Aspects of Gauge-Strings Duality

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Outline

1. I will comment on recent work in the area of non-Abelian T-duality. Focus on the general ideas and outcomes.
2. The knowledge of field theory results at strong coupling allows us to say things about a geometry (ranges of coordinates, smoothing-out of singularities, etc).
3. All I will discuss today will be in the context of $\mathcal{N} = 2$ SCFT in four dimensions. The results can be (and in some cases have been) extended to cases with less SUSY and no conformality.
4. This talk is a very schematic selection of topics taken from a paper written with Yolanda Lozano, in late March 2016. But also draws and is inspired by results obtained in different papers in collaboration with: G. Itsios, N. MacPherson, S. Zacarias, Y. Bea, A. Sierra, C. Whitting, J. Edelstein, K. Sfetsos, D. Thompson, E. Caceres, V. Rodgers, L. Pando Zayas.

This audience is very used to work with proposed or conjectured dualities:

AdS/CFT, Seiberg-Witten, Seiberg duality, etc. Many tests for these proposals and Physics predictions have been produced.

Other dualities can be proven. Characteristically in 2d.

Given an Action, a four-steps procedure is followed:

- Detect a global symmetry. Gauge it. $\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi \rightsquigarrow i\bar{\Psi}(\not{\partial}+\not{A})\Psi$
- Impose the gauge field has no dynamics using a Lagrange multiplier. $\mathcal{L} = i\bar{\Psi}(\not{\partial}+\not{A})\Psi + \Lambda F_{\mu\nu}\epsilon_{\mu\nu}$
- Integrate out the original degrees of freedom and gauge field. (2d is important) The Action in terms of the Lagrange multiplier is the 'Dual Action' $\mathcal{L} = i\bar{\Psi}\not{\partial}\Psi \longleftrightarrow \mathcal{L}_D = (\partial_\mu \Lambda)^2$

There are many relevant examples, interesting in Physics

ISING MODEL　　BOSONISATION　　T-DUALITY

A non-Abelian extension of T-duality exist. It follows the procedure above.

de la Ossa + Quevedo (1994), Rocek + Verlinde + Greene (1994) + Alvarez-Gaume, Lozano, Borlon (1996) + ... (~100 papers) + Sfetsos, Thompson (2011) + ...

Allow me NOT to pass you through the whole formalism

But the idea is that given a background with—for example– an $SU(2)$-isometry,

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \sum_{i=1}^{3} f_i(x)(\omega_i - A_i(x))^2, \quad \Phi(x).$$

$$C_{p,RR} \sim B_2 \sim a_{\mu\nu} dx^\mu \wedge dx^\nu + b_{\mu i} dx^\mu \wedge \omega^i + c_{ij}\omega_i \wedge \omega_j.$$

ω_i = Left invariant forms of SU(2)

$\omega_1 = \cos\psi \, d\theta + \sin\psi \sin\theta \, d\varphi$
$\omega_2 = -\sin\psi \, d\theta + \cos\psi \sin\theta \, d\varphi$
$\omega_3 = d\psi + \cos\theta \, d\varphi$

Non-Abelian T-duality can be applied and a new background (solution of Type II eqs. of motion) is generated.

In terms of the new coordinates or Lagrange multipliers.

$\omega_i(\theta, \varphi, \psi) \to z_a(\eta, \chi, \xi)$.

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu + \sum_{i=1}^{3} g_{ab} dz_a dz_b, \quad \tilde{\Phi}, \quad \tilde{B}_2, \quad \tilde{C}_p.$$

Various things are unclear about this non-Abelian version of T-duality.

- What is the range of the dual coordinates (Lagrange multipliers)?
- Can we dualise again? $(\eta, \chi, \xi) \to (\theta, \varphi, \psi)$?
- What happens if we perform this duality on a world-sheet with genus $g > 0$?
- When is the generated background singular? Can these singularities be resolved?

