N=2* Holography and Localization

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X. Chen-Lin, A. Dekel, K.Z. 1512.06420

X. Chen-Lin, K.Z. 1502.01942

K.Z. 1410.6114

X. Chen-Lin, J. Gordon, K.Z. 1408.6040

A. Buchel, J. Russo, K.Z. 1301.1597

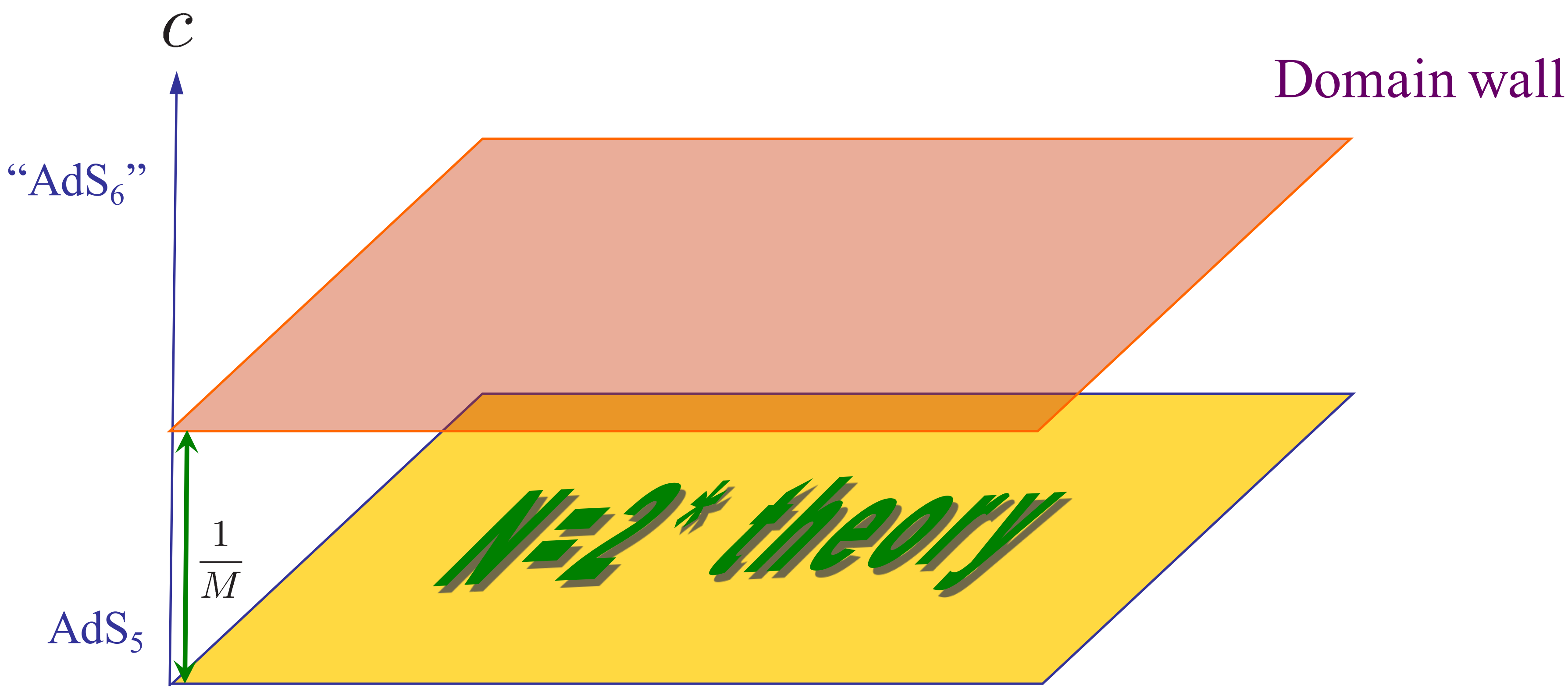
Current Themes in Holography, Copenhagen, 29.04.16

N=2* theory

vector multiplet	hypermultiplets
A_μ	Z
$\psi \qquad \lambda$	$\chi \qquad \tilde{\chi} \qquad + \text{conj.}$
$\Phi,\ \Phi'$	$\tilde{Z}$
mass = 0	mass = $\pm M$

- relevant perturbation of N=4 super-Yang-Mills

Holographic dual



Pilch,Warner'00

$$ds^2 = \frac{A}{c^2-1} M^2 dx_\mu^2 + \frac{1}{A\left(c^2-1\right)^2} dc^2$$

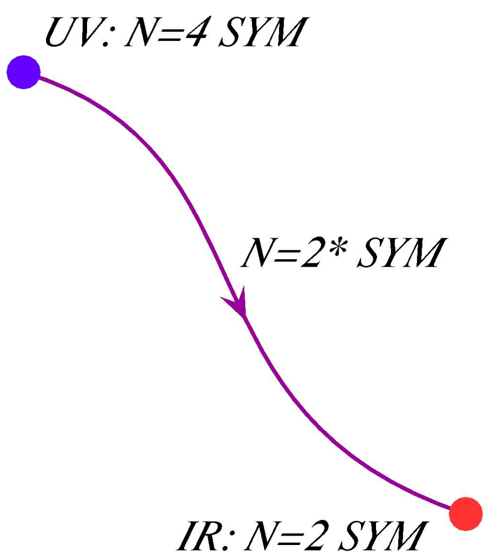
$$A = c + \frac{c^2-1}{2} \ln \frac{c-1}{c+1}$$

