

Soft pomeron in Holographic QCD

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arXiv:1508.00008 (with A. Bayona, M. Djuric, R. Quevedo)

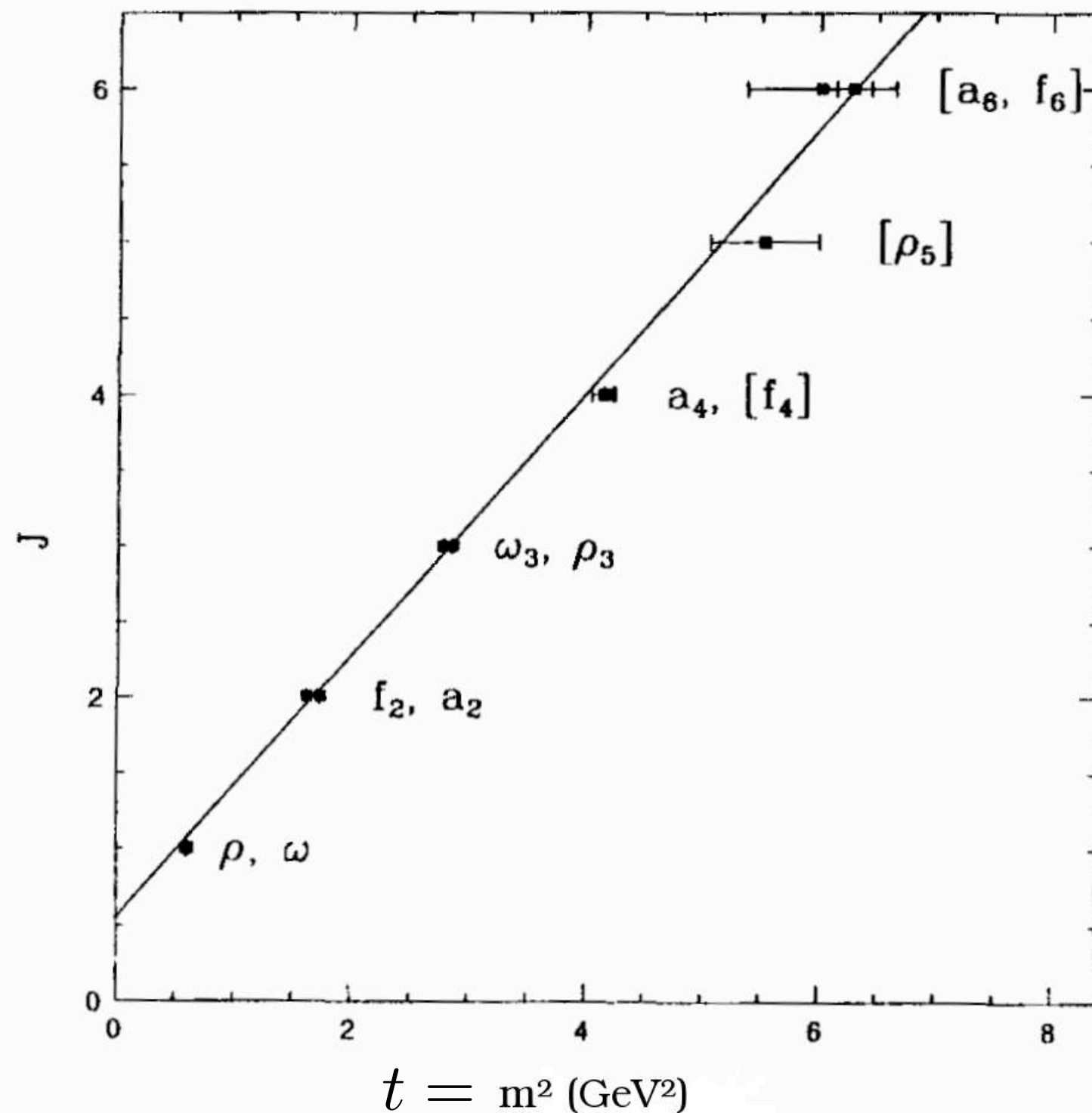
Current Themes in Holography:
Exact results, applications, extensions and fundamentals

Niels Bohr Institute, Copenhagen, April 25-29, 2016

Regge behaviour in QCD

- Hadronic resonances fall in linear trajectories

$I = 1$ even parity mesons

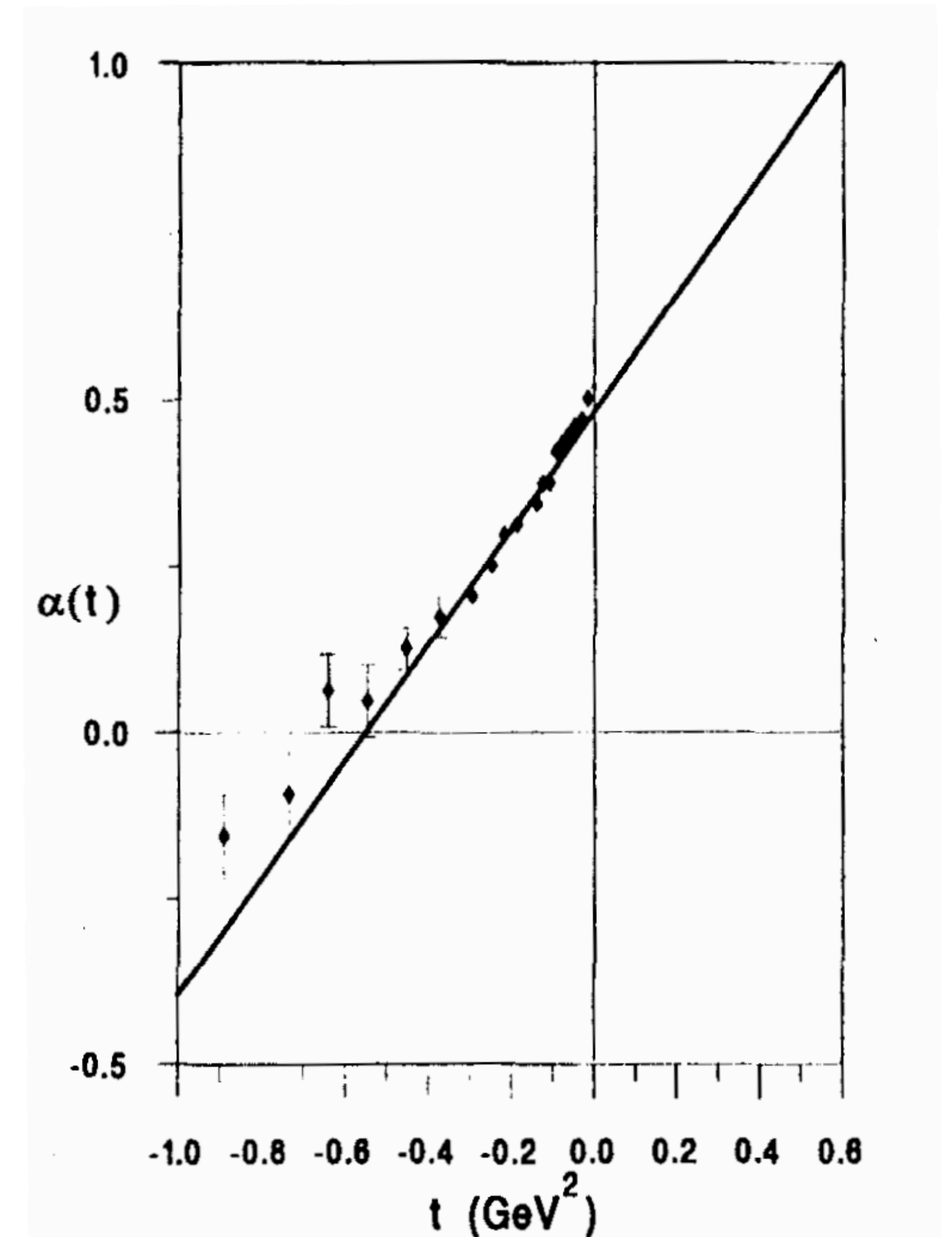
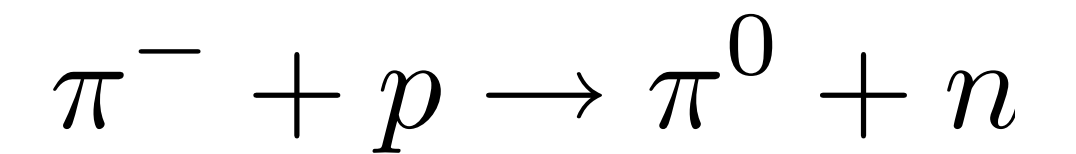


$$J = j(t) = j(0) + \alpha' t$$

$$A(s, t) \sim \beta(t) s^{j(t)} \quad (s \gg t)$$

Total cross section

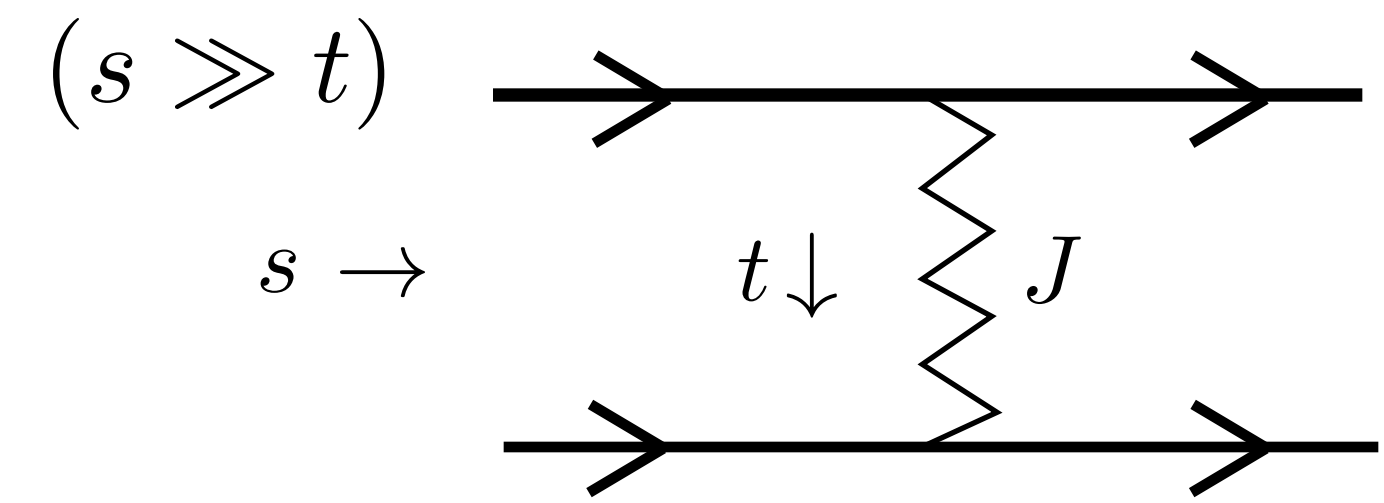
$$\sigma \sim s^{j(0)-1}$$



Regge theory

- Scattering dominated by t-channel exchange of a Regge trajectory

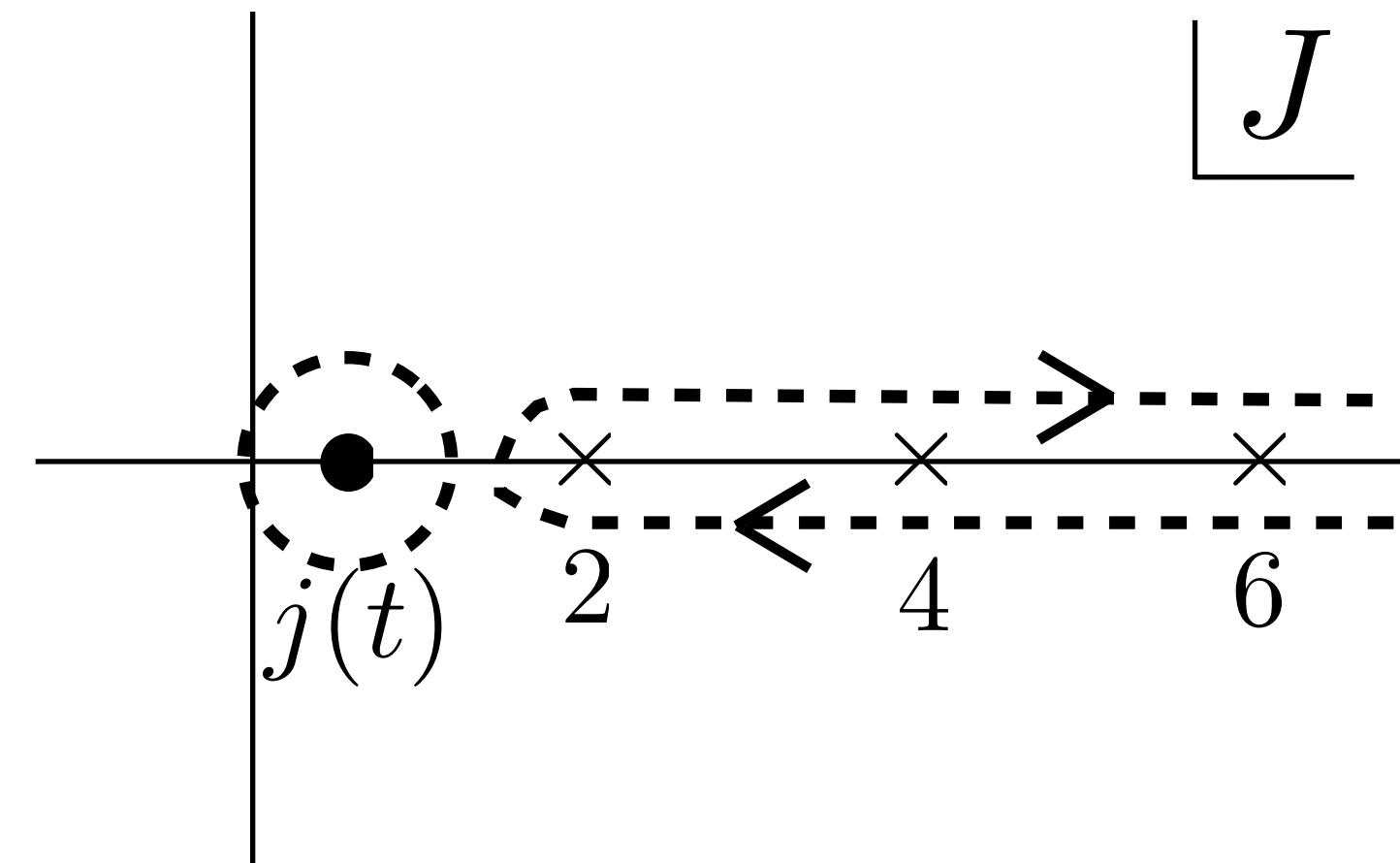
$$A(s, t) \approx \sum_J g_J \frac{s^J}{t - m^2(J)} \sim \sum_J g_J \frac{s^J}{J - j(t)}$$



- Sommerfeld-Watson transform:

$$\sum_J \rightarrow \int \frac{dJ}{\sin \pi J}$$

$$A(s, t) \sim \beta(t) s^{j(t)}$$



Soft Pomeron trajectory [Donnachie, Landshoff]

- Trajectory selected by exchanged quantum numbers. For elastic scattering these are the vacuum quantum numbers.

