Measuring the Scrambling of Quantum Information

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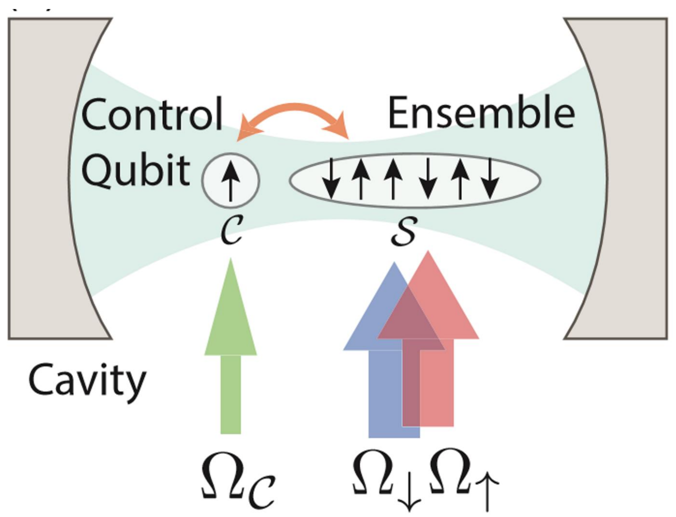
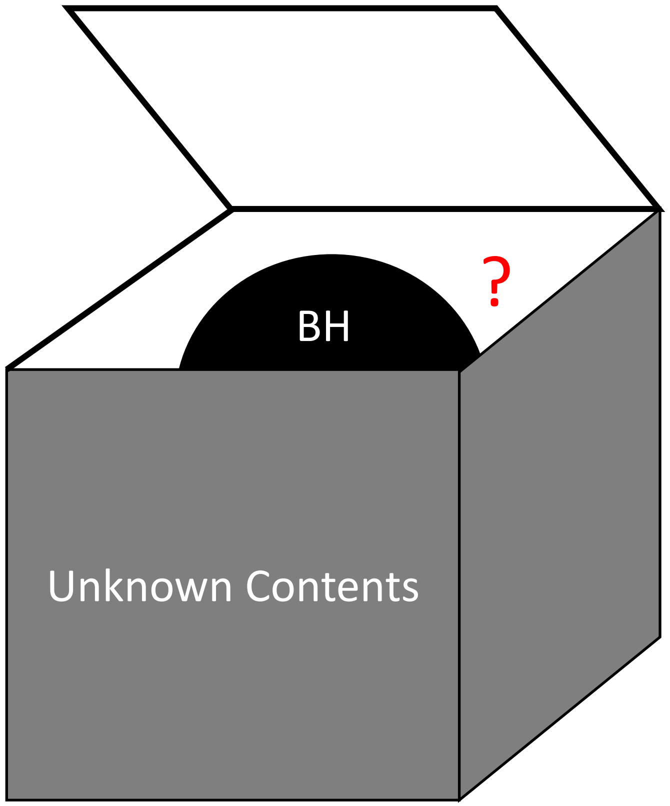
SIMONS FOUNDATION

$$F(t) = \langle W_t^\dagger V^\dagger W_t V \rangle$$





Scrambling

- Throw a qubit into a black hole; scrambling occurs when the quantum information in that qubit is spread over all the degrees of freedom of the black hole and becomes inaccessible to local measurements
- In a random circuit model with one circuit layer per thermal time, the scrambling time is

$$t_* \sim \beta \log S$$

[conjectured: Hayden-Preskill, rigorous: Brown-Fawzi CMP ‘15]

- Black holes conjectured to have the same scrambling time and to be the fastest scramblers [Sekino-Susskind]

Scrambling and out-of-time-order correlators

- By considering various shockwave geometries, Shenker-Stanford showed that the scrambling time also appears in certain out-of-time-order correlators (V op, W_t Heisenberg op), related to commutator

$$F(t) = \langle W_t^\dagger V^\dagger W_t V \rangle$$

[Stanford-Shenker ‘13]

- This correlator can also be interpreted (up to thermal regulators) as a correlator between the two sides of a two-sided black hole
- Shenker-Stanford and Kitaev made a connection between this correlator and chaos in the form of the “butterfly effect”: a small perturbation generates a huge shockwave at the horizon

Out-of-time-order correlators and chaos

- Given a chaotic mechanical system with position q and momentum p, consider the operators

$$V = e^{iq/a},\ W = e^{ip/b}$$

$$F(t) \approx e^{-\langle [q_t, p] \rangle / ab + \dots}$$

early times

- Correspondence principle: [Larkin-Ovchinnikov JETP '69]

$$\langle [q_t, p] \rangle \approx i\hbar \{q_t, p\}_{\mathrm{PB}} = i\hbar \frac{\partial q_t}{\partial q} = i\hbar e^{\lambda_L t}$$

Lyapunov exponent, butterfly effect

$$t_* = \lambda_L^{-1} \log \left(\frac{ab}{\hbar} \right)$$

Ehrenfest time, time-scale for significant decay of F

Chaos (MSS) bound

- With local interactions and many DOF so that there is a separation of time-scales between the dissipation (TO) and scrambling times (OTO), Maldacena-Shenker-Stanford argued for a bound:

$$F = 1 - \epsilon e^{\lambda_L t} + \ldots \qquad \epsilon \sim \frac{1}{N^2} \sim G_N$$

$$\lambda_L \leq \frac{2\pi T}{\hbar}$$ [Maldacena-Shenker-Stanford '15]

- Black holes in Einstein gravity saturate the bound, MSS suggested that saturation of the bound might correspond to Einstein gravity in the near horizon region, closely related to absence of higher spins

$$\text{gravitational scattering: } G_N s, \; s \sim e^{2\pi T t}$$

Measuring Scrambling

Why out-of-time-order correlators?