$$c = 1 + \frac{M^2 z^2}{2} + \dots$$

- UV regularization of pure N=2 SYM

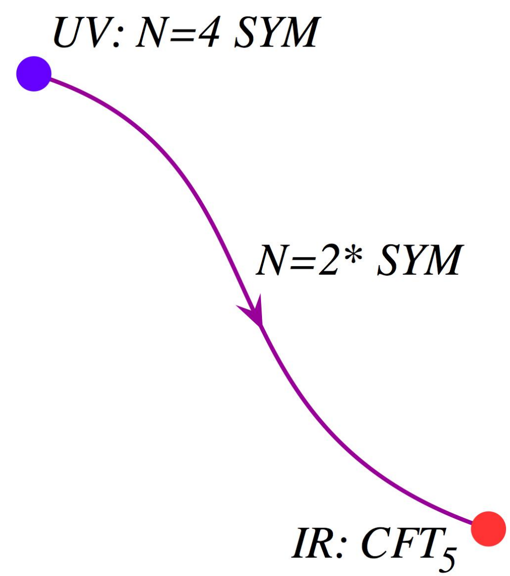
In this talk:
$N=\infty$
$\lambda = g_{YM}^2 N$

Weak coupling



$$\Lambda = M\,e^{-\frac{4\pi^2}{\lambda}}$$

Strong coupling



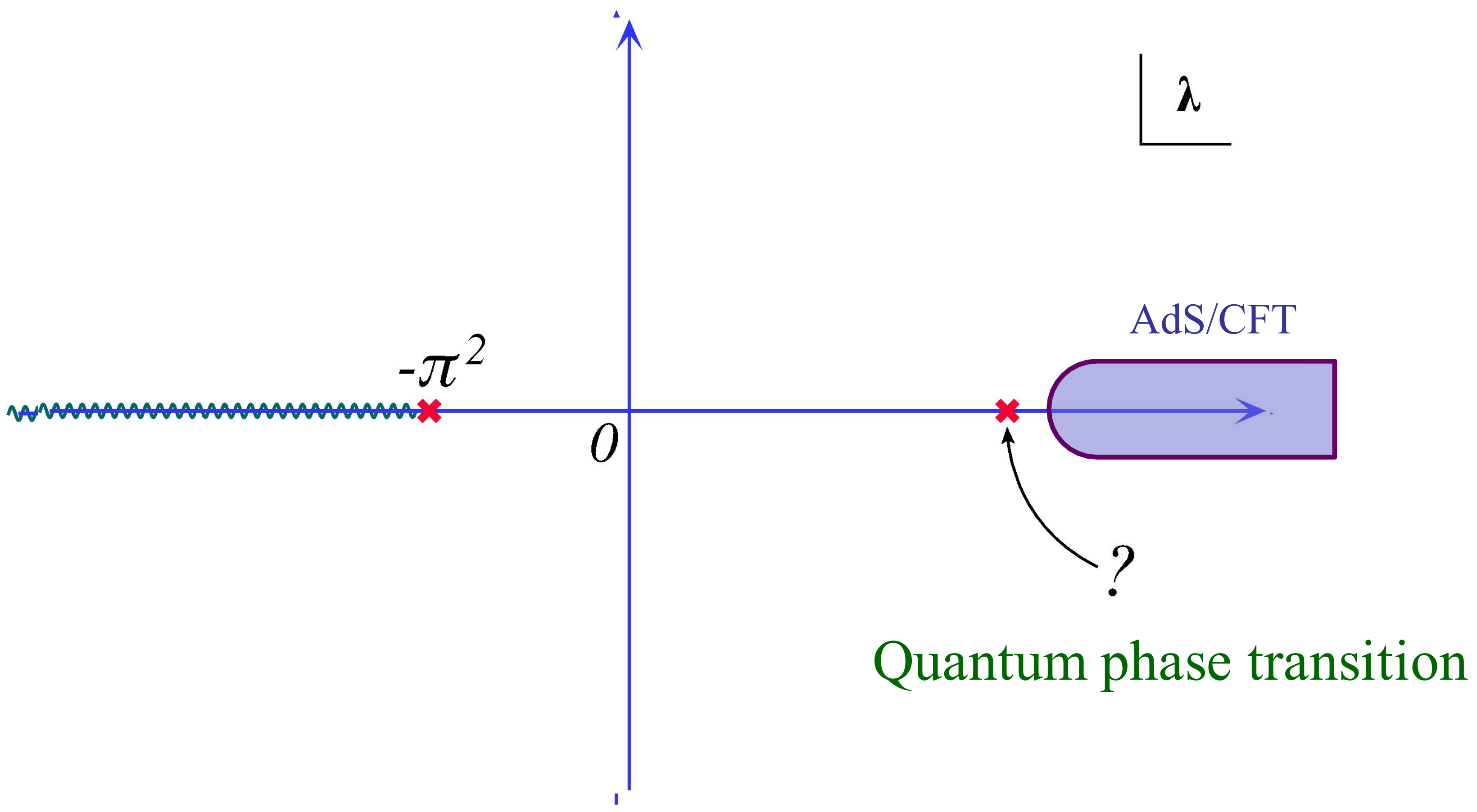
Hoyos'10

Dimensional crossover
by Eguchi-Kawai mechanism

Young,Z.'14

Phase transitions

In N=4 super-Yang-Mills:



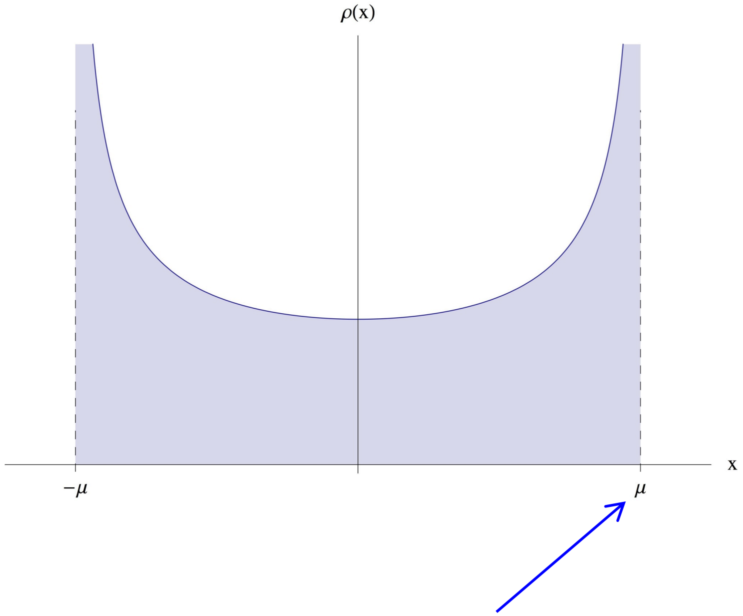
Master field

Symmetry breaking: $SU(N) \to U(1)^{N-1}$

$$\langle \Phi \rangle = \mathrm{diag}\,(a_1, \ldots, a_N)$$

Large-N master field:

$$\rho(x) = \left\langle \frac{1}{N}\,\mathrm{tr}\,\delta\,(x - \Phi) \right\rangle = \frac{1}{N}\sum_{i=1}^{N}\delta\,(x - a_i)$$



typical scale of symmetry breaking

Exact solution

Localization Pestun'07

SW theory Seiberg,Witten'94

Russo,Z.'13

Billo,Frau,Fucito,Lerda,Morales,Poghossian,Ricci Pacifici'14

Hollowood,Prem Kumar'15

$$\fint_{-\mu}^{\mu} dy\,\rho(y)\left(\frac{2}{x-y}-\frac{1}{x-y+M}-\frac{1}{x-y-M}\right)=0$$

$$\int_{-\mu}^{\mu} dx\,\rho(x)\ln\frac{\left|M^2-x^2\right|}{x^2}=\frac{8\pi^2}{\lambda}$$

$$\int_{-\mu}^{\mu} dx\,\rho(x)=1$$

RG log

Weak Coupling

$$\lambda \ll 1 \quad \Longrightarrow \quad \mu \ll M$$

$$⨍_{-\mu}^{\mu} dy\, \rho(y) \left(\frac{2}{x-y} - \frac{1}{x-y+M} - \frac{1}{x-y-M} \right) = 0$$