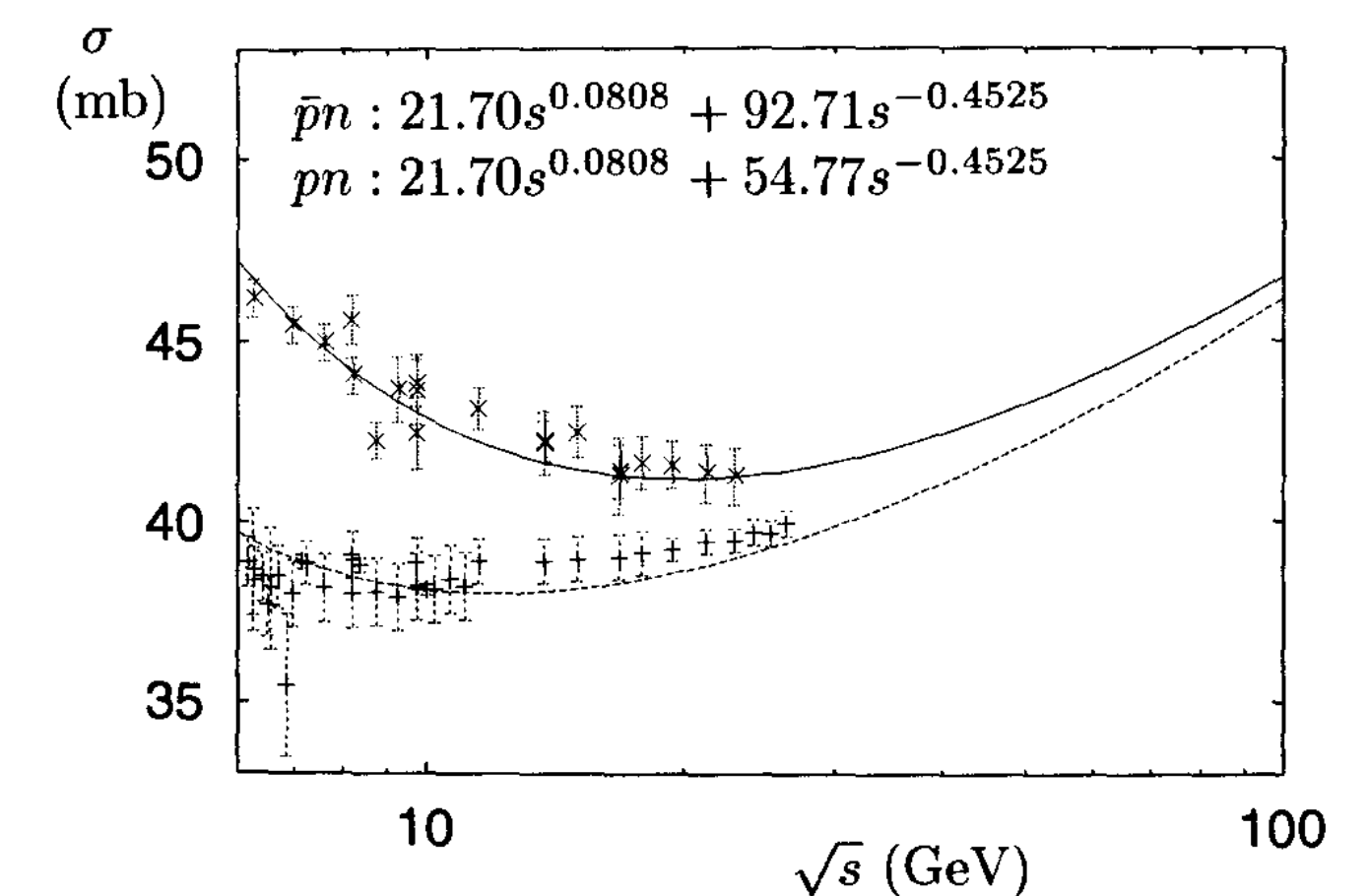
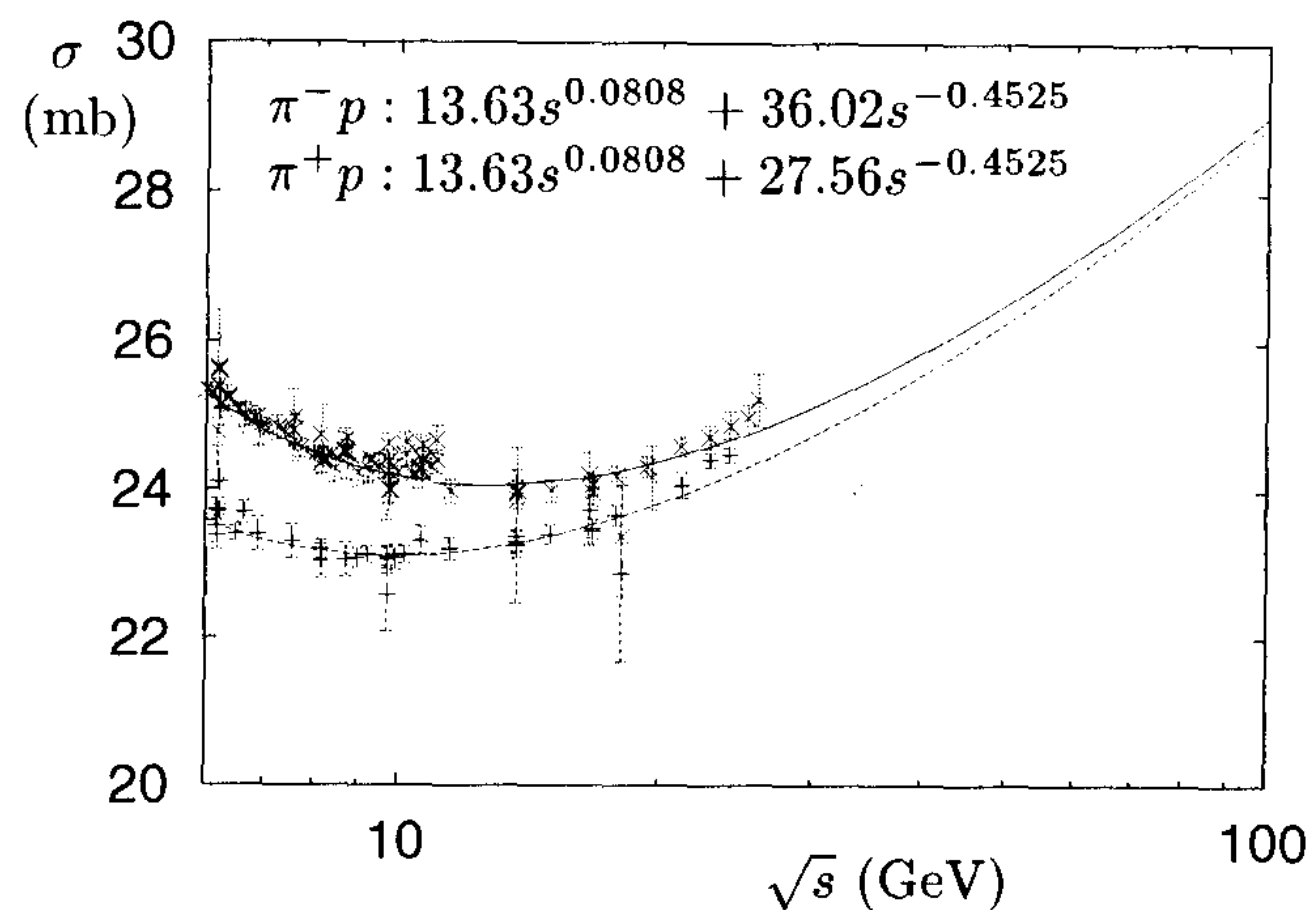
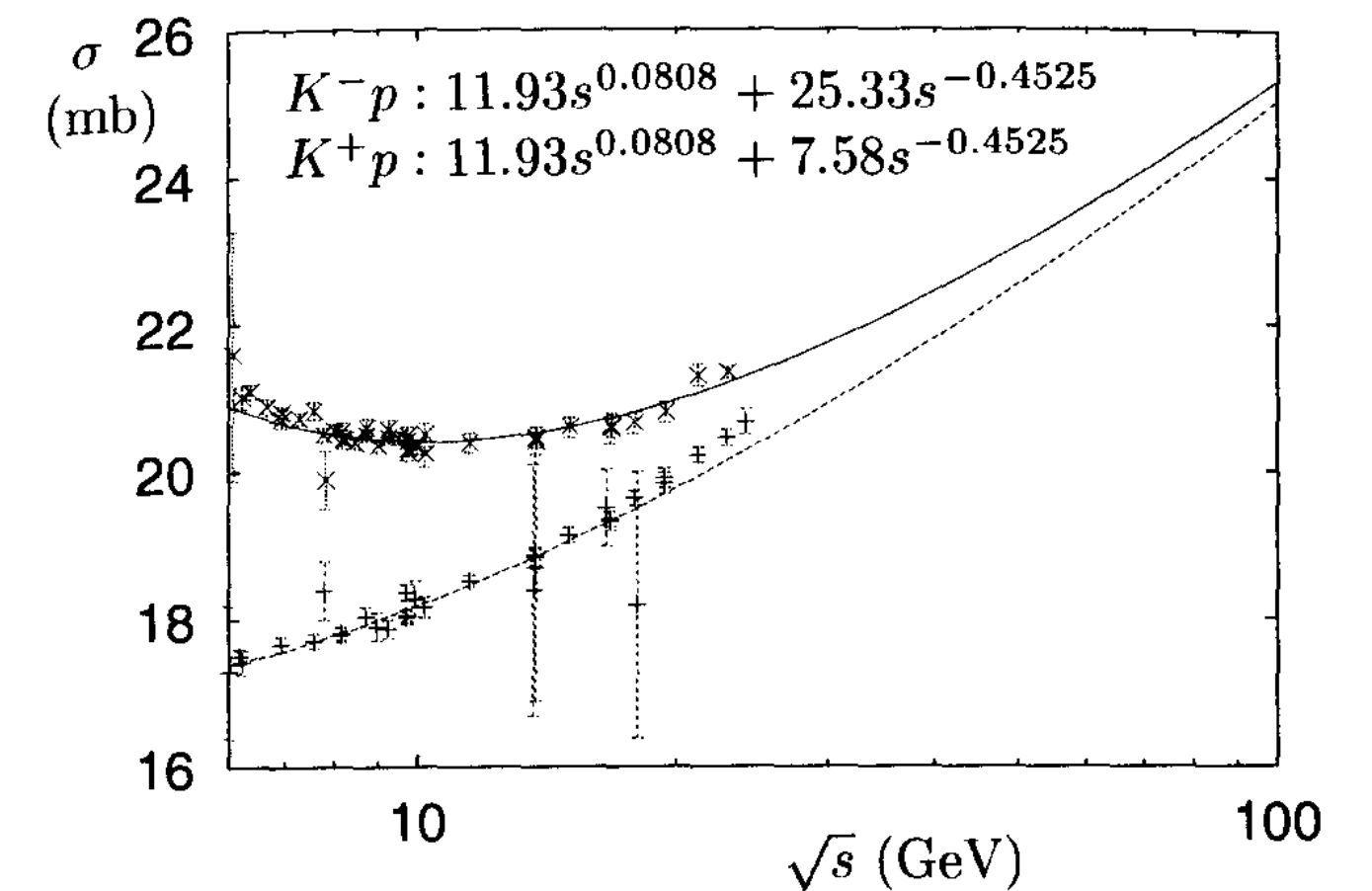
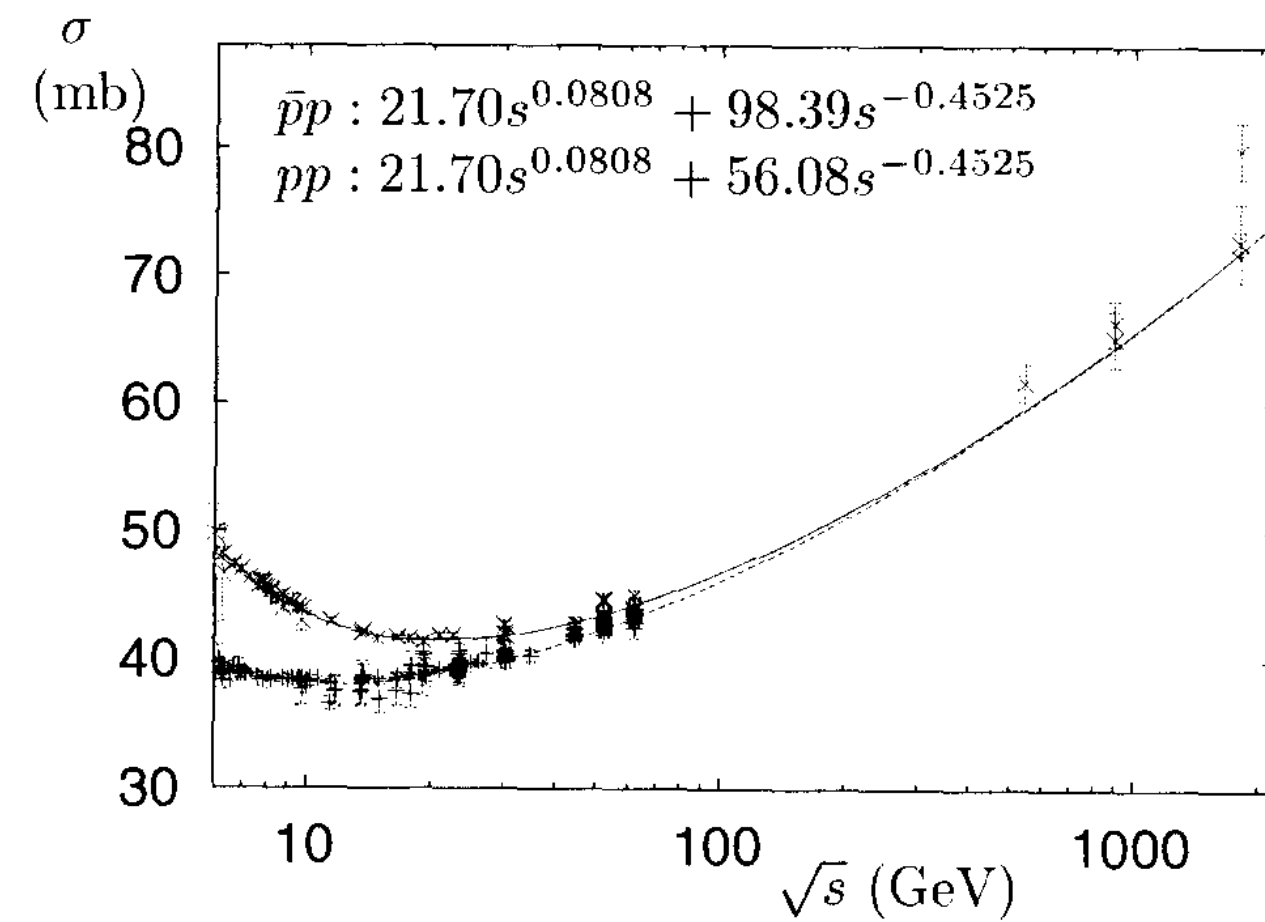
$$j_P(t) \approx 1.08 + 0.25t \quad (\text{GeV units})$$

Total elastic cross sections

$$\sigma \sim s^{j_P(0)-1} \sim s^{0.08}$$

Exchange of even spin glueballs ($J \geq 2$)

$$\mathcal{O}_J \sim \text{Tr} \left(F_{\alpha[\beta_1} D_{\beta_2} \dots D_{\beta_{J-1}} F_{\beta_J]}^{\alpha} \right)$$



Deep Inelastic Scattering

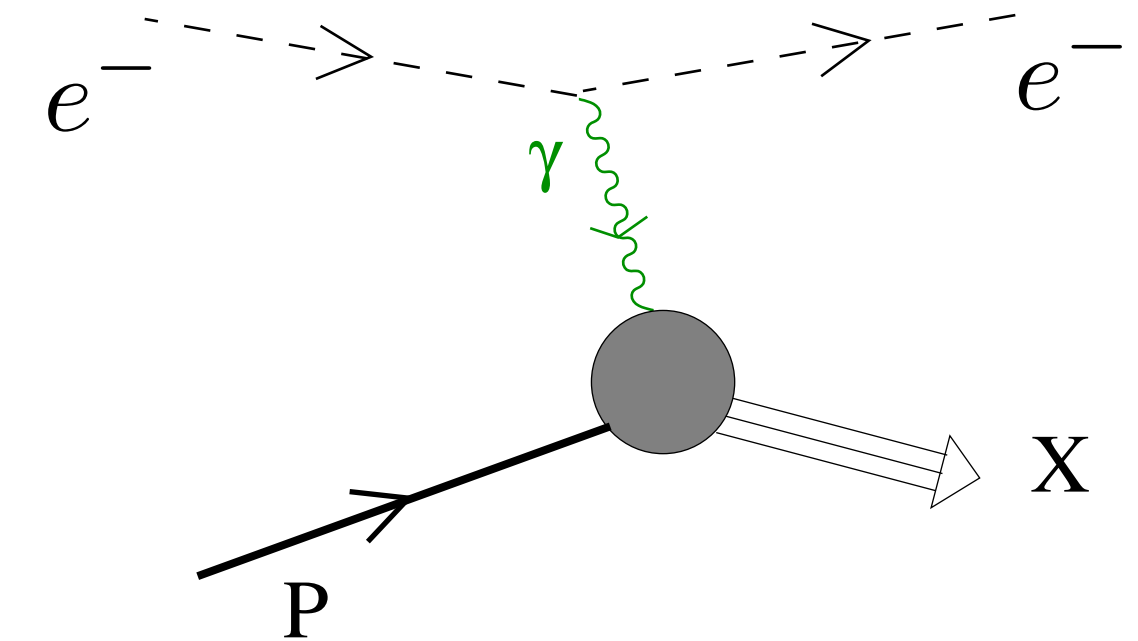
- Pomeron enters also in diffractive processes. For example DIS, where electron interacts with proton via exchange of off-shell photon
- Optical theorem

$$\sum_X \left| \begin{array}{c} \gamma \\ \nearrow P \\ \bullet \\ \longrightarrow X \end{array} \right|^2 = \text{Im}_{(t=0)} \begin{array}{c} \gamma \quad \gamma \\ \nearrow P \quad \searrow P \\ \bullet \end{array}$$