- A test for black hole horizons? $\lambda_L = \frac{2\pi}{\beta}$ and MSS bound
- Probe of quantum chaos, access finite N effects
- Probe of thermalization, localization vs thermalization
- Bounds on transport? Other bounds on quantum dynamics?
- Precision measurement? [Davis-Bentsen-Schleier-Smith PRL ‘16]

Out-of-time-order correlators (again)

Given two unitary operators V and W, define the OTO correlator:

$$F(t) = \langle W_t^\dagger V^\dagger W_t V \rangle$$

$$W_t = e^{iHt} W e^{-iHt}$$ Heisenberg operator

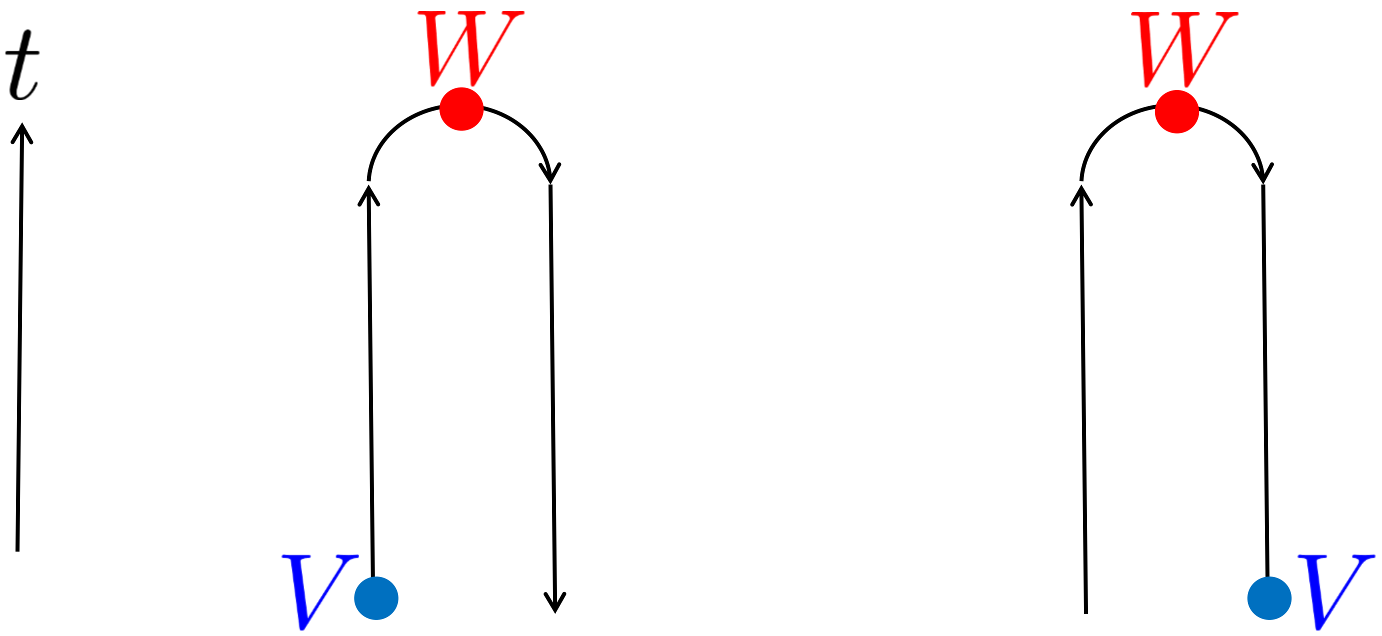
F measures the degree of non-commutativity of V and the time evolved version of W:

$$\langle [W_t, V]^\dagger [W_t, V] \rangle = 2 - 2\Re(F)$$

Physical meaning of OTO correlator

Measures the overlap between two quantum states:

$$W_t V|\psi\rangle \qquad\qquad V W_t|\psi\rangle$$

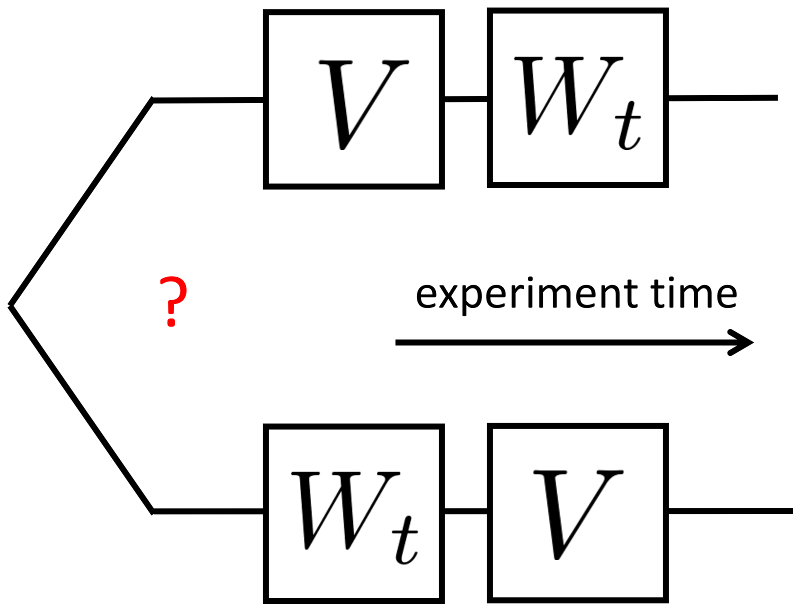


At a minimum we need the ability to evolve with H and –H; similar to echo measurements

[echo: Hahn PR ’50, Rhim et al PRB ’71, Zhang et al PRL ’92, Jalabert-Pastawski PRL ‘01, …]

Interferometric Protocol

Can we design an interferometer to measure the overlap?



