BMS Holography

Geoffrey Compère

Université Libre de Bruxelles

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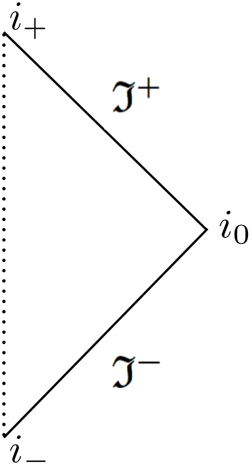
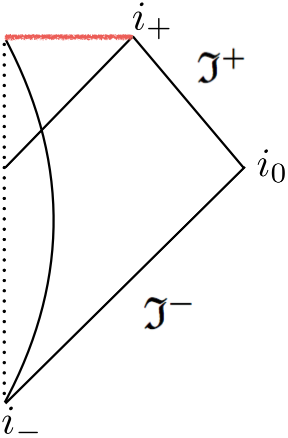
Asymptotically flat spacetimes

Conformal compatification $\Rightarrow$ Penrose diagram (may $\nexists$)





Pletora of boundaries : $\mathfrak{I}^+$, $\mathfrak{I}^-$, "i_0", i_-, i_+ ($\neq AdS$)

Ingoing and outgoing energy flux at $\mathfrak{I}^+$, $\mathfrak{I}^-$ ($\neq AdS$)

What does BMS holography may mean ?

- Reformulate the S-matrix and the Hamiltonian from a non-gravitational field theory
- Use dual description to microscopically count Kerr black hole entropy

Attempts :

- Asymptotic BMS quantization [Ashtekar, 1987] [Arcioni, Dappiaggi, 2003]
- Limit from AdS/CFT [Polchinski, 1999]
- Slicing with dS/AdS [de Boer, Solodukhin, 2003]
- $3d$ Einstein $\leftrightarrow$ Chern-Simons $\leftrightarrow$ Liouville [Barnich, Gonzalez, 2013]
- BMS on black hole horizons [Strominger, Zhiboedov, 2014] [Hawking, Perry, Strominger, 2016] [. . .]

Step zero : fully understand the (semi-)classical structure of $4d$ asymptotically flat spacetimes.

Recent renewal [Barnich, Troessaert, 2009] [Strominger, 2013] led to rethinking about classical symmetries and memory effects.

Outline

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1. The BMS group
2. Extending the BMS group into the bulk
3. Poincaré invariant vacua with BMS charges
4. Black holes with classical BMS hair

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1. The BMS group

The space of solutions to Einstein gravity with “reasonable” asymptotically flat boundary conditions can be expanded close to null infinity in a fixed gauge.

$$\begin{aligned} ds^2 &= -du^2 - 2dudr + r^2 d^2\Omega + \dots \\ &= -dv^2 + 2dvdr + r^2 d^2\Omega + \dots \end{aligned}$$

The group of diffeomorphisms which

- preserve the form of the asymptotic metric, mapping one metric to another but preserving the gauge,
- are associated with finite and non-trivial canonical charges

is the asymptotic symmetry group.

Using “reasonable” boundary conditions, the asymptotic symmetry group was found to be the *BMS* group [Bondi, van der Burg, Metzner, 1962] [Sachs, 1962].

A translation in Minkowski spacetime

- (t, x, y, z)

$$\partial_z$$

- (t, r, θ, ϕ)

$$\cos\theta\partial_r - \frac{1}{r}\sin\theta\partial_\theta$$

- (u, r, θ, ϕ), retarded time $u = t - r$

$$-\cos\theta\partial_u + \cos\theta\partial_r - \frac{1}{r}\sin\theta\partial_\theta$$

The bms algebra

$$bms \simeq so(3,1) \oplus \text{Supertranslations}$$

Supertranslations are either translations or pure supertranslations. Pure supertranslations are (abelian) “higher harmonic angle-dependent translations”

$$T(\theta,\phi)\partial_u + \frac{1}{2}\nabla^2 T\partial_r - \frac{1}{r}(\partial_\theta T\partial_\theta + \frac{1}{\sin^2\theta}\partial_\phi T\partial_\phi) + \dots$$

The solutions to $\nabla^2(\nabla^2+2)T = 0$ are the translations. Those are the $\ell = 0$ and $\ell = 1$ spherical harmonics, $T = 1$, $T = \cos\theta$, $T = \sin\theta\cos\phi$, $T = \sin\theta\sin\phi$.