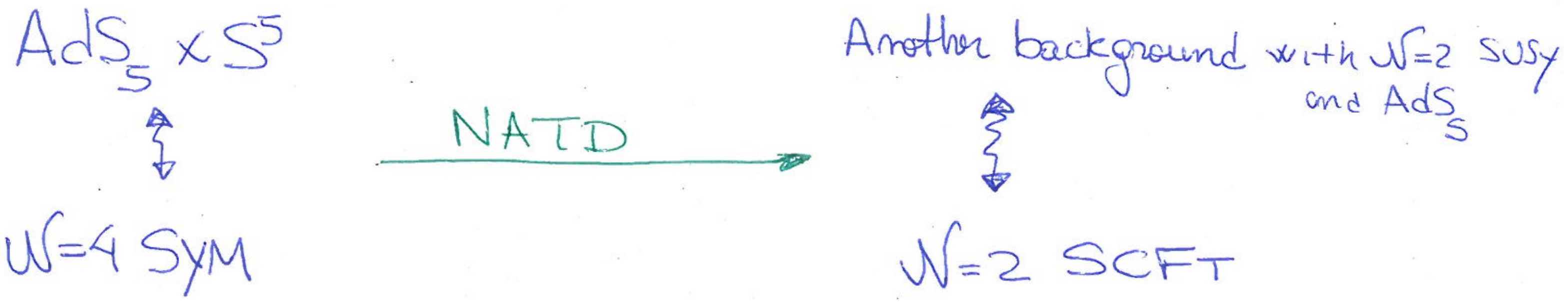
These issues are not well understood. There were some papers in the 1990's trying to address them, but the results are not conclusive.

The point of the papers on which this talk is based is to contribute to the understanding of the problems above.

To do so, we will focus on a very simple example dual to a CFT and use the CFT to 'inform' the background.

The idea is to take a background whose dual QFT is well understood. Transform the background with non-Abelian T-duality.

The example chosen for this talk is that of $AdS_5 \times S^5$. Studying the non-Abelian T dual background, we will find it can be put in correspondence with $\mathcal{N} = 2$ SCFT. We have a powerful formalism to study the CFT and learn things about the background.



Let us then consider $AdS_5 \times S^5$ in $g_s = 1$ units, written as

$$ds^2 = AdS_5 + 4L^2\Big[\frac{d\sigma^2}{(1-\sigma^2)} + \sigma^2 d\beta^2 + \frac{(1-\sigma^2)}{8}(\omega_1^2+\omega_2^2+\omega_3^2)\Big],$$

$$F_5 = \frac{2}{L}(e^{tx_1x_2x_3R} + e^{\sigma\beta 123}), \quad N_{D3} = \frac{\int F_5}{2\kappa_{10}^2 T_{D3}} \to \frac{L^4}{\alpha'^2} = \frac{\pi N_{D3}}{4}.$$

$$\sum_{i=1}^{3}\frac{\omega_i^2}{2} = (d\psi + \cos\theta d\varphi)^2 + d\theta^2 + \sin^2\theta d\varphi^2.$$

We perform the (usual) T-dual and non-Abelian T-dual.

- T-duality in ψ: $\psi \to \eta$.
- non-Abelian T-duality:$(\theta,\varphi,\psi) \to (\eta,\chi,\xi)$

The generated backgrounds read,

T-dual on ψ

$$ds^2_{IIA,st} = AdS_5 + 4L^2\Big[\frac{d\sigma^2 + d\eta^2}{1-\sigma^2} + \sigma^2 d\beta^2\Big] + $$
$$+ L^2(1-\sigma^2)d\Omega_2(\theta,\varphi).$$
$$B_2 = 2L^2\eta d\Omega_2(\theta,\varphi), \quad e^{-2\Phi} = \frac{L^2}{\alpha'}(1-\sigma^2),$$
$$F_4 = \frac{8L^4}{\sqrt{\alpha'}}\sigma(1-\sigma^2)d\sigma\wedge d\beta\wedge d\Omega_2(\theta,\varphi).$$

The T-dual background is singular at $\sigma = 1$.
Which is understandable, since the size of the ψ-cycle before the duality vanishes at $\sigma = 1$.

Non-abelian T-duality on $(\theta, \varphi, \psi) \to (\eta, \chi, \xi)$.

$$ds^2_{IIA,st} = AdS_5 + 4L^2\Big[\frac{d\sigma^2 + d\eta^2}{1-\sigma^2} + \sigma^2 d\beta^2\Big] +$$
$$L^2(1-\sigma^2)\frac{4\eta^2}{4\eta^2 + (1-\sigma^2)} d\Omega_2(\theta,\varphi).$$
$$B_2 = \frac{8L^2\eta^3}{4\eta^2 + 1 - \sigma^2} d\Omega_2(\theta,\varphi), \quad e^{-2\Phi} = \frac{L^2}{\alpha'}(1-\sigma^2)(4\eta^2 + 1 - \sigma^2),$$
$$F_2 = \frac{8L^4}{\sqrt{\alpha'^3}}\sigma(1-\sigma^2) d\sigma \wedge d\beta, \quad F_4 = B_2 \wedge F_2.$$

Again, the non-Abelian T-dual background is singular at $\sigma = 1$.
Which is again understandable, since the size of the (θ, φ, ψ)-cycle before the duality vanishes at $\sigma = 1$.