$$\rho(x) = \frac{1}{\pi \sqrt{\mu^2 - x^2}}$$

Douglas,Shenker'95

$$\mu \simeq 2M\, \mathrm{e}^{-\frac{4\pi^2}{\lambda}}$$

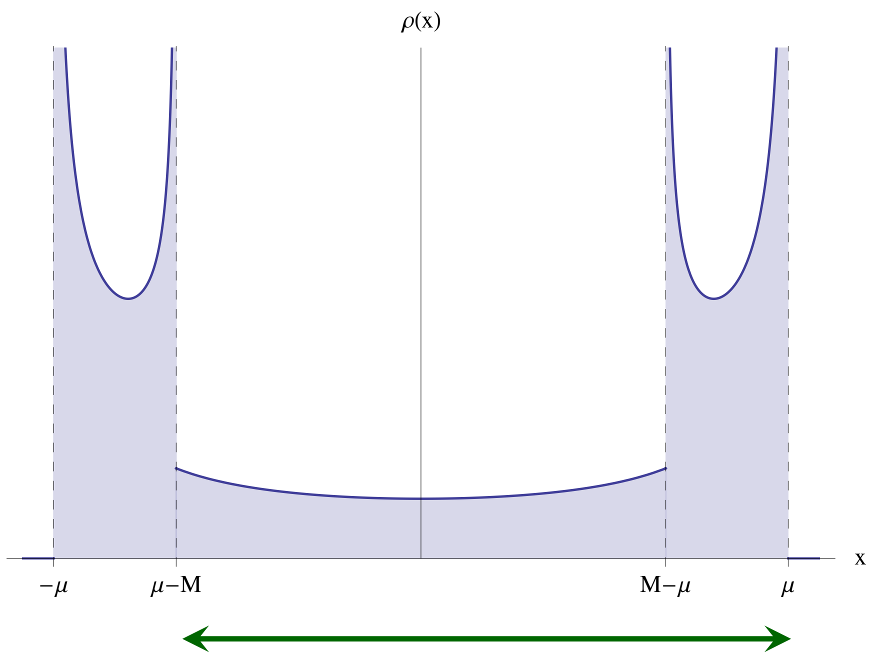
$$\frac{\mu}{M} = 2\,\mathrm{e}^{-\frac{4\pi^2}{\lambda}} - 4\,\mathrm{e}^{-\frac{12\pi^2}{\lambda}} - 16\,\mathrm{e}^{-\frac{\cdots}{\lambda}} - 58\,\mathrm{e}^{-\frac{\cdots\pi^2}{\lambda}} - 324\,\mathrm{e}^{-\frac{44\pi^2}{\lambda}} - 1856\,\mathrm{e}^{-\frac{52\pi^2}{\lambda}} + \ldots$$

OPE in condensates

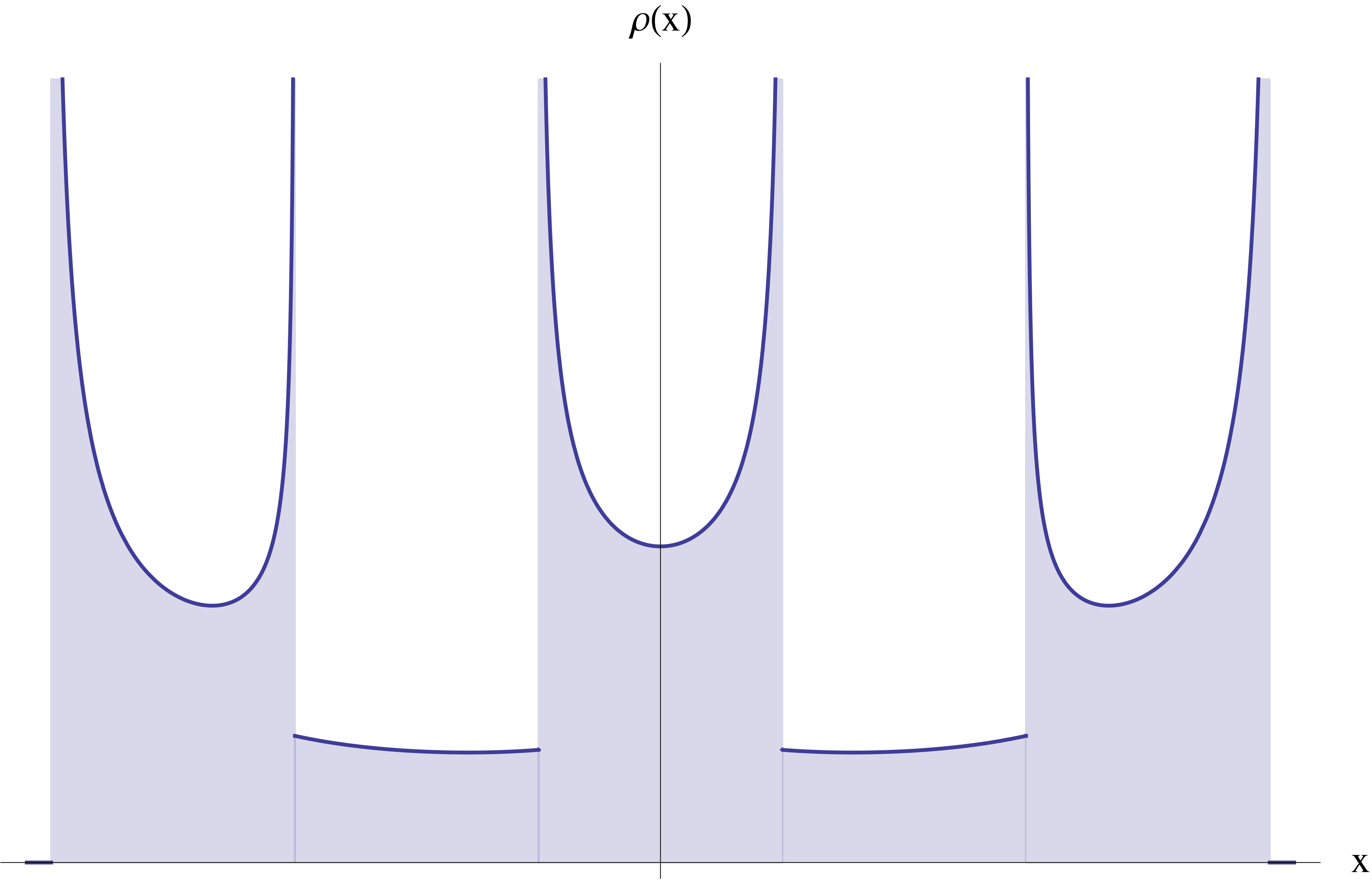
β-function of N=2 SYM!