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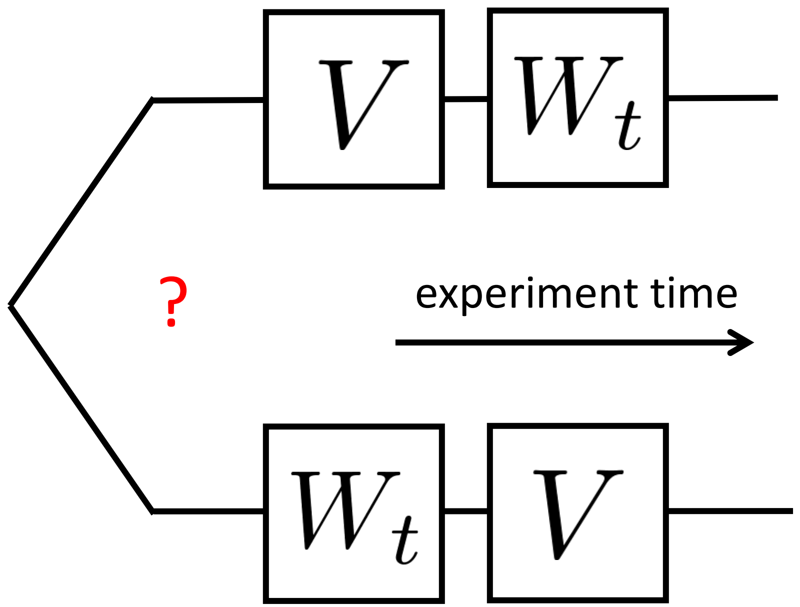
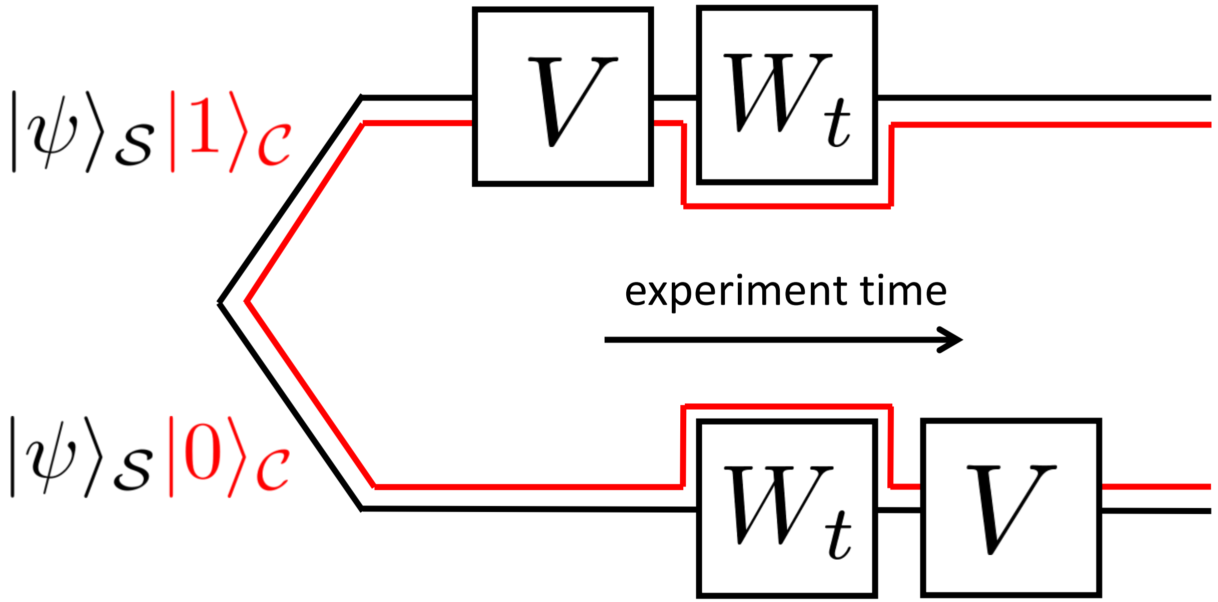
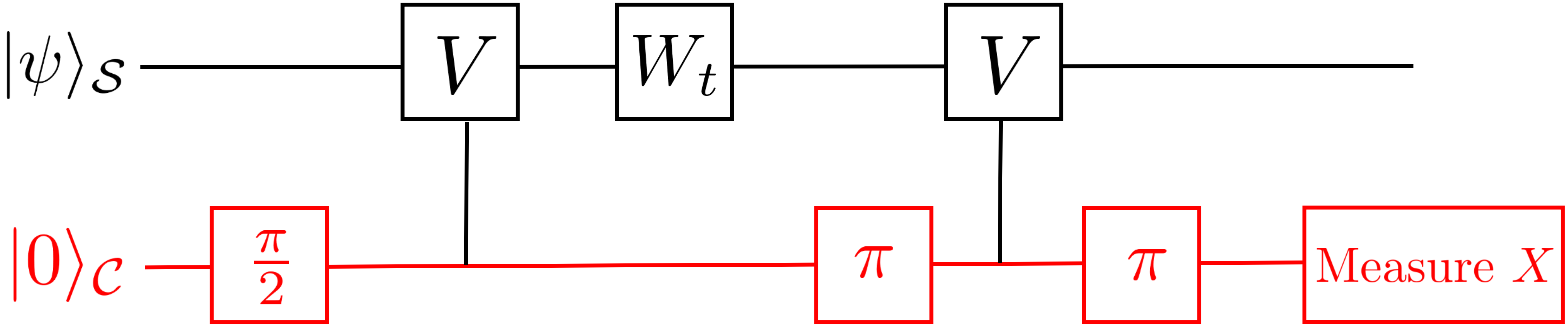