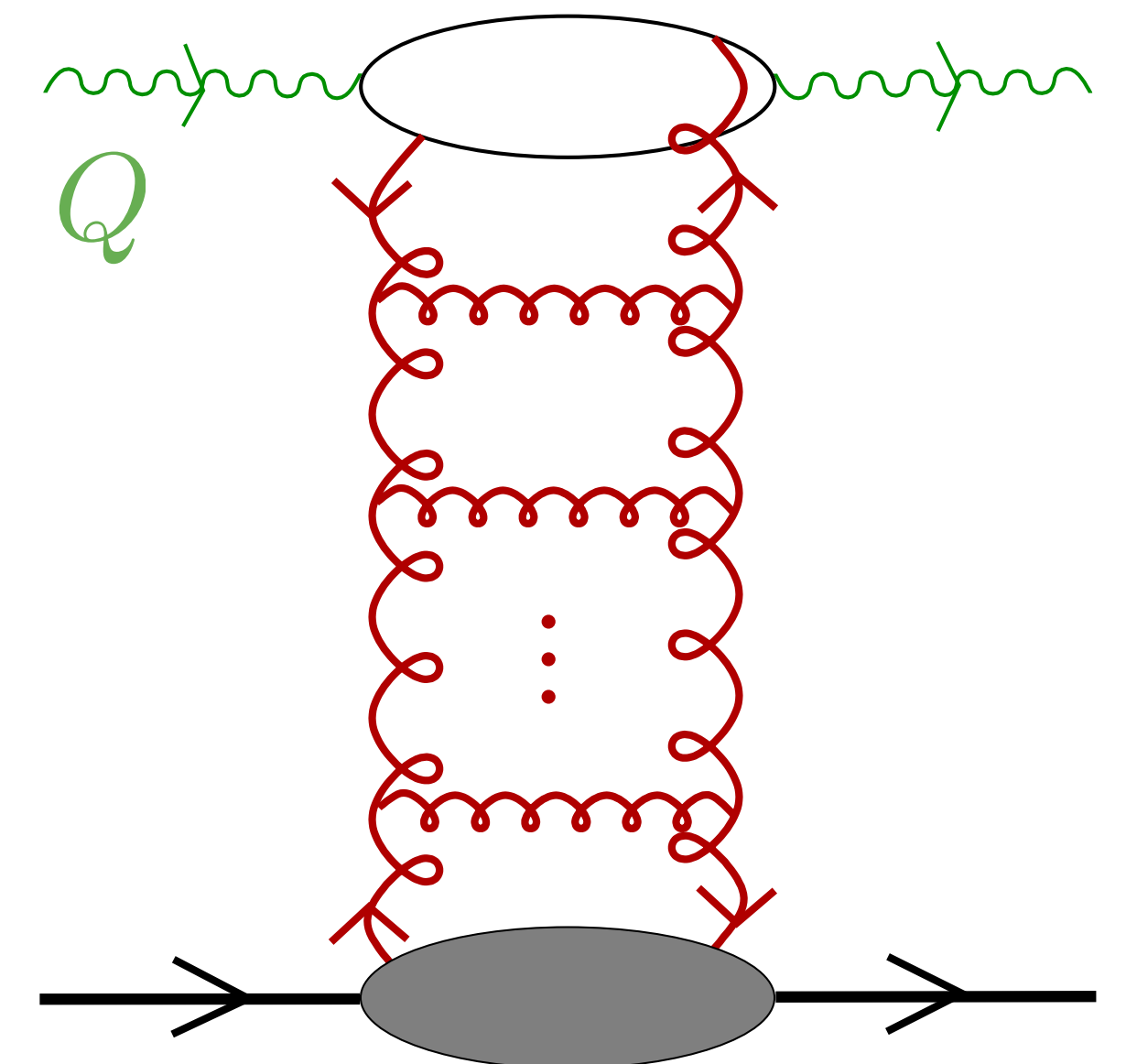
- Regge limit corresponds to low x ($s \sim Q^2/x$)

BFKL pomeron explains well DIS data outside the confining region $Q \sim \Lambda_{QCD}$
[Kowalski, Lipatov, Ross, Watt 10]

**Is it the same Regge trajectory?
One or two pomerons (soft and hard)?**



$$j_0 \sim 1.1 - 1.4$$



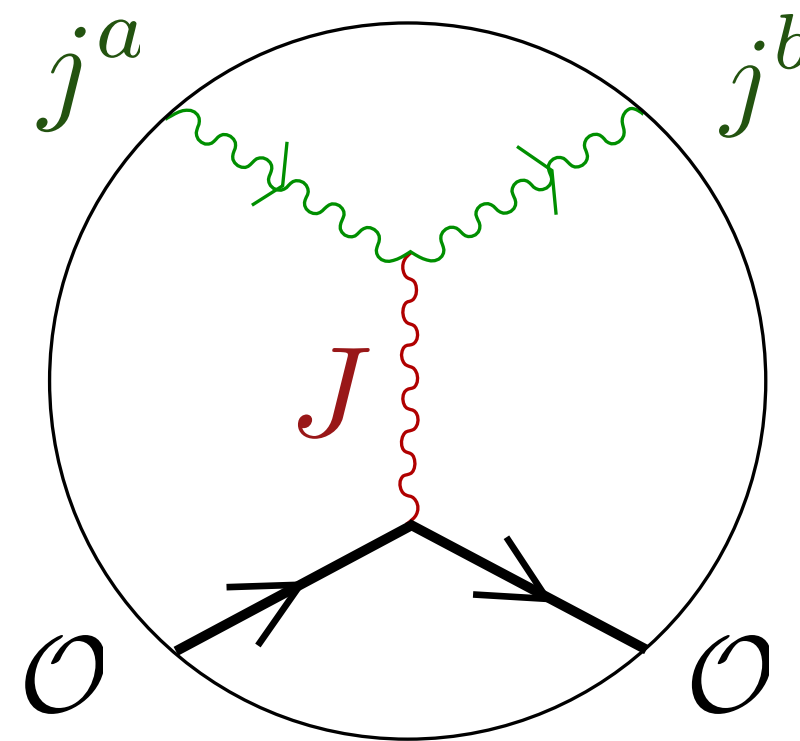
Graviton/Pomeron Regge trajectory [Brower, Polchinski, Strassler, Tan 06]

- At strong coupling pomeron trajectory described by graviton Regge trajectory of string theory in Anti-de Sitter space (large N, conformal theory $\mathcal{N} = 4$ SYM)

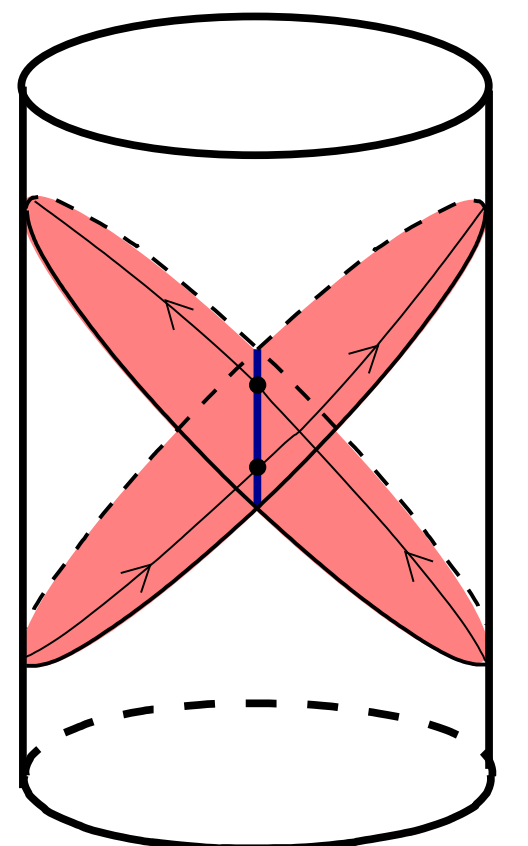
Exchange of spin J field in AdS
(symmetric, traceless and transverse)

$$(D^2 - m^2) h_{a_1 \dots a_J} = 0$$

$$\text{with } m^2 = \Delta(\Delta - 4) - J$$



AdS scattering process



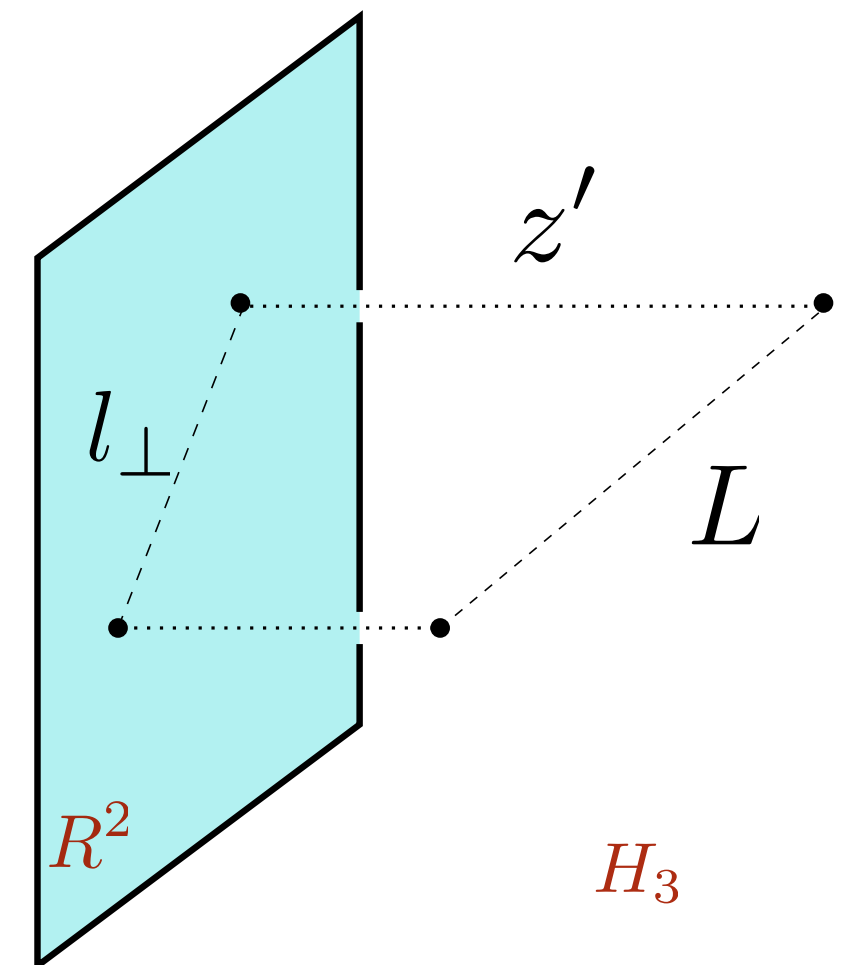
- AdS impact parameter representation. In Regge limit

[Cornalba, MSC, Penedones, Schiappa 07]

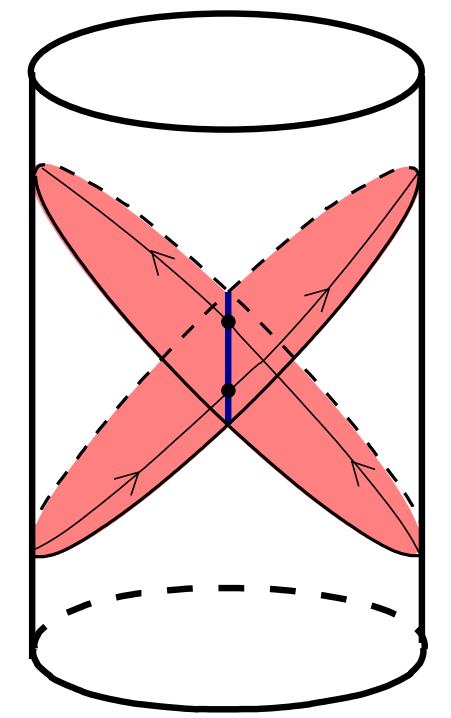
$$A_J(s, t) \approx iV \kappa_J \kappa'_J s \int dl_{\perp} e^{iq_{\perp} \cdot l_{\perp}} \int \frac{dz}{z^3} \frac{dz'}{z'^3} \Phi_1(z) \Phi_3(z) \Phi_2(z') \Phi_4(z') S^{J-1} G_J(L)$$

$S = zz' s$, AdS energy squared

$$\cosh L = \frac{z^2 + z'^2 + l_{\perp}^2}{2zz'} , \text{ impact parameter}$$



$$A_J(s, t) \approx iV \kappa_J \kappa'_J s \int dl_\perp e^{iq_\perp \cdot l_\perp} \int \frac{dz}{z^3} \frac{dz'}{z'^3} \Phi_1(z) \Phi_3(z) \Phi_2(z') \Phi_4(z) S^{J-1} G_J(L)$$

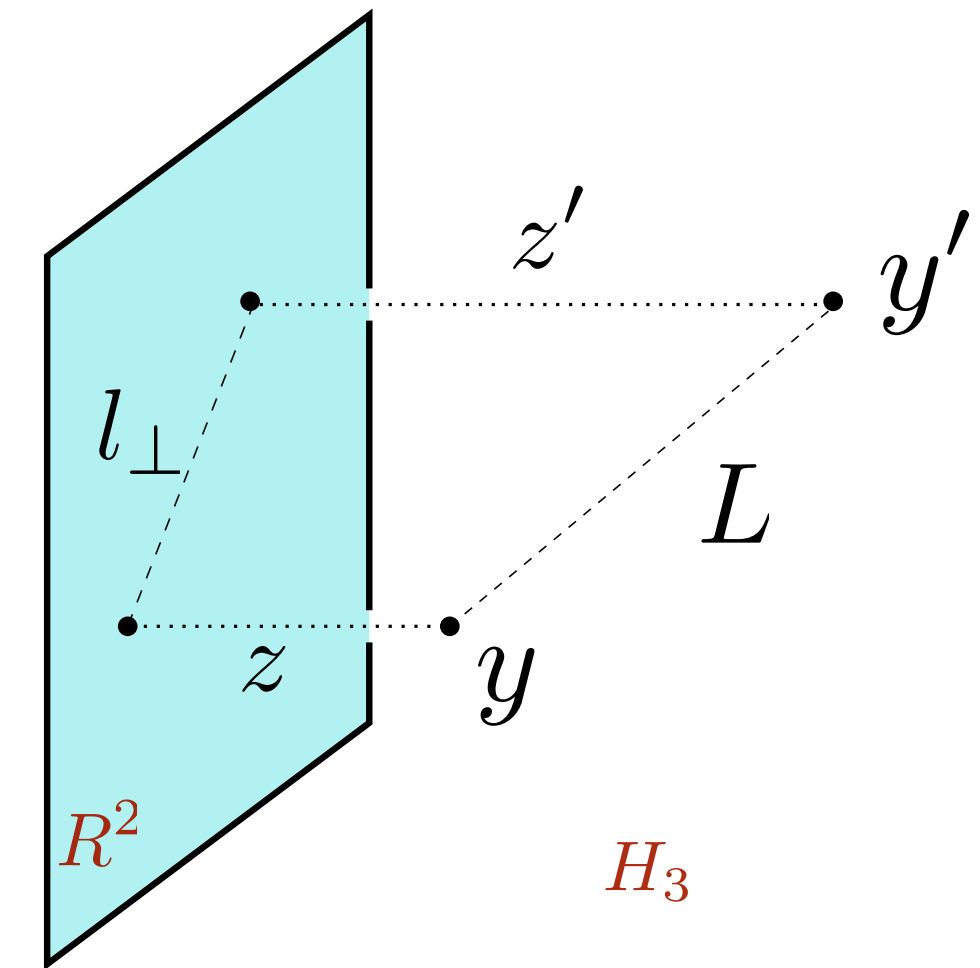


- $G_J(L)$ is the integrated propagator ($w = x - x' = (w^+, w^-, l_\perp)$)

$$G_J(L) \sim i (zz')^{(J-1)} \int dw^+ dw^- \Pi_{+ \dots +, - \dots -}(z, z', w)$$

and obeys scalar propagator equation in transverse space

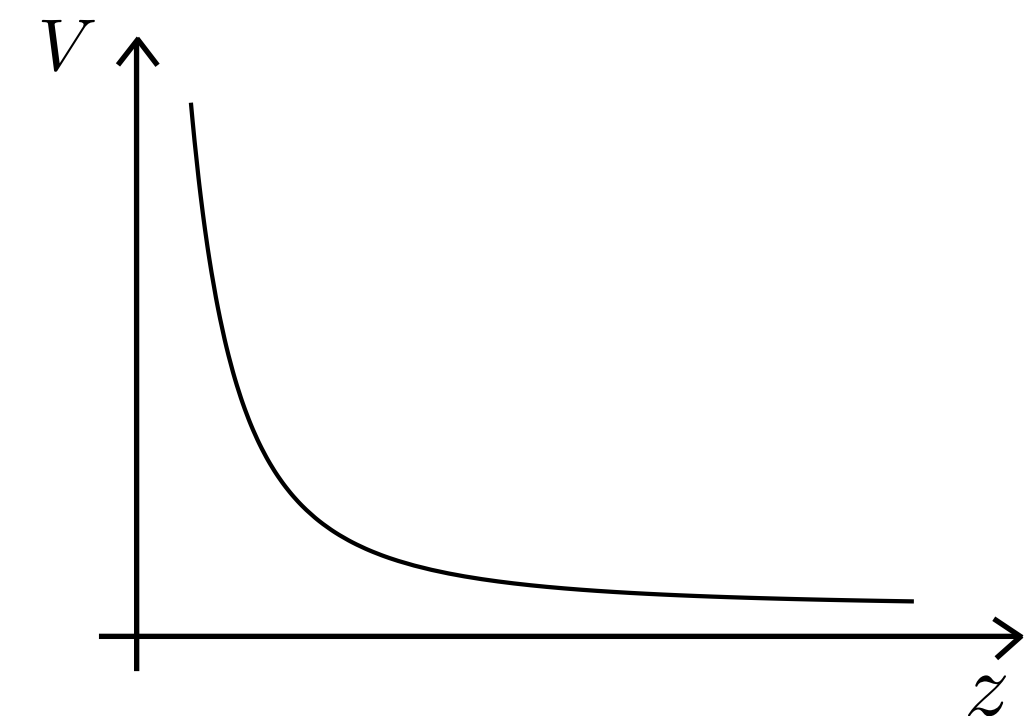
$$\left[\square_{H_3} - 3 - \Delta(\Delta - 4) \right] G_J(L) = -\delta_{H_3}(y, y')$$