We can calculate the charges associated to each of these backgrounds

In the Abelian T-dual case

$$Q_{D4} = \frac{1}{2\kappa_{10}^2 T_{D4}} \int F_4 = N_4 \to \frac{L^4}{\alpha'^2} = \frac{\pi N_4}{2}.$$

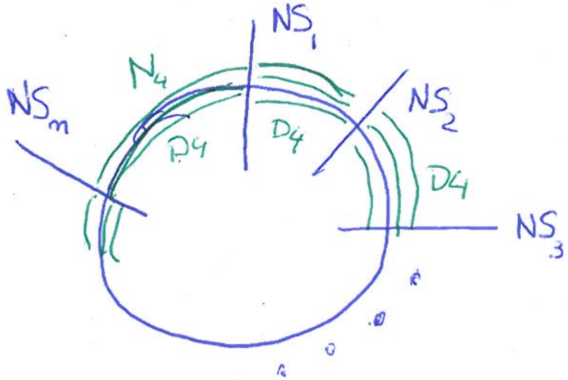
$$Q_{NS5} = \frac{1}{2\kappa_{10}^2 T_{NS5}} \int H_3 = 1 \quad (0 < \eta < \pi).$$

On the other hand, if $0 < \eta < n\pi$ we would have had $Q_{NS5} = n$.

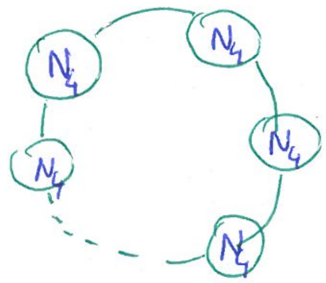
This corresponds to a Hanany-Witten set-up containing D4 and NS-five branes,

$D4 : (R^{1,3}, \eta)$

$NS_5 : (R^{1,3}, \alpha, \beta)$



T dual of $AdS_5 \times S^5/Z_N$



quiver.

For the non-abelian T-dual, the charges are,

$$Q_{D6} = \frac{1}{2\kappa_{10}^2 T_{D6}} \int F_2 = N_6 \to \frac{L^4}{\alpha'^2} = \frac{N_6}{2}.$$

$$Q_{D4} = \frac{1}{2\kappa_{10}^2 T_{D4}} \int F_4 - B_2 \wedge F_2 = 0.$$

$$Q_{NS5} = \frac{1}{2\kappa_{10}^2 T_{NS5}} \int H_3 = \frac{1}{4\pi^2\alpha'} \int_0^{2\pi} d\xi \int_0^{\pi} d\chi \int_0^{\eta_*} d\eta.$$

How to determine η_*? Macpherson and Lozano (2015), proposed to take a look at the B_2-field on a shrinking Σ_2, and the integral

$$b_0 = \frac{1}{4\pi^2\alpha'} \oint_{\Sigma_2} B_2 = \frac{\eta}{\pi}. \quad \Sigma_2 = [\chi, \xi]_{2\alpha=\pi}.$$

This suggest that every time we cross a position $\eta = n\pi$, we should large-gauge-transform the B_2-field, hence changing the charges