Final state: $\dfrac{VW_t|\psi\rangle_{\mathcal{S}}|0\rangle_{\mathcal{C}} + W_tV|\psi\rangle_{\mathcal{S}}|1\rangle_{\mathcal{C}}}{\sqrt{2}}$

Measure Pauli X of the control: $\langle X_{\mathcal{C}} \rangle = \Re(F)$

[BGS-Bentsen-Schleier-Smith-Hayden '16]

Requirements

- Evolve with both H and –H (forward/backward time evolution of a many-body Hamiltonian)
- Controlled unitary operations

If the control is not available, it is still possible to measure |F| using the *distinguishability protocol:*

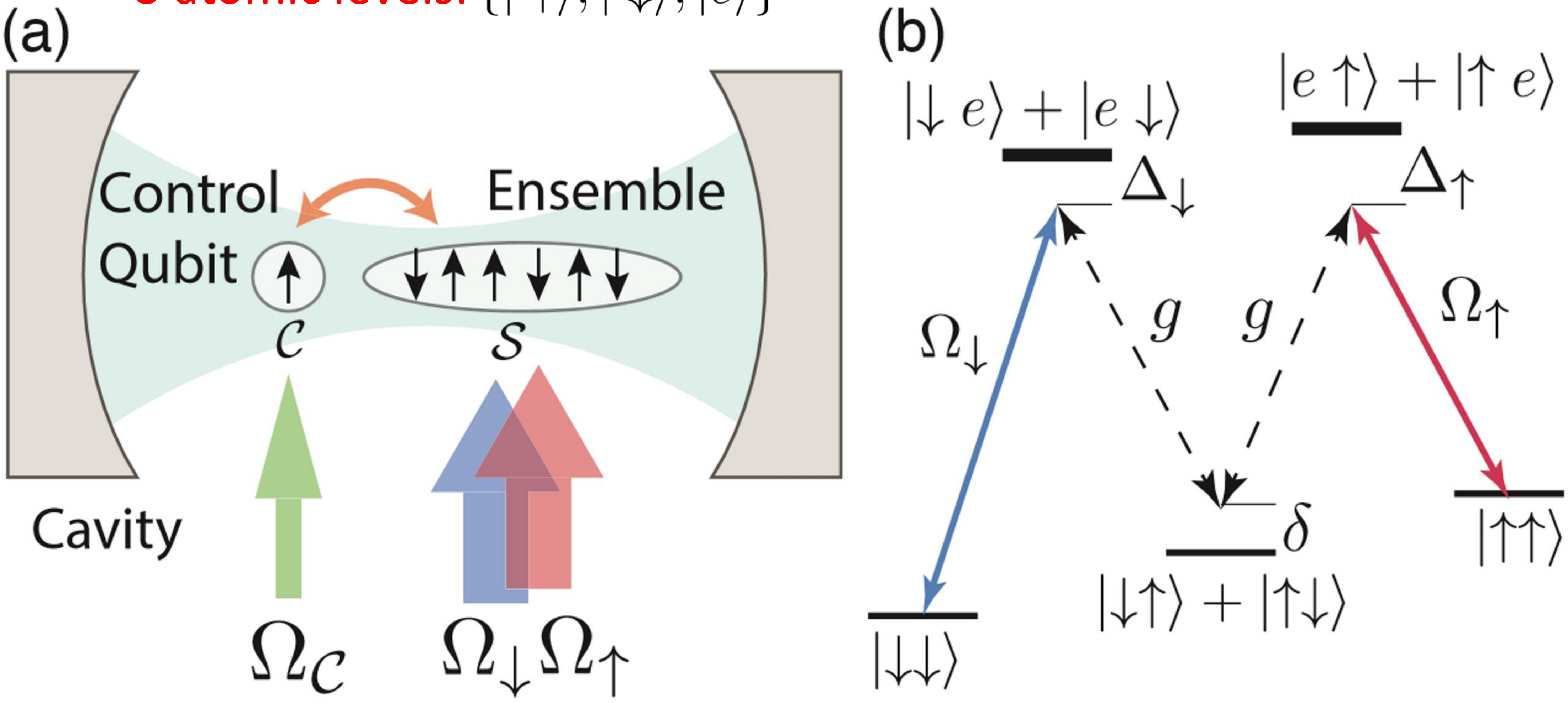
$$|\psi_f\rangle = W_t^\dagger V^\dagger W_t V |\psi\rangle$$

$$|F|^2 = \langle \psi_f | \Pi | \psi_f \rangle \qquad \Pi = |\psi\rangle\langle\psi|$$

projector onto the initial state, can be hard to measure

Model system: atoms coupled to a cavity

3 atomic levels: $\{|\uparrow\rangle, |\downarrow\rangle, |e\rangle\}$



Effective dynamics

- Consider three level atoms $\{|\uparrow\rangle, |\downarrow\rangle, |e\rangle\}$ coupled to the cavity field
- "Integrate out" the high-energy degrees of freedom (atomic excited states, cavity photons):

atom positions

$$H = \sum_{i,j,\alpha} \frac{\Omega_\uparrow(r_i)\Omega^*_\downarrow(r_j)}{\Delta_\uparrow \Delta_\downarrow} \frac{g_\alpha(r_i) g^*_\alpha(r_j)}{\delta} s_{x,i} s_{x,j}$$

atom "spin" op

[Sørenson-Mølmer PRA '02,
Gopalakrishnan-Lev-Goldbart PRL '11,
Strack-Sachdev PRL '11]