Phase transtion

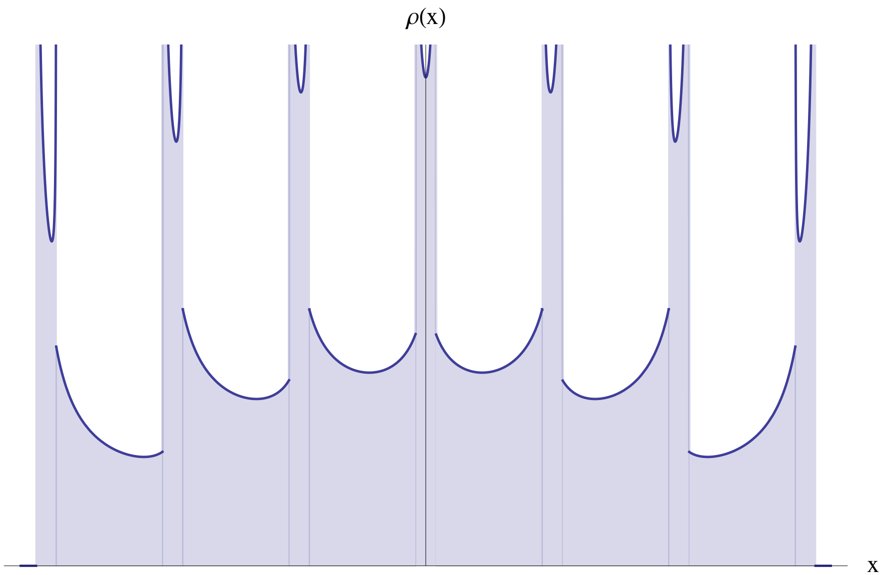
Weak-coupling solution is valid up to $\mu_c = \frac{M}{2} \Longleftrightarrow \lambda < \lambda_c = 35.42...$