What are reasonable boundary conditions ?

- Admit Kerr, gravitational waves and electromagnetic fields
- All canonical charges finite
- Positive energy
- Allow to describe memory effects [Zeldovich, Polnarev, 1974] [Christodoulou, 1991]
- Allow for small perturbations to decay (non-linear stability) [Christodoulou, Klainerman, 1993]
- Allow to describe a semi-classical S-matrix which obeys all known theorems [Weinberg, 1965] [Cachazo, Strominger, 2014]

The list has been evolving over time.

The memory effect

After the passage of either gravitational waves or null matter between two detectors placed in the asymptotic null region, or after a change of Bondi mass, the detectors generically acquire a finite relative displacement and a finite time shift. This is the *memory effect*.

Historically, it is refered to as the linear memory effect for changes of Bondi mass or null radiation [Zeldovich, Polnarev, 1974] and the non-linear memory or Christodoulou effect for gravitational waves [Christodoulou, 1991].

Memory effects are a 2.5PN effect. [Damour, Blanchet, 1988]

It is an unobserved prediction of GR.

Supertranslations and memories

Memory effects can be interpreted from the existence of the supertranslation field $C(\theta, \phi)$ which is effectively shifted by a supertranslation after the passage of radiation as [Geroch, Winicour, 1981]

$$\delta_T C(\theta, \phi) = T(\theta, \phi).$$

The supertranslation field is the Goldstone boson of spontaneously broken BMS invariance which labels inequivalent Poincaré vacua. [He, Lysov, Mitra, Strominger, 2014]

More precisely, supertranslation memories follow from an angle-dependent energy conservation law deduced from Einstein's equations integrated over a finite retarded time interval of $\mathfrak{I}^+$: [Frauendiener, 1992] [Strominger, Zhiboedov, 2014]

$$-\frac{1}{4}\nabla^2(\nabla^2+2)(C|_{u_2}-C|_{u_1}) = m|_{u_2}-m|_{u_1}+\int_{u_2}^{u_1} du T_{uu},$$
$$T_{uu} \equiv \frac{1}{4}N_{zz}N^{zz}+4\pi G \lim_{r\to\infty}[r^2 T^{matter}_{uu}].$$

The supertranslation shift can be constructed from the radiation flux history. It allows to compute the shift of the geodesic deviation vector s^A, $A=\theta,\phi$

$$s^A|_{u_2}-s^A|_{u_1} \sim \frac{1}{r}\partial^A\partial_B(C|_{u_2}-C|_{u_1})s^B$$

This is a classical effect of Einstein gravity, $O(\hbar^0)$.

The extended bms algebra

[Barnich, Troessaert, 2010]

$$\textit{ext bms} \simeq \text{Superrotations}^* \oplus \text{Supertranslations}^*$$

where

$$\begin{aligned} \text{Superrotations}^* &\simeq \text{Vir}^* \oplus \text{Vir}^*, \\ \text{Supertranslations}^* &\simeq \text{Regular supert.} \oplus \text{Meromorphic supert.} \end{aligned}$$

The Lorentz subalgebra

$$so(3,1) \simeq sl(2,\mathbb{R}) \oplus sl(2,\mathbb{R}) \subset \text{Vir}^* \oplus \text{Vir}^*$$

is generated by global conformal transformations on the sphere. The rest of the algebra has generators which contain meromorphic functions, $\delta z = R^z(z)$, with poles on S^2.