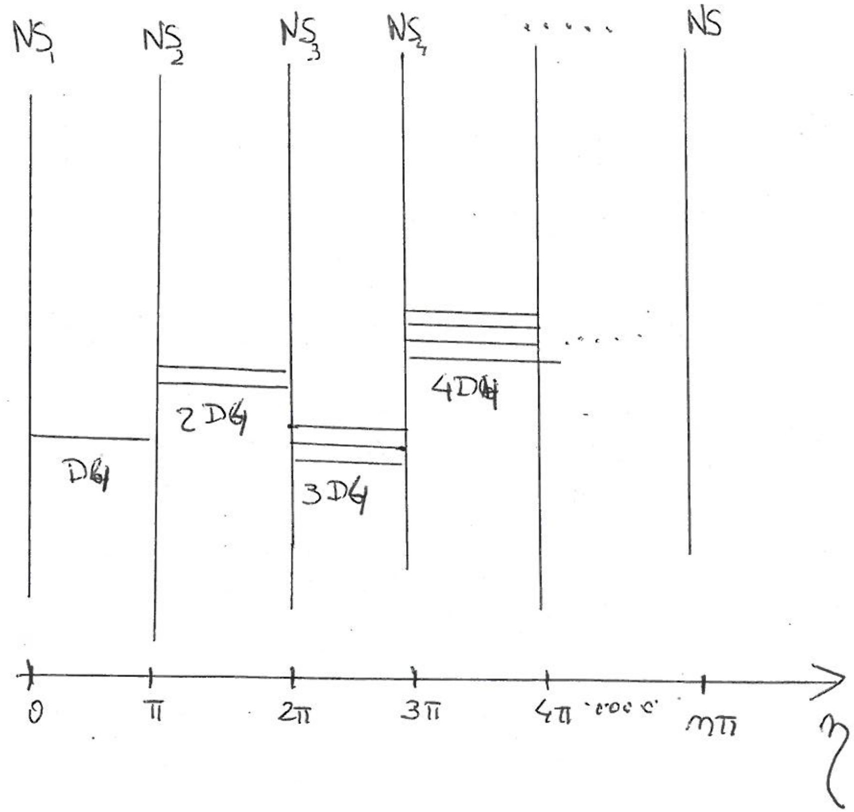
$$\Delta B_2 = -n\pi\alpha' sin\chi d\chi \wedge d\xi \to \Delta Q_{D6} = 0, \;\; \Delta Q_{D4} = nN_{D6}, \;\; Q_{NS5} = n.$$

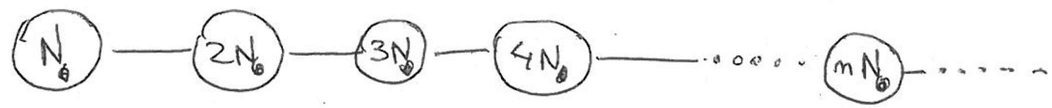
The Hanany-Witten set up in this case is

$D4: R^{1,3}; \eta$

$NS_5: R^{1,3}; \sigma, \beta$

$D6: R^{1,3}; \eta, \chi, \xi$





quiver

Let us try to put these backgrounds in a more general context.
There is a generic background geometry dual to an $\mathcal{N}=2$ SCFT.
After some simplifications are made, the configuration is described in terms of a function $V(\sigma,\eta)$, solving a Laplace equation.
Defining $\dot{V}=\sigma\partial_\sigma V$, $V'=\partial_\eta V$ It reads,

$$ds^2_{IIA,st}=\alpha'\left(\frac{2\dot{V}-\ddot{V}}{V''}\right)^{1/2}\Big[4AdS_5+\mu^2\frac{2V''\dot{V}}{\Delta}d\Omega_2^2(\chi,\xi)+$$
$$\mu^2\frac{2V''}{\dot{V}}(d\sigma^2+d\eta^2)+\mu^2\frac{4V''\sigma^2}{2\dot{V}-\ddot{V}}d\beta^2\Big],$$
$$A_1=2\mu^4\sqrt{\alpha'}\frac{2\dot{V}\dot{V}'}{2\dot{V}-\ddot{V}}d\beta,\quad e^{4\Phi}=4\frac{(2\dot{V}-\ddot{V})^3}{\mu^4V''\dot{V}^2\Delta^2},$$
$$B_2=2\mu^2\alpha'\left(\frac{\dot{V}\dot{V}'}{\Delta}-\eta\right)d\Omega_2,\quad C_3=-4\mu^6\alpha'^{3/2}\frac{\dot{V}^2V''}{\Delta}d\beta\wedge d\Omega_2.$$
$$\Delta=(2\dot{V}-\ddot{V})V''+(\dot{V}')^2.$$

Lin Lunin Maldacena (2004)
Gaiotto-Maldacena (2009)
Stefanski + Reid Edwards (2010)
Aharony, Berdichevsky, Berkooz (2011)

A quantity that proves quite useful to define is

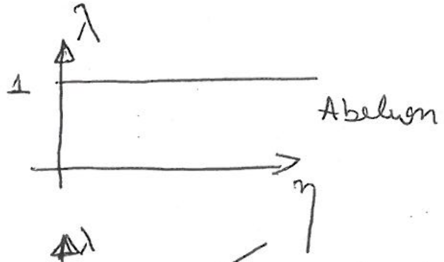
$$\lambda(\eta)=\sigma\partial_\sigma V(\sigma,\eta)|_{\sigma=0}.$$

What is the point of all this?

