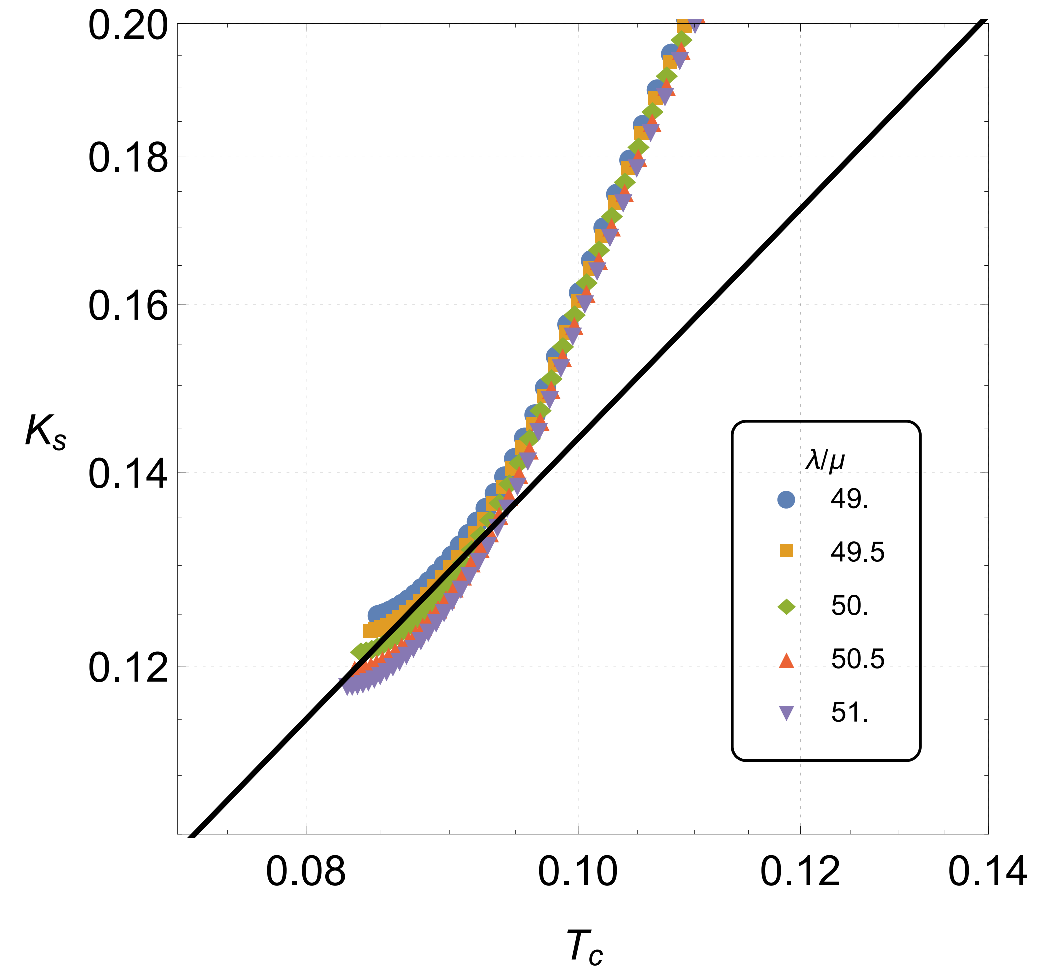





λ/μ
1.
5.
10.
30.
50.
70.
90.
$\frac{K_s}{T_c}$
k/μ

~~Uemura's law~~


λ/μ
49.
49.5
50.
50.5
51.
$\frac{K_s}{T_c}$
k/μ


λ/μ
49.
49.5
50.
50.5
51.
K_s
T_c

Contents

Summary and outlook

- Homes' law $\rho_s(T=0) = C\sigma_{DC}(T_c)T_c$
- Uemura's law $\rho_s(T=0) = BT_c$

$$S_{HHH} = \int d^4x\sqrt{-g}\left[R + 6 - \frac{1}{4}F^2 - |(\partial - iqA)\Phi|^2 - m_\Phi^2\Phi\Phi^*\right]$$