Later, effects of dissipation …

Key features

$$H = \sum_{i,j,\alpha} \frac{\Omega_\uparrow(r_i)\Omega_\downarrow^*(r_j)}{\Delta_\uparrow \Delta_\downarrow} \frac{g_\alpha(r_i) g_\alpha^*(r_j)}{\delta} s_{x,i} s_{x,j}$$

- Sign of H can be controlled by detuning δ
- H can be switched off by turning off control fields
- Rich pattern of non-local interactions, cousin of a model that scrambles like a black hole; can add fields in any direction and time dependence (random circuit models)
- Controlled Sz rotation possible: (with interactions off) coherently map control atom state to presence or absence of cavity photon

[Jiang et al Nat Phys '08]

A benchmark: single mode limit

- Suppose the cavity only has a single relevant mode which couples identically to all N atoms; "one-axis twisting" Hamiltonian:

$$H = \chi \left(\sum_{i=1}^{N} s_{x,i} \right)^2 = \chi S_x^2$$

- If the initial state is symmetric then it remains so at all times; the problem is reduced to an effective single particle problem of a large spin S = N/2 (easier to analyze)
- Related? Near Heisenberg limited measurement by reversing time

[Davis-Bentsen-Schleier-Smith PRL '16]

A chaotic model: the kicked top

- One-axis twisting Hamiltonian + periodic external “kick” (Sz rotation)

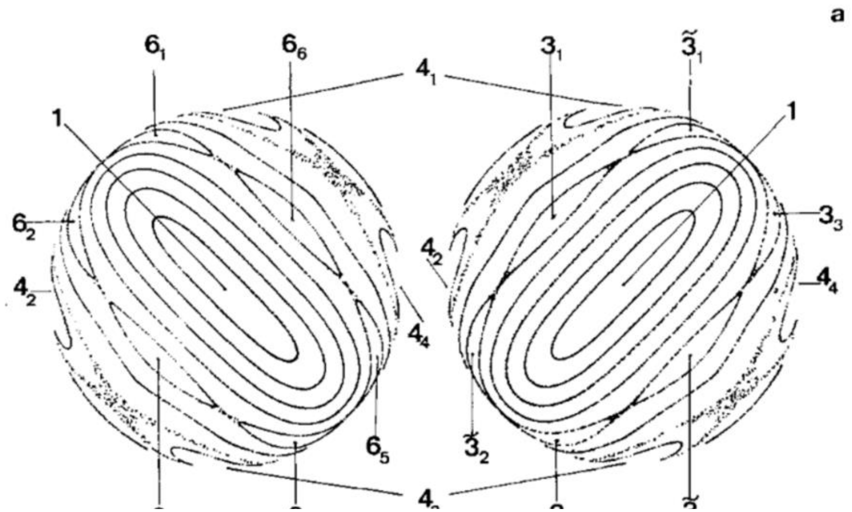
$$H(t) = \frac{1}{\tau}\frac{k}{2S}S_x^2 + pS_z \sum_n \delta(t - n\tau)$$

$$U = e^{-i\frac{k}{2S}S_x^2} e^{-ipS_z}$$

- Convenient to set $p = \pi/2$
- Regular to chaotic motion as k increases [Haake et al. 1987]
- Studied experimentally for S=3 [Chaudhurry et al. Nature ‘09]

Kicked top phase space portrait [Haake et al. 1987]

k=2

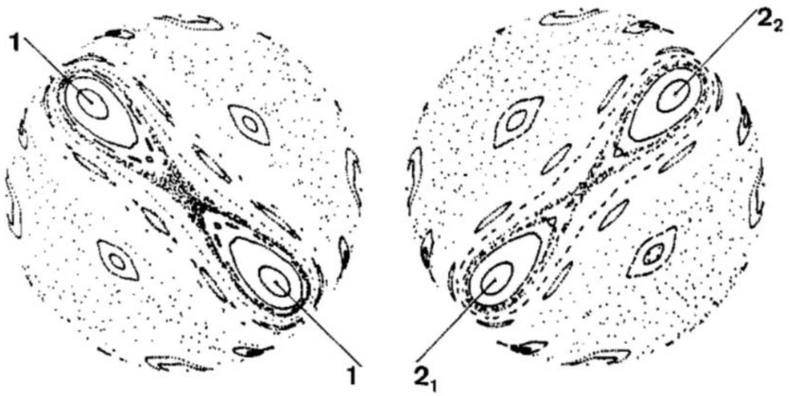


$$x = \frac{S_x}{S} \qquad y = \frac{S_y}{S} \qquad z = \frac{S_z}{S}$$

$$x^2 + y^2 + z^2 = 1$$

Analog of “Planck’s constant” = 1/S
Analog of Ehrenfest time = log(S)