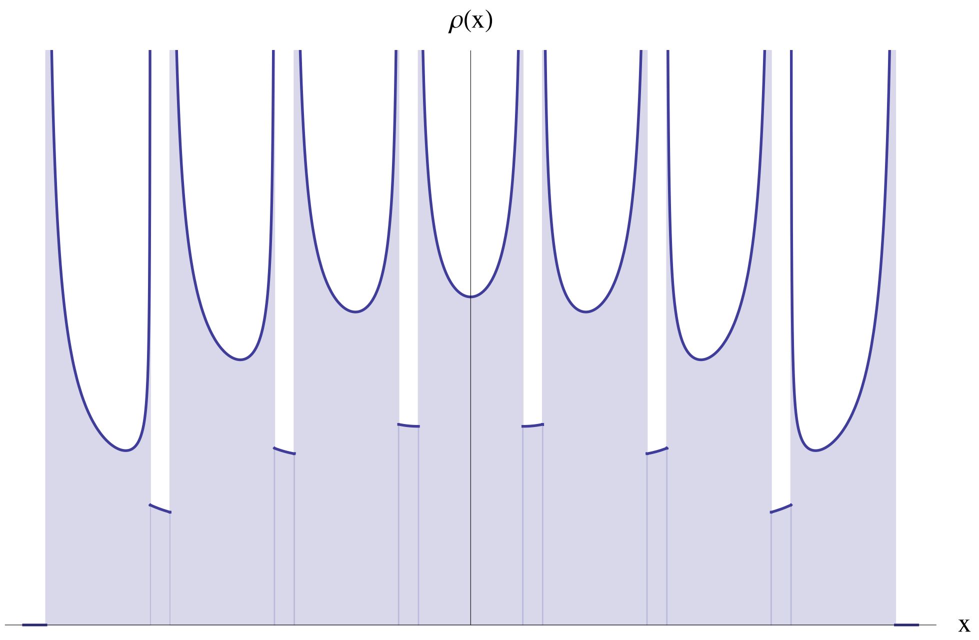


resonance on massless hyper: $m_{ij}^{\mathrm{h}} = |a_i - a_j \pm M|$

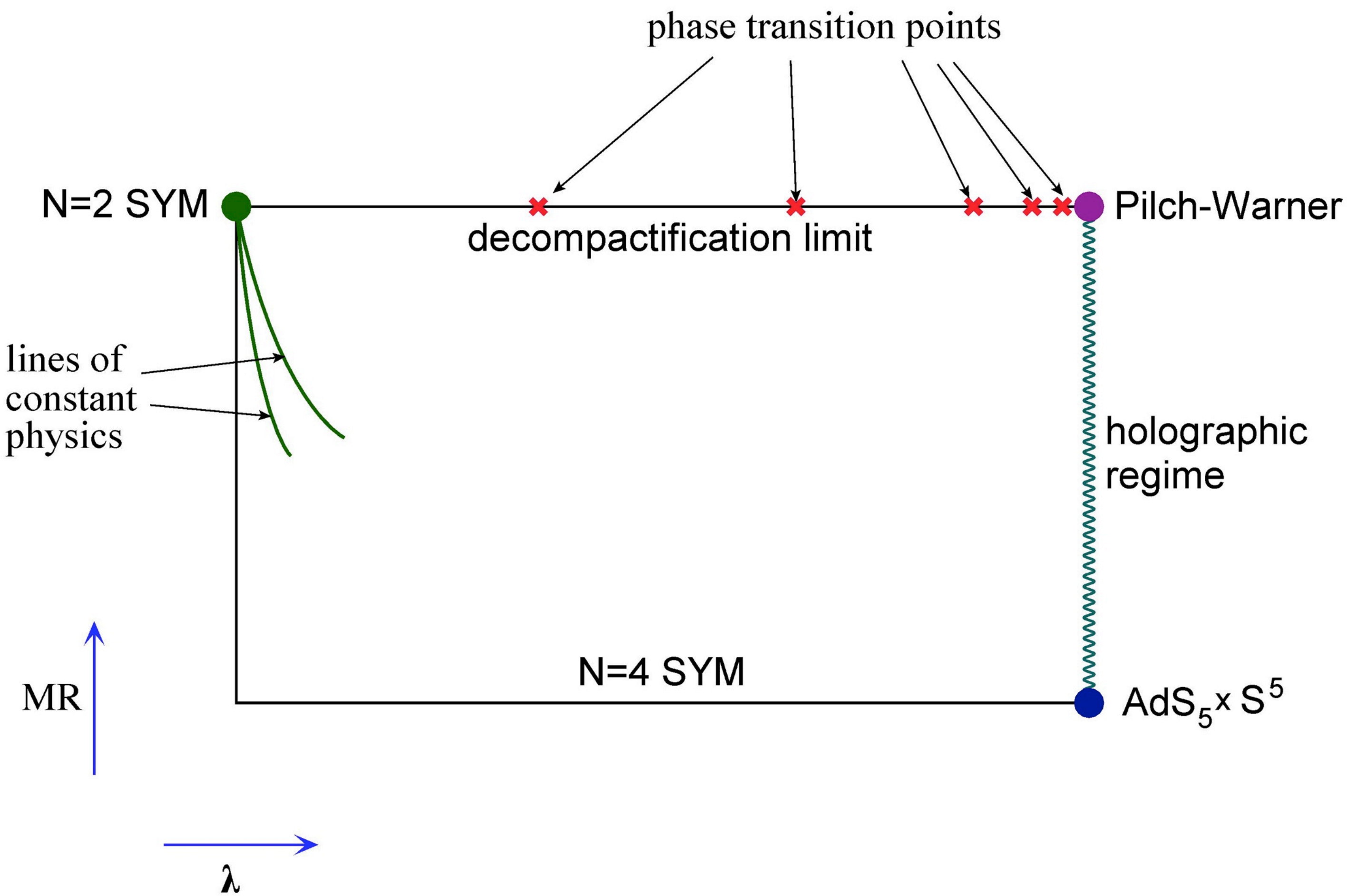


ρ(x)
x


ρ(x)
x

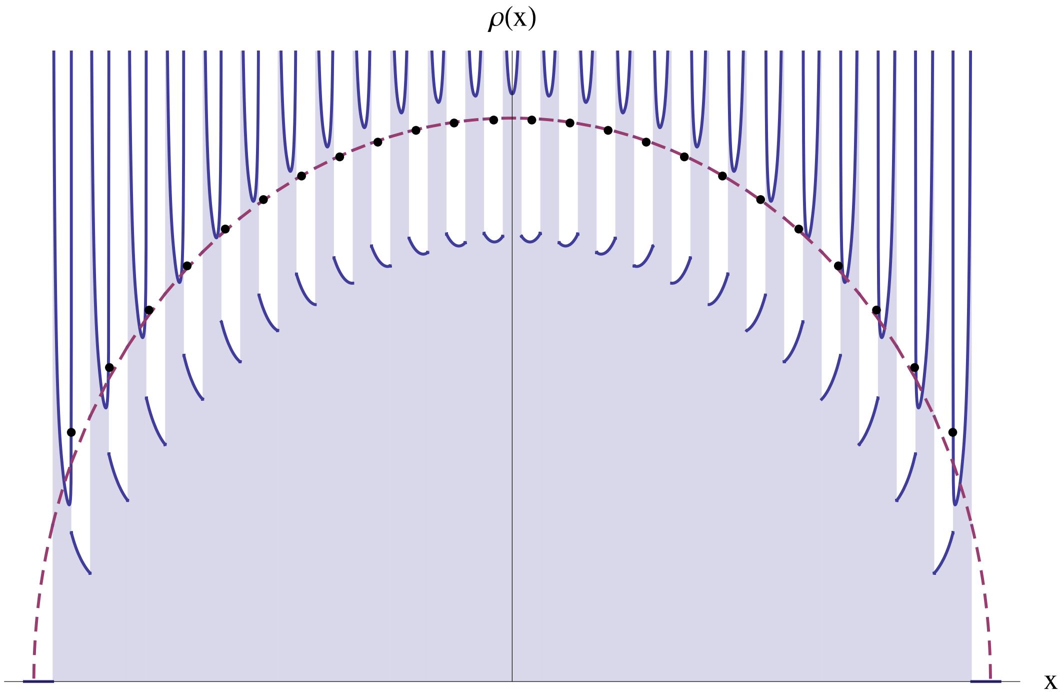

ρ(x)
x

Phase diagram



R: radius of S^4

Strong coupling



$$\mu = \frac{\sqrt{\lambda}M}{2\pi}$$

$$\rho(x) \simeq \frac{2}{\pi\mu^2}\sqrt{\mu^2 - x^2}$$

$\mathcal{O}\left(\frac{1}{\mu}\right)$

$$+\frac{1}{\pi}\sqrt{\frac{M}{2\mu^5}}\left[(\mu - x)\,\zeta\left(\frac{1}{2},\left\{\frac{\mu+x}{M}\right\}\right) + (\mu + x)\,\zeta\left(\frac{1}{2},\left\{\frac{\mu-x}{M}\right\}\right)\right],$$

$\mathcal{O}\left(\frac{M^{1/2}}{\mu^{3/2}}\right) = \mathcal{O}\left(\frac{\lambda^{-1/4}}{\mu}\right)$

Perimeter law

$$W(C)=\left\langle \frac{1}{N}\,\mathrm{tr}\,\mathrm{P}\exp\oint_C ds\,\left(i\dot{x}^{\mu}A_{\mu}+|\dot{x}|\Phi\right)\right\rangle$$

substitute <Φ>

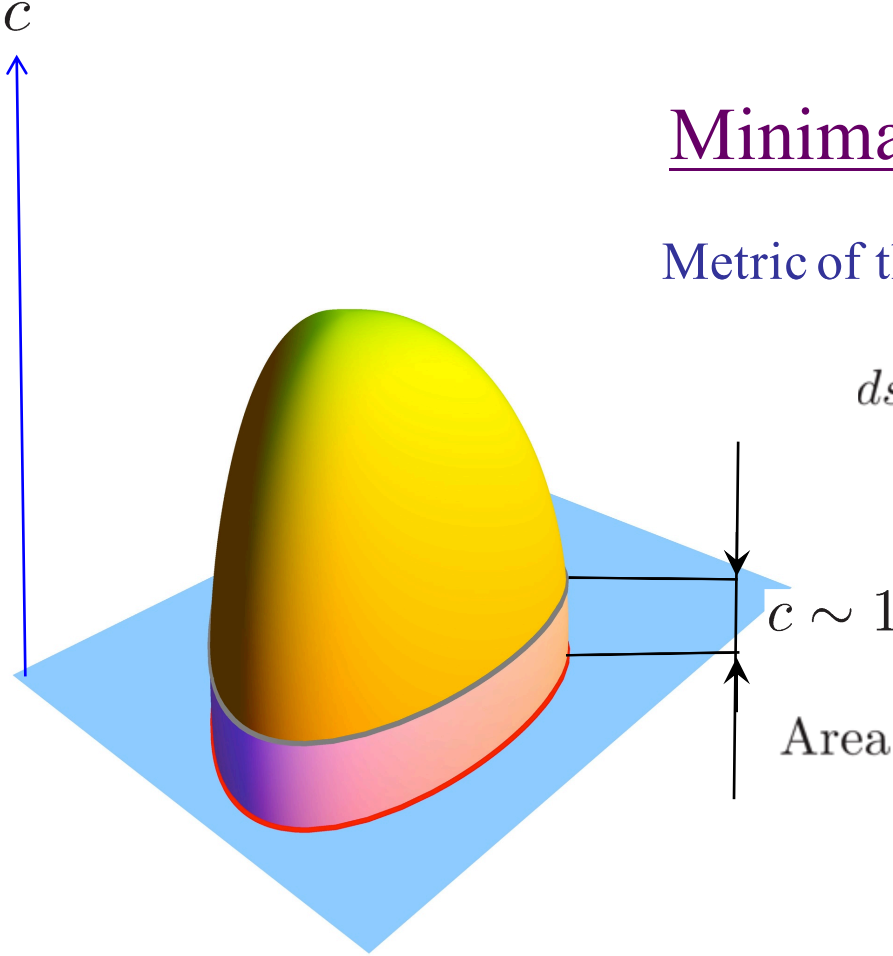
$$W(C)=\frac{1}{N}\sum_i \mathrm{e}^{\,La_i}\overset{L\to\infty}{\Longrightarrow}\mathrm{e}^{\,L\mu}$$