The extended bms algebra : comments

The algebra is not realized as asymptotic symmetry algebra, at least in the standard sense :

- The Kerr black hole has infinite meromorphic supertranslation momenta. [Barnich, Troessaert, 2010]
- Minkowski acted upon with a finite superrotation diffeomorphism has negative energy. [G.C., Long, 2016]

The superrotations still have a role to play :

- Superrotation charges are finite and can be non-trivial [Barnich, Troessaert, 2011] [Flanagan, Nichols, 2015] [G.C., Long, 2016]
- The subleading soft graviton theorem has been related to the Ward identity of the superrotation algebra [Kapec, Lysov, Pasterski, Strominger, 2014] [Campiglia, Laddha, 2015]

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2. Mapping Symmetries from $\mathfrak{I}^+$ to $\mathfrak{I}^-$

Requiring BMS invariance of the S-matrix and consistency with Weinberg soft graviton theorem requires a map between BMS supertranslations at $\mathfrak{I}^-$ and $\mathfrak{I}^+$

$$BMS_+ \times BMS_- \to BMS_{diagonal}$$

[Strominger, 2013] [He, Lysov, Mitra, Strominger, 2014]

Similarly to Poincaré symmetry, there is only one BMS symmetry asymptotically.

Symmetries away from $\mathfrak{I}^+$ and $\mathfrak{I}^-$

Either : Symmetries are defined only asymptotically

Either : Symmetries can be extended into the bulk

Only one statement is right, which one ?

Symmetries away from $\mathfrak{I}^+$ and $\mathfrak{I}^-$

If : Symmetries are defined only asymptotically

Then : One can defines various symmetries at different places : black hole horizon versus $\mathfrak{I}^+$

[Carlip, 1998] [Guica, Hartman, Song, Strominger, 2008] [. . .]

If : Symmetries can be extended into the bulk

Then : Background structure in the bulk induced from the asymptotic boundary conditions
Then : Symmetries at black hole horizon dictated by symmetries at $\mathfrak{I}^+$

Symmetries can be extended into the bulk

Scenario explicitly realized in 3d in non-radiating spacetimes. [Barnich, Troessaert, 2010] [Compère, Donnay, Lambert, Schulgin, 2014]

Example : BTZ black hole with Virasoro hair [Bañados, 1999]

$$ds^2 = \ell^2 \frac{dr^2}{r^2} - \left(r dx^+ - \ell^2 \frac{L_-(x^-) dx^-}{r} \right) \left(r dx^- - \ell^2 \frac{L_+(x^+) dx^+}{r} \right)$$

admits two copies of the Virasoro algebra everywhere in the bulk. In the extremal case, one Virasoro algebra acts everywhere from infinity to the near-horizon region.

[Compère, Mao, Seraj, Sheikh-Jabbari, 2015]

In the asymptotically flat, one gets cosmological solutions with bulk BMS hair. [Barnich, Gomberoff, González, 2012]

Bulk symmetries = Symplectic symmetries

Underlying structure : asymptotic symmetries are *symplectic symmetries* : It exists a presymplectic structure such that

$$\boldsymbol{\omega}[\mathcal{L}_\xi g_{\mu\nu}, \delta g_{\mu\nu}; g_{\mu\nu}] \approx 0$$

[Compère, Donnay, Lambert, Schulgin, 2014] [Compère, Hajian, Seraj, Sheikh-Jabbari, 2015]

This is a generalization of Killing symmetries which obey $\mathcal{L}_\xi g_{\mu\nu} \approx 0$. Here roughly : $\boldsymbol{\omega}^\mu \sim \delta g \wedge D^\mu \delta g$.

The generalized Noether theorem for diffeomorphism invariant theories [Iyer, Wald , 1993] [Barnich, Brandt, Henneaux, 1995] [Barnich, Brandt, 2001]

$$\boldsymbol{\omega}[\mathcal{L}_\xi g_{\mu\nu}, \delta g_{\mu\nu}; g_{\mu\nu}] \approx d\boldsymbol{k}_\xi[\delta g_{\mu\nu}; g_{\mu\nu}]$$

and Stokes' theorem then leads to conserved charges everywhere in the bulk

$$Q_\xi[g] = \int_{\bar{g}}^{g} \int_S \boldsymbol{k}_\xi[\delta g_{\mu\nu}; g_{\mu\nu}]$$

$4d$ BMS Symmetries in the bulk ?