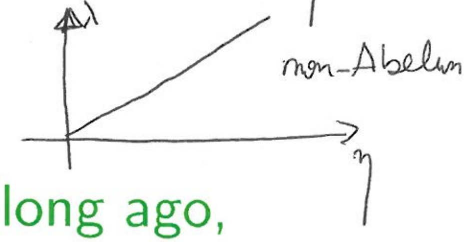
We have encoded in two functions $V(\sigma, \eta)$ and $\lambda(\eta)$ all the geometric (and QFT) information

We can then check that the Abelian and non-Abelian T-dual backgrounds are just a particular example of this generic configurations. Indeed,

$$V_{ATD} = \log\sigma - \frac{\sigma^2}{2} + \eta^2, \quad \lambda(\eta) = 1.$$



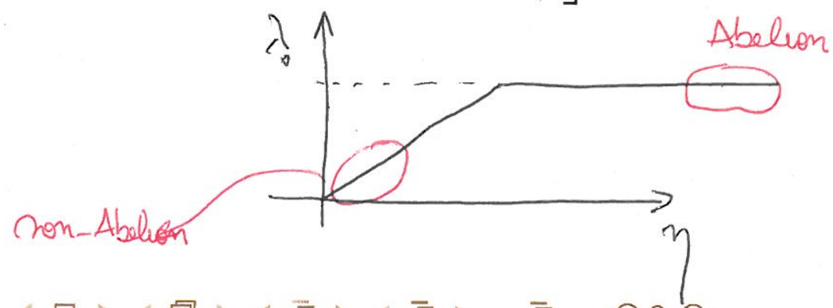
$$V_{NATD} = \eta\left(\log\sigma - \frac{\sigma^2}{2} + \frac{\eta^2}{2}\right), \; \lambda(\eta) = \eta.$$



There is also a nice relation with another solution written long ago,

$$\sigma\partial_\sigma V_0 = \dot{V}_0(\sigma, \eta) = \frac{1}{2}\left[\sqrt{(N_c+\eta)^2+\sigma^2} - \sqrt{(N_c-\eta)^2+\sigma^2}\right],$$

$$\lambda_0(\eta) = \frac{1}{2}\Big(|\eta + N_c| - |\eta - N_c|\Big).$$



What we can do with all these? how can we get information about the range of η or resolve the singularities of the T-dual examples? Take the non-Abelian T-dual,for example. We calculate the central charge using the background (let me use $\sim$ symbols, though we worked with precise expressions!)

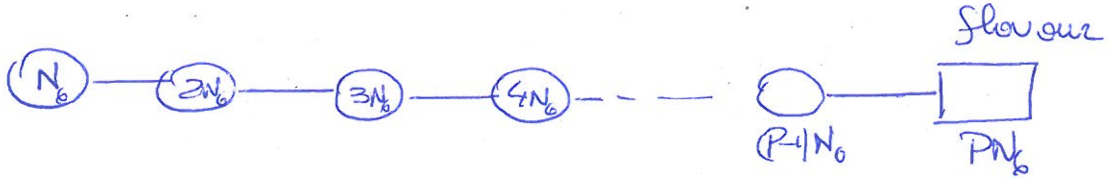
$$c_{NATD} \sim V_{int} \sim \int_0^{\eta_*} f(\eta) d\eta = \frac{N_{D6}^2 N_{NS5}^3}{12}.$$

We have 'cut' the coordinate at $\eta = \eta_* = n\pi$.

Then we can ask: what is the CFT that has the same central charge?

We 'design' a quiver for a $\mathcal{N} = 2$ SCFT that calculates the same central charge (and other observables). We write the associated geometry and fluxes (has same charges).