(perimeter law)

$$\ln W(C)\simeq\frac{\sqrt{\lambda}ML}{2\pi}\left(1-\frac{\pi}{\sqrt{\lambda}}+\ldots\right)$$

Buchel,Russo,Z.’13
Chen-Lin,Gordon,Z.’14
Z.’14

Minimal area



Metric of the gravity dual:

$$ds^2 = \frac{\rho^6}{c^2-1} M^2 dl^2 + \frac{1}{\rho^6 \left(c^2-1\right)^2} dc^2$$

Pilch, Warner'00

$$\text{Area} = ML \int_{1+\frac{\varepsilon^2 M^2}{2}}^{\infty} \frac{dc}{\left(c^2-1\right)^{\frac{3}{2}}} = \frac{L}{\varepsilon} - ML$$

renormalized away

$$\ln W(C) \simeq \frac{\sqrt{\lambda} ML}{2\pi} \qquad \left(\lambda \to \infty,\ ML \gg 1\right)$$

agrees!

- free enegry also agrees

Bobev, Elvang, Freedman, Pufu'13

Higher representations

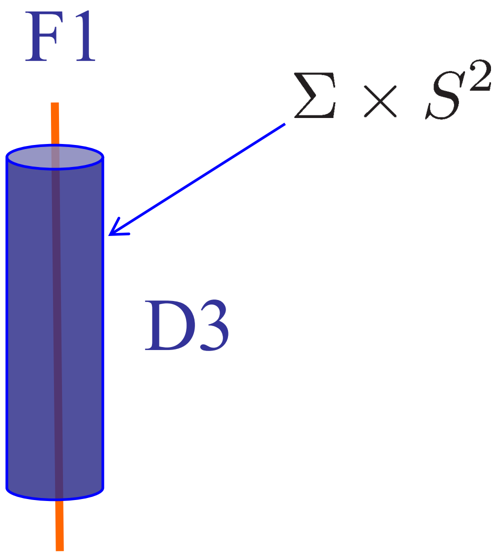
$$W_{\mathcal{R}}(C) = \left\langle \frac{1}{N}\,\mathrm{tr}_{\mathcal{R}}\,\mathrm{P}\exp\oint_C ds\,\left(i\dot{x}^{\mu}A_{\mu} + |\dot{x}|\Phi\right)\right\rangle$$

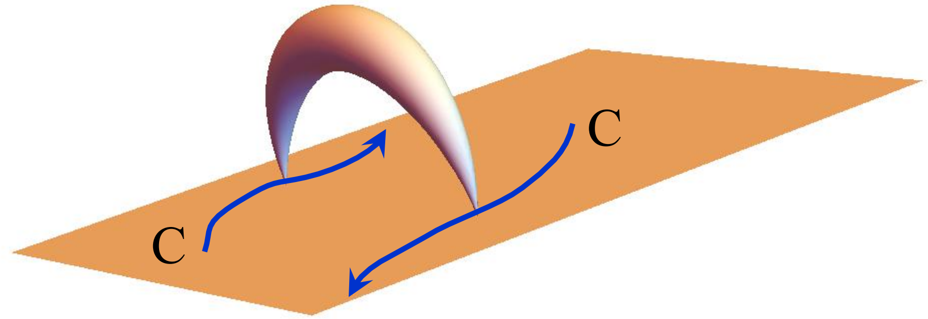
$$\mathcal{R}_k = \overbrace{\square\square\square\square\square}^{k}$$

$$W_{\mathcal{R}_k} \simeq \mathrm{e}^{\frac{NM^2L^2}{2\pi^2}\,f\left(\frac{\pi\sqrt{\lambda}k}{2NML}\right)} \overset{ML\gg 1}{\simeq} \mathrm{e}^{\frac{\sqrt{\lambda}kML}{2\pi}}$$

Chen-Lin,Z.'15

$$f(\kappa) = \kappa\sqrt{1+\kappa^2} + \mathrm{arcsinh}\,\kappa$$





D3-brane embedding (for straight Wilson line):

$$r = \frac{\sqrt{\lambda}k}{N}\sqrt{c^2-1}, \qquad F = -\frac{i\mathrm{Vol}(S^2)}{(c^2-1)^{\frac{3}{2}}}$$

$$W_{\mathcal{R}_k} = \mathrm{e}^{-S_{\mathrm{D3}}} = \mathrm{e}^{\frac{\sqrt{\lambda}kML}{2\pi}}$$

Chen-Lin,Dekel,Z.'15

agrees w. QFT

Conclusions

- Basic agreement between first-principles QFT calculations and non-conformal holography:

Strong coupling <---> Classical gravity

- Quantum corrections? $1/\sqrt{\lambda}$

- Strong-weak coupling phase transitions do not obstruct holography

- What is holographic interpretation of these phase transitions?