Motivated by these $3d$ results, we looked whether or not the $4d$ supertranslations and superrotation symmetries could be extended in the bulk spacetime.

After overcoming technical problems, we found that they can. [G.Compère, J. Long, 2016]

It leads to explicit metrics for Poincaré vacua and black holes with BMS hair.

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3. Building BMS vacua/black holes

Algorithm :

- Start with Minkowski/Schwarzschild spacetime.
- Write a generic change of coordinates $x^\mu \to x'^\mu$ which exponentiates the infinitesimal change of coordinates $x'^\mu = x^\mu + \xi^\mu + \dots$ where ξ^μ is a generic supertranslation + superrotation vector field at $\mathfrak{I}^+$
- Solve for the finite change of coordinates at each order in the asymptotic radial expansion such that $g_{\mu\nu} = \frac{\partial x^\beta}{\partial x'^\mu}\eta_{\alpha\beta}\frac{\partial x^\alpha}{\partial x'^\nu}$ fits in BMS gauge.
- Resum the infinite radial expansion.
- Rewrite the final metric in closed and beautiful form.

The result is the BMS orbit of Minkowski/Schwarzschild spacetime. It is the representation of the BMS group on the bulk metric.

The Poincaré vacua of Einstein gravity

The vacuum metric with supertranslation field only is

$$ds^2 = -dt^2 + dx_s^2 + dy_s^2 + dz_s^2 = -dt^2 + d\rho^2 + g_{AB} d\theta^A d\theta^B,$$

where $\theta^A = \theta, \phi$ and

$$\begin{aligned} g_{AB} &= (\rho - C)^2 \gamma_{AB} - 2(\rho - C) D_A D_B C + D_A D_E C D_B D^E C, \\ &= (\rho \gamma_{AC} - D_A D_C C - \gamma_{AC} C) \gamma^{CD} (\rho \gamma_{DB} - D_D D_B C - \gamma_{DB} C) \end{aligned}$$

Under a supertranslation,

$$\delta_T C(\theta, \phi) = T(\theta, \phi).$$

It admits 10 Killing vectors. We checked that the 10 Poincaré charges are zero $\Rightarrow$ Poincaré vacua.

All supertranslation charges are zero.

The Poincaré vacua of Einstein gravity

The vacua are non-trivial in the sense that they admit canonical charges : superrotation charges

$$\mathcal{Q}_R = -\frac{1}{4G}\int_S d^2\Omega R^A\left(\frac{1}{8}D_A(C_{EF}C^{EF}) + \frac{1}{2}C_{AB}D_E C^{EB}\right)$$

where $C_{AB} = -2D_A D_B C + \gamma_{AB}D^2 C$. [Barnich, Troessaert, 2011]

Even thought the superrotation transformation $\delta z = R^z(z)$, $\delta\bar{z} = R^{\bar{z}}(\bar{z})$ admit poles, the superrotation charges are finite.

There is therefore an obstruction in the bulk at shrinking the surface of integration $\Rightarrow$ Bulk defect.

Maybe our universe is patched with such vacua, originating from a pregeometric phase. Are there bulk defects ?

Supertranslation horizon

The static coordinates (t, ρ, θ, ϕ) of the vacua

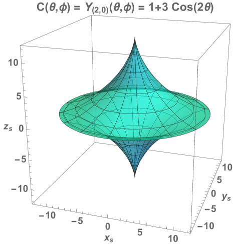
$$ds^2 = -dt^2 + d\rho^2 + g_{AB} d\theta^A d\theta^B,$$

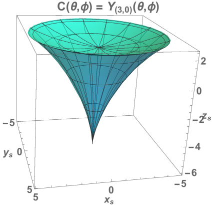
break down where $Det(g_{AB}) = 0$.