k=2.5



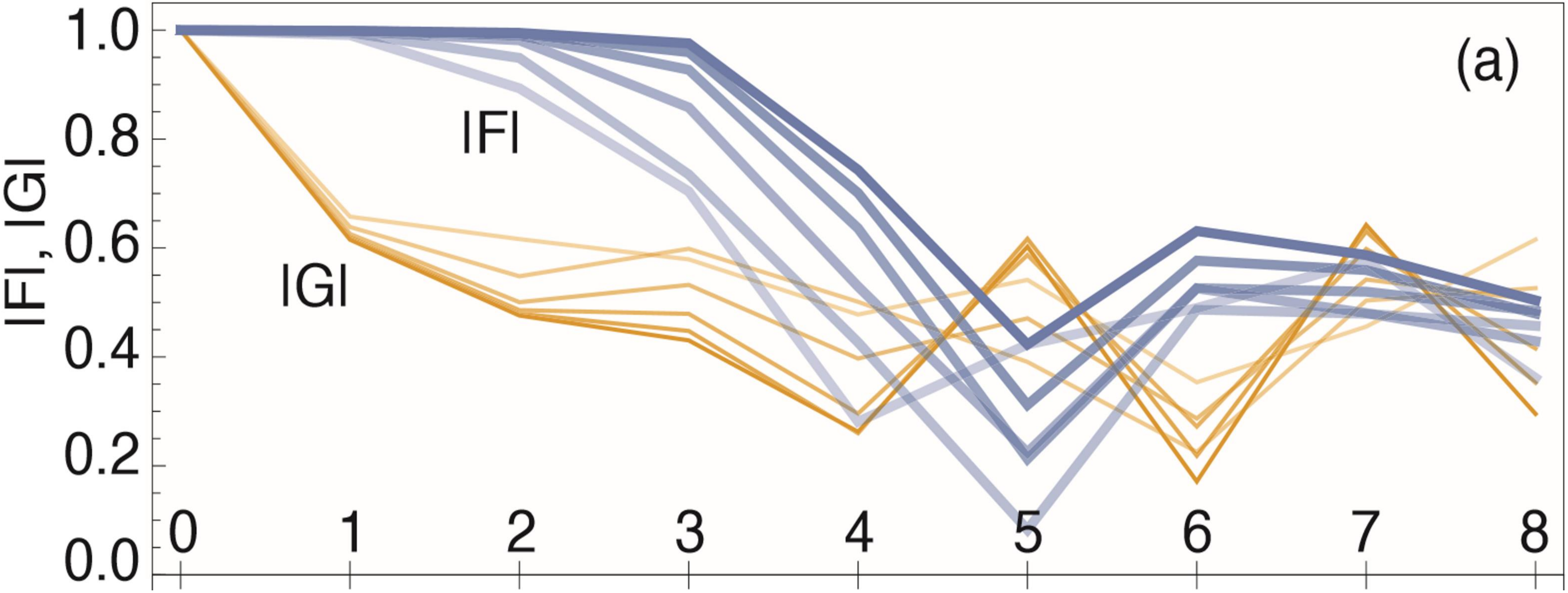
k=3

Kicked top

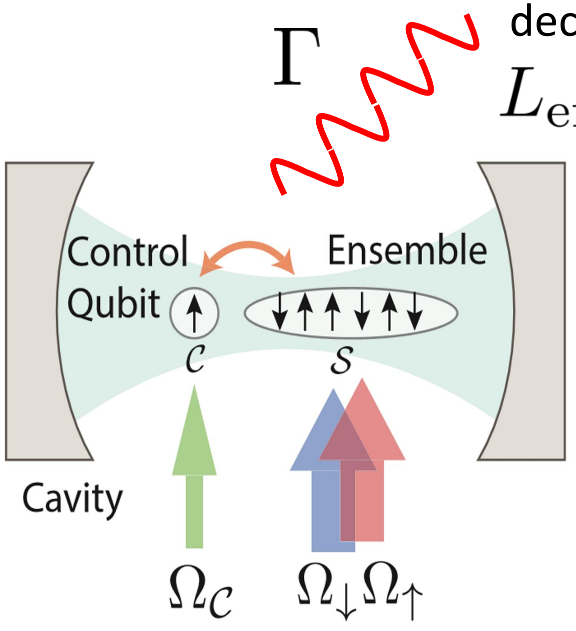
N=50, 100, ..., 500 (lighter to darker); k=3

$V = W = e^{-iS_z/\sqrt{N}}$ analogy to mechanical q, p

$F(t) = \langle W_t^\dagger V^\dagger W_t V \rangle \quad G(t) = \langle V_t^\dagger V \rangle$

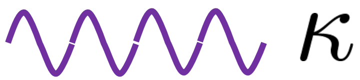


Dissipation



decay of atomic excited state

$$L_{\text{eff},i} = \sqrt{\gamma_1}|\downarrow\rangle\langle\uparrow|_i$$



loss of cavity photon

$$L_{\text{eff}} = \sqrt{\gamma_2}S_x$$

Cooperativity:

$$\eta = \frac{4g^2}{\kappa\Gamma}$$

Conditions:

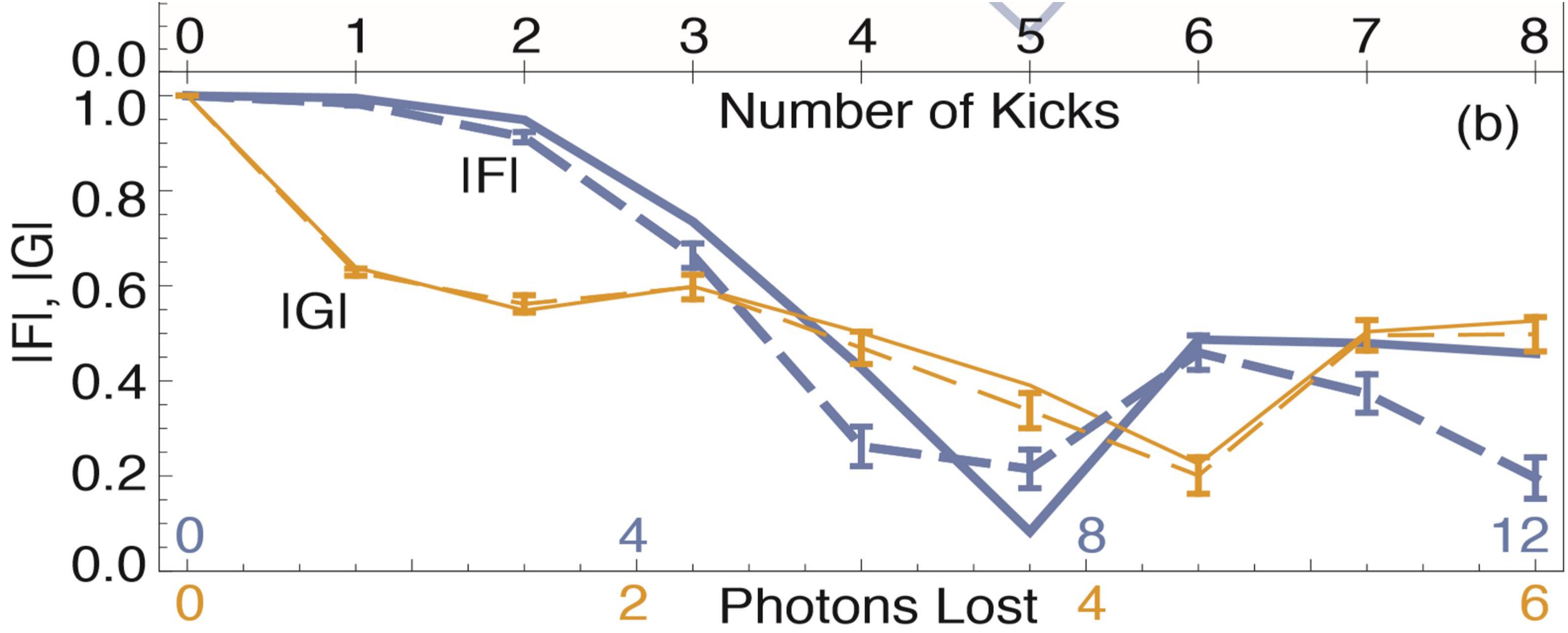
1. $\phi \leq \sqrt{\frac{\eta}{4N}}$
2. $\eta \geq (k \ln N)^2$

Convenient to organize Hilbert space into sectors of total spin; describe physics with master equation

- Cavity decay keeps the system within a given total spin sector
- Spontaneous emission moves between different total spin sectors, breaks single particle picture

With dissipation (quantum trajectories)

N=100; k=3; $\eta = 100$ a large cooperativity, but feasible with state-of-the-art cavity



What can we learn?

Chaos and the physics of F

- Chaos in the kicked top model, longer time effects of dissipation (many-body effects)? We are already at the edge of our ability to directly simulate the physics on a computer.
- Detailed study of the physics of F in a well-controlled model, e.g. finite size (finite N) structure, sensitivity to imperfections
  - Necessary to inform future experimental efforts, data interpretation
- Chaos in simple many-body models, e.g. few mode cavity with position dependent atom couplings; with 100 atoms this is outside our ability to directly simulate and would be new territory

Towards models dual to black holes?

- Kitaev, building on work of Sachdev-Ye, proposed a 4-fermion model of a black hole

$$H_{SYK} = \sum_{ijkl} J_{ijkl} \chi_i \chi_j \chi_k \chi_l$$

[Kitaev ‘15, Sachdev-Ye ‘93]

- This model originates in the quantum spin glass literature, e.g. random long-range quantum spin models as in the cavity setup:

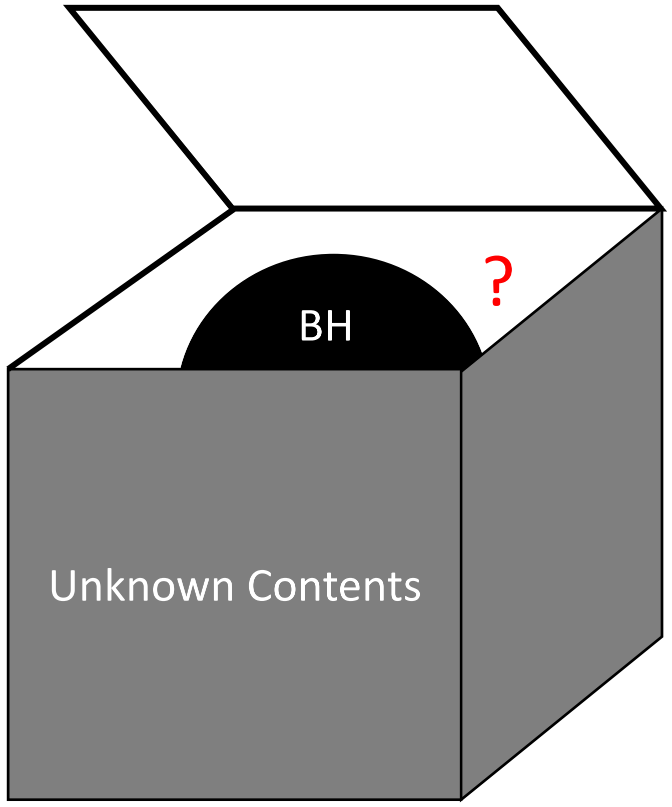
$$H = \sum_{i,j,\alpha} \frac{\Omega_\uparrow \Omega_\downarrow}{\Delta_\uparrow \Delta_\downarrow} \frac{g_\alpha(r_i) g_\alpha(r_j)}{\delta} s_{x,i} s_{x,j}$$

+ help avoiding glassy physics from external drive? random circuit models?

Connection: write spin operator in terms of fermions + constraint, Hubbard-Stratonovich transformation gives fermions + gauge field; gauge field is quenched at large N

A test for black hole horizons?