Quiver



$$P = N_5$$

$$C = \frac{2 n_v + n_h}{12} \qquad \text{(Shapere + Tachikawa 2008)}$$

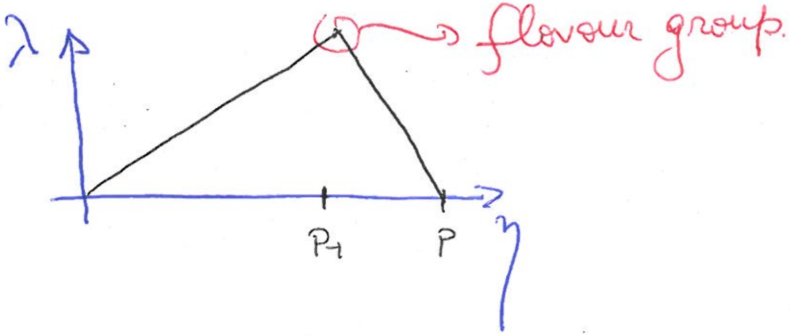
$$n_v = \sum_{k=1}^{P-1} k^2 N_6^2 - 1 = N_6^2 \left[\frac{P^3}{3} - \frac{P^2}{2} + \frac{P}{6} + \frac{1-P}{N_6^2} \right]$$

$$n_h = \sum_{k=1}^{P-1} k(k+1) N_6 = N_6^2 \left(\frac{P^3 - P}{3} \right)$$

$$C = \frac{N_6^2 N_5^3}{12} \left\{ 1 + \mathcal{O}\left(\frac{1}{P}, \frac{1}{N_6} \right) \right\}$$

The gravity solution is characterised by

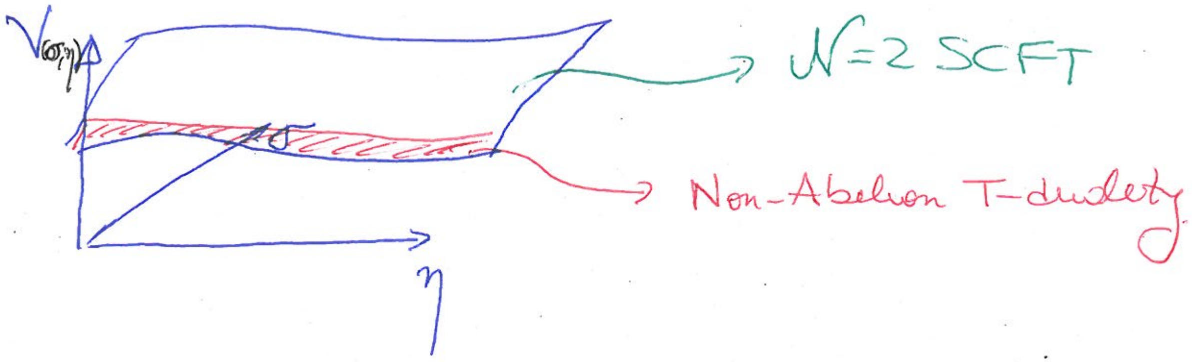
$$\lambda(\eta) = \begin{cases} N_6 \eta & \eta \in [0, P-1] \\ (1-P)\eta + (P^2 - P) & \eta \in [P-1, P] \end{cases}$$



$$2\dot{V}(\sigma, \eta) = \sum_{m=-\infty}^{\infty} (P-1) N_6 \left\{ \sqrt{\sigma^2 + (\eta - 2mP - P)^2} - \sqrt{\sigma^2 + (\eta - 2mP + P)^2} \right\} - P N_6 \left\{ \sqrt{\sigma^2 + (\eta - 2mP - N_6)^2} - \sqrt{\sigma^2 + (\eta - 2mP + N_6)^2} \right\}$$

In this way we:

- Give a range for the η-coordinate from CFT considerations.
- Resolve the singularity at $\sigma = 1$, by finding a manifold for which the non-Abelian T-dual of $AdS_5 \times S^5$ is just a 'patch' of it.



In a very different form, this "picture" is already in {Sfetsos (1994), Sfetsos-Polychronakos (2011)}

Summary, Conclusions and Final Comments

- The usual T-duality and its non-Abelian version can be seen as "solution generating techniques".
- In the case of T-duality, the stringy character of the transformation is well understood. This is not the case in the non-abelian generalisation.
- Using CFT information, we got certain hints about the manifold generated by non-Abelian T-duality acting on $AdS_5 \times S^5$.
- Other examples should follow a similar logic, the non-Abelian T-duality, 'focuses' on a patch of a more generic manifold.
  On the other hand, non-Abelian T-duality can be applied to a large variety of examples and it has proven very useful to:
  - Find new backgrounds, avoiding known classifications.
  - Generate new backgrounds with 'dynamic' $SU(2)$-structure and other G-structures.
  - Use these new backgrounds to 'define' new QFTs at strong coupling, by the calculation of the QFT observables.
- The "CFT perspective" promoted in this talk may help to clarify problems and issues present in the points above.