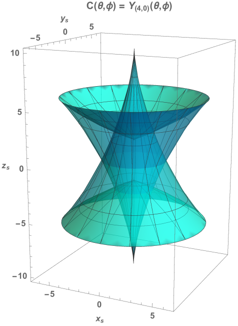
This location $\rho = \rho_{SH}(\theta, \phi)$ defines the *supertranslation horizon*.

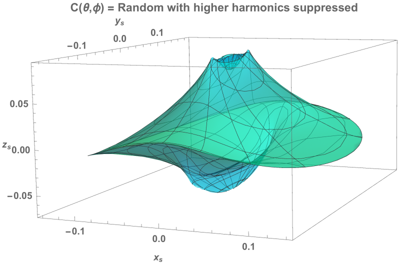
The defect should be at or beyond the supertranslation horizon.

Isometric embedding of the supertranslation horizon









Finite supertranslation diffeomorphism

$$ds^2 = -dt_s^2 + d\rho_s^2 + \gamma_{AB} d\theta_s^A d\theta_s^B = -dt^2 + d\rho^2 + g_{AB} d\theta^A d\theta^B$$

The finite diffeomorphism from Minkowski to the vacua is

$$\begin{aligned} t_s &= t + C_{(0,0)}, & & (z \equiv \cot \frac{\theta}{2} e^{i\phi}) \\ \rho_s &= \sqrt{(\rho - C + C_{(0,0)})^2 + D_A C D^A C}, & & (Pythagoras'\, rule) \\ z_s &= \frac{(z - \bar{z}^{-1})(\rho - C + C_{(0,0)}) + (z + \bar{z}^{-1})(\rho_s - z\partial_z C - \bar{z}\partial_{\bar{z}} C)}{2(\rho - C + C_{(0,0)}) + (1 + z\bar{z})(\bar{z}\partial_{\bar{z}} C - \bar{z}^{-1}\partial_z C)}. \end{aligned}$$

When C is a combination of the 4 lowest spherical harmonics, it is the change between spherical coordinates to other spherical coordinates at a translated origin.

Supertranslation diffeomorphisms are generalization of “spatially translating the origin of coordinates” which introduce a spacetime distortion.

Vacua with superrotation field

The metric with supertranslation and superrotation fields can also be constructed from a combined finite supertranslation and superrotation diffeomorphism from Minkowski with in particular $z \to G(z) + O(r^{-1})$.

The Bondi mass decreases with retarded time u,

$$\partial_u M = -\frac{1}{8} T^{AB} T_{AB}$$

where the traceless, divergence-free tensor is

$$T_{zz} = \frac{\partial_z^3 G}{\partial_z G} - \frac{3(\partial_z^2 G)^2}{2(\partial_z G)^2}$$

$\Rightarrow$ Unbounded negative energy.

$\Rightarrow$ Discard by imposing the Dirichlet boundary condition $T_{zz} = 0$.

$\Rightarrow$ Only Lorentz transformations are asymptotic symmetries

Remark on the canonical structure

The symplectic structure at $\mathfrak{I}^+$ for the vacua is

$$\Omega_{\mathfrak{I}^+} \equiv -\frac{1}{4G}\int_{\mathfrak{I}^+} du d^2\Omega\, \delta C_{AB} \wedge \delta T^{AB}.$$

The supertranslation field and superrotation field are canonically conjugate. Imposing a Dirichlet boundary condition on T^{AB} select the superrotation field as the source and the supertranslation field as the vev.

Yet, conserved superrotation charges for the physical vacua exist while the supertranslation charges are zero. This might be interpreted as in AdS as “Turning on a source infinitesimally to compute a vev”.