- Given a complex quantum many-body system sitting in your lab, what is a good way to tell if it has a black hole in its dual description?
- Using the experimental protocols described in this talk, out-of-time-ordered correlators offer a good probe: prepare a thermal-like state, measure the Lyapunov exponent (it is universal, requires only relatively early times), if the result hits the MSS bound then perhaps you have a black hole horizon



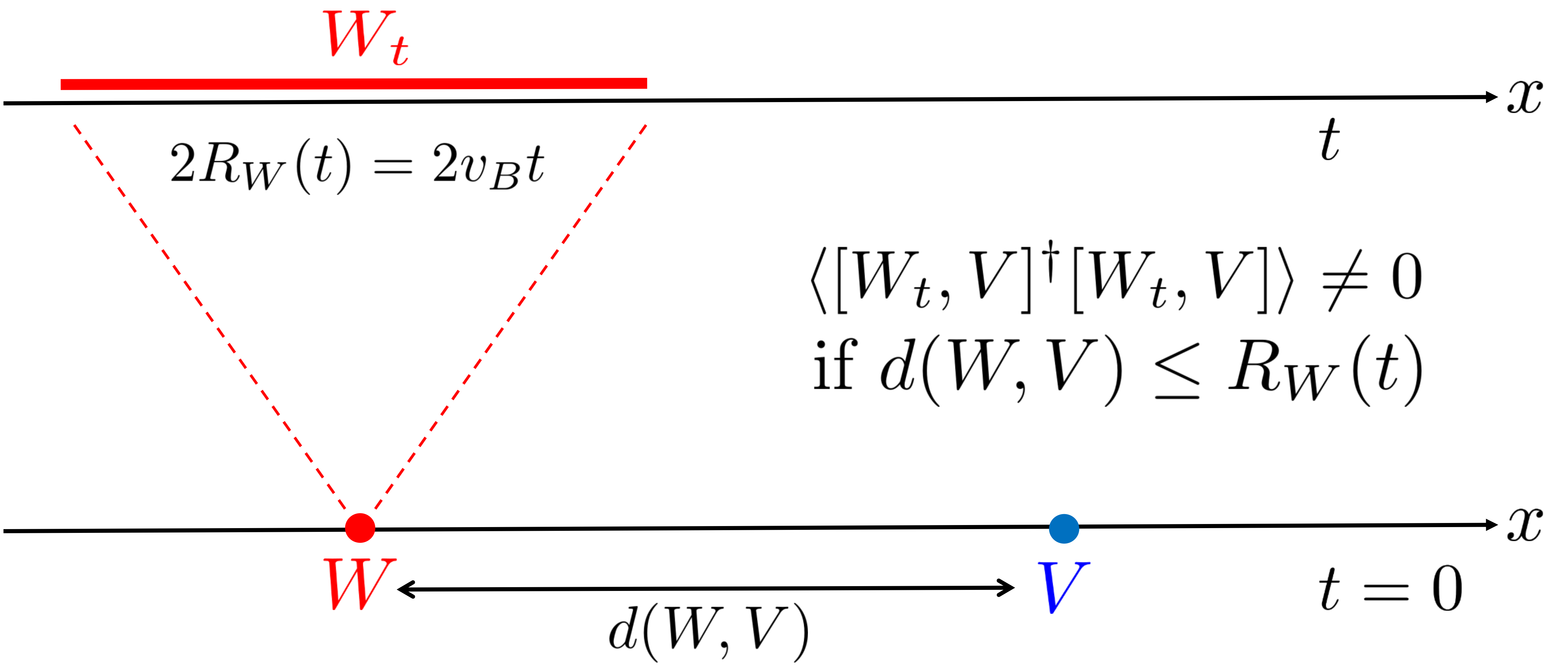
Local models: Hubbard model

$$H = -w \sum_{rr'} b_r^\dagger b_{r'} + U \sum_r b_r^\dagger b_r (b_r^\dagger b_r - 1)$$

- Time can also be reversed in optical lattice implementations of the Hubbard model: Feshbach interaction → sign of interaction; lattice modulation → sign of hopping
- Control operations using local impurities
- All components have been demonstrated in experiments

  [lattice modulation Struck et al PRL ’12, Struck et al Science ‘11, Aidelsburger et al PRL ’11; impurities Knap et al PRX ’12, Cetina et al PRL ‘15]

Butterfly velocity [Shenker-Stanford ‘13, Roberts-Stanford-Susskind ‘15]



Butterfly velocity in non-conformal theories

- To get some handle on non-conformal theories, can consider as an effective theory Einstein-Maxwell-Dilaton gravity

  [Iizuka et al '11, Gouteraux et al '12, Ogawa et al '12, BGS et al '12, Dong et al '12, ...]

- A class of solutions characterized by two thermodynamic exponents, a dynamical exponent z and a hyperscaling violation exponent θ

$$\xi(T) \sim T^{-1/z} \qquad S_T \sim T^{\frac{d-\theta}{z}}$$

Example: a Fermi gas
z=1, $\theta = 1$

- Compute using shockwaves, IR version of Lieb-Robinson velocity

$$v_B = \left(\frac{T}{T_0}\right)^{\frac{z-1}{z}} \sqrt{\frac{d+z-\theta}{2(d-\theta)}}$$

[BGS-Roberts '16, Blake '16]

Summary

- Scrambling and out-of-time-order correlators are broadly interesting, lots of calculations to perform and models to explore
- Possible to measure out-of-time-order correlators using a general protocol; detailed proposal in cavity experiment; ingredients available in a wide variety of systems – Rydberg atoms, superconducting circuits, trapped ions, optical lattices, …
- Learn about bounds on quantum dynamics; Clear open problems, both theoretically and experimentally, of increasing complexity; Long term goal is detection of black hole horizons in table top experiments