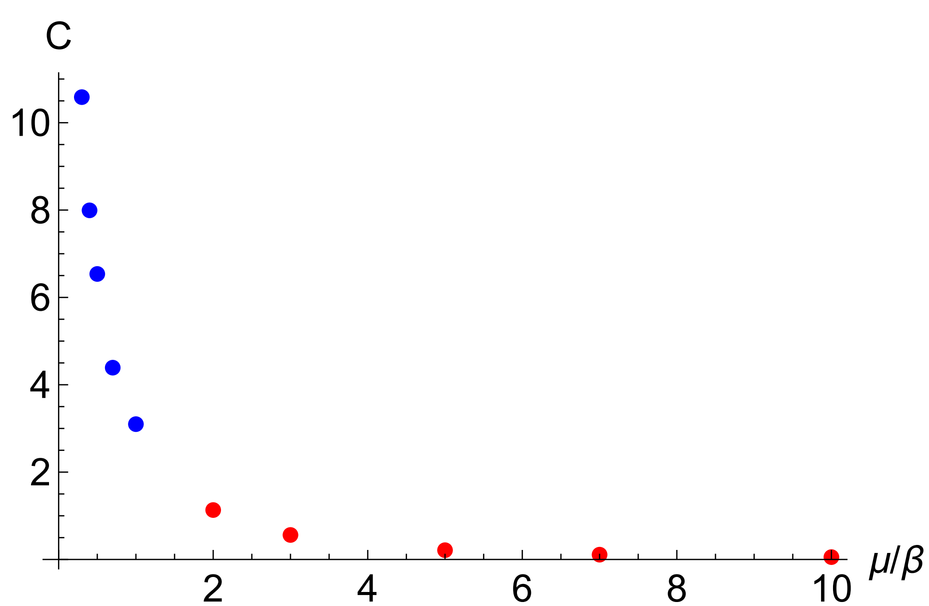
$$S_{MS} = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\sum_{I=1,2}(\partial\psi_I)^2\right] \quad \psi_I = (\beta x, \beta y)$$

~~Homes' law~~ Uemura's law

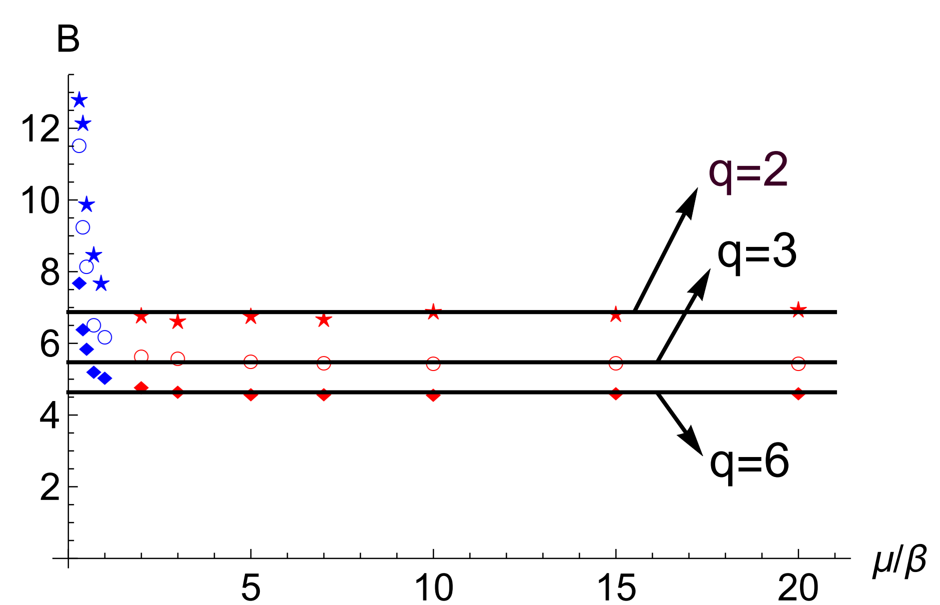
$$S_Q = \int d^4x\sqrt{-g}\left[-|\partial\Psi|^2 - m_\Psi^2|\Psi|^2\right] \quad \Psi = e^{ikx}z\psi(z) \quad \psi(0) = \lambda$$