4. Memories from 4d Gravitational Collapse

The final static ($J = 0$) metric after spherical gravitational collapse, if analytic, is diffeomorphic to the Schwarzschild metric. [No hair theorems]

[Carter, Hawking, Robinson, 1971-1975] [Chrusciel, Costa, 2008] [Alexakis, Ionescu, Klainerman, 2009]

But memory effects accumulate before and during collapse, so the final metric is in a different BMS vacuum that the global vacuum.

A loophole of no hair theorems is that the diffeomorphism might be singular inside the event horizon, so the black hole can carry superrotation charges which characterize the classical vacuum.

Two questions :

- What is the metric $g_{\mu\nu}(M, C(\theta,\phi))$ of the final state of collapse ?
- How does the supertranslation field $C(\theta,\phi)$ compares to the final mass M ?

The Schwarzschild metric

It admits Weyl conformally flat sections. This is manifest in isotropic coordinates $(t, \rho_s, \theta_s, \phi_s)$:

$$ds^2 = -\frac{\left(1-\frac{M}{2\rho_s}\right)^2}{\left(1+\frac{M}{2\rho_s}\right)^2}dt^2 + \left(1+\frac{M}{2\rho_s}\right)^4\left(d\rho_s^2 + \gamma_{AB}d\theta^A d\theta^B\right)$$

where

$$\begin{aligned} \gamma_{AB}d\theta^A d\theta^B &= d\theta_s^2 + \sin^2\theta_s d\phi_s^2, \\ \rho_s &= \infty \text{ at spatial infinity} \\ \rho_s &= \frac{M}{2} \text{ at the event horizon} \end{aligned}$$

The Schwarzschild metric embedded in the BMS supertranslation vacuum

$$ds^2 = -\frac{\left(1-\frac{M}{2\rho_s}\right)^2}{\left(1+\frac{M}{2\rho_s}\right)^2}dt^2 + \left(1+\frac{M}{2\rho_s}\right)^4\left(d\rho^2 + g_{AB}d\theta^A d\theta^B\right)$$

where

$$\begin{aligned} g_{AB} &= (\rho\gamma_{AC} - D_A D_C C - \gamma_{AC}C)\gamma^{CD}(\rho\gamma_{DB} - D_D D_B C - \gamma_{DB}C) \\ \rho_s^2 &= (\rho - C)^2 + D_A C D^A C \end{aligned}$$

Remarks :

- When $C = 0$, this is Schwarzschild
- Obtained by finite supertranslation diffeomorphism
- The non-trivial Poincaré charges are just the energy M
- There are superrotation charges quadratic in C

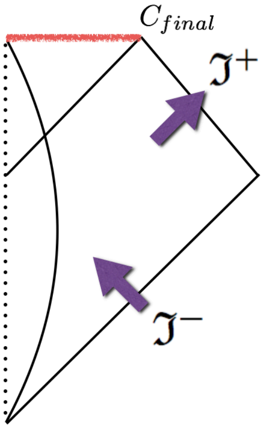
The Schwarzschild metric with BMS hair

In comparison with ["Soft hair on black holes", Hawking, Perry, Strominger, 2016]

- Agree : The hair is soft (zero energy). Supermomenta commute with the Hamiltonian.
- $O(\hbar^0)$, not $O(\hbar^1)$. The classical nature of the BMS hair is rooted in the classical memory effect. The metric are angles/distances which are classically observable (on the contrary electromagnetic hair is encoded in phases measurable only by a quantum apparatus). $O(\hbar^0)$ correction is compatible with quantum theory arguments allowing for a resolution of Hawking's paradox [Mathur, 2009]
- Non-linear, not linear. A linearized diffeomorphism would give only the linearized metric, valid close to $\mathfrak{I}^+$ or $\mathfrak{I}^-$. But non-linear effects in the bulk follow from Einstein's equations.

How much supertranslation hair ?

What is the final value of $C(\theta, \phi)$?



It depends upon the fluxes and Bondi mass at $\mathfrak{I}^+$ and $\mathfrak{I}^-$.

How much supertranslation hair?