$G_J(L) = e^{iq_\perp \cdot l_\perp} \sqrt{z} \psi(z)$, reduces to Schrodinger problem

$$\left(-\frac{d}{dz^2} + V(z) \right) = t \psi(z), \text{ with } V = \left(\frac{15}{4} + \Delta(\Delta - 4) \right) \frac{1}{z^2}$$

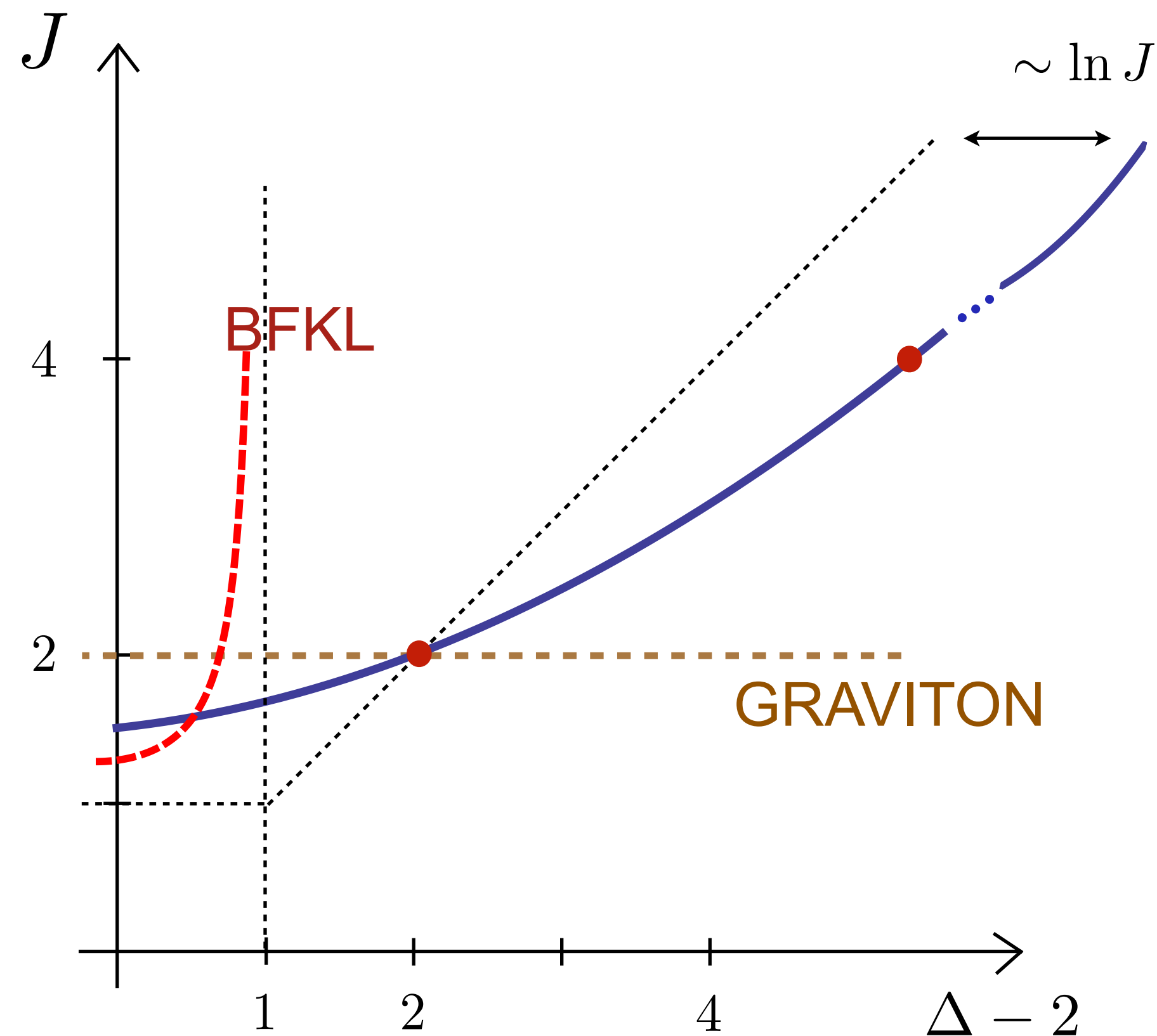
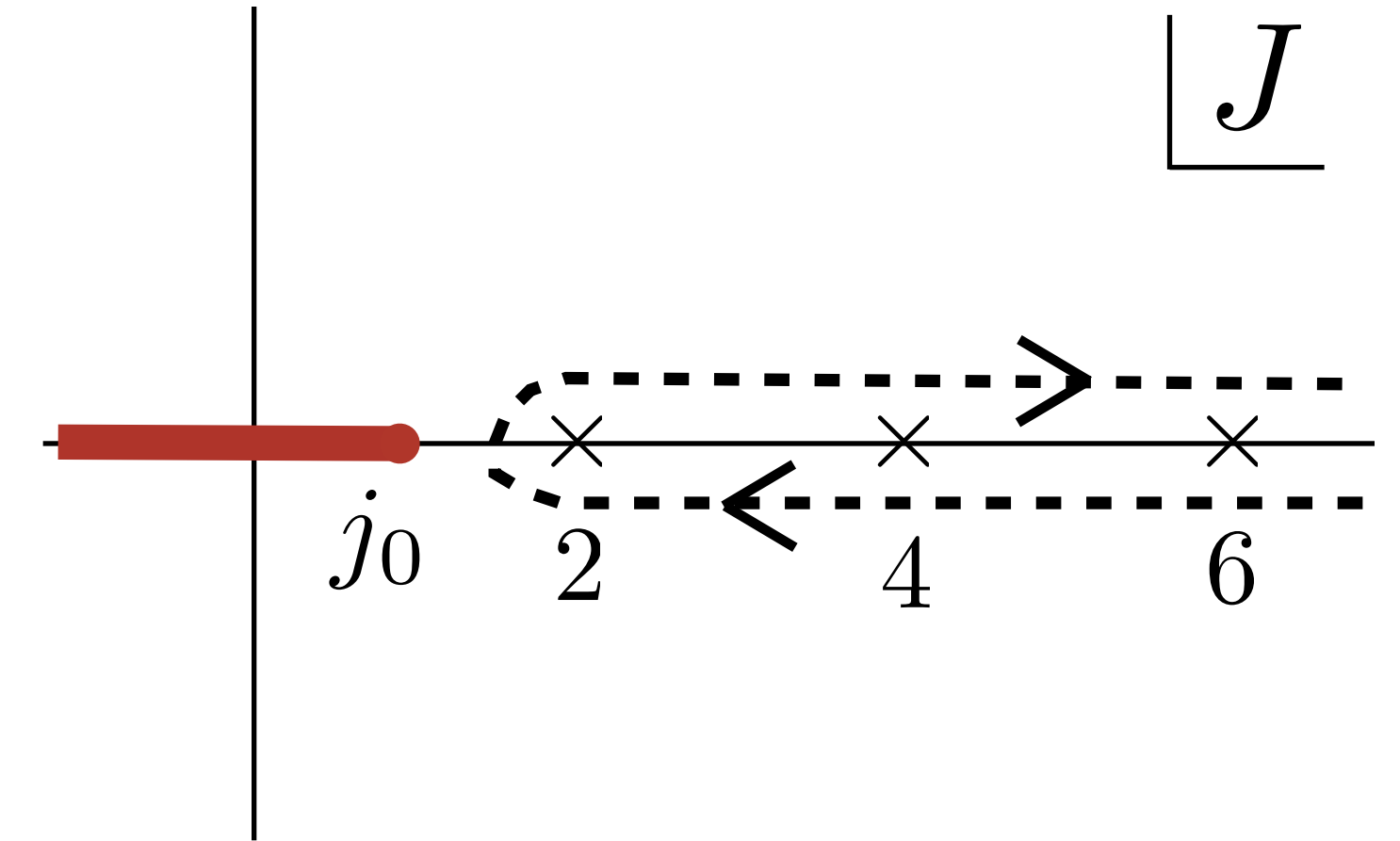
$$\Delta = \Delta(J)$$



- Sommerfeld-Watson transform $\sum_J \rightarrow \int \frac{dJ}{\sin \pi J}$

Operators that contribute have twist 2 and even spin

$$\mathcal{O}_J \sim \text{Tr} (F_{\alpha\beta_1} D_{\beta_2} \dots D_{\beta_{J-1}} F_{\beta_J}{}^\alpha)$$



At strong coupling, $\sqrt{\lambda} = \frac{1}{\alpha'} \gg 1$, we have

$$\Delta(\Delta - 4) \approx \frac{2}{\alpha'} (J - 2)$$

$$G_J(z, q_\perp) = \sqrt{zz'} I_\alpha(q_\perp z) K_\alpha(q_\perp z')$$

$$\alpha = \sqrt{4 + \Delta(\Delta - 4)}$$

Branch cut starts for $J < j_0 = 2 - \frac{2}{\sqrt{\lambda}}$

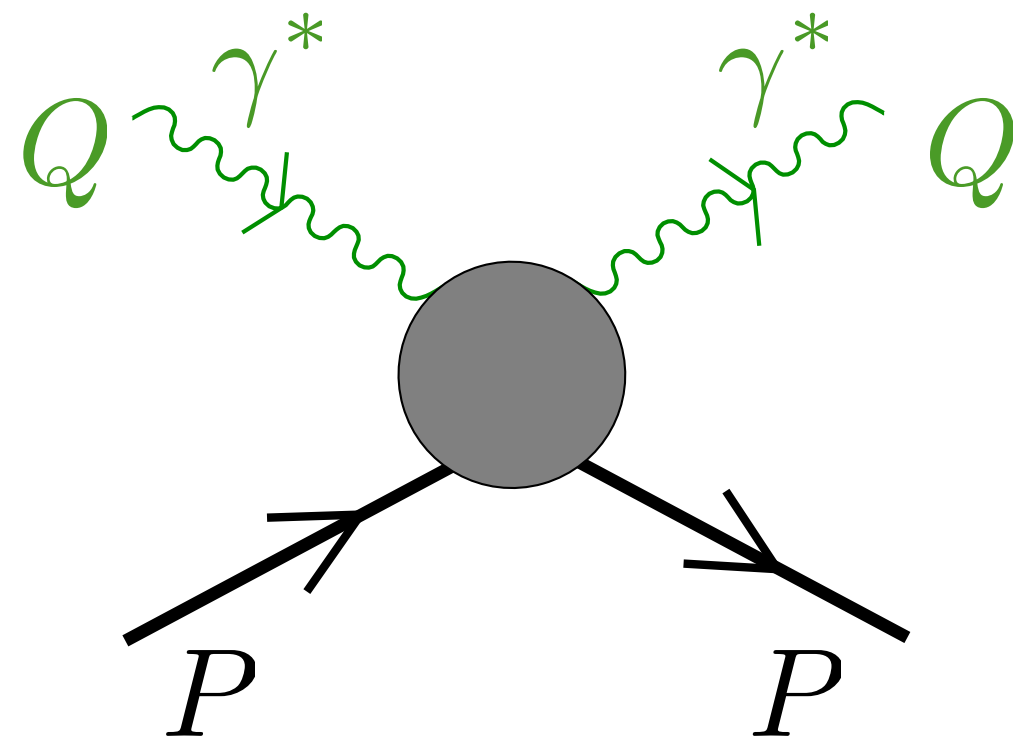
Applications to low-x QCD

- Fits low x data for DIS, DVCS, VMP including confining region $Q \sim \Lambda_{QCD}$

DIS [Cornalba, MSC 08; Levin, Potashnikova 10; Brower, Djuric, Sarcevic, Tan 10];

DVSC [MSC, Djuric 12]; VMP [MSC, Djuric, Evans 13]

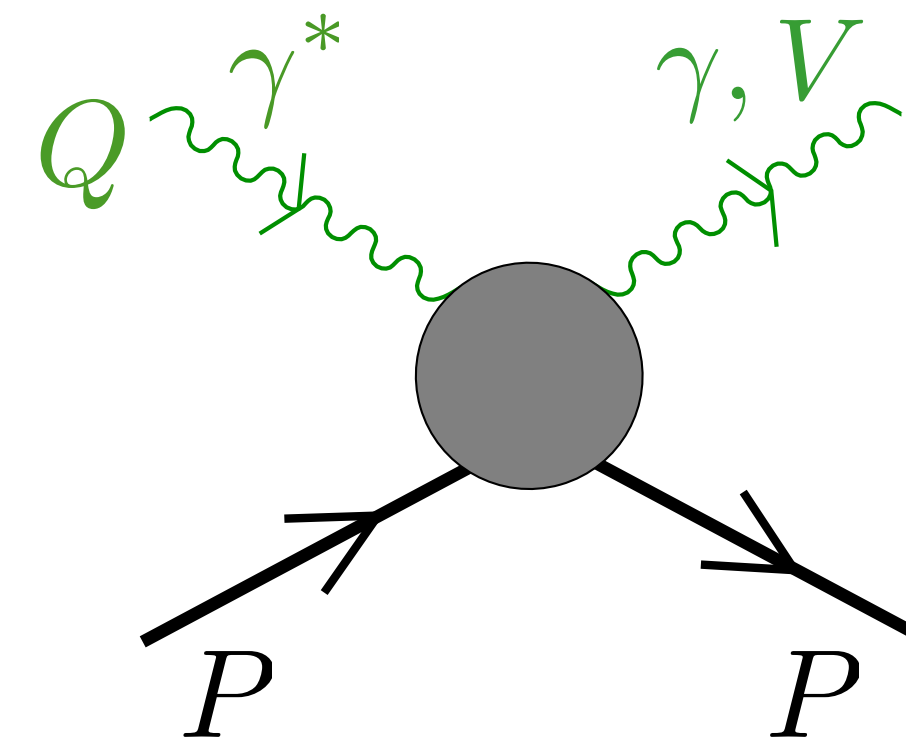
- DIS



$$\sigma(Q, x) \propto \text{Im } W \quad \rightarrow \quad F_2$$

$(t = 0)$

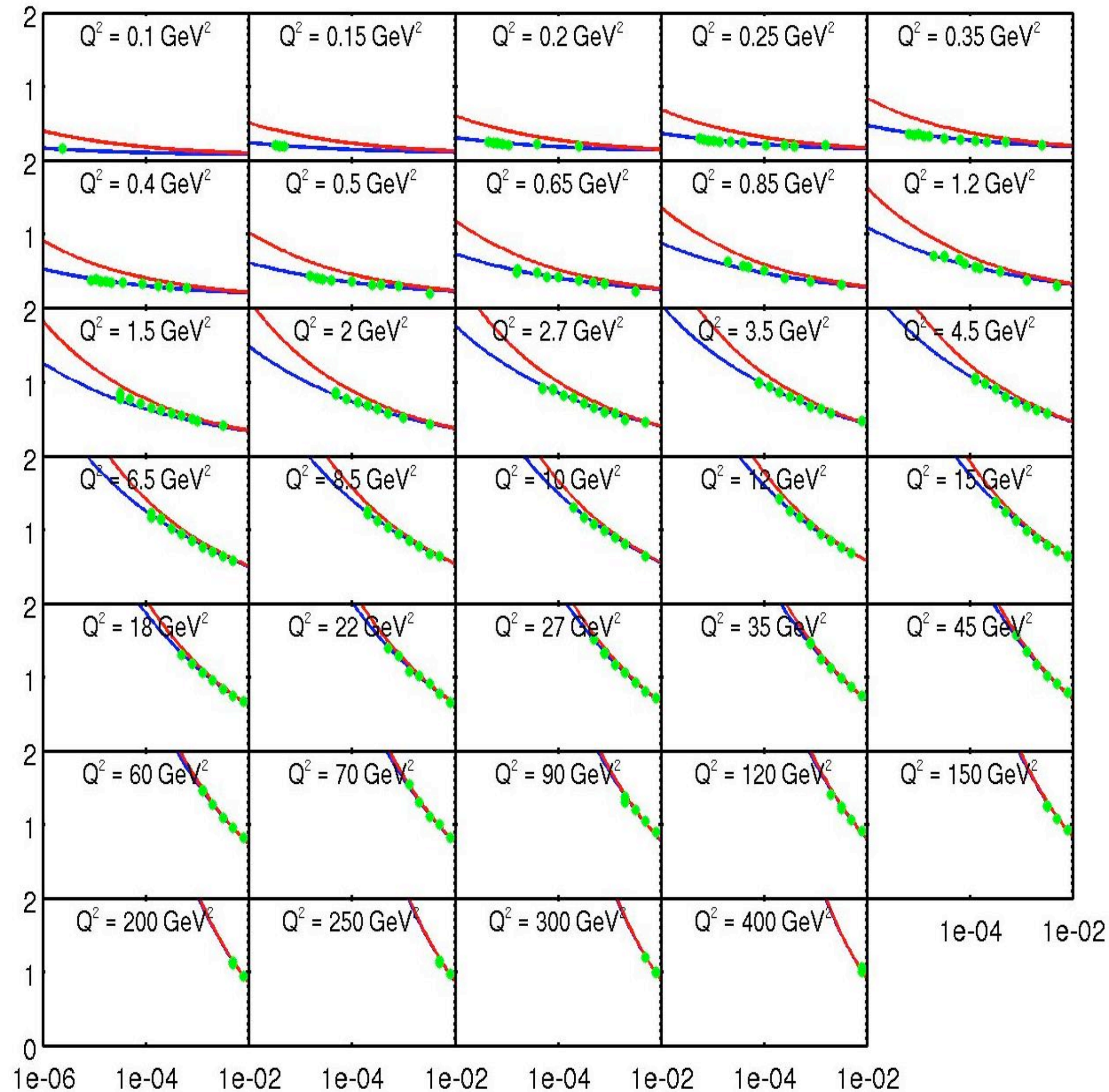
- DVCS & VMP



$$\frac{d\sigma}{dt}(Q, x, t) \propto |W|^2 \quad \text{and} \quad \sigma_{tot}(Q, x)$$

- Fits for hard-wall model have 4 parameters j_0, κ^2, z_*, z_0

DIS - AdS Pomeron [Brower, Djuric, Sarcevic, Tan 10]



HERA combined data by H1 and ZEUS experiments [Aaron et al 10] with

$$0.10 < Q^2 < 400 \text{ GeV}^2, x < 10^{-2}$$

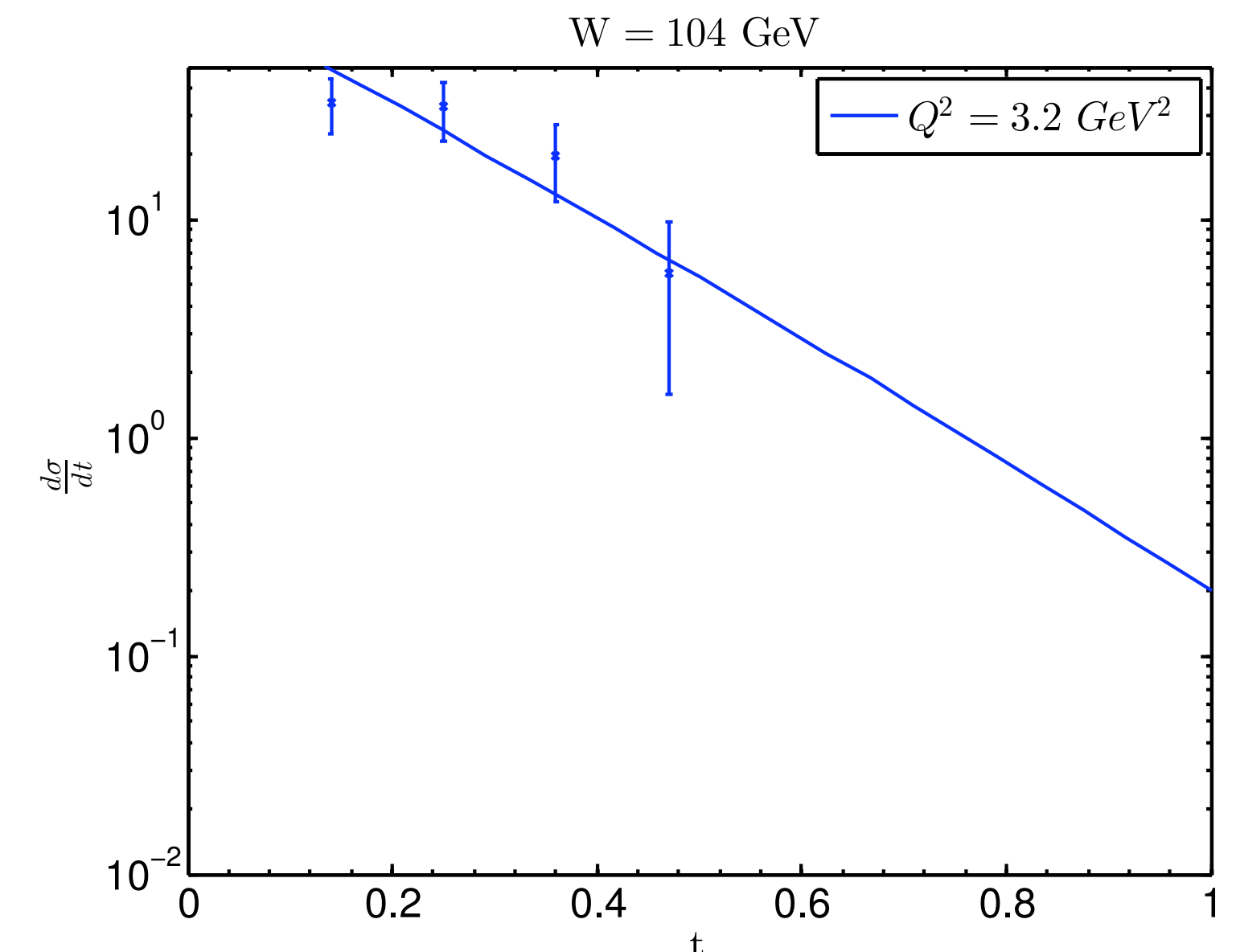
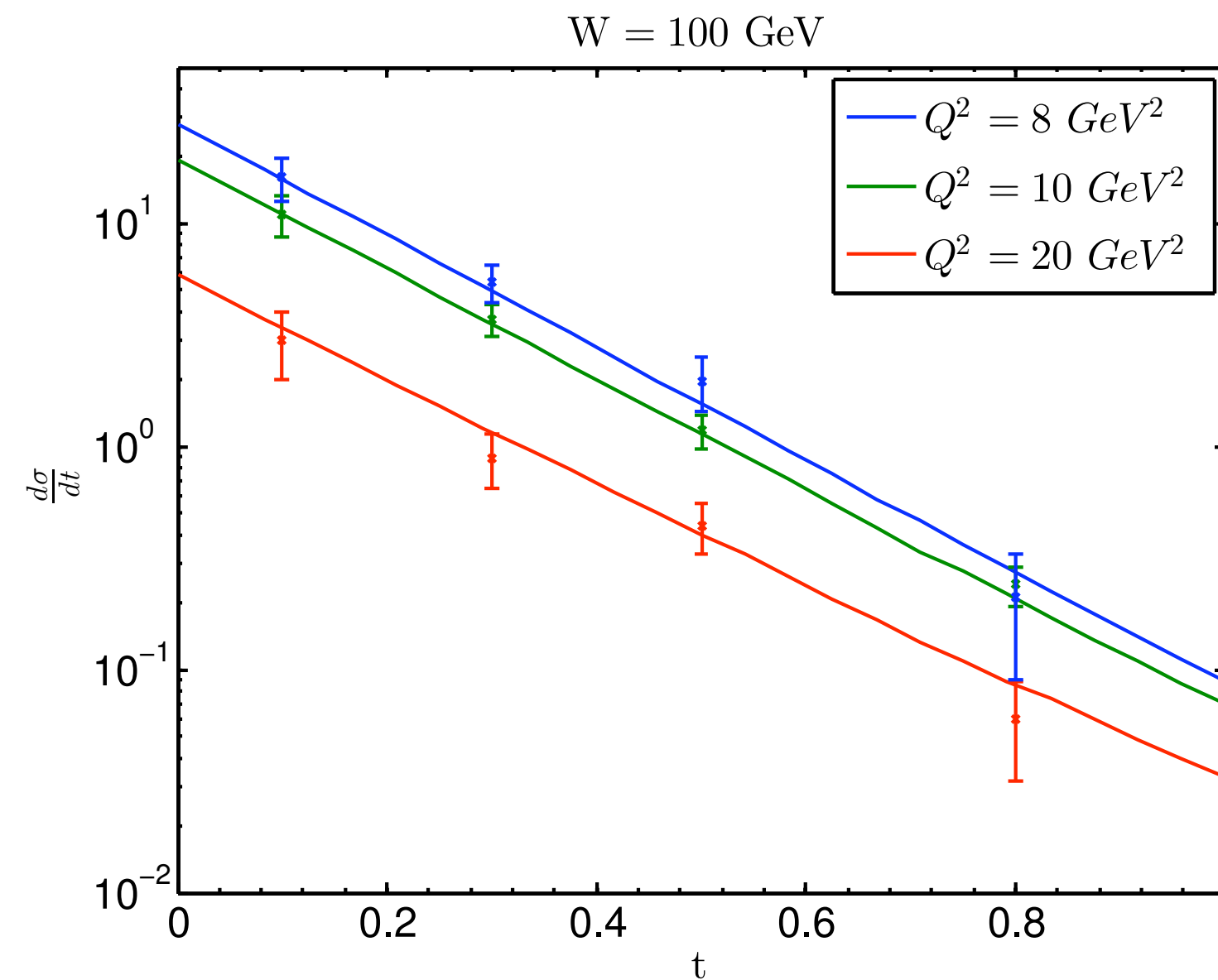
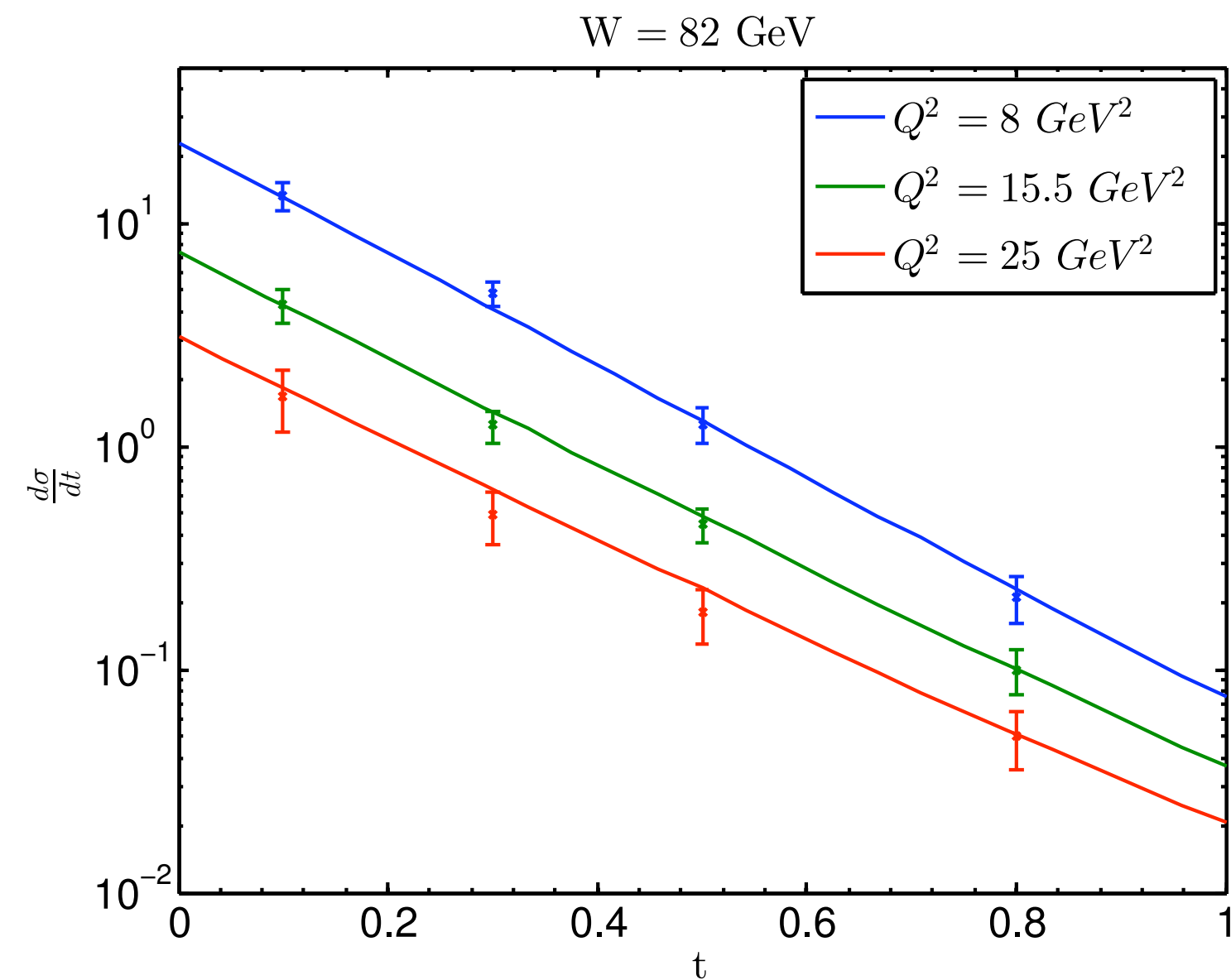
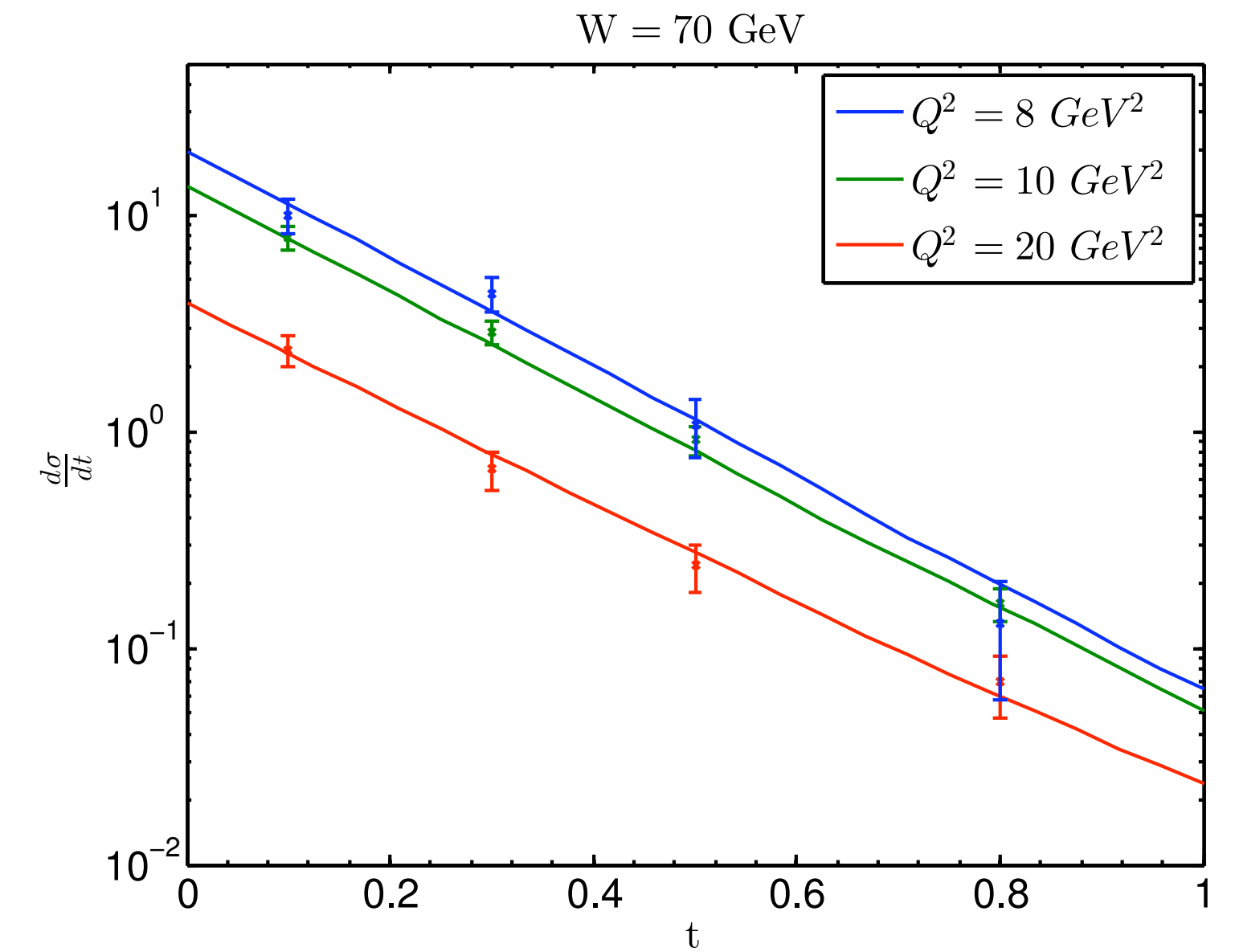
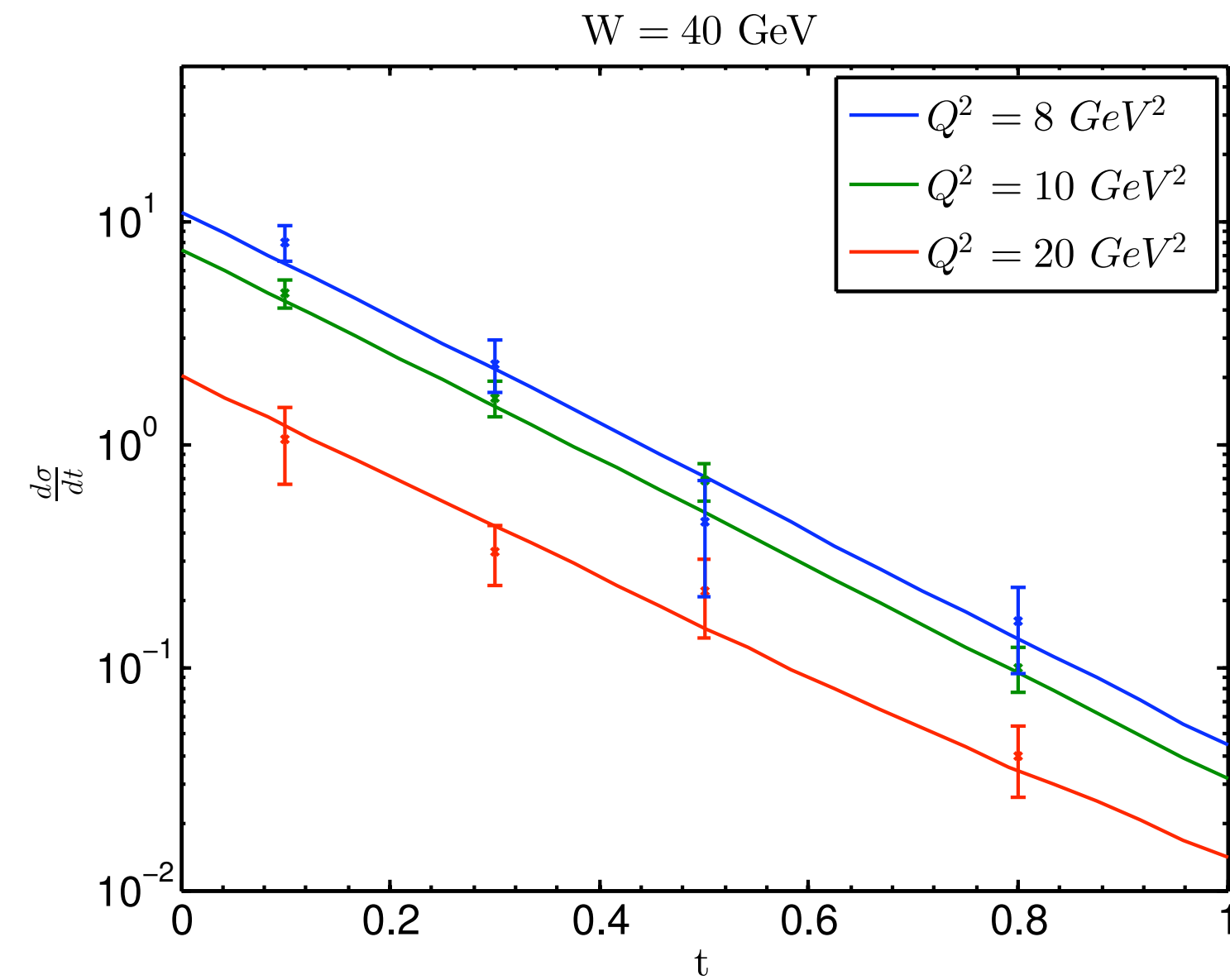
For hard wall model obtained excellent fit with (249 points)

$$\chi^2_{d.o.f.} = 1.07$$

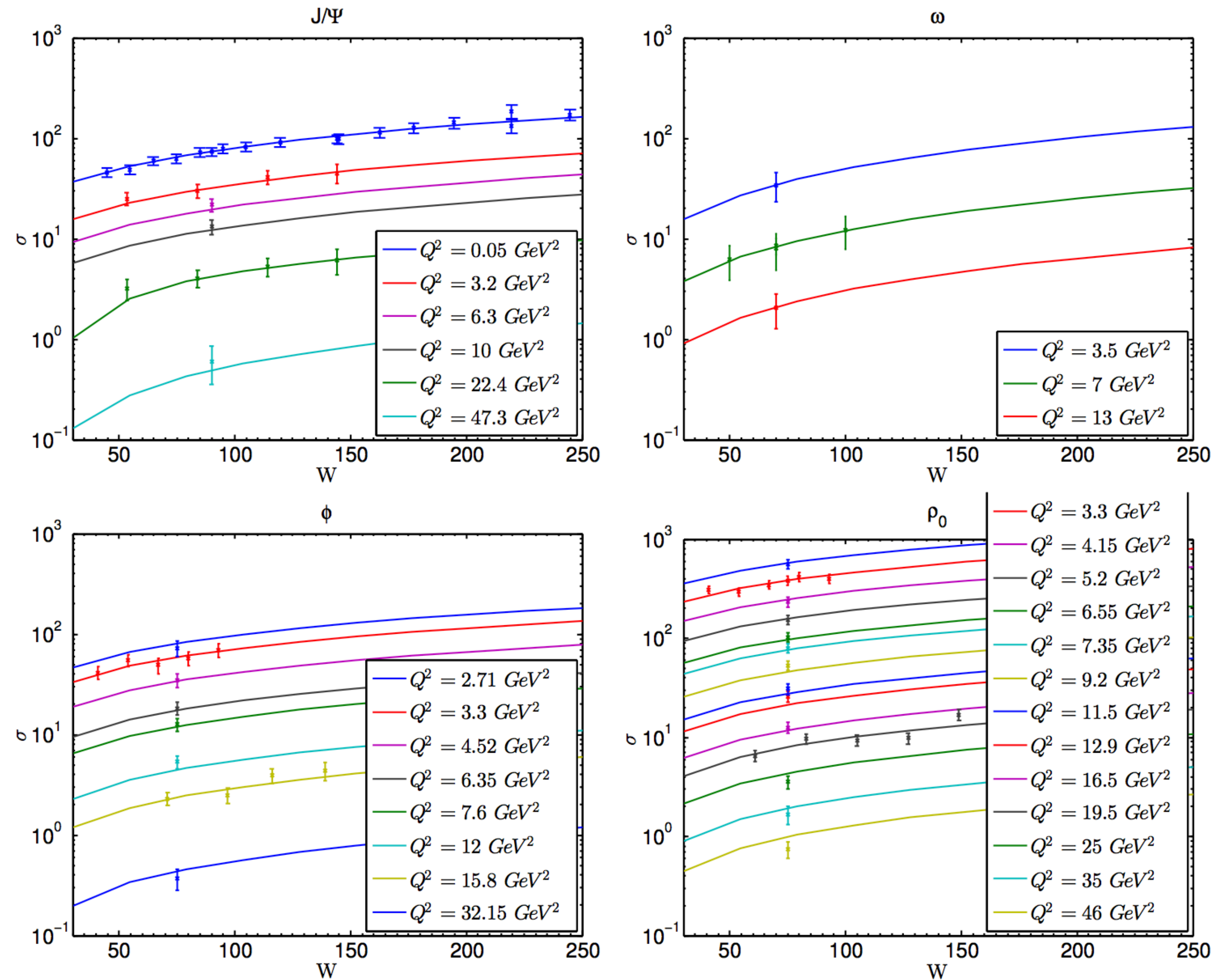
DVSC (differential cross section) [MSC, Djuric 12]

All data (52 points)

$$\chi^2_{d.o.f.} = 0.51$$



VMP (J/Ψ , ω , ϕ , ρ_0) [MSC, Djuric, Evans 13]

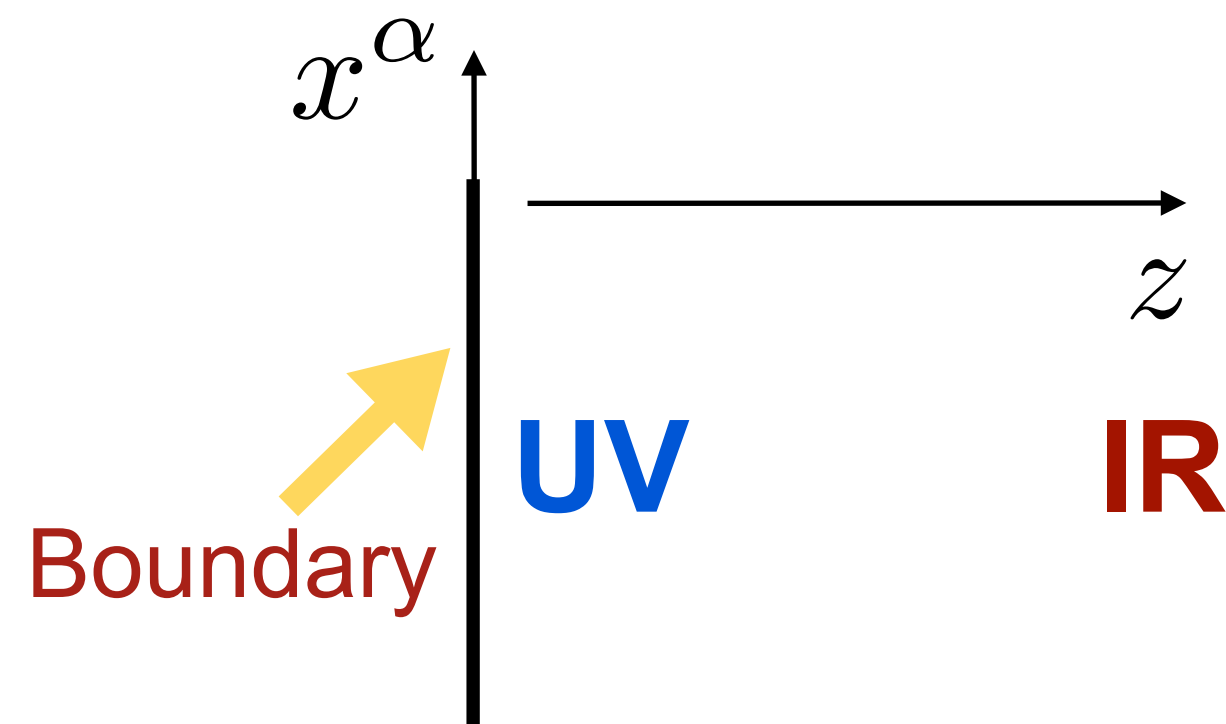


Soft pomeron in holographic QCD [Bayona, MSC, Djuric, Quevedo 15]

- 5D dilaton-gravity model constructed to reproduce QCD [Gursoy, Kiritsis, Nitti 07]

$$S = \frac{1}{2\kappa^2} \int d^5x \sqrt{-g} e^{-2\Phi} \left[R + 4(\partial\phi)^2 + V(\phi) \right]$$

Judicious choice of potential with only 2 free parameters



Constructed to match QCD perturbative beta function

Reproduces: - heavy quark-antiquark linear potential

- glueball spectrum from lattice simulations

- thermodynamic properties of QGP (bulk viscosity, drag force and jet quenching parameters)

$$ds^2 = e^{2A(z)} (dz^2 + \eta_{\alpha\beta} dx^\alpha dx^\beta)$$

$$\Phi = \Phi(z)$$

Spin J field in holographic QCD

- Construct spin J field dual to gluon operator $\mathcal{O}_J \sim \text{Tr} (F_{\alpha\beta_1} D_{\beta_2} \dots D_{\beta_{J-1}} F_{\beta_J}{}^\alpha)$

Decompose symmetric, traceless and transverse field $h_{a_1 \dots a_J}$ with respect to global $SO(1, 3)$ boundary symmetry. Propagating modes have boundary indices $h_{\alpha_1 \dots \alpha_J}$

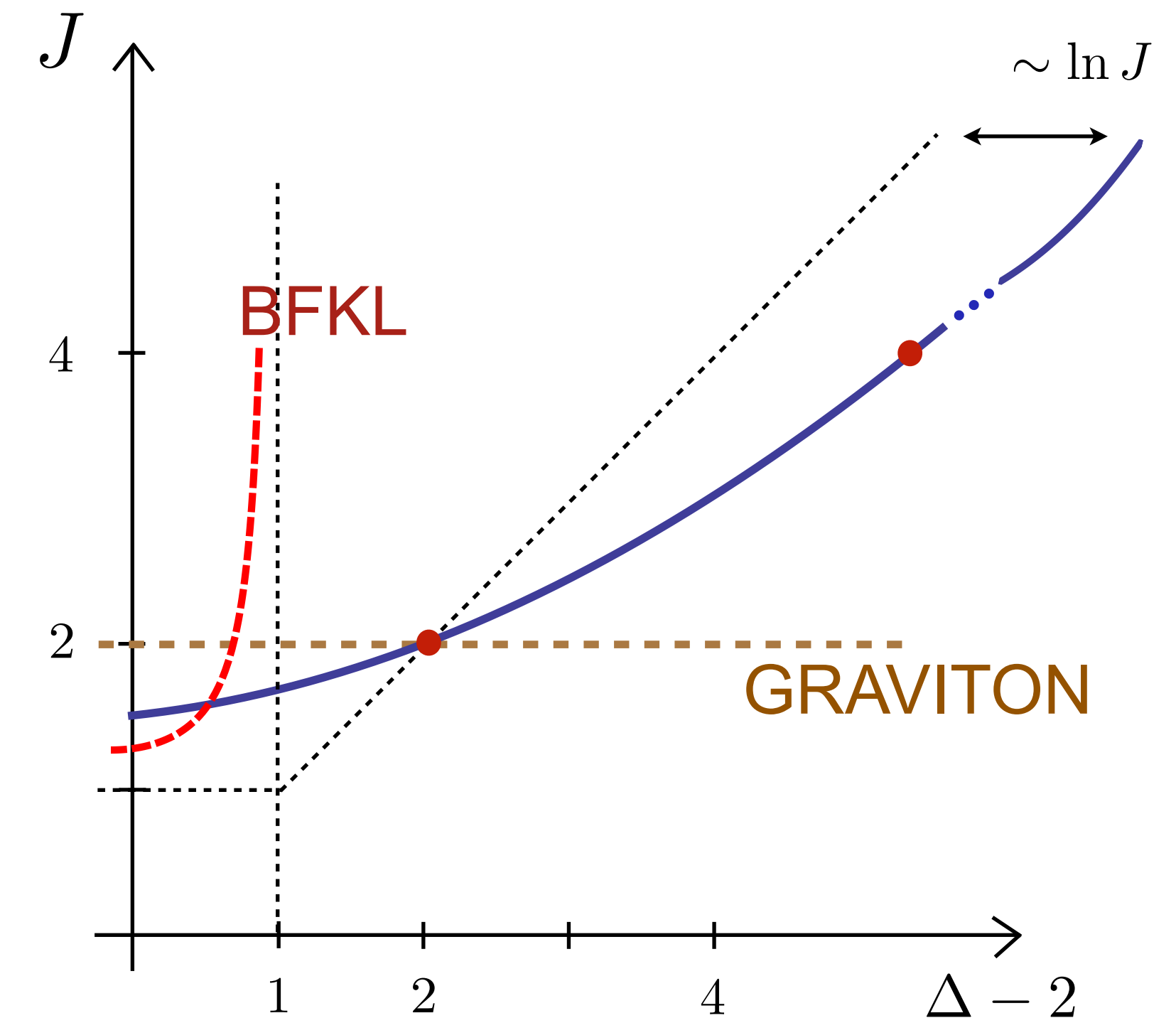
Spin J equation must:

- In AdS limit reduce to $(D^2 - m^2) h_{a_1 \dots a_J} = 0$
 $m^2 = \Delta(\Delta - 4) - J, \quad \Delta = \Delta(J)$
- For $J = 2$ reproduce TT metric fluctuations

$$(\nabla^2 - 2\dot{\Phi}\nabla_z + 2\dot{A}^2 e^{-2A(z)}) h_{\alpha\beta} = 0$$

Proposed equation for propagating mode

$$(\nabla^2 - 2\dot{\Phi}\nabla_z - \Delta(\Delta - 4) + J\dot{A}^2 e^{-2A}) h_{\alpha_1 \dots \alpha_J} = 0$$

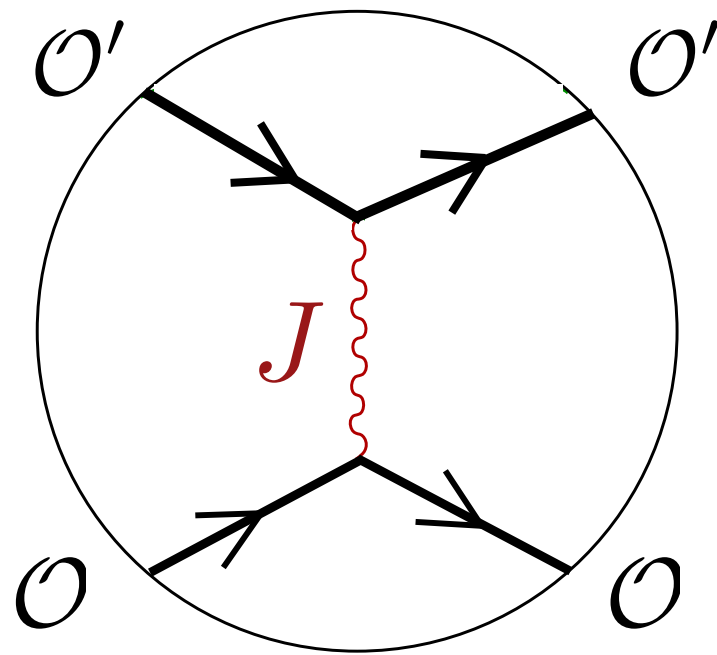


- In the region $J < 2$ we take

$$\Delta(\Delta - 4) \approx \frac{2}{l_s^2} (J - 2)$$

Soft pomeron Regge trajectories

- Consider 5D exchange of spin J field in the Regge limit



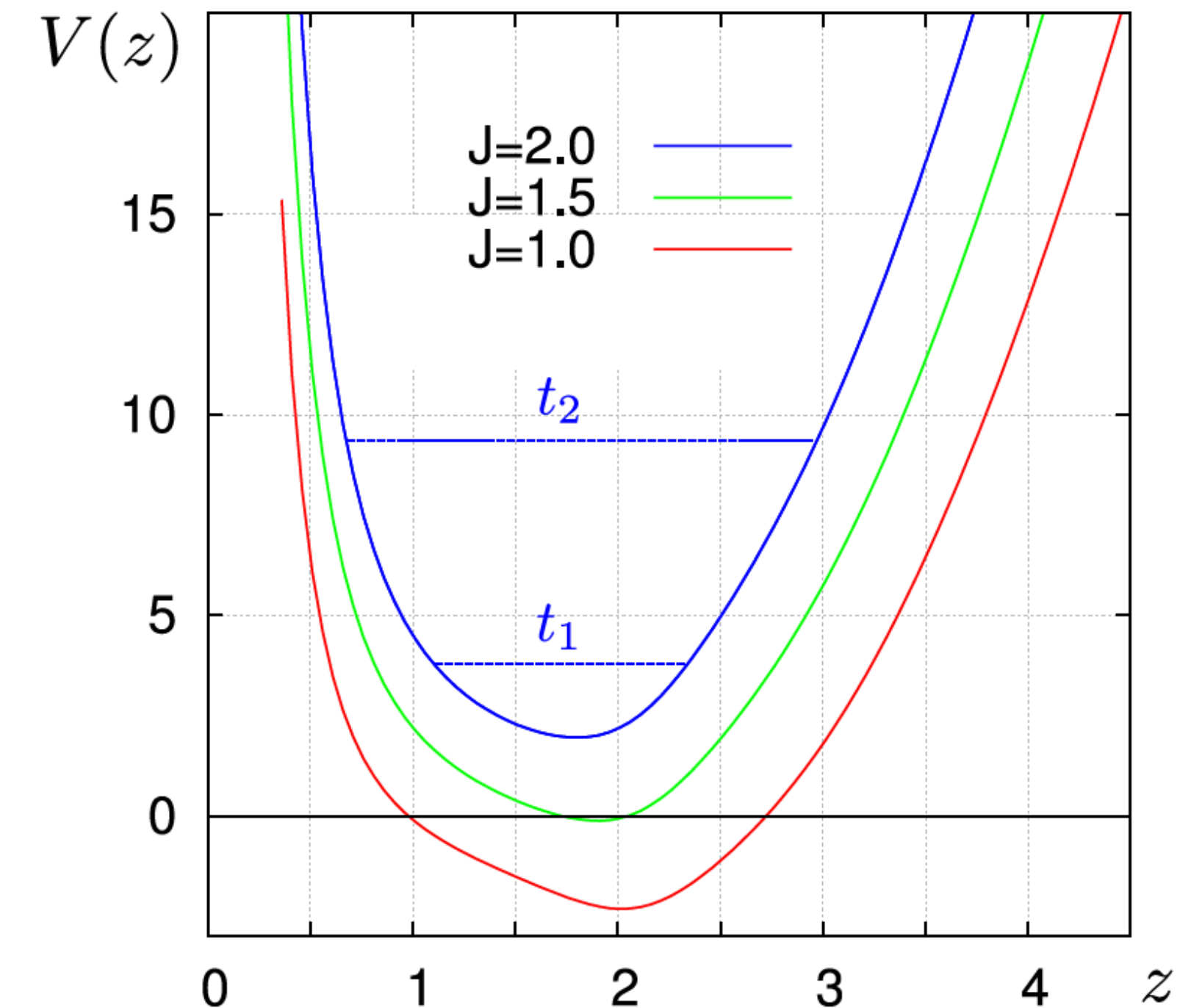
$$A_J(s, t) = iV \frac{\kappa_J \kappa'_J}{(-2)^J} s \int dz dz' e^{3A+3A'-\Phi-\Phi'}$$

$$|v_1|^2 |v'_2|^2 \left(s e^{-A-A'} \right)^{J-1} G_J(z, z', t)$$

Problem reduces to a Schrodinger problem

$$G_J(z, z', t) = e^{\Phi - \frac{A}{2} + \Phi' - \frac{A'}{2}} \sum_n \frac{\psi_n(z) \psi_n^*(z')}{t_n(J) - t}$$

$$V(z) = \frac{15}{4} \dot{A}^2 - 5 \dot{A} \dot{\Phi} + \dot{\Phi}^2 + \Delta(\Delta - 4) e^{2A(z)}$$

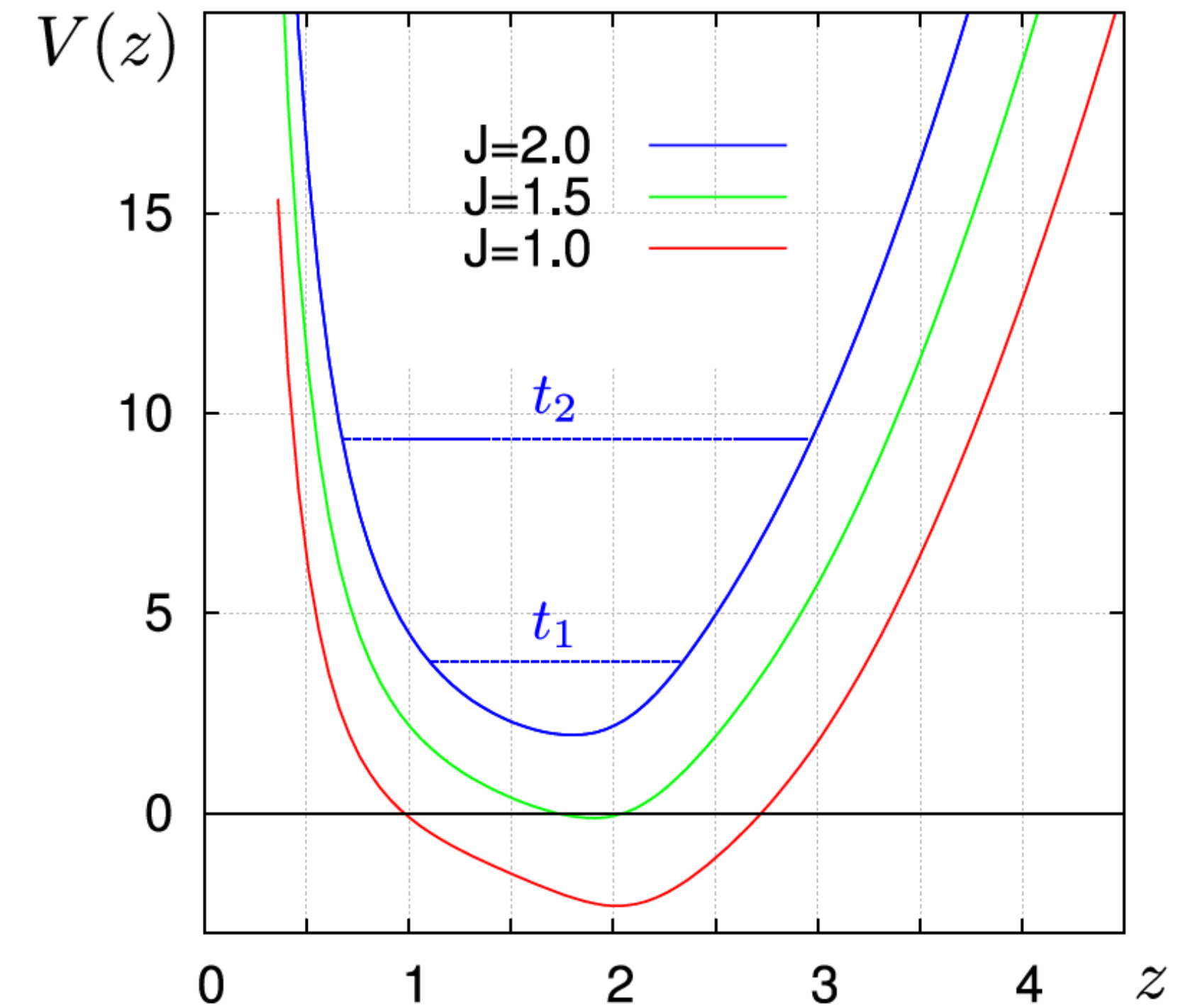
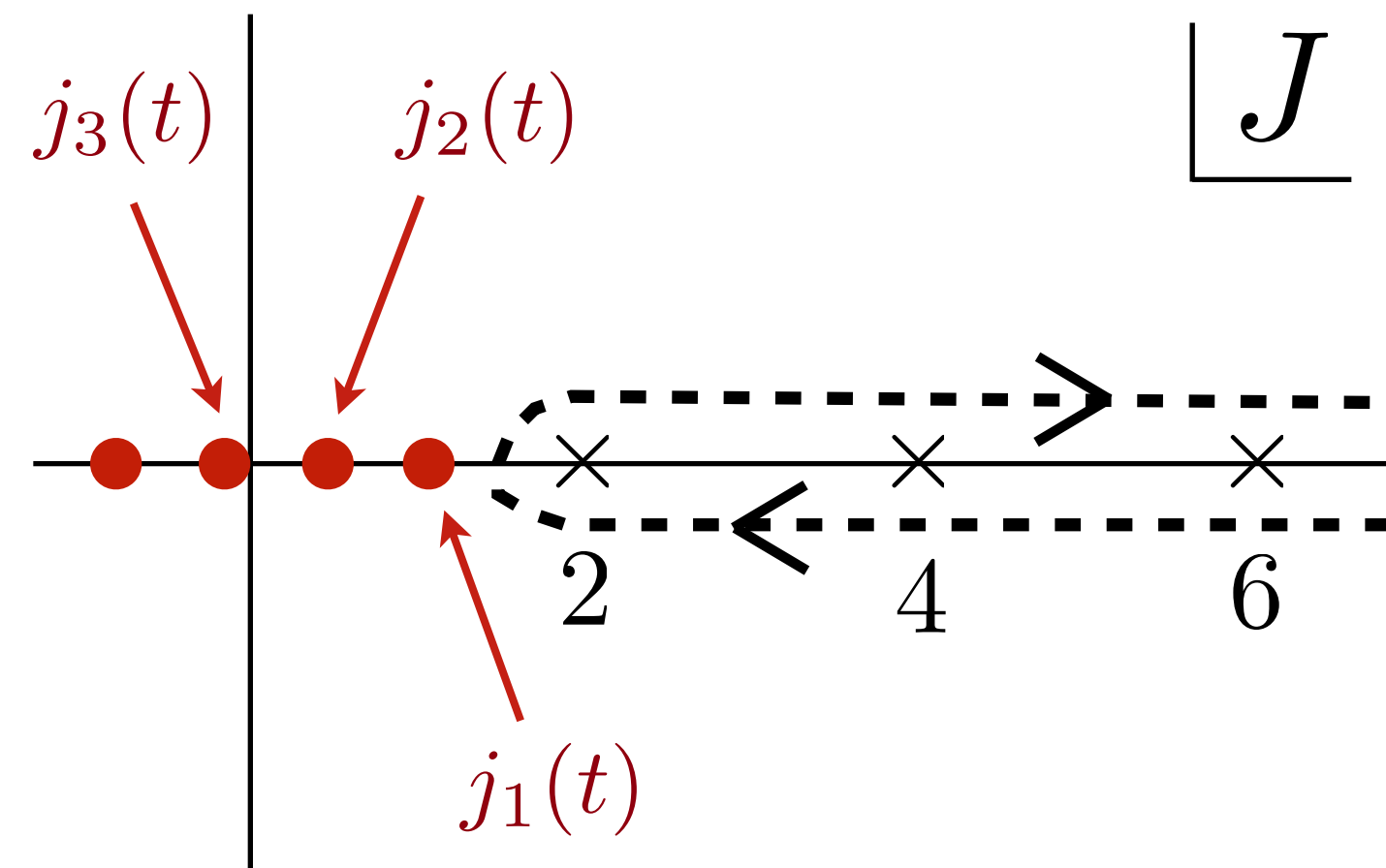


- Sum over spin J exchanges in 5D dual theory

$$\sum_J \rightarrow \int \frac{dJ}{\sin \pi J}$$

$$G_J(z, z', t) = e^{\Phi - \frac{A}{2} + \Phi' - \frac{A'}{2}} \sum_n \frac{\psi_n(z) \psi_n^*(z')}{t_n(J) - t}$$

Poles in the J-plane at $t = t_n(J) \Rightarrow J = j_n(t)$

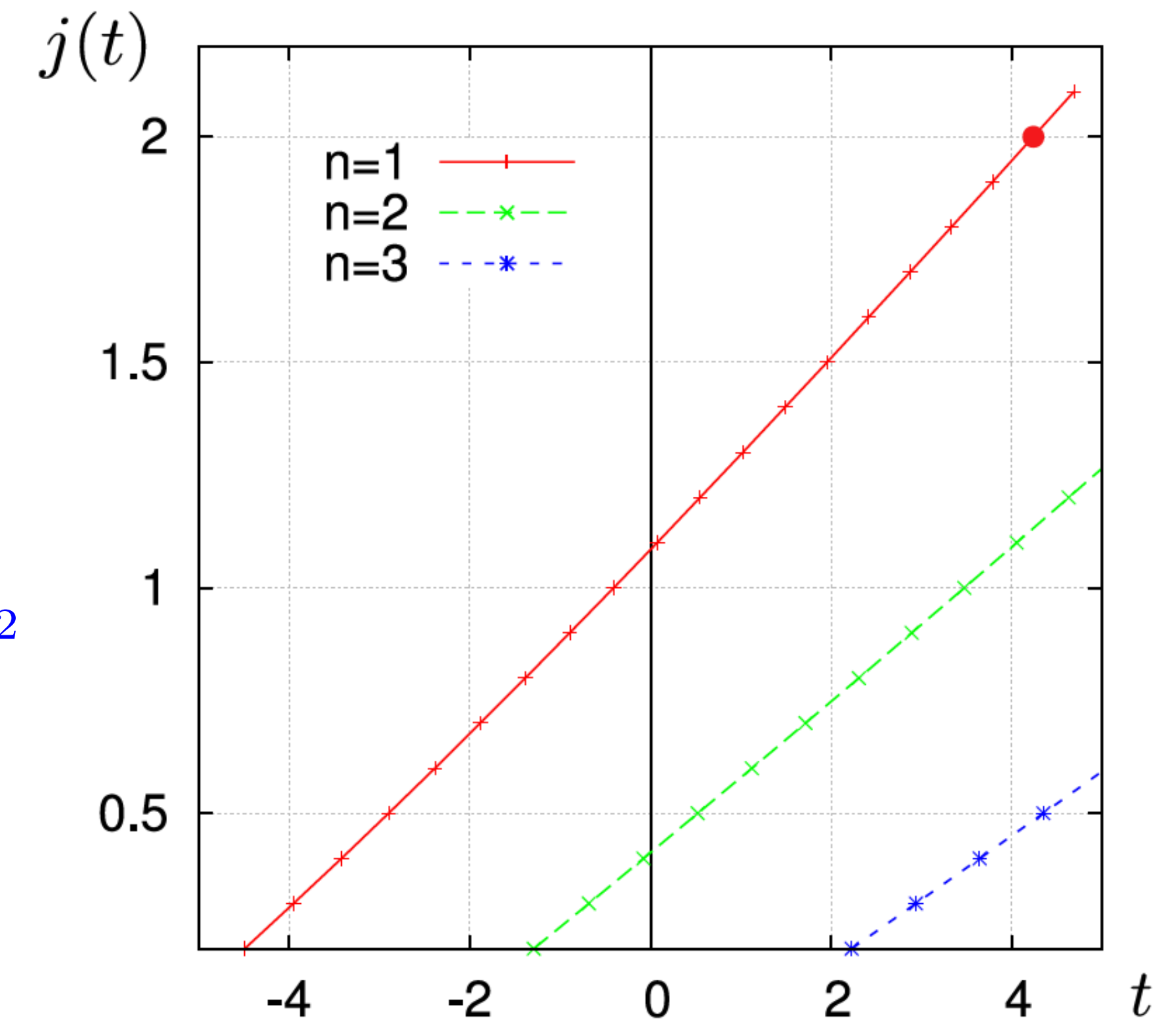


- Obtained approximate linear Regge trajectories. Spin J equation has one free parameter, l_s , to fit soft pomeron intercept and slop.

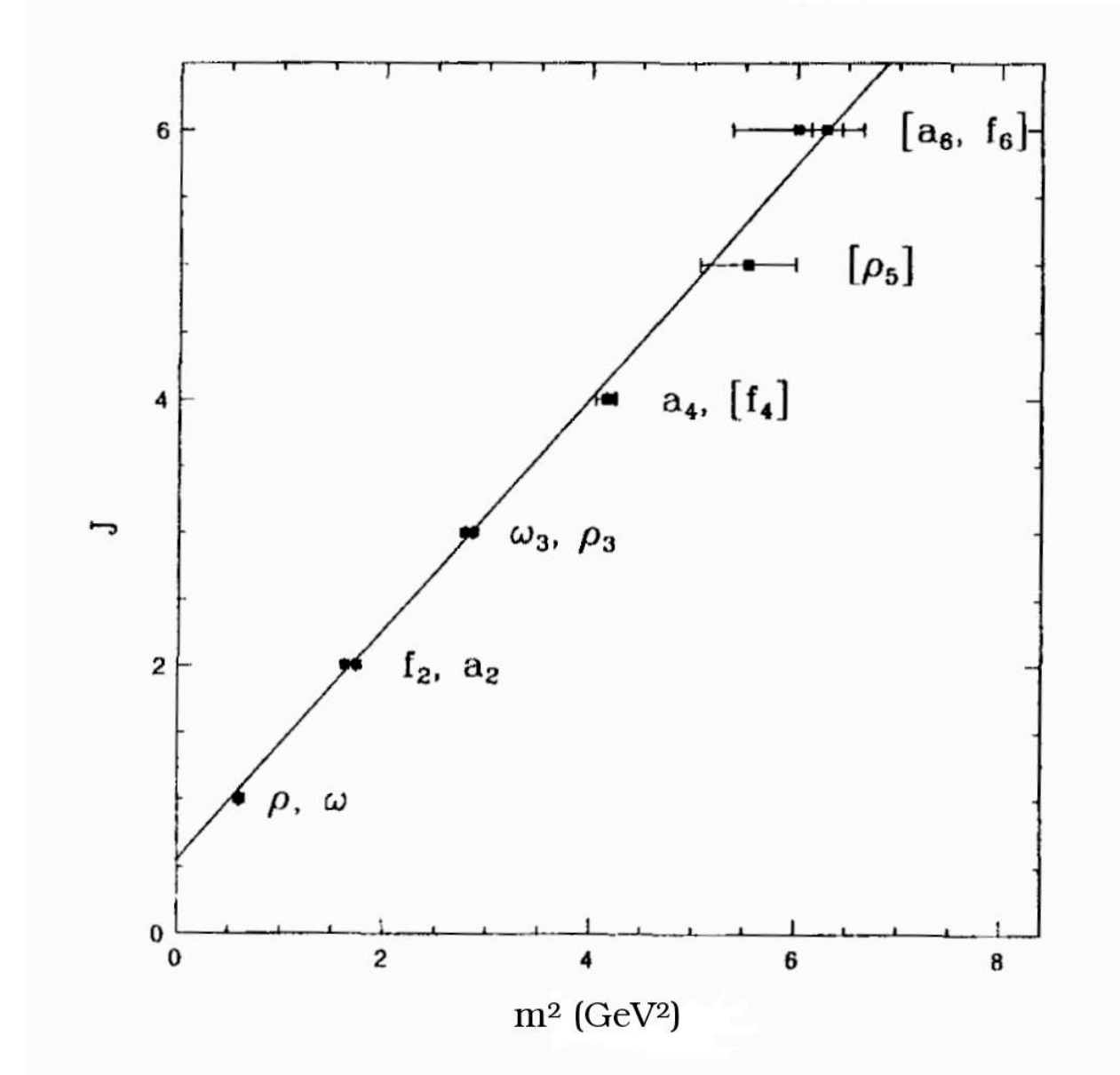
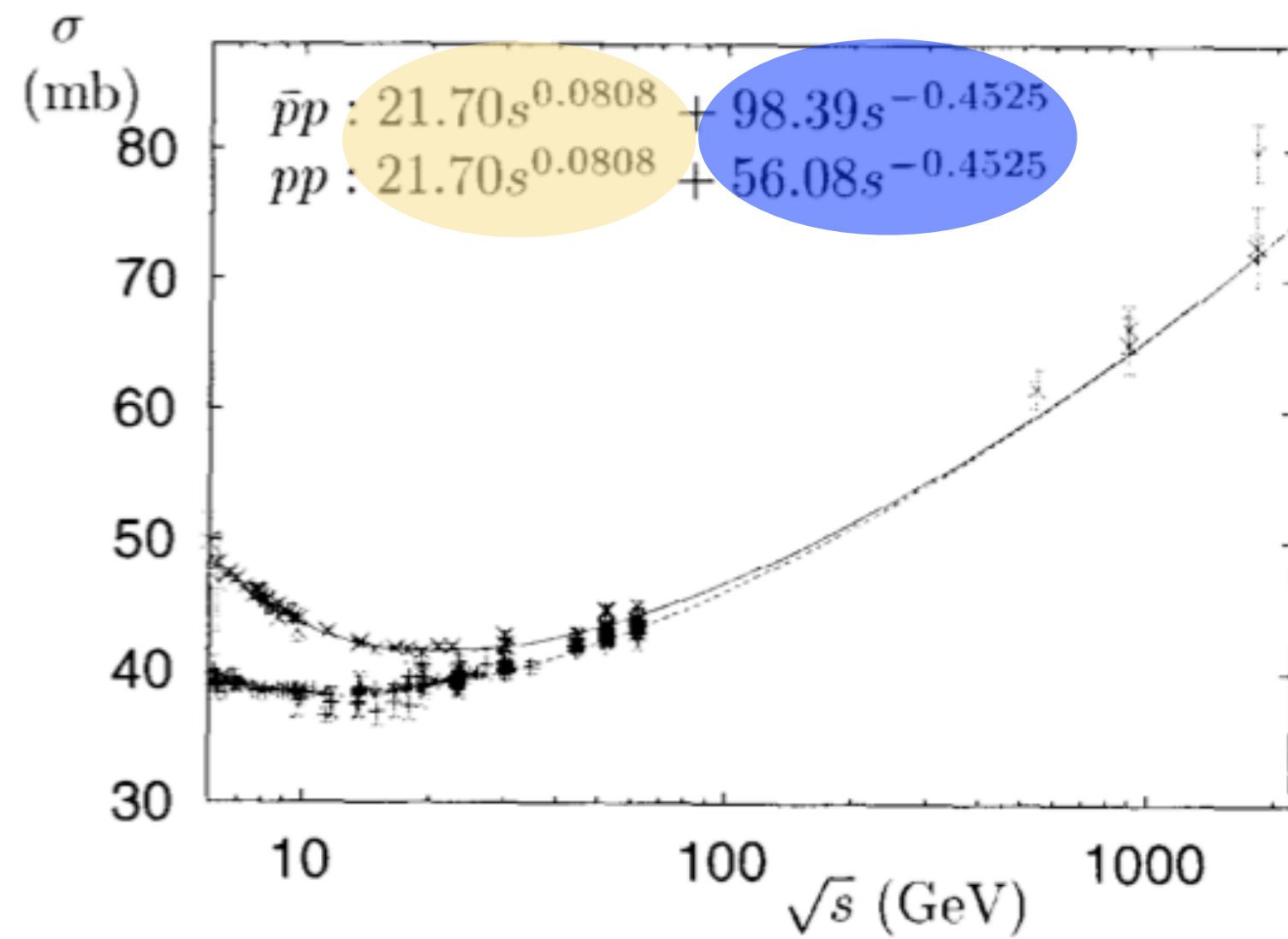
$$l_s = 0.178 \text{ GeV}^{-1} \Rightarrow \begin{aligned} j_0 &= 1.08 \\ \alpha' \Lambda_{QCD}^2 &= 0.018 \end{aligned}$$

where Λ_{QCD} is a parameter of the holographic QCD model

- $\Lambda_{QCD} = 0.292 \text{ GeV}$ to match m_{0++} from lattice then $\alpha' = 0.21 \text{ GeV}^{-2}$
- $\Lambda_{QCD} = 0.265 \text{ GeV}$ to match possible value of m_{2++} at pomeron trajectory then $\alpha' = 0.25 \text{ GeV}^{-2}$



- Recall that Donnachie-Landshoff fits require the ρ , ω , f_2 , a_2 meson trajectories that are subleading

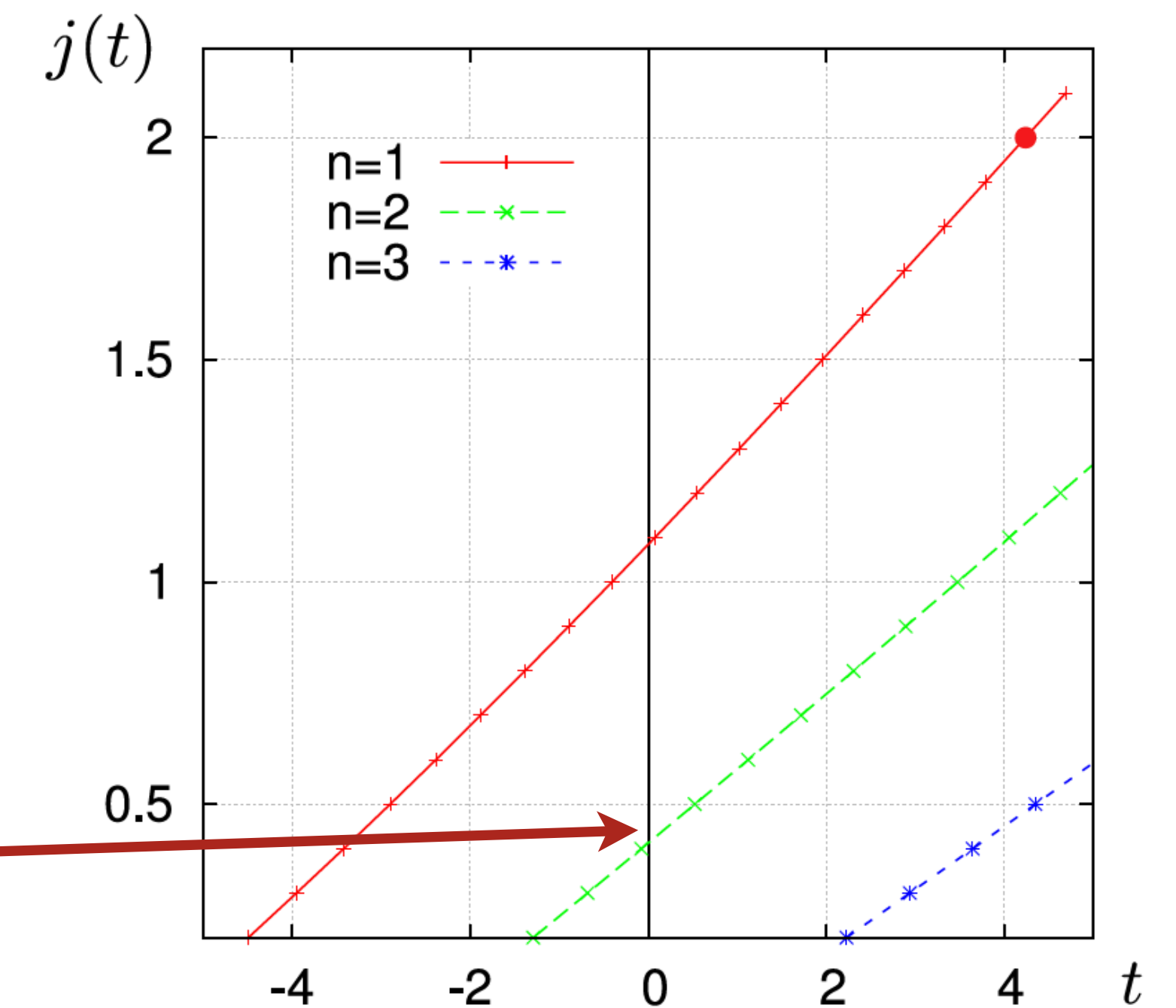


$$\sigma = X s^{j_0-1} + Y s^{j'_0-1}$$

Fits allow for intercept j'_0 in the range $0.35 - 0.55$

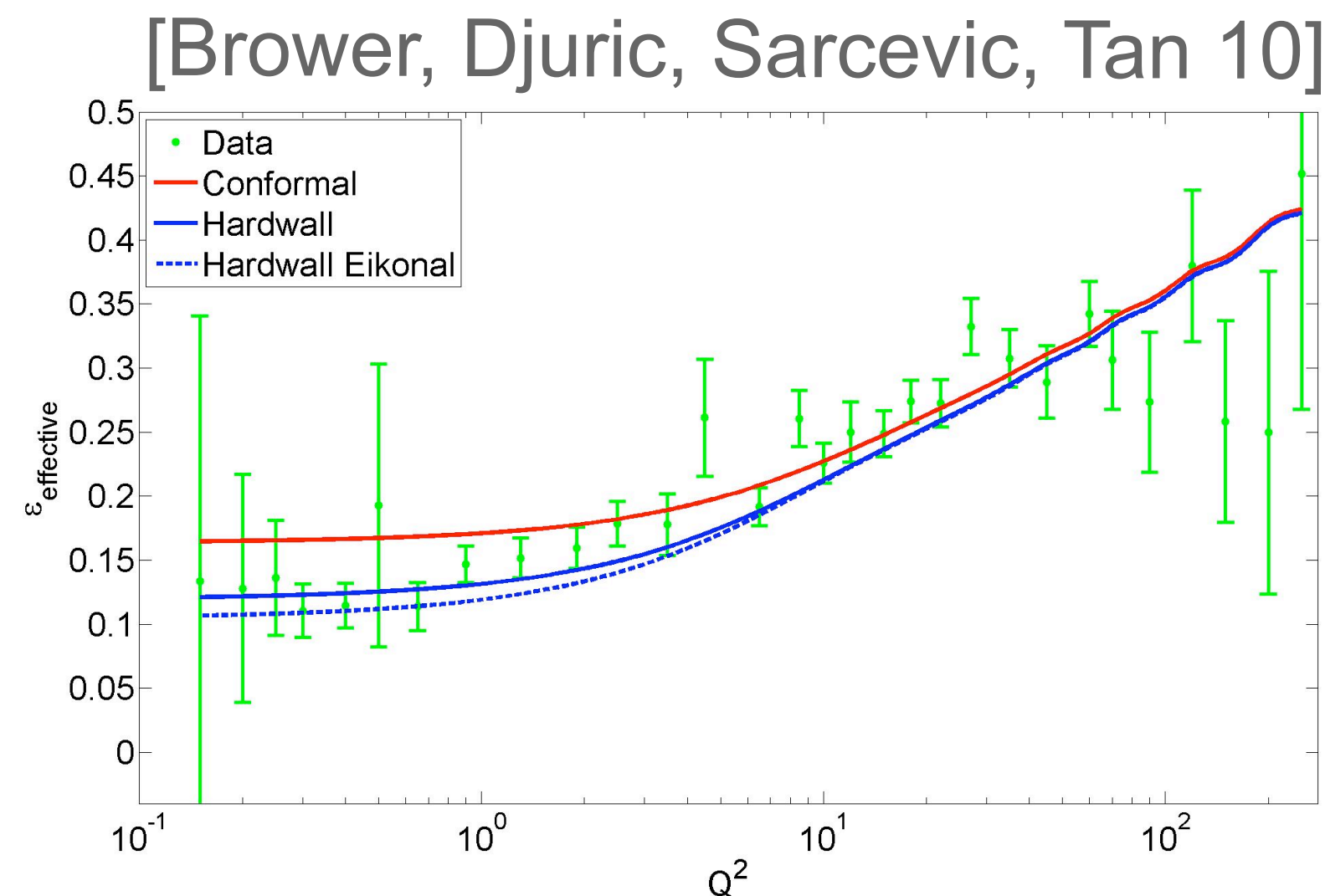
The second pomeron trajectory is precisely in this range!!

0.433



Concluding Remarks

- Holographic QCD model reproduces physics of Donnachie-Landshoff pomeron, with sensible results for intercept and slope. Importance of second pomeron trajectory?
- Connection with hard-pomeron, i.e. **unify hard and soft pomerons**. Understand running of effective exponent



$$\sigma \sim f(Q) \left(\frac{1}{x} \right)^{\epsilon_{eff}(Q)}$$

- Add next sub-leading poles and study behavior with a varying probe scale Q
- Understand better asymptotics of spin J field near the boundary to reproduce QCD anomalous dimension. What is analytic structure in J -plane?

- Ultimate goal: a single holographic model for scattering of soft probes, as well as DIS, DVSC and VMP based, on the graviton Regge trajectory.

THANK YOU