Homes' law ~~Uemura's law~~

Massless scalar model



(a) $C(=\tilde{\rho}_s/(\sigma_{DC}\tilde{T}_c))$, $q=3$

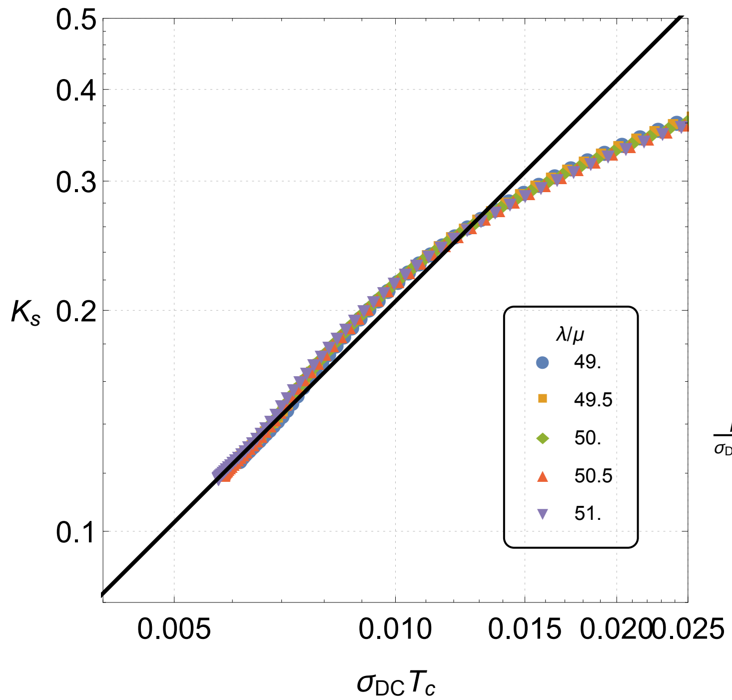


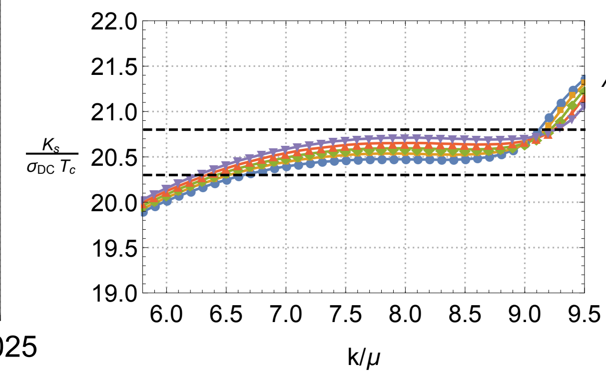
(b) $B(=\tilde{\rho}_s/\tilde{T}_c)$ for $q = 2, 3, 6$

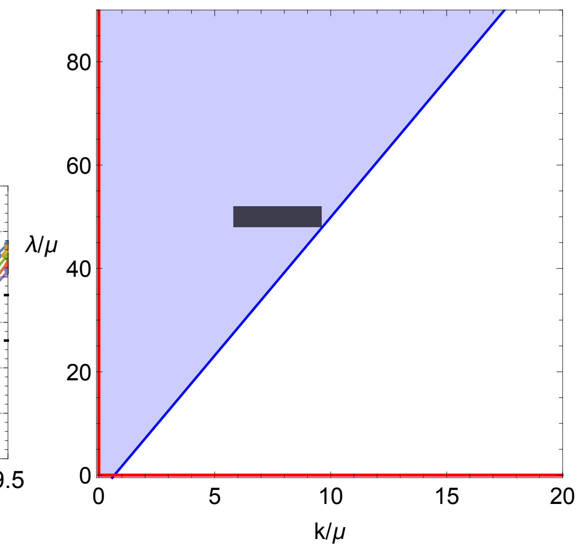
Summary and outlook

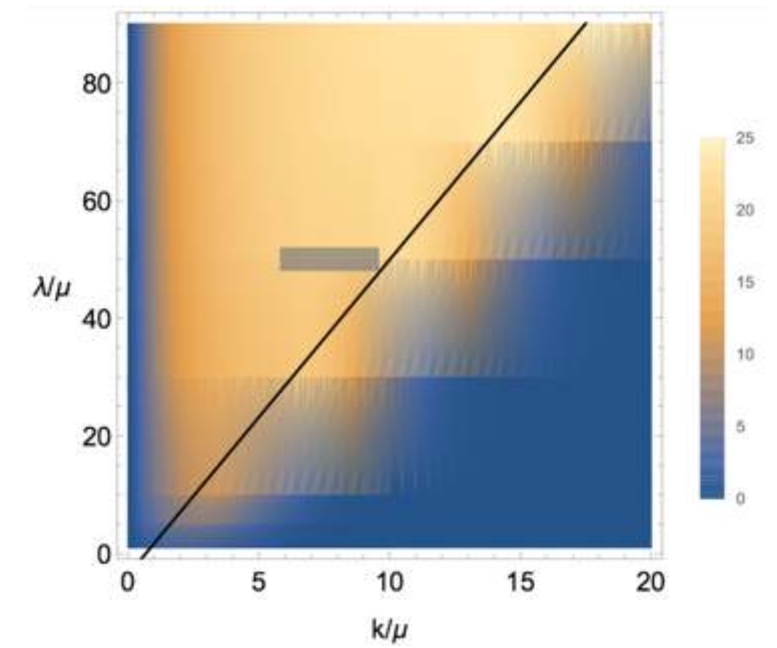
Q-lattice model

- Physical understanding
- Numerical issue near Zero T









Helical lattice model