Assuming particular junction conditions joining $\mathcal{I}^+_-$ and $\mathcal{I}^-_+$ and boundary conditions on radiation [Christodoulou, Klainerman, 1993], Einstein's equations give

$$-\frac{1}{4}\nabla^2(\nabla^2+2)(C|_{final}(\theta,\phi)-C|_{in}(\pi-\theta,\phi+\pi))$$
$$= m|_{final} - m|_{in} + \int_{-\infty}^{+\infty} du T_{uu}(\theta,\phi) - \int_{-\infty}^{+\infty} dv T_{vv}(\pi-\theta,\phi+\pi)$$

This is the global angle-dependent energy conservation law for asymptotically flat spacetimes. [Geroch, Winicour, 1980] [Strominger, Zhiboedov, 2014] [G.C., Long, 2016]

Spherically symmetric collapse of a null shell
$\Rightarrow C|_{final} = 0$ (metric described by Vaidya metric).

How much supertranslation hair?

Non-spherically symmetric collapse of a null shell is constrained by the null energy condition

$$T_{vv}(\theta,\phi) \geqslant 0.$$

Assuming all matter arrives at $v = 0$,

$$T_{vv} = \left(\frac{M + M \sum P_{l,m} Y_{l,m}(\theta,\phi)}{4\pi r^2} + O(r^{-3}) \right) \delta(v)$$

we get the complicated constraint

$$\sum P_{l,m} Y_{l,m}(\theta,\phi) \geqslant -1.$$

How much supertranslation hair ?

In the ideal case (no outgoing radiation, no initial mass, only ingoing collapsing radiation), the solution to the global energy conservation law is

$$C(\theta,\phi) = M \sum_{\ell \geqslant 2, m} \frac{4(-1)^{\ell}}{(\ell-1)\ell(\ell+1)(\ell+2)} P_{l,m} Y_{l,m}(\theta,\phi)$$

with the constraint

$$\sum P_{l,m} Y_{l,m}(\theta,\phi) \geqslant -1.$$

which bounds C from above and below (from compactness). So, for a general non-spherically symmetric collapse we expect (think binary black hole merger or accretion)

$$|C(\theta,\phi)| \simeq M \qquad \text{(leading order classical effect)}$$

Competition between supertranslation horizon and infinite redshift surface

$$ds^2 = -\frac{\left(1-\frac{M}{2\rho_s}\right)^2}{\left(1+\frac{M}{2\rho_s}\right)^2}dt^2 + \left(1+\frac{M}{2\rho_s}\right)^4 \left(d\rho^2 + g_{AB}d\theta^A d\theta^B\right)$$

where $\rho_s^2 = (\rho - C)^2 + D_A C D^A C$. The infinite redshift surface is located at $\rho = \rho_H(\theta, \phi)$ solution to

$$\frac{M^2}{4} = (\rho_H - C)^2 + D_A C D^A C.$$

- When $C \ll M$, this is a black hole with event horizon
- When $D_A C D^A C > \frac{M^2}{4}$, there is no infinite redshift surface.
  $\Rightarrow$ Probable violation of the weak cosmic censorship
- But it turns out that for all cases studied, $D_A C D^A C \leqslant \frac{M^2}{4}$ from the weak energy condition bound !
  $\Rightarrow$ New test of the weak cosmic censorship

Summary

- We constructed the supertranslation orbit of Minkowski and recognized that these are Poincaré vacua with a defect carrying superrotation charge which is hidden behind the supertranslation horizon.
- Superrotations do not belong to the asymptotic symmetry group since they would lead to unbounded negative energy but superrotation charges are well-defined.
- In the center-of-mass frame, supertranslations are spatial, except the zero mode ($=$ time translation).
- Memory effects lead to a different final state of collapse : the Schwarzschild black hole with supertranslation hair. The hair is a large non-linear $O(\hbar^0)$ effect. Assuming suitable boundary conditions at spatial infinity, the final hair is computable from past history of evolution and collapse and is $O(M)$.
- Much physics and maths remains